Formalizing Statecharts using Hierarchical Automata

Steffen Helke and Florian Kammüller

April 18, 2024

Abstract

We formalize in Isabelle/HOL the abtract syntax and a synchronous step semantics for the specification language Statecharts [HN96]. The formalization is based on Hierarchical Automata [MLS97] which allow a structural decomposition of Statecharts into Sequential Automata. To support the composition of Statecharts, we introduce calculating operators to construct a Hierarchical Automaton in a stepwise manner [HK01]. Furthermore, we present a complete semantics of Statecharts including a theory of data spaces, which enables the modelling of racing effects [HK05]. We also adapt CTL for Statecharts to build a bridge for future combinations with model checking. However the main motivation of this work is to provide a sound and complete basis for reasoning on Statecharts. As a central meta theorem we prove that the well-formedness of a Statechart is preserved by the semantics [Hel07].

Contents

1 Contributions to the Standard Library of HOL **1**
- 1.1 Basic definitions and lemmas . . . 1
  - 1.1.1 Lambda expressions . . . 1
  - 1.1.2 Maps . . . 2
  - 1.1.3 *rtrancl* . . . 2
  - 1.1.4 *finite* . . . 4
  - 1.1.5 *override* . . . 4
  - 1.1.6 *Part* . . . 5
  - 1.1.7 Set operators . . . 6
  - 1.1.8 One point rule . . . 7

2 Partitoned Data Spaces for Statecharts **8**
- 2.1 Definitions . . . 8
- 2.2 Lemmas . . . 9
  - 2.2.1 *DataSpace* . . . 9
  - 2.2.2 *PartNum* . . . 10

3 Data Space Assignments **11**
3.1 Total data space assignments . . . 11
3.2 Partial data space assignments . . . 12
3.2.1 *DefaultPData* . . . 14
3.2.2 *Data2PData* . . . 15
3.2.3 *DataOverride* . . . 16
3.2.4 *OptionOverride* . . . 17

4 Update-Functions on Data Spaces **18**
4.1 Total update-functions . . . 18
4.1.1 Basic lemmas . . . 18
4.1.2 *DefaultUpdate* . . . 19
4.2 Partial update-functions . . . 19
4.2.1 Basic lemmas . . . 19
4.2.2 *Data2PData* . . . 20
4.2.3 *PUpdate* . . . 20
4.2.4 *SequentialRacing* . . . 20

5 Label Expressions **21**

6 Sequential Automata **26**

7 Syntax of Hierarchical Automata **30**
7.1 Definitions . . . 30
7.1.1 Well-formedness for the syntax of HA . . . 31
7.1.2 State successor function . . . 32
7.1.3 Configurations . . . 33
7.2 Lemmas . . . 34
7.2.1 *HAStates* . . . 34
7.2.2 *HAEvents* . . . 35
7.2.3 *NoCycles* . . . 35
7.2.4 *OneAncestor* . . . 35
7.2.5 *MutuallyDistinct* . . . 35
7.2.6 *RootEx* . . . 36
7.2.7 *HARoot* . . . 37
7.2.8 *CompFun* . . . 38
7.2.9 *SAs* . . . 40
7.2.10 *HAInitState* . . . 41
7.2.11 *Chi* . . . 43
7.2.12 *ChiRel* . . . 45
7.2.13 *ChiPlus* . . . 50
7.2.14 *ChiStar* . . . 58
7.2.15 *InitConf* . . . 59
7.2.16 *StepConf* . . . 63

8 Semantics of Hierarchical Automata 63
8.1 Definitions . . . 63
8.1.1 *Status* . . . 64
8.2 Lemmas . . . 68
8.2.1 *IsConfSet* . . . 68
8.2.2 *InitStatus* . . . 68
8.2.3 *Events* . . . 69
8.2.4 *StepStatus* . . . 70
8.2.5 Enabled Transitions *ET* . . . 71
8.2.6 Finite Transition Set . . . 71
8.2.7 *PUpdate* . . . 71
8.2.8 Higher Priority Transitions *HPT* . . . 71
8.2.9 Delta Transition Set . . . 72
8.2.10 Target Transition Set . . . 74
8.2.11 Source Transition Set . . . 75
8.2.12 *StepActEvents* . . . 77
8.2.13 *UniqueSucStates* . . . 78
8.2.14 *RootState* . . . 78
8.2.15 Configuration *Conf* . . . 80
8.2.16 *RootExSem* . . . 86
8.2.17 *StepConf* . . . 87
8.2.18 *StepStatus* . . . 90
8.3 Meta Theorem: Preservation for Statecharts . . . 96

9 Kripke Structures and CTL 97

10 Kripke Structures as Hierarchical Automata 98

11 Constructing Hierarchical Automata 102
11.1 Constructing a Composition Function for a PseudoHA . . . 102
11.2 Extending a Composition Function by a SA . . . 104
11.3 Constructing a PseudoHA . . . 111
11.4 Extending a HA by a SA (*AddSA*) . . . 112
11.5 Theorems for Calculating Wellformedness of HA . . . 121

12 Example Specification for a Car Audio System 125
12.1 Definitions . . . 125
12.1.1 Data space for two Integer-Variables . . . 125
12.1.2 Sequential Automaton *Root-CTRL* . . . 126
12.1.3 Sequential Automaton *CDPlayer-CTRL* . . . 126
12.1.4 Sequential Automaton *AudioPlayer-CTRL* . . . 127
12.1.5 Sequential Automaton *On-CTRL* . . . 128
12.1.6 Sequential Automaton *TunerMode-CTRL* . . . 129
12.1.7 Sequential Automaton *CDMode-CTRL* . . . 129

12.1.8 Hierarchical Automaton *CarAudioSystem* 130
12.2 Lemmas 131
12.2.1 Sequential Automaton *CDMode-CTRL* 131
12.2.2 Sequential Automaton *CDPlayer-CTRL* 132
12.2.3 Sequential Automaton *AudioPlayer-CTRL* 133
12.2.4 Sequential Automaton *On-CTRL* 134
12.2.5 Sequential Automaton *TunerMode-CTRL* 134
12.2.6 Sequential Automaton *CDMode-CTRL* 135
12.2.7 Hierarchical Automaton *CarAudioSystem* 136

1 Contributions to the Standard Library of HOL

theory *Contrib*
imports *Main*
begin

1.1 Basic definitions and lemmas

1.1.1 Lambda expressions

definition *restrict* :: ['*a* => '*b*, '*a set*] => ('*a* => '*b*) **where**
restrict f A = (%*x*. *if x* : *A then f x else* (@ *y*. *True*))

syntax (*ASCII*)
-lambda-in :: [*pttrn*, '*a set*, '*a* => '*b*] => ('*a* => '*b*) ((*3*%-:-./ -) [*0*, *0*, *3*] *3*)
syntax
-lambda-in :: [*pttrn*, '*a set*, '*a* => '*b*] => ('*a* => '*b*) ((*3*λ-∈-./ -) [*0*, *0*, *3*] *3*)
translations
λ*x*∈*A*. *f* == *CONST restrict* (%*x*. *f*) *A*

1.1.2 Maps

definition *chg-map* :: ('*b* => '*b*) => '*a* => ('*a* ⇀ '*b*) => ('*a* ⇀ '*b*) **where**
chg-map f a m = (*case m a of None* => *m* | *Some b* => *m*(*a*|->*f b*))

lemma *map-some-list* [*simp*]:
map the (*map Some L*) = *L*
apply (*induct-tac L*)
apply *auto*
done

lemma *dom-ran-the*:
⟦ *ran G* = {*y*}; *x* ∈ (*dom G*) ⟧ ⟹ (*the* (*G x*)) = *y*
apply (*unfold ran-def dom-def*)
apply *auto*
done

lemma *dom-None*:

$(S \notin dom\ F) \Longrightarrow (F\ S = None)$
by (*unfold dom-def*, *auto*)

lemma *ran-dom-the*:
$[\![\ y \notin Union\ (ran\ G);\ x \in dom\ G\ ]\!] \Longrightarrow y \notin the\ (G\ x)$
by (*unfold ran-def dom-def*, *auto*)

lemma *dom-map-upd*: $dom(m(a|{->}b)) = insert\ a\ (dom\ m)$
apply *auto*
done

1.1.3 *rtrancl*

lemma *rtrancl-Int*:
$[\![\ (a,b) \in A;\ (a,b) \in B\ ]\!] \Longrightarrow (a,b) \in (A \cap\ B)\hat{}*$
by *auto*

lemma *rtrancl-mem-Sigma*:
$[\![\ a \neq b;\ (a,\ b) \in (A \times A)\hat{}*\ ]\!] \Longrightarrow b \in A$
apply (*frule rtranclD*)
apply (*cut-tac r=A* $\times$ *A* **and** *A=A* **in** *trancl-subset-Sigma*)
apply *auto*
done

lemma *help-rtrancl-Range*:
$[\![\ a \neq b;\ (a,b) \in R\ \hat{}*\ ]\!] \Longrightarrow b \in Range\ R$
apply (*erule rtranclE*)
apply *auto*
done

lemma *rtrancl-Int-help*:
$(a,b) \in (A \cap B)\ \hat{}*\ ==> (a,b) \in A\hat{}* \wedge (a,b) \in B\hat{}*$
apply (*unfold Int-def*)
apply *auto*
apply (*rule-tac b=b* **in** *rtrancl-induct*)
apply *auto*
apply (*rule-tac b=b* **in** *rtrancl-induct*)
apply *auto*
done

lemmas *rtrancl-Int1* = *rtrancl-Int-help* [*THEN conjunct1*]
lemmas *rtrancl-Int2* = *rtrancl-Int-help* [*THEN conjunct2*]

lemma *tranclD3* [*rule-format*]:
$(S,T) \in R\hat{}+ \Longrightarrow (S,T) \notin R \longrightarrow (\exists\ U.\ (S,U) \in R \wedge (U,T) \in R\hat{}+)$
apply (*rule-tac a=S* **and** *b=T* **and** *r=R* **in** *trancl-induct*)
apply *auto*
done

lemma *trancID4* [*rule-format*]:
$(S,T) \in R\hat{}+ \Longrightarrow (S,T) \notin R \longrightarrow (\exists\ U.\ (S,U) \in R\hat{}+ \wedge (U,T) \in R)$
apply (*rule-tac* $a{=}S$ **and** $b{=}T$ **and** $r{=}R$ **in** *trancl-induct*)
apply *auto*
done

lemma *trancl-collect* [*rule-format*]:
$[\![(x,y) \in R\hat{}*;\ S \notin Domain\ R]\!] \Longrightarrow y \neq S \longrightarrow (x,y) \in \{p.\ fst\ p \neq S \wedge snd\ p \neq S \wedge p \in R\}\hat{}*$
apply (*rule-tac* $b{=}y$ **in** *rtrancl-induct*)
apply *auto*
apply (*rule rtrancl-into-rtrancl*)
apply *fast*
apply *auto*
done

lemma *trancl-subseteq*:
$[\![R \subseteq Q;\ S \in R\hat{}*]\!] \Longrightarrow S \in Q\hat{}*$
apply (*frule rtrancl-mono*)
apply *fast*
done

lemma *trancl-Int-subset*:
$(R \cap (A \times A))^+ \subseteq R^+ \cap (A \times A)$
apply (*rule subsetI*)
apply (*rename-tac S*)
apply (*case-tac S*)
apply (*rename-tac T V*)
apply *auto*
apply (*rule-tac* $a{=}T$ **and** $b{=}V$ **and** $r{=}(R\ \cap A \times A)$ **in** *converse-trancl-induct*, *auto*)+
done

lemma *trancl-Int-mem*:
$(S,T) \in (R \cap (A \times A))^+ \Longrightarrow (S,T)\ \in R^+ \cap A \times A$
by (*rule trancl-Int-subset* [*THEN subsetD*], *assumption*)

lemma *Int-expand*:
$\{(S,S').\ P\ S\ S' \wedge\ \ Q\ S\ S'\} = (\{(S,S').\ P\ S\ S'\} \cap \{(S,S').\ Q\ S\ S'\})$
by *auto*

1.1.4 *finite*

lemma *finite-conj*:
finite $(\{(S,S').\ P\ S\ S'\}::(('a*'b)set)) \longrightarrow$
finite $\{(S,S').\ P\ (S::'a)\ (S'::'b) \wedge Q\ (S::'a)\ (S'::'b)\}$
apply (*rule impI*)
apply (*subst Int-expand*)
apply (*rule finite-Int*)

apply *auto*
done

lemma *finite-conj2*:
⟦ *finite A*; *finite B* ⟧ ⟹ *finite* ({(*S*,*S'*). *S*: *A* ∧ *S'* : *B*})
by *auto*

1.1.5 *override*

lemma *dom-override-the*:
(*x* ∈ (*dom G2*)) ⟶ ((*G1* ++ *G2*) *x*) = (*G2 x*)
by (*auto*)

lemma *dom-override-the2* [*simp*]:
⟦ *dom G1* ∩ *dom G2* = {}; *x* ∈ (*dom G1*) ⟧ ⟹ ((*G1* ++ *G2*) *x*) = (*G1 x*)
apply (*unfold dom-def map-add-def*)
apply *auto*
apply (*drule sym*)
apply (*erule equalityE*)
apply (*unfold Int-def*)
apply *auto*
apply (*erule-tac x=x* **in** *allE*)
apply *auto*
done

lemma *dom-override-the3* [*simp*]:
⟦ *x* ∉ *dom G2*; *x* ∈ *dom G1* ⟧ ⟹ ((*G1* ++ *G2*) *x*) = (*G1 x*)
apply (*unfold dom-def map-add-def*)
apply *auto*
done

lemma *Union-ran-override* [*simp*]:
S ∈ *dom G* ⟹ ⋃ (*ran* (*G* ++ *Map.empty*(*S* ↦ *insert SA* (*the*(*G S*))))) =
(*insert SA* (*Union* (*ran G*)))
apply (*unfold dom-def ran-def*)
apply *auto*
apply (*rename-tac T*)
apply (*case-tac T* = *S*)
apply *auto*
done

lemma *Union-ran-override2* [*simp*]:
S ∈ *dom G* ⟹ ⋃ (*ran* (*G*(*S* ↦ *insert SA* (*the*(*G S*))))) = (*insert SA* (*Union* (*ran G*)))
apply (*unfold dom-def ran-def*)
apply *auto*
apply (*rename-tac T*)
apply (*case-tac T* = *S*)
apply *auto*

done

lemma *ran-override* [*simp*]:
(*dom A* ∩ *dom B*) = {} ⟹ *ran* (*A* ++ *B*) = (*ran A*) ∪ (*ran B*)
apply (*unfold Int-def ran-def*)
apply (*simp add*: *map-add-Some-iff*)
apply *auto*
done

lemma *chg-map-new* [*simp*]:
m a = *None* ⟹ *chg-map f a m* = *m*
by (*unfold chg-map-def*, *auto*)

lemma *chg-map-upd* [*simp*]:
m a = *Some b* ⟹ *chg-map f a m* = *m*(*a*|−>*f b*)
by (*unfold chg-map-def*, *auto*)

lemma *ran-override-chg-map*:
A ∈ *dom G* ⟹ *ran* (*G* ++ *Map.empty*(*A*|−>*B*)) = (*ran* (*chg-map* (λ *x*. *B*) *A G*))
apply (*unfold dom-def ran-def*)
apply (*subst map-add-Some-iff* [*THEN ext*])
apply *auto*
apply (*rename-tac T*)
apply (*case-tac T* = *A*)
apply *auto*
done

1.1.6 *Part*

definition *Part* :: ['*a set*, '*b* => '*a*] => '*a set* **where**
Part A h = *A* ∩ {*x*. ∃ *z*. *x* = *h*(*z*)}

lemma *Part-UNIV-Inl-comp*:
((*Part UNIV* (*Inl o f*)) = (*Part UNIV* (*Inl o g*))) = ((*Part UNIV f*) = (*Part UNIV g*))
apply (*unfold Part-def*)
apply *auto*
apply (*erule equalityE*)
apply (*erule subsetCE*)
apply *auto*
apply (*erule equalityE*)
apply (*erule subsetCE*)
apply *auto*
done

lemma *Part-eqI* [*intro*]: ⟦ *a* ∈ *A*; *a*=*h*(*b*) ⟧ ⟹ *a* ∈ *Part A h*
by (*auto simp add*: *Part-def*)

lemmas *PartI* = *Part-eqI* [*OF - refl*]

lemma *PartE* [*elim*!]: ⟦ $a \in Part\ A\ h$; !!z. ⟦ $a \in A$; $a{=}h(z)$ ⟧ ⟹ P ⟧ ⟹ P
by (*auto simp add*: *Part-def*)

lemma *Part-subset*: $Part\ A\ h \subseteq A$
by (*auto simp add*: *Part-def*)

lemma *Part-mono*: $A \subseteq B \Longrightarrow Part\ A\ h \subseteq Part\ B\ h$
by *blast*

lemmas *basic-monos* = *basic-monos Part-mono*

lemma *PartD1*: $a \in Part\ A\ h \Longrightarrow a \in A$
by (*simp add*: *Part-def*)

lemma *Part-id*: $Part\ A\ (\lambda\ x.\ x) = A$
by *blast*

lemma *Part-Int*: $Part\ (A \cap B)\ h = (Part\ A\ h) \cap (Part\ B\ h)$
by *blast*

lemma *Part-Collect*: $Part\ (A \cap \{x.\ P\ x\})\ h = (Part\ A\ h) \cap \{x.\ P\ x\}$
by *blast*

1.1.7 Set operators

lemma *subset-lemma*:
⟦ $A \cap B = \{\}$; $A \subseteq B$ ⟧ ⟹ $A = \{\}$
by *auto*

lemma *subset-lemma2*:
⟦ $B \cap A = \{\}$; $C \subseteq A$ ⟧ ⟹ $C \cap B = \{\}$
by *auto*

lemma *insert-inter*:
⟦ $a \notin A$; $(A \cap B) = \{\}$ ⟧ ⟹ $(A \cap (insert\ a\ B)) = \{\}$
by *auto*

lemma *insert-notmem*:
⟦ $a \neq b$; $a \notin B$ ⟧ ⟹ $a \notin (insert\ b\ B)$
by *auto*

lemma *insert-union*:
$A \cup (insert\ a\ B) = insert\ a\ A \cup B$
by *auto*

lemma *insert-or*:

$\{s.\ s = t1 \vee (P\ s)\} = insert\ t1\ \{s.\ P\ s\ \}$
by *auto*

lemma *Collect-subset*:
$\{x\ .\ x \subseteq A \wedge P\ x\} = \{x \in Pow\ A.\ P\ x\}$
by *auto*

lemma *OneElement-Card* [*simp*]:
$[\![\ finite\ M;\ card\ M <= Suc\ 0;\ t \in M\ ]\!] \Longrightarrow M = \{t\}$
apply *auto*
apply (*rename-tac s*)
apply (*rule-tac P=finite M* **in** *mp*)
apply (*rule-tac P=card M <= Suc 0* **in** *mp*)
apply (*rule-tac P=t ∈ M* **in** *mp*)
apply (*rule-tac F=M* **in** *finite-induct*)
apply *auto*
apply (*rule-tac P=finite M* **in** *mp*)
apply (*rule-tac P=card M <= Suc 0* **in** *mp*)
apply (*rule-tac P=s ∈ M* **in** *mp*)
apply (*rule-tac P=t ∈ M* **in** *mp*)
apply (*rule-tac F=M* **in** *finite-induct*)
apply *auto*
done

1.1.8 One point rule

lemma *Ex1-one-point* [*simp*]:
$(\exists!\ x.\ P\ x \wedge x = a) = P\ a$
by *auto*

lemma *Ex1-one-point2* [*simp*]:
$(\exists!\ x.\ P\ x \wedge Q\ x \wedge x = a) = (P\ a \wedge Q\ a)$
by *auto*

lemma *Some-one-point* [*simp*]:
$P\ a \Longrightarrow (SOME\ x.\ P\ x \wedge x = a) = a$
by *auto*

lemma *Some-one-point2* [*simp*]:
$[\![\ Q\ a;\ P\ a\ ]\!] \Longrightarrow (SOME\ x.\ P\ x \wedge Q\ x \wedge x = a) = a$
by *auto*

end

2 Partitoned Data Spaces for Statecharts

theory *DataSpace*
imports *Contrib*
begin

2.1 Definitions

definition
DataSpace :: (*'d set*) *list*
=> *bool* **where**
DataSpace L = ((*distinct L*) ∧
(∀ *D1*∈(*set L*). ∀ *D2*∈(*set L*).
D1 ≠ *D2* ⟶ ((*D1* ∩ *D2*) = {})) ∧
((⋃ (*set L*)) = *UNIV*))

lemma *DataSpace-EmptySet*:
[*UNIV*] ∈ { *L* | *L*. *DataSpace L* }
by (*unfold DataSpace-def*, *auto*)

definition *dataspace* = { *L* | (*L*::(*'d set*) *list*). *DataSpace L*}

typedef *'d dataspace* = *dataspace* :: *'d set list set*
unfolding *dataspace-def*
apply (*rule exI*)
apply (*rule DataSpace-EmptySet*)
done

definition
PartNum :: (*'d*)*dataspace* => *nat* **where**
PartNum = *length o Rep-dataspace*

definition
PartDom :: [*'d dataspace*, *nat*] => (*'d set*) (**infixl** !*D*! *101*) **where**
PartDom d n = (*Rep-dataspace d*) ! *n*

2.2 Lemmas

2.2.1 *DataSpace*

lemma *DataSpace-UNIV* [*simp*]:
DataSpace [*UNIV*]
by (*unfold DataSpace-def*, *auto*)

lemma *DataSpace-select*:
DataSpace (*Rep-dataspace L*)
apply (*cut-tac x*=*L* **in** *Rep-dataspace*)
apply (*unfold dataspace-def*)
apply *auto*
done

lemma *UNIV-dataspace* [*simp*]:
[*UNIV*] ∈ *dataspace*
by (*unfold dataspace-def*, *auto*)

lemma *Inl-Inr-DataSpace* [*simp*]:

DataSpace [*Part UNIV Inl*, *Part UNIV Inr*]
apply (*unfold DataSpace-def*)
apply *auto*
apply (*rename-tac d*)
apply (*rule-tac b=*(*inv Inl*) *d* **in** *Part-eqI*)
apply *auto*
apply (*rule sym*)
apply (*case-tac d*)
apply *auto*
done

lemma *Inl-Inr-dataspace* [*simp*]:
[*Part UNIV Inl*, *Part UNIV Inr*] ∈ *dataspace*
by (*unfold dataspace-def*, *auto*)

lemma *InlInr-InlInl-Inr-DataSpace* [*simp*]:
DataSpace [*Part UNIV* (*Inl o Inr*), *Part UNIV* (*Inl o Inl*), *Part UNIV Inr*]
apply (*unfold DataSpace-def*)
apply *auto*
apply (*unfold Part-def*)
apply *auto*
apply (*rename-tac x*)
apply (*case-tac x*)
apply *auto*
apply (*rename-tac a*)
apply (*case-tac a*)
apply *auto*
done

lemma *InlInr-InlInl-Inr-dataspace* [*simp*]:
[*Part UNIV* (*Inl o Inr*), *Part UNIV* (*Inl o Inl*), *Part UNIV Inr*] : *dataspace*
by (*unfold dataspace-def*, *auto*)

2.2.2 *PartNum*

lemma *PartDom-PartNum-distinct*:
⟦ *i* < *PartNum d*; *j* < *PartNum d*;
i ≠ *j*; *p* ∈ (*d* !*D*! *i*) ⟧ ⟹
p ∉ (*d* !*D*! *j*)
apply *auto*
apply (*cut-tac L=d* **in** *DataSpace-select*)
apply (*unfold DataSpace-def*)
apply *auto*
apply (*erule-tac x=Rep-dataspace d* ! *i* **in** *ballE*)
apply (*erule-tac x=Rep-dataspace d* ! *j* **in** *ballE*)
apply *auto*
apply (*simp add:distinct-conv-nth PartNum-def*)
apply (*unfold PartDom-def PartNum-def*)
apply *auto*

done

lemma *PartDom-PartNum-distinct2*:
⟦ $i < PartNum\ d$; $j < PartNum\ d$;
$i \neq j$; $p \in (d\ !D!\ j)$ ⟧ ⟹
$p \notin (d\ !D!\ i)$
apply *auto*
apply (*cut-tac L=d* **in** *DataSpace-select*)
apply (*unfold DataSpace-def*)
apply *auto*
apply (*erule-tac x=Rep-dataspace d ! i* **in** *ballE*)
apply (*erule-tac x=Rep-dataspace d ! j* **in** *ballE*)
apply *auto*
apply (*simp add:distinct-conv-nth PartNum-def*)
apply (*unfold PartDom-def PartNum-def*)
apply *auto*
done

lemma *PartNum-length* [*simp*]:
$(DataSpace\ L) \Longrightarrow (PartNum\ (Abs\text{-}dataspace\ L) = (length\ L))$
apply (*unfold PartNum-def*)
apply *auto*
apply (*subst Abs-dataspace-inverse*)
apply (*unfold dataspace-def*)
apply *auto*
done

end

3 Data Space Assignments

theory *Data*
imports *DataSpace*
begin

3.1 Total data space assignments

definition
$Data :: ['d\ list, 'd\ dataspace]$
$\Rightarrow bool$ **where**
$Data\ L\ D = (((length\ L) = (PartNum\ D)) \wedge$
$(\forall\ i \in \{n.\ n < (PartNum\ D)\}.\ (L!i) \in (PartDom\ D\ i)))$

lemma *Data-EmptySet*:
$([@\ t.\ True], Abs\text{-}dataspace\ [UNIV]) \in \{\ (L,D) \mid L\ D.\ Data\ L\ D\ \}$
apply (*unfold Data-def PartDom-def*)
apply *auto*
apply (*subst Abs-dataspace-inverse*)
apply *auto*

done

definition
data =
{ (*L*,*D*) |
(*L*::('*d list*))
(*D*::('*d dataspace*)).
Data L D }

typedef '*d data* = *data* :: ('*d list* * '*d dataspace*) *set*
unfolding *data-def*
apply (*rule exI*)
apply (*rule Data-EmptySet*)
done

definition
DataValue :: ('*d data*) => ('*d list*) **where**
DataValue = *fst o Rep-data*

definition
DataSpace :: ('*d data*) => ('*d dataspace*) **where**
DataSpace = *snd o Rep-data*

definition
DataPart :: ['*d data*, *nat*] => '*d* ((- !*P*!/ -) [*10*,*11*]*10*) **where**
DataPart d n = (*DataValue d*) ! *n*

lemma *Rep-data-tuple*:
Rep-data D = (*DataValue D*, *DataSpace D*)
by (*unfold DataValue-def DataSpace-def*, *simp*)

lemma *Rep-data-select*:
(*DataValue D*, *DataSpace D*) ∈ *data*
apply (*subst Rep-data-tuple* [*THEN sym*])
apply (*rule Rep-data*)
done

lemma *Data-select*:
Data (*DataValue D*) (*DataSpace D*)
apply (*cut-tac D*=*D* **in** *Rep-data-select*)
apply (*unfold data-def*)
apply *auto*
done

lemma *length-DataValue-PartNum* [*simp*]:
length (*DataValue D*) = *PartNum* (*Data.DataSpace D*)
apply (*cut-tac D*=*D* **in** *Data-select*)
apply (*unfold Data-def*)
apply *auto*

done

lemma *DataValue-PartDom* [*simp*]:
i < *PartNum* (*Data.DataSpace D*) ⟹
DataValue D ! *i* ∈ *PartDom* (*Data.DataSpace D*) *i*
apply (*cut-tac D=D* **in** *Data-select*)
apply (*unfold Data-def*)
apply *auto*
done

lemma *DataPart-PartDom* [*simp*]:
i < *PartNum* (*Data.DataSpace d*) ⟶ (*d* !*P*! *i*) ∈ ((*Data.DataSpace d*) !*D*! *i*)
apply (*unfold DataPart-def*)
apply *auto*
done

3.2 Partial data space assignments

definition
PData :: [*'d option list*, *'d dataspace*] => *bool* **where**
PData L D == ((*length L*) = (*PartNum D*)) ∧
(∀ *i* ∈ {*n*. *n* < (*PartNum D*)}.
(*L*!*i*) ≠ *None* ⟶ *the* (*L*!*i*) ∈ (*PartDom D i*))

lemma *PData-EmptySet*:
([*Some* (@ *t*. *True*)], *Abs-dataspace* [*UNIV*]) ∈ { (*L*,*D*) | *L D*. *PData L D* }
apply (*unfold PData-def PartDom-def*)
apply *auto*
apply (*subst Abs-dataspace-inverse*)
apply *auto*
done

definition
pdata =
{ (*L*,*D*) |
(*L*::(*'d option list*))
(*D*::(*'d dataspace*)).
PData L D }

typedef *'d pdata* = *pdata* :: (*'d option list* ∗ *'d dataspace*) *set*
unfolding *pdata-def*
apply (*rule exI*)
apply (*rule PData-EmptySet*)
done

definition
PDataValue :: (*'d pdata*) => (*'d option list*) **where**
PDataValue = *fst o Rep-pdata*

definition
PDataSpace :: (*'d pdata*) => (*'d dataspace*) **where**
PDataSpace = *snd o Rep-pdata*

definition
Data2PData :: (*'d data*) => (*'d pdata*) **where**
Data2PData D = (*let*
(*L*,*DP*) = *Rep-data D*;
OL = *map Some L*
in
Abs-pdata (*OL*,*DP*))

definition
PData2Data :: (*'d pdata*) => (*'d data*) **where**
PData2Data D = (*let*
(*OL*,*DP*) = *Rep-pdata D*;
L = *map the OL*
in
Abs-data (*L*,*DP*))

definition
DefaultPData :: (*'d dataspace*) => (*'d pdata*) **where**
DefaultPData D = *Abs-pdata* (*replicate* (*PartNum D*) *None*, *D*)

definition
OptionOverride :: (*'d option* ∗ *'d*) => *'d* **where**
OptionOverride P = (*if* (*fst P*) = *None then* (*snd P*) *else* (*the* (*fst P*)))

definition
DataOverride :: [*'d pdata*, *'d data*] => *'d data* ((- [*D*+]/ -) [*10*,*11*]*10*) **where**
DataOverride D1 D2 =
(*let*
(*L1*,*DP1*) = *Rep-pdata D1*;
(*L2*,*DP2*) = *Rep-data D2*;
L = *map OptionOverride* (*zip L1 L2*)
in
Abs-data (*L*,*DP2*))

lemma *Rep-pdata-tuple*:
Rep-pdata D = (*PDataValue D*, *PDataSpace D*)
apply (*unfold PDataValue-def PDataSpace-def*)
apply (*simp*)
done

lemma *Rep-pdata-select*:
(*PDataValue D*, *PDataSpace D*) ∈ *pdata*
apply (*subst Rep-pdata-tuple* [*THEN sym*])
apply (*rule Rep-pdata*)
done

lemma *PData-select*:
PData (*PDataValue D*) (*PDataSpace D*)
apply (*cut-tac D=D* **in** *Rep-pdata-select*)
apply (*unfold pdata-def*)
apply *auto*
done

3.2.1 *DefaultPData*

lemma *PData-DefaultPData* [*simp*]:
PData (*replicate* (*PartNum D*) *None*) *D*
apply (*unfold PData-def*)
apply *auto*
done

lemma *pdata-DefaultPData* [*simp*]:
(*replicate* (*PartNum D*) *None*, *D*) ∈ *pdata*
apply (*unfold pdata-def*)
apply *auto*
done

lemma *PDataSpace-DefaultPData* [*simp*]:
PDataSpace (*DefaultPData D*) = *D*
apply (*unfold DataSpace-def PDataSpace-def DefaultPData-def*)
apply *auto*
apply (*subst Abs-pdata-inverse*)
apply *auto*
done

lemma *length-PartNum-PData* [*simp*]:
length (*PDataValue P*) = *PartNum* (*PDataSpace P*)
apply (*cut-tac D=P* **in** *Rep-pdata-select*)
apply (*unfold pdata-def PData-def*)
apply *auto*
done

3.2.2 *Data2PData*

lemma *PData-Data2PData* [*simp*]:
PData (*map Some* (*DataValue D*)) (*Data.DataSpace D*)
apply (*unfold PData-def*)
apply *auto*
done

lemma *pdata-Data2PData* [*simp*]:
(*map Some* (*DataValue D*), *Data.DataSpace D*) ∈ *pdata*
apply (*unfold pdata-def*)
apply *auto*
done

lemma *DataSpace-Data2PData* [*simp*]:
(*PDataSpace* (*Data2PData D*)) = (*Data.DataSpace D*)
apply (*unfold DataSpace-def PDataSpace-def Data2PData-def Let-def*)
apply *auto*
apply (*cut-tac D=D* **in** *Rep-data-tuple*)
apply *auto*
apply (*subst Abs-pdata-inverse*)
apply *auto*
done

lemma *PDataValue-Data2PData-DataValue* [*simp*]:
(*map the* (*PDataValue* (*Data2PData D*))) = *DataValue D*
apply (*unfold DataValue-def PDataValue-def Data2PData-def Let-def*)
apply *auto*
apply (*cut-tac D=D* **in** *Rep-data-tuple*)
apply *auto*
apply (*subst Abs-pdata-inverse*)
apply *simp*
apply (*simp del*: *map-map*)
done

lemma *DataSpace-PData2Data*:
Data (*map the* (*PDataValue D*)) (*PDataSpace D*) ⟹
(*Data.DataSpace* (*PData2Data D*) = (*PDataSpace D*))
apply (*unfold DataSpace-def PDataSpace-def PData2Data-def Let-def*)
apply *auto*
apply (*cut-tac D=D* **in** *Rep-pdata-tuple*)
apply *auto*
apply (*subst Abs-data-inverse*)
apply (*unfold data-def*)
apply *auto*
done

lemma *PartNum-PDataValue-PartDom* [*simp*]:
⟦ *i* < *PartNum* (*PDataSpace Q*);
PDataValue Q ! *i* = *Some y* ⟧ ⟹
y ∈ *PartDom* (*PDataSpace Q*) *i*
apply (*cut-tac D=Q* **in** *Rep-pdata-select*)
apply (*unfold pdata-def PData-def*)
apply *auto*
done

