The pi-calculus

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Abstract

Contents

1 Overview **1**

2 Formalisation **2**

1 Overview

These theories formalise the following results for psi-calculi. Note that there is only an early semantics for psi-calculi, although a late one may appear later.

- strong bisimilarity is preserved by all operators except the input-prefix
- strong equivalence is a congruence
- weak bisimilarity is preserved by all operators except case and the input-prefix
- weak congruence is a congruence
- strong equivalence respect the laws of structural congruence
- all strongly equivalent agents are also weakly congruent which in turn are weakly bisimilar. Moreover, strongly equivalent agents are also strongly bisimilar
- as a corollary of the last two points, all mentioned equivalences respect the law of structural congruence
- for instances of psi-calculi where assertion composition satisfies weakening, the definition of weak bisimilarity can be simplified significantly and proven equivalent to the version that applies when weakening does not hold

- for certain versions of psi-calculi, sum can be encoded
- for certain versions of psi-calculi, the tau-prefix can be encoded and when weakening is satisfied, all of the tau-laws hold.

The file naming convention is hopefully self explanatory, where the prefixes *Strong* and *Weak* denote that the file covers theories required to formalise properties of strong and weak bisimilarity respectively; files with the prefix *Weaken* cover theories where weakening holds for the static implication; if the file name contains *Sim* the theories cover simulation, file names containing *Bisim* cover bisimulation, and file names containing *Cong* cover weak congruence; files with the suffix *Pres* deal with theories that reason about preservation properties of operators such as a simulation or bisimulation being preserved by a certain operator; files with the suffix *StructCong* reason about structural congruence.

For a complete exposition of all of theories, please consult Bengtson's Ph. D. thesis [1]. A shorter presentation can be found in our TPHOLs paper 'Psi-calculi in Isabelle' from 2009 [3]. There are also two LICS-papers that focus on the mathematical theories, rather than the Isabelle formalisations [2, 4].

2 Formalisation

theory *Chain*
imports *HOL−Nominal.Nominal*
begin

lemma *pt-set-nil*:
fixes *Xs* :: *'a set*
assumes *pt*: *pt TYPE('a) TYPE ('x)*
and *at*: *at TYPE('x)*

shows ([]::*'x prm*)•*Xs* = *Xs*
by(*auto simp add*: *perm-set-def pt1*[*OF pt*])

lemma *pt-set-append*:
fixes *pi1* :: *'x prm*
and *pi2* :: *'x prm*
and *Xs* :: *'a set*
assumes *pt*: *pt TYPE('a) TYPE ('x)*
and *at*: *at TYPE('x)*

shows (*pi1*@*pi2*)•*Xs* = *pi1*•(*pi2*•*Xs*)
by(*auto simp add*: *perm-set-def pt2*[*OF pt*])

lemma *pt-set-prm-eq*:

fixes *pi1* :: *'x prm*
and *pi2* :: *'x prm*
and *Xs* :: *'a set*
assumes *pt*: *pt TYPE('a) TYPE ('x)*
and *at*: *at TYPE('x)*

shows *pi1* $\triangleq$ *pi2* $\Longrightarrow$ *pi1*$\cdot$*Xs* = *pi2*$\cdot$*Xs*
by(*auto simp add*: *perm-set-def pt3*[*OF pt*])

lemma *pt-set-inst*:
assumes *pt*: *pt TYPE('a) TYPE ('x)*
and *at*: *at TYPE('x)*

shows *pt TYPE('a set) TYPE('x)*
apply(*simp add*: *pt-def pt-set-nil*[*OF pt*, *OF at*] *pt-set-append*[*OF pt*, *OF at*])
apply(*clarify*)
by(*rule pt-set-prm-eq*[*OF pt*, *OF at*])

lemma *pt-ball-eqvt*:
fixes *pi* :: *'a prm*
and *Xs* :: *'b set*
and *P* :: *'b* $\Rightarrow$ *bool*

assumes *pt*: *pt TYPE('b) TYPE('a)*
and *at*: *at TYPE('a)*

shows (*pi* $\cdot$ ($\forall$ *x* $\in$ *Xs*. *P x*)) = ($\forall$ *x* $\in$ (*pi* $\cdot$ *Xs*). *pi* $\cdot$ *P* (*rev pi* $\cdot$ *x*))
apply(*auto simp add*: *perm-bool*)
apply(*drule-tac pi=rev pi* **in** *pt-set-bij2*[*OF pt*, *OF at*])
apply(*simp add*: *pt-rev-pi*[*OF pt-set-inst*[*OF pt*, *OF at*], *OF at*])
apply(*drule-tac pi=pi* **in** *pt-set-bij2*[*OF pt*, *OF at*])
apply(*erule-tac x=pi* $\cdot$ *x* **in** *ballE*)
apply(*simp add*: *pt-rev-pi*[*OF pt*, *OF at*])
by *simp*

lemma *perm-cart-prod*:
fixes *Xs* :: *'b set*
and *Ys* :: *'c set*
and *p* :: *'a prm*

assumes *pt1*: *pt TYPE('b) TYPE('a)*
and *pt2*: *pt TYPE('c) TYPE('a)*
and *at*: *at TYPE('a)*

shows (*p* $\cdot$ (*Xs* $\times$ *Ys*)) = (((*p* $\cdot$ *Xs*) $\times$ (*p* $\cdot$ *Ys*))::(('*b* $\times$ '*c*) *set*))
by(*auto simp add*: *perm-set-def*)

lemma *supp-member*:
fixes *Xs* :: *'b set*

```
  and   x :: 'a

  assumes pt: pt TYPE('b) TYPE('a)
  and     at: at TYPE('a)
  and     fs: fs TYPE('b) TYPE('a)
  and     finite Xs
  and     x ∈ ((supp Xs)::'a set)

  obtains X where (X::'b) ∈ Xs and x ∈ supp X
proof −
  from ‹finite Xs› ‹x ∈ supp Xs› have ∃ X::'b. (X ∈ Xs) ∧ (x ∈ (supp X))
  proof(induct rule: finite-induct)
    case empty
    from ‹x ∈ ((supp {})::'a set)› have False
      by(simp add: supp-set-empty)
    thus ?case by simp
  next
    case(insert y Xs)
    show ?case
    proof(case-tac x ∈ supp y)
      assume x ∈ supp y
      thus ?case by force
    next
      assume x ∉ supp y
      with ‹x ∈ supp(insert y Xs)› have x ∈ supp Xs
        by(simp add: supp-fin-insert[OF pt, OF at, OF fs, OF ‹finite Xs›])
      with ‹x ∈ supp Xs ⟹ ∃ X. X ∈ Xs ∧ x ∈ supp X›
      show ?case by force
    qed
  qed
  with that show ?thesis by blast
qed

lemma supp-cart-prod-empty[simp]:
  fixes Xs :: 'b set

  shows supp (Xs × {}) = ({}::'a set)
  and   supp ({} × Xs) = ({}::'a set)
by(auto simp add: supp-set-empty)

lemma supp-cart-prod:
  fixes Xs :: 'b set
  and   Ys :: 'c set

  assumes pt1: pt TYPE('b) TYPE('a)
  and     pt2: pt TYPE('c) TYPE('a)
  and     fs1: fs TYPE('b) TYPE('a)
  and     fs2: fs TYPE('c) TYPE('a)
  and     at:  at TYPE('a)
```

and *f1*: *finite Xs*
and *f2*: *finite Ys*
and *a*: $Xs \neq \{\}$
and *b*: $Ys \neq \{\}$

shows $((supp\ (Xs \times Ys))::'a\ set) = ((supp\ Xs) \cup (supp\ Ys))$
proof −
from *f1 f2* **have** *f3*: *finite*$(Xs \times Ys)$ **by** *simp*
show *?thesis*
apply(*simp add*: *supp-of-fin-sets*[*OF pt-prod-inst*[*OF pt1*, *OF pt2*], *OF at*, *OF fs-prod-inst*[*OF fs1*, *OF fs2*], *OF f3*] *supp-prod*)
apply(*rule equalityI*)
using *Union-included-in-supp*[*OF pt1*, *OF at*, *OF fs1*, *OF f1*] *Union-included-in-supp*[*OF pt2*, *OF at*, *OF fs2*, *OF f2*]
apply(*force simp add*: *supp-prod*)
using *a b*
apply(*auto simp add*: *supp-prod*)
using *supp-member*[*OF pt1*, *OF at*, *OF fs1*, *OF f1*]
apply *blast*
using *supp-member*[*OF pt2*, *OF at*, *OF fs2*, *OF f2*]
by *blast*
qed

lemma *fresh-cart-prod*:
fixes *x* :: *'a*
and *Xs* :: *'b set*
and *Ys* :: *'c set*

assumes *pt1*: *pt TYPE('b) TYPE('a)*
and *pt2*: *pt TYPE('c) TYPE('a)*
and *fs1*: *fs TYPE('b) TYPE('a)*
and *fs2*: *fs TYPE('c) TYPE('a)*
and *at*: *at TYPE('a)*
and *f1*: *finite Xs*
and *f2*: *finite Ys*
and *a*: $Xs \neq \{\}$
and *b*: $Ys \neq \{\}$

shows $(x \sharp (Xs \times Ys)) = (x \sharp Xs \wedge x \sharp Ys)$
using *assms*
by(*simp add*: *supp-cart-prod fresh-def*)

lemma *fresh-star-cart-prod*:
fixes *Zs* :: *'a set*
and *xvec* :: *'a list*
and *Xs* :: *'b set*
and *Ys* :: *'c set*

assumes *pt1*: *pt TYPE('b) TYPE('a)*

and *pt2*: *pt TYPE('c) TYPE('a)*
and *fs1*: *fs TYPE('b) TYPE('a)*
and *fs2*: *fs TYPE('c) TYPE('a)*
and *at*: *at TYPE('a)*
and *f1*: *finite Xs*
and *f2*: *finite Ys*
and *a*: $Xs \neq \{\}$
and *b*: $Ys \neq \{\}$

shows $(Zs \sharp* (Xs \times Ys)) = (Zs \sharp* Xs \wedge Zs \sharp* Ys)$
and $(xvec \sharp* (Xs \times Ys)) = (xvec \sharp* Xs \wedge xvec \sharp* Ys)$
using *assms*
by(*force simp add*: *fresh-cart-prod fresh-star-def*)+

lemma *permCommute*:
fixes p :: *'a prm*
and q :: *'a prm*
and P :: *'x*
and Xs :: *'a set*
and Ys :: *'a set*

assumes *pt*: *pt TYPE('x) TYPE('a)*
and *at*: *at TYPE('a)*
and *a*: $(set\ p) \subseteq Xs \times Ys$
and *b*: $Xs \sharp* q$
and *c*: $Ys \sharp* q$

shows $p \cdot q \cdot P = q \cdot p \cdot P$
proof −
have $p \cdot q \cdot P = (p \cdot q) \cdot p \cdot P$
by(*rule pt-perm-compose*[*OF pt*, *OF at*])
moreover from *at* **have** *pt TYPE('a) TYPE('a)*
by(*rule at-pt-inst*)
hence *pt TYPE(('a × 'a) list) TYPE('a)*
by(*force intro*: *pt-prod-inst pt-list-inst*)
hence $p \cdot q = q$ **using** *at a b c*
by(*rule pt-freshs-freshs*)
ultimately show *?thesis* **by** *simp*
qed

definition
distinctPerm :: *'a prm* ⇒ *bool* **where**
distinctPerm p ≡ *distinct*((*map fst p*)@(*map snd p*))

lemma *at-set-avoiding-aux'*:
fixes *Xs*::*'a set*
and *As*::*'a set*
assumes *at*: *at TYPE('a)*

```
  and      a: finite Xs
  and      b: Xs ⊆ As
  and      c: finite As
  and      d: finite ((supp c)::'a set)
  shows ∃(Ys::'a set) (pi::'a prm). Ys♯*c ∧ Ys ∩ As = {} ∧ (pi•Xs=Ys) ∧
        set pi ⊆ Xs × Ys ∧ finite Ys ∧ (distinctPerm pi)
using a b c
proof (induct)
  case empty
  have ({}::'a set)♯*c by (simp add: fresh-star-def)
  moreover
  have ({}::'a set) ∩ As = {} by simp
  moreover
  have ([]::'a prm)•{} = ({}::'a set)
    by(rule pt1) (metis Nominal.pt-set-inst at at-pt-inst)
  moreover
  have set ([]::'a prm) ⊆ {} × {} by simp
  moreover
  have finite ({}::'a set) by simp
  moreover have distinctPerm([]::'a prm)
    by(simp add: distinctPerm-def)
  ultimately show ?case by blast
next
  case (insert x Xs)
  then have ih: ∃ Ys pi. Ys♯*c ∧ Ys ∩ As = {} ∧ pi•Xs = Ys ∧ set pi ⊆ Xs ×
Ys ∧ finite Ys ∧ distinctPerm pi by simp
  then obtain Ys pi where a1: Ys♯*c and a2: Ys ∩ As = {} and a3: (pi::'a
prm)•Xs = Ys and
                        a4: set pi ⊆ Xs × Ys and a5: finite Ys
                    and a6: distinctPerm pi by blast
  have b: x∉Xs by fact
  have d1: finite As by fact
  have d2: finite Xs by fact
  have d3: insert x Xs ⊆ As by fact
  have d4: finite((supp pi)::'a set)
    by(induct pi)
      (auto simp add: supp-list-nil supp-prod at-fin-set-supp[OF at]
                      supp-list-cons at-supp[OF at])
  have ∃ y::'a. y♯(c,x,Ys,As,pi) using d d1 a5 d4
    by (rule-tac at-exists-fresh'[OF at])
       (simp add: supp-prod at-supp[OF at] at-fin-set-supp[OF at])
  then obtain y::'a where  e: y♯(c,x,Ys,As,pi) by blast
  have ({y}∪Ys)♯*c using a1 e by (simp add: fresh-star-def)
  moreover
 have ({y}∪Ys)∩As = {} using a2 d1 e by (simp add: fresh-prod at-fin-set-fresh[OF
at])
  moreover
  have (((pi•x,y)#pi)•(insert x Xs)) = {y}∪Ys
  proof −
```

```
    have eq: [(pi•x,y)]• Ys = Ys
    proof −
      have (pi•x)♯Ys using a3[symmetric] b d2
        by(simp add: pt-fresh-bij[OF pt-set-inst, OF at-pt-inst[OF at], OF at, OF
at]
                     at-fin-set-fresh[OF at d2])
      moreover
      have y♯Ys using e by simp
      ultimately show [(pi•x,y)]• Ys = Ys
        by (simp add: pt-fresh-fresh[OF pt-set-inst, OF at-pt-inst[OF at], OF at,
OF at])
    qed
    have (((pi•x,y)#pi)•({x}∪Xs)) = ([(pi•x,y)]•(pi•({x}∪Xs)))
      by (simp add: pt2[symmetric, OF pt-set-inst, OF at-pt-inst[OF at], OF at])
    also have ... = {y}∪([(pi•x,y)]•(pi•Xs))
      by (simp only: union-eqvt perm-set-def at-calc[OF at])(auto)
    also have ... = {y}∪([(pi•x,y)]• Ys) using a3 by simp
    also have ... = {y}∪Ys using eq by simp
    finally show (((pi•x,y)#pi)•(insert x Xs)) = {y}∪Ys by auto
  qed
  moreover
  have pi•x=x using a4 b a2 a3 d3 by (rule-tac at-prm-fresh2[OF at]) (auto)
  then have set ((pi•x,y)#pi) ⊆ (insert x Xs) × ({y}∪Ys) using a4 by auto
  moreover
  have finite ({y}∪Ys) using a5 by simp
  moreover from ‹Ys ∩ As = {}› ‹(insert x Xs) ⊆ As› ‹finite Ys› have x ∉ Ys
    by(auto simp add: fresh-def at-fin-set-supp[OF at])
  with a6 ‹pi • x = x› ‹x ∉ Xs› ‹(set pi) ⊆ Xs × Ys› e have distinctPerm((pi •
x, y)#pi)
    apply(auto simp add: distinctPerm-def fresh-prod at-fresh[OF at])
    proof −
      fix a b
      assume b ♯ pi and (a, b) ∈ set pi
      thus False
        by(induct pi)
          (auto simp add: supp-list-cons supp-prod at-supp[OF at] fresh-list-cons
fresh-prod at-fresh[OF at])
    next
      fix a b
      assume a ♯ pi and (a, b) ∈ set pi
      thus False
        by(induct pi)
          (auto simp add: supp-list-cons supp-prod at-supp[OF at] fresh-list-cons
fresh-prod at-fresh[OF at])
    qed
  ultimately
  show ?case by blast
qed
```