3.2.3 *DataOverride*

lemma *Data-DataOverride*:
((*PDataSpace P*) = (*Data.DataSpace Q*)) ⟹
Data (*map OptionOverride* (*zip* (*PDataValue P*) (*Data.DataValue Q*))) (*Data.DataSpace Q*)

apply (*unfold Data-def*)
apply *auto*
apply (*unfold OptionOverride-def*)
apply *auto*
apply (*rename-tac i D*)
apply (*case-tac PDataValue P ! i = None*)
apply *auto*
apply (*drule sym*)
apply *auto*
done

lemma *data-DataOverride*:
((*PDataSpace P*) = (*Data.DataSpace Q*)) ⟹
(*map OptionOverride* (*zip* (*PDataValue P*) (*Data.DataValue Q*)), *Data.DataSpace Q*) ∈ *data*
apply (*unfold data-def*)
apply *auto*
apply (*rule Data-DataOverride*)
apply *fast*
done

lemma *DataSpace-DataOverride* [*simp*]:
((*Data.DataSpace D*) = (*PDataSpace E*)) ⟹
Data.DataSpace (*E* [*D*+] *D*) = (*Data.DataSpace D*)
apply (*unfold DataSpace-def DataOverride-def Let-def*)
apply *auto*
apply (*cut-tac D=D* **in** *Rep-data-tuple*)
apply (*cut-tac D=E* **in** *Rep-pdata-tuple*)
apply *auto*
apply (*subst Abs-data-inverse*)
apply *auto*
apply (*drule sym*)
apply *simp*
apply (*rule data-DataOverride*)
apply *auto*
done

lemma *DataValue-DataOverride* [*simp*]:
((*PDataSpace P*) = (*Data.DataSpace Q*)) ⟹
(*DataValue* (*P* [*D*+] *Q*)) = (*map OptionOverride* (*zip* (*PDataValue P*) (*Data.DataValue Q*)))
apply (*unfold DataValue-def DataOverride-def Let-def*)
apply *auto*
apply (*cut-tac D=P* **in** *Rep-pdata-tuple*)
apply (*cut-tac D=Q* **in** *Rep-data-tuple*)
apply *auto*
apply (*subst Abs-data-inverse*)
apply *auto*
apply (*rule data-DataOverride*)

apply *auto*
done

3.2.4 *OptionOverride*

lemma *DataValue-OptionOverride-nth*:
⟦ ((*PDataSpace P*) = (*DataSpace Q*));
i < *PartNum* (*DataSpace Q*) ⟧ ⟹
(*DataValue* (*P* [*D*+] *Q*) ! *i*) =
OptionOverride (*PDataValue P* ! *i*, *DataValue Q* ! *i*)
apply *auto*
done

lemma *None-OptionOverride* [*simp*]:
(*fst P*) = *None* ⟹ *OptionOverride P* = (*snd P*)
apply (*unfold OptionOverride-def*)
apply *auto*
done

lemma *Some-OptionOverride* [*simp*]:
(*fst P*) ≠ *None* ⟹ *OptionOverride P* = *the* (*fst P*)
apply (*unfold OptionOverride-def*)
apply *auto*
done

end

4 Update-Functions on Data Spaces

theory *Update*
imports *Data*
begin

4.1 Total update-functions

definition
Update :: ((*'d data*) => (*'d data*)) => *bool* **where**
Update U = (∀ *d*. *Data.DataSpace d* = *DataSpace* (*U d*))

lemma *Update-EmptySet*:
(% *d*. *d*) ∈ { *L* | *L*. *Update L*}
by (*unfold Update-def*, *auto*)

definition
update = { *L* | (*L*::((*'d data*) => (*'d data*))). *Update L*}

typedef *'d update* = *update* :: (*'d data* => *'d data*) *set*
unfolding *update-def*
apply (*rule exI*)

apply (*rule Update-EmptySet*)
done

definition
UpdateApply :: [*'d update*, *'d data*] => *'d data* ((- !!!/ -) [*10*,*11*]*10*) **where**
UpdateApply U D == *Rep-update U D*

definition
DefaultUpdate :: (*'d update*) **where**
DefaultUpdate == *Abs-update* (λ *D*. *D*)

4.1.1 Basic lemmas

lemma *Update-select*:
Update (*Rep-update U*)
apply (*cut-tac x*=*U* **in** *Rep-update*)
apply (*unfold update-def*)
apply *auto*
done

lemma *DataSpace-DataSpace-Update* [*simp*]:
Data.DataSpace (*Rep-update U DP*) = *Data.DataSpace DP*
apply (*cut-tac U*=*U* **in** *Update-select*)
apply (*unfold Update-def*)
apply *auto*
done

4.1.2 *DefaultUpdate*

lemma *Update-DefaultUpdate* [*simp*]:
Update (λ *D*. *D*)
by (*unfold Update-def*, *auto*)

lemma *update-DefaultUpdate* [*simp*]:
(λ *D*. *D*) ∈ *update*
by (*unfold update-def*, *auto*)

lemma *DataSpace-UpdateApply* [*simp*]:
Data.DataSpace (*U* !!! *D*) = *Data.DataSpace D*
by (*unfold UpdateApply-def*, *auto*)

4.2 Partial update-functions

definition
PUpdate :: ((*'d data*) => (*'d pdata*)) => *bool* **where**
PUpdate U = (∀ *d*. *Data.DataSpace d* = *PDataSpace* (*U d*))

lemma *PUpdate-EmptySet*:
(% *d*. *Data2PData d*) ∈ { *L* | *L*. *PUpdate L*}
by (*unfold PUpdate-def*, *auto*)

definition *pupdate* = { *L* | (*L*::((*'d data*) => (*'d pdata*))). *PUpdate L*}

typedef *'d pupdate* = *pupdate* :: (*'d data* => *'d pdata*) *set*
unfolding *pupdate-def*
apply (*rule exI*)
apply (*rule PUpdate-EmptySet*)
done

definition
PUpdateApply :: [*'d pupdate*, *'d data*] => *'d pdata* ((- !!/ -) [*10*,*11*]*10*) **where**
PUpdateApply U D = *Rep-pupdate U D*

definition
DefaultPUpdate :: (*'d pupdate*) **where**
DefaultPUpdate = *Abs-pupdate* (λ *D*. *DefaultPData* (*Data.DataSpace D*))

4.2.1 Basic lemmas

lemma *PUpdate-select*:
PUpdate (*Rep-pupdate U*)
apply (*cut-tac x*=*U* **in** *Rep-pupdate*)
apply (*unfold pupdate-def*)
apply *auto*
done

lemma *DataSpace-PDataSpace-PUpdate* [*simp*]:
PDataSpace (*Rep-pupdate U DP*) = *Data.DataSpace DP*
apply (*cut-tac U*=*U* **in** *PUpdate-select*)
apply (*unfold PUpdate-def*)
apply *auto*
done

4.2.2 *Data2PData*

lemma *PUpdate-Data2PData* [*simp*]:
PUpdate Data2PData
by (*unfold PUpdate-def*, *auto*)

lemma *pupdate-Data2PData* [*simp*]:
Data2PData ∈ *pupdate*
by (*unfold pupdate-def*, *auto*)

4.2.3 *PUpdate*

lemma *PUpdate-DefaultPUpdate* [*simp*]:
PUpdate (λ *D*. *DefaultPData* (*Data.DataSpace D*))
apply (*unfold PUpdate-def*)
apply *auto*
done

lemma *pupdate-DefaultPUpdate* [*simp*]:
(λ *D*. *DefaultPData* (*Data.DataSpace D*)) ∈ *pupdate*
apply (*unfold pupdate-def*)
apply *auto*
done

lemma *DefaultPUpdate-None* [*simp*]:
(*DefaultPUpdate* !! *D*) = *DefaultPData* (*DataSpace D*)
apply (*unfold DefaultPUpdate-def PUpdateApply-def*)
apply (*subst Abs-pupdate-inverse*)
apply *auto*
done

4.2.4 *SequentialRacing*

definition
UpdateOverride :: [*'d pupdate*, *'d update*] =>
'd update ((- [*U*+]/ -) [*10*,*11*]*10*) **where**
UpdateOverride U P = *Abs-update* (λ *DA* . (*U* !! *DA*) [*D*+] (*P* !!! *DA*))

inductive
FoldSet :: (*'b* => *'a* => *'a*) => *'a* => *'b set* => *'a* => *bool*
for *h* :: *'b* => *'a* => *'a*
and *z* :: *'a*
where
emptyI [*intro*]: *FoldSet h z* {} *z*
| *insertI* [*intro*]:
⟦ *x* ∉ *A*; *FoldSet h z A y* ⟧
⟹ *FoldSet h z* (*insert x A*) (*h x y*)

definition
SequentialRacing :: (*'d pupdate set*) => (*'d update set*) **where**
SequentialRacing U =
{*u*. *FoldSet UpdateOverride DefaultUpdate U u*}

lemma *FoldSet-imp-finite*:
FoldSet h z A x ⟹ *finite A*
by (*induct set*: *FoldSet*) *auto*

lemma *finite-imp-FoldSet*:
finite A ⟹ ∃ *x*. *FoldSet h z A x*

by (*induct set*: *finite*) *auto*

lemma *finite-SequentialRacing*:
  finite US $\Longrightarrow$ (*SOME u*. *u* $\in$ *SequentialRacing US*) $\in$ *SequentialRacing US*
apply (*unfold SequentialRacing-def*)
apply *auto*
apply (*drule-tac h*=*UpdateOverride* **and** *z*=*DefaultUpdate* **in** *finite-imp-FoldSet*)
apply *auto*
apply (*rule someI*)
apply *auto*
done

end

5 Label Expressions

theory *Expr*
imports *Update*
begin

no-notation *not* (*NOT*)

datatype ('*s*,'*e*)*expr* = *true*
  | *In* '*s*
  | *En* '*e*
  | *NOT* ('*s*,'*e*)*expr*
  | *And* ('*s*,'*e*)*expr* ('*s*,'*e*)*expr*
  | *Or* ('*s*,'*e*)*expr* ('*s*,'*e*)*expr*

type-synonym '*d guard* = ('*d data*) => *bool*
type-synonym ('*e*,'*d*)*action* = ('*e set* * '*d pupdate*)
type-synonym ('*s*,'*e*,'*d*)*label* = (('*s*,'*e*)*expr* * '*d guard* * ('*e*,'*d*)*action*)
type-synonym ('*s*,'*e*,'*d*)*trans* = ('*s* * ('*s*,'*e*,'*d*)*label* * '*s*)

primrec
  eval-expr :: [('*s set* * '*e set*), ('*s*,'*e*)*expr*] $\Rightarrow$ *bool* **where**
    eval-expr sc true = *True*
  | *eval-expr sc* (*En ev*) = (*ev* $\in$ *snd sc*)
  | *eval-expr sc* (*In st*) = (*st* $\in$ *fst sc*)
  | *eval-expr sc* (*NOT e1*) = ($\neg$ (*eval-expr sc e1*))
  | *eval-expr sc* (*And e1 e2*) = ((*eval-expr sc e1*) $\wedge$ (*eval-expr sc e2*))
  | *eval-expr sc* (*Or e1 e2*) = ((*eval-expr sc e1*) $\vee$ (*eval-expr sc e2*))

primrec
  ExprEvents :: ('*s*,'*e*)*expr* $\Rightarrow$ '*e set* **where**
    ExprEvents true = {}
  | *ExprEvents* (*En ev*) = {*ev*}
  | *ExprEvents* (*In st*) = {}

```
| ExprEvents (NOT e)     = (ExprEvents e)
| ExprEvents (And e1 e2) = ((ExprEvents e1) ∪ (ExprEvents e2))
| ExprEvents (Or  e1 e2) = ((ExprEvents e1) ∪ (ExprEvents e2))


datatype ('s, 'e, dead 'd)atomar =
                        TRUE
                        | FALSE
                        | IN 's
                        | EN 'e
                        | VAL 'd data => bool

definition
  source     :: ('s,'e,'d)trans => 's where
  source t  = fst t

definition
   Source     :: ('s,'e,'d)trans set => 's set where
  Source T == source ` T

definition
   target     :: ('s,'e,'d)trans => 's where
  target t  = snd(snd t)

definition
   Target     :: ('s,'e,'d)trans set => 's set where
  Target T = target ` T

definition
   label      :: ('s,'e,'d)trans => ('s,'e,'d)label where
  label t  = fst (snd t)

definition
   Label      :: ('s,'e,'d)trans set => ('s,'e,'d)label set where
  Label T = label ` T

definition
   expr       :: ('s,'e,'d)label => ('s,'e)expr where
  expr  = fst

definition
   action     :: ('s,'e,'d)label  => ('e,'d)action where
  action = snd o snd

definition
   Action     :: ('s,'e,'d)label set => ('e,'d)action set where
  Action L = action ` L
```

definition
pupdate :: ('s,'e,'d)*label* => 'd *pupdate* **where**
pupdate = *snd o action*

definition
PUpdate :: ('s,'e,'d)*label set* => ('d *pupdate*) *set* **where**
PUpdate L = *pupdate* ‘ *L*

definition
actevent :: ('s,'e,'d)*label* => 'e *set* **where**
actevent = *fst o action*

definition
Actevent :: ('s,'e,'d)*label set* => ('e *set*) *set* **where**
Actevent L = *actevent* ‘ *L*

definition
guard :: ('s,'e,'d)*label* => 'd *guard* **where**
guard = *fst o snd*

definition
Guard :: ('s,'e,'d)*label set* => ('d *guard*) *set* **where**
Guard L = *guard* ‘ *L*

definition
defaultexpr :: ('s,'e)*expr* **where**
defaultexpr = *true*

definition
defaultaction :: ('e,'d)*action* **where**
defaultaction = ({},*DefaultPUpdate*)

definition
defaultguard :: ('d *guard*) **where**
defaultguard = (λ *d*. *True*)

definition
defaultlabel :: ('s,'e,'d)*label* **where**
defaultlabel = (*defaultexpr*, *defaultguard*, *defaultaction*)

definition
eval :: [('s *set* * 'e *set* * 'd *data*), ('s,'e,'d)*label*] => *bool*
(- |= - [*91*,*90*]*90*) **where**
eval scd l = (*let* (*s*,*e*,*d*) = *scd*
in
((*eval-expr* (*s*,*e*) (*expr l*)) ∧ ((*guard l*) *d*)))

lemma *Source-EmptySet* [*simp*]:
Source {} = {}

by (*unfold Source-def*, *auto*)

lemma *Target-EmptySet* [*simp*]:
Target {} = {}
by (*unfold Target-def*, *auto*)

lemma *Label-EmptySet* [*simp*]:
Label {} = {}
by (*unfold Label-def*, *auto*)

lemma *Action-EmptySet* [*simp*]:
Action {} = {}
by (*unfold Action-def*, *auto*)

lemma *PUpdate-EmptySet* [*simp*]:
PUpdate {} = {}
by (*unfold PUpdate-def*, *auto*)

lemma *Actevent-EmptySet* [*simp*]:
Actevent {} = {}
by (*unfold Actevent-def*, *auto*)

lemma *Union-Actevent-subset*:
⟦ $m \in M$; ((⋃ (*Actevent* (*Label* (*Union M*)))) ⊆ (N::'a *set*)) ⟧ ⟹
((⋃ (*Actevent* (*Label m*))) ⊆ N)
by (*unfold Actevent-def Label-def*, *auto*)

lemma *action-select* [*simp*]:
action (a,b,c) = c
by (*unfold action-def*, *auto*)

lemma *label-select* [*simp*]:
label (a,b,c) = b
by (*unfold label-def*, *auto*)

lemma *target-select* [*simp*]:
target (a,b,c) = c
by (*unfold target-def*, *auto*)

lemma *actevent-select* [*simp*]:
actevent (a,b,(c,d)) = c
by (*unfold actevent-def*, *auto*)

lemma *pupdate-select* [*simp*]:
pupdate (a,b,c,d) = d
by (*unfold pupdate-def*, *auto*)

lemma *source-select* [*simp*]:
source (a,b) = a

by (*unfold source-def*, *auto*)

lemma *finite-PUpdate* [*simp*]:
finite S $\Longrightarrow$ *finite*(*PUpdate S*)
by (*unfold PUpdate-def*, *auto*)

lemma *finite-Label* [*simp*]:
finite S $\Longrightarrow$ *finite*(*Label S*)
by (*unfold Label-def*, *auto*)

lemma *fst-defaultaction* [*simp*]:
fst defaultaction = {}
by (*unfold defaultaction-def*, *auto*)

lemma *action-defaultlabel* [*simp*]:
(*action defaultlabel*) = *defaultaction*
by (*unfold defaultlabel-def action-def*, *auto*)

lemma *fst-defaultlabel* [*simp*]:
(*fst defaultlabel*) = *defaultexpr*
by (*unfold defaultlabel-def*, *auto*)

lemma *ExprEvents-defaultexpr* [*simp*]:
(*ExprEvents defaultexpr*) = {}
by (*unfold defaultexpr-def*, *auto*)

lemma *defaultlabel-defaultexpr* [*simp*]:
expr defaultlabel = *defaultexpr*
by (*unfold defaultlabel-def expr-def*, *auto*)

lemma *target-Target* [*simp*]:
$t \in T \Longrightarrow$ *target* $t \in$ *Target* T
by (*unfold Target-def*, *auto*)

lemma *Source-union* : *Source* $s \cup$ *Source* t = *Source* $(s \cup t)$
apply (*unfold Source-def*)
apply *auto*
done

lemma *Target-union* : *Target* $s \cup$ *Target* t = *Target* $(s \cup t)$
apply (*unfold Target-def*)
apply *auto*
done

end

6 Sequential Automata

theory *SA*

imports *Expr*
begin

definition
SeqAuto :: ['*s set*,
'*s*,
(('*s*,'*e*,'*d*)*label*) *set*,
(('*s*,'*e*,'*d*)*trans*) *set*]
=> *bool* **where**
SeqAuto S I L D = (*I* ∈ *S* ∧ *S* ≠ {} ∧ *finite S* ∧ *finite D* ∧
(∀ (*s*,*l*,*t*) ∈ *D*. *s* ∈ *S* ∧ *t* ∈ *S* ∧ *l* ∈ *L*))

lemma *SeqAuto-EmptySet*:
({@*x* .*True*}, (@*x* .*True*), {}, {}) ∈ {(*S*,*I*,*L*,*D*) | *S I L D*. *SeqAuto S I L D*}
by (*unfold SeqAuto-def*, *auto*)

definition
seqauto =
{ (*S*,*I*,*L*,*D*) |
(*S*::'*s set*)
(*I*::'*s*)
(*L*::(('*s*,'*e*,'*d*)*label*) *set*)
(*D*::(('*s*,'*e*,'*d*)*trans*) *set*).
SeqAuto S I L D}

typedef ('*s*,'*e*,'*d*) *seqauto* =
seqauto :: ('*s set* * '*s* * (('*s*,'*e*,'*d*)*label*) *set* * (('*s*,'*e*,'*d*)*trans*) *set*) *set*
unfolding *seqauto-def*
apply (*rule exI*)
apply (*rule SeqAuto-EmptySet*)
done

definition
States :: (('*s*,'*e*,'*d*)*seqauto*) => '*s set* **where**
States = *fst o Rep-seqauto*

definition
InitState :: (('*s*,'*e*,'*d*)*seqauto*) => '*s* **where**
InitState = *fst o snd o Rep-seqauto*

definition
Labels :: (('*s*,'*e*,'*d*)*seqauto*) => (('*s*,'*e*,'*d*)*label*) *set* **where**
Labels = *fst o snd o snd o Rep-seqauto*

definition
Delta :: (('*s*,'*e*,'*d*)*seqauto*) => (('*s*,'*e*,'*d*)*trans*) *set* **where**
Delta = *snd o snd o snd o Rep-seqauto*

definition

SAEvents :: $(('s,'e,'d)seqauto) => 'e\ set$ **where**
SAEvents SA = $(\bigcup\ l \in Label\ (Delta\ SA).\ (fst\ (action\ l)) \cup (ExprEvents\ (expr\ l)))$

lemma *Rep-seqauto-tuple*:
Rep-seqauto SA = (*States SA*, *InitState SA*, *Labels SA*, *Delta SA*)
by (*unfold States-def InitState-def Labels-def Delta-def*, *auto*)

lemma *Rep-seqauto-select*:
(*States SA*,*InitState SA*,*Labels SA*,*Delta SA*) ∈ *seqauto*
by (*rule Rep-seqauto-tuple* [*THEN subst*], *rule Rep-seqauto*)

lemma *SeqAuto-select*:
SeqAuto (*States SA*) (*InitState SA*) (*Labels SA*) (*Delta SA*)
by (*cut-tac SA=SA* **in** *Rep-seqauto-select*, *unfold seqauto-def*, *auto*)

lemma *neq-States* [*simp*]:
States SA ≠ {}
apply (*cut-tac Rep-seqauto-select*)
apply *auto*
apply (*unfold seqauto-def SeqAuto-def*)
apply *auto*
done

lemma *SA-States-disjunct* :
(*States A*) ∩ (*States A'*) = {} ⟹ *A'* ≠ *A*
by *auto*

lemma *SA-States-disjunct2* :
⟦ (*States A*) ∩ *C* = {}; *States B* ⊆ *C* ⟧ ⟹ *B* ≠ *A*
apply (*rule SA-States-disjunct*)
apply *auto*
done

lemma *SA-States-disjunct3* :
⟦ *C* ∩ *States A* = {}; *States B* ⊆ *C* ⟧ ⟹ *States A* ∩ *States B* = {}
apply (*cut-tac SA=B* **in** *neq-States*)
apply *fast*
done

lemma *EX-State-SA* [*simp*]:
∃ *S*. *S* ∈ *States SA*
apply (*cut-tac Rep-seqauto-select*)
apply (*unfold seqauto-def SeqAuto-def*)
apply *auto*
done

lemma *finite-States* [*simp*]:
finite (*States A*)

apply (*cut-tac Rep-seqauto-select*)
apply (*unfold seqauto-def SeqAuto-def*)
apply *auto*
done

lemma *finite-Delta* [*simp*]:
  finite (*Delta A*)
apply (*cut-tac Rep-seqauto-select*)
apply (*unfold seqauto-def SeqAuto-def*)
apply *auto*
done

lemma *InitState-States* [*simp*]:
  InitState A ∈ *States A*
apply (*cut-tac Rep-seqauto-select*)
apply (*unfold seqauto-def SeqAuto-def*)
apply *auto*
done

lemma *SeqAuto-EmptySet-States* [*simp*]:
 (*States* (*Abs-seqauto* ({@*x*. *True*}, (@*x*. *True*), {}, {}))) = {(@*x*. *True*)}
apply (*unfold States-def*)
apply (*simp*)
apply (*subst Abs-seqauto-inverse*)
apply (*unfold seqauto-def*)
apply (*rule SeqAuto-EmptySet*)
apply *auto*
done

lemma *SeqAuto-EmptySet-SAEvents* [*simp*]:
 (*SAEvents* (*Abs-seqauto* ({@*x*. *True*}, (@*x*. *True*), {}, {}))) = {}
apply (*unfold SAEvents-def Delta-def*)
apply *simp*
apply (*subst Abs-seqauto-inverse*)
apply (*unfold seqauto-def*)
apply (*rule SeqAuto-EmptySet*)
apply *auto*
done

lemma *Label-Delta-subset* [*simp*]:
  (*Label* (*Delta SA*)) ⊆ *Labels SA*
apply (*unfold Label-def label-def*)
apply *auto*
apply (*cut-tac SA*=*SA* **in** *SeqAuto-select*)
apply (*unfold SeqAuto-def*)
apply *auto*
done

lemma *Target-SAs-Delta-States*:

Target ($\bigcup$(*Delta* ‘ (*SAs HA*))) $\subseteq$ $\bigcup$(*States* ‘ (*SAs HA*))
apply (*unfold image-def Target-def target-def*)
apply *auto*
apply (*rename-tac SA Source Trigger Guard Action Update Target*)
apply (*cut-tac SA=SA* **in** *SeqAuto-select*)
apply (*unfold SeqAuto-def*)
apply *auto*
done

lemma *States-Int-not-mem*:
($\bigcup$(*States* ‘ *F*) *Int States SA*) = {} $\Longrightarrow$ *SA* $\notin$ *F*
apply (*unfold Int-def*)
apply *auto*
apply (*subgoal-tac* $\exists$ *S*. *S* $\in$ *States SA*)
prefer *2*
apply (*rule EX-State-SA*)
apply (*erule exE*)
apply (*rename-tac T*)
apply (*erule-tac x=T* **in** *allE*)
apply *auto*
done

lemma *Delta-target-States* [*simp*]:
⟦ *T* $\in$ *Delta A*⟧ $\Longrightarrow$ *target T* $\in$ *States A*
apply (*cut-tac SA=A* **in** *SeqAuto-select*)
apply (*unfold SeqAuto-def source-def target-def*)
apply *auto*
done

lemma *Delta-source-States* [*simp*]:
⟦ *T* $\in$ *Delta A* ⟧ $\Longrightarrow$ *source T* $\in$ *States A*
apply (*cut-tac SA=A* **in** *SeqAuto-select*)
apply (*unfold SeqAuto-def source-def target-def*)
apply *auto*
done

end

7 Syntax of Hierarchical Automata

theory *HA*
imports *SA*
begin

7.1 Definitions

definition
RootEx :: [(('*s*,'*e*,'*d*)*seqauto*) *set*,
'*s* $\rightharpoonup$ ('*s*,'*e*,'*d*) *seqauto set*] => *bool* **where**

$RootEx\ F\ G = (\exists!\ A.\ A \in F \wedge A \notin \bigcup\ (ran\ G))$

definition
$Root :: [(('s,'e,'d)seqauto)\ set,$
$\quad 's \rightharpoonup ('s,'e,'d)\ seqauto\ set] \Rightarrow ('s,'e,'d)\ seqauto$ **where**
$Root\ F\ G = (@\ A.\ A \in F \wedge\ A \notin \bigcup\ (ran\ G))$

definition
$MutuallyDistinct :: (('s,'e,'d)seqauto)\ set \Rightarrow bool$ **where**
$MutuallyDistinct\ F =$
$\quad (\forall\ a \in F.\ \forall\ b \in F.\ a \neq b \longrightarrow (States\ a) \cap (States\ b) = \{\})$

definition
$OneAncestor :: [(('s,'e,'d)seqauto)\ set,$
$\quad 's \rightharpoonup ('s,'e,'d)\ seqauto\ set] \Rightarrow bool$ **where**
$OneAncestor\ F\ G =$
$\quad (\forall\ A \in F - \{Root\ F\ G\}\ .$
$\quad\quad \exists!\ s.\ s \in (\bigcup\ A' \in F - \{A\}\ .\ States\ A') \wedge$
$\quad\quad\quad A \in the\ (G\ s))$

definition
$NoCycles :: [(('s,'e,'d)seqauto)\ set,$
$\quad 's \rightharpoonup ('s,'e,'d)\ seqauto\ set] \Rightarrow bool$ **where**
$NoCycles\ F\ G =$
$\quad (\forall\ S \in Pow\ (\bigcup\ A \in F.\ States\ A).$
$\quad\quad S \neq \{\} \longrightarrow\ (\exists\ s \in S.\ S \cap (\bigcup\ A \in the\ (G\ s).\ States\ A) = \{\}))$

definition
$IsCompFun :: [(('s,'e,'d)seqauto)\ set,$
$\quad 's \rightharpoonup ('s,'e,'d)\ seqauto\ set] \Rightarrow bool$ **where**
$IsCompFun\ F\ G = ((dom\ G = (\bigcup\ A \in F.\ States\ A)) \wedge$
$\quad (\bigcup\ (ran\ G) = (F - \{Root\ F\ G\})) \wedge$
$\quad (RootEx\ F\ G) \wedge$
$\quad (OneAncestor\ F\ G) \wedge$
$\quad (NoCycles\ F\ G))$

7.1.1 Well-formedness for the syntax of HA

definition
$HierAuto :: ['d\ data,$

```
                (('s,'e,'d)seqauto) set,
               'e set,
               's ⇀ (('s,'e,'d)seqauto) set]
               => bool where
  HierAuto D F E G = ((⋃ A ∈ F. SAEvents A) ⊆ E ∧
                      MutuallyDistinct F ∧
                      finite F ∧
                      IsCompFun F G)

lemma HierAuto-EmptySet:
 ((@x. True),{Abs-seqauto ({@x. True}, (@x. True), {}, {})}, {},
   Map.empty ( @x. True ↦ {})) ∈ {(D,F,E,G) | D F E G. HierAuto D F E G}
apply (unfold HierAuto-def IsCompFun-def Root-def RootEx-def MutuallyDistinct-def
         OneAncestor-def NoCycles-def)
apply auto
done

definition
  hierauto =
    {(D,F,E,G) |
       (D::'d data)
       (F::(('s,'e,'d) seqauto) set)
       (E::('e set))
       (G::('s ⇀ (('s,'e,'d) seqauto) set)).
                              HierAuto D F E G}

typedef ('s,'e,'d) hierauto =
    hierauto :: ('d data * ('s,'e,'d) seqauto set * 'e set * ('s ⇀ ('s,'e,'d) seqauto
set)) set
  unfolding hierauto-def
  apply (rule exI)
  apply (rule HierAuto-EmptySet)
  done

definition
  SAs :: (('s,'e,'d) hierauto)  => (('s,'e,'d) seqauto) set where
  SAs = fst o snd o Rep-hierauto

definition
  HAEvents :: (('s,'e,'d) hierauto) => ('e set) where
  HAEvents = fst o snd o snd o Rep-hierauto

definition
  CompFun :: (('s,'e,'d) hierauto) => ('s ⇀ ('s,'e,'d) seqauto set) where
  CompFun = (snd o snd o snd o Rep-hierauto)

definition
  HAStates :: (('s,'e,'d) hierauto) => ('s set) where
  HAStates HA = (⋃  A ∈ (SAs HA). States A)
```

definition
HADelta :: $((\prime s,\prime e,\prime d)\ hierauto) \Rightarrow ((\prime s,\prime e,\prime d)trans)set$ **where**
HADelta HA = $(\bigcup\ F \in (SAs\ HA).\ Delta\ F)$

definition
HAInitValue :: $((\prime s,\prime e,\prime d)\ hierauto) \Rightarrow \prime d\ data$ **where**
HAInitValue == *fst o Rep-hierauto*

definition
HAInitStates :: $((\prime s,\prime e,\prime d)\ hierauto) \Rightarrow \prime s\ set$ **where**
HAInitStates HA == $\bigcup\ A \in (SAs\ HA).\ \{\ InitState\ A\ \}$

definition
HARoot :: $((\prime s,\prime e,\prime d)\ hierauto) \Rightarrow (\prime s,\prime e,\prime d)seqauto$ **where**
HARoot HA == *Root* (*SAs HA*) (*CompFun HA*)

definition
HAInitState :: $((\prime s,\prime e,\prime d)\ hierauto) \Rightarrow \prime s$ **where**
HAInitState HA == *InitState* (*HARoot HA*)

7.1.2 State successor function

definition
Chi :: $(\prime s,\prime e,\prime d)hierauto \Rightarrow \prime s \Rightarrow \prime s\ set$ **where**
Chi A == $(\lambda\ S \in (HAStates\ A)\ .$
$\{S'.\ \exists\ SA \in (SAs\ A)\ .\ SA \in the\ ((CompFun\ A)\ S) \wedge S' \in States\ SA$
$\})$

definition
ChiRel :: $(\prime s,\prime e,\prime d)hierauto \Rightarrow (\prime s * \prime s)\ set$ **where**
ChiRel A == $\{\ (S,S').\ S \in HAStates\ A \wedge S' \in HAStates\ A \wedge S' \in (Chi\ A)\ S\ \}$

definition
ChiPlus :: $(\prime s,\prime e,\prime d)hierauto \Rightarrow (\prime s * \prime s)\ set$ **where**
ChiPlus A == $(ChiRel\ A)\ \hat{}+$

definition
ChiStar :: $(\prime s,\prime e,\prime d)hierauto \Rightarrow (\prime s * \prime s)\ set$ **where**
ChiStar A == $(ChiRel\ A)\ \hat{}*$

definition
HigherPriority :: $[(\prime s,\prime e,\prime d)hierauto,$

$('s,'e,'d)trans * ('s,'e,'d)trans] => bool$ **where**
HigherPriority A ==
$\lambda\ (t,t') \in (HADelta\ A) \times (HADelta\ A).$
$(source\ t', source\ t) \in\ ChiPlus\ A$

7.1.3 Configurations

definition
InitConf :: $('s,'e,'d)hierauto => 's\ set$ **where**
InitConf A == $(((((HAInitStates\ A) \times (HAInitStates\ A)) \cap (ChiRel\ A))^{*})$
$``\ \{HAInitState\ A\})$

definition
StepConf :: $[('s,'e,'d)hierauto,\ 's\ set,$
$('s,'e,'d)trans\ set] => 's\ set$ **where**

StepConf A C TS ==
$(C - ((ChiStar\ A)\ ``\ (Source\ TS))) \cup$
$(Target\ TS) \cup$
$((ChiRel\ A)\ ``\ (Target\ TS)) \cap (HAInitStates\ A) \cup$
$((((ChiRel\ A) \cap ((HAInitStates\ A) \times (HAInitStates\ A)))^{+})$
$``\ (((ChiRel\ A) ``\ (Target\ TS)) \cap (HAInitStates\ A)))$

7.2 Lemmas

lemma *Rep-hierauto-tuple*:
Rep-hierauto HA = (*HAInitValue HA*, *SAs HA*, *HAEvents HA*, *CompFun HA*)
by (*unfold SAs-def HAEvents-def CompFun-def HAInitValue-def*, *simp*)

lemma *Rep-hierauto-select*:
(*HAInitValue HA*, *SAs HA*, *HAEvents HA*, *CompFun HA*): *hierauto*
by (*rule Rep-hierauto-tuple* [*THEN subst*], *rule Rep-hierauto*)

lemma *HierAuto-select* [*simp*]:
HierAuto (*HAInitValue HA*) (*SAs HA*) (*HAEvents HA*) (*CompFun HA*)

by (*cut-tac Rep-hierauto-select*, *unfold hierauto-def*, *simp*)

7.2.1 *HAStates*

lemma *finite-HAStates* [*simp*]:
finite (*HAStates HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *auto*
apply (*simp add*: *HAStates-def*)
apply (*rule finite-UN-I*)
apply *fast*
apply (*rule finite-States*)
done

lemma *HAStates-SA-mem*:
⟦ *SA* ∈ *SAs A*; *S* ∈ *States SA* ⟧ ⟹ *S* ∈ *HAStates A*
by (*unfold HAStates-def*, *auto*)

lemma *ChiRel-HAStates* [*simp*]:
(*a*,*b*) ∈ *ChiRel A* ⟹ *a* ∈ *HAStates A*
apply (*unfold ChiRel-def*)
apply *auto*
done

lemma *ChiRel-HAStates2* [*simp*]:
(*a*,*b*) ∈ *ChiRel A* ⟹ *b* ∈ *HAStates A*
apply (*unfold ChiRel-def*)
apply *auto*
done

7.2.2 *HAEvents*

lemma *HAEvents-SAEvents-SAs*:
⋃(*SAEvents* ‘ (*SAs HA*)) ⊆ *HAEvents HA*
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *fast*
done

7.2.3 *NoCycles*

lemma *NoCycles-EmptySet* [*simp*]:
NoCycles {} *S*
by (*unfold NoCycles-def*, *auto*)

lemma *NoCycles-HA* [*simp*]:
NoCycles (*SAs HA*) (*CompFun HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def IsCompFun-def*)

apply *auto*
done

7.2.4 *OneAncestor*

lemma *OneAncestor-HA* [*simp*]:
OneAncestor (*SAs HA*) (*CompFun HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def IsCompFun-def*)
apply *auto*
done

7.2.5 *MutuallyDistinct*

lemma *MutuallyDistinct-Single* [*simp*]:
MutuallyDistinct {*SA*}
by (*unfold MutuallyDistinct-def*, *auto*)

lemma *MutuallyDistinct-EmptySet* [*simp*]:
MutuallyDistinct {}
by (*unfold MutuallyDistinct-def*, *auto*)

lemma *MutuallyDistinct-Insert*:
⟦ *MutuallyDistinct S*; (*States A*) ∩ (⋃ *B* ∈ *S*. *States B*) = {} ⟧
⟹ *MutuallyDistinct* (*insert A S*)
by (*unfold MutuallyDistinct-def*, *safe*, *fast+*)

lemma *MutuallyDistinct-Union*:
⟦ *MutuallyDistinct A*; *MutuallyDistinct B*;
(⋃ *C* ∈ *A*. *States C*) ∩ (⋃ *C* ∈ *B*. *States C*) = {} ⟧
⟹ *MutuallyDistinct* (*A* ∪ *B*)
by (*unfold MutuallyDistinct-def*, *safe*, *blast+*)

lemma *MutuallyDistinct-HA* [*simp*]:
MutuallyDistinct (*SAs HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def IsCompFun-def*)
apply *auto*
done

7.2.6 *RootEx*

lemma *RootEx-Root* [*simp*]:
RootEx F G ⟹ *Root F G* ∈ *F*
apply (*unfold RootEx-def Root-def*)
apply (*erule ex1E*)
apply (*erule conjE*)
apply (*rule someI2*)
apply *blast+*
done

lemma *RootEx-Root-ran* [*simp*]:
RootEx F G $\Longrightarrow$ *Root F G* $\notin \bigcup$ (*ran G*)
apply (*unfold RootEx-def Root-def*)
apply (*erule ex1E*)
apply (*erule conjE*)
apply (*rule someI2*)
apply *blast+*
done

lemma *RootEx-States-Subset* [*simp*]:
(*RootEx F G*) $\Longrightarrow$ *States* (*Root F G*) $\subseteq$ ($\bigcup$ *x* $\in$ *F* . *States x*)
apply (*unfold RootEx-def Root-def*)
apply (*erule ex1E*)
apply (*erule conjE*)
apply (*rule someI2*)
apply *fast*
apply (*unfold UNION-eq*)
apply (*simp add*: *subset-eq*)
apply *auto*
done

lemma *RootEx-States-notdisjunct* [*simp*]:
RootEx F G $\Longrightarrow$ *States* (*Root F G*) $\cap$ ($\bigcup$ *x* $\in$ *F* . *States x*) $\neq$ {}
apply (*frule RootEx-States-Subset*)
apply (*case-tac States* (*Root F G*)={})
prefer *2*
apply *fast*
apply *simp*
done

lemma *Root-neq-SA* [*simp*]:
$[\![$ *RootEx F G*; ($\bigcup$ *x* $\in$ *F* . *States x*) $\cap$ *States SA* = {} $]\!]$ $\Longrightarrow$ *Root F G* $\neq$ *SA*
apply (*rule SA-States-disjunct*)
apply (*frule RootEx-States-Subset*)
apply *fast*
done

lemma *RootEx-HA* [*simp*]:
RootEx (*SAs HA*) (*CompFun HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def IsCompFun-def*)
apply *fast*
done