lemma *at-set-avoiding*:
fixes *Xs*::*'a set*
assumes *at*: *at TYPE*(*'a*)
and *a*: *finite Xs*
and *b*: *finite* ((*supp c*)::*'a set*)
obtains *pi*::*'a prm* **where** (*pi* ∙ *Xs*) ♯* *c* **and** *set pi* ⊆ *Xs* × (*pi* ∙ *Xs*) **and** *distinctPerm pi*
using *a b*
by (*frule-tac As=Xs* **in** *at-set-avoiding-aux'*[*OF at*]) *auto*

lemma *pt-swap*:
fixes *x* :: *'a*
and *a* :: *'x*
and *b* :: *'x*

assumes *pt*: *pt TYPE*(*'a*) *TYPE*(*'x*)
and *at*: *at TYPE*(*'x*)

shows [(*a*, *b*)] ∙ *x* = [(*b*, *a*)] ∙ *x*
proof −
show *?thesis* **by**(*simp add*: *pt3*[*OF pt*] *at-ds5*[*OF at*])
qed

atom-decl *name*

lemma *supp-subset*:
fixes *Xs* :: *'a*::*fs-name set*
and *Ys* :: *'a*::*fs-name set*

assumes *Xs* ⊆ *Ys*
and *finite Xs*
and *finite Ys*

shows (*supp Xs*) ⊆ ((*supp Ys*)::*name set*)
proof(*rule subsetI*)
fix *x*
assume *x* ∈ ((*supp Xs*)::*name set*)
with ‹*finite Xs*› **obtain** *X* **where** *X* ∈ *Xs* **and** *x* ∈ *supp X*
by(*rule supp-member*[*OF pt-name-inst*, *OF at-name-inst*, *OF fs-name-inst*])
from ‹*X* ∈ *Xs*› ‹*Xs* ⊆ *Ys*› **have** *X* ∈ *Ys* **by** *auto*
with ‹*finite Ys*› ‹*x* ∈ *supp X*› **show** *x* ∈ *supp Ys*
by(*induct rule*: *finite-induct*)
(*auto simp add*: *supp-fin-insert*[*OF pt-name-inst*, *OF at-name-inst*, *OF fs-name-inst*])
qed

abbreviation *mem-def* :: *'a* ⇒ *'a list* ⇒ *bool* (- *mem* - [*80*, *80*] *80*) **where**
x mem xs ≡ *x* ∈ *set xs*

lemma *memFresh*:
  fixes x :: *name*
  and p :: *'a*::*fs-name*
  and l :: (*'a*::*fs-name*) *list*

  assumes $x \sharp l$
  and p *mem* l

  shows $x \sharp p$
using *assms*
by(*induct l*, *auto simp add*: *fresh-list-cons*)

lemma *memFreshChain*:
  fixes *xvec* :: *name list*
  and p :: *'a*::*fs-name*
  and l :: *'a*::*fs-name list*
  and *Xs* :: *name set*

  assumes p *mem* l

  shows *xvec* $\sharp*$ $l \Longrightarrow$ *xvec* $\sharp*$ p
  and *Xs* $\sharp*$ $l \Longrightarrow$ *Xs* $\sharp*$ p
using *assms*
by(*auto simp add*: *fresh-star-def intro*: *memFresh*)

lemma *fresh-star-list-append*[*simp*]:
  fixes A :: *name list*
  and B :: *name list*
  and C :: *name list*

  shows $(A \sharp* (B @ C)) = ((A \sharp* B) \wedge (A \sharp* C))$
by(*auto simp add*: *fresh-star-def fresh-list-append*)

lemma *unionSimps*[*simp*]:
  fixes *Xs* :: *name set*
  and *Ys* :: *name set*
  and C :: *'a*::*fs-name*

  shows $((Xs \cup Ys) \sharp* C) = ((Xs \sharp* C) \wedge (Ys \sharp* C))$
by(*auto simp add*: *fresh-star-def*)

lemma *substFreshAux*[*simp*]:
  fixes C :: *'a*::*fs-name*
  and *xvec* :: *name list*

  shows *xvec* $\sharp*$ (*supp* C − *set xvec*)
by(*auto simp add*: *fresh-star-def fresh-def at-fin-set-supp*[*OF at-name-inst*] *fs-name1*)

lemma *fresh-star-perm-app*[*simp*]:
  fixes *Xs* :: *name set*
  and *xvec* :: *name list*
  and *p* :: *name prm*
  and *C* :: *'d*::*fs-name*

  shows $[\![Xs \sharp* p; Xs \sharp* C]\!] \Longrightarrow Xs \sharp* (p \cdot C)$
  and $[\![xvec \sharp* p; xvec \sharp* C]\!] \Longrightarrow xvec \sharp* (p \cdot C)$
by(*auto simp add*: *fresh-star-def fresh-perm-app*)

lemma *freshSets*[*simp*]:
  fixes *x* :: *name*
  and *y* :: *name*
  and *xvec* :: *name list*
  and *X* :: *name set*
  and *C* :: *'a*

  shows ([]::*name list*) $\sharp*$ *C*
  and ([]::*name list*) $\sharp*$ [*y*].*C*
  and ({}::*name set*) $\sharp*$ *C*
  and ({}::*name set*) $\sharp*$ [*y*].*C*
  and $((x\#xvec) \sharp* C) = (x \sharp C \wedge xvec \sharp* C)$
  and $((x\#xvec) \sharp* ([y].C)) = (x \sharp ([y].C) \wedge xvec \sharp* ([y].C))$
  and $((insert\ x\ X) \sharp* C) = (x \sharp C \wedge X \sharp* C)$
  and $((insert\ x\ X) \sharp* ([y].C)) = (x \sharp ([y].C) \wedge X \sharp* ([y].C))$
by(*auto simp add*: *fresh-star-def*)

lemma *freshStarAtom*[*simp*]: (*xvec*::*name list*) $\sharp*$ (*x*::*name*) = $x \sharp xvec$
by(*induct xvec*)
  (*auto simp add*: *fresh-list-nil fresh-list-cons fresh-atm*)

lemma *name-list-set-fresh*[*simp*]:
  fixes *xvec* :: *name list*
  and *x* :: *'a*::*fs-name*

  shows (*set xvec*) $\sharp*$ *x* = *xvec* $\sharp*$ *x*
by(*auto simp add*: *fresh-star-def*)

lemma *name-list-supp*:
  fixes *xvec* :: *name list*

  shows *set xvec* = *supp xvec*
proof −
  have *set xvec* = *supp*(*set xvec*)
    by(*simp add*: *at-fin-set-supp*[*OF at-name-inst*])
  moreover have ... = *supp xvec*
   by(*simp add*: *pt-list-set-supp*[*OF pt-name-inst*, *OF at-name-inst*, *OF fs-name-inst*])
  ultimately show *?thesis*
    by *simp*

qed

lemma *abs-fresh-list-star*:
  fixes *xvec* :: *name list*
  and *a* :: *name*
  and *P* :: *'a::fs-name*

  shows (*xvec* $\sharp*$ [*a*].*P*) = ((*set xvec*) − {*a*}) $\sharp*$ *P*
by(*induct xvec*) (*auto simp add*: *fresh-star-def abs-fresh*)

lemma *abs-fresh-set-star*:
  fixes *X* :: *name set*
  and *a* :: *name*
  and *P* :: *'a::fs-name*

  shows (*X* $\sharp*$ [*a*].*P*) = (*X* − {*a*}) $\sharp*$ *P*
by(*auto simp add*: *fresh-star-def abs-fresh*)

lemmas *abs-fresh-star* = *abs-fresh-list-star abs-fresh-set-star*

lemma *abs-fresh-list-star'*[*simp*]:
  fixes *xvec* :: *name list*
  and *a* :: *name*
  and *P* :: *'a::fs-name*

  assumes *a* $\sharp$ *xvec*

  shows *xvec* $\sharp*$ [*a*].*P* = *xvec* $\sharp*$ *P*
using *assms*
by(*induct xvec*) (*auto simp add*: *abs-fresh fresh-list-cons fresh-atm*)

lemma *freshChainSym*[*simp*]:
  fixes *xvec* :: *name list*
  and *yvec* :: *name list*

  shows *xvec* $\sharp*$ *yvec* = *yvec* $\sharp*$ *xvec*
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)

lemmas [*eqvt*] = *perm-cart-prod*[*OF pt-name-inst*, *OF pt-name-inst*, *OF at-name-inst*]

lemma *name-set-avoiding*:
  fixes *c* :: *'a::fs-name*
  and *X* :: *name set*

  assumes *finite X*
  and $\bigwedge$*pi*::*name prm*. ⟦(*pi* ∙ *X*) $\sharp*$ *c*; *distinctPerm pi*; *set pi* ⊆ *X* × (*pi* ∙ *X*)⟧
⟹ *thesis*

  shows *thesis*

using *assms*
by(*rule-tac c=c* **in** *at-set-avoiding*[*OF at-name-inst*]) (*simp-all add*: *fs-name1*)

lemmas *simps*[*simp*] = *fresh-atm fresh-prod*
pt3[*OF pt-name-inst*, *OF at-ds1*, *OF at-name-inst*]
pt-fresh-fresh[*OF pt-name-inst*, *OF at-name-inst*]
pt-rev-pi[*OF pt-name-inst*, *OF at-name-inst*]
pt-pi-rev[*OF pt-name-inst*, *OF at-name-inst*]

lemmas *name-supp-cart-prod* = *supp-cart-prod*[*OF pt-name-inst*, *OF pt-name-inst*, *OF fs-name-inst*, *OF fs-name-inst*, *OF at-name-inst*]
lemmas *name-fresh-cart-prod* = *fresh-cart-prod*[*OF pt-name-inst*, *OF pt-name-inst*, *OF fs-name-inst*, *OF fs-name-inst*, *OF at-name-inst*]
lemmas *name-fresh-star-cart-prod* = *fresh-star-cart-prod*[*OF pt-name-inst*, *OF pt-name-inst*, *OF fs-name-inst*, *OF fs-name-inst*, *OF at-name-inst*]

lemmas *name-swap-bij*[*simp*] = *pt-swap-bij*[*OF pt-name-inst*, *OF at-name-inst*]
lemmas *name-swap* = *pt-swap*[*OF pt-name-inst*, *OF at-name-inst*]
lemmas *name-set-fresh-fresh*[*simp*] = *pt-freshs-freshs*[*OF pt-name-inst*, *OF at-name-inst*]
lemmas *list-fresh*[*simp*] = *fresh-list-nil fresh-list-cons fresh-list-append*

definition *eqvt* :: *'a*::*fs-name set* ⇒ *bool* **where**
eqvt X ≡ ∀ *x* ∈ *X*. ∀ *p*::*name prm*. *p* ∙ *x* ∈ *X*

lemma *eqvtUnion*[*intro*]:
fixes *Rel* :: (*'d*::*fs-name*) *set*
and *Rel'* :: *'d set*

assumes *EqvtRel*: *eqvt Rel*
and *EqvtRel'*: *eqvt Rel'*

shows *eqvt* (*Rel* ∪ *Rel'*)
using *assms*
by(*force simp add*: *eqvt-def*)

lemma *eqvtPerm*[*simp*]:
fixes *X* :: (*'d*::*fs-name*) *set*
and *x* :: *name*
and *y* :: *name*

assumes *eqvt X*

shows ([(*x*, *y*)] ∙ *X*) = *X*
using *assms*
apply(*auto simp add*: *eqvt-def*)
apply(*erule-tac x*=[(*x*, *y*)] ∙ *xa* **in** *ballE*)
apply(*erule-tac x*=[(*x*, *y*)] **in** *allE*)
apply *simp*

apply(*drule-tac pi*=[(*x*, *y*)] **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
apply *simp*
apply(*erule-tac x*=*xa* **in** *ballE*)
apply(*erule-tac x*=[(*x*, *y*)] **in** *allE*)
apply(*drule-tac pi*=[(*x*, *y*)] **and** *x*=[(*x*, *y*)] ∙ *xa* **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
apply *simp*
by *simp*

lemma *eqvtI*:
  fixes *X* :: *'d*::*fs-name set*
  and *x* :: *'d*
  and *p* :: *name prm*

  assumes *eqvt X*
  and *x* ∈ *X*

  shows (*p* ∙ *x*) ∈ *X*
using *assms*
by(*unfold eqvt-def*) *auto*

lemma *fresh-star-list-nil*[*simp*]:
  fixes *xvec* :: *name list*
  and *Xs* :: *name set*

  shows *xvec* ♯* []
  and *Xs* ♯* []
by(*auto simp add*: *fresh-star-def*)

lemma *fresh-star-list-cons*[*simp*]:
  fixes *xvec* :: *name list*
  and *Xs* :: *name set*
  and *x* :: *'a*::*fs-name*
  and *xs* :: *'a list*

  shows (*xvec* ♯* (*x*#*xs*)) = ((*xvec* ♯* *x*) ∧ *xvec* ♯* *xs*)
  and (*Xs* ♯* (*x*#*xs*)) = ((*Xs* ♯* *x*) ∧ (*Xs* ♯* *xs*))
by(*auto simp add*: *fresh-star-def*)

lemma *freshStarPair*[*simp*]:
  fixes *X* :: *name set*
  and *xvec* :: *name list*
  and *x* :: *'a*::*fs-name*
  and *y* :: *'b*::*fs-name*

  shows (*X* ♯* (*x*, *y*)) = (*X* ♯* *x* ∧ *X* ♯* *y*)
  and (*xvec* ♯* (*x*, *y*)) = (*xvec* ♯* *x* ∧ *xvec* ♯* *y*)
by(*auto simp add*: *fresh-star-def*)
lemma *name-list-avoiding*:

fixes c :: $'a$::*fs-name*
and *xvec* :: *name list*

assumes $\bigwedge$*pi*::*name prm*. $[\![(pi \cdot xvec) \sharp* c; distinctPerm\ pi; set\ pi \subseteq (set\ xvec) \times (set\ (pi \cdot xvec))]\!] \Longrightarrow thesis$

shows *thesis*
proof −
have *finite*(*set xvec*) **by** *simp*
thus *?thesis* **using** *assms*
by(*rule name-set-avoiding*) (*auto simp add*: *eqvts fresh-star-def*)
qed

lemma *distinctPermSimps*[*simp*]:
fixes *p* :: *name prm*
and *a* :: *name*
and *b* :: *name*

shows *distinctPerm*([]::*name prm*)
and (*distinctPerm*((*a*, *b*)#*p*)) = (*distinctPerm p* ∧ *a* ≠ *b* ∧ *a* ♯ *p* ∧ *b* ♯ *p*)
apply(*simp add*: *distinctPerm-def*)
apply(*induct p*)
apply(*unfold distinctPerm-def*)
apply(*clarsimp*)
apply(*rule iffI*, *erule iffE*)
by(*clarsimp*)+

lemma *map-eqvt*[*eqvt*]:
fixes *p* :: *name prm*
and *lst* :: $'a$::*pt-name list*

shows (*p* ∙ (*map f lst*)) = *map* (*p* ∙ *f*) (*p* ∙ *lst*)
apply(*induct lst*, *auto*)
by(*simp add*: *pt-fun-app-eq*[*OF pt-name-inst*, *OF at-name-inst*])

lemma *consPerm*:
fixes *x* :: *name*
and *y* :: *name*
and *p* :: *name prm*
and *C* :: $'a$::*pt-name*

shows ((*x*, *y*)#*p*) ∙ *C* = [(*x*, *y*)] ∙ *p* ∙ *C*
by(*simp add*: *pt2*[*OF pt-name-inst*, *THEN sym*])

simproc-setup *consPerm* (((*x*, *y*)#*p*) ∙ *C*) = ‹
fn - => *fn* - => *fn ct* =>
case Thm.term-of ct of
Const (@{*const-name perm*}, -) $ (*Const* (@{*const-name Cons*}, -) $ - $ *p*)
$ - =>

```
      (case p of
        Const (@{const-name Nil}, -) => NONE
      | - => SOME(mk-meta-eq @{thm consPerm}))
  | - => NONE
›

lemma distinctEqvt[eqvt]:
  fixes p  :: name prm
  and   xs :: 'a::pt-name list

  shows (p • (distinct xs)) = distinct (p • xs)
by(induct xs) (auto simp add: eqvts)

lemma distinctClosed[simp]:
  fixes p  :: name prm
  and   xs :: 'a::pt-name list

  shows distinct (p • xs) = distinct xs
apply(induct xs)
apply(auto simp add: eqvts)
apply(drule-tac pi=p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
apply(simp add: eqvts)
apply(drule-tac pi=rev p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
by(simp add: eqvts)

lemma lengthEqvt[eqvt]:
  fixes p  :: name prm
  and   xs :: 'a::pt-name list

  shows p • (length xs) = length (p • xs)
by(induct xs) (auto simp add: eqvts)

lemma lengthClosed[simp]:
  fixes p  :: name prm
  and   xs :: 'a::pt-name list

  shows length (p • xs) = length xs
by(induct xs) (auto simp add: eqvts)

lemma subsetEqvt[eqvt]:
  fixes p :: name prm
  and   S :: ('a::pt-name) set
  and   T :: ('a::pt-name) set

  shows (p • (S ⊆ T)) = ((p • S) ⊆ (p • T))
by(rule pt-subseteq-eqvt[OF pt-name-inst, OF at-name-inst])

lemma subsetClosed[simp]:
  fixes p :: name prm
```

```
  and   S :: ('a::pt-name) set
  and   T :: ('a::pt-name) set

  shows ((p • S) ⊆ (p • T)) = (S ⊆ T)
apply auto
apply(drule-tac pi=p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
apply(insert pt-set-bij[OF pt-name-inst, OF at-name-inst])
apply auto
apply(drule-tac pi=rev p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
apply auto
apply(subgoal-tac rev p • x ∈ T)
apply auto
apply(drule-tac pi=p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
apply auto
apply(drule-tac pi=p in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
by auto

lemma subsetClosed'[simp]:
  fixes p    :: name prm
  and   xvec :: name list
  and   P    :: 'a::fs-name

  shows (set (p • xvec) ⊆ supp (p • P)) = (set xvec ⊆ supp P)
by(simp add: eqvts[THEN sym])

lemma memEqvt[eqvt]:
  fixes p  :: name prm
  and   x  :: 'a::pt-name
  and   xs :: ('a::pt-name) list

  shows (p • (x mem xs)) = ((p • x) mem (p • xs))
by(induct xs)
  (auto simp add: pt-bij[OF pt-name-inst, OF at-name-inst] eqvts)

lemma memClosed[simp]:
  fixes p  :: name prm
  and   x  :: 'a::pt-name
  and   xs :: ('a::pt-name) list

  shows (p • x) mem (p • xs) = (x mem xs)
proof −
  have x mem xs = p • (x mem xs)
    by(case-tac x mem xs) auto
  thus ?thesis by(simp add: eqvts)
qed

lemma memClosed'[simp]:
  fixes p  :: name prm
  and   x  :: 'a::pt-name
```

and $y :: 'b::pt\text{-}name$
and $xs :: ('a \times 'b)\ list$

shows $((p \cdot x, p \cdot y)\ mem\ (p \cdot xs)) = ((x, y)\ mem\ xs)$
apply(*subgoal-tac* $((x, y)\ mem\ xs) = (p \cdot (x, y))\ mem\ (p \cdot xs)$)
apply *force*
by(*force simp del*: *eqvts*)

lemma *freshPerm*:
fixes $x :: name$
and $p :: name\ prm$

assumes $x \sharp p$

shows $p \cdot x = x$
using *assms*
apply(*rule-tac pt-pi-fresh-fresh*[*OF pt-name-inst*, *OF at-name-inst*])
by(*induct p*, *auto simp add*: *fresh-list-cons fresh-prod*)

lemma *freshChainPermSimp*:
fixes $xvec :: name\ list$
and $p :: name\ prm$

assumes $xvec \sharp* p$

shows $p \cdot xvec = xvec$
and $rev\ p \cdot xvec = xvec$
using *assms*
by(*induct xvec*) (*auto simp add*: *freshPerm pt-bij1*[*OF pt-name-inst*, *OF at-name-inst*, *THEN sym*])

lemma *freshChainAppend*[*simp*]:
fixes $xvec :: name\ list$
and $yvec :: name\ list$
and $C :: 'a::fs\text{-}name$

shows $(xvec@yvec) \sharp* C = ((xvec \sharp* C) \wedge (yvec \sharp* C))$
by(*force simp add*: *fresh-star-def*)

lemma *subsetFresh*:
fixes $xvec :: name\ list$
and $yvec :: name\ list$
and $C :: 'd::fs\text{-}name$

assumes $set\ xvec \subseteq set\ yvec$
and $yvec \sharp* C$

shows $xvec \sharp* C$

using *assms*
by(*auto simp add*: *fresh-star-def*)

lemma *distinctPermCancel*[*simp*]:
  fixes *p* :: *name prm*
  and *T* :: *'a*::*pt-name*

  assumes *distinctPerm p*

  shows (*p* ∙ (*p* ∙ *T*)) = *T*
using *assms*
proof(*induct p*)
  case *Nil*
  show *?case* **by** *simp*
next
  case(*Cons a p*)
  thus *?case*
  proof(*case-tac a*, *auto*)
    fix *a b*
    assume (*a*::*name*) ♯ (*p*::*name prm*) *b* ♯ *p p* ∙ *p* ∙ *T* = *T a* ≠ *b*
    thus [(*a*, *b*)] ∙ *p* ∙ [(*a*, *b*)] ∙ *p* ∙ *T* = *T*
      by(*subst pt-perm-compose*[*OF pt-name-inst*, *OF at-name-inst*]) *simp*
  qed
qed

fun *composePerm* :: *name list* ⇒ *name list* ⇒ *name prm*
where
  Base: *composePerm* [] [] = []
| *Step*: *composePerm* (*x*#*xs*) (*y*#*ys*) = (*x*, *y*)#(*composePerm xs ys*)
| *Empty*: *composePerm* - -= []

lemma *composePermInduct*[*consumes 1*, *case-names cBase cStep*]:
  fixes *xvec* :: *name list*
  and *yvec* :: *name list*
  and *P* :: *name list* ⇒ *name list* ⇒ *bool*

  assumes *L*: *length xvec* = *length yvec*
  and *rBase*: *P* [] []
  and *rStep*: ⋀*x xvec y yvec*. ⟦*length xvec* = *length yvec*; *P xvec yvec*⟧ ⟹ *P* (*x* # *xvec*) (*y* # *yvec*)

  shows *P xvec yvec*
using *assms*
by(*induct rule*: *composePerm.induct*) *auto*

lemma *composePermEqvt*[*eqvt*]:
  fixes *p* :: *name prm*
  and *xvec* :: *name list*
  and *yvec* :: *name list*

shows $(p \cdot (composePerm\ xvec\ yvec)) = composePerm\ (p \cdot xvec)\ (p \cdot yvec)$
by(*induct xvec yvec rule*: *composePerm.induct*) *auto*

abbreviation
composePermJudge ([- -] $\cdot_v$ - [*80*, *80*, *80*] *80*) **where** $[xvec\ yvec] \cdot_v p \equiv (composePerm\ xvec\ yvec) \cdot p$

abbreviation
composePermInvJudge ([- -]$^-$ $\cdot_v$ - [*80*, *80*, *80*] *80*) **where** $[xvec\ yvec]^- \cdot_v p \equiv (rev\ (composePerm\ xvec\ yvec)) \cdot p$

lemma *permChainSimps*[*simp*]:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *perm* :: *name prm*
and *p* :: *'a::pt-name*

shows $((composePerm\ xvec\ yvec) @ perm) \cdot p = [xvec\ yvec] \cdot_v (perm \cdot p)$
by(*simp add*: *pt2*[*OF pt-name-inst*])

lemma *permChainEqvt*[*eqvt*]:
fixes *p* :: *name prm*
and *xvec* :: *name list*
and *yvec* :: *name list*
and *x* :: *'a::pt-name*

shows $(p \cdot ([xvec\ yvec] \cdot_v x)) = [(p \cdot xvec)\ (p \cdot yvec)] \cdot_v (p \cdot x)$
and $(p \cdot ([xvec\ yvec]^- \cdot_v x)) = [(p \cdot xvec)\ (p \cdot yvec)]^- \cdot_v (p \cdot x)$
by(*subst pt-perm-compose*[*OF pt-name-inst*, *OF at-name-inst*], *simp add*: *eqvts rev-eqvt*)+

lemma *permChainBij*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *p* :: *'a::pt-name*
and *q* :: *'a::pt-name*

assumes *length xvec* = *length yvec*

shows $(([xvec\ yvec] \cdot_v p) = ([xvec\ yvec] \cdot_v q)) = (p = q)$
and $(([xvec\ yvec]^- \cdot_v p) = ([xvec\ yvec]^- \cdot_v q)) = (p = q)$
using *assms*
by(*induct rule*: *composePermInduct*)
(*auto simp add*: *pt-bij*[*OF pt-name-inst*, *OF at-name-inst*])

lemma *permChainAppend*:
fixes *xvec1* :: *name list*
and *yvec1* :: *name list*

and *xvec2* :: *name list*
and *yvec2* :: *name list*
and *p* :: *'a::pt-name*

assumes *length xvec1* = *length yvec1*

shows $([(xvec1@xvec2)\ (yvec1@yvec2)] \cdot_v p) = [xvec1\ yvec1] \cdot_v [xvec2\ yvec2] \cdot_v p$
and $([(xvec1@xvec2)\ (yvec1@yvec2)]^- \cdot_v p) = [xvec2\ yvec2]^- \cdot_v [xvec1\ yvec1]^- \cdot_v p$
using *assms*
by(*induct arbitrary*: *p rule*: *composePermInduct*, *auto*) (*simp add*: *pt2*[*OF pt-name-inst*])

lemma *calcChainAtom*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *x* :: *name*

assumes *length xvec* = *length yvec*
and $x \sharp xvec$
and $x \sharp yvec$

shows $[xvec\ yvec] \cdot_v x = x$
using *assms*
by(*induct rule*: *composePermInduct*, *auto*)

lemma *calcChainAtomRev*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *x* :: *name*

assumes *length xvec* = *length yvec*
and $x \sharp xvec$
and $x \sharp yvec$

shows $[xvec\ yvec]^- \cdot_v x = x$
using *assms*
by(*induct rule*: *composePermInduct*, *auto*)
(*auto simp add*: *pt2*[*OF pt-name-inst*] *fresh-list-cons calc-atm*)

lemma *permChainFresh*[*simp*]:
fixes *x* :: *name*
and *xvec* :: *name list*
and *yvec* :: *name list*
and *p* :: *'a::pt-name*

assumes $x \sharp xvec$
and $x \sharp yvec$
and *length xvec* = *length yvec*

shows $x \sharp [xvec\ yvec] \cdot_v p = x \sharp p$
and $x \sharp [xvec\ yvec]^- \cdot_v p = x \sharp p$
using *assms*
by(*auto simp add*: *fresh-left calcChainAtomRev calcChainAtom*)

lemma *chainFreshFresh*:
fixes x :: *name*
and y :: *name*
and $xvec$ :: *name list*
and p :: $'a$::*pt-name*

assumes $x \sharp xvec$
and $y \sharp xvec$

shows $xvec \sharp* ([(x, y)] \cdot p) = (xvec \sharp* p)$
using *assms*
by(*induct xvec*) (*auto simp add*: *fresh-list-cons fresh-left*)

lemma *permChainFreshFresh*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and p :: $'a$::*pt-name*

assumes $xvec \sharp* p$
and $yvec \sharp* p$
and *length xvec* = *length yvec*

shows $[xvec\ yvec] \cdot_v p = p$
and $[xvec\ yvec]^- \cdot_v p = p$
using *assms*
by(*induct rule*: *composePerm.induct*, *auto*) (*simp add*: *pt2*[*OF pt-name-inst*])