7.2.7 *HARoot*

lemma *HARoot-SAs* [*simp*]:
(*HARoot HA*) $\in$ *SAs HA*

apply (*unfold HARoot-def*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *auto*
done

lemma *States-HARoot-HAStates*:
States (*HARoot HA*) ⊆ *HAStates HA*
apply (*unfold HAStates-def*)
apply *auto*
apply (*rule-tac x=HARoot HA* **in** *bexI*)
apply *auto*
done

lemma *SAEvents-HARoot-HAEvents*:
SAEvents (*HARoot HA*) ⊆ *HAEvents HA*
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *auto*
apply (*rename-tac S*)
apply (*unfold UNION-eq*)
apply (*simp add*: *subset-eq*)
apply (*erule-tac x=S* **in** *allE*)
apply *auto*
done

lemma *HARoot-ran-CompFun*:
HARoot HA ∉ *Union* (*ran* (*CompFun HA*))
apply (*unfold HARoot-def*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold IsCompFun-def hierauto-def HierAuto-def*)
apply *fast*
done

lemma *HARoot-ran-CompFun2*:
S ∈ *ran* (*CompFun HA*) ⟶ *HARoot HA* ∉ *S*
apply (*unfold HARoot-def*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold IsCompFun-def hierauto-def HierAuto-def*)
apply *fast*
done

7.2.8 *CompFun*

lemma *IsCompFun-HA* [*simp*]:
IsCompFun (*SAs HA*) (*CompFun HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *auto*

done

lemma *dom-CompFun* [*simp*]:
dom (*CompFun HA*) = *HAStates HA*
apply (*cut-tac HA=HA* **in** *IsCompFun-HA*)
apply (*unfold IsCompFun-def HAStates-def*)
apply *auto*
done

lemma *ran-CompFun* [*simp*]:
Union (*ran* (*CompFun HA*)) = ((*SAs HA*) − {*Root* (*SAs HA*)(*CompFun HA*)})
apply (*cut-tac HA=HA* **in** *IsCompFun-HA*)
apply (*unfold IsCompFun-def*)
apply *fast*
done

lemma *ran-CompFun-subseteq*:
Union (*ran* (*CompFun HA*)) ⊆ (*SAs HA*)
apply (*cut-tac HA=HA* **in** *IsCompFun-HA*)
apply (*unfold IsCompFun-def*)
apply *fast*
done

lemma *ran-CompFun-is-not-SA*:
¬ *Sas* ⊆ (*SAs HA*) ⟹ *Sas* ∉ (*ran* (*CompFun HA*))
apply (*cut-tac HA=HA* **in** *IsCompFun-HA*)
apply (*unfold IsCompFun-def*)
apply *fast*
done

lemma *HAStates-HARoot-CompFun* [*simp*]:
S ∈ *HAStates HA* ⟹ *HARoot HA* ∉ *the* (*CompFun HA S*)
apply (*rule ran-dom-the*)
back
apply (*simp add*: *HARoot-ran-CompFun2 HARoot-def HAStates-def*)+
done

lemma *HAStates-CompFun-SAs*:
S ∈ *HAStates A* ⟹ *the* (*CompFun A S*) ⊆ *SAs A*
apply *auto*
apply (*rename-tac T*)
apply (*cut-tac HA=A* **in** *ran-CompFun*)
apply (*erule equalityE*)
apply (*erule-tac c=T* **in** *subsetCE*)
apply (*drule ran-dom-the*)
apply *auto*
done

lemma *HAStates-CompFun-notmem* [*simp*]:

⟦ $S \in HAStates\ A$; $SA \in the\ (CompFun\ A\ S)$ ⟧ ⟹ $S \notin States\ SA$
apply (*unfold HAStates-def*)
apply *auto*
apply (*rename-tac T*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=T* **in** *ballE*)
apply *auto*
prefer *2*
apply (*cut-tac A=A* **and** *S=S* **in** *HAStates-CompFun-SAs*)
apply (*unfold HAStates-def*)
apply *simp*
apply *fast*
apply *fast*
apply (*cut-tac HA=A* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x={S}* **in** *ballE*)
apply *auto*
done

lemma *CompFun-Int-disjoint*:
⟦ $S \neq T$; $S \in HAStates\ A$; $T \in HAStates\ A$ ⟧ ⟹ $the\ (CompFun\ A\ T) \cap the\ (CompFun\ A\ S) = \{\}$
apply *auto*
apply (*rename-tac U*)
apply (*cut-tac HA=A* **in** *OneAncestor-HA*)
apply (*unfold OneAncestor-def*)
apply (*erule-tac x=U* **in** *ballE*)
prefer *2*
apply *simp*
apply (*fold HARoot-def*)
apply (*frule HAStates-HARoot-CompFun*)
apply *simp*
apply (*frule HAStates-CompFun-SAs*)
apply *auto*
apply (*erule-tac x=S* **in** *allE*)
apply (*erule-tac x=T* **in** *allE*)
apply *auto*
apply (*cut-tac HA=A* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*simp only*: *HAStates-def*)
apply *safe*
apply (*erule-tac x={S}* **in** *ballE*)
apply *simp*
apply *fast*
apply *simp*
apply (*cut-tac HA=A* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)

apply (*simp only*: *HAStates-def*)
apply *safe*
apply (*erule-tac x*={ *T*} **in** *ballE*)
apply *simp*
apply *fast*
apply *simp*
done

7.2.9 *SAs*

lemma *finite-SAs* [*simp*]:
finite (*SAs HA*)
apply (*cut-tac Rep-hierauto-select*)
apply (*unfold hierauto-def HierAuto-def*)
apply *fast*
done

lemma *HAStates-SAs-disjunct*:
HAStates HA1 ∩ *HAStates HA2* = {} ⟹ *SAs HA1* ∩ *SAs HA2* = {}
apply (*unfold UNION-eq HAStates-def Int-def*)
apply *auto*
apply (*rename-tac SA*)
apply (*cut-tac SA*=*SA* **in** *EX-State-SA*)
apply (*erule exE*)
apply *auto*
done

lemma *HAStates-CompFun-SAs-mem* [*simp*]:
⟦ *S* ∈ *HAStates A*; *T* ∈ *the* (*CompFun A S*) ⟧ ⟹ *T* ∈ *SAs A*
apply (*cut-tac A*=*A* **and** *S*=*S* **in** *HAStates-CompFun-SAs*)
apply *auto*
done

lemma *SAs-States-HAStates*:
SA ∈ *SAs A* ⟹ *States SA* ⊆ *HAStates A*
by (*unfold HAStates-def*, *auto*)

7.2.10 *HAInitState*

lemma *HAInitState-HARoot* [*simp*]:
HAInitState A ∈ *States* (*HARoot A*)
by (*unfold HAInitState-def*, *auto*)

lemma *HAInitState-HARoot2* [*simp*]:
HAInitState A ∈ *States* (*Root* (*SAs A*) (*CompFun A*))
by (*fold HARoot-def*, *simp*)

lemma *HAInitStates-HAStates* [*simp*]:
HAInitStates A ⊆ *HAStates A*
apply (*unfold HAInitStates-def HAStates-def*)

apply *auto*
done

lemma *HAInitStates-HAStates2* [*simp*]:
  $S \in HAInitStates\ A \Longrightarrow S \in HAStates\ A$
apply (*cut-tac A=A* **in** *HAInitStates-HAStates*)
apply *fast*
done

lemma *HAInitState-HAStates* [*simp*]:
  $HAInitState\ A \in HAStates\ A$
apply (*unfold HAStates-def*)
apply *auto*
apply (*rule-tac x=HARoot A* **in** *bexI*)
apply *auto*
done

lemma *HAInitState-HAInitStates* [*simp*]:
  $HAInitState\ A \in HAInitStates\ A$
by (*unfold HAInitStates-def HAInitState-def*, *auto*)

lemma *CompFun-HAInitStates-HAStates* [*simp*]:
$[\![S \in HAStates\ A; SA \in the\ (CompFun\ A\ S)]\!] \Longrightarrow (InitState\ SA) \in HAInitStates\ A$
apply (*unfold HAInitStates-def*)
apply *auto*
done

lemma *CompFun-HAInitState-HAInitStates* [*simp*]:
$[\![SA \in the\ (CompFun\ A\ (HAInitState\ A))]\!] \Longrightarrow (InitState\ SA) \in HAInitStates\ A$
apply (*unfold HAInitStates-def*)
apply *auto*
apply (*rule-tac x=SA* **in** *bexI*)
apply *auto*
apply (*cut-tac A=A* **and** *S=HAInitState A* **in** *HAStates-CompFun-SAs*)
apply *auto*
done

lemma *HAInitState-notmem-States* [*simp*]:
  $[\![S \in HAStates\ A; SA \in the\ (CompFun\ A\ S)]\!] \Longrightarrow HAInitState\ A \notin States\ SA$
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=HARoot A* **in** *ballE*)
apply *auto*
done

lemma *InitState-notmem-States* [*simp*]:
⟦ *S* ∈ *HAStates A*; *SA* ∈ *the* (*CompFun A S*);
T ∈ *HAInitStates A*; *T* ≠ *InitState SA* ⟧
⟹ *T* ∉ *States SA*
apply (*unfold HAInitStates-def*)
apply *auto*
apply (*rename-tac SAA*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply *auto*
done

lemma *InitState-States-notmem* [*simp*]:
⟦ *B* ∈ *SAs A*; *C* ∈ *SAs A*; *B* ≠ *C* ⟧ ⟹ *InitState B* ∉ *States C*
apply *auto*
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply *force*
done

lemma *OneHAInitState-SAStates*:
⟦ *S* ∈ *HAInitStates A*; *T* ∈ *HAInitStates A*;
S ∈ *States SA*; *T* ∈ *States SA*; *SA* ∈ *SAs A* ⟧ ⟹
S = *T*
apply (*unfold HAInitStates-def*)
apply *auto*
apply (*rename-tac AA AAA*)
apply (*case-tac AA = SA*)
apply *auto*
apply (*case-tac AAA = SA*)
apply *auto*
done

7.2.11 *Chi*

lemma *HARootStates-notmem-Chi* [*simp*]:
⟦ *S* ∈ *HAStates A*; *T* ∈ *States* (*HARoot A*) ⟧ ⟹ *T* ∉ *Chi A S*
apply (*unfold Chi-def restrict-def*, *auto*)
apply (*rename-tac SA*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=HARoot A* **in** *ballE*)
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
done

lemma *SAStates-notmem-Chi* [*simp*]:

$[\![S \in States\ SA;\ T \in States\ SA;$
$SA \in SAs\ A\ ]\!] \Longrightarrow T \notin Chi\ A\ S$
apply (*unfold Chi-def restrict-def*, *auto*)
apply (*rename-tac SAA*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *HAInitState-notmem-Chi* [*simp*]:
$S \in HAStates\ A \Longrightarrow HAInitState\ A \notin Chi\ A\ S$
by (*unfold Chi-def restrict-def*, *auto*)

lemma *Chi-HAStates* [*simp*]:
$T \in HAStates\ A \Longrightarrow (Chi\ A\ T) \subseteq HAStates\ A$
apply (*unfold Chi-def restrict-def*)
apply (*auto*)
apply (*cut-tac A=A* **and** *S=T* **in** *HAStates-CompFun-SAs*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *Chi-HAStates-Self* [*simp*]:
$s \in HAStates\ a \Longrightarrow s \notin (Chi\ a\ s)$
by (*unfold Chi-def restrict-def*, *auto*)

lemma *ChiRel-HAStates-Self* [*simp*]:
$(s,s) \notin (ChiRel\ a)$
by(*unfold ChiRel-def*, *auto*)

lemma *HAStates-Chi-NoCycles*:
$[\![s \in HAStates\ a;\ t \in HAStates\ a;\ s \in Chi\ a\ t\ ]\!] \Longrightarrow t \notin Chi\ a\ s$
apply (*unfold Chi-def restrict-def*)
apply *auto*
apply (*cut-tac HA=a* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x={s,t}* **in** *ballE*)
apply *auto*
done

lemma *HAStates-Chi-NoCycles-trans*:
$[\![s \in HAStates\ a;\ t \in HAStates\ a;\ u \in HAStates\ a;$
$t \in Chi\ a\ s;\ u \in Chi\ a\ t\ ]\!] \Longrightarrow s \notin Chi\ a\ u$
apply (*unfold Chi-def restrict-def*)
apply *auto*

apply (*cut-tac HA=a* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x={s,t,u}* **in** *ballE*)
prefer *2*
apply *simp*
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *Chi-range-disjoint*:
⟦ $S \neq T$; $T \in HAStates\ A$; $S \in HAStates\ A$; $U \in Chi\ A\ S$ ⟧ ⟹ $U \notin Chi\ A\ T$
apply (*frule CompFun-Int-disjoint*)
apply *auto*
apply (*unfold Chi-def restrict-def*)
apply *auto*
apply (*rename-tac SA SAA*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply *auto*
done

lemma *SAStates-Chi-trans* [*rule-format*]:
⟦ $U \in Chi\ A\ T$; $S \in Chi\ A\ U$; $T \in States\ SA$;
$SA \in SAs\ A$; $U \in HAStates\ A$ ⟧ ⟹ $S \notin States\ SA$
apply (*frule HAStates-SA-mem*)
apply *auto*
apply (*unfold Chi-def restrict-def*)
apply *auto*
apply (*rename-tac SAA SAAA*)
apply (*cut-tac HA=A* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x={U,T}* **in** *ballE*)
prefer *2*
apply (*simp only*: *HAStates-def*)
apply *auto*
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*rotate-tac −1*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*rotate-tac −1*)
apply (*erule-tac x=SAAA* **in** *ballE*)
apply *auto*
done

7.2.12 *ChiRel*

lemma *finite-ChiRel* [*simp*]:

finite (*ChiRel A*)
apply (*rule-tac B=HAStates A* × *HAStates A* **in** *finite-subset*)
apply *auto*
done

lemma *ChiRel-HAStates-subseteq* [*simp*]:
(*ChiRel A*) ⊆ (*HAStates A* × *HAStates A*)
apply (*unfold ChiRel-def Chi-def restrict-def*)
apply *auto*
done

lemma *ChiRel-CompFun*:
s ∈ *HAStates a* ⟹ *ChiRel a* `` {*s*} = (⋃ *x* ∈ *the* (*CompFun a s*). *States x*)
apply (*unfold ChiRel-def Chi-def restrict-def Image-def*)
apply *simp*
apply *auto*
apply (*frule HAStates-CompFun-SAs-mem*)
apply *fast*
apply (*unfold HAStates-def*)
apply *fast*
done

lemma *ChiRel-HARoot*:
⟦ (*x*,*y*) ∈ *ChiRel A* ⟧ ⟹ *y* ∉ *States* (*HARoot A*)
apply (*unfold ChiRel-def Chi-def*)
apply *auto*
apply (*unfold restrict-def*)
apply *auto*
apply (*rename-tac SA*)
apply (*frule HAStates-HARoot-CompFun*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply *auto*
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=HARoot A* **in** *ballE*)
apply *auto*
done

lemma *HAStates-CompFun-States-ChiRel*:
S ∈ *HAStates A* ⟹ ⋃ (*States* ` *the* (*CompFun A S*)) = *ChiRel A* `` {*S*}
apply (*unfold ChiRel-def Chi-def restrict-def*)
apply *auto*
apply (*drule HAStates-CompFun-SAs*)
apply (*subst HAStates-def*)
apply *fast*
done

lemma *HAInitState-notmem-Range-ChiRel* [*simp*]:
HAInitState A ∉ *Range* (*ChiRel A*)

by (*unfold ChiRel-def*, *auto*)

lemma *HAInitState-notmem-Range-ChiRel2* [*simp*]:
(*S*,*HAInitState A*) ∉ (*ChiRel A*)
by (*unfold ChiRel-def*, *auto*)

lemma *ChiRel-OneAncestor-notmem*:
⟦ *S* ≠ *T*; (*S*,*U*) ∈ *ChiRel A*⟧ ⟹ (*T*,*U*) ∉ *ChiRel A*
apply (*unfold ChiRel-def*)
apply *auto*
apply (*simp only*: *Chi-range-disjoint*)
done

lemma *ChiRel-OneAncestor*:
⟦ (*S1*,*U*) ∈ *ChiRel A*; (*S2*,*U*) ∈ *ChiRel A* ⟧ ⟹ *S1* = *S2*
apply (*rule notnotD*, *rule notI*)
apply (*simp add*: *ChiRel-OneAncestor-notmem*)
done

lemma *CompFun-ChiRel*:
⟦ *S1* ∈ *HAStates A*; *SA* ∈ *the* (*CompFun A S1*);
S2 ∈ *States SA* ⟧ ⟹ (*S1*,*S2*) ∈ *ChiRel A*
apply (*unfold ChiRel-def Chi-def restrict-def*)
apply *auto*
apply (*cut-tac A=A* **and** *S=S1* **in** *HAStates-CompFun-SAs*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *CompFun-ChiRel2*:
⟦ (*S*,*T*) ∈ *ChiRel A*; *T* ∈ *States SA*; *SA* ∈ *SAs A* ⟧ ⟹ *SA* ∈ *the* (*CompFun A S*)
apply (*unfold ChiRel-def Chi-def restrict-def*)
apply *auto*
apply (*rename-tac SAA*)
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*rotate-tac −1*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply *auto*
done

lemma *ChiRel-HAStates-NoCycles*:
(*s*,*t*) ∈ (*ChiRel a*) ⟹ (*t*,*s*) ∉ (*ChiRel a*)
apply (*unfold ChiRel-def*)
apply *auto*
apply (*frule HAStates-Chi-NoCycles*)
apply *auto*

done

lemma *HAStates-ChiRel-NoCycles-trans*:
⟦ (*s*,*t*) ∈ (*ChiRel a*); (*t*,*u*) ∈ (*ChiRel a*) ⟧ ⟹ (*u*,*s*) ∉ (*ChiRel a*)
apply (*unfold ChiRel-def*)
apply *auto*
apply (*frule HAStates-Chi-NoCycles-trans*)
apply *fast*
back
back
prefer *3*
apply *fast*
apply *auto*
done

lemma *SAStates-ChiRel*:
⟦ *S* ∈ *States SA*; *T* ∈ *States SA*;
SA ∈ *SAs A* ⟧ ⟹ (*S*,*T*) ∉ (*ChiRel A*)
by (*unfold ChiRel-def*, *auto*)

lemma *ChiRel-SA-OneAncestor*:
⟦ (*S*,*T*) ∈ *ChiRel A*; *T* ∈ *States SA*;
U ∈ *States SA*; *SA* ∈ *SAs A* ⟧ ⟹
(*S*,*U*) ∈ *ChiRel A*
apply (*frule CompFun-ChiRel2*)
apply *auto*
apply (*rule CompFun-ChiRel*)
apply *auto*
done

lemma *ChiRel-OneAncestor2*:
⟦ *S* ∈ *HAStates A*; *S* ∉ *States* (*HARoot A*) ⟧ ⟹
∃! *T*. (*T*,*S*) ∈ *ChiRel A*
apply (*unfold ChiRel-def*)
apply *auto*
prefer *2*
apply (*rename-tac T U*)
prefer *2*
apply (*unfold Chi-def restrict-def*)
apply *auto*
prefer *2*
apply (*rename-tac SA SAA*)
prefer *2*
apply (*cut-tac HA=A* **in** *OneAncestor-HA*)
apply (*unfold OneAncestor-def*)
apply (*fold HARoot-def*)
apply *auto*
apply (*simp cong*: *rev-conj-cong*)
apply (*unfold HAStates-def*)

apply *auto*
apply (*rename-tac SA*)
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
apply (*case-tac T = U*)
apply *auto*
apply (*frule CompFun-Int-disjoint*)
apply (*unfold HAStates-def*)
apply *auto*
apply (*case-tac SA=SAA*)
apply *auto*
apply (*cut-tac HA=A* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
apply (*cut-tac S=T* **and** *A=A* **in** *HAStates-CompFun-SAs*)
apply (*unfold HAStates-def*)
apply *fast*
apply *fast*
apply (*cut-tac S=U* **and** *A=A* **in** *HAStates-CompFun-SAs*)
apply (*unfold HAStates-def*)
apply *fast*
apply *fast*
done

lemma *HARootStates-notmem-Range-ChiRel* [*simp*]:
S ∈ *States* (*HARoot A*) ⟹ *S* ∉ *Range* (*ChiRel A*)
by (*unfold ChiRel-def*, *auto*)

lemma *ChiRel-int-disjoint*:
S ≠ *T* ⟹ (*ChiRel A* `` {*S*}) ∩ (*ChiRel A* `` {*T*}) = {}
apply (*unfold ChiRel-def*)
apply *auto*
apply (*simp only*: *Chi-range-disjoint*)
done

lemma *SAStates-ChiRel-trans* [*rule-format*]:
⟦ (*S*,*U*) ∈ (*ChiRel A*); (*U*,*T*) ∈ *ChiRel A*;
S ∈ *States SA*; *SA* ∈ *SAs A* ⟧ ⟹ *T* ∉ *States SA*
apply *auto*
apply (*unfold ChiRel-def*)
apply *auto*
apply (*frule SAStates-Chi-trans*)
back
apply *fast+*
done

lemma *HAInitStates-InitState-trancl*:

⟦ $S \in HAInitStates\ (HA\ ST)$; $A \in the\ (CompFun\ (HA\ ST)\ S)$ ⟧ ⟹
$(S, InitState\ A) \in (ChiRel\ (HA\ ST) \cap HAInitStates\ (HA\ ST) \times HAInitStates\ (HA\ ST))^+$
apply (*case-tac* $S \in HAStates\ (HA\ ST)$)
apply (*frule CompFun-ChiRel*)
apply *fast+*
apply (*rule InitState-States*)
apply *auto*
apply (*rule r-into-trancl'*)
apply *auto*
apply (*rule CompFun-HAInitStates-HAStates*)
apply *auto*
done

lemma *HAInitStates-InitState-trancl2*:
⟦ $S \in HAStates\ (HA\ ST)$; $A \in the\ (CompFun\ (HA\ ST)\ S)$;
$(x, S) \in (ChiRel\ (HA\ ST) \cap HAInitStates\ (HA\ ST) \times HAInitStates\ (HA\ ST))^+$
⟧
⟹ $(x, InitState\ A) \in (ChiRel\ (HA\ ST) \cap HAInitStates\ (HA\ ST) \times HAInitStates\ (HA\ ST))^+$
apply (*rule-tac* $a{=}x$ **and** $b{=}S$ **and** $r{=}ChiRel\ (HA\ ST) \cap HAInitStates\ (HA\ ST) \times HAInitStates\ (HA\ ST)$ **in** *converse-trancl-induct*)
apply *auto*
prefer *2*
apply (*rename-tac T U*)
prefer *2*
apply (*case-tac* $S \in HAStates\ (HA\ ST)$)
apply (*frule CompFun-ChiRel*)
apply *fast*
apply (*rule InitState-States*)
apply *simp*
apply (*rule trancl-trans* [*of* - *S*])
apply (*rule r-into-trancl'*)
apply *auto*
apply (*rule r-into-trancl'*)
apply *auto*
apply (*rule CompFun-HAInitStates-HAStates*)
prefer *2*
apply *fast*
apply (*cut-tac* $A{=}HA\ ST$ **in** *HAInitStates-HAStates*, *fast*)
apply (*rule-tac* $y = U$ **in** *trancl-trans*)
apply (*rule r-into-trancl'*)
apply *auto*
done

7.2.13 *ChiPlus*

lemma *ChiPlus-ChiRel* [*simp*]:
$(S,T) \in ChiRel\ A \Longrightarrow (S,T) \in ChiPlus\ A$

apply (*unfold ChiPlus-def*)
apply (*frule r-into-trancl*)
apply *auto*
done

lemma *ChiPlus-HAStates* [*simp*]:
(*ChiPlus A*) ⊆ (*HAStates A* × *HAStates A*)
apply (*unfold ChiPlus-def*)
apply (*rule trancl-subset-Sigma*)
apply *auto*
done

lemma *ChiPlus-subset-States*:
ChiPlus a `` {t} ⊆ ⋃(*States ` (SAs a)*)
apply (*cut-tac A=a* **in** *ChiPlus-HAStates*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *finite-ChiPlus* [*simp*]:
finite (*ChiPlus A*)
apply (*rule-tac B=HAStates A* × *HAStates A* **in** *finite-subset*)
apply *auto*
done

lemma *ChiPlus-OneAncestor*:
⟦ *S* ∈ *HAStates A*; *S* ∉ *States* (*HARoot A*) ⟧ ⟹
∃ *T*. (*T*,*S*) ∈ *ChiPlus A*
apply (*unfold ChiPlus-def*)
apply (*frule ChiRel-OneAncestor2*)
apply *auto*
done

lemma *ChiPlus-HAStates-Left*:
(*S*,*T*) ∈ *ChiPlus A* ⟹ *S* ∈ *HAStates A*
apply (*cut-tac A=A* **in** *ChiPlus-HAStates*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *ChiPlus-HAStates-Right*:
(*S*,*T*) ∈ *ChiPlus A* ⟹ *T* ∈ *HAStates A*
apply (*cut-tac A=A* **in** *ChiPlus-HAStates*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *ChiPlus-ChiRel-int* [*rule-format*]:
⟦ (*T*,*S*) ∈ (*ChiPlus A*)⟧ ⟹ (*ChiPlus A `` {T}*) ∩ (*ChiRel A `` {S}*) = (*ChiRel*

A `` $\{S\}$)
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply *auto*
done

lemma *ChiPlus-ChiPlus-int* [*rule-format*]:
⟦ (T,S) ∈ (*ChiPlus A*)⟧ ⟹ (*ChiPlus A* `` $\{T\}$) ∩ (*ChiPlus A* `` $\{S\}$) = (*ChiPlus A* `` $\{S\}$)
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply *auto*
done

lemma *ChiPlus-ChiRel-NoCycle-1* [*rule-format*]:
⟦ (T,S) ∈ *ChiPlus A*⟧ ⟹
(*insert S* (*insert T* ({U. (T,U) ∈ *ChiPlus A* ∧ (U,S) ∈ *ChiPlus A*}))) ∩ (*ChiRel A* `` $\{T\}$) ≠ {}
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply (*unfold Image-def Int-def*)
apply *auto*
done

lemma *ChiPlus-ChiRel-NoCycle-2* [*rule-format*]:
⟦ (T,S) ∈ *ChiPlus A*⟧ ⟹ (S,T) ∈ (*ChiRel A*) ⟶
(*insert S* (*insert T* ({U. (T,U) ∈ *ChiPlus A* ∧ (U,S) ∈ *ChiPlus A*}))) ∩ (*ChiRel A* `` $\{S\}$) ≠ {}
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply (*unfold Image-def Int-def*)
apply *auto*
done

lemma *ChiPlus-ChiRel-NoCycle-3* [*rule-format*]:
⟦ (T,S) ∈ *ChiPlus A*⟧ ⟹ (S,T) ∈ (*ChiRel A*) ⟶ (T,U) ∈ *ChiPlus A* ⟶ (U, S) ∈ *ChiPlus A* ⟶
(*insert S* (*insert T* ({U. (T,U) ∈ *ChiPlus A* ∧ (U,S) ∈ *ChiPlus A*}))) ∩ (*ChiRel A* `` $\{U\}$) ≠ {}
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply (*unfold Image-def Int-def*, *simp*)
apply (*rename-tac V*)
prefer *2*
apply (*rename-tac V W*)
prefer *2*
apply (*simp*, *safe*)
apply (*simp only*: *ChiRel-HAStates-NoCycles*)
apply *simp*

apply (*case-tac* $(U,W) \in (ChiRel\ A)$, *fast*, *rotate-tac 5*, *frule tranclD3*, *fast*, *blast intro*: *trancl-into-trancl*)+
done

lemma *ChiPlus-ChiRel-NoCycle-4* [*rule-format*]:
$[\![(T,S) \in ChiPlus\ A]\!] \Longrightarrow (S,T) \in (ChiRel\ A) \longrightarrow ((ChiPlus\ A\ ``\{T\}) \cap (ChiRel\ A\ ``\{S\})) \neq \{\}$
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=T **and** *b*=S **and** *r*=$(ChiRel\ A)$ **in** *trancl-induct*)
apply (*unfold Image-def Int-def*)
apply *auto*
apply (*simp only*: *ChiRel-HAStates-NoCycles*)
apply (*rule-tac x*=T **in** *exI*)
apply *simp*
apply (*rule-tac x*=T **in** *exI*)
apply *simp*
done

lemma *ChiRel-ChiPlus-NoCycles*:
$(S,T) \in (ChiRel\ A) \Longrightarrow (T,S) \notin (ChiPlus\ A)$
apply (*cut-tac HA*=A **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x*=*insert S* (*insert T* ($\{U.\ (T,U) \in ChiPlus\ A \wedge (U,S) \in ChiPlus\ A\}$)) **in** *ballE*)
prefer *2*
apply (*simp add*: *ChiPlus-subset-States*)
apply (*cut-tac A*=A **in** *ChiPlus-HAStates*)
apply (*unfold HAStates-def*)
apply *auto*
apply (*frule ChiPlus-ChiRel-NoCycle-2*)
apply *fast*
apply (*simp add*:*ChiRel-CompFun*)
apply (*frule ChiPlus-ChiRel-NoCycle-1*)
apply (*simp add*:*ChiRel-CompFun*)
apply (*frule ChiPlus-ChiRel-NoCycle-3*)
apply *fast*
apply *fast*
back
apply *fast*
apply (*rename-tac V*)
apply (*case-tac* $V \in HAStates\ A$)
apply (*simp add*: *ChiRel-CompFun*)
apply (*simp only*: *ChiPlus-HAStates-Right*)
apply *fast*
done

lemma *ChiPlus-ChiPlus-NoCycles*:
$(S,T) \in (ChiPlus\ A) \Longrightarrow (T,S) \notin (ChiPlus\ A)$
apply (*unfold ChiPlus-def*)

apply (*rule-tac a*=S **and** *b*=T **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply *fast*
apply (*frule ChiRel-ChiPlus-NoCycles*)
apply (*auto intro*: *trancl-into-trancl2 simp add*:*ChiPlus-def*)
done

lemma *ChiPlus-NoCycles* [*rule-format*]:
$(S,T) \in (ChiPlus\ A) \Longrightarrow S \neq T$
apply (*frule ChiPlus-ChiPlus-NoCycles*)
apply *auto*
done

lemma *ChiPlus-NoCycles-2* [*simp*]:
$(S,S) \notin (ChiPlus\ A)$
apply (*rule notI*)
apply (*frule ChiPlus-NoCycles*)
apply *fast*
done

lemma *ChiPlus-ChiPlus-NoCycles-2*:
$[\![(S,U) \in ChiPlus\ A;\ (U,T) \in ChiPlus\ A\ ]\!] \Longrightarrow (T,S) \notin ChiPlus\ A$
apply (*rule ChiPlus-ChiPlus-NoCycles*)
apply (*auto intro*: *trancl-trans simp add*: *ChiPlus-def*)
done

lemma *ChiRel-ChiPlus-trans*:
$[\![(U,S) \in ChiPlus\ A;\ (S,T) \in ChiRel\ A]\!] \Longrightarrow (U,T) \in ChiPlus\ A$
apply (*unfold ChiPlus-def*)
apply *auto*
done

lemma *ChiRel-ChiPlus-trans2*:
$[\![(U,S) \in ChiRel\ A;\ (S,T) \in ChiPlus\ A\ ]\!] \Longrightarrow (U,T) \in ChiPlus\ A$
apply (*unfold ChiPlus-def*)
apply *auto*
done

lemma *ChiPlus-ChiRel-Ex* [*rule-format*]:
$[\![(S,T) \in ChiPlus\ A\ ]\!] \Longrightarrow (S,T) \notin ChiRel\ A \longrightarrow$
$(\exists\ U.\ (S,U) \in ChiPlus\ A \wedge (U,T) \in ChiRel\ A)$
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=S **and** *b*=T **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply *auto*
apply (*rename-tac U*)
apply (*rule-tac x*=U **in** *exI*)
apply *auto*
done

lemma *ChiPlus-ChiRel-Ex2* [*rule-format*]:

⟦ (*S*,*T*) ∈ *ChiPlus A* ⟧ ⟹ (*S*,*T*) ∉ *ChiRel A* ⟶
(∃ *U*. (*S*,*U*) ∈ *ChiRel A* ∧ (*U*,*T*) ∈ *ChiPlus A*)
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*T* **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply *auto*
done

lemma *HARootStates-Range-ChiPlus* [*simp*]:
⟦ *S* ∈ *States* (*HARoot A*) ⟧ ⟹ *S* ∉ *Range* (*ChiPlus A*)
by (*unfold ChiPlus-def*, *auto*)

lemma *HARootStates-Range-ChiPlus2* [*simp*]:
⟦ *S* ∈ *States* (*HARoot A*) ⟧ ⟹ (*x*,*S*) ∉ (*ChiPlus A*)
by (*frule HARootStates-Range-ChiPlus*, *unfold Domain-converse* [*symmetric*], *fast*)

lemma *SAStates-ChiPlus-ChiRel-NoCycle-1* [*rule-format*]:
⟦ (*S*,*U*) ∈ *ChiPlus A*; *SA* ∈ *SAs A* ⟧ ⟹ (*U*,*T*) ∈ (*ChiRel A*) ⟶ *S* ∈ *States SA*
⟶ *T* ∈ *States SA* ⟶
(*insert S* (*insert U* ({*V*. (*S*,*V*) ∈ *ChiPlus A* ∧ (*V*,*U*) ∈ *ChiPlus A*}))) ∩ (*ChiRel*
A `` {*U*}) ≠ {}
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply (*simp*, *safe*)
apply (*simp only*: *SAStates-ChiRel-trans*)
apply (*simp add*:*ChiRel-CompFun*)
apply *safe*
apply (*erule-tac x*=*SA* **in** *ballE*)
apply (*simp add*: *CompFun-ChiRel2*)+
apply (*simp add*:*Int-def*, *fast*)
apply *auto*
apply (*fold ChiPlus-def*)
apply (*rename-tac W*)
apply (*frule-tac U*=*U* **and** *T*=*U* **and** *S*=*W* **in** *ChiRel-ChiPlus-trans2*)
apply *auto*
done

lemma *SAStates-ChiPlus-ChiRel-NoCycle-2* [*rule-format*]:
⟦ (*S*,*U*) ∈ *ChiPlus A* ⟧ ⟹ (*U*,*T*) ∈ (*ChiRel A*) ⟶
(*insert S* (*insert U* ({*V*. (*S*,*V*) ∈ *ChiPlus A* ∧ (*V*,*U*) ∈ *ChiPlus A*}))) ∩
(*ChiRel A* `` {*S*}) ≠ {}
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply (*unfold Image-def Int-def*)
apply *auto*
done

lemma *SAStates-ChiPlus-ChiRel-NoCycle-3* [*rule-format*]:

$[\![(S,U) \in \ ChiPlus\ A\ ]\!] \Longrightarrow\ (U,T) \in (ChiRel\ A) \longrightarrow (S,s) \in ChiPlus\ A \longrightarrow (s,U) \in ChiPlus\ A \longrightarrow$
$(insert\ S\ (insert\ U\ (\{V.\ (S,V) \in ChiPlus\ A \wedge (V,U) \in ChiPlus\ A\}))) \cap (ChiRel\ A\ ``\ \{s\}) \neq \{\}$
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply *fast*
apply (*rename-tac W*)
prefer *2*
apply (*rename-tac W X*)
prefer *2*
apply (*unfold Image-def Int-def*)
apply (*simp*, *safe*)
apply (*fold ChiPlus-def*)
apply (*case-tac* $(s,W) \in ChiRel\ A$)
apply *fast*
apply (*frule-tac S*=*s* **and** *T*=*W* **in** *ChiPlus-ChiRel-Ex2*)
apply *simp*
apply *safe*
apply (*rename-tac X*)
apply (*rule-tac x*=*X* **in** *exI*)
apply (*fast intro*: *ChiRel-ChiPlus-trans*)
apply *simp*
apply (*case-tac* $(s,X) \in ChiRel\ A$)
apply *force*
apply (*frule-tac S*=*s* **and** *T*=*X* **in** *ChiPlus-ChiRel-Ex2*)
apply *simp*
apply *safe*
apply (*rename-tac Y*)
apply (*erule-tac x*=*Y* **in** *allE*)
apply *simp*
apply (*fast intro*: *ChiRel-ChiPlus-trans*)
apply *simp*
apply (*case-tac* $(s,X) \in ChiRel\ A$)
apply *force*
apply (*frule-tac S*=*s* **and** *T*=*X* **in** *ChiPlus-ChiRel-Ex2*)
apply *simp*
apply *safe*
apply (*rename-tac Y*)
apply (*erule-tac x*=*Y* **in** *allE*)
apply *simp*
apply (*fast intro*: *ChiRel-ChiPlus-trans*)
apply *fastforce*
apply *simp*
apply (*erule-tac x*=*W* **in** *allE*)
apply *simp*
apply *simp*
apply (*rename-tac Y*)
apply (*erule-tac x*=*Y* **in** *allE*)

apply *simp*
apply (*fast intro*: *ChiRel-ChiPlus-trans*)
done

lemma *SAStates-ChiPlus-ChiRel-trans* [*rule-format*]:
⟦ $(S,U) \in (ChiPlus\ A)$; $(U,T) \in (ChiRel\ A)$; $S \in States\ SA$; $SA \in SAs\ A$ ⟧ ⟹ $T \notin States\ SA$
apply (*cut-tac HA=A* **in** *NoCycles-HA*)
apply (*unfold NoCycles-def*)
apply (*erule-tac x=insert S* (*insert U* ({ V. $(S,V) \in ChiPlus\ A \wedge (V,U) \in ChiPlus\ A$})) **in** *ballE*)
prefer *2*
apply (*simp add*: *ChiPlus-subset-States*)
apply (*cut-tac A=A* **in** *ChiPlus-HAStates*)
apply (*unfold HAStates-def*)
apply *auto*[*1*]
apply *safe*
apply *fast*
apply (*frule SAStates-ChiPlus-ChiRel-NoCycle-2*)
apply *fast*
apply (*frule HAStates-SA-mem*)
apply *fast*
apply (*simp only*:*ChiRel-CompFun*)
apply (*frule SAStates-ChiPlus-ChiRel-NoCycle-1*)
apply *auto*[*3*]
apply *fast*
apply (*simp add*:*ChiRel-CompFun*)
apply (*frule SAStates-ChiPlus-ChiRel-NoCycle-3*)
apply *fast*
apply *fast*
back
apply *fast*
apply (*simp only*:*ChiPlus-HAStates-Left ChiRel-CompFun*)
done

lemma *SAStates-ChiPlus2* [*rule-format*]:
⟦ $(S,T) \in ChiPlus\ A$; $SA \in SAs\ A$ ⟧ ⟹ $S \in States\ SA \longrightarrow T \notin States\ SA$
apply (*unfold ChiPlus-def*)
apply (*rule-tac a=S* **and** *b=T* **and** *r=*(*ChiRel A*) **in** *trancl-induct*)
apply *auto*
apply (*rename-tac U*)
apply (*frule-tac S=S* **and** *T=U* **in** *SAStates-ChiRel*)
apply *auto*
apply (*fold ChiPlus-def*)
apply (*simp only*: *SAStates-ChiPlus-ChiRel-trans*)
done

lemma *SAStates-ChiPlus* [*rule-format*]:
⟦ $S \in States\ SA$; $T \in States\ SA$; $SA \in SAs\ A$ ⟧ ⟹ $(S,T) \notin ChiPlus\ A$

apply *auto*
apply (*simp only*: *SAStates-ChiPlus2*)
done

lemma *SAStates-ChiPlus-ChiRel-OneAncestor* [*rule-format*]:
⟦ *T* ∈ *States SA*; *SA* ∈ *SAs A*; (*S*,*U*) ∈ *ChiPlus A*⟧ ⟹ *S* ≠ *T* ⟶ *S* ∈ *States SA* ⟶ (*T*,*U*) ∉ *ChiRel A*
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply *auto*
apply (*simp add*: *ChiRel-OneAncestor-notmem*)
apply (*rename-tac V W*)
apply (*fold ChiPlus-def*)
apply (*case-tac V*=*T*)
apply (*simp add*: *ChiRel-OneAncestor-notmem SAStates-ChiPlus*)+
done

lemma *SAStates-ChiPlus-OneAncestor* [*rule-format*]:
⟦ *T* ∈ *States SA*; *SA* ∈ *SAs A*; (*S*,*U*) ∈ *ChiPlus A* ⟧ ⟹ *S* ≠ *T* ⟶ *S* ∈ *States SA* ⟶ (*T*,*U*) ∉ *ChiPlus A*
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply *auto*
apply (*fold ChiPlus-def*)
apply (*rename-tac V*)
apply (*frule-tac T*=*S* **and** *S*=*T* **and** *U*=*V* **in** *SAStates-ChiPlus-ChiRel-OneAncestor*)
apply *auto*
apply (*rename-tac V W*)
apply (*frule-tac S*=*T* **and** *T*=*W* **in** *ChiPlus-ChiRel-Ex*)
apply *auto*
apply (*frule-tac T*=*T* **and** *S*=*S* **and** *U*=*W* **in** *SAStates-ChiPlus-ChiRel-OneAncestor*)
apply *auto*
apply (*rule ChiRel-ChiPlus-trans*)
apply *auto*
apply (*rename-tac X*)
apply (*case-tac V*=*X*)
apply *simp*
apply (*simp add*: *ChiRel-OneAncestor-notmem*)
done

lemma *ChiRel-ChiPlus-OneAncestor* [*rule-format*]:
⟦ (*T*,*U*) ∈ *ChiPlus A* ⟧ ⟹ *T* ≠ *S* ⟶ (*S*,*U*) ∈ *ChiRel A* ⟶ (*T*,*S*) ∈ *ChiPlus A*
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*T* **and** *b*=*U* **and** *r*=(*ChiRel A*) **in** *trancl-induct*)
apply *auto*
apply (*fast intro*:*ChiRel-OneAncestor*)
apply (*rename-tac V W*)
apply (*case-tac S*=*V*)

apply *auto*
apply (*fast intro*:*ChiRel-OneAncestor*)
done

lemma *ChiPlus-SA-OneAncestor* [*rule-format*]:
⟦ $(S,T) \in$ *ChiPlus A*;
$U \in$ *States SA*; *SA* $\in$ *SAs A* ⟧ $\Longrightarrow$ $T \in$ *States SA* $\longrightarrow$
$(S,U) \in$ *ChiPlus A*
apply (*unfold ChiPlus-def*)
apply (*rule-tac a*=*S* **and** *b*=*T* **and** *r*=(*ChiRel A*) **in** *converse-trancl-induct*)
apply *auto*
apply (*frule ChiRel-SA-OneAncestor*)
apply *fast*+
done

7.2.14 *ChiStar*

lemma *ChiPlus-ChiStar* [*simp*]:
⟦ $(S,T) \in$ *ChiPlus A* ⟧ $\Longrightarrow$ $(S,T) \in$ *ChiStar A*
by (*unfold ChiPlus-def ChiStar-def*, *auto*)

lemma *HARootState-Range-ChiStar* [*simp*]:
⟦ $x \neq S$; $S \in$ *States* (*HARoot A*) ⟧ $\Longrightarrow$ $(x,S) \notin$ (*ChiStar A*)
apply (*unfold ChiStar-def*)
apply (*subst rtrancl-eq-or-trancl*)
apply (*fold ChiPlus-def*)
apply *auto*
done

lemma *ChiStar-Self* [*simp*]:
$(S,S) \in$ *ChiStar A*
apply (*unfold ChiStar-def*)
apply *simp*
done

lemma *ChiStar-Image* [*simp*]:
$S \in M \Longrightarrow S \in$ (*ChiStar A* `` *M*)
apply (*unfold Image-def*)
apply (*auto intro*: *ChiStar-Self*)
done

lemma *ChiStar-ChiPlus-noteq*:
⟦ $S \neq T$; $(S,T) \in$ *ChiStar A* ⟧ $\Longrightarrow$ $(S,T) \in$ *ChiPlus A*
apply (*unfold ChiPlus-def ChiStar-def*)
apply (*simp add*: *rtrancl-eq-or-trancl*)
done

lemma *ChiRel-ChiStar-trans*:
⟦ $(S,U) \in$ *ChiStar A*; $(U,T) \in$ *ChiRel A* ⟧ $\Longrightarrow$ $(S,T) \in$ *ChiStar A*

apply (*unfold ChiStar-def*)
apply *auto*
done

7.2.15 *InitConf*

lemma *InitConf-HAStates* [*simp*]:
InitConf A ⊆ *HAStates A*
apply (*unfold InitConf-def HAStates-def*)
apply *auto*
apply (*rule rtrancl-induct*)
back
apply *auto*
apply (*rule-tac x=HARoot A* **in** *bexI*)
apply *auto*
apply (*unfold HAStates-def ChiRel-def*)
apply *auto*
done

lemma *InitConf-HAStates2* [*simp*]:
S ∈ *InitConf A* ⟹ *S* ∈ *HAStates A*
apply (*cut-tac A=A* **in** *InitConf-HAStates*)
apply *fast*
done

lemma *HAInitState-InitConf* [*simp*]:
HAInitState A ∈ *InitConf A*
by (*unfold HAInitState-def InitConf-def*, *auto*)

lemma *InitConf-HAInitState-HARoot*:
[| *S* ∈ *InitConf A*; *S* ≠ *HAInitState A* |] ==> *S* ∉ *States* (*HARoot A*)
apply (*unfold InitConf-def*)
apply *auto*
apply (*rule mp*)
prefer *2*
apply *fast*
back
apply (*rule mp*)
prefer *2*
apply *fast*
back
back
apply (*rule-tac b=S* **in** *rtrancl-induct*)
apply *auto*
apply (*simp add*: *ChiRel-HARoot*)+
done

lemma *InitConf-HARoot-HAInitState* [*simp*]:
⟦ *S* ∈ *InitConf A*; *S* ∈ *States* (*HARoot A*) ⟧ ⟹ *S* = *HAInitState A*

apply (*subst not-not* [*THEN sym*])
apply (*rule notI*)
apply (*simp add*:*InitConf-HAInitState-HARoot*)
done

lemma *HAInitState-CompFun-InitConf* [*simp*]:
[|*SA* ∈ *the* (*CompFun A* (*HAInitState A*)) |] ==> (*InitState SA*) ∈ *InitConf A*
apply (*unfold InitConf-def HAStates-def*)
apply *auto*
apply (*rule rtrancl-Int*)
apply *auto*
apply (*cut-tac A*=*A* **and** *S*=*HAInitState A* **in** *HAStates-CompFun-States-ChiRel*)
apply *auto*
apply (*rule Image-singleton-iff* [*THEN subst*])
apply (*rotate-tac* −*1*)
apply (*drule sym*)
apply *simp*
apply (*rule-tac x*=*SA* **in** *bexI*)
apply *auto*
done

lemma *InitState-CompFun-InitConf*:
[| *S* ∈ *HAStates A*; *SA* ∈ *the* (*CompFun A S*); *S* ∈ *InitConf A* |] ==> (*InitState SA*) ∈ *InitConf A*
apply (*unfold InitConf-def*)
apply *auto*
apply (*rule-tac b*=*S* **in** *rtrancl-into-rtrancl*)
apply *fast*
apply (*frule rtrancl-Int1*)
apply *auto*
apply (*case-tac S* = *HAInitState A*)
apply *simp*
apply (*rule rtrancl-mem-Sigma*)
apply *auto*
apply (*cut-tac A*=*A* **and** *S*=*S* **in** *HAStates-CompFun-States-ChiRel*)
apply *auto*
apply (*rule Image-singleton-iff* [*THEN subst*])
apply (*rotate-tac* −*1*)
apply (*drule sym*)
apply *simp*
apply (*rule-tac x*=*SA* **in** *bexI*)
apply *auto*
done

lemma *InitConf-HAInitStates*:
InitConf A ⊆ *HAInitStates A*
apply (*unfold InitConf-def*)
apply (*rule subsetI*)
apply *auto*

apply (*frule rtrancl-Int1*)
apply (*case-tac* $x = HAInitState\ A$)
apply *simp*
apply (*rule rtrancl-mem-Sigma*)
apply *auto*
done

lemma *InitState-notmem-InitConf*:
[| $SA \in the\ (CompFun\ A\ S)$; $S \in InitConf\ A$; $T \in States\ SA$;
$T \neq InitState\ SA$ |] ==> $T \notin InitConf\ A$
apply (*frule InitConf-HAStates2*)
apply (*unfold InitConf-def*)
apply *auto*
apply (*rule mp*)
prefer *2*
apply *fast*
apply (*rule mp*)
prefer *2*
apply *fast*
back
apply (*rule mp*)
prefer *2*
apply *fast*
back
back
apply (*rule mp*)
prefer *2*
apply *fast*
back
back
back
apply (*rule mp*)
prefer *2*
apply *fast*
back
back
back
back
apply (*rule mp*)
prefer *2*
apply *fast*
back
back
back
back
back
apply (*rule-tac* $b{=}T$ **in** *rtrancl-induct*)
apply *auto*
done

lemma *InitConf-CompFun-InitState* [*simp*]:
⟦ *SA* ∈ *the* (*CompFun A S*); *S* ∈ *InitConf A*; *T* ∈ *States SA*;
T ∈ *InitConf A* ⟧ ⟹ *T* = *InitState SA*
apply (*subst not-not* [*THEN sym*])
apply (*rule notI*)
apply (*frule InitState-notmem-InitConf*)
apply *auto*
done

lemma *InitConf-ChiRel-Ancestor*:
⟦ *T* ∈ *InitConf A*; (*S*, *T*) ∈ *ChiRel A* ⟧ ⟹ *S* ∈ *InitConf A*
apply (*unfold InitConf-def*)
apply *auto*
apply (*erule rtranclE*)
apply *auto*
apply (*rename-tac U*)
apply (*cut-tac A*=*A* **in** *HAInitState-notmem-Range-ChiRel*)
apply *auto*
apply (*case-tac U* = *S*)
apply (*auto simp add*: *ChiRel-OneAncestor*)
done

lemma *InitConf-CompFun-Ancestor*:
⟦ *S* ∈ *HAStates A*; *SA* ∈ *the* (*CompFun A S*); *T* ∈ *InitConf A*; *T* ∈ *States SA* ⟧
⟹ *S* ∈ *InitConf A*
apply (*rule InitConf-ChiRel-Ancestor*)
apply *auto*
apply (*rule CompFun-ChiRel*)
apply *auto*
done

7.2.16 *StepConf*

lemma *StepConf-EmptySet* [*simp*]:
StepConf A C {} = *C*
by (*unfold StepConf-def*, *auto*)

end

8 Semantics of Hierarchical Automata

theory *HASem*
imports *HA*
begin

8.1 Definitions

definition

RootExSem :: [(('s,'e,'d)*seqauto*) *set*, 's ⇀ ('s,'e,'d)*seqauto set*,
's *set*] => *bool* **where**
RootExSem F G C == (∃! *S*. *S* ∈ *States* (*Root F G*) ∧ *S* ∈ *C*)

definition
UniqueSucStates :: [(('s,'e,'d)*seqauto*) *set*, 's ⇀ ('s,'e,'d)*seqauto set*,
's *set*] ⇒ *bool* **where**
UniqueSucStates F G C == ∀ *S* ∈ (⋃(*States* ' *F*)).
∀ *A* ∈ *the* (*G S*).
if (*S* ∈ *C*) *then*
∃! *S'* . *S'* ∈ *States A* ∧ *S'* ∈ *C*
else
∀ *S* ∈ *States A*. *S* ∉ *C*

definition
IsConfSet :: [(('s,'e,'d)*seqauto*) *set*, 's ⇀ ('s,'e,'d)*seqauto set*,
's *set*] => *bool* **where**
IsConfSet F G C ==
C ⊆ (⋃(*States* ' *F*)) &
RootExSem F G C &
UniqueSucStates F G C

definition
Status :: [('s,'e,'d)*hierauto*,
's *set*,
'e *set*,
'd *data*] => *bool* **where**
Status HA C E D == *E* ⊆ *HAEvents HA* ∧
IsConfSet (*SAs HA*) (*CompFun HA*) *C* ∧
Data.DataSpace (*HAInitValue HA*) = *Data.DataSpace D*

8.1.1 *Status*

lemma *Status-EmptySet*:
(*Abs-hierauto* ((@ *x* . *True*),
{*Abs-seqauto* ({ @ *x* . *True*}, (@ *x* . *True*), {}, {})}, {}, *Map.empty*(@ *x* . *True*
↦ {})),
{@*x*. *True*},{}, @*x*. *True*) ∈
{(*HA*,*C*,*E*,*D*) | *HA C E D*. *Status HA C E D*}
apply (*unfold Status-def CompFun-def SAs-def*)
apply *auto*
apply (*subst Abs-hierauto-inverse*)
apply (*subst hierauto-def*)
apply (*rule HierAuto-EmptySet*)
apply (*subst Abs-hierauto-inverse*)
apply (*subst hierauto-def*)
apply (*rule HierAuto-EmptySet*)
apply *auto*
apply (*unfold IsConfSet-def UniqueSucStates-def RootExSem-def*)

apply *auto*
apply (*unfold States-def*)
apply *auto*
apply (*unfold Root-def*)
apply (*rule someI2*)
apply (*rule conjI*)
apply *fast*
apply (*simp add*: *ran-def*)
apply *simp*
apply (*subst Abs-seqauto-inverse*)
apply (*subst seqauto-def*)
apply (*rule SeqAuto-EmptySet*)
apply *simp*
apply (*unfold HAInitValue-def*)
apply *auto*
apply (*subst Abs-hierauto-inverse*)
apply (*subst hierauto-def*)
apply (*rule HierAuto-EmptySet*)
apply *simp*
done

definition
status =
{(*HA*,*C*,*E*,*D*) |
(*HA*::('*s*,'*e*,'*d*)*hierauto*)
(*C*::('*s set*))
(*E*::('*e set*))
(*D*::'*d data*). *Status HA C E D*}

typedef ('*s*,'*e*,'*d*) *status* =
status :: (('*s*,'*e*,'*d*)*hierauto* * '*s set* * '*e set* * '*d data*) *set*
unfolding *status-def*
apply (*rule exI*)
apply (*rule Status-EmptySet*)
done

definition
HA :: ('*s*,'*e*,'*d*) *status* => ('*s*,'*e*,'*d*) *hierauto* **where**
HA == *fst o Rep-status*

definition
Conf :: ('*s*,'*e*,'*d*) *status* => '*s set* **where**
Conf == *fst o snd o Rep-status*

definition
Events :: ('*s*,'*e*,'*d*) *status* => '*e set* **where**
Events == *fst o snd o snd o Rep-status*

definition
Value :: $('s,'e,'d)$ *status* => $'d$ *data* **where**
Value == *snd o snd o snd o Rep-status*

definition
RootState :: $('s,'e,'d)$ *status* => $'s$ **where**
RootState ST == @ *S*. *S* ∈ *Conf ST* ∧ *S* ∈ *States* (*HARoot* (*HA ST*))

definition
EnabledTrans :: $(('s,'e,'d)status * ('s,'e,'d)seqauto *$
$('s,'e,'d)trans)$ *set* **where**
EnabledTrans == {(*ST*,*SA*,*T*) .
SA ∈ *SAs* (*HA ST*) ∧
T ∈ *Delta SA* ∧
source T ∈ *Conf ST* ∧
(*Conf ST*, *Events ST*, *Value ST*) |= (*label T*) }

definition
ET :: $('s,'e,'d)$ *status* => $(('s,'e,'d)$ *trans*$)$ *set* **where**
ET ST == ⋃ *SA* ∈ *SAs* (*HA ST*). (*EnabledTrans* `` {*ST*}) `` {*SA*}

definition
MaxNonConflict :: $[('s,'e,'d)status,$
$('s,'e,'d)trans\ set] \Rightarrow bool$ **where**
MaxNonConflict ST T ==
(*T* ⊆ *ET ST*) ∧
(∀ *A* ∈ *SAs* (*HA ST*). *card* (*T Int Delta A*) ≤ *1*) ∧
(∀ *t* ∈ (*ET ST*). (*t* ∈ *T*) = (¬ (∃ t' ∈ *ET ST*. *HigherPriority* (*HA ST*)
(t',*t*))))

definition
ResolveRacing :: $('s,'e,'d)trans\ set$
=> $('d\ update\ set)$ **where**
ResolveRacing TS ==
let
U = *PUpdate* (*Label TS*)

```
in
    SequentialRacing U
```

definition

```
HPT :: ('s,'e,'d)status => (('s,'e,'d)trans set) set where
HPT ST == { T. MaxNonConflict ST T}
```

definition

```
InitStatus :: ('s,'e,'d)hierauto => ('s,'e,'d)status where
InitStatus A ==
   Abs-status (A,InitConf A,{}, HAInitValue A)
```

definition

```
StepActEvent :: ('s,'e,'d)trans set => 'e set where
StepActEvent TS == Union (Actevent (Label TS))
```

definition

```
StepStatus :: [('s,'e,'d)status, ('s,'e,'d)trans set, 'd update]
              => ('s,'e,'d)status where
StepStatus ST TS U =
            (let
              (A,C,E,D) = Rep-status ST;
              C'        = StepConf A C TS;
              E'        = StepActEvent TS;
              D'        = U !!! D
             in
              Abs-status (A,C',E',D'))
```

definition
StepRelSem :: $('s,'e,'d)hierauto$
$\Rightarrow$ $(('s,'e,'d)status * ('s,'e,'d)status)$ *set* **where**
StepRelSem A == $\{(ST,ST').$ $(HA\ ST) = A \wedge$
$((HPT\ ST \neq \{\}) \longrightarrow$
$(\exists\, TS \in HPT\ ST.$
$\exists\, U \in ResolveRacing\ TS.$
$ST' = StepStatus\ ST\ TS\ U))$ &
$((HPT\ ST = \{\}) \longrightarrow$
$(ST' = StepStatus\ ST\ \{\}\ DefaultUpdate))\}$

inductive-set
ReachStati :: $('s,'e,'d)hierauto$ => $('s,'e,'d)$ *status set*
for *A* :: $('s,'e,'d)hierauto$
where
Status0 : $InitStatus\ A \in ReachStati\ A$
| *StatusStep* :
$[\![\ ST \in ReachStati\ A;\ \ TS \in HPT\ ST;\ U \in ResolveRacing\ TS\ ]\!]$
$\Longrightarrow StepStatus\ ST\ TS\ U \in ReachStati\ A$

8.2 Lemmas

lemma *Rep-status-tuple*:
$Rep\text{-}status\ ST = (HA\ ST,\ Conf\ ST,\ Events\ ST,\ Value\ ST)$
by (*unfold HA-def Conf-def Events-def Value-def*, *simp*)

lemma *Rep-status-select*:
$(HA\ ST,\ Conf\ ST,\ Events\ ST,\ Value\ ST) \in status$
by (*rule Rep-status-tuple* [*THEN subst*], *rule Rep-status*)

lemma *Status-select* [*simp*]:
$Status\ (HA\ ST)\ (Conf\ ST)\ (Events\ ST)\ (Value\ ST)$
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def*)
apply *simp*
done

8.2.1 *IsConfSet*

lemma *IsConfSet-Status* [*simp*]:
$IsConfSet\ (SAs\ (HA\ ST))\ (CompFun\ (HA\ ST))\ (Conf\ ST)$
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def*)

apply *auto*
done

8.2.2 *InitStatus*

lemma *IsConfSet-InitConf* [*simp*]:
IsConfSet (*SAs A*) (*CompFun A*) (*InitConf A*)
apply (*unfold IsConfSet-def RootExSem-def UniqueSucStates-def*, *fold HARoot-def*)
apply (*rule conjI*)
apply (*fold HAStates-def*, *simp*)
apply (*rule conjI*)
apply (*rule-tac a=HAInitState A* **in** *ex1I*)
apply *auto*
apply (*rename-tac S SA*)
apply (*case-tac S* ∈ *InitConf A*)
apply *auto*
apply (*rule-tac x=InitState SA* **in** *exI*)
apply *auto*
apply (*rule InitState-CompFun-InitConf*)
apply *auto*
apply (*rename-tac S SA T U*)
apply (*case-tac U = InitState SA*)
apply *auto*
apply (*simp only*:*InitConf-CompFun-Ancestor HAStates-SA-mem*, *simp*)+
done

lemma *InitConf-status* [*simp*]:
(*A*, *InitConf A*, {}, *HAInitValue A*) ∈ *status*
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def*)
apply *auto*
done

lemma *Conf-InitStatus-InitConf* [*simp*]:
Conf (*InitStatus A*) = *InitConf A*
apply (*unfold Conf-def InitStatus-def*)
apply *simp*
apply (*subst Abs-status-inverse*)
apply *auto*
done

lemma *HAInitValue-Value-DataSpace-Status* [*simp*]:
Data.DataSpace (*HAInitValue* (*HA ST*)) = *Data.DataSpace* (*Value ST*)
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def*)
apply *fast*
done

lemma *Value-InitStatus-HAInitValue* [*simp*]:

Value (*InitStatus A*) = *HAInitValue A*
apply (*unfold Value-def InitStatus-def*)
apply *simp*
apply (*subst Abs-status-inverse*)
apply *auto*
done

lemma *HA-InitStatus* [*simp*]:
HA (*InitStatus A*) = *A*
apply (*unfold InitStatus-def HA-def*)
apply *auto*
apply (*subst Abs-status-inverse*)
apply *auto*
done

8.2.3 *Events*

lemma *Events-HAEvents-Status*:
(*Events ST*) ⊆ *HAEvents* (*HA ST*)
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def*)
apply *fast*
done

lemma *TS-EventSet*:
TS ⊆ *ET ST* ⟹ ⋃ (*Actevent* (*Label TS*)) ⊆ *HAEvents* (*HA ST*)
apply (*unfold Actevent-def actevent-def ET-def EnabledTrans-def Action-def Label-def*)
apply (*cut-tac HA=HA ST* **in** *HAEvents-SAEvents-SAs*)
apply *auto*
apply (*rename-tac Event Source Trigger Guard Action Update Target*)
apply (*unfold SAEvents-def*)
apply (*erule subsetCE*)
apply *auto*
apply (*rename-tac SA*)
apply (*erule subsetCE*)
apply *auto*
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
apply (*erule-tac x=*(*Trigger*, *Guard*, *Action*, *Update*) **in** *ballE*)
apply *auto*
apply (*cut-tac SA=SA* **in** *Label-Delta-subset*)
apply (*erule subsetCE*)
apply (*unfold Label-def image-def*)
apply *auto*
done

8.2.4 *StepStatus*

lemma *StepStatus-empty*:

Abs-status (*HA ST*, *Conf ST*, {}, *U* !!! (*Value ST*)) = *StepStatus ST* {} *U*
apply (*unfold StepStatus-def Let-def*)
apply *auto*
apply (*subst Rep-status-tuple*)
apply *auto*
apply (*unfold StepActEvent-def*)
apply *auto*
done

lemma *status-empty-eventset* [*simp*]:
(*HA ST*, *Conf ST*, {}, *U* !!! (*Value ST*)) ∈ *status*
apply (*unfold status-def Status-def*)
apply *auto*
done

lemma *HA-StepStatus-emptyTS* [*simp*]:
HA (*StepStatus ST* {} *U*) = *HA ST*
apply (*subst StepStatus-empty* [*THEN sym*])
apply (*unfold HA-def*)
apply *auto*
apply (*subst Abs-status-inverse*)
apply *auto*
apply (*subst Rep-status-tuple*)
apply *auto*
done

8.2.5 Enabled Transitions *ET*

lemma *HPT-ETI*:
TS ∈ *HPT ST* ⟹ *TS* ⊆ *ET ST*
by (*unfold HPT-def MaxNonConflict-def*, *auto*)

lemma *finite-ET* [*simp*]:
finite (*ET ST*)
by (*unfold ET-def Image-def EnabledTrans-def*, *auto*)

8.2.6 Finite Transition Set

lemma *finite-MaxNonConflict* [*simp*]:
MaxNonConflict ST TS ⟹ *finite TS*
apply (*unfold MaxNonConflict-def*)
apply *auto*
apply (*subst finite-subset*)
apply *auto*
done

lemma *finite-HPT* [*simp*]:
TS ∈ *HPT ST* ⟹ *finite TS*
by (*unfold HPT-def*, *auto*)

8.2.7 *PUpdate*

lemma *finite-Update*:
finite TS $\Longrightarrow$ *finite* ((*λ F*. (*Rep-pupdate F*) (*Value ST*)) ‘ (*PUpdate* (*Label TS*)))
by (*rule finite-imageI*, *auto*)

lemma *finite-PUpdate*:
TS $\in$ *HPT S* $\Longrightarrow$ *finite* (*Expr.PUpdate* (*Label TS*))
apply *auto*
done

lemma *HPT-ResolveRacing-Some* [*simp*]:
TS $\in$ *HPT S* $\Longrightarrow$ (*SOME u*. *u* $\in$ *ResolveRacing TS*) $\in$ *ResolveRacing TS*
apply (*unfold ResolveRacing-def Let-def*)
apply (*rule finite-SequentialRacing*)
apply *auto*
done

8.2.8 Higher Priority Transitions *HPT*

lemma *finite-HPT2* [*simp*]:
finite (*HPT ST*)
apply (*cut-tac ST=ST* **in** *finite-ET*)
apply (*unfold HPT-def MaxNonConflict-def*)
apply (*subst Collect-subset*)
apply (*frule finite-Collect-subsets*)
apply *auto*
done

lemma *HPT-target-StepConf* [*simp*]:
⟦ *TS* $\in$ *HPT ST*; *T* $\in$ *TS* ⟧ $\Longrightarrow$ *target T* $\in$ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*unfold StepConf-def*)
apply *auto*
done

lemma *HPT-target-StepConf2* [*simp*]:
⟦ *TS* $\in$ *HPT ST*; (*S*,*L*,*T*) $\in$ *TS* ⟧ $\Longrightarrow$ *T* $\in$ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*unfold StepConf-def Target-def Source-def source-def target-def image-def*)
apply *auto*
apply *auto*
done

8.2.9 Delta Transition Set

lemma *ET-Delta*:
⟦ *TS* $\subseteq$ *ET ST*; *t* $\in$ *TS*; *source t* $\in$ *States A*; *A* $\in$ *SAs* (*HA ST*)⟧ $\Longrightarrow$ *t* $\in$ *Delta A*
apply (*unfold ET-def EnabledTrans-def*)
apply *simp*
apply (*erule subsetCE*)

apply *auto*
apply (*rename-tac SA*)
apply (*case-tac A = SA*)
apply *auto*
apply (*cut-tac HA=HA ST* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply *force*
done

lemma *ET-Delta-target*:
⟦ *TS* ⊆ *ET ST*; *t* ∈ *TS*; *target t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*) ⟧ ⟹ *t* ∈ *Delta A*
apply (*unfold ET-def EnabledTrans-def*)
apply *simp*
apply (*erule subsetCE*)
apply *auto*
apply (*rename-tac SA*)
apply (*case-tac A = SA*)
apply *auto*
apply (*cut-tac HA=HA ST* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply *force*
done

lemma *ET-HADelta*:
⟦ *TS* ⊆ *ET ST*; *t* ∈ *TS* ⟧ ⟹ *t* ∈ *HADelta* (*HA ST*)
apply (*unfold HADelta-def*)
apply *auto*
apply (*unfold ET-def EnabledTrans-def Image-def*)
apply *auto*
done

lemma *HPT-HADelta*:
⟦ *TS* ∈ *HPT ST*; *t* ∈ *TS* ⟧ ⟹ *t* ∈ *HADelta* (*HA ST*)
apply (*rule ET-HADelta*)
apply (*unfold HPT-def MaxNonConflict-def*)
apply *auto*
done

lemma *HPT-Delta*:
⟦ *TS* ∈ *HPT ST*; *t* ∈ *TS*; *source t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*)⟧ ⟹ *t* ∈ *Delta A*
apply (*rule ET-Delta*)
apply *auto*
apply (*unfold HPT-def MaxNonConflict-def*)
apply *fast*
done

lemma *HPT-Delta-target*:

⟦ *TS* ∈ *HPT ST*; *t* ∈ *TS*; *target t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*)⟧ ⟹ *t* ∈ *Delta A*
apply (*rule ET-Delta-target*)
apply *auto*
apply (*unfold HPT-def MaxNonConflict-def*)
apply *fast*
done

lemma *OneTrans-HPT-SA*:
⟦ *TS* ∈ *HPT ST*; *T* ∈ *TS*; *source T* ∈ *States SA*;
U ∈ *TS*; *source U* ∈ *States SA*; *SA* ∈ *SAs* (*HA ST*) ⟧ ⟹ *T* = *U*
apply (*unfold HPT-def MaxNonConflict-def Source-def*)
apply *auto*
apply (*erule-tac x=SA* **in** *ballE*)
apply (*case-tac finite* (*TS* ∩ *Delta SA*))
apply (*frule-tac t=T* **in** *OneElement-Card*)
apply *fast*
apply (*frule-tac t=T* **and** *A=SA* **in** *ET-Delta*)
apply *assumption+*
apply *fast*
apply (*frule-tac t=U* **in** *OneElement-Card*)
apply *fast*
apply (*frule-tac t=U* **and** *A=SA* **in** *ET-Delta*)
apply *auto*
done

lemma *OneTrans-HPT-SA2*:
⟦ *TS* ∈ *HPT ST*; *T* ∈ *TS*; *target T* ∈ *States SA*;
U ∈ *TS*; *target U* ∈ *States SA*; *SA* ∈ *SAs* (*HA ST*) ⟧ ⟹ *T* = *U*
apply (*unfold HPT-def MaxNonConflict-def Target-def*)
apply *auto*
apply (*erule-tac x=SA* **in** *ballE*)
apply (*case-tac finite* (*TS* ∩ *Delta SA*))
apply (*frule-tac t=T* **in** *OneElement-Card*)
apply *fast*
apply (*frule-tac t=T* **and** *A=SA* **in** *ET-Delta-target*)
apply *assumption+*
apply *fast*
apply (*frule-tac t=U* **in** *OneElement-Card*)
apply *fast*
apply (*frule-tac t=U* **and** *A=SA* **in** *ET-Delta-target*)
apply *auto*
done

8.2.10 Target Transition Set

lemma *ET-Target-HAStates*:
TS ⊆ *ET ST* ⟹ *Target TS* ⊆ *HAStates* (*HA ST*)
apply (*unfold HAStates-def Target-def target-def ET-def EnabledTrans-def Action-def*

Label-def)
apply (*cut-tac HA=HA ST* **in** *Target-SAs-Delta-States*)
apply *auto*
apply (*rename-tac Source Trigger Guard Action Update Target*)
apply (*unfold Target-def*)
apply (*erule subsetCE*)
apply *auto*
apply (*rename-tac SA*)
apply (*erule subsetCE*)
apply *auto*
apply (*unfold image-def*)
apply *auto*
apply (*metis target-select*)
done

lemma *HPT-Target-HAStates*:
TS ∈ *HPT ST* ⟹ *Target TS* ⊆ *HAStates* (*HA ST*)
apply (*rule HPT-ETI* [*THEN ET-Target-HAStates*])
apply *assumption*
done

lemma *HPT-Target-HAStates2* [*simp*]:
⟦*TS* ∈ *HPT ST*; *S* ∈ *Target TS*⟧ ⟹ *S* ∈ *HAStates* (*HA ST*)
apply (*cut-tac HPT-Target-HAStates*)
apply *fast+*
done

lemma *OneState-HPT-Target*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *Target TS*;
T ∈ *Target TS*; *S* ∈ *States SA*;
T ∈ *States SA*; *SA* ∈ *SAs* (*HA ST*) ⟧
⟹ *S* = *T*
apply (*unfold Target-def*)
apply (*auto dest*: *OneTrans-HPT-SA2*[*rotated* −*1*])
done