lemma *setFresh*[*simp*]:
fixes x :: *name*
and *xvec* :: *name list*

shows $x \notin set\ xvec = x \sharp xvec$
by(*simp add*: *name-list-supp fresh-def*)

lemma *calcChain*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*

assumes $yvec \sharp* xvec$
and *length xvec* = *length yvec*
and *distinct xvec*
and *distinct yvec*

```
  shows [xvec yvec] ∙v xvec = yvec
using assms
by(induct xvec yvec rule: composePerm.induct, auto)
  (subst consPerm, simp add: calcChainAtom calc-atm name-list-supp fresh-def[symmetric])+

lemma freshChainPerm:
  fixes xvec :: name list
  and   yvec :: name list
  and   x    :: name
  and   C    :: 'a::pt-name

  assumes length xvec = length yvec
  and     yvec ♯* C
  and     xvec ♯* yvec
  and     x mem xvec
  and     distinct yvec

  shows x ♯ [xvec yvec] ∙v C
using assms
proof(induct rule: composePermInduct)
  case cBase
  have x mem [] by fact
  hence False by simp
  thus ?case by simp
next
  case(cStep x' xvec y yvec)
  have (y # yvec) ♯* C by fact
  hence yFreshC: y ♯ C and yvecFreshp: yvec ♯* C by simp+
  have (x' # xvec) ♯* (y # yvec) by fact
  hence x'ineqy: x' ≠ y and xvecFreshyvec: xvec ♯* yvec
    and x'Freshyvec: x' ♯ yvec and yFreshxvec: y ♯ xvec
    by(auto simp add: fresh-list-cons)
  have distinct (y#yvec) by fact
  hence yFreshyvec: y ♯ yvec and yvecDist: distinct yvec
    by simp+
  have L: length xvec = length yvec by fact
  have x ♯ [(x', y)] ∙ [xvec yvec] ∙v C
  proof(case-tac x = x')
    assume xeqx': x = x'
    moreover from yFreshxvec yFreshyvec yFreshC L have y ♯ [xvec yvec] ∙v C
      by simp
    hence ([(x, y)] ∙ y) ♯ [(x, y)] ∙ [xvec yvec] ∙v C
      by(rule pt-fresh-bij1[OF pt-name-inst, OF at-name-inst])
    with x'ineqy xeqx' show ?thesis by(simp add: calc-atm)
  next
    assume xineqx': x ≠ x'
    have x mem (x' # xvec) by fact
    with xineqx' have xmemxvec: x mem xvec by simp
    moreover have ⟦yvec ♯* C; xvec ♯* yvec; x mem xvec; distinct yvec⟧ ⟹ x ♯
```

```
[xvec yvec] •v C by fact
    ultimately have x ♯ [xvec yvec] •v C using yvecFreshp xvecFreshyvec yvecDist
      by simp
    hence ([(x', y)] • x) ♯ [(x', y)] • [xvec yvec] •v C
      by(rule pt-fresh-bij1[OF pt-name-inst, OF at-name-inst])
    moreover from xmemxvec yFreshxvec have x ≠ y
      by(induct xvec) (auto simp add: fresh-list-cons)
    ultimately show ?thesis using xineqx' x'ineqy by(simp add: calc-atm)
  qed
  thus ?case by simp
qed

lemma memFreshSimp[simp]:
  fixes y    :: name
  and   yvec :: name list

  shows (¬(y mem yvec)) = y ♯ yvec
by(induct yvec)
  (auto simp add: fresh-list-nil fresh-list-cons)

lemma freshChainPerm':
  fixes xvec :: name list
  and   yvec :: name list
  and   p    :: 'a::pt-name

  assumes length xvec = length yvec
  and     yvec ♯* p
  and     xvec ♯* yvec
  and     distinct yvec

  shows xvec ♯* ([xvec yvec] •v p)
using assms
proof(induct rule: composePermInduct)
  case cBase
  show ?case by simp
next
  case(cStep x xvec y yvec)
  have (y # yvec) ♯* p by fact
  hence yFreshp: y ♯ p and yvecFreshp: yvec ♯* p
    by simp+
  moreover have (x # xvec) ♯* (y # yvec) by fact
  hence xineqy: x ≠ y and xvecFreshyvec: xvec ♯* yvec
    and xFreshyvec: x ♯ yvec and yFreshxvec: y ♯ xvec
    by(auto simp add: fresh-list-cons)
  have distinct (y # yvec) by fact
  hence yFreshyvec: y ♯ yvec and yvecDist: distinct yvec
    by simp+
  have L: length xvec = length yvec by fact
  have ⟦yvec ♯* p; xvec ♯* yvec; distinct yvec⟧ ⟹ xvec ♯* ([xvec yvec] •v p) by fact
```

```
  with yvecFreshp xvecFreshyvec yvecDist have IH: xvec ♯* ([xvec yvec] •v p) by
simp
  show ?case
  proof(auto)
   from L yFreshp yvecFreshp xineqy xvecFreshyvec yvecDist yFreshyvec yFreshxvec
xFreshyvec
    have x ♯ [(x # xvec) (y # yvec)] •v p
      by(rule-tac freshChainPerm) (auto simp add: fresh-list-cons)
    thus x ♯ [(x, y)] • [xvec yvec] •v p by simp
  next
    show xvec ♯* ([(x, y)] • [xvec yvec] •v p)
    proof(case-tac x mem xvec)
      assume x mem xvec
      with L yvecFreshp xvecFreshyvec yvecDist xFreshyvec
      havex ♯ [xvec yvec] •v p
        by(rule-tac freshChainPerm) (auto simp add: fresh-list-cons)
      moreover from yFreshxvec yFreshyvec yFreshp L
      have y ♯ [xvec yvec] •v p by simp
      ultimately show ?thesis using IH
        by(subst consPerm) (simp add: perm-fresh-fresh)
    next
      assume ¬(x mem xvec)
      hence xFreshxvec: x ♯ xvec by simp
      from IH have ([(x, y)] • xvec) ♯* ([(x, y)] • [xvec yvec] •v p)
        by(simp add: pt-fresh-star-bij[OF pt-name-inst, OF at-name-inst])
      with xFreshxvec yFreshxvec show ?thesis by simp
    qed
  qed
qed

lemma permSym:
  fixes x    :: name
  and   y    :: name
  and   xvec :: name list
  and   yvec :: name list
  and   p    :: 'a::pt-name

  assumes x ♯ xvec
  and     x ♯ yvec
  and     y ♯ xvec
  and     y ♯ yvec
  and     length xvec = length yvec

  shows ([(x, y)] • [xvec yvec] •v p) = [xvec yvec] •v [(x, y)] • p
using assms
apply(induct rule: composePerm.induct, auto)
by(subst pt-perm-compose[OF pt-name-inst, OF at-name-inst]) simp

lemma distinctPermClosed[simp]:
```

fixes p :: *name prm*
and q :: *name prm*

assumes *distinctPerm p*

shows *distinctPerm*($q \cdot p$)
using *assms*
by(*induct p*) (*auto simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*] *dest*: *pt-bij4*[*OF pt-name-inst*, *OF at-name-inst*])

lemma *freshStarSimps*:
fixes x :: *name*
and Xs :: *name set*
and Ys :: *name set*
and C :: *'a::fs-name*
and p :: *name prm*

assumes *set* $p \subseteq Xs \times Ys$
and $Xs \sharp* x$
and $Ys \sharp* x$

shows $x \sharp (p \cdot C) = x \sharp C$
using *assms*
by(*subst pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, *of - C p*]) *simp*

lemma *freshStarChainSimps*:
fixes *xvec* :: *name list*
and Xs :: *name set*
and Ys :: *name set*
and C :: *'a::fs-name*
and p :: *name prm*

assumes *set* $p \subseteq Xs \times Ys$
and $Xs \sharp* xvec$
and $Ys \sharp* xvec$

shows $xvec \sharp* (p \cdot C) = xvec \sharp* C$
using *assms*
by(*induct xvec*) (*auto simp add*: *freshStarSimps*)

lemma *permStarFresh*:
fixes *xvec* :: *name list*
and p :: *name prm*
and T :: *'a::pt-name*

assumes $xvec \sharp* p$

shows $xvec \sharp* (p \cdot T) = xvec \sharp* T$

using *assms*
by(*induct p*) (*auto simp add*: *chainFreshFresh*)

lemma *swapStarFresh*:
  fixes x :: *name*
  and p :: *name prm*
  and T :: $'a$::*pt-name*

  assumes $x \sharp p$

  shows $x \sharp (p \cdot T) = x \sharp T$
proof −
  from *assms* **have** $[x] \sharp* (p \cdot T) = [x] \sharp* T$
    by(*rule-tac permStarFresh*) *auto*
  thus *?thesis* **by** *simp*
qed

lemmas *freshChainSimps* = *freshStarSimps freshStarChainSimps permStarFresh swapStarFresh chainFreshFresh freshPerm subsetFresh*

lemma *freshAlphaPerm*:
  fixes *xvec* :: *name list*
  and *Xs* :: *name set*
  and *Ys* :: *name set*
  and p :: *name prm*

  assumes *S*: *set p* $\subseteq$ *Xs* $\times$ *Ys*
  and *Xs* $\sharp*$ *xvec*
  and *Ys* $\sharp*$ *xvec*

  shows *xvec* $\sharp*$ p
using *assms*
apply(*induct p*)
by *auto* (*simp add*: *fresh-star-def fresh-def name-list-supp supp-list-nil*)+

lemma *freshAlphaSwap*:
  fixes x :: *name*
  and *Xs* :: *name set*
  and *Ys* :: *name set*
  and p :: *name prm*

  assumes *S*: *set p* $\subseteq$ *Xs* $\times$ *Ys*
  and *Xs* $\sharp*$ x
  and *Ys* $\sharp*$ x

  shows $x \sharp p$
proof −
  from *assms* **have** $[x] \sharp* p$
    apply(*rule-tac freshAlphaPerm*)

apply *assumption*
by *auto*
thus *?thesis* **by** *simp*
qed

lemma *setToListFresh*[*simp*]:
fixes *xvec* :: *name list*
and *C* :: *'a*::*fs-name*
and *yvec* :: *name list*
and *Xs* :: *name set*
and *x* :: *name*

shows *xvec* $\sharp*$ (*set yvec*) = *xvec* $\sharp*$ *yvec*
and *Xs* $\sharp*$ (*set yvec*) = *Xs* $\sharp*$ *yvec*
and *x* $\sharp$ (*set yvec*) = *x* $\sharp$ *yvec*
and *set xvec* $\sharp*$ *Xs* = *xvec* $\sharp*$ *Xs*
by(*auto simp add*: *fresh-star-def name-list-supp fresh-def fs-name1 at-fin-set-supp*[*OF at-name-inst*])

end

theory *Subst-Term*
imports *Chain*
begin

locale *substType* =
fixes *subst* :: *'a*::*fs-name* ⇒ *name list* ⇒ *'b*::*fs-name list* ⇒ *'a* (-[-::=-] [*80*, *80*,*80*] *130*)

assumes *eq*[*eqvt*]: ⋀*p*::*name prm*. (*p* ∙ (*M*[*xvec*::=*Tvec*])) = ((*p* ∙ *M*)[(*p* ∙ *xvec*)::=(*p* ∙ *Tvec*)])

and *subst3*: ⋀*xvec Tvec T x*. ⟦*length xvec* = *length Tvec*; *distinct xvec*; *set*(*xvec*) ⊆ *supp*(*T*); (*x*::*name*) $\sharp$ *T*[*xvec*::=*Tvec*]⟧ ⟹ *x* $\sharp$ *Tvec*

and *renaming*: ⋀*xvec Tvec p T*. ⟦*length xvec* = *length Tvec*; (*set p*) ⊆ *set xvec* × *set* (*p* ∙ *xvec*);
distinctPerm p; (*p* ∙ *xvec*) $\sharp*$ *T*⟧ ⟹
T[*xvec*::=*Tvec*] = (*p* ∙ *T*)[(*p* ∙ *xvec*)::=*Tvec*]
begin

lemma *suppSubst*:
fixes *M* :: *'a*
and *xvec* :: *name list*
and *Tvec* :: *'b list*

shows (*supp*(*M*[*xvec*::=*Tvec*])::*name set*) ⊆ ((*supp M*) ∪ (*supp xvec*) ∪ (*supp Tvec*))
proof(*auto simp add*: *eqvts supp-def*)

fix x::*name*
let *?P* = $\lambda y.\ ([(x,\ y)] \cdot M)[([(x,\ y)] \cdot xvec)::=([(x,\ y)] \cdot Tvec)] \neq M[xvec::=Tvec]$
let *?Q* = $\lambda y\ M.\ ([(x,\ y)] \cdot M) \neq (M::'a)$
let *?R* = $\lambda y\ xvec.\ ([(x,\ y)] \cdot xvec) \neq (xvec::name\ list)$
let *?S* = $\lambda y\ Tvec.\ ([(x,\ y)] \cdot Tvec) \neq (Tvec::'b\ list)$
assume *A*: *finite* {*y*. *?Q y M*} **and** *B*: *finite* {*y*. *?R y xvec*} **and** *C*: *finite* {*y*. *?S y Tvec*} **and** *D*: *infinite* {*y*. *?P*(*y*)}
hence *infinite*({*y*. *?P*(*y*)} − {*y*. *?Q y M*} − {*y*. *?R y xvec*} − {*y*. *?S y Tvec*})
by(*auto intro*: *Diff-infinite-finite*)
hence *infinite*({*y*. *?P*(*y*) ∧ ¬(*?Q y M*) ∧ ¬ (*?R y xvec*) ∧ ¬ (*?S y Tvec*)})
by(*simp add*: *set-diff-eq*)
moreover have {*y*. *?P*(*y*) ∧ ¬(*?Q y M*) ∧ ¬ (*?R y xvec*) ∧ ¬ (*?S y Tvec*)} = {} **by** *auto*
ultimately have *infinite* {} **by**(*drule-tac Infinite-cong*) *auto*
thus *False* **by** *simp*
qed

lemma *subst2*[*intro*]:
fixes x :: *name*
and M :: $'a$
and *xvec* :: *name list*
and *Tvec* :: $'b$ *list*

assumes $x \sharp M$
and $x \sharp xvec$
and $x \sharp Tvec$

shows $x \sharp M[xvec::=Tvec]$
using *assms suppSubst*
by(*auto simp add*: *fresh-def*)

lemma *subst2Chain*[*intro*]:
fixes *yvec* :: *name list*
and M :: $'a$
and *xvec* :: *name list*
and *Tvec* :: $'b$ *list*

assumes $yvec \sharp* M$
and $yvec \sharp* xvec$
and $yvec \sharp* Tvec$

shows $yvec \sharp* M[xvec::=Tvec]$
using *assms*
by(*induct yvec*) *auto*

lemma *fs*[*simp*]: *finite* ((*supp subst*)::*name set*)
by(*simp add*: *supp-def perm-fun-def eqvts*)

lemma *subst3Chain*:

```
  fixes xvec :: name list
  and   Tvec :: 'b list
  and   Xs   :: name set
  and   T    :: 'a

  assumes length xvec = length Tvec
  and     distinct xvec
  and     set xvec ⊆ supp T
  and     Xs ♯* T[xvec::=Tvec]

  shows Xs ♯* Tvec
using assms
by(auto intro: subst3 simp add: fresh-star-def)

lemma subst4Chain:
  fixes xvec :: name list
  and   Tvec :: 'b list
  and   T    :: 'a

  assumes length xvec = length Tvec
  and     distinct xvec
  and     xvec ♯* Tvec

  shows xvec ♯* T[xvec::=Tvec]
proof −
  obtain p where ((p::name prm) · (xvec::name list)) ♯* T and (p · xvec) ♯* xvec
            and S: (set p) ⊆ set xvec × set (p · xvec)
            and distinctPerm p
    by(rule-tac xvec=xvec and c=(T, xvec) in name-list-avoiding) auto

  from ‹length xvec = length Tvec› have length(p · xvec) = length Tvec by simp
  moreover from ‹(p · xvec) ♯* T› have (p · p · xvec) ♯* (p · T)
    by(simp add: pt-fresh-star-bij[OF pt-name-inst, OF at-name-inst])
  with ‹distinctPerm p› have xvec ♯* (p · T) by simp
  ultimately have (set xvec) ♯* (p · T)[(p · xvec)::=Tvec] using ‹xvec ♯* Tvec›
‹(p · xvec) ♯* xvec›
    by auto

  thus ?thesis using ‹length xvec = length Tvec› ‹distinct xvec› S ‹(p · xvec) ♯*
T› ‹distinctPerm p›
    by(simp add: renaming)
qed

definition seqSubst :: 'a ⇒ (name list × 'b list) list ⇒ 'a (-[<->] [80, 80] 130)
  where M[<σ>] ≡ foldl (λN. λ(xvec, Tvec). N[xvec::=Tvec]) M σ

lemma seqSubstNil[simp]:
  seqSubst M [] = M
by(simp add: seqSubst-def)
```

lemma *seqSubstCons*[*simp*]:
shows *seqSubst M* ((*xvec*, *Tvec*)#σ) = *seqSubst*(*M*[*xvec*::=*Tvec*]) σ
by(*simp add*: *seqSubst-def*)

lemma *seqSubstTermAppend*[*simp*]:
shows *seqSubst M* (σ@σ') = *seqSubst* (*seqSubst M* σ) σ'
by(*induct* σ) (*auto simp add*: *seqSubst-def*)

definition *wellFormedSubst* :: ((*'d*::*fs-name*) *list* × (*'e*::*fs-name*) *list*) *list* ⇒ *bool*
where *wellFormedSubst* σ = ((*filter* (λ(*xvec*, *Tvec*). ¬(*length xvec* = *length Tvec* ∧ *distinct xvec*)) σ) = [])

lemma *wellFormedSubstEqvt*[*eqvt*]:
fixes σ :: ((*'d*::*fs-name*) *list* × (*'e*::*fs-name*) *list*) *list*
and *p* :: *name prm*

shows *p* ∙ (*wellFormedSubst* σ) = *wellFormedSubst*(*p* ∙ σ)
by(*induct* σ *arbitrary*: *p*) (*auto simp add*: *eqvts wellFormedSubst-def*)

lemma *wellFormedSimp*[*simp*]:
fixes σ :: ((*'d*::*fs-name*) *list* × (*'e*::*fs-name*) *list*) *list*
and *p* :: *name prm*

shows *wellFormedSubst*(*p* ∙ σ) = *wellFormedSubst* σ
by(*induct* σ) (*auto simp add*: *eqvts wellFormedSubst-def*)

lemma *wellFormedNil*[*simp*]:
wellFormedSubst []
by(*simp add*: *wellFormedSubst-def*)

lemma *wellFormedCons*[*simp*]:
shows *wellFormedSubst*((*xvec*, *Tvec*)#σ) = (*length xvec* = *length Tvec* ∧ *distinct xvec* ∧ *wellFormedSubst* σ)
by(*simp add*: *wellFormedSubst-def*) *auto*

lemma *wellFormedAppend*[*simp*]:
fixes σ :: ((*'d*::*fs-name*) *list* × (*'e*::*fs-name*) *list*) *list*
and σ' :: ((*'d*::*fs-name*) *list* × (*'e*::*fs-name*) *list*) *list*

shows *wellFormedSubst*(σ@σ') = (*wellFormedSubst* σ ∧ *wellFormedSubst* σ')
by(*simp add*: *wellFormedSubst-def*)

lemma *seqSubst2*[*intro*]:
fixes σ :: (*name list* × *'b list*) *list*
and *T* :: *'a*
and *x* :: *name*

assumes *x* ♯ σ

and $x \sharp T$

shows $x \sharp T[<\sigma>]$
using *assms*
by(*induct* σ *arbitrary*: T) (*clarsimp* | *blast*)+

lemma *seqSubst2Chain*[*intro*]:
fixes σ :: (*name list* × *'b list*) *list*
and T :: *'a*
and *xvec* :: *name list*

assumes *xvec* $\sharp*$ σ
and *xvec* $\sharp*$ T

shows *xvec* $\sharp*$ $T[<\sigma>]$
using *assms*
by(*induct xvec*) *auto*

end

end

theory *Agent*
imports *Subst-Term*
begin

nominal-datatype (*'term*, *'assertion*, *'condition*) *psi* =
PsiNil (**0** *190*)

| *Output 'term*::*fs-name 'term* (*'term*, *'assertion*::*fs-name*, *'condition*::*fs-name*) *psi*
(-⟨-⟩.- [*120*, *120*, *110*] *110*)
| *Input 'term* (*'term*, *'assertion*, *'condition*) *input* (-(|-
[*120*, *120*] *110*)
| *Case* ((*'term*, *'assertion*, *'condition*) *psiCase*) (*Case*
- [*120*] *120*)
| *Par* (*'term*, *'assertion*, *'condition*) *psi* (*'term*, *'assertion*, *'condition*) *psi* (**infixl**
|| *90*)
| *Res* «*name*»((*'term*, *'assertion*, *'condition*) *psi*) ((|ν-|)-
[*120*, *120*] *110*)
| *Assert 'assertion* ({|-|}
[*120*] *120*)
| *Bang* (*'term*, *'assertion*, *'condition*) *psi* (!-
[*110*] *110*)

and (*'term*, *'assertion*, *'condition*) *input* =
Trm 'term ((*'term*, *'assertion*, *'condition*) *psi*) (|)-.-
[*130*, *130*] *130*)
| *Bind* «*name*»((*'term*, *'assertion*, *'condition*) *input*) (ν--

[*120*, *120*] *120*)

and ('*term*, '*assertion*, '*condition*) *psiCase* =
  EmptyCase (⊥$_c$ *120*)
| *Cond* '*condition* (('*term*, '*assertion*, '*condition*) *psi*)
  (('*term*, '*assertion*, '*condition*) *psiCase*) (□ -
⇒ - - [*120*, *120*, *120*] *120*)

lemma *psiFreshSet*[*simp*]:
  fixes X :: *name set*
  and M :: '*a*::*fs-name*
  and N :: '*a*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
  and I :: ('*a*, '*b*, '*c*) *input*
  and C :: ('*a*, '*b*, '*c*) *psiCase*
  and Q :: ('*a*, '*b*, '*c*) *psi*
  and x :: *name*
  and Ψ :: '*b*
  and Φ :: '*c*

  shows $X \sharp* (M\langle N\rangle.P) = (X \sharp* M \wedge X \sharp* N \wedge X \sharp* P)$
  and $X \sharp* M(\!|I = (X \sharp* M \wedge X \sharp* I)$
  and $X \sharp* Case\ C = X \sharp* C$
  and $X \sharp* (P \parallel Q) = (X \sharp* P \wedge X \sharp* Q)$
  and $X \sharp* (|\nu x|)P = (X \sharp* [x].P)$
  and $X \sharp* \{\!|\Psi|\!\} = X \sharp* \Psi$
  and $X \sharp* !P = X \sharp* P$
  and $X \sharp* \mathbf{0}$
  and $X \sharp* Trm\ N\ P = (X \sharp* N \wedge X \sharp* P)$
  and $X \sharp* Bind\ x\ I = X \sharp* ([x].I)$

  and $X \sharp* \bot_c$
  and $X \sharp* \Box\ \Phi \Rightarrow P\ C = (X \sharp* \Phi \wedge X \sharp* P \wedge X \sharp* C)$
by(*auto simp add*: *fresh-star-def psi.fresh*)+

lemma *psiFreshVec*[*simp*]:
  fixes *xvec* :: *name list*

  shows $xvec \sharp* (M\langle N\rangle.P) = (xvec \sharp* M \wedge xvec \sharp* N \wedge xvec \sharp* P)$
  and $xvec \sharp* M(\!|I = (xvec \sharp* M \wedge xvec \sharp* I)$
  and $xvec \sharp* Case\ C = xvec \sharp* C$
  and $xvec \sharp* (P \parallel Q) = (xvec \sharp* P \wedge xvec \sharp* Q)$
  and $xvec \sharp* (|\nu x|)P = (xvec \sharp* [x].P)$
  and $xvec \sharp* \{\!|\Psi|\!\} = xvec \sharp* \Psi$
  and $xvec \sharp* !P = xvec \sharp* P$
  and $xvec \sharp* \mathbf{0}$

  and $xvec \sharp* Trm\ N\ P = (xvec \sharp* N \wedge xvec \sharp* P)$
  and $xvec \sharp* Bind\ x\ I = xvec \sharp* ([x].I)$

and *xvec* $\sharp*$ $\bot_c$
and *xvec* $\sharp*$ $\Box$ Φ $\Rightarrow$ *P C* = (*xvec* $\sharp*$ Φ $\wedge$ *xvec* $\sharp*$ *P* $\wedge$ *xvec* $\sharp*$ *C*)
by(*auto simp add*: *fresh-star-def*)

fun *psiCases* :: ('*c*::*fs-name* × ('*a*::*fs-name*, '*b*::*fs-name*, '*c*) *psi*) *list* ⇒ ('*a*, '*b*, '*c*) *psiCase*
where
base: *psiCases* [] = $\bot_c$
| *step*: *psiCases* ((Φ, *P*)#*xs*) = *Cond* Φ *P* (*psiCases xs*)

lemma *psiCasesEqvt*[*eqvt*]:
fixes *p* :: *name prm*
and *Cs* :: ('*c*::*fs-name* × ('*a*::*fs-name*, '*b*::*fs-name*, '*c*) *psi*) *list*

shows (*p* ∙ (*psiCases Cs*)) = *psiCases*(*p* ∙ *Cs*)
by(*induct Cs*) *auto*

lemma *psiCasesFresh*[*simp*]:
fixes *x* :: *name*
and *Cs* :: ('*c*::*fs-name* × ('*a*::*fs-name*, '*b*::*fs-name*, '*c*) *psi*) *list*

shows *x* $\sharp$ *psiCases Cs* = *x* $\sharp$ *Cs*
by(*induct Cs*)
(*auto simp add*: *fresh-list-nil fresh-list-cons*)

lemma *psiCasesFreshChain*[*simp*]:
fixes *xvec* :: *name list*
and *Cs* :: ('*c*::*fs-name* × ('*a*::*fs-name*, '*b*::*fs-name*, '*c*) *psi*) *list*
and *Xs* :: *name set*

shows (*xvec* $\sharp*$ *psiCases Cs*) = *xvec* $\sharp*$ *Cs*
and (*Xs* $\sharp*$ *psiCases Cs*) = *Xs* $\sharp*$ *Cs*
by(*auto simp add*: *fresh-star-def*)

abbreviation
psiCasesJudge (*Cases* - [*80*] *80*) **where** *Cases Cs* ≡ *Case*(*psiCases Cs*)

primrec *resChain* :: *name list* ⇒ ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi* ⇒ ('*a*, '*b*, '*c*) *psi* **where**
base: *resChain* [] *P* = *P*
| *step*: *resChain* (*x*#*xs*) *P* = (ν*x*)(*resChain xs P*)

notation *resChain* ((ν*-)- [*80*, *80*] *80*)

lemma *resChainEqvt*[*eqvt*]:
fixes *perm* :: *name prm*
and *lst* :: *name list*
and *P* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

shows $perm \cdot ((\!|\nu{*}xvec|\!)P) = (\!|\nu{*}(perm \cdot xvec)|\!)(perm \cdot P)$
by(*induct-tac xvec*, *auto*)

lemma *resChainSupp*:
fixes $xvec :: name\ list$
and $P :: ('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$

shows $supp((\!|\nu{*}xvec|\!)P) = (supp\ P) - set\ xvec$
by(*induct xvec*) (*auto simp add*: *psi.supp abs-supp*)

lemma *resChainFresh*:
fixes $x :: name$
and $xvec :: name\ list$
and $P :: ('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$

shows $x \sharp (\!|\nu{*}xvec|\!)P = (x \in set\ xvec \vee x \sharp P)$
by (*induct xvec*) (*simp-all add*: *abs-fresh*)

lemma *resChainFreshSet*:
fixes $Xs :: name\ set$
and $xvec :: name\ list$
and $yvec :: name\ list$
and $P :: ('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$

shows $Xs \sharp* ((\!|\nu{*}xvec|\!)P) = (\forall x{\in}Xs.\ x \in set\ xvec \vee x \sharp P)$
and $yvec \sharp* ((\!|\nu{*}xvec|\!)P) = (\forall x{\in}(set\ yvec).\ x \in set\ xvec \vee x \sharp P)$
by (*simp add*: *fresh-star-def resChainFresh*)+

lemma *resChainFreshSimps*[*simp*]:
fixes $Xs :: name\ set$
and $xvec :: name\ list$
and $P :: ('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$
and $yvec :: name\ list$

shows $Xs \sharp* xvec \Longrightarrow Xs \sharp* ((\!|\nu{*}xvec|\!)P) = (Xs \sharp* P)$
and $yvec \sharp* xvec \Longrightarrow yvec \sharp* ((\!|\nu{*}xvec|\!)P) = (yvec \sharp* P)$
and $xvec \sharp* ((\!|\nu{*}xvec|\!)P)$
apply(*simp add*: *resChainFreshSet*) **apply**(*force simp add*: *fresh-star-def name-list-supp fresh-def*)
apply(*simp add*: *resChainFreshSet*) **apply**(*force simp add*: *fresh-star-def name-list-supp fresh-def*)
by(*simp add*: *resChainFreshSet*)

lemma *resChainAlpha*:
fixes $p :: name\ prm$
and $xvec :: name\ list$
and $P :: ('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psi$

assumes *xvecFreshP*: $(p \cdot xvec) \sharp* P$
and S: $set\ p \subseteq set\ xvec \times set\ (p \cdot xvec)$