8.2.11 Source Transition Set

lemma *ET-Source-Conf*:
TS ⊆ *ET ST* ⟹ (*Source TS*) ⊆ *Conf ST*
apply (*unfold Source-def ET-def EnabledTrans-def*)
apply *auto*
done

lemma *HPT-Source-Conf* [*simp*]:
TS ∈ *HPT ST* ⟹ (*Source TS*) ⊆ *Conf ST*
apply (*unfold HPT-def MaxNonConflict-def*)
apply (*rule ET-Source-Conf*)
apply *auto*

done

lemma *ET-Source-Target* [*simp*]:
⟦ *SA* ∈ *SAs* (*HA ST*); *TS* ⊆ *ET ST*; *States SA* ∩ *Source TS* = {} ⟧ ⟹ *States SA* ∩ *Target TS* = {}
apply (*unfold ET-def EnabledTrans-def Source-def Target-def*)
apply *auto*
apply (*rename-tac Source Trigger Guard Action Update Target*)
apply (*erule subsetCE*)
apply *auto*
apply (*rename-tac SAA*)
apply (*unfold image-def source-def Int-def*)
apply *auto*
apply (*erule-tac x=Source* **in** *allE*)
apply *auto*
apply (*frule Delta-source-States*)
apply (*unfold source-def*)
apply *auto*
apply (*case-tac SA=SAA*)
apply *auto*
apply (*cut-tac HA=HA ST* **in** *MutuallyDistinct-HA*)
apply (*unfold MutuallyDistinct-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*erule-tac x=SAA* **in** *ballE*)
apply *auto*
apply (*frule Delta-target-States*)
apply (*unfold target-def*)
apply *force*
done

lemma *HPT-Source-Target* [*simp*]:
⟦ *TS* ∈ *HPT ST*; *States SA* ∩ *Source TS* = {}; *SA* ∈ *SAs* (*HA ST*) ⟧ ⟹ *States SA* ∩ *Target TS* = {}
apply (*unfold HPT-def MaxNonConflict-def*)
apply *auto*
done

lemma *ET-target-source*:
⟦ *TS* ⊆ *ET ST*; *t* ∈ *TS*; *target t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*) ⟧ ⟹ *source t* ∈ *States A*
apply (*frule ET-Delta-target*)
apply *auto*
done

lemma *ET-source-target*:
⟦ *TS* ⊆ *ET ST*; *t* ∈ *TS*; *source t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*) ⟧ ⟹ *target t* ∈ *States A*
apply (*frule ET-Delta*)
apply *auto*

done

lemma *HPT-target-source*:
⟦ *TS* ∈ *HPT ST*; *t* ∈ *TS*; *target t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*)⟧ ⟹ *source t* ∈ *States A*
apply (*rule ET-target-source*)
apply *auto*
apply (*unfold HPT-def MaxNonConflict-def*)
apply *fast*
done

lemma *HPT-source-target*:
⟦ *TS* ∈ *HPT ST*; *t* ∈ *TS*; *source t* ∈ *States A*; *A* ∈ *SAs* (*HA ST*) ⟧ ⟹ *target t* ∈ *States A*
apply (*rule ET-source-target*)
apply *auto*
apply (*unfold HPT-def MaxNonConflict-def*)
apply *fast*
done

lemma *HPT-source-target2* [*simp*]:
⟦ *TS* ∈*HPT ST*; (*s*,*l*,*t*) ∈ *TS*; *s* ∈ *States A*; *A* ∈ *SAs* (*HA ST*)⟧ ⟹ *t* ∈*States A*
apply (*cut-tac ST*=*ST* **and** *TS*=*TS* **and** *t*=(*s*,*l*,*t*) **in** *HPT-source-target*)
apply *auto*
done

lemma *ChiRel-ChiStar-Source-notmem*:
⟦ *TS* ∈ *HPT ST*; (*S*, *T*) ∈ *ChiRel* (*HA ST*); *S* ∈ *Conf ST*;
T ∉ *ChiStar* (*HA ST*) `` *Source TS* ⟧ ⟹
S ∉ *ChiStar* (*HA ST*) `` *Source TS*
apply *auto*
apply (*rename-tac U*)
apply (*simp only*: *Image-def*)
apply *auto*
apply (*erule-tac x*=*U* **in** *ballE*)
apply (*fast intro*: *ChiRel-ChiStar-trans*)+
done

lemma *ChiRel-ChiStar-notmem*:
⟦ *TS* ∈ *HPT ST*; (*S*,*T*) ∈ *ChiRel* (*HA ST*);
S ∈ *ChiStar* (*HA ST*) `` *Source TS* ⟧ ⟹ *T* ∉ *Source TS*
using [[*hypsubst-thin* = *true*]]
apply (*unfold HPT-def MaxNonConflict-def HigherPriority-def restrict-def*)
apply *auto*
apply (*rename-tac U*)
apply (*unfold Source-def image-def*)
apply *auto*
apply (*rename-tac SSource STrigger SGuard SAction SUpdate STarget*

TSource TTrigger TGuard TAction TUpdate TTarget)
apply (*erule-tac x*=(*SSource*, (*STrigger*, *SGuard*, *SAction*, *SUpdate*), *STarget*) **in** *ballE*)
apply *auto*
apply (*erule-tac x*=(*TSource*, (*TTrigger*, *TGuard*, *TAction*, *TUpdate*), *TTarget*) **in** *ballE*)
apply *auto*
apply (*simp add*: *ET-HADelta*)
apply (*case-tac SSource*=*S*)
apply *auto*
apply (*frule ChiStar-ChiPlus-noteq*)
apply *fast*
apply (*fast intro*: *ChiRel-ChiPlus-trans*)
done

8.2.12 *StepActEvents*

lemma *StepActEvent-empty* [*simp*]:
StepActEvent {} = {}
by (*unfold StepActEvent-def*, *auto*)

lemma *StepActEvent-HAEvents*:
TS ∈ *HPT ST* ⟹ *StepActEvent TS* ⊆ *HAEvents* (*HA ST*)
apply (*unfold StepActEvent-def image-def*)
apply (*rule HPT-ETI* [*THEN TS-EventSet*])
apply *assumption*
done

8.2.13 *UniqueSucStates*

lemma *UniqueSucStates-Status* [*simp*]:
UniqueSucStates (*SAs* (*HA ST*)) (*CompFun* (*HA ST*)) (*Conf ST*)
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def IsConfSet-def*)
apply *auto*
done

8.2.14 *RootState*

lemma *RootExSem-Status* [*simp*]:
RootExSem (*SAs* (*HA ST*)) (*CompFun* (*HA ST*)) (*Conf ST*)
apply (*cut-tac Rep-status-select*)
apply (*unfold status-def Status-def IsConfSet-def*)
apply *auto*
done

lemma *RootState-HARootState* [*simp*]:
(*RootState ST*) ∈ *States* (*HARoot* (*HA ST*))
apply (*unfold RootState-def*)
apply (*cut-tac ST*=*ST* **in** *RootExSem-Status*)

apply (*unfold RootExSem-def HARoot-def HAStates-def*)
apply *auto*
apply (*subst some1-equality*)
apply *auto*
done

lemma *RootState-Conf* [*simp*]:
(*RootState ST*) ∈ (*Conf ST*)
apply (*unfold RootState-def*)
apply (*cut-tac ST=ST* **in** *RootExSem-Status*)
apply (*unfold RootExSem-def HARoot-def HAStates-def*)
apply *auto*
apply (*subst some1-equality*)
apply *auto*
done

lemma *RootState-notmem-Chi* [*simp*]:
S ∈ *HAStates* (*HA ST*) ⟹ (*RootState ST*) ∉ *Chi* (*HA ST*) *S*
by *auto*

lemma *RootState-notmem-Range-ChiRel* [*simp*]:
RootState ST ∉ *Range* (*ChiRel* (*HA ST*))
by *auto*

lemma *RootState-Range-ChiPlus* [*simp*]:
RootState ST ∉ *Range* (*ChiPlus* (*HA ST*))
by *auto*

lemma *RootState-Range-ChiStar* [*simp*]:
⟦ *x* ≠ *RootState ST* ⟧ ⟹ (*x*,*RootState ST*) ∉ (*ChiStar* (*HA ST*))
by *auto*

lemma *RootState-notmem-ChiRel* [*simp*]:
(*x*,*RootState ST*) ∉ (*ChiRel* (*HA ST*))
by (*unfold ChiRel-def*, *auto*)

lemma *RootState-notmem-ChiRel2* [*simp*]:
⟦ *S* ∈ *States* (*HARoot* (*HA ST*)) ⟧ ⟹ (*x*,*S*) ∉ (*ChiRel* (*HA ST*))
by (*unfold ChiRel-def*, *auto*)

lemma *RootState-Conf-StepConf* [*simp*]:
⟦ *RootState ST* ∉ *Source TS* ⟧ ⟹ *RootState ST* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*unfold StepConf-def*)
apply *auto*
apply (*rename-tac S*)
apply (*case-tac S=RootState ST*)
apply *fast*
apply *auto*

apply (*rename-tac S*)
apply (*case-tac S=RootState ST*)
apply *fast*
apply *auto*
done

lemma *OneRootState-Conf* [*simp*]:
⟦ *S* ∈ *States* (*HARoot* (*HA ST*)); *S* ∈ *Conf ST* ⟧ ⟹ *S* = *RootState ST*
apply (*cut-tac ST=ST* **in** *IsConfSet-Status*)
apply (*unfold IsConfSet-def RootExSem-def*)
apply (*fold HARoot-def*)
apply *auto*
done

lemma *OneRootState-Source*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *Source TS*; *S* ∈ *States* (*HARoot* (*HA ST*)) ⟧ ⟹ *S* = *RootState ST*
apply (*cut-tac ST=ST* **and** *TS=TS* **in** *HPT-Source-Conf*, *assumption*)
apply (*cut-tac ST=ST* **in** *OneRootState-Conf*)
apply *fast+*
done

lemma *OneState-HPT-Target-Source*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *States SA*; *SA* ∈ *SAs* (*HA ST*);
States SA ∩ *Source TS* = {} ⟧
⟹ *S* ∉ *Target TS*
apply (*unfold Target-def*)
apply *auto*
apply (*unfold Source-def Image-def Int-def*)
apply *auto*
apply (*frule HPT-target-source*)
apply *auto*
done

lemma *RootState-notmem-Target* [*simp*]:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *States* (*HARoot* (*HA ST*)); *RootState ST* ∉ *Source TS* ⟧
⟹ *S* ∉ *Target TS*
apply *auto*
apply (*frule OneState-HPT-Target-Source*)
prefer *4*
apply *fast+*
apply *simp*
apply (*unfold Int-def*)
apply *auto*
apply (*frule OneRootState-Source*)
apply *fast+*
done

8.2.15 Configuration *Conf*

lemma *Conf-HAStates*:
Conf ST $\subseteq$ *HAStates* (*HA ST*)
apply (*cut-tac Rep-status-select*)
apply (*unfold IsConfSet-def status-def Status-def HAStates-def*)
apply *fast*
done

lemma *Conf-HAStates2* [*simp*]:
$S \in$ *Conf ST* $\Longrightarrow S \in$ *HAStates* (*HA ST*)
apply (*cut-tac ST=ST* **in** *Conf-HAStates*)
apply *fast*
done

lemma *OneState-Conf* [*intro*]:
$[\![S \in$ *Conf ST*; $T \in$ *Conf ST*; $S \in$ *States SA*; $T \in$ *States SA*;
SA $\in$ *SAs* (*HA ST*)$]\!] \Longrightarrow T = S$
apply (*cut-tac ST=ST* **in** *IsConfSet-Status*)
apply (*unfold IsConfSet-def UniqueSucStates-def*)
apply (*case-tac SA = HARoot* (*HA ST*))
apply (*cut-tac ST=ST* **and** *S=S* **in** *OneRootState-Conf*)
apply *fast+*
apply (*simp only:OneRootState-Conf*)
apply (*erule conjE*)+
apply (*cut-tac HA=HA ST* **in** *OneAncestor-HA*)
apply (*unfold OneAncestor-def*)
apply (*fold HARoot-def*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*drule ex1-implies-ex*)
apply (*erule exE*)
apply (*rename-tac U*)
apply (*erule-tac x=U* **in** *ballE*)
apply (*erule-tac x=SA* **in** *ballE*)
apply (*case-tac U* $\in$ *Conf ST*)
apply *simp*
apply *safe*
apply *fast+*
apply *simp*
apply *fast*
done

lemma *OneState-HPT-SA*:
$[\![TS \in$ *HPT ST*; $S \in$ *Source TS*; $T \in$ *Source TS*;
$S \in$ *States SA*; $T \in$ *States SA*;
SA $\in$ *SAs* (*HA ST*) $]\!] \Longrightarrow S = T$
apply (*rule OneState-Conf*)
apply *auto*
apply (*frule HPT-Source-Conf*, *fast*)+
done

lemma *HPT-SAStates-Target-Source*:
⟦*TS* ∈ *HPT ST*; *A* ∈ *SAs* (*HA ST*); *S* ∈ *States A*; *T* ∈ *States A*; *S* ∈ *Conf ST*;
T ∈ *Target TS* ⟧ ⟹ *S* ∈ *Source TS*
apply (*case-tac States A* ∩ *Source TS* ={})
apply (*frule OneState-HPT-Target-Source*)
apply *fast*
back
apply *simp+*
apply *auto*
apply (*rename-tac U*)
apply (*cut-tac ST=ST* **in** *HPT-Source-Conf*)
apply *fast*
apply (*frule-tac S=S* **and** *T=U* **in** *OneState-Conf*)
apply *fast+*
done

lemma *HPT-Conf-Target-Source*:
⟦*TS* ∈ *HPT ST*; *S* ∈ *Conf ST*;
S ∈ *Target TS* ⟧ ⟹ *S* ∈ *Source TS*
apply (*frule Conf-HAStates2*)
apply (*unfold HAStates-def*)
apply *auto*
apply (*simp only*:*HPT-SAStates-Target-Source*)
done

lemma *Conf-SA*:
S ∈ *Conf ST* ⟹ ∃ *A* ∈ *SAs* (*HA ST*). *S* ∈ *States A*
apply (*cut-tac ST=ST* **in** *IsConfSet-Status*)
apply (*unfold IsConfSet-def*)
apply *fast*
done

lemma *HPT-Source-HAStates* [*simp*]:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *Source TS* ⟧ ⟹ *S* ∈ *HAStates* (*HA ST*)
apply (*frule HPT-Source-Conf*)
apply (*rule Conf-HAStates2*)
apply *fast*
done

lemma *Conf-Ancestor*:
⟦ *S* ∈ *Conf ST*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*) ⟧ ⟹ ∃! *T* ∈ *States A*. *T* ∈ *Conf ST*
apply (*cut-tac ST=ST* **in** *IsConfSet-Status*)
apply (*unfold IsConfSet-def UniqueSucStates-def*)
apply *safe*
apply (*erule-tac x=S* **in** *ballE*)
prefer *2*
apply *blast*

apply (*erule-tac x=A* **in** *ballE*)
prefer *2*
apply *fast*
apply *simp*
apply (*fast intro*: *HAStates-CompFun-SAs-mem Conf-HAStates2*)+
done

lemma *Conf-ChiRel*:
⟦ (*S*,*T*) ∈ *ChiRel* (*HA ST*); *T* ∈ *Conf ST* ⟧ ⟹ *S* ∈ *Conf ST*
apply (*unfold ChiRel-def Chi-def restrict-def*)
apply *simp*
apply *safe*
apply *simp*
apply *safe*
apply (*rename-tac SA*)
apply (*unfold HAStates-def*)
apply *simp*
apply *safe*
apply (*rename-tac U*)
apply (*cut-tac ST=ST* **in** *UniqueSucStates-Status*)
apply (*unfold UniqueSucStates-def*)
apply (*erule-tac x=S* **in** *ballE*)
apply (*erule-tac x=SA* **in** *ballE*)
apply *auto*
apply (*case-tac S* ∈ *Conf ST*)
apply *simp+*
done

lemma *Conf-ChiPlus*:
⟦ (*T*,*S*) ∈ *ChiPlus* (*HA ST*) ⟧ ⟹ *S* ∈ *Conf ST* ⟶ *T* ∈ *Conf ST*
apply (*unfold ChiPlus-def*)
apply (*rule-tac a=T* **and** *b=S* **and** *r=*(*ChiRel* (*HA ST*)) **in** *trancl-induct*)
apply (*fast intro*: *Conf-ChiRel*)+
done

lemma *HPT-Conf-Target-Source-ChiPlus*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *Conf ST*; *S* ∈ *ChiPlus* (*HA ST*) `` *Target TS* ⟧
⟹ *S* ∈ *ChiStar* (*HA ST*) `` *Source TS*
apply *auto*
apply (*rename-tac T*)
apply (*simp add*: *Image-def*)
apply (*frule HPT-Target-HAStates2*)
apply *fast*
apply (*unfold HAStates-def*)
apply *auto*
apply (*rename-tac SA*)
apply (*case-tac States SA* ∩ *Source TS* = {})
apply (*simp only*:*OneState-HPT-Target-Source*)

apply *auto*
apply (*rename-tac U*)
apply (*erule-tac x=U* **in** *ballE*)
apply *auto*
apply (*case-tac U=T*)
apply *auto*
apply (*frule Conf-ChiPlus*)
apply *simp*
apply (*frule HPT-Conf-Target-Source*)
apply *fast*
back
apply *fast*
apply (*simp add:OneState-HPT-SA*)
done

lemma *OneState-HPT-Target-ChiRel*:
⟦ *TS* ∈ *HPT ST*; (*U*, *T*) ∈ *ChiRel* (*HA ST*);
U ∈ *Target TS*; *A* ∈ *SAs* (*HA ST*); *T* ∈ *States A*;
S ∈ *States A* ⟧ ⟹ *S* ∉ *Target TS*
using [[*hypsubst-thin* = *true*]]
apply *auto*
apply (*unfold HigherPriority-def restrict-def HPT-def MaxNonConflict-def Target-def*)
apply *auto*
apply (*rename-tac SSource STrigger SGuard SAction SUpdate STarget TSource TTrigger TGuard TAction TUpdate TTarget*)
apply (*cut-tac t=*(*TSource*, (*TTrigger*, *TGuard*, *TAction*, *TUpdate*), *TTarget*) **and** *TS=TS* **and** *ST=ST* **and** *A=A* **in** *ET-target-source*)
apply *assumption+*
apply *simp*
apply *assumption*
apply (*frule ChiRel-HAStates*)
apply (*unfold HAStates-def*)
apply *safe*
apply (*cut-tac t=*(*SSource*, (*STrigger*, *SGuard*, *SAction*, *SUpdate*), *STarget*) **and** *A=x* **and** *ST=ST* **and** *TS=TS* **in** *ET-target-source*)
apply *assumption+*
apply *simp*
apply *assumption*
apply *simp*
apply (*erule-tac x=*(*SSource*, (*STrigger*, *SGuard*, *SAction*, *SUpdate*), *STarget*) **in** *ballE*)
apply *simp*
apply (*erule-tac x=*(*TSource*, (*TTrigger*, *TGuard*, *TAction*, *TUpdate*), *TTarget*) **in** *ballE*)
apply (*simp add*: *ET-HADelta*)
apply (*cut-tac A=HA ST* **and** *S=STarget* **and** *T=T* **and** *U=TSource* **in** *ChiRel-SA-OneAncestor*)
apply *fast+*
apply (*frule ET-Source-Conf*)

apply (*unfold Source-def image-def*)
apply (*case-tac SSource* ∈*Conf ST*)
prefer *2*
apply (*erule subsetCE*)
back
apply *fast*
back
apply *simp*
apply (*case-tac TSource* ∈*Conf ST*)
prefer *2*
apply (*erule subsetCE*)
back
apply *fast*
apply *simp*
apply (*case-tac STarget=SSource*)
apply *simp*
apply (*fast intro*:*Conf-ChiRel*)+
done

lemma *OneState-HPT-Target-ChiPlus* [*rule-format*]:
⟦ *TS* ∈ *HPT ST*; (*U*,*T*) ∈ *ChiPlus* (*HA ST*);
S ∈ *Target TS*; *A* ∈ *SAs* (*HA ST*);
S ∈ *States A* ⟧ ⟹ *T* ∈ *States A* ⟶ *U* ∉ *Target TS*
using [[*hypsubst-thin* = *true*]]
apply (*unfold ChiPlus-def*)
apply (*rule-tac a=U* **and** *b=T* **and** *r=*(*ChiRel* (*HA ST*)) **in** *converse-trancl-induct*)
apply *auto*
apply (*simp only*:*OneState-HPT-Target-ChiRel*)
apply (*rename-tac V W*)
apply (*fold ChiPlus-def*)
apply (*unfold HPT-def MaxNonConflict-def Target-def HigherPriority-def restrict-def*)
apply *auto*
apply (*rename-tac SSource STrigger SGuard SAction SUpdate STarget*
TSource TTrigger TGuard TAction TUpdate TTarget)
apply (*cut-tac t=*(*SSource*, (*STrigger*, *SGuard*, *SAction*, *SUpdate*), *STarget*) **and**
ST=ST **and** *TS=TS* **and** *A=A* **in** *ET-target-source*)
apply *assumption*+
apply *simp*
apply *assumption*
apply *simp*
apply (*frule ChiRel-HAStates*)
apply (*unfold HAStates-def*)
apply *safe*
apply (*cut-tac t=*(*TSource*, (*TTrigger*, *TGuard*, *TAction*, *TUpdate*), *TTarget*)
and *A=x* **and** *TS=TS* **and** *ST=ST* **in** *ET-target-source*)
apply *assumption*+
apply *simp*
apply *assumption*
apply *simp*

apply (*erule-tac x*=(*TSource*, (*TTrigger*, *TGuard*, *TAction*, *TUpdate*), *TTarget*)
in *ballE*)
apply *simp*
apply (*erule-tac x*=(*SSource*, (*STrigger*, *SGuard*, *SAction*, *SUpdate*), *STarget*) **in**
ballE)
apply (*simp add*: *ET-HADelta*)
apply (*cut-tac A*=*HA ST* **and** *S*=*TTarget* **and** *T*=*T* **and** *U*=*SSource* **in** *ChiPlus-SA-OneAncestor*)
apply (*fast intro*: *ChiRel-ChiPlus-trans2*)
apply *fast*+
apply (*frule ET-Source-Conf*)
apply (*unfold Source-def image-def*)
apply (*case-tac SSource* ∈*Conf ST*)
prefer *2*
apply (*erule subsetCE*)
back
apply *fast*
apply *simp*
apply (*case-tac TSource* ∈*Conf ST*)
prefer *2*
apply (*erule subsetCE*)
back
apply *fast*
back
apply *simp*
apply (*case-tac TTarget*=*SSource*)
apply *simp*
apply (*frule-tac T*=*TTarget* **and** *S*=*SSource* **in** *Conf-ChiPlus*)
apply *simp*
apply (*frule-tac T*=*TSource* **and** *S*=*TTarget* **in** *OneState-Conf*)
apply *fast*+
done

8.2.16 *RootExSem*

lemma *RootExSem-StepConf*:
⟦ *TS* ∈ *HPT ST* ⟧ ⟹
RootExSem (*SAs* (*HA ST*)) (*CompFun* (*HA ST*)) (*StepConf* (*HA ST*) (*Conf*
ST) *TS*)
apply (*unfold RootExSem-def*)
apply (*fold HARoot-def*)
apply *auto*
apply (*case-tac RootState ST* ∉ *Source TS*)
apply (*rule-tac x*=*RootState ST* **in** *exI*)
apply *simp*
apply *simp*
apply (*unfold Source-def image-def*)
apply *simp*
apply (*erule bexE*)
apply (*rename-tac T*)

apply (*rule-tac x=target T* **in** *exI*)
apply *simp*
apply (*rule HPT-source-target*)
apply *auto*
apply (*rename-tac S T*)
apply (*case-tac S ∈ Conf ST*)
apply (*case-tac T ∈ Conf ST*)
apply (*frule OneRootState-Conf*)
apply *auto*
apply (*frule OneRootState-Conf*)
apply *auto*
apply (*frule OneRootState-Conf*)
apply *auto*
apply (*case-tac RootState ST ∈ Source TS*)
apply (*case-tac T ∈ Source TS*)
apply (*frule HPT-Source-Conf*)
apply *fast*
apply (*unfold StepConf-def*)
apply *auto*
apply (*frule OneState-HPT-Target*)
apply (*frule-tac SA=HARoot (HA ST)* **and** *TS=TS* **and** *S=T* **and** *T=RootState ST* **in** *OneState-HPT-Target*)
apply *fast+*
apply *simp+*
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *force*)+
prefer *2*
apply (*frule OneState-HPT-Target*)
apply *fast+*
back
apply *simp+*
apply (*case-tac RootState ST ∈ Source TS*)
apply (*case-tac T = RootState ST*)
apply *auto*
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *force*)+
done

8.2.17 *StepConf*

lemma *Target-StepConf*:
  $S \in Target\ TS \Longrightarrow S \in StepConf\ (HA\ ST)\ (Conf\ ST)\ TS$
apply (*unfold StepConf-def*)
apply *auto*
done

lemma *Target-ChiRel-HAInit-StepConf*:
  $[\![S \in Target\ TS; (S,T) \in ChiRel\ A;$
    $T \in HAInitStates\ A\ ]\!] \Longrightarrow T \in StepConf\ A\ C\ TS$
apply (*unfold StepConf-def*)
apply *auto*

done

lemma *StepConf-HAStates*:
TS ∈ *HPT ST* ⟹ *StepConf* (*HA ST*) (*Conf ST*) *TS* ⊆ *HAStates* (*HA ST*)
apply (*unfold StepConf-def*)
apply *auto*
apply (*frule tranclD2*)
apply *auto*
done

lemma *RootState-Conf-StepConf2* [*simp*]:
⟦ *source T* = *RootState ST*; *T* ∈ *TS* ⟧ ⟹ *target T* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*unfold StepConf-def*)
apply *auto*
done

lemma *HPT-StepConf-HAStates* [*simp*]:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS* ⟧ ⟹ *S* ∈ *HAStates* (*HA ST*)
apply (*unfold StepConf-def*)
apply *auto*
apply (*frule tranclD2*)
apply *auto*
done

lemma *StepConf-Target-HAInitStates*:
⟦ *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *S* ∉ *Target TS*; *S* ∉ *Conf ST*⟧ ⟹ *S* ∈ *HAInitStates* (*HA ST*)
apply (*unfold StepConf-def*)
apply *auto*
apply (*frule tranclD2*)
apply *auto*
done

lemma *InitSucState-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∉ *Target TS*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*);
S ∉ *Conf ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS* ⟧ ⟹
InitState A ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*frule StepConf-HAStates* [*THEN subsetD*, *THEN CompFun-HAInitStates-HAStates*])
apply *fast+*
apply (*subst* (*asm*) *StepConf-def*)
apply *safe*
apply (*unfold StepConf-def*)
apply (*fast intro*: *HAInitStates-InitState-trancl*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*)
apply (*fast intro*:*ChiPlus-HAStates-Right* [*THEN HAInitStates-InitState-trancl2*])

done

lemma *InitSucState-Target-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *Target TS*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*)⟧ ⟹
InitState A ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*frule HPT-Target-HAStates2* [*THEN CompFun-HAInitStates-HAStates*])
apply *fast*+
apply (*frule HPT-Target-HAStates2* [*THEN CompFun-ChiRel*])
apply (*fast intro*:*InitState-States*)+
apply (*unfold StepConf-def*)
apply *auto*
done

lemma *InitSucState-Conf-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*;
S ∉ *Target TS*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*);
S ∈ *Conf ST*; *S* ∈ *ChiStar* (*HA ST*) `` (*Source TS*) ⟧ ⟹
InitState A ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*frule Conf-HAStates2* [*THEN CompFun-HAInitStates-HAStates*])
apply *fast*
apply (*subst* (*asm*) *StepConf-def*)
apply *safe*
apply *fast*
apply (*unfold StepConf-def*)
apply (*fast intro*:*HAInitStates-InitState-trancl*)
apply (*rename-tac T U V*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *safe*)
apply (*subst* (*asm*) *Image-def*, *safe*)
apply (*rule-tac x*=*U* **in** *bexI*)
apply (*simp only*: *ChiPlus-HAStates-Right* [*THEN HAInitStates-InitState-trancl2*])
apply *fast*
apply (*subst* (*asm*) *Image-def*, *safe*)
apply (*rule-tac x*=*U* **in** *bexI*)
apply (*simp only*: *ChiPlus-HAStates-Right* [*THEN HAInitStates-InitState-trancl2*])
apply *fast*
done

lemma *SucState-Conf-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*;
S ∉ *Target TS*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*);
S ∈ *Conf ST*; *States A* ∩ *ChiStar* (*HA ST*) `` (*Source TS*) = {} ⟧ ⟹
∃ *x*. *x* ∈ *States A* ∧ *x* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*unfold StepConf-def*)
apply (*cut-tac ST*=*ST* **in** *UniqueSucStates-Status*)
apply (*unfold UniqueSucStates-def*)
apply (*cut-tac ST*=*ST* **in** *Conf-HAStates2*)
apply *fast*
apply (*fold HAStates-def*)
apply (*erule-tac x*=*S* **in** *ballE*)
apply (*erule-tac x*=*A* **in** *ballE*)

apply *simp*
apply *fast+*
done

lemma *SucState-Conf-Source-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*;
S ∉ *Target TS*; *A* ∈ *the* (*CompFun* (*HA ST*) *S*);
S ∈ *Conf ST*; *States A* ∩ *ChiStar* (*HA ST*) `` (*Source TS*) ≠ {};
S ∉ *ChiStar* (*HA ST*) `` (*Source TS*)⟧ ⟹
∃ *x*. *x* ∈ *States A* ∧ *x* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply *safe*
apply (*rename-tac T U*)
apply (*frule Conf-HAStates2* [*THEN CompFun-ChiRel*])
apply *fast+*
apply *simp*
apply (*case-tac U=T*)
apply *simp*
apply (*rotate-tac −5*)
apply (*simp only*:*Source-def Target-def image-def*)
apply *safe*
apply (*rename-tac Source Trigger Guard Action Update Target*)
apply (*erule-tac x=Target* **in** *allE*)
apply *simp*
apply (*frule HPT-source-target2*)
apply *fast+*
apply (*rule HAStates-CompFun-SAs-mem*)
apply (*rule Conf-HAStates2*)
apply *fast+*
apply (*frule ChiStar-ChiPlus-noteq*)
apply *fast*
apply (*case-tac U=S*)
apply (*fast intro*:*ChiStar-Self ChiRel-ChiPlus-OneAncestor ChiPlus-ChiStar*)+
done

lemma *SucState-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*;
A ∈ *the* (*CompFun* (*HA ST*) *S*) ⟧ ⟹
∃ *x*. *x* ∈ *States A* ∧ *x* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*case-tac S* ∈ *Target TS*)
apply (*fast intro*: *InitSucState-Target-StepConf InitState-States*)
apply (*case-tac S* ∈ *Conf ST*)
prefer *2*
apply (*fast intro*: *InitSucState-StepConf InitState-States*)
apply (*case-tac S* ∈ *ChiStar* (*HA ST*) `` (*Source TS*))
apply (*fast intro*: *InitSucState-Conf-StepConf InitState-States*)
apply (*case-tac States A* ∩ *ChiStar* (*HA ST*) `` (*Source TS*) = {})
apply (*fast intro*: *SucState-Conf-StepConf SucState-Conf-Source-StepConf*)+
done