shows $(\!|\nu* xvec|\!)P = (\!|\nu*(p \cdot xvec)|\!)(p \cdot P)$
proof −
note *pt-name-inst at-name-inst S*
moreover have *set xvec* $\sharp*$ $((\!|\nu* xvec|\!)P)$
by (*simp add*: *resChainFreshSet*)
moreover from *xvecFreshP* **have** *set* $(p \cdot xvec) \sharp* ((\!|\nu* xvec|\!)P)$
by (*simp add*: *resChainFreshSet*) (*simp add*: *fresh-star-def*)
ultimately have $(\!|\nu* xvec|\!)P = p \cdot ((\!|\nu* xvec|\!)P)$
by (*rule-tac pt-freshs-freshs* [*symmetric*])
then show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *resChainAppend*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and P :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

shows $(\!|\nu*(xvec@yvec)|\!)P = (\!|\nu* xvec|\!)((\!|\nu* yvec|\!)P)$
by(*induct xvec*) *auto*

lemma *resChainSimps*[*dest*]:
fixes *xvec* :: *name list*
and P :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
and Q :: ('*a*, '*b*, '*c*) *psi*
and P' :: ('*a*, '*b*, '*c*) *psi*
and Q' :: ('*a*, '*b*, '*c*) *psi*

shows $((\!|\nu* xvec|\!)(P \parallel Q)) = P' \parallel Q' \Longrightarrow (P = P' \wedge Q = Q')$
and $(P \parallel Q = (\!|\nu* xvec|\!)(P' \parallel Q')) \Longrightarrow (P = P' \wedge Q = Q')$
by(*case-tac xvec*, *simp-all add*: *psi.inject*)+

primrec *inputChain* :: *name list* ⇒ '*a*::*fs-name* ⇒ ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi* ⇒ ('*a*, '*b*, '*c*) *input* **where**
base: *inputChain* [] N $P = (\!|N|\!).P$
| *step*: *inputChain* $(x\#xs)$ N $P = \nu\ x$ (*inputChain xs N P*)

abbreviation
inputChainJudge (-(|λ*- -|).- [*80*, *80*, *80*, *80*] *80*) **where** $M(\!|\lambda* xvec\ N|\!).P \equiv M(\!|(inputChain\ xvec\ N\ P)|\!)$

lemma *inputChainEqvt*[*eqvt*]:
fixes p :: *name prm*
and *xvec* :: *name list*
and N :: '*a*::*fs-name*
and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

shows $p \cdot (inputChain\ xvec\ N\ P) = inputChain\ (p \cdot xvec)\ (p \cdot N)\ (p \cdot P)$
by(*induct-tac xvec*) *auto*

lemma *inputChainFresh*:
fixes x :: *name*
and *xvec* :: *name list*
and N :: $'a$::*fs-name*
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

shows $x \sharp (inputChain\ xvec\ N\ P) = (x \in set\ xvec \vee (x \sharp N \wedge x \sharp P))$
by (*induct xvec*) (*simp-all add*: *abs-fresh*)

lemma *inductChainSimps*[*simp*]:
fixes *xvec* :: *name list*
and N :: $'a$::*fs-name*
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

shows *xvec* $\sharp*$ (*inputChain xvec N P*)
by(*induct xvec*) (*auto simp add*: *abs-fresh abs-fresh-star fresh-star-def*)

lemma *inputChainFreshSet*:
fixes *Xs* :: *name set*
and *xvec* :: *name list*
and N :: $'a$::*fs-name*
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

shows *Xs* $\sharp*$ (*inputChain xvec N P*) = ($\forall x \in Xs.\ x \in set\ xvec \vee (x \sharp N \wedge x \sharp P)$)
by (*simp add*: *fresh-star-def inputChainFresh*)

lemma *inputChainAlpha*:
fixes p :: *name prm*
and *Xs* :: *name set*
and *Ys* :: *name set*

assumes *XsFreshP*: *Xs* $\sharp*$ (*inputChain xvec N P*)
and *YsFreshN*: *Ys* $\sharp*$ *N*
and *YsFreshP*: *Ys* $\sharp*$ *P*
and *S*: *set p* $\subseteq$ *Xs* $\times$ *Ys*

shows (*inputChain xvec N P*) = (*inputChain* ($p \cdot xvec$) ($p \cdot N$) ($p \cdot P$))
proof −
note *pt-name-inst at-name-inst XsFreshP S*
moreover from *YsFreshN YsFreshP* **have** *Ys* $\sharp*$ (*inputChain xvec N P*)
by (*simp add*: *inputChainFreshSet*) (*simp add*: *fresh-star-def*)
ultimately have (*inputChain xvec N P*) = $p \cdot$ (*inputChain xvec N P*)
by (*rule-tac pt-freshs-freshs* [*symmetric*])
then show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *inputChainAlpha'*:
  fixes p :: *name prm*
  and *xvec* :: *name list*
  and N :: *'a::fs-name*
  and P :: *('a, 'b::fs-name, 'c::fs-name) psi*

  assumes *xvecFreshP*: $(p \cdot xvec) \sharp* P$
  and *xvecFreshN*: $(p \cdot xvec) \sharp* N$
  and S: *set* $p \subseteq$ *set xvec* $\times$ *set* $(p \cdot xvec)$

  shows $(inputChain\ xvec\ N\ P) = (inputChain\ (p \cdot xvec)\ (p \cdot N)\ (p \cdot P))$
proof −
  note *pt-name-inst at-name-inst S*
  moreover have *set xvec* $\sharp*$ $(inputChain\ xvec\ N\ P)$
    by (*simp add*: *inputChainFreshSet*)
  ultimately show *?thesis* **using** *xvecFreshN xvecFreshP*
    by(*rule-tac inputChainAlpha*) (*simp add*: *fresh-star-def*)+
qed

lemma *alphaRes*:
  fixes M :: *'a::fs-name*
  and x :: *name*
  and P :: *('a, 'b::fs-name, 'c::fs-name) psi*
  and y :: *name*

  assumes *yFreshP*: $y \sharp P$

  shows $(\!|\nu x|\!)P = (\!|\nu y|\!)([(x, y)] \cdot P)$
proof(*cases* $x = y$)
  assume *x=y*
  thus *?thesis* **by** *simp*
next
  assume $x \neq y$
  with *yFreshP* **show** *?thesis*
    by(*perm-simp add*: *psi.inject alpha calc-atm fresh-left*)
qed

lemma *alphaInput*:
  fixes x :: *name*
  and I :: *('a::fs-name, 'b::fs-name, 'c::fs-name) input*
  and c :: *name*

  assumes *A1*: $c \sharp I$

  shows $\nu\ x\ I = \nu\ c([(x, c)] \cdot I)$
proof(*cases* $x = c$)
  assume *x=c*
  thus *?thesis* **by** *simp*
next

assume $x \neq c$
with *A1* **show** *?thesis*
by(*perm-simp add*: *input.inject alpha calc-atm fresh-left*)
qed

lemma *inputChainLengthEq*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *M* :: *'a*::*fs-name*
and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

assumes *length xvec* = *length yvec*
and *xvec* $\sharp*$ *yvec*
and *distinct yvec*
and *yvec* $\sharp*$ *M*
and *yvec* $\sharp*$ *P*

obtains *N Q* **where** *inputChain xvec M P* = *inputChain yvec N Q*
proof −
assume $\bigwedge$*N Q*. *inputChain xvec M P* = *inputChain yvec N Q* $\Longrightarrow$ *thesis*
moreover obtain *n* **where** *n* = *length xvec* **by** *auto*
with *assms* **have** $\exists$ *N Q*. *inputChain xvec M P* = *inputChain yvec N Q*
proof(*induct n arbitrary*: *xvec yvec M P*)
case *0*
thus *?case* **by** *auto*
next
case(*Suc n xvec yvec M P*)
from ‹*Suc n* = *length xvec*›
obtain *x xvec'* **where** *xvec* = *x*#*xvec'* **and** *length xvec'* = *n*
by(*case-tac xvec*) *auto*
with ‹*length xvec* = *length yvec*›
obtain *y yvec'* **where** *yvec* = *y*#*yvec'* **by**(*case-tac yvec*) *auto*
from ‹*yvec* = *y*#*yvec'*› ‹*xvec*=*x*#*xvec'*› ‹*xvec* $\sharp*$ *yvec*› ‹*distinct yvec*› ‹*length xvec* = *length yvec*› ‹*yvec* $\sharp*$ *M*› ‹*yvec* $\sharp*$ *P*›
have *length xvec'* = *length yvec'* **and** *xvec'* $\sharp*$ *yvec'* **and** *distinct yvec'* **and** *yvec'* $\sharp*$ *M* **and** *yvec'* $\sharp*$ *P*
by *simp*+
then obtain *N Q* **where** *Eq*: *inputChain xvec' M P* = *inputChain yvec' N Q* **using** ‹*length xvec'* = *n*›
by(*drule-tac Suc*) *auto*
moreover from ‹*distinct yvec*› ‹*yvec* = *y*#*yvec'*› **have** *y* $\sharp$ *yvec'* **by** *auto*
moreover from ‹*xvec* $\sharp*$ *yvec*› ‹*xvec* = *x*#*xvec'*› ‹*yvec*=*y*#*yvec'*› **have** *x* $\neq$ *y* **and** *x* $\sharp$ *yvec'*
by *auto*
moreover from ‹*yvec* $\sharp*$ *M*› ‹*yvec* $\sharp*$ *P*› ‹*yvec* = *y*#*yvec'*› **have** *y* $\sharp$ *M* **and** *y* $\sharp$ *P* **by** *auto*
hence *y* $\sharp$ *inputChain xvec' M P* **by**(*simp add*: *inputChainFresh*)
with *Eq* **have** *y* $\sharp$ *inputChain yvec' N Q* **by**(*simp add*: *inputChainFresh*)
ultimately have ν *x* (*inputChain xvec' M P*) = ν *y* (*inputChain yvec'* ([(*x*,

```
y)] • N) ([(x, y)] • Q))
      by(simp add: input.inject alpha' eqvts name-swap)
    thus ?case using ‹xvec = x#xvec'› ‹yvec=y#yvec'› by force
  qed
  ultimately show ?thesis
    by blast
qed

lemma inputChainEq:
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a, 'b, 'c) psi

  assumes inputChain xvec M P = inputChain yvec N Q
  and     xvec ♯* yvec
  and     distinct xvec
  and     distinct yvec

  obtains p where (set p) ⊆ (set xvec) × set (p • xvec) and distinctPerm p and
yvec = p • xvec and N = p • M and Q = p • P
proof −
  assume ⋀p. ⟦set p ⊆ set xvec × set (p • xvec); distinctPerm p; yvec = p • xvec;
N = p • M; Q = p • P⟧ ⟹ thesis
  moreover obtain n where n = length xvec by auto
  with assms have ∃p. (set p) ⊆ (set xvec) × set (yvec) ∧ distinctPerm p ∧  yvec
= p • xvec ∧ N = p • M ∧ Q = p • P
  proof(induct n arbitrary: xvec yvec M N P Q)
    case(0 xvec yvec M N P Q)
    have Eq: inputChain xvec M P = inputChain yvec N Q by fact
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with Eq have yvec = []
      by(case-tac yvec) auto
    ultimately show ?case using Eq
      by(simp add: input.inject)
  next
    case(Suc n xvec yvec M N P Q)
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹inputChain xvec M P = inputChain yvec N Q› ‹xvec = x # xvec'›
    obtain y yvec' where inputChain (x#xvec') M P = inputChain (y#yvec') N
Q
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence EQ: ν x (inputChain xvec' M P) = ν y (inputChain yvec' N Q)
      by simp
```

from ‹*xvec* = *x*#*xvec'*› ‹*yvec*=*y*#*yvec'*› ‹*xvec* ♯* *yvec*›
have $x \neq y$ **and** $xvec' \sharp^* yvec'$ **and** $x \sharp yvec'$ **and** $y \sharp xvec'$
by(*auto simp add*: *fresh-list-cons*)
from ‹*distinct xvec*› ‹*distinct yvec*› ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*› **have** $x \sharp xvec'$ **and** $y \sharp yvec'$ **and** *distinct xvec'* **and** *distinct yvec'*
by *simp*+
have *IH*: $\bigwedge xvec\ yvec\ M\ N\ P\ Q.\ [\![inputChain\ xvec\ (M::'a)\ (P::('a, 'b, 'c)\ psi) = inputChain\ yvec\ (N::'a)\ (Q::('a, 'b, 'c)\ psi);\ xvec \sharp^* yvec;\ distinct\ xvec;\ distinct\ yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p.\ (set\ p) \subseteq (set\ xvec) \times (set\ yvec) \wedge distinctPerm\ p \wedge\ yvec = p \cdot xvec \wedge N = p \cdot M \wedge Q = p \cdot P$
by *fact*
from *EQ* ‹$x \neq y$› ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› **have** $inputChain\ xvec'\ M\ P = inputChain\ yvec'\ ([(x, y)] \cdot N)\ ([(x, y)] \cdot Q)$
by(*simp add*: *input.inject alpha eqvts*)
with ‹$xvec' \sharp^* yvec'$› ‹*distinct xvec'*› ‹*distinct yvec'*› ‹*length xvec'* = *n*› *IH*
obtain *p* **where** *S*: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm p* **and** $yvec' = p \cdot xvec'$ **and** $([(x, y)] \cdot N) = p \cdot M$ **and** $([(x, y)] \cdot Q) = p \cdot P$
by *metis*
from *S* **have** $set((x, y)\#p) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover from ‹$x \sharp xvec'$› ‹$x \sharp yvec'$› ‹$y \sharp xvec'$› ‹$y \sharp yvec'$› *S* **have** $x \sharp p$ **and** $y \sharp p$
apply(*induct p*)
by(*auto simp add*: *fresh-list-nil fresh-list-cons fresh-prod name-list-supp*) (*auto simp add*: *fresh-def*)