8.2.18 *StepStatus*

lemma *StepStatus-expand*:
Abs-status (*HA ST*, *StepConf* (*HA ST*) (*Conf ST*) *TS*,
StepActEvent TS, *U* !!! (*Value ST*))
= (*StepStatus ST TS U*)
apply (*unfold StepStatus-def Let-def*)
apply (*subst Rep-status-tuple*)
apply *auto*
done

lemma *UniqueSucState-Conf-Source-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *A* ∈ *SAs* (*HA ST*);
A ∈ *the* (*CompFun* (*HA ST*) *S*); *T* ∈ *States A*; *U* ∈ *States A*;
T ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *T* ≠ *U*; *U* ∈ *Conf ST* ⟧ ⟹
U ∈ *ChiStar* (*HA ST*) `` *Source TS*
apply (*frule-tac ?S2.0=T* **in** *StepConf-HAStates* [*THEN subsetD*, *THEN Comp-Fun-ChiRel*])
apply *fast+*
apply (*frule-tac ?S2.0=U* **in** *StepConf-HAStates* [*THEN subsetD*, *THEN Comp-Fun-ChiRel*])
apply *fast+*
apply (*frule-tac S=S* **and** *T=U* **in** *Conf-ChiRel*, *fast*)
apply (*case-tac S* ∈ *ChiStar* (*HA ST*) `` *Source TS*)
apply (*fast intro*: *ChiRel-ChiStar-trans*)
apply (*case-tac U* ∈ *Source TS*)
apply *force*
apply (*unfold StepConf-def*)
apply *simp*
apply *safe*
apply (*fast intro*: *HPT-SAStates-Target-Source*)+
apply (*rename-tac V*)
apply (*case-tac V=S*)
apply (*frule-tac S=S* **in** *HPT-Conf-Target-Source*, *fast+*)
apply (*fast intro*: *ChiStar-Image ChiRel-OneAncestor*)+
apply (*rename-tac V W*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *safe*)
apply (*cut-tac ST=ST* **and** *S=S* **in** *HPT-Conf-Target-Source-ChiPlus*)
apply *fast+*
apply (*simp only*:*Image-def*, *safe*)
apply (*case-tac* (*V*, *T*) ∉ *ChiRel* (*HA ST*))
apply (*frule-tac S=V* **and** *T=T* **in** *ChiPlus-ChiRel-Ex*)
apply (*fast*, *safe*)
apply (*rename-tac X*)
apply (*case-tac X=S*)
apply (*rule-tac x=W* **in** *bexI*)
prefer *4*
apply (*case-tac V=S*)
prefer *2*
apply *simp*

apply (*fast intro*: *ChiPlus-ChiRel ChiRel-ChiPlus-trans2 ChiRel-OneAncestor*)+
done

lemma *UniqueSucState-Target-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *A* ∈ *SAs* (*HA ST*);
A ∈ *the* (*CompFun* (*HA ST*) *S*); *T* ∈ *States A*; *U* ∈ *States A*;
T ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *T* ≠ *U* ⟧ ⟹
U ∉ *Target TS*
apply *auto*
apply (*frule-tac ST=ST* **in** *Target-StepConf*)
apply (*subst* (*asm*) (*2*) *StepConf-def*)
apply *simp*
apply *safe*
apply (*cut-tac TS=TS* **and** *ST=ST* **and** *S=S* **and** *T=U* **in** *UniqueSucState-Conf-Source-StepConf*)
apply *fast+*
apply (*simp add*: *OneState-HPT-Target*)
apply (*simp only*:*OneState-HPT-Target-ChiRel*)
apply (*rename-tac V W*)
apply (*frule-tac U=W* **and** *S=V* **and** *T=T* **in** *ChiRel-ChiPlus-trans2*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *force*)
apply (*simp only*:*OneState-HPT-Target-ChiPlus*)
done

lemma *UniqueSucState-Target-ChiRel-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *A* ∈ *SAs* (*HA ST*);
A ∈ *the* (*CompFun* (*HA ST*) *S*); *T* ∈ *States A*; *U* ∈ *States A*;
T ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *T* ≠ *U*; (*V*,*U*) ∈ *ChiRel* (*HA ST*);
U ∈ *HAInitStates* (*HA ST*) ⟧
⟹ *V* ∉ *Target TS*
apply *auto*
apply (*frule-tac A=HA ST* **and** *C=Conf ST* **in** *Target-ChiRel-HAInit-StepConf*)
apply *fast+*
apply (*subst* (*asm*) (*2*) *StepConf-def*, *safe*)
apply (*fast intro*:*UniqueSucState-Conf-Source-StepConf*)
apply (*simp only*:*OneState-HPT-Target-ChiRel*)
apply (*fast intro*:*OneHAInitState-SAStates*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*)
apply (*fast intro*:*OneHAInitState-SAStates*)
done

lemma *UniqueSucState-Target-ChiPlus-StepConf* [*rule-format*]:
⟦ *TS* ∈ *HPT ST*; (*S*,*T*) ∈ *ChiRel* (*HA ST*); (*S*,*U*) ∈ *ChiRel* (*HA ST*);
V ∈ *Target TS*; (*V*,*W*) ∈ *ChiRel* (*HA ST*); *T* ∉ *ChiStar* (*HA ST*) `` *Source TS*;
(*W*,*U*) ∈ (*ChiRel* (*HA ST*) ∩ *HAInitStates* (*HA ST*) × *HAInitStates* (*HA ST*))$^+$;
T ∈ *Conf ST* ⟧ ⟹ (*S*,*U*) ∈ *ChiRel* (*HA ST*) ⟶ *T=U*
apply (*frule-tac S=S* **and** *T=T* **in** *Conf-ChiRel*)
apply *fast*

apply (*rule-tac a=W* **and** *b=U* **and** *r=ChiRel* (*HA ST*) ∩ *HAInitStates* (*HA ST*) × *HAInitStates* (*HA ST*) **in** *trancl-induct*)
apply *safe*
apply (*rename-tac X*)
apply (*case-tac W=S*)
apply *simp*
prefer *2*
apply (*simp add*: *ChiRel-OneAncestor*)
prefer *2*
apply (*rename-tac X Y*)
apply (*case-tac X=S*)
apply *simp*
prefer *2*
apply (*simp add*: *ChiRel-OneAncestor*)
prefer *2*
apply (*frule-tac a=V* **in** *ChiRel-HAStates*)
apply (*unfold HAStates-def*)
apply (*simp,safe*)
apply (*rename-tac Y*)
apply (*case-tac States Y* ∩ *Source TS* = {})
apply (*simp add:OneState-HPT-Target-Source*)
apply (*subst* (*asm*) *Int-def*, *safe*)
apply (*rename-tac Z*)
apply (*frule-tac S=V* **and** *T=S* **in** *Conf-ChiRel*)
apply *fast*
apply (*frule HPT-Conf-Target-Source*)
prefer *2*
apply *fast*
apply *fast*
apply (*frule-tac S=Z* **and** *T=V* **in** *OneState-HPT-SA*)
apply *fast+*
apply *simp*
apply (*fast intro*: *ChiPlus-ChiRel ChiRel-ChiPlus-trans ChiPlus-ChiStar*)
apply (*simp add*: *Image-def*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *simp*, *safe*)
back
apply (*frule-tac T=W* **and** *S=S* **in** *Conf-ChiPlus*)
apply *simp*
apply (*frule-tac S=V* **and** *T=W* **in** *Conf-ChiRel*)
apply *fast*
apply (*frule-tac a=V* **in** *ChiRel-HAStates*)
apply (*unfold HAStates-def*)
apply (*simp*, *safe*)
apply (*rename-tac Z*)
apply (*case-tac States Z* ∩ *Source TS* = {})
apply (*simp add:OneState-HPT-Target-Source*)
apply (*subst* (*asm*) *Int-def*, *safe*)
apply (*frule-tac S=V* **in** *HPT-Conf-Target-Source*)
apply *fast+*

apply (*rename-tac P*)
apply (*frule-tac S=P* **and** *T=V* **in** *OneState-HPT-SA*)
apply *fast+*
apply (*frule-tac U=V* **and** *S=W* **and** *T=S* **in** *ChiRel-ChiPlus-trans2*)
apply *fast+*
apply (*fast intro*: *ChiPlus-ChiRel ChiRel-ChiPlus-trans ChiPlus-ChiStar*)
apply (*case-tac T=S*)
apply (*simp add*: *ChiRel-OneAncestor*)+
done

lemma *UniqueSucStates-SAStates-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *A* ∈ *SAs* (*HA ST*);
A ∈ *the* (*CompFun* (*HA ST*) *S*); *T* ∈ *States A*; *U* ∈ *States A*;
T ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*; *T* ≠ *U* ⟧ ⟹
U ∉ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*subst StepConf-def*)
apply (*simp*, *safe*)
apply (*rule UniqueSucState-Conf-Source-StepConf*)
apply *fast+*
apply (*frule-tac U=U* **in** *UniqueSucState-Target-StepConf*)
apply *fast+*
apply (*frule-tac U=U* **in** *UniqueSucState-Target-ChiRel-StepConf*)
apply *fast+*
apply (*rename-tac V W*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*)
apply (*simp*, *safe*)
apply (*frule-tac ?S2.0=T* **in** *StepConf-HAStates* [*THEN subsetD*, *THEN Comp-Fun-ChiRel*])
apply *fast+*
apply (*frule-tac ?S2.0=U* **in** *StepConf-HAStates* [*THEN subsetD*, *THEN Comp-Fun-ChiRel*])
apply *fast+*
apply (*subst* (*asm*) (*2*) *StepConf-def*)
apply (*simp*, *safe*)
apply (*fast intro*: *UniqueSucState-Target-ChiPlus-StepConf*)
apply (*frule-tac U=W* **and** *T=U* **and** *S=T* **in** *OneState-HPT-Target-ChiPlus*)
apply (*fast intro*: *ChiPlus-ChiRel ChiRel-ChiPlus-trans2 OneHAInitState-SAStates*)+
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *simp*, *safe*)
apply (*fast intro*:*OneHAInitState-SAStates*)
done

lemma *UniqueSucStates-Ancestor-StepConf*:
⟦ *TS* ∈ *HPT ST*; *S* ∈ *HAStates* (*HA ST*); *SA* ∈ *the* (*CompFun* (*HA ST*) *S*);
T ∈ *States SA*; *T* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS* ⟧
⟹ *S* ∈ *StepConf* (*HA ST*) (*Conf ST*) *TS*
apply (*rule notnotD*, *rule notI*)
apply (*subst* (*asm*) *StepConf-def*)
apply (*simp*, *safe*)
apply (*frule CompFun-ChiRel*, *fast+*)

apply (*frule Conf-ChiRel*, *fast*)
apply (*frule ChiRel-ChiStar-Source-notmem*, *fast+*)
apply (*subst* (*asm*) *StepConf-def*)
apply *force*
apply (*case-tac States SA ∩ Source TS = {}*)
apply (*simp add:OneState-HPT-Target-Source*)
apply (*subst* (*asm*) *Int-def*)
apply (*simp*, *safe*)
apply (*rename-tac U*)
apply (*frule-tac ?S2.0=U* **in** *CompFun-ChiRel*, *fast+*)
apply (*frule Conf-ChiRel*)
apply (*frule HPT-Source-Conf*, *fast*)
apply (*case-tac S ∈ ChiStar* (*HA ST*) *`` Source TS*)
apply (*subst* (*asm*) *StepConf-def*)
apply *simp*
apply (*frule ChiRel-ChiStar-notmem*)
apply *fast+*
apply (*case-tac U=S*)
apply (*subst* (*asm*) *StepConf-def*)
apply *force*
apply (*subst* (*asm*) *StepConf-def*)
apply *force*
apply (*rename-tac U*)
apply (*case-tac U=S*)
apply (*subst* (*asm*) *StepConf-def*)
apply *force*
apply (*frule CompFun-ChiRel*, *fast+*)
apply (*simp add*: *ChiRel-OneAncestor*)
apply (*rename-tac U V*)
apply (*frule trancl-Int-mem*, *fold ChiPlus-def*, *simp*, *safe*)
apply (*frule tranclD2*)
apply *safe*
apply (*rename-tac W*)
apply (*case-tac W=S*)
apply *simp*
prefer *2*
apply (*frule CompFun-ChiRel*, *fast+*)
apply (*simp only*: *ChiRel-OneAncestor*)
apply (*subst* (*asm*) *StepConf-def*)
apply *safe*
apply (*simp add*: *Image-def*)
apply (*erule-tac x=U* **in** *ballE*)
apply (*case-tac U=S*)
apply *fast*
apply (*simp add*: *rtrancl-eq-or-trancl*)
apply *fast*
apply (*simp add*: *Image-def*)
apply (*rename-tac W*)
apply (*erule-tac x=U* **in** *ballE*)

apply (*simp add*: *rtrancl-eq-or-trancl*)
apply *fast*+
done

lemma *UniqueSucStates-StepConf*:
⟦ *TS* ∈ *HPT ST* ⟧ ⟹
UniqueSucStates (*SAs* (*HA ST*)) (*CompFun* (*HA ST*)) (*StepConf* (*HA ST*) (*Conf ST*) *TS*)
apply (*unfold UniqueSucStates-def*)
apply *auto*
apply (*simp only*: *SucState-StepConf*)
apply (*rule notnotD*, *rule notI*)
apply (*frule UniqueSucStates-SAStates-StepConf*)
apply *fast*
prefer *2*
apply *fast*
apply (*rule HAStates-CompFun-SAs-mem*)
prefer *2*
apply *fast*
apply (*simp only*: *HAStates-def*, *fast*)
apply *fast*+
back
apply (*frule UniqueSucStates-Ancestor-StepConf*)
prefer *2*
apply *fast*
apply (*simp only*:*HAStates-def*, *fast*)
apply *fast*+
done

lemma *Status-Step*:
⟦ *TS* ∈ *HPT ST*; *U* ∈ *ResolveRacing TS* ⟧ ⟹
(*HA ST*, *StepConf* (*HA ST*) (*Conf ST*) *TS*, *StepActEvent TS*, *U* !!! (*Value ST*)) ∈ *status*
apply (*unfold status-def Status-def*)
apply *auto*
apply (*frule StepActEvent-HAEvents*)
apply *blast*
apply (*unfold IsConfSet-def*)
apply (*rule conjI*, *frule StepConf-HAStates*, *unfold HAStates-def*,*assumption*)
apply (*rule conjI*, *rule RootExSem-StepConf*, *assumption*)
apply (*rule UniqueSucStates-StepConf*, *assumption*)
done

8.3 Meta Theorem: Preservation for Statecharts

lemma *IsConfSet-StepConf*:
TS ∈ *HPT ST* ⟹ *IsConfSet* (*SAs* (*HA ST*)) (*CompFun* (*HA ST*))
(*StepConf* (*HA ST*) (*Conf ST*) *TS*)
apply (*unfold IsConfSet-def*)

apply *auto*
apply (*frule StepConf-HAStates*)
apply (*unfold HAStates-def*, *fast*)
apply (*rule RootExSem-StepConf*, *assumption*)
apply (*rule UniqueSucStates-StepConf*, *assumption*)
done

lemma *HA-StepStatus-HPT-ResolveRacing* [*simp*]:
⟦ *TS* ∈ *HPT ST*; *U* ∈ *ResolveRacing TS* ⟧ ⟹
HA (*StepStatus ST TS U*) = *HA ST*
apply (*subst StepStatus-expand* [*THEN sym*])
apply (*subst HA-def*)
apply *auto*
apply (*subst Abs-status-inverse*)
apply *auto*
apply (*rule Status-Step*)
apply *auto*
done

end

9 Kripke Structures and CTL

theory *Kripke*
imports *Main*
begin

definition
Kripke :: ['*s set*,
'*s set*,
('*s* ∗ '*s*) *set*,
('*s* ⇀ '*a set*)]
=> *bool* **where**

Kripke S S0 R L =
(*S0* ⊆ *S* ∧
R <= *S* × *S* ∧
(*Domain R*) = *S* ∧
(*dom L*) = *S*)

lemma *Kripke-EmptySet*:
({@*x*. *True*}, {@*x*. *True*},{(@*x*. *True*, @*x*. *True*)}, *Map.empty*(@*x*. *True* ↦ {@*x*. *True*})) ∈
{(*S*,*S0*,*R*,*L*) | *S S0 R L*. *Kripke S S0 R L*}
by (*unfold Kripke-def Domain-unfold*, *auto*)

definition
kripke =
{(*S*,*S0*,*T*,*L*) |

```
        (S::('s set))
        (S0::('s set))
        (T::(('s * 's) set))
        (L::('s ⇀ ('a  set))).
                    Kripke S S0 T L}

typedef ('s,'a) kripke =
    kripke :: ('s set * 's set * ('s * 's) set * ('s ⇀ 'a set)) set
  unfolding kripke-def
  apply (rule exI)
  apply (rule Kripke-EmptySet)
  done

definition
  Statuses :: ('s,'a) kripke => 's set where
  Statuses = fst o Rep-kripke

definition
   InitStatuses :: ('s,'a) kripke  => 's set where
  InitStatuses == fst o snd o Rep-kripke

definition
   StepRel :: ('s,'a) kripke => ('s * 's) set where
  StepRel == fst o snd o snd o Rep-kripke

definition
   LabelFun :: ('s,'a) kripke => ('s ⇀ 'a set) where
  LabelFun == snd o snd o snd o Rep-kripke

definition
   Paths :: [('s,'a) kripke, 's] =>
            (nat => 's) set where
  Paths M S == { p . S = p (0::nat) ∧ (∀ i. (p i, p (i+1)) ∈ (StepRel M))}

datatype ('s,'a) ctl =  Atom 'a
                      | AND ('s,'a) ctl ('s,'a) ctl
                      | OR  ('s,'a) ctl ('s,'a) ctl
                      | IMPLIES ('s,'a) ctl ('s,'a) ctl
                      | CAX ('s,'a) ctl
                      | AF  ('s,'a) ctl
                      | AG  ('s,'a) ctl
                      | AU  ('s,'a) ctl ('s,'a) ctl
                      | AR  ('s,'a) ctl ('s,'a) ctl


primrec
   eval-ctl :: [('s,'a) kripke, 's, ('s,'a) ctl] => bool  (-,- |=c= - [92,91,90]90)
   where
    (M,S |=c= (Atom P))        = (P ∈ the ((LabelFun M) S))
```

| $(M,S \models c= (AND\ F1\ F2)) = ((M,S \models c= F1) \wedge (M,S \models c= F2))$
| $(M,S \models c= (OR\ F1\ F2)) = ((M,S \models c= F1) \vee (M,S \models c= F2))$
| $(M,S \models c= (IMPLIES\ F1\ F2)) = ((M,S \models c= F1) \longrightarrow (M,S \models c= F2))$
| $(M,S \models c= (CAX\ F)) = (\forall\ T.\ (S,T) \in (StepRel\ M) \longrightarrow (M,T \models c= F))$
| $(M,S \models c= (AF\ F)) = (\forall\ P \in Paths\ M\ S.\ \exists\ i.\ (M,(P\ i) \models c= F))$
| $(M,S \models c= (AG\ F)) = (\forall\ P \in Paths\ M\ S.\ \forall\ i.\ (M,(P\ i) \models c= F))$
| $(M,S \models c= (AU\ F\ G)) = (\forall\ P \in Paths\ M\ S.\ \exists\ i.\ (M,(P\ i) \models c= G) \wedge (\forall\ j.\ j < i \longrightarrow (M,(P\ j) \models c= F)))$
| $(M,S \models c= (AR\ F\ G)) = (\forall\ P \in Paths\ M\ S.\ \forall\ i.\ (M,(P\ i) \models c= G) \vee (\exists\ j.\ j < i \wedge (M,(P\ j) \models c= F)))$

end

10 Kripke Structures as Hierarchical Automata

theory *HAKripke*
imports *HASem Kripke*
begin

type-synonym $('s,'e,'d)hakripke = (('s,'e,'d)status,('s,'e,'d)atomar)kripke$
type-synonym $('s,'e,'d)hactl = (('s,'e,'d)status,('s,'e,'d)atomar)ctl$

definition
LabelFunSem :: $('s,'e,'d)hierauto$
$\Rightarrow (('s,'e,'d)status \rightharpoonup (((('s,'e,'d)\ atomar)\ set))$ **where**
LabelFunSem a = $(\lambda\ ST.$
(*if* $(HA\ ST = a)$ *then*
(*let*
In-preds = $(\lambda\ s.\ (IN\ s))\ `\ (Conf\ ST)$;
En-preds = $(\lambda\ e.\ (EN\ e))\ `\ (Events\ ST)$;
Val-preds = $\{\ x\ .\ (\exists\ P.\ (x = (VAL\ P)) \wedge P\ (Value\ ST))\ \}$
in
Some $(In\text{-}preds \cup En\text{-}preds \cup Val\text{-}preds \cup \{atomar.TRUE\})$)
else
None))

definition
HA2Kripke :: $('s,'e,'d)hierauto \Rightarrow ('s,'e,'d)hakripke$ **where**
HA2Kripke a =
Abs-kripke $(\{ST.\ HA\ ST = a\},$
$\{InitStatus\ a\},$
StepRelSem a,
LabelFunSem a)

definition
eval-ctl-HA :: $[('s,'e,'d)hierauto,\ ('s,'e,'d)hactl]$

```
                => bool  (- |=H= - [92,91]90) where

  eval-ctl-HA a f = ((HA2Kripke a), (InitStatus a) |=c= f)

lemma Kripke-HA [simp]:
  Kripke {ST. HA ST = a} {InitStatus a} (StepRelSem a) (LabelFunSem a)
apply (unfold Kripke-def)
apply auto
apply (unfold StepRelSem-def)
apply auto
apply (unfold LabelFunSem-def Let-def If-def dom-def)
apply auto
prefer 2
apply (rename-tac ST S)
apply (case-tac HA ST = a)
apply auto
apply (rename-tac ST)
apply (case-tac HPT ST = {})
apply auto
apply (rename-tac TSS)
apply (erule-tac x=StepStatus ST TSS (@ u. u : ResolveRacing TSS) in allE)
apply (erule-tac x=TSS in ballE)
apply auto
done

lemma LabelFun-LabelFunSem [simp]:
  (LabelFun (HA2Kripke a)) = (LabelFunSem a)
apply (unfold HA2Kripke-def LabelFun-def)
apply auto
apply (subst Abs-kripke-inverse)
apply auto
apply (unfold kripke-def)
apply auto
done

lemma InitStatuses-InitStatus [simp]:
   (InitStatuses (HA2Kripke a)) = {(InitStatus a)}
apply (unfold HA2Kripke-def InitStatuses-def)
apply simp
apply (subst Abs-kripke-inverse)
apply (unfold kripke-def)
apply auto
done

lemma Statuses-StatusesOfHA [simp]:
   (Statuses (HA2Kripke a)) = {ST. HA ST = a}
apply (unfold HA2Kripke-def Statuses-def)
apply simp
apply (subst Abs-kripke-inverse)
```

apply (*unfold kripke-def*)
apply *auto*
done

lemma *StepRel-StepRelSem* [*simp*]:
(*StepRel* (*HA2Kripke a*)) = (*StepRelSem a*)
apply (*unfold HA2Kripke-def StepRel-def*)
apply *simp*
apply (*subst Abs-kripke-inverse*)
apply (*unfold kripke-def*)
apply *auto*
done

lemma *TRUE-LabelFunSem* [*simp*]:
atomar.TRUE ∈ *the* (*LabelFunSem* (*HA ST*) *ST*)
apply (*unfold LabelFunSem-def Let-def*)
apply *auto*
done

lemma *FALSE-LabelFunSem* [*simp*]:
atomar.FALSE ∉ *the* (*LabelFunSem* (*HA ST*) *ST*)
apply (*unfold LabelFunSem-def Let-def*)
apply *auto*
done

lemma *Conf-LabelFunSem* [*simp*]:
((*IN S*) ∈ *the* (*LabelFunSem* (*HA ST*) *ST*)) = (*S* ∈ (*Conf ST*))
apply (*unfold LabelFunSem-def Let-def*)
apply *auto*
done

lemma *Events-LabelFunSem* [*simp*]:
((*EN S*) ∈ *the* (*LabelFunSem* (*HA ST*) *ST*)) = (*S* ∈ (*Events ST*))
apply (*unfold LabelFunSem-def Let-def*)
apply *auto*
done

lemma *Value-LabelFunSem* [*simp*]:
((*VAL P*) ∈ *the* (*LabelFunSem* (*HA ST*) *ST*)) = (*P* (*Value ST*))
apply (*unfold LabelFunSem-def Let-def*)
apply *auto*
done

lemma *AtomTRUE-EvalCTLHA* [*simp*]:
a |=*H*= (*Atom* (*atomar.TRUE*))
apply (*unfold eval-ctl-HA-def*)
apply *auto*
apply (*subst HA-InitStatus* [*THEN sym*])
apply (*rule TRUE-LabelFunSem*)

done

lemma *AtomFalse-EvalCTLHA* [*simp*]:
  ¬ *a* |=*H*= (*Atom* (*atomar.FALSE*))
apply (*unfold eval-ctl-HA-def*)
apply *auto*
apply (*subst* (*asm*) *HA-InitStatus* [*THEN sym*])
apply (*simp only*: *FALSE-LabelFunSem*)
done

lemma *Events-InitStatus-EvalCTLHA* [*simp*]:
  (*a* |=*H*= (*Atom* (*EN S*))) = (*S* ∈ (*Events* (*InitStatus a*)))
apply (*unfold eval-ctl-HA-def*)
apply *simp*
apply (*subst HA-InitStatus* [*THEN sym*])
apply (*rule Events-LabelFunSem*)
done

lemma *Conf-InitStatus-EvalCTLHA* [*simp*]:
  (*a* |=*H*= (*Atom* (*IN S*))) = (*S* ∈ (*Conf* (*InitStatus a*)))
apply (*unfold eval-ctl-HA-def*)
apply *simp*
apply (*subst HA-InitStatus* [*THEN sym*])
apply (*subst Conf-LabelFunSem*)
apply *simp*
done

lemma *HAInitValue-EvalCTLHA* [*simp*]:
  (*a* |=*H*= (*Atom* (*VAL P*))) = (*P* (*HAInitValue a*))
apply (*unfold eval-ctl-HA-def*)
apply *simp*
apply (*subst HA-InitStatus* [*THEN sym*])
apply (*subst Value-LabelFunSem*)
apply *auto*
done

end

11 Constructing Hierarchical Automata

theory *HAOps*
imports *HA*
begin

11.1 Constructing a Composition Function for a PseudoHA

definition
  EmptyMap :: *'s set* => (*'s* ⇀ ((*'s*,*'e*,*'d*)*seqauto*) *set*) **where**
  EmptyMap S = (λ *a* . *if a* ∈ *S then Some* {} *else None*)

lemma *EmptyMap-dom* [*simp*]:
dom (*EmptyMap S*) = *S*
by (*unfold dom-def EmptyMap-def*,*auto*)

lemma *EmptyMap-ran* [*simp*]:
S ≠ {} ⟹ *ran* (*EmptyMap S*) = {{}}
by (*unfold ran-def EmptyMap-def*, *auto*)

lemma *EmptyMap-the* [*simp*]:
x ∈ *S* ⟹ *the* ((*EmptyMap S*) *x*) = {}
by (*unfold ran-def EmptyMap-def*, *auto*)

lemma *EmptyMap-ran-override*:
⟦ *S* ≠ {}; (*S* ∩ (*dom G*)) = {} ⟧ ⟹
ran (*G* ++ *EmptyMap S*) = *insert* {} (*ran G*)
apply (*subst ran-override*)
apply (*simp add*: *Int-commute*)
apply *simp*
done

lemma *EmptyMap-Union-ran-override*:
⟦ *S* ≠ {};
S ∩ *dom G* = {} ⟧ ⟹
(*Union* (*ran* (*G* ++ (*EmptyMap S*)))) = (*Union* (*ran G*))
apply (*subst EmptyMap-ran-override*)
apply *auto*
done

lemma *EmptyMap-Union-ran-override2*:
⟦ *S* ≠ {}; *S* ∩ *dom G1* = {};
dom G1 ∩ *dom G2* = {} ⟧ ⟹
⋃ (*ran* (*G1* ++ *EmptyMap S* ++ *G2*)) = (⋃ (*ran G1* ∪ *ran G2*))
apply (*unfold Union-eq UNION-eq EmptyMap-def Int-def ran-def*)
apply (*simp add*: *map-add-Some-iff*)
apply (*unfold dom-def*)
apply *simp*
apply (*rule equalityI*)
apply (*rule subsetI*)
apply *simp*
apply *fast*
apply (*rule subsetI*)
apply (*rename-tac t*)
apply *simp*
apply (*erule bexE*)
apply (*rename-tac U*)
apply *simp*
apply (*erule disjE*)
apply (*erule exE*)

apply (*rename-tac v*)
apply (*rule-tac x=U* **in** *exI*)
apply *simp*
apply (*rule-tac x=v* **in** *exI*)
apply *auto*
done

lemma *EmptyMap-Root* [*simp*]:
Root {*SA*} (*EmptyMap* (*States SA*)) = *SA*
by (*unfold Root-def*, *auto*)

lemma *EmptyMap-RootEx* [*simp*]:
RootEx {*SA*} (*EmptyMap* (*States SA*))
by (*unfold RootEx-def*, *auto*)

lemma *EmptyMap-OneAncestor* [*simp*]:
OneAncestor {*SA*} (*EmptyMap* (*States SA*))
by (*unfold OneAncestor-def*, *auto*)

lemma *EmptyMap-NoCycles* [*simp*]:
NoCycles {*SA*} (*EmptyMap* (*States SA*))
by (*unfold NoCycles-def EmptyMap-def* , *auto*)

lemma *EmptyMap-IsCompFun* [*simp*]:
IsCompFun {*SA*} (*EmptyMap* (*States SA*))
by (*unfold IsCompFun-def*, *auto*)

lemma *EmptyMap-hierauto* [*simp*]:
(*D*,{*SA*}, *SAEvents SA*, *EmptyMap* (*States SA*)) ∈ *hierauto*
by (*unfold hierauto-def HierAuto-def*, *auto*)

11.2 Extending a Composition Function by a SA

definition
FAddSA :: [('*s* ⇀ (('*s*,'*e*,'*d*)*seqauto*) *set*), '*s* ∗ ('*s*,'*e*,'*d*)*seqauto*]
=> ('*s* ⇀ (('*s*,'*e*,'*d*)*seqauto*) *set*)
((- [*f*+]/ -) [*10*,*11*]*10*) **where**
FAddSA G SSA = (*let* (*S*,*SA*) = *SSA*
in
(*if* ((*S* ∈ *dom G*) ∧ (*S* ∉ *States SA*)) *then*
(*G* ++ (*Map.empty*(*S* ↦ (*insert SA* (*the* (*G S*)))))
++ *EmptyMap* (*States SA*))
else G))

lemma *FAddSA-dom* [*simp*]:
(*S* ∉ (*dom* (*A*::('*a* => ('*a*,'*c*,'*d*)*seqauto set option*)))) ⟹
((*A* [*f*+] (*S*,(*SA*::('*a*,'*c*,'*d*)*seqauto*))) = *A*)
by (*unfold FAddSA-def Let-def*, *auto*)

lemma *FAddSA-States* [*simp*]:
$(S \in (States\ (SA::('a,'c,'d)seqauto)))\ \Longrightarrow$
$(((A::('a => ('a,'c,'d)seqauto\ set\ option))\ [f+]\ (S,SA)) = A)$
by (*unfold FAddSA-def Let-def*, *auto*)

lemma *FAddSA-dom-insert* [*simp*]:
$[\![\ S \in (dom\ A);\ S \notin\ States\ SA\ ]\!] \Longrightarrow$
$(((A\ [f+]\ (S,SA))\ S) = Some\ (insert\ SA\ (the\ (A\ S))))$
by (*unfold FAddSA-def Let-def restrict-def*, *auto*)

lemma *FAddSA-States-neq* [*simp*]:
$[\![\ S' \notin States\ (SA::('a,'c,'d)seqauto);\ S \neq\ S'\ ]\!] \Longrightarrow$
$((((A::('a => ('a,'c,'d)seqauto\ set\ option))\ [f+]\ (S,SA))\ S') = (A\ S'))$
apply (*case-tac* $S \in dom\ A$)
apply (*case-tac* $S \in States\ SA$)
apply *auto*
apply (*case-tac* $S' \in dom\ A$)
apply (*unfold FAddSA-def Let-def*)
apply *auto*
apply (*simp add*: *dom-None*)
done

lemma *FAddSA-dom-emptyset* [*simp*]:
$[\![\ S \in (dom\ A);\ S \notin States\ SA;\ S' \in States\ (SA::('a,'c,'d)seqauto)\ ]\!] \Longrightarrow$
$((((A::('a => ('a,'c,'d)seqauto\ set\ option)))\ [f+]\ (S,SA))\ S') = (Some\ \{\})$
apply (*unfold FAddSA-def Let-def*)
apply *auto*
apply (*unfold EmptyMap-def*)
apply *auto*
done

lemma *FAddSA-dom-dom-States* [*simp*]:
$[\![\ S \in (dom\ F);\ S \notin States\ SA\ ]\!] \Longrightarrow$
$(dom\ ((F::('a \rightharpoonup (('a,'b,'d)seqauto)\ set))\ [f+]\ (S,\ SA))) =$
$((dom\ F) \cup (States\ (SA::('a,'b,'d)seqauto)))$
by (*unfold FAddSA-def Let-def*, *auto*)

lemma *FAddSA-dom-dom* [*simp*]:
$S \notin (dom\ F) \Longrightarrow$
$(dom\ ((F::('a \rightharpoonup (('a,'b,'d)seqauto)\ set))\ [f+]$
$(S,(SA::('a,'b,'d)seqauto)))) = (dom\ F)$
by (*unfold FAddSA-def Let-def*, *auto*)

lemma *FAddSA-States-dom* [*simp*]:
$S \in (States\ SA) \Longrightarrow$
$(dom\ ((F::('a \rightharpoonup (('a,'b,'d)seqauto)\ set))\ [f+]$
$(S,(SA::('a,'b,'d)seqauto)))) = (dom\ F)$
by (*unfold FAddSA-def Let-def*, *auto*)

lemma *FAddSA-dom-insert-dom-disjunct* [*simp*]:
⟦ *S* ∈ *dom G*; *States SA* ∩ *dom G* = {} ⟧ ⟹ ((*G* [*f*+] (*S*,*SA*)) *S*) = *Some* (*insert SA* (*the* (*G S*)))
apply (*rule FAddSA-dom-insert*)
apply *auto*
done

lemma *FAddSA-Union-ran*:
⟦ *S* ∈ *dom G*; (*States SA*) ∩ (*dom G*) = {} ⟧ ⟹
(⋃ (*ran* (*G* [*f*+] (*S*,*SA*)))) = (*insert SA* (⋃ (*ran G*)))
apply (*unfold FAddSA-def Let-def*)
apply *simp*
apply (*rule conjI*)
prefer *2*
apply (*rule impI*)
apply (*unfold Int-def*)
apply *simp*
apply (*fold Int-def*)
apply (*rule impI*)
apply (*subst EmptyMap-Union-ran-override*)
apply *auto*
done

lemma *FAddSA-Union-ran2*:
⟦ *S* ∈ *dom G1*; (*States SA*) ∩ (*dom G1*) = {}; (*dom G1* ∩ *dom G2*) = {} ⟧ ⟹
(⋃ (*ran* ((*G1* [*f*+] (*S*,*SA*)) ++ *G2*))) = (*insert SA* (⋃ ((*ran G1*) ∪ (*ran G2*))))
apply (*unfold FAddSA-def Let-def*)
apply (*simp* (*no-asm-simp*))
apply (*rule conjI*)
apply (*rule impI*)
apply (*subst EmptyMap-Union-ran-override2*)
apply *simp*
apply *simp*
apply *simp*
apply *fast*
apply (*subst Union-Un-distrib*)
apply (*subst Union-ran-override2*)
apply *auto*
done

lemma *FAddSA-ran*:
⟦ ∀ *T* ∈ *dom G* . *T* ≠ *S* ⟶ (*the* (*G T*) ∩ *the* (*G S*)) = {};
S ∈ *dom G*; (*States SA*) ∩ (*dom G*) = {} ⟧ ⟹
ran (*G* [*f*+] (*S*,*SA*)) = *insert* {} (*insert* (*insert SA* (*the* (*G S*))) (*ran G* − {*the* (*G S*)}))
apply (*unfold FAddSA-def Let-def*)
apply *simp*
apply (*rule conjI*)
apply (*rule impI*)+

prefer *2*
apply *fast*
apply (*simp add*: *EmptyMap-ran-override*)
apply (*unfold ran-def*)
apply *auto*
apply (*rename-tac Y X a xa xb*)
apply (*erule-tac x=a* **in** *allE*)
apply *simp*
apply (*erule-tac x=a* **in** *allE*)
apply *simp*
done

lemma *FAddSA-RootEx-def*:
⟦ $S \in dom\ G$; $(States\ SA) \cap (dom\ G) = \{\}$ ⟧ ⟹
$RootEx\ F\ (G\ [f+]\ (S,SA)) = (\exists!\ A\ .\ A \in F \wedge A \notin insert\ SA\ (\bigcup\ (ran\ G)))$
apply (*unfold RootEx-def*)
apply (*simp only*: *FAddSA-Union-ran Int-commute*)
done

lemma *FAddSA-RootEx*:
⟦ $\bigcup\ (ran\ G) = F - \{Root\ F\ G\}$;
$dom\ G = \bigcup(States\ `\ F)$;
$(dom\ G \cap States\ SA) = \{\}$; $S \in dom\ G$;
$RootEx\ F\ G$ ⟧ ⟹ $RootEx\ (insert\ SA\ F)\ (G\ [f+]\ (S,SA))$
apply (*simp add*: *FAddSA-RootEx-def Int-commute cong*: *rev-conj-cong*)
apply (*auto cong*: *conj-cong*)
done

lemma *FAddSA-Root-def*:
⟦ $S \in dom\ G$; $(States\ SA) \cap (dom\ G) = \{\}$ ⟧ ⟹
$(Root\ F\ (G\ [f+]\ (S,SA)) = (@\ A\ .\ A \in F \wedge A \notin insert\ SA\ (\bigcup\ (ran\ G))))$
apply (*unfold Root-def*)
apply (*simp only*: *FAddSA-Union-ran Int-commute*)
done

lemma *FAddSA-RootEx-Root*:
⟦ $Union\ (ran\ G) = F - \{Root\ F\ G\}$;
$\bigcup(States\ `\ F) = dom\ G$;
$(dom\ G \cap States\ SA) = \{\}$; $S \in dom\ G$;
$RootEx\ F\ G$ ⟧ ⟹ $(Root\ (insert\ SA\ F)\ (G\ [f+]\ (S,SA))) = (Root\ F\ G)$
apply (*simp add*: *FAddSA-Root-def Int-commute cong*: *rev-conj-cong*)
apply (*simp cong*:*conj-cong*)
done

lemma *FAddSA-OneAncestor*:
⟦ $\bigcup\ (ran\ G) = F - \{Root\ F\ G\}$;
$(dom\ G \cap States\ SA) = \{\}$; $S \in dom\ G$;
$\bigcup(States\ `\ F) = dom\ G$; $RootEx\ F\ G$;
$OneAncestor\ F\ G$ ⟧ ⟹ $OneAncestor\ (insert\ SA\ F)\ (G\ [f+]\ (S,SA))$

apply (*subst OneAncestor-def*)
apply *simp*
apply (*rule ballI*)
apply (*rename-tac SAA*)
apply (*case-tac SA = SAA*)
apply (*rule-tac a=S* **in** *ex1I*)
apply (*rule conjI*)
apply *simp*
apply *fast*
apply (*subst FAddSA-dom-insert*)
apply *simp*
apply (*simp add:Int-def*)
apply *simp*
apply (*rename-tac T*)
apply (*erule conjE bexE exE disjE*)+
apply (*rename-tac SAAA*)
apply *simp*
apply (*erule conjE*)
apply (*subst not-not* [*THEN sym*])
apply (*rule notI*)
apply (*case-tac T ∈ States SAA*)
apply *blast*
apply (*drule-tac A=G* **and** *S=S* **and** *SA=SAA* **in** *FAddSA-States-neq*)
apply *fast*
apply *simp*
apply (*case-tac SAA ∉ Union* (*ran G*))
apply (*frule ran-dom-the*)
prefer *2*
apply *fast*
apply *blast*
apply *simp*
apply (*erule conjE*)
apply (*simp add*: *States-Int-not-mem*)
apply (*unfold OneAncestor-def*)
apply (*drule-tac G=G* **and** *S=S* **and** *SA=SA* **in** *FAddSA-RootEx-Root*)
apply *simp*
apply *simp*
apply *simp*
apply *simp*
apply (*erule-tac x=SAA* **in** *ballE*)
prefer *2*
apply *simp*
apply *simp*
apply (*erule conjE bexE ex1E exE disjE*)+
apply (*rename-tac T SAAA*)
apply (*rule-tac a=T* **in** *ex1I*)
apply (*rule conjI*)
apply *fast*
apply (*case-tac T = S*)

apply *simp*
apply (*case-tac* $S \notin States\ SA$)
apply *simp*
apply *simp*
apply (*subst FAddSA-States-neq*)
apply *blast*
apply (*rule not-sym*)
apply *simp*
apply *simp*
apply (*rename-tac U*)
apply *simp*
apply (*erule conjE bexE*)+
apply (*rename-tac SAAAA*)
apply *simp*
apply (*erule conjE disjE*)+
apply (*frule FAddSA-dom-emptyset*)
prefer *2*
apply *fast*
back
back
apply *simp*
apply *blast*
apply *simp*
apply (*erule-tac x=U* **in** *allE*)
apply (*erule impE*)
prefer *2*
apply *simp*
apply (*rule conjI*)
apply *fast*
apply (*case-tac* $S \neq U$)
apply (*subgoal-tac* $U \notin States\ SA$)
apply (*drule-tac A=G* **in** *FAddSA-States-neq*)
apply *fast*
apply *simp*
apply *blast*
apply (*drule-tac A=G* **and** *SA=SA* **in** *FAddSA-dom-insert*)
apply *simp*
apply *blast*
apply *auto*
done

lemma *FAddSA-NoCycles*:
$[\![(States\ SA \cap dom\ G) = \{\};\ S \in dom\ G;$
$dom\ G = \bigcup (States\ `\ F);\ NoCycles\ F\ G\ ]\!] \Longrightarrow$
$NoCycles\ \ (insert\ SA\ F)\ (G\ [f+]\ (S,SA))$
apply (*unfold NoCycles-def*)
apply (*rule ballI impI*)+
apply (*rename-tac SAA*)
apply (*case-tac* $\exists\ s \in SAA.\ s \in States\ SA$)

apply *simp*
apply (*erule bexE*)+
apply (*rename-tac SAAA T*)
apply (*rule-tac x=T* **in** *bexI*)
apply *simp*
apply (*subst FAddSA-dom-emptyset*)
apply *simp*
apply *fast*
apply *blast*
apply *simp*
apply *simp*
apply *simp*
apply *simp*
apply (*erule-tac x=SAA* **in** *ballE*)
prefer *2*
apply *simp*
apply *auto*[*1*]
apply (*unfold UNION-eq Pow-def*)
apply *simp*
apply (*case-tac SAA = {}*)
apply *fast*
apply *simp*
apply (*erule bexE*)+
apply (*rename-tac SAAA T*)
apply (*rule-tac x=T* **in** *bexI*)
prefer *2*
apply *simp*
apply (*case-tac T=S*)
apply *simp*
apply (*subst FAddSA-dom-insert*)
apply *auto*
done

lemma *FAddSA-IsCompFun*:
⟦ (*States SA* ∩ (⋃(*States* ‘ *F*))) = {};
S ∈ (⋃(*States* ‘ *F*));
IsCompFun F G ⟧ ⟹ *IsCompFun* (*insert SA F*) (*G* [*f*+] (*S*,*SA*))
apply (*unfold IsCompFun-def*)
apply (*erule conjE*)+
apply (*simp add*: *Int-commute FAddSA-RootEx-Root FAddSA-RootEx FAddSA-OneAncestor FAddSA-NoCycles*)
apply (*rule conjI*)
apply (*subst FAddSA-dom-dom-States*)
apply *simp*
apply *blast*
apply (*simp add*: *Un-commute*)
apply (*simp add*: *FAddSA-Union-ran*)
apply (*case-tac SA = Root F G*)
prefer *2*

apply *blast*
apply (*subgoal-tac States* (*Root F G*) $\subseteq$ $\bigcup$(*States* ‘ *F*))
apply *simp*
apply (*frule subset-lemma*)
apply *auto*
done

lemma *FAddSA-HierAuto*:
⟦ (*States SA* ∩ ($\bigcup$(*States* ‘ *F*))) = {};
S ∈ ($\bigcup$(*States* ‘ *F*));
HierAuto D F E G ⟧ ⟹ *HierAuto D* (*insert SA F*) (*E* ∪ *SAEvents SA*) (*G* [*f*+] (*S*,*SA*))
apply (*unfold HierAuto-def*)
apply *auto*
apply (*simp add*: *MutuallyDistinct-Insert*)
apply (*rule FAddSA-IsCompFun*)
apply *auto*
done

lemma *FAddSA-HierAuto-insert* [*simp*]:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
HierAuto (*HAInitValue HA*)
(*insert SA* (*SAs HA*))
(*HAEvents HA* ∪ *SAEvents SA*)
(*CompFun HA* [*f*+] (*S*,*SA*))
apply (*unfold HAStates-def*)
apply (*rule FAddSA-HierAuto*)
apply *auto*
done