with *S* ‹*distinctPerm p*› ‹$x \neq y$› **have** $distinctPerm((x, y)\#p)$ **by** *auto*
moreover from ‹$yvec' = p \cdot xvec'$› ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $(y\#yvec') = ((x, y)\#p) \cdot (x\#xvec')$
by(*simp add*: *calc-atm freshChainSimps*)
moreover from ‹$([(x, y)] \cdot N) = p \cdot M$› **have** $([(x, y)] \cdot [(x, y)] \cdot N) = [(x, y)] \cdot p \cdot M$
by(*simp add*: *pt-bij*)
hence $N = ((x, y)\#p) \cdot M$ **by** *simp*
moreover from ‹$([(x, y)] \cdot Q) = p \cdot P$› **have** $([(x, y)] \cdot [(x, y)] \cdot Q) = [(x, y)] \cdot p \cdot P$
by(*simp add*: *pt-bij*)
hence $Q = ((x, y)\#p) \cdot P$ **by** *simp*
ultimately show *?case* **using** ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*›
by *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *inputChainEqLength*:
fixes *xvec* :: *name list*
and *M* :: *'a*::*fs-name*
and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
and *yvec* :: *name list*
and *N* :: *'a*

```
  and    Q    :: ('a, 'b, 'c) psi

  assumes inputChain xvec M P = inputChain yvec N Q

  shows length xvec = length yvec
proof −
  obtain n where n = length xvec by auto
  with assms show ?thesis
  proof(induct n arbitrary: xvec yvec M P N Q)
    case(0 xvec yvec M P N Q)
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with ‹inputChain xvec M P = inputChain yvec N Q› have yvec =
[]
      by(case-tac yvec) auto
    ultimately show ?case by simp
  next
    case(Suc n xvec yvec M P N Q)
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹inputChain xvec M P = inputChain yvec N Q› ‹xvec = x # xvec'›
    obtain y yvec' where inputChain (x#xvec') M P = inputChain (y#yvec') N
Q
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence EQ: ν x (inputChain xvec' M P) = ν y (inputChain yvec' N Q)
      by simp
    have IH: ⋀xvec yvec M P N Q. ⟦inputChain xvec (M::'a) (P::('a, 'b, 'c) psi)
= inputChain yvec N Q; n = length xvec⟧ ⟹ length xvec = length yvec
      by fact
    show ?case
    proof(case-tac x = y)
      assume x = y
      with EQ have inputChain xvec' M P = inputChain yvec' N Q
        by(simp add: alpha input.inject)
      with IH ‹length xvec' = n› have length xvec' = length yvec'
        by blast
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
      show ?case by simp
    next
      assume x ≠ y
      with EQ have inputChain xvec' M P = inputChain ([(x, y)] • yvec') ([(x, y)]
• N) ([(x, y)] • Q)
        by(simp add: alpha input.inject eqvts)
      with IH ‹length xvec' = n› have length xvec' = length ([(x, y)] • yvec')
        by blast
      hence length xvec' = length yvec'
        by simp
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
```

```
    show ?case by simp
   qed
  qed
qed

lemma alphaInputChain:
  fixes yvec :: name list
  and   xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi

  assumes length xvec = length yvec
  and     yvec ♯* M
  and     yvec ♯* P
  and     yvec ♯* xvec
  and     distinct yvec

  shows inputChain xvec M P = inputChain yvec ([xvec yvec] •v M) ([xvec yvec]
•v P)
using assms
proof(induct rule: composePermInduct)
  case cBase
  show ?case by simp
next
  case(cStep x xvec y yvec)
  thus ?case
   apply auto
   by(subst alphaInput[of y]) (auto simp add: inputChainFresh eqvts)
qed

lemma inputChainInject[simp]:

  shows (inputChain xvec M P = inputChain xvec N Q) = ((M = N) ∧ (P = Q))
by(induct xvec) (auto simp add: input.inject alpha)

lemma alphaInputDistinct:
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a, 'b, 'c) psi

  assumes Eq: inputChain xvec M P = inputChain yvec N Q
  and     xvecDist: distinct xvec
  and     Mem: ⋀x. x ∈ set xvec ⟹ x ∈ supp M
  and     xvecFreshyvec: xvec ♯* yvec
  and     xvecFreshN: xvec ♯* N
  and     xvecFreshQ: xvec ♯* Q
```

shows *distinct yvec*
proof −
from *Eq* **have** *length xvec = length yvec*
by(*rule inputChainEqLength*)
with *assms* **show** *?thesis*
proof(*induct n==length xvec arbitrary*: *xvec yvec N Q rule*: *nat.induct*)
case(*zero xvec yvec N Q*)
thus *?case* **by** *simp*
next
case(*Suc n xvec yvec N Q*)
have *L*: *length xvec = length yvec* **and** *Suc n = length xvec* **by** *fact+*
then obtain *x xvec′ y yvec′* **where** *xEq*: *xvec = x#xvec′* **and** *yEq*: *yvec = y#yvec′*
and *L′*: *length xvec′ = length yvec′*
by(*cases xvec*, *auto*, *cases yvec*, *auto*)
have *xvecFreshyvec*: *xvec ♯∗ yvec* **and** *xvecDist*: *distinct xvec* **by** *fact+*
with *xEq yEq* **have** *xineqy*: *x ≠ y* **and** *xvec′Freshyvec′*: *xvec′ ♯∗ yvec′*
and *xvec′Dist*: *distinct xvec′* **and** *xFreshxvec′*: *x ♯ xvec′*
and *xFreshyvec′*: *x ♯ yvec′* **and** *yFreshxvec′*: *y ♯ xvec′*
by(*auto simp add*: *fresh-list-cons*)
have *Eq*: *inputChain xvec M P = inputChain yvec N Q* **by** *fact*
with *xEq yEq xineqy* **have** *Eq′*: *inputChain xvec′ M P = inputChain* ([(*x*, *y*)] · *yvec′*) ([(*x*, *y*)] · *N*) ([(*x*, *y*)] · *Q*)
by(*simp add*: *input.inject alpha eqvts*)
moreover have *Mem*:⋀*x*. *x ∈ set xvec ⟹ x ∈ supp M* **by** *fact*
with *xEq* **have** ⋀*x*. *x ∈ set xvec′ ⟹ x ∈ supp M* **by** *simp*
moreover have *xvecFreshN*: *xvec ♯∗ N* **by** *fact*
with *xEq xFreshxvec′ yFreshxvec′* **have** *xvec′ ♯∗* ([(*x*, *y*)] · *N*) **by** *simp*
moreover have *xvecFreshQ*: *xvec ♯∗ Q* **by** *fact*
with *xEq xFreshxvec′ yFreshxvec′* **have** *xvec′ ♯∗* ([(*x*, *y*)] · *Q*) **by** *simp*
moreover have *Suc n = length xvec* **by** *fact*
with *xEq* **have** *n = length xvec′* **by** *simp*
moreover from *xvec′Freshyvec′ xFreshxvec′ yFreshxvec′* **have** *xvec′ ♯∗* ([(*x*, *y*)] · *yvec′*)
by *simp*
moreover from *L′* **have** *length xvec′ = length*([(*x*, *y*)] · *yvec′*) **by** *simp*
ultimately have *distinct*([(*x*, *y*)] · *yvec′*) **using** *xvec′Dist*
by(*rule-tac Suc*)
hence *distinct yvec′* **by** *simp*
from *Mem xEq* **have** *xSuppM*: *x ∈ supp M* **by** *simp*
from *L xvecFreshyvec xvecDist xvecFreshN xvecFreshQ*
have *inputChain yvec N Q = inputChain xvec* ([*yvec xvec*] ·$_v$ *N*) ([*yvec xvec*] ·$_v$ *Q*)
by(*simp add*: *alphaInputChain*)
with *Eq* **have** *M* = [*yvec xvec*] ·$_v$ *N* **by** *auto*
with *xEq yEq* **have** *M* = [(*y*, *x*)] · [*yvec′ xvec′*] ·$_v$ *N*
by *simp*
with *xSuppM* **have** *ySuppN*: *y ∈ supp*([*yvec′ xvec′*] ·$_v$ *N*)

```
    by(drule-tac pi=[(x, y)] in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
      (simp add: calc-atm eqvts name-swap)
   have y ♯ yvec'
   proof(simp add: fresh-def, rule notI)
    assume y ∈ supp yvec'
    hence y mem yvec'
     by(induct yvec') (auto simp add: supp-list-nil supp-list-cons supp-atm)
    moreover from xvecFreshN xEq xFreshxvec' have xvec' ♯* N by simp
    ultimately have y ♯ [yvec' xvec'] •v N using L' xvec'Freshyvec' xvec'Dist
     by(force intro: freshChainPerm simp add: freshChainSym)
    with ySuppN show False by(simp add: fresh-def)
   qed
   with ‹distinct yvec'› yEq show ?case by simp
  qed
qed

lemma psiCasesInject[simp]:
  fixes CsP :: ('c::fs-name × ('a::fs-name, 'b::fs-name, 'c) psi) list
  and   CsQ :: ('c × ('a, 'b, 'c) psi) list

  shows (psiCases CsP = psiCases CsQ) = (CsP = CsQ)
proof(induct CsP arbitrary: CsQ)
  case(Nil CsQ)
  thus ?case by(case-tac CsQ) (auto)
next
  case(Cons a CsP CsQ)
  thus ?case
    by(case-tac a, case-tac CsQ) (clarsimp simp add: psiCase.inject)+
qed

lemma casesInject[simp]:
  fixes CsP :: ('c::fs-name × ('a::fs-name, 'b::fs-name, 'c) psi) list
  and   CsQ :: ('c × ('a, 'b, 'c) psi) list

  shows (Cases CsP = Cases CsQ) = (CsP = CsQ)
apply(induct CsP)
apply(auto simp add: psiCase.inject)
apply(case-tac CsQ)
apply(simp add: psiCase.inject psi.inject)
apply(force simp add: psiCase.inject psi.inject)
apply(case-tac CsQ)
apply(force simp add: psiCase.inject psi.inject)
apply(auto simp add: psiCase.inject psi.inject)
apply(simp only: psiCases.simps[symmetric])
apply(simp only: psiCasesInject)
apply simp
apply(case-tac CsQ)
by(auto simp add: psiCase.inject psi.inject)
```

nominal-primrec
  guarded :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psi* ⇒ *bool*
  and *guarded'* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *input* ⇒ *bool*
  and *guarded''* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psiCase* ⇒ *bool*

where
  guarded (**0**) = *True*
| *guarded* (*M*⟨*N*⟩.*P*) = *True*
| *guarded* (*M*(⦇*I*) = *True*
| *guarded* (*Case C*) = *guarded''* *C*
| *guarded* (*P* ∥ *Q*) = ((*guarded P*) ∧ (*guarded Q*))
| *guarded* (⦇*νx*⦈*P*) = (*guarded P*)
| *guarded* ({⦇Ψ⦈}) = *False*
| *guarded* (!*P*) = *guarded P*

| *guarded'* (*Trm M P*) = *False*
| *guarded'* (*ν y I*) = *False*

| *guarded''* (⊥$_c$) = *True*
| *guarded''* (□*φ* ⇒ *P C*) = (*guarded P* ∧ *guarded''* *C*)
apply(*finite-guess*)+
apply(*rule TrueI*)+
by(*fresh-guess add*: *fresh-bool*)+

lemma *guardedEqvt*[*eqvt*]:
  fixes *p* :: *name prm*
  and *P* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
  and *I* :: (*'a*, *'b*, *'c*) *input*
  and *C* :: (*'a*, *'b*, *'c*) *psiCase*

  shows (*p* ∙ (*guarded P*)) = *guarded* (*p* ∙ *P*)
  and (*p* ∙ (*guarded' I*)) = *guarded'* (*p* ∙ *I*)
  and (*p* ∙ (*guarded'' C*)) = *guarded''* (*p* ∙ *C*)
by(*nominal-induct P* **and** *I* **and** *C rule*: *psi-input-psiCase.strong-inducts*)
  (*simp add*: *eqvts*)+

lemma *guardedClosed*[*simp*]:
  fixes *P* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
  and *p* :: *name prm*

  assumes *guarded P*

  shows *guarded*(*p* ∙ *P*)
proof −
  from ‹*guarded P*› **have** *p* ∙ (*guarded P*)
    by(*simp add*: *perm-bool*)
  thus *?thesis* **by**(*simp add*: *eqvts*)
qed

locale *substPsi* =
  substTerm?: *substType substTerm* +
  substAssert?: *substType substAssert* +
  substCond?: *substType substCond*

  for *substTerm* :: ('*a*::*fs-name*) ⇒ *name list* ⇒ '*a*::*fs-name list* ⇒ '*a*
  and *substAssert* :: ('*b*::*fs-name*) ⇒ *name list* ⇒ '*a*::*fs-name list* ⇒ '*b*
  and *substCond* :: ('*c*::*fs-name*) ⇒ *name list* ⇒ '*a*::*fs-name list* ⇒ '*c*
begin

nominal-primrec
    subs :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi* ⇒ *name list* ⇒ '*a list* ⇒ ('*a*, '*b*, '*c*) *psi*
and *subs'* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *input* ⇒ *name list* ⇒ '*a list* ⇒ ('*a*, '*b*, '*c*) *input*
and *subs''* :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psiCase* ⇒ *name list* ⇒ '*a list* ⇒ ('*a*, '*b*, '*c*) *psiCase*

where
  subs (**0**) *xvec Tvec* = **0**
| (*subs* (*M*⟨*N*⟩.*P*) *xvec Tvec*) = (*substTerm M xvec Tvec*)⟨(*substTerm N xvec Tvec*)⟩.(*subs P xvec Tvec*)
| (*subs* (*M*(⌊*I*) *xvec Tvec*) = (*substTerm M xvec Tvec*)(⌊(*subs' I xvec Tvec*)

| (*subs* (*Case C*) *xvec Tvec*) = (*Case* (*subs'' C xvec Tvec*))
| (*subs* (*P* ∥ *Q*) *xvec Tvec*) = (*subs P xvec Tvec*) ∥ (*subs Q xvec Tvec*)
| ⟦*y* ♯ *xvec*; *y* ♯ *Tvec*⟧ ⟹ (*subs* ((⌊*ν y*⌋)*P*) *xvec Tvec*) = (⌊*ν y*⌋)(*subs P xvec Tvec*)
| (*subs* (⦃Ψ⦄) *xvec Tvec*) = ⦃(*substAssert* Ψ *xvec Tvec*)⦄
| (*subs* (!*P*) *xvec Tvec*) = !(*subs P xvec Tvec*)

| (*subs'* ((*Trm M P*)::('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *input*) *xvec Tvec*) = (⌋)(*substTerm M xvec Tvec*).(*subs P xvec Tvec*))
| ⟦*y* ♯ *xvec*; *y* ♯ *Tvec*⟧ ⟹ (*subs'* (*ν y I*) *xvec Tvec*) = (*ν y* (*subs' I xvec Tvec*))

| (*subs''* (⊥$_c$::('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psiCase*) *xvec Tvec*) = ⊥$_c$
| (*subs''* (□Φ ⇒ *P C*) *xvec Tvec*) = (□(*substCond* Φ *xvec Tvec*) ⇒ (*subs P xvec Tvec*) (*subs'' C xvec Tvec*))
apply(*finite-guess add*: *substTerm.fs substAssert.fs substCond.fs*)+
apply(*rule TrueI*)+
apply(*simp add*: *abs-fresh*)
apply(*simp add*: *abs-fresh*)
apply(*simp add*: *abs-fresh*)
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)

apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess*)+
apply(*rule supports-fresh*[*of supp*(*xvec*, *Tvec*)])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
done

lemma *substEqvt*[*eqvt*]:
fixes *p* :: *name prm*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *xvec* :: *name list*
and *Tvec* :: '*a list*
and *I* :: ('*a*, '*b*, '*c*) *input*
and *C* :: ('*a*, '*b*, '*c*) *psiCase*