11.3 Constructing a PseudoHA

definition
PseudoHA :: [('*s*,'*e*,'*d*)*seqauto*,'*d data*] => ('*s*,'*e*,'*d*)*hierauto* **where**
PseudoHA SA D = *Abs-hierauto*(*D*,{*SA*}, *SAEvents SA* ,*EmptyMap* (*States SA*))

lemma *PseudoHA-SAs* [*simp*]:
SAs (*PseudoHA SA D*) = {*SA*}
by (*unfold PseudoHA-def SAs-def*, *simp add*: *Abs-hierauto-inverse*)

lemma *PseudoHA-Events* [*simp*]:
HAEvents (*PseudoHA SA D*) = *SAEvents SA*
by (*unfold PseudoHA-def HAEvents-def*, *simp add*: *Abs-hierauto-inverse*)

lemma *PseudoHA-CompFun* [*simp*]:
CompFun (*PseudoHA SA D*) = *EmptyMap* (*States SA*)
by (*unfold PseudoHA-def CompFun-def*, *simp add*: *Abs-hierauto-inverse*)

lemma *PseudoHA-HAStates* [*simp*]:
HAStates (*PseudoHA SA D*) = (*States SA*)
by (*unfold HAStates-def*, *auto*)

lemma *PseudoHA-HAInitValue* [*simp*]:
(*HAInitValue* (*PseudoHA SA D*)) = *D*
by (*unfold PseudoHA-def Let-def HAInitValue-def*, *simp add*: *Abs-hierauto-inverse*)

lemma *PseudoHA-CompFun-the* [*simp*]:
S ∈ *States A* ⟹ (*the* (*CompFun* (*PseudoHA A D*) *S*)) = {}
by *simp*

lemma *PseudoHA-CompFun-ran* [*simp*]:
(*ran* (*CompFun* (*PseudoHA SA D*))) = {{}}
by *auto*

lemma *PseudoHA-HARoot* [*simp*]:
(*HARoot* (*PseudoHA SA D*)) = *SA*
by (*unfold HARoot-def*, *auto*)

lemma *PseudoHA-HAInitState* [*simp*]:
HAInitState (*PseudoHA A D*) = *InitState A*
apply (*unfold HAInitState-def*)
apply *simp*
done

lemma *PseudoHA-HAInitStates* [*simp*]:
HAInitStates (*PseudoHA A D*) = {*InitState A*}
apply (*unfold HAInitStates-def*)
apply *simp*
done

lemma *PseudoHA-Chi* [*simp*]:
S ∈ *States A* ⟹ *Chi* (*PseudoHA A D*) *S* = {}
apply (*unfold Chi-def restrict-def*)
apply *auto*
done

lemma *PseudoHA-ChiRel* [*simp*]:
ChiRel (*PseudoHA A D*) = {}
apply (*unfold ChiRel-def*)
apply *simp*
done

lemma *PseudoHA-InitConf* [*simp*]:
InitConf (*PseudoHA A D*) = {*InitState A*}
apply (*unfold InitConf-def*)
apply *simp*
done

11.4 Extending a HA by a SA (*AddSA*)

definition
AddSA :: [('s,'e,'d)*hierauto*, 's * ('s,'e,'d)*seqauto*]
=> ('s,'e,'d)*hierauto*
((- [++]/ -) [*10*,*11*]*10*) **where**
AddSA HA SSA = (*let* (*S*,*SA*) = *SSA*;
DNew = *HAInitValue HA*;
FNew = *insert SA* (*SAs HA*);
ENew = *HAEvents HA* ∪ *SAEvents SA*;
GNew = *CompFun HA* [*f*+] (*S*,*SA*)
in
Abs-hierauto(*DNew*,*FNew*,*ENew*,*GNew*))

definition
AddHA :: [('s,'e,'d)*hierauto*, 's * ('s,'e,'d)*hierauto*]
=> ('s,'e,'d)*hierauto*
((- [**]/ -) [*10*,*11*]*10*) **where**
AddHA HA1 SHA =
(*let* (*S*,*HA2*) = *SHA*;
(*D1*,*F1*,*E1*,*G1*) = *Rep-hierauto* (*HA1* [++] (*S*,*HARoot HA2*));
(*D2*,*F2*,*E2*,*G2*) = *Rep-hierauto HA2*;
FNew = *F1* ∪ *F2*;
ENew = *E1* ∪ *E2*;
GNew = *G1* ++ *G2*
in
Abs-hierauto(*D1*,*FNew*,*ENew*,*GNew*))

lemma *AddSA-SAs*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹ (*SAs* (*HA* [++] (*S*,*SA*))) = *insert SA* (*SAs HA*)
apply (*unfold Let-def AddSA-def*)
apply (*subst SAs-def*)
apply (*simp add*: *hierauto-def Abs-hierauto-inverse*)
done

lemma *AddSA-Events*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
HAEvents (*HA* [++] (*S*,*SA*)) = (*HAEvents HA*) ∪ (*SAEvents SA*)
apply (*unfold Let-def AddSA-def*)
apply (*subst HAEvents-def*)
apply (*simp add*: *hierauto-def Abs-hierauto-inverse*)
done

lemma *AddSA-CompFun*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
CompFun (*HA* [++] (*S*,*SA*)) = (*CompFun HA* [*f*+] (*S*,*SA*))
apply (*unfold Let-def AddSA-def*)

apply (*subst CompFun-def*)
apply (*simp add*: *hierauto-def Abs-hierauto-inverse*)
done

lemma *AddSA-HAStates*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
HAStates (*HA* [++] (*S*,*SA*)) = (*HAStates HA*) ∪ (*States SA*)
apply (*unfold HAStates-def*)
apply (*subst AddSA-SAs*)
apply (*unfold HAStates-def*)
apply *auto*
done

lemma *AddSA-HAInitValue*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
(*HAInitValue* (*HA* [++] (*S*,*SA*))) = (*HAInitValue HA*)
apply (*unfold Let-def AddSA-def*)
apply (*subst HAInitValue-def*)
apply (*simp add*: *hierauto-def Abs-hierauto-inverse*)
done

lemma *AddSA-HARoot*:
⟦ (*States SA* ∩ *HAStates HA*) = {};
S ∈ *HAStates HA* ⟧ ⟹
(*HARoot* (*HA* [++] (*S*,*SA*))) = (*HARoot HA*)
apply (*unfold HARoot-def*)
apply (*simp add*: *AddSA-CompFun AddSA-SAs*)
apply (*subst FAddSA-RootEx-Root*)
apply *auto*
apply (*simp only*: *HAStates-SA-mem*)
apply (*unfold HAStates-def*)
apply *fast*
done

lemma *AddSA-CompFun-the*:
⟦ (*States SA* ∩ *HAStates A*) = {};
S ∈ *HAStates A* ⟧ ⟹
(*the* ((*CompFun* (*A* [++] (*S*,*SA*))) *S*)) = *insert SA* (*the* ((*CompFun A*) *S*))
by (*simp add*: *AddSA-CompFun*)

lemma *AddSA-CompFun-the2*:
⟦ *S'* ∈ *States* (*SA*::('*a*,'*c*,'*d*)*seqauto*);
(*States SA* ∩ *HAStates A*) = {};
S ∈ *HAStates A* ⟧ ⟹
the ((*CompFun* (*A* [++] (*S*,*SA*))) *S'*) = {}
apply (*simp add*: *AddSA-CompFun*)
apply (*subst FAddSA-dom-emptyset*)

apply *auto*
done

lemma *AddSA-CompFun-the3*:
⟦ $S' \notin$ *States* (*SA*::('*a*,'*c*,'*d*)*seqauto*);
$S \neq S'$;
(*States SA* ∩ *HAStates A*) = {};
$S \in$ *HAStates A* ⟧ ⟹
(*the* ((*CompFun* (*A* [++] (*S*,*SA*))) *S'*)) = (*the* ((*CompFun A*) *S'*))
by (*simp add*: *AddSA-CompFun*)

lemma *AddSA-CompFun-ran*:
⟦ (*States SA* ∩ *HAStates A*) = {};
S ∈ *HAStates A* ⟧ ⟹
ran (*CompFun* (*A* [++] (*S*,*SA*))) =
insert {} (*insert* (*insert SA* (*the* ((*CompFun A*) *S*))) (*ran* (*CompFun A*) −
{*the* ((*CompFun A*) *S*)}))
apply (*simp add*: *AddSA-CompFun*)
apply (*subst FAddSA-ran*)
apply *auto*
apply (*fast dest*: *CompFun-Int-disjoint*)
done

lemma *AddSA-CompFun-ran2*:
⟦ (*States SA1* ∩ *HAStates A*) = {};
(*States SA2* ∩ (*HAStates A* ∪ *States SA1*)) = {};
S ∈ *HAStates A*;
T ∈ *States SA1* ⟧ ⟹
ran (*CompFun* ((*A* [++] (*S*,*SA1*)) [++] (*T*,*SA2*))) =
insert {} (*insert* {*SA2*} (*ran* (*CompFun* (*A* [++] (*S*,*SA1*))))))
apply (*simp add*: *AddSA-HAStates AddSA-CompFun*)
apply (*subst FAddSA-ran*)
apply (*rule ballI*)
apply (*rule impI*)
apply (*subst AddSA-CompFun* [*THEN sym*])
apply *simp*
apply *simp*
apply (*subst AddSA-CompFun* [*THEN sym*])
apply *simp*
apply *simp*
apply (*rule CompFun-Int-disjoint*)
apply *simp*
apply (*simp add*: *AddSA-HAStates*)
apply (*simp add*: *AddSA-HAStates*)
apply (*case-tac S* ∈ *States SA1*)
apply *simp*
apply (*simp only*: *dom-CompFun* [*THEN sym*])
apply (*frule FAddSA-dom-dom-States*)
apply *fast*

apply *simp*
apply (*case-tac S ∈ States SA1*)
apply *simp*
apply *fast*
apply (*subst FAddSA-dom-dom-States*)
apply *simp*
apply *simp*
apply *simp*
apply (*case-tac S ∈ States SA1*)
apply *simp*
apply *fast*
apply (*subst FAddSA-dom-dom-States*)
apply *simp*
apply *simp*
apply *simp*
apply (*case-tac S ∈ States SA1*)
apply *simp*
apply *fast*
apply *simp*
apply *fast*
done

lemma *AddSA-CompFun-ran-not-mem*:
⟦ *States SA2* ∩ (*HAStates A* ∪ *States SA1*) = {};
States SA1 ∩ *HAStates A* = {};
S ∈ *HAStates A* ⟧ ⟹
{*SA2*} ∉ *ran* (*CompFun A* [*f*+] (*S*, *SA1*))
apply (*cut-tac HA*=*A* [++] (*S*,*SA1*) **and** *Sas*={*SA2*} **in** *ran-CompFun-is-not-SA*)
apply (*metis AddSA-HAStates SA-States-disjunct2 SAs-States-HAStates insert-subset*)
apply (*simp add*: *AddSA-HAStates AddSA-CompFun*)
done

lemma *AddSA-CompFun-ran3*:
⟦ (*States SA1* ∩ *HAStates A*) = {};
(*States SA2* ∩ (*HAStates A* ∪ *States SA1*)) = {};
(*States SA3* ∩ (*HAStates A* ∪ *States SA1* ∪ *States SA2*)) = {};
S ∈ *HAStates A*;
T ∈ *States SA1* ⟧ ⟹
ran (*CompFun* ((*A* [++] (*S*,*SA1*)) [++] (*T*,*SA2*) [++] (*T*,*SA3*))) =
insert {} (*insert* {*SA3*,*SA2*} (*ran* (*CompFun* (*A* [++] (*S*,*SA1*))))))
apply (*simp add*: *AddSA-HAStates AddSA-CompFun*)
apply (*subst FAddSA-ran*)
apply (*metis AddSA-CompFun AddSA-HAStates CompFun-Int-disjoint UnCI dom-CompFun*)
apply (*metis AddSA-CompFun AddSA-HAStates UnCI dom-CompFun*)
apply (*metis AddSA-CompFun AddSA-HAStates UnCI dom-CompFun*)

apply (*subst AddSA-CompFun* [*THEN sym*])
back

apply *simp*
apply *simp*

apply (*subst AddSA-CompFun* [*THEN sym*])
back
apply (*simp add*: *AddSA-HAStates*)
apply (*simp add*: *AddSA-HAStates*)
apply (*subst AddSA-CompFun-ran2*)
apply *fast*
apply *fast*
apply *fast*
apply *fast*
apply (*simp add*: *AddSA-CompFun*)
apply (*subst FAddSA-dom-insert*)
apply (*subst FAddSA-dom-dom-States*)
apply *simp*
apply *fast*
apply *simp*
apply *fast*
apply (*subst FAddSA-dom-emptyset*)
apply *simp*
apply *fast*
apply *simp*
apply *simp*
apply (*subst FAddSA-dom-insert*)
apply (*subst FAddSA-dom-dom-States*)
apply *simp*
apply *fast*
apply *simp*
apply *fast*
apply (*subst FAddSA-dom-emptyset*)
apply *simp*
apply *fast*
apply *simp*
apply *simp*
by (*simp add*: *AddSA-CompFun-ran-not-mem insert-Diff-if insert-commute*)

lemma *AddSA-CompFun-PseudoHA-ran*:
⟦ *S* ∈ *States RootSA*;
States RootSA ∩ *States SA* = {} ⟧ ⟹
(*ran* (*CompFun* ((*PseudoHA RootSA D*) [++] (*S*,*SA*)))) = (*insert* {} {{*SA*}})
apply (*subst AddSA-CompFun-ran*)
apply *auto*
done

lemma *AddSA-CompFun-PseudoHA-ran2*:
⟦ *States SA1* ∩ *States RootSA* = {};
States SA2 ∩ (*States RootSA* ∪ *States SA1*) = {};
S ∈ *States RootSA* ⟧ ⟹

(*ran* (*CompFun* ((*PseudoHA RootSA D*) [++] (*S*,*SA1*) [++] (*S*,*SA2*)))) =
(*insert* {} {{*SA2*,*SA1*}})
apply (*subst AddSA-CompFun-ran*)
prefer *3*
apply (*subst AddSA-CompFun-the*)
apply *simp*
apply *simp*
apply (*subst AddSA-CompFun-PseudoHA-ran*)
apply *fast*
apply *fast*
apply (*subst AddSA-CompFun-the*)
apply *simp*
apply *simp*
apply *simp*
apply *fast*
apply (*simp add*: *AddSA-HAStates*)
apply (*simp add*: *AddSA-HAStates*)
done

lemma *AddSA-HAInitStates* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A* ⟧ ⟹
HAInitStates (*A* [++] (*S*,*SA*)) = *insert* (*InitState SA*) (*HAInitStates A*)
apply (*unfold HAInitStates-def*)
apply (*simp add*: *AddSA-SAs*)
done

lemma *AddSA-HAInitState* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A* ⟧ ⟹
HAInitState (*A* [++] (*S*,*SA*)) = (*HAInitState A*)
apply (*unfold HAInitState-def*)
apply (*simp add*: *AddSA-HARoot*)
done

lemma *AddSA-Chi* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A* ⟧ ⟹
Chi (*A* [++] (*S*,*SA*)) *S* = (*States SA*) ∪ (*Chi A S*)
apply (*unfold Chi-def restrict-def*)
apply (*simp add*: *AddSA-SAs AddSA-HAStates AddSA-CompFun-the*)
apply *auto*
done

lemma *AddSA-Chi2* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A*;
T ∈ *States SA* ⟧ ⟹
Chi (*A* [++] (*S*,*SA*)) *T* = {}

apply (*unfold Chi-def restrict-def*)
apply (*simp add*: *AddSA-SAs AddSA-HAStates AddSA-CompFun-the2*)
done

lemma *AddSA-Chi3* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A*;
T ∉ *States SA*; *T* ≠ *S* ⟧ ⟹
Chi (*A* [++] (*S*,*SA*)) *T* = *Chi A T*
apply (*unfold Chi-def restrict-def*)
apply (*simp add*: *AddSA-SAs AddSA-HAStates AddSA-CompFun-the3*)
apply *auto*
done

lemma *AddSA-ChiRel* [*simp*]:
⟦ *States SA* ∩ *HAStates A* = {};
S ∈ *HAStates A* ⟧ ⟹
ChiRel (*A* [++] (*S*,*SA*)) = { (*T*,*T'*) . *T* = *S* ∧ *T'* ∈ *States SA* } ∪ (*ChiRel A*)
apply (*unfold ChiRel-def*)
apply (*simp add*: *AddSA-HAStates*)
apply *safe*
apply (*rename-tac T U*)
apply (*case-tac T* ∈ *States SA*)
apply *simp*
apply *simp*
apply (*rename-tac T U*)
apply (*case-tac T* ≠ *S*)
apply (*case-tac T* ∈ *States SA*)
apply *simp*
apply *simp*
apply *simp*
apply (*rename-tac T U*)
apply (*case-tac T* ∈ *States SA*)
apply *simp*
apply *simp*
apply (*cut-tac A*=*A* **and** *T*=*T* **in** *Chi-HAStates*)
apply *fast*
apply (*case-tac T* ∈ *States SA*)
apply *simp*
apply *simp*
apply (*cut-tac A*=*A* **and** *T*=*T* **in** *Chi-HAStates*)
apply *fast*
apply *fast*
apply (*rename-tac T U*)
apply (*case-tac T* ≠ *S*)
apply (*case-tac T* ∈ *States SA*)
apply *simp*
apply *simp*
apply *simp*

apply (*rename-tac T U*)
apply (*case-tac T ∈ States SA*)
apply *auto*
apply (*metis AddSA-Chi AddSA-Chi3 Int-iff Un-iff empty-iff*)
done

lemma *help-InitConf*:
⟦*States SA ∩ HAStates A = {}* ⟧ ⟹ {*p. fst p ≠ InitState SA ∧ snd p ≠ InitState SA ∧*
p ∈ insert (*InitState SA*) (*HAInitStates A*) × *insert* (*InitState SA*) (*HAInitStates A*) ∧
(*p ∈ {S} × States SA ∨ p ∈ ChiRel A*)} =
(*HAInitStates A × HAInitStates A ∩ ChiRel A*)
apply *auto*
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*cut-tac A=A* **in** *HAInitStates-HAStates*, *fast*)
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*cut-tac A=A* **in** *HAInitStates-HAStates*, *fast*)
done

lemma *AddSA-InitConf* [*simp*]:
⟦ *States SA ∩ HAStates A = {}*;
S ∈ InitConf A ⟧ ⟹
InitConf (*A* [++] (*S*,*SA*)) = *insert* (*InitState SA*) (*InitConf A*)
apply (*frule InitConf-HAStates2*)
apply (*unfold InitConf-def*)
apply (*simp del*: *insert-Times-insert*)
apply *auto*
apply (*rename-tac T*)
apply (*case-tac T=S*)
apply *auto*
prefer *3*
apply (*rule-tac R=*(*HAInitStates A*) × (*HAInitStates A*) ∩ *ChiRel A* **in** *trancl-subseteq*)
apply *auto*
apply (*rotate-tac 3*)
apply (*frule trancl-collect*)
prefer *2*
apply *fast*
apply *auto*
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*frule ChiRel-HAStates*)
apply *fast*
apply (*frule ChiRel-HAStates*)
apply (*cut-tac A=SA* **in** *InitState-States*)
apply *fast*
apply (*subst help-InitConf* [*THEN sym*])
apply *fast*
apply *auto*
apply (*rule-tac b=S* **in** *rtrancl-into-rtrancl*)

apply *auto*
prefer *2*
apply (*erule rtranclE*)
apply *auto*
prefer *2*
apply (*erule rtranclE*)
apply *auto*
apply (*rule-tac R=(HAInitStates A) × (HAInitStates A) ∩ ChiRel A* **in** *trancl-subseteq*)
apply *auto*
done

lemma *AddSA-InitConf2* [*simp*]:
⟦ *States SA ∩ HAStates A = {}*;
S ∉ InitConf A;
S ∈ HAStates A ⟧ ⟹
InitConf (*A* [++] (*S*,*SA*)) = *InitConf A*
apply (*unfold InitConf-def*)
apply *simp*
apply *auto*
apply (*rename-tac T*)
prefer *2*
apply (*rule-tac R=(HAInitStates A) × (HAInitStates A) ∩ ChiRel A* **in** *trancl-subseteq*)
apply *auto*
apply (*case-tac T=InitState SA*)
apply *auto*
prefer *2*
apply (*rotate-tac 3*)
apply (*frule trancl-collect*)
prefer *2*
apply *fast*
apply *auto*
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*frule ChiRel-HAStates*)
apply *fast*
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*frule ChiRel-HAStates*)
apply *fast*
apply (*cut-tac A=SA* **in** *InitState-States*)
apply (*cut-tac A=A* **in** *HAInitStates-HAStates*)
apply (*subst help-InitConf* [*THEN sym*])
apply *fast*
apply *auto*
apply (*rule-tac b=InitState SA* **in** *rtrancl-induct*)
apply *auto*
apply (*frule ChiRel-HAStates2*)
apply (*cut-tac A=SA* **in** *InitState-States*)
apply *fast*
prefer *2*
apply (*frule ChiRel-HAStates*)

apply (*cut-tac A=SA* **in** *InitState-States*)
apply *fast*
apply (*rule rtrancl-into-rtrancl*)
apply *auto*
done

11.5 Theorems for Calculating Wellformedness of HA

lemma *PseudoHA-HAStates-IFF*:
(*States SA*) = *X* ⟹ (*HAStates* (*PseudoHA SA D*)) = *X*
apply *simp*
done

lemma *AddSA-SAs-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*SAs HA*) = *X* ⟧ ⟹ (*SAs* (*HA* [++] (*S*, *SA*))) = (*insert SA X*)
apply (*subst AddSA-SAs*)
apply *auto*
done

lemma *AddSA-Events-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*HAEvents HA*) = *HAE*;
(*SAEvents SA*) = *SAE*;
(*HAE* ∪ *SAE*) = *X* ⟧ ⟹ (*HAEvents* (*HA* [++] (*S*, *SA*))) = *X*
apply (*subst AddSA-Events*)
apply *auto*
done

lemma *AddSA-CompFun-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*CompFun HA*) = *HAG*;
(*HAG* [*f*+] (*S*, *SA*)) = *X* ⟧ ⟹ (*CompFun* (*HA* [++] (*S*, *SA*))) = *X*
apply (*subst AddSA-CompFun*)
apply *auto*
done

lemma *AddSA-HAStates-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*HAStates HA*) = *HAS*;
(*States SA*) = *SAS*;
(*HAS* ∪ *SAS*) = *X* ⟧ ⟹ (*HAStates* (*HA* [++] (*S*, *SA*))) = *X*
apply (*subst AddSA-HAStates*)
apply *auto*
done

lemma *AddSA-HAInitValue-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*HAInitValue HA*) = *X* ⟧ ⟹ (*HAInitValue* (*HA* [++] (*S*, *SA*))) = *X*
apply (*subst AddSA-HAInitValue*)
apply *auto*
done

lemma *AddSA-HARoot-IFF*:
⟦ *States SA* ∩ *HAStates HA* = {};
S ∈ *HAStates HA*;
(*HARoot HA*) = *X* ⟧ ⟹ (*HARoot* (*HA* [++] (*S*, *SA*))) = *X*
apply (*subst AddSA-HARoot*)
apply *auto*
done

lemma *AddSA-InitConf-IFF*:
⟦ *InitConf A* = *Y*;
States SA ∩ *HAStates A* = {};
S ∈ *HAStates A*;
(*if S* ∈ *Y then insert* (*InitState SA*) *Y else Y*) = *X* ⟧ ⟹
InitConf (*A* [++] (*S*,*SA*)) = *X*
apply (*case-tac S* ∈ *Y*)
apply *auto*
done

lemma *AddSA-CompFun-ran-IFF*:
⟦ (*States SA* ∩ *HAStates A*) = {};
S ∈ *HAStates A*;
(*insert* {} (*insert* (*insert SA* (*the* ((*CompFun A*) *S*))) (*ran* (*CompFun A*) − {*the* ((*CompFun A*) *S*)}))) = *X* ⟧ ⟹
ran (*CompFun* (*A* [++] (*S*,*SA*))) = *X*
apply (*subst AddSA-CompFun-ran*)
apply *auto*
done

lemma *AddSA-CompFun-ran2-IFF*:
⟦ (*States SA1* ∩ *HAStates A*) = {};
(*States SA2* ∩ (*HAStates A* ∪ *States SA1*)) = {};
S ∈ *HAStates A*;
T ∈ *States SA1*;
insert {} (*insert* {*SA2*} (*ran* (*CompFun* (*A* [++] (*S*,*SA1*))))) = *X* ⟧ ⟹
ran (*CompFun* ((*A* [++] (*S*,*SA1*)) [++] (*T*,*SA2*))) = *X*
apply (*subst AddSA-CompFun-ran2*)
apply *auto*
done

lemma *AddSA-CompFun-ran3-IFF*:

⟦ (*States SA1* ∩ *HAStates A*) = {};
(*States SA2* ∩ (*HAStates A* ∪ *States SA1*)) = {};
(*States SA3* ∩ (*HAStates A* ∪ *States SA1* ∪ *States SA2*)) = {};
S ∈ *HAStates A*;
T ∈ *States SA1*;
insert {} (*insert* {*SA3*,*SA2*} (*ran* (*CompFun* (*A* [++] (*S*,*SA1*))))) = *X* ⟧ ⟹
ran (*CompFun* ((*A* [++] (*S*,*SA1*)) [++] (*T*,*SA2*) [++] (*T*,*SA3*))) = *X*
apply (*subst AddSA-CompFun-ran3*)
apply *auto*
done

lemma *AddSA-CompFun-PseudoHA-ran-IFF*:
⟦ *S* ∈ *States RootSA*;
States RootSA ∩ *States SA* = {};
(*insert* {} {{*SA*}}) = *X* ⟧ ⟹
(*ran* (*CompFun* ((*PseudoHA RootSA D*) [++] (*S*,*SA*)))) = *X*
apply (*subst AddSA-CompFun-PseudoHA-ran*)
apply *auto*
done

lemma *AddSA-CompFun-PseudoHA-ran2-IFF*:
⟦ *States SA1* ∩ *States RootSA* = {};
States SA2 ∩ (*States RootSA* ∪ *States SA1*) = {};
S ∈ *States RootSA*;
(*insert* {} {{*SA2*,*SA1*}}) = *X* ⟧ ⟹
(*ran* (*CompFun* ((*PseudoHA RootSA D*) [++] (*S*,*SA1*) [++] (*S*,*SA2*)))) = *X*
apply (*subst AddSA-CompFun-PseudoHA-ran2*)
apply *auto*
done

ML ‹

val AddSA-SAs-IFF = @{*thm AddSA-SAs-IFF*};
val AddSA-Events-IFF = @{*thm AddSA-Events-IFF*};
val AddSA-CompFun-IFF = @{*thm AddSA-CompFun-IFF*};
val AddSA-HAStates-IFF = @{*thm AddSA-HAStates-IFF*};
val PseudoHA-HAStates-IFF = @{*thm PseudoHA-HAStates-IFF*};
val AddSA-HAInitValue-IFF = @{*thm AddSA-HAInitValue-IFF*};
val AddSA-CompFun-ran-IFF = @{*thm AddSA-CompFun-ran-IFF*};
val AddSA-HARoot-IFF = @{*thm AddSA-HARoot-IFF*};
val insert-inter = @{*thm insert-inter*};
val insert-notmem = @{*thm insert-notmem*};
val PseudoHA-CompFun = @{*thm PseudoHA-CompFun*};
val PseudoHA-Events = @{*thm PseudoHA-Events*};
val PseudoHA-SAs = @{*thm PseudoHA-SAs*};
val PseudoHA-HARoot = @{*thm PseudoHA-HARoot*};
val PseudoHA-HAInitValue = @{*thm PseudoHA-HAInitValue*};
val PseudoHA-CompFun-ran = @{*thm PseudoHA-CompFun-ran*};

val Un-empty-right = @{*thm Un-empty-right*};
val insert-union = @{*thm insert-union*};

fun wellformed-tac ctxt L i =
  FIRST[*resolve-tac ctxt* [*AddSA-SAs-IFF*] *i*,
      resolve-tac ctxt [*AddSA-Events-IFF*] *i*,
      resolve-tac ctxt [*AddSA-CompFun-IFF*] *i*,
      resolve-tac ctxt [*AddSA-HAStates-IFF*] *i*,
      resolve-tac ctxt [*PseudoHA-HAStates-IFF*] *i*,
      resolve-tac ctxt [*AddSA-HAInitValue-IFF*] *i*,
      resolve-tac ctxt [*AddSA-HARoot-IFF*] *i*,
      resolve-tac ctxt [*AddSA-CompFun-ran-IFF*] *i*,
      resolve-tac ctxt [*insert-inter*] *i*,
      resolve-tac ctxt [*insert-notmem*] *i*,
      CHANGED (*simp-tac* (*put-simpset HOL-basic-ss ctxt addsimps*
      [*PseudoHA-HARoot*, *PseudoHA-CompFun*, *PseudoHA-CompFun-ran*,*PseudoHA-Events*,*PseudoHA-SAs*
          PseudoHA-HAInitValue,*Un-empty-right*]@ *L*) *i*),
      fast-tac ctxt i,
      CHANGED (*simp-tac ctxt i*)];
›

method-setup *wellformed* = ‹*Attrib.thms* >> (*fn thms* => *fn ctxt* => (*METHOD*
(*fn facts* =>
                                    (*HEADGOAL* (*wellformed-tac ctxt* (*facts* @
thms))))))›

end

12 Example Specification for a Car Audio System

theory *CarAudioSystem*
imports *HAKripke HAOps*
begin

12.1 Definitions

12.1.1 Data space for two Integer-Variables

datatype *d* = *V0 int*
          | *V1 int*

primrec
 Sel0 :: *d* => *int* **where**
  Sel0 (*V0 i*) = *i*

primrec
 Sel1 :: *d* => *int* **where**
  Sel1 (*V1 i*) = *i*

definition
Select0 :: [*d data*] => *int* **where**
Select0 d = *Sel0* (*DataPart d 0*)

definition
Select1 :: [*d data*] => *int* **where**
Select1 d = *Sel1* (*DataPart d 1*)

definition
DSpace :: *d dataspace* **where**
DSpace = *Abs-dataspace* ([*range V0*, *range V1*])

definition
LiftInitData :: (*d list*) => *d data* **where**
LiftInitData L = *Abs-data* (*L*,*DSpace*)

definition
LiftPUpdate :: (*d data* => ((*d option*) *list*)) => *d pupdate* **where**
LiftPUpdate L = *Abs-pupdate*
(λ *d*. *if* ((*DataSpace d*) = *DSpace*) *then*
Abs-pdata (*L d*, *DSpace*)
else (*Data2PData d*))

12.1.2 Sequential Automaton *Root-CTRL*

definition
Root-CTRL-States :: *string set* **where**
Root-CTRL-States = {''*CarAudioSystem*''}

definition
Root-CTRL-Init :: *string* **where**
Root-CTRL-Init = ''*CarAudioSystem*''

definition
Root-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
Root-CTRL-Labels = {}

definition
Root-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**
Root-CTRL-Delta = {}

definition
Root-CTRL :: (*string*,*string*,*d*)*seqauto* **where**

Root-CTRL = *Abs-seqauto* (*Root-CTRL-States*, *Root-CTRL-Init*,
Root-CTRL-Labels, *Root-CTRL-Delta*)

12.1.3 Sequential Automaton *CDPlayer-CTRL*

definition
CDPlayer-CTRL-States :: *string set* **where**
CDPlayer-CTRL-States = {''*ReadTracks*'',''*CDFull*'',''*CDEmpty*''}

definition
CDPlayer-CTRL-Init :: *string* **where**
CDPlayer-CTRL-Init = ''*CDEmpty*''

definition
CDPlayer-CTRL-Update1 :: *d pupdate* **where**
CDPlayer-CTRL-Update1 = *LiftPUpdate* (% *d*. [*None*, *Some* (*V1* ((*Select0 d*) + *1*))])

definition
CDPlayer-CTRL-Action1 :: (*string*,*d*)*action* **where**
CDPlayer-CTRL-Action1 = ({},*CDPlayer-CTRL-Update1*)

definition
CDPlayer-CTRL-Update2 :: *d pupdate* **where**
CDPlayer-CTRL-Update2 = *LiftPUpdate* (% *d*. [*Some* (*V0* ((*Select0 d*) + *1*)), *None*])

definition
CDPlayer-CTRL-Action2 :: (*string*,*d*)*action* **where**
CDPlayer-CTRL-Action2 = ({},*CDPlayer-CTRL-Update2*)

definition
CDPlayer-CTRL-Update3 :: *d pupdate* **where**
CDPlayer-CTRL-Update3 = *LiftPUpdate* (% *d*. [*Some* (*V0 0*), *None*])

definition
CDPlayer-CTRL-Action3 :: (*string*,*d*)*action* **where**
CDPlayer-CTRL-Action3 = ({},*CDPlayer-CTRL-Update3*)

definition
CDPlayer-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
CDPlayer-CTRL-Labels = {(*En* ''*LastTrack*'', *defaultguard*, *CDPlayer-CTRL-Action1*),
(*En* ''*NewTrack*'', *defaultguard*, *CDPlayer-CTRL-Action2*),
(*And* (*En* ''*CDEject*'') (*In* ''*On*''), *defaultguard*, *CDPlayer-CTRL-Action3*),
(*En* ''*CDIn*'', *defaultguard*, *defaultaction*)}

definition
CDPlayer-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**

CDPlayer-CTRL-Delta = {(''*ReadTracks*'',(*En* ''*LastTrack*'', *defaultguard*, *CD-Player-CTRL-Action1*),
''*CDFull*''),
(''*CDFull*'',(*And* (*En* ''*CDEject*'') (*In* ''*On*''), *defaultguard*, *CDPlayer-CTRL-Action3*),
''*CDEmpty*''),
(''*ReadTracks*'',(*En* ''*NewTrack*'', *defaultguard*, *CD-Player-CTRL-Action2*),
''*ReadTracks*''),
(''*CDEmpty*'',(*En* ''*CDIn*'', *defaultguard*, *defaultaction*),
''*ReadTracks*'')}

definition
CDPlayer-CTRL :: (*string*,*string*,*d*)*seqauto* **where**
CDPlayer-CTRL = *Abs-seqauto* (*CDPlayer-CTRL-States*, *CDPlayer-CTRL-Init*, *CDPlayer-CTRL-Labels*, *CDPlayer-CTRL-Delta*)

12.1.4 Sequential Automaton *AudioPlayer-CTRL*

definition
AudioPlayer-CTRL-States :: *string set* **where**
AudioPlayer-CTRL-States = {''*Off*'',''*On*''}

definition
AudioPlayer-CTRL-Init :: *string* **where**
AudioPlayer-CTRL-Init = ''*Off*''

definition
AudioPlayer-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
AudioPlayer-CTRL-Labels = {(*En* ''*O*'', *defaultguard*, *defaultaction*)}

definition
AudioPlayer-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**
AudioPlayer-CTRL-Delta = {(''*Off*'', (*En* ''*O*'', *defaultguard*, *defaultaction*), ''*On*''),
(''*On*'', (*En* ''*O*'', *defaultguard*, *defaultaction*), ''*Off*'')}

definition
AudioPlayer-CTRL :: (*string*,*string*,*d*)*seqauto* **where**
AudioPlayer-CTRL = *Abs-seqauto* (*AudioPlayer-CTRL-States*, *AudioPlayer-CTRL-Init*, *AudioPlayer-CTRL-Labels*, *AudioPlayer-CTRL-Delta*)

12.1.5 Sequential Automaton *On-CTRL*

definition
On-CTRL-States :: *string set* **where**
On-CTRL-States = {''*TunerMode*'',''*CDMode*''}

definition
On-CTRL-Init :: *string* **where**
On-CTRL-Init = ''*TunerMode*''

definition
On-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
On-CTRL-Labels = {(*And* (*En* ''*Src*'') (*In* ''*CDFull*''), *defaultguard*, *defaultaction*),
(*En* ''*Src*'', *defaultguard*, *defaultaction*),
(*En* ''*CDEject*'', *defaultguard*, *defaultaction*),
(*En* ''*EndOfTitle*'', (λ *d*. (*DataPart d 0*) = (*DataPart d 1*)), *defaultaction*)}

definition
On-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**
On-CTRL-Delta = {(''*TunerMode*'',(*And* (*En* ''*Src*'') (*In* ''*CDFull*''), *defaultguard*, *defaultaction*),''*CDMode*''),
(''*CDMode*'', (*En* ''*Src*'', *defaultguard*, *defaultaction*), ''*TunerMode*''),
(''*CDMode*'', (*En* ''*CDEject*'', *defaultguard*, *defaultaction*), ''*TunerMode*''),
(''*CDMode*'', (*En* ''*EndOfTitle*'', (λ *d*. (*DataPart d 0*) = (*DataPart d 1*)), *defaultaction*),
''*TunerMode*'')}

definition
On-CTRL :: (*string*,*string*,*d*)*seqauto* **where**
On-CTRL = *Abs-seqauto* (*On-CTRL-States*, *On-CTRL-Init*,
On-CTRL-Labels, *On-CTRL-Delta*)

12.1.6 Sequential Automaton *TunerMode-CTRL*

definition
TunerMode-CTRL-States :: *string set* **where**
TunerMode-CTRL-States = {''*1*'',''*2*'',''*3*'',''*4*''}

definition
TunerMode-CTRL-Init :: *string* **where**
TunerMode-CTRL-Init = ''*1*''

definition
TunerMode-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
TunerMode-CTRL-Labels = {(*En* ''*Next*'', *defaultguard*, *defaultaction*),
(*En* ''*Back*'', *defaultguard*, *defaultaction*)}

definition
TunerMode-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**
TunerMode-CTRL-Delta = {(''*1*'', (*En* ''*Next*'', *defaultguard*, *defaultaction*), ''*2*''),
(''*2*'', (*En* ''*Next*'', *defaultguard*, *defaultaction*), ''*3*''),
(''*3*'', (*En* ''*Next*'', *defaultguard*, *defaultaction*), ''*4*''),
(''*4*'', (*En* ''*Next*'', *defaultguard*, *defaultaction*), ''*1*''),
(''*1*'', (*En* ''*Back*'', *defaultguard*, *defaultaction*), ''*4*''),
(''*4*'', (*En* ''*Back*'', *defaultguard*, *defaultaction*), ''*3*''),

$(''3'', (En\ ''Back'', defaultguard, defaultaction), ''2'')$,
$(''2'', (En\ ''Back'', defaultguard, defaultaction), ''1'')\}$

definition
TunerMode-CTRL :: (*string*,*string*,*d*)*seqauto* **where**
TunerMode-CTRL = *Abs-seqauto* (*TunerMode-CTRL-States*, *TunerMode-CTRL-Init*, *TunerMode-CTRL-Labels*, *TunerMode-CTRL-Delta*)

12.1.7 Sequential Automaton *CDMode-CTRL*

definition
CDMode-CTRL-States :: *string set* **where**
CDMode-CTRL-States = {''*Playing*'',''*SelectingNextTrack*'', ''*SelectingPreviousTrack*''}

definition
CDMode-CTRL-Init :: *string* **where**
CDMode-CTRL-Init = ''*Playing*''

definition
CDMode-CTRL-Update1 :: *d pupdate* **where**
CDMode-CTRL-Update1 = *LiftPUpdate* (% *d*. [*Some* (*V0* ((*Select0 d*) + *1*)), *None*])

definition
CDMode-CTRL-Action1 :: (*string*,*d*)*action* **where**
CDMode-CTRL-Action1 = ({},*CDMode-CTRL-Update1*)

definition
CDMode-CTRL-Update2 :: *d pupdate* **where**
CDMode-CTRL-Update2 = *LiftPUpdate* (% *d*. [*Some* (*V0* ((*Select0 d*) − *1*)), *None*])

definition
CDMode-CTRL-Action2 :: (*string*,*d*)*action* **where**
CDMode-CTRL-Action2 = ({},*CDMode-CTRL-Update2*)

definition
CDMode-CTRL-Labels :: (*string*,*string*,*d*)*label set* **where**
CDMode-CTRL-Labels = {(*En* ''*Next*'', *defaultguard*, *defaultaction*), (*En* ''*Back*'', *defaultguard*, *defaultaction*), (*En* ''*Ready*'', *defaultguard*, *CDMode-CTRL-Action1*), (*En* ''*Ready*'', *defaultguard*, *CDMode-CTRL-Action2*), (*En* ''*EndOfTitle*'',(λ (*d*:: *d data*). (*Select0 d*) < (*Select1 d*)), *defaultaction*)}

definition
CDMode-CTRL-Delta :: (*string*,*string*,*d*)*trans set* **where**
CDMode-CTRL-Delta = {(''*Playing*'', (*En* ''*Next*'', *defaultguard*, *defaultaction*) ,

''SelectingNextTrack''),
(*''SelectingNextTrack''*, (*En ''Ready''*, *defaultguard*, *CD-Mode-CTRL-Action1*) ,*''Playing''*),
(*''Playing''*, (*En ''Back''*, *defaultguard*, *defaultaction*) ,*''SelectingPreviousTrack''*),
(*''SelectingPreviousTrack''*, (*En ''Ready''*, *defaultguard*, *CDMode-CTRL-Action2*),
''Playing''),
(*''Playing''*, (*En ''EndOfTitle''*, (λ (*d*:: *d data*). (*Select0 d*) < (*Select1 d*)), *defaultaction*),
''SelectingNextTrack'')}

definition
CDMode-CTRL :: (*string*,*string*,*d*)*seqauto* **where**
CDMode-CTRL = *Abs-seqauto* (*CDMode-CTRL-States*, *CDMode-CTRL-Init*, *CDMode-CTRL-Labels*, *CDMode-CTRL-Delta*)

12.1.8 Hierarchical Automaton *CarAudioSystem*

definition
CarAudioSystem :: (*string*,*string*,*d*)*hierauto* **where**
CarAudioSystem = ((*PseudoHA Root-CTRL* (*LiftInitData* [*V0 0*, *V1 0*]))
[++] (*''CarAudioSystem''*,*CDPlayer-CTRL*)
[++] (*''CarAudioSystem''*,*AudioPlayer-CTRL*)
[++] (*''On''*, *TunerMode-CTRL*)
[++] (*''On''*, *CDMode-CTRL*))

12.2 Lemmas

12.2.1 Sequential Automaton *CDMode-CTRL*

lemma *check-Root-CTRL*:
(*Root-CTRL-States*,*Root-CTRL-Init*,*Root-CTRL-Labels*,*Root-CTRL-Delta*) : *seqauto*
apply (*unfold seqauto-def SeqAuto-def Root-CTRL-States-def Root-CTRL-Init-def Root-CTRL-Labels-def Root-CTRL-Delta-def*)
apply *simp*
done

lemma *States-Root-CTRL*:
States Root-CTRL = *Root-CTRL-States*
apply (*simp add*: *Root-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-Root-CTRL*)
done

lemma *Init-State-Root-CTRL*:
InitState Root-CTRL = *Root-CTRL-Init*
apply (*simp add*: *Root-CTRL-def*)
apply (*unfold InitState-def*)

apply (*simp add*: *Abs-seqauto-inverse check-Root-CTRL*)
done

lemma *Labels-Root-CTRL*:
Labels Root-CTRL = Root-CTRL-Labels
apply (*simp add*: *Root-CTRL-def*)
apply (*unfold Labels-def*)
apply (*simp add*: *Abs-seqauto-inverse check-Root-CTRL*)
done

lemma *Delta-Root-CTRL*:
Delta Root-CTRL = Root-CTRL-Delta
apply (*simp add*: *Root-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-Root-CTRL*)
done

schematic-goal *Events-Root-CTRL*:
SAEvents Root-CTRL = ?X
apply (*unfold SAEvents-def expr-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Delta-Root-CTRL Root-CTRL-Delta-def*)
apply (*rule refl*)
done

12.2.2 Sequential Automaton *CDPlayer-CTRL*

lemma *check-CDPlayer-CTRL*:
(*CDPlayer-CTRL-States*,*CDPlayer-CTRL-Init*,*CDPlayer-CTRL-Labels*,*CDPlayer-CTRL-Delta*)
: *seqauto*
apply (*unfold seqauto-def SeqAuto-def CDPlayer-CTRL-States-def CDPlayer-CTRL-Init-def CDPlayer-CTRL-Labels-def CDPlayer-CTRL-Delta-def*)
apply *simp*
done

lemma *States-CDPlayer-CTRL*:
States CDPlayer-CTRL = CDPlayer-CTRL-States
apply (*simp add*: *CDPlayer-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDPlayer-CTRL*)
done

lemma *Init-State-CDPlayer-CTRL*:
InitState CDPlayer-CTRL = CDPlayer-CTRL-Init
apply (*simp add*: *CDPlayer-CTRL-def*)
apply (*unfold InitState-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDPlayer-CTRL*)
done

lemma *Labels-CDPlayer-CTRL*:
Labels CDPlayer-CTRL = CDPlayer-CTRL-Labels
apply (*simp add*: *CDPlayer-CTRL-def*)
apply (*unfold Labels-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDPlayer-CTRL*)
done

lemma *Delta-CDPlayer-CTRL*:
Delta CDPlayer-CTRL = CDPlayer-CTRL-Delta
apply (*simp add*: *CDPlayer-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDPlayer-CTRL*)
done

schematic-goal *Events-CDPlayer-CTRL*:
SAEvents CDPlayer-CTRL = ?X
apply (*unfold SAEvents-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Delta-CDPlayer-CTRL CDPlayer-CTRL-Delta-def CD-Player-CTRL-Action1-def CDPlayer-CTRL-Action2-def CDPlayer-CTRL-Action3-def Label-def*)
apply (*rule refl*)
done

12.2.3 Sequential Automaton *AudioPlayer-CTRL*

lemma *check-AudioPlayer-CTRL*:
(*AudioPlayer-CTRL-States*,*AudioPlayer-CTRL-Init*,*AudioPlayer-CTRL-Labels*,*AudioPlayer-CTRL-Delta*)
: *seqauto*
apply (*unfold seqauto-def SeqAuto-def AudioPlayer-CTRL-States-def AudioPlayer-CTRL-Init-def AudioPlayer-CTRL-Labels-def AudioPlayer-CTRL-Delta-def*)
apply *simp*
done

lemma *States-AudioPlayer-CTRL*:
States AudioPlayer-CTRL = AudioPlayer-CTRL-States
apply (*simp add*: *AudioPlayer-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-AudioPlayer-CTRL*)
done

lemma *Init-State-AudioPlayer-CTRL*:
InitState AudioPlayer-CTRL = AudioPlayer-CTRL-Init
apply (*simp add*: *AudioPlayer-CTRL-def*)
apply (*unfold InitState-def*)
apply (*simp add*: *Abs-seqauto-inverse check-AudioPlayer-CTRL*)
done

lemma *Labels-AudioPlayer-CTRL*:

Labels AudioPlayer-CTRL = AudioPlayer-CTRL-Labels
apply (*simp add*: *AudioPlayer-CTRL-def*)
apply (*unfold Labels-def*)
apply (*simp add*: *Abs-seqauto-inverse check-AudioPlayer-CTRL*)
done

lemma *Delta-AudioPlayer-CTRL*:
Delta AudioPlayer-CTRL = AudioPlayer-CTRL-Delta
apply (*simp add*: *AudioPlayer-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-AudioPlayer-CTRL*)
done

schematic-goal *Events-AudioPlayer-CTRL*:
SAEvents AudioPlayer-CTRL = ?X
apply (*unfold SAEvents-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Delta-AudioPlayer-CTRL AudioPlayer-CTRL-Delta-def Label-def*)
apply (*rule refl*)
done

12.2.4 Sequential Automaton *On-CTRL*

lemma *check-On-CTRL*:
(*On-CTRL-States*,*On-CTRL-Init*,*On-CTRL-Labels*,*On-CTRL-Delta*) : *seqauto*
apply (*unfold seqauto-def SeqAuto-def On-CTRL-States-def On-CTRL-Init-def On-CTRL-Labels-def On-CTRL-Delta-def*)
apply *simp*
done

lemma *States-On-CTRL*:
States On-CTRL = On-CTRL-States
apply (*simp add*: *On-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-On-CTRL*)
done

lemma *Init-State-On-CTRL*:
InitState On-CTRL = On-CTRL-Init
apply (*simp add*: *On-CTRL-def*)
apply (*unfold InitState-def*)
apply (*simp add*: *Abs-seqauto-inverse check-On-CTRL*)
done

lemma *Labels-On-CTRL*:
Labels On-CTRL = On-CTRL-Labels
apply (*simp add*: *On-CTRL-def*)
apply (*unfold Labels-def*)

apply (*simp add*: *Abs-seqauto-inverse check-On-CTRL*)
done

lemma *Delta-On-CTRL*:
Delta On-CTRL = On-CTRL-Delta
apply (*simp add*: *On-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-On-CTRL*)
done

schematic-goal *Events-On-CTRL*:
SAEvents On-CTRL = ?X
apply (*unfold SAEvents-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Delta-On-CTRL On-CTRL-Delta-def Label-def*)
apply (*rule refl*)
done

12.2.5 Sequential Automaton *TunerMode-CTRL*

lemma *check-TunerMode-CTRL*:
(*TunerMode-CTRL-States*, *TunerMode-CTRL-Init*, *TunerMode-CTRL-Labels*, *TunerMode-CTRL-Delta*)
: *seqauto*
apply (*unfold seqauto-def SeqAuto-def TunerMode-CTRL-States-def TunerMode-CTRL-Init-def TunerMode-CTRL-Labels-def TunerMode-CTRL-Delta-def*)
apply *simp*
done

lemma *States-TunerMode-CTRL*:
States TunerMode-CTRL = TunerMode-CTRL-States
apply (*simp add*: *TunerMode-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-TunerMode-CTRL*)
done

lemma *Init-State-TunerMode-CTRL*:
InitState TunerMode-CTRL = TunerMode-CTRL-Init
apply (*simp add*: *TunerMode-CTRL-def*)
apply (*unfold InitState-def*)
apply (*simp add*: *Abs-seqauto-inverse check-TunerMode-CTRL*)
done

lemma *Labels-TunerMode-CTRL*:
Labels TunerMode-CTRL = TunerMode-CTRL-Labels
apply (*simp add*: *TunerMode-CTRL-def*)
apply (*unfold Labels-def*)
apply (*simp add*: *Abs-seqauto-inverse check-TunerMode-CTRL*)
done

lemma *Delta-TunerMode-CTRL*:
Delta TunerMode-CTRL = TunerMode-CTRL-Delta
apply (*simp add*: *TunerMode-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-TunerMode-CTRL*)
done

schematic-goal *Events-TunerMode-CTRL*:
SAEvents TunerMode-CTRL = ?X
apply (*unfold SAEvents-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Delta-TunerMode-CTRL TunerMode-CTRL-Delta-def Label-def*)
apply (*rule refl*)
done

12.2.6 Sequential Automaton *CDMode-CTRL*

lemma *check-CDMode-CTRL*:
(*CDMode-CTRL-States*,*CDMode-CTRL-Init*,*CDMode-CTRL-Labels*,*CDMode-CTRL-Delta*) : *seqauto*
apply (*unfold seqauto-def SeqAuto-def CDMode-CTRL-States-def CDMode-CTRL-Init-def CDMode-CTRL-Labels-def CDMode-CTRL-Delta-def*)
apply *simp*
done

lemma *States-CDMode-CTRL*:
States CDMode-CTRL = CDMode-CTRL-States
apply (*simp add*: *CDMode-CTRL-def*)
apply (*unfold States-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDMode-CTRL*)
done

lemma *Init-State-CDMode-CTRL*:
InitState CDMode-CTRL = CDMode-CTRL-Init
apply (*simp add*: *CDMode-CTRL-def*)
apply (*unfold InitState-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDMode-CTRL*)
done

lemma *Labels-CDMode-CTRL*:
Labels CDMode-CTRL = CDMode-CTRL-Labels
apply (*simp add*: *CDMode-CTRL-def*)
apply (*unfold Labels-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDMode-CTRL*)
done

lemma *Delta-CDMode-CTRL*:
Delta CDMode-CTRL = CDMode-CTRL-Delta

apply (*simp add*: *CDMode-CTRL-def*)
apply (*unfold Delta-def*)
apply (*simp add*: *Abs-seqauto-inverse check-CDMode-CTRL*)
done

schematic-goal *Events-CDMode-CTRL*:
SAEvents CDMode-CTRL = ?X
apply (*unfold SAEvents-def*)
apply (*rule trans*)
apply (*simp add*: *expr-def Label-def Delta-CDMode-CTRL CDMode-CTRL-Delta-def CDMode-CTRL-Action1-def CDMode-CTRL-Action2-def*)
apply (*rule refl*)
done

12.2.7 Hierarchical Automaton *CarAudioSystem*

lemmas *CarAudioSystemStates = States-Root-CTRL States-CDPlayer-CTRL States-AudioPlayer-CTRL States-On-CTRL*
States-TunerMode-CTRL States-CDMode-CTRL
Root-CTRL-States-def CDPlayer-CTRL-States-def
AudioPlayer-CTRL-States-def
On-CTRL-States-def TunerMode-CTRL-States-def
CDMode-CTRL-States-def

lemmas *CarAudioSystemInitState = Init-State-Root-CTRL Init-State-CDPlayer-CTRL Init-State-AudioPlayer-CTRL*
Init-State-On-CTRL Init-State-TunerMode-CTRL
Init-State-CDMode-CTRL
Root-CTRL-Init-def CDPlayer-CTRL-Init-def
AudioPlayer-CTRL-Init-def
On-CTRL-Init-def TunerMode-CTRL-Init-def
CDMode-CTRL-Init-def

lemmas *CarAudioSystemEvents = Events-Root-CTRL Events-CDPlayer-CTRL Events-AudioPlayer-CTRL Events-On-CTRL*
Events-TunerMode-CTRL Events-CDMode-CTRL

lemmas *CarAudioSystemthms = CarAudioSystemStates CarAudioSystemEvents CarAudioSystemInitState*

schematic-goal *CarAudioSystem-StatesRoot*:
HAStates (*PseudoHA Root-CTRL* (*LiftInitData* [*V0 0*, *V1 0*])) = *?X*
apply (*wellformed CarAudioSystemthms*)+
done

lemmas *CarAudioSystemthms-1 = CarAudioSystemthms CarAudioSystem-StatesRoot*

schematic-goal *CarAudioSystem-StatesCDPlayer*:
HAStates (PseudoHA Root-CTRL (LiftInitData [V0 0, V1 0]) [++]
("CarAudioSystem",CDPlayer-CTRL)) = ?X
apply (*wellformed CarAudioSystemthms-1*)+
done

lemmas *CarAudioSystemthms-2 = CarAudioSystemthms-1 CarAudioSystem-StatesCDPlayer*

schematic-goal *CarAudioSystem-StatesAudioPlayer*:
HAStates (PseudoHA Root-CTRL (LiftInitData [V0 0, V1 0])
[++] ("CarAudioSystem",CDPlayer-CTRL)
[++] ("CarAudioSystem",AudioPlayer-CTRL)) = ?X
apply (*wellformed CarAudioSystemthms-2*)+
done

lemmas *CarAudioSystemthms-3 = CarAudioSystemthms-2 CarAudioSystem-StatesAudioPlayer*

schematic-goal *CarAudioSystem-StatesTunerMode*:
HAStates (PseudoHA Root-CTRL (LiftInitData [V0 0, V1 0])
[++] ("CarAudioSystem",CDPlayer-CTRL)
[++] ("CarAudioSystem",AudioPlayer-CTRL)
[++] ("On", TunerMode-CTRL)) = ?X
apply (*wellformed CarAudioSystemthms-3*)+
done

lemmas *CarAudioSystemthms-4 = CarAudioSystemthms-3 CarAudioSystem-StatesTunerMode*

schematic-goal *CarAudioSystem-StatesCDMode*:
HAStates (PseudoHA Root-CTRL (LiftInitData [V0 0, V1 0])
[++] ("CarAudioSystem",CDPlayer-CTRL)
[++] ("CarAudioSystem",AudioPlayer-CTRL)
[++] ("On", TunerMode-CTRL)
[++] ("On", CDMode-CTRL)) = ?X
apply (*wellformed CarAudioSystemthms-4*)+
done

lemmas *CarAudioSystemthms-5 = CarAudioSystemthms-4 CarAudioSystem-StatesCDMode*

schematic-goal *SAsCarAudioSystem*:
SAs CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *EventsCarAudioSystem*:
HAEvents CarAudioSystem = ?X

apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *CompFunCarAudioSystem*:
CompFun CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *StatesCarAudioSystem*:
HAStates CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *ValueCarAudioSystem*:
HAInitValue CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *HAInitStatesCarAudioSystem*:
HAInitStates CarAudioSystem = ?X
by (*simp add*: *HAInitStates-def SAsCarAudioSystem CarAudioSystemInitState*)

schematic-goal *HARootCarAudioSystem*:
HARoot CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *HAInitStateCarAudioSystem*:
HAInitState CarAudioSystem = ?X
by (*simp add*: *HARootCarAudioSystem HAInitState-def CarAudioSystemInitState*)

lemma *check-DataSpace* [*simp*]:
[*range V0*, *range V1*] ∈ *dataspace*
apply (*unfold dataspace-def DataSpace.DataSpace-def*)
apply *auto*
apply (*rename-tac D*)
apply (*case-tac D*)
apply *auto*
done

lemma *PartNum-DataSpace* [*simp*]:
PartNum (*DSpace*) = *2*
apply (*unfold PartNum-def DSpace-def*)
apply (*simp add*: *Abs-dataspace-inverse*)
done

lemma *PartDom-DataSpace-V0* [*simp*]:
(*PartDom DSpace 0*) = *range V0*
apply (*unfold PartDom-def DSpace-def*)
apply (*simp add*: *Abs-dataspace-inverse*)
done

lemma *PartDom-DataSpace-V1* [*simp*]:
(*PartDom DSpace* (*Suc 0*)) = *range V1*
apply (*unfold PartDom-def DSpace-def*)
apply (*simp add*: *Abs-dataspace-inverse*)
done

lemma *check-InitialData* [*simp*]:
([*V0 0*, *V1 0*],*DSpace*) ∈ *data*
apply (*unfold data-def Data.Data-def*)
apply *auto*
apply (*rename-tac d*)
apply (*case-tac d=0* ∨ *d* = *1*)
apply *auto*
done

lemma *Select0-InitData* [*simp*]:
Select0 (*LiftInitData* [*V0 0*, *V1 0*]) = *0*
apply (*unfold LiftInitData-def Select0-def DataPart-def DataValue-def*)
apply (*simp add*: *Abs-data-inverse*)
done

lemma *Select1-InitData* [*simp*]:
Select1 (*LiftInitData* [*V0 0*, *V1 0*]) = *0*
apply (*unfold LiftInitData-def Select1-def DataPart-def DataValue-def*)
apply (*simp add*: *Abs-data-inverse*)
done

lemma *HAInitValue1-CarAudioSystem*:
CarAudioSystem |=*H*= *Atom* (*VAL* (λ *d*. (*Select0 d*) = *0*))
apply (*simp add*: *ValueCarAudioSystem*)
done

lemma *HAInitValue2-CarAudioSystem*:
CarAudioSystem |=*H*= *Atom* (*VAL* (λ *d*. (*Select1 d*) = *0*))
apply (*simp add*: *ValueCarAudioSystem*)
done

lemma *HAInitValue-DSpace-CarAudioSystem* [*simp*]:
Data.DataSpace (*LiftInitData* [*V0 0*, *V1 0*]) = *DSpace*
apply (*unfold LiftInitData-def Data.DataSpace-def*)
apply (*simp add*: *Abs-data-inverse*)
done

lemma *check-InitStatus* [*simp*]:
(*CarAudioSystem*, *InitConf CarAudioSystem*, {},*LiftInitData* [*V0 0*, *V1 0*]) ∈ *status*
apply (*unfold status-def Status-def*)
apply (*simp add*: *ValueCarAudioSystem*)
done

lemma *InitData-InitStatus* [*simp*]:
Value (*InitStatus CarAudioSystem*) = *LiftInitData* [*V0 0*, *V1 0*]
apply (*simp add*: *ValueCarAudioSystem*)
done

lemma *Events-InitStatus* [*simp*]:
Events (*InitStatus CarAudioSystem*) = {}
apply (*unfold InitStatus-def Events-def*)
apply (*simp add*: *Abs-status-inverse*)
done

lemma *Conf-InitStatus* [*simp*]:
Conf (*InitStatus CarAudioSystem*) = *InitConf CarAudioSystem*
apply (*unfold InitStatus-def Conf-def*)
apply (*simp add*: *Abs-status-inverse*)
done

lemma *CompFunCarAudioSystem-the*:
the (*CompFun CarAudioSystem ''On''*) = {*CDMode-CTRL*,*TunerMode-CTRL*}
apply (*unfold CarAudioSystem-def*)
apply (*subst AddSA-CompFun-the*)
prefer *3*
apply (*subst AddSA-CompFun-the*)
prefer *3*
apply (*subst AddSA-CompFun-the2*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *CompFunCarAudioSystem-the2*:
the (*CompFun CarAudioSystem ''CarAudioSystem''*) = {*AudioPlayer-CTRL*, *CDPlayer-CTRL*}
apply (*unfold CarAudioSystem-def*)
apply (*subst AddSA-CompFun-the3*)
prefer *5*
apply (*subst AddSA-CompFun-the3*)

prefer *5*
apply (*subst AddSA-CompFun-the*)
prefer *3*
apply (*subst AddSA-CompFun-the*)
prefer *3*
apply (*subst PseudoHA-CompFun-the*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *CompFunCarAudioSystem-the3*:
the (*CompFun CarAudioSystem ''Off''*) = {}
apply (*unfold CarAudioSystem-def*)
apply (*subst AddSA-CompFun-the3*)
prefer *5*
apply (*subst AddSA-CompFun-the3*)
prefer *5*
apply (*subst AddSA-CompFun-the2*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *CompFunCarAudioSystem-ran*:
ran (*CompFun CarAudioSystem*) = *?X*
apply (*unfold CarAudioSystem-def*)
apply (*rule AddSA-CompFun-ran3-IFF*)
prefer *6*
apply (*subst AddSA-CompFun-PseudoHA-ran2*)
prefer *4*
apply (*simp add*: *insert-commute*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *Root-CTRL-CDPlayer-CTRL-noteq* [*simp*]:
Root-CTRL ≠ *CDPlayer-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *Root-CTRL-AudioPlayer-CTRL-noteq* [*simp*]:
Root-CTRL ≠ *AudioPlayer-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *Root-CTRL-TunerMode-CTRL-noteq* [*simp*]:
Root-CTRL ≠ *TunerMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *Root-CTRL-CDMode-CTRL-noteq* [*simp*]:
Root-CTRL ≠ *CDMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *CDPlayer-CTRL-AudioPlayer-CTRL-noteq* [*simp*]:
CDPlayer-CTRL ≠ *AudioPlayer-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *CDPlayer-CTRL-TunerMode-CTRL-noteq* [*simp*]:
CDPlayer-CTRL ≠ *TunerMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *CDPlayer-CTRL-CDMode-CTRL-noteq* [*simp*]:
CDPlayer-CTRL ≠ *CDMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *AudioPlayer-CTRL-TunerMode-CTRL-noteq* [*simp*]:
AudioPlayer-CTRL ≠ *TunerMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *AudioPlayer-CTRL-CDMode-CTRL-noteq* [*simp*]:
AudioPlayer-CTRL ≠ *CDMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

lemma *TunerMode-CTRL-CDMode-CTRL-noteq* [*simp*]:
TunerMode-CTRL ≠ *CDMode-CTRL*
apply (*rule SA-States-disjunct*)
apply (*wellformed CarAudioSystemthms-5*)+
done

schematic-goal *Chi-CarAudioSystem*:
Chi CarAudioSystem ''CarAudioSystem'' = ?X
apply (*unfold Chi-def*)
apply (*rule trans*)
apply (*simp add*: *SAsCarAudioSystem StatesCarAudioSystem restrict-def Comp-FunCarAudioSystem-the2*)
apply (*rule trans*)

apply (*simp add*: *not-sym*)
apply (*simp add*: *CarAudioSystemStates insert-or*)
done

schematic-goal *Chi-CarAudioSystem-On*:
Chi CarAudioSystem ''On'' = ?X
apply (*unfold Chi-def*)
apply (*rule trans*)
apply (*simp add*: *SAsCarAudioSystem StatesCarAudioSystem restrict-def Comp-FunCarAudioSystem-the*)
apply (*rule trans*)
apply (*simp add*: *not-sym*)
apply (*simp add*: *CarAudioSystemStates insert-or*)
done

schematic-goal *Chi-CarAudioSystem-Off*:
Chi CarAudioSystem ''Off'' = ?X
apply (*unfold Chi-def*)
apply (*simp add*: *SAsCarAudioSystem StatesCarAudioSystem restrict-def Comp-FunCarAudioSystem-the3*)
done

schematic-goal *InitConf-CarAudioSystem*:
InitConf CarAudioSystem = ?X
apply (*unfold CarAudioSystem-def*)
apply (*rule AddSA-InitConf-IFF*)+
apply *simp*
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply (*wellformed CarAudioSystemthms-5*)
apply *simp*
apply (*simp add*: *CarAudioSystemthms-5*)
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply *simp*
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply *simp*
apply (*wellformed CarAudioSystemthms-5*)
apply *force*
apply (*wellformed CarAudioSystemthms-5*)
apply *fast*
apply (*simp add*: *CarAudioSystemthms-5*)
done

lemma *Initial-State-CarAudioSystem*:
CarAudioSystem |=*H*= *Atom* (*IN ''Off''*)
apply (*simp add*: *InitConf-CarAudioSystem*)
done

end

References

[HN96] D. Harel and D. Naamad. A STATEMATE semantics for statecharts. *ACM Transactions on Software Engineering and Methodology*, 5(4):293–333, Oct 1996.

[MLS97] E. Mikk, Y. Lakhnech, and M. Siegel. Hierarchical automata as model for statecharts. In *Asian Computing Science Conference (ASIAN'97)*, *Springer LNCS*, **1345**, 1997.

[HK01] S. Helke and F. Kammüller. Representing Hierarchical Automata in Interactive Theorem Provers. In R. J. Boulton, P. B. Jackson, editors, *Theorem Proving in Higher Order Logics, TPHOLs 2001*, *Springer LNCS*, **2152**, 2001.

[HK05] S. Helke and F. Kammüller. Structure Preserving Data Abstractions for Statecharts. In F. Wang, editors, *Formal Techniques for Networked and Distributed Systems, FORTE 2005*, *Springer LNCS*, **3731**, 2005.

[Hel07] S. Helke. *Verification of Statecharts using Structure- and Property-Preserving Data Abstraction* [*german*] . PhD thesis, Fakultät IV, Technische Universität Berlin, Germany, 2007.