shows $(p \cdot (subs\ P\ xvec\ Tvec)) = subs\ (p \cdot P)\ (p \cdot xvec)\ (p \cdot Tvec)$
and $(p \cdot (subs'\ I\ xvec\ Tvec)) = subs'\ (p \cdot I)\ (p \cdot xvec)\ (p \cdot Tvec)$
and $(p \cdot (subs''\ C\ xvec\ Tvec)) = subs''\ (p \cdot C)\ (p \cdot xvec)\ (p \cdot Tvec)$
apply(*nominal-induct P* **and** *I* **and** *C avoiding*: *xvec Tvec rule*: *psi-input-psiCase.strong-inducts*)
apply(*auto simp add*: *eqvts*)
apply(*drule-tac pi=p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi=p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply *simp*
apply(*drule-tac pi=p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi=p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
by *simp*

lemma *subst2*[*intro*]:
fixes *xvec* :: *name list*
and *Tvec* :: *'a list*
and *x* :: *name*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *I* :: (*'a*, *'b*, *'c*) *input*
and *C* :: (*'a*, *'b*, *'c*) *psiCase*

assumes $x \sharp Tvec$
and $x \sharp xvec$

shows $x \sharp P \Longrightarrow x \sharp (subs\ P\ xvec\ Tvec)$
and $x \sharp I \Longrightarrow x \sharp (subs'\ I\ xvec\ Tvec)$
and $x \sharp C \Longrightarrow x \sharp (subs''\ C\ xvec\ Tvec)$
using *assms*
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *xvec Tvec rule*: *psi-input-psiCase.strong-inducts*)
(*auto intro*: *substTerm.subst2 substCond.subst2 substAssert.subst2 simp add*: *abs-fresh*)

lemma *subst2Chain*[*intro*]:
fixes *xvec* :: *name list*
and *Tvec* :: *'a list*
and *Xs* :: *name set*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *I* :: (*'a*, *'b*, *'c*) *input*
and *C* :: (*'a*, *'b*, *'c*) *psiCase*

assumes $Xs \sharp* xvec$
and $Xs \sharp* Tvec$

shows $Xs \sharp* P \Longrightarrow Xs \sharp* (subs\ P\ xvec\ Tvec)$
and $Xs \sharp* I \Longrightarrow Xs \sharp* (subs'\ I\ xvec\ Tvec)$
and $Xs \sharp* C \Longrightarrow Xs \sharp* (subs''\ C\ xvec\ Tvec)$
using *assms*
by(*auto intro*: *subst2 simp add*: *fresh-star-def*)

lemma *renaming*:

fixes *xvec* :: *name list*
and *Tvec* :: *'a list*
and *p* :: *name prm*
and *P* :: *('a, 'b, 'c) psi*
and *I* :: *('a ,'b, 'c) input*
and *C* :: *('a, 'b, 'c) psiCase*

assumes *length xvec = length Tvec*
and *set p ⊆ set xvec × set (p • xvec)*
and *distinctPerm p*

shows *⟦(p • xvec) ♯* P⟧ ⟹ (subs P xvec Tvec) = subs (p • P) (p • xvec) Tvec*
and *⟦(p • xvec) ♯* I⟧ ⟹ (subs' I xvec Tvec) = subs' (p • I) (p • xvec) Tvec*
and *⟦(p • xvec) ♯* C⟧ ⟹ (subs'' C xvec Tvec) = subs'' (p • C) (p • xvec) Tvec*
using *assms*
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *xvec p Tvec rule*: *psi-input-psiCase.strong-inducts*)
(*auto intro*: *substTerm.renaming substCond.renaming substAssert.renaming simp add*: *freshChainSimps psi.inject input.inject psiCase.inject*)

lemma *subst4Chain*:
fixes *xvec* :: *name list*
and *Tvec* :: *'a list*
and *P* :: *('a, 'b, 'c) psi*
and *I* :: *('a, 'b, 'c) input*
and *C* :: *('a, 'b, 'c) psiCase*

assumes *length xvec = length Tvec*
and *distinct xvec*
and *xvec ♯* Tvec*

shows *xvec ♯* (subs P xvec Tvec)*
and *xvec ♯* (subs' I xvec Tvec)*
and *xvec ♯* (subs'' C xvec Tvec)*
using *assms*
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *xvec Tvec rule*: *psi-input-psiCase.strong-inducts*)
(*auto intro*: *substTerm.subst4Chain substCond.subst4Chain substAssert.subst4Chain simp add*: *abs-fresh*)

lemma *guardedSubst*[*simp*]:
fixes *P* :: *('a, 'b, 'c) psi*
and *I* :: *('a, 'b, 'c) input*
and *C* :: *('a, 'b, 'c) psiCase*
and *xvec* :: *name list*
and *Tvec* :: *'a list*

assumes *length xvec = length Tvec*
and *distinct xvec*

shows *guarded P ⟹ guarded*(subs P xvec Tvec)*

and *guarded′ I* $\Longrightarrow$ *guarded′*(*subs′ I xvec Tvec*)
and *guarded″ C* $\Longrightarrow$ *guarded″*(*subs″ C xvec Tvec*)
using *assms*
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *xvec Tvec rule*: *psi-input-psiCase.strong-inducts*) *auto*

definition *seqSubs* :: (′*a*, ′*b*, ′*c*) *psi* ⇒ (*name list* × ′*a list*) *list* ⇒ (′*a*, ′*b*, ′*c*) *psi* (-[<->] [*80*, *80*] *130*)
where *P*[<*σ*>] ≡ *foldl* (λ*Q*. λ(*xvec*, *Tvec*). *subs Q xvec Tvec*) *P σ*

definition *seqSubs′* :: (′*a*, ′*b*, ′*c*) *input* ⇒ (*name list* × ′*a list*) *list* ⇒ (′*a*, ′*b*, ′*c*) *input*
where *seqSubs′ I σ* ≡ *foldl* (λ*Q*. λ(*xvec*, *Tvec*). *subs′ Q xvec Tvec*) *I σ*

definition *seqSubs″* :: (′*a*, ′*b*, ′*c*) *psiCase* ⇒ (*name list* × ′*a list*) *list* ⇒ (′*a*, ′*b*, ′*c*) *psiCase*
where *seqSubs″ C σ* ≡ *foldl* (λ*Q*. λ(*xvec*, *Tvec*). *subs″ Q xvec Tvec*) *C σ*

lemma *substInputChain*[*simp*]:
fixes *xvec* :: *name list*
and *N* :: ′*a*
and *P* :: (′*a*, ′*b*, ′*c*) *psi*
and *yvec* :: *name list*
and *Tvec* :: ′*a list*

assumes *xvec* ♯* *yvec*
and *xvec* ♯* *Tvec*

shows *subs′* (*inputChain xvec N P*) *yvec Tvec* = *inputChain xvec* (*substTerm N yvec Tvec*) (*subs P yvec Tvec*)
using *assms*
by(*induct xvec*) (*auto simp add*: *psi.inject*)

fun *caseListSubst* :: (′*c* × (′*a*, ′*b*, ′*c*) *psi*) *list* ⇒ *name list* ⇒ ′*a list* ⇒ (′*c* × (′*a*, ′*b*, ′*c*) *psi*) *list*
where
caseListSubst [] - - = []
| *caseListSubst* ((*φ*, *P*)#*Cs*) *xvec Tvec* = (*substCond φ xvec Tvec*, (*subs P xvec Tvec*))#(*caseListSubst Cs xvec Tvec*)

lemma *substCases*[*simp*]:
fixes *Cs* :: (′*c* × (′*a*, ′*b*, ′*c*) *psi*) *list*
and *xvec* :: *name list*
and *Tvec* :: ′*a list*

shows *subs* (*Cases Cs*) *xvec Tvec* = *Cases*(*caseListSubst Cs xvec Tvec*)
by(*induct Cs*) (*auto simp add*: *psi.inject*)

lemma *substCases′*[*simp*]:

fixes Cs :: $('c \times ('a, 'b, 'c)\ psi)\ list$
and $xvec$ :: $name\ list$
and $Tvec$ :: $'a\ list$

shows $(subs''\ (psiCases\ Cs)\ xvec\ Tvec) = psiCases(caseListSubst\ Cs\ xvec\ Tvec)$
by$(induct\ Cs)\ auto$

lemma *seqSubstSimps*[*simp*]:
shows $seqSubs\ (\mathbf{0})\ \sigma = \mathbf{0}$
and $(seqSubs\ (M\langle N\rangle.P)\ \sigma) = (substTerm.seqSubst\ M\ \sigma)\langle(substTerm.seqSubst\ N\ \sigma)\rangle.(seqSubs\ P\ \sigma)$
and $(seqSubs\ (M(\!|I|\!)\ \sigma) = (substTerm.seqSubst\ M\ \sigma)(\!|(seqSubs'\ I\ \sigma)$

and $(seqSubs\ (Case\ C)\ \sigma) = (Case\ (seqSubs''\ C\ \sigma))$
and $(seqSubs\ (P \parallel Q)\ \sigma) = (seqSubs\ P\ \sigma) \parallel (seqSubs\ Q\ \sigma)$
and $[\![y \sharp \sigma]\!] \Longrightarrow (seqSubs\ ((\!|\nu y|\!)P)\ \sigma) = (\!|\nu y|\!)(seqSubs\ P\ \sigma)$
and $(seqSubs\ (\{\!|\Psi|\!\})\ \sigma) = \{\!|(substAssert.seqSubst\ \Psi\ \sigma)|\!\}$
and $(seqSubs\ (!P)\ \sigma) = !(seqSubs\ P\ \sigma)$

and $(seqSubs'\ ((Trm\ M\ P)::('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ input)\ \sigma) = (\!|)(substTerm.seqSubst\ M\ \sigma).(seqSubs\ P\ \sigma))$
and $[\![y \sharp \sigma]\!] \Longrightarrow (seqSubs'\ (\nu\ y\ I)\ \sigma) = (\nu\ y\ (seqSubs'\ I\ \sigma))$

and $(seqSubs''\ (\bot_c::('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psiCase)\ \sigma) = \bot_c$
and $(seqSubs''\ (\Box\Phi \Rightarrow P\ C)\ \sigma) = (\Box(substCond.seqSubst\ \Phi\ \sigma) \Rightarrow (seqSubs\ P\ \sigma)\ (seqSubs''\ C\ \sigma))$
by$(induct\ \sigma\ arbitrary: M\ N\ P\ I\ C\ Q\ \Psi\ \Phi, auto\ simp\ add: seqSubs\text{-}def\ seqSubs'\text{-}def\ seqSubs''\text{-}def)$

lemma *seqSubsNil*[*simp*]:
$seqSubs\ P\ [] = P$
by$(simp\ add: seqSubs\text{-}def)$

lemma *seqSubsCons*[*simp*]:
shows $seqSubs\ P\ ((xvec, Tvec)\#\sigma) = seqSubs(subs\ P\ xvec\ Tvec)\ \sigma$
by$(simp\ add: seqSubs\text{-}def)$

lemma *seqSubsTermAppend*[*simp*]:
shows $seqSubs\ P\ (\sigma@\sigma') = seqSubs\ (seqSubs\ P\ \sigma)\ \sigma'$
by$(induct\ \sigma)\ (auto\ simp\ add: seqSubs\text{-}def)$

fun $caseListSeqSubst :: ('c \times ('a, 'b, 'c)\ psi)\ list \Rightarrow (name\ list \times 'a\ list)\ list \Rightarrow ('c \times ('a, 'b, 'c)\ psi)\ list$
where
$caseListSeqSubst\ []\ \text{-} = []$
$|\ caseListSeqSubst\ ((\varphi, P)\#Cs)\ \sigma = (substCond.seqSubst\ \varphi\ \sigma, (seqSubs\ P\ \sigma))\#(caseListSeqSubst\ Cs\ \sigma)$

lemma *seqSubstCases*[*simp*]:

fixes $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$
and $\sigma :: (name\ list \times 'a\ list)\ list$

shows $seqSubs\ (Cases\ Cs)\ \sigma = Cases(caseListSeqSubst\ Cs\ \sigma)$
by(*induct Cs*) (*auto simp add*: *psi.inject*)

lemma *seqSubstCases'*[*simp*]:
fixes $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$
and $\sigma :: (name\ list \times 'a\ list)\ list$

shows $(seqSubs''\ (psiCases\ Cs)\ \sigma) = psiCases(caseListSeqSubst\ Cs\ \sigma)$
by(*induct Cs*) *auto*

lemma *seqSubstEqvt*[*eqvt*]:
fixes $P :: ('a, 'b, 'c)\ psi$
and $\sigma :: (name\ list \times 'a\ list)\ list$
and $p :: name\ prm$

shows $(p \cdot (P[<\sigma>])) = (p \cdot P)[<(p \cdot \sigma)>]$
by(*induct* σ *arbitrary*: *P*) (*auto simp add*: *eqvts seqSubs-def*)

lemma *guardedSeqSubst*:
assumes *guarded P*
and *wellFormedSubst* σ

shows *guarded*(*seqSubs P* σ)
using *assms*
by(*induct* σ *arbitrary*: *P*) (*auto dest*: *guardedSubst*)

end

lemma *inter-eqvt*:
shows $(pi::name\ prm) \cdot ((X::name\ set) \cap Y) = (pi \cdot X) \cap (pi \cdot Y)$
by(*auto simp add*: *perm-set-def perm-bij*)

lemma *delete-eqvt*:
fixes $p :: name\ prm$
and $X :: name\ set$
and $Y :: name\ set$

shows $p \cdot (X - Y) = (p \cdot X) - (p \cdot Y)$
by(*auto simp add*: *perm-set-def perm-bij*)

lemma *perm-singleton*[*simp*]:
shows $(p::name\ prm) \cdot \{(x::name)\} = \{p \cdot x\}$
by(*auto simp add*: *perm-set-def*)

end

theory *Frame*
  imports *Agent*
begin

lemma *permLength*[*simp*]:
  fixes p :: *name prm*
  and *xvec* :: $'a$::*pt-name list*

  shows *length*(p $\cdot$ *xvec*) = *length xvec*
by(*induct xvec*) *auto*

nominal-datatype $'$*assertion frame* =
    FAssert $'$*assertion*::*fs-name*
  | *FRes* «*name*» ($'$*assertion frame*) ((⦇ν-⦈- [*80*, *80*] *80*)

primrec *frameResChain* :: *name list* ⇒ ($'a$::*fs-name*) *frame* ⇒ $'a$ *frame* **where**
  base: *frameResChain* [] *F* = *F*
| *step*: *frameResChain* (*x*#*xs*) *F* = (⦇ν*x*⦈)(*frameResChain xs F*)

notation *frameResChain* ((⦇ν*-⦈- [*80*, *80*] *80*)
notation *FAssert* (⟨ε, -⟩ [*80*] *80*)
abbreviation *FAssertJudge* (⟨-, -⟩ [*80*, *80*] *80*) **where** ⟨A_F, Ψ_F⟩ ≡ *frameResChain* A_F (*FAssert* Ψ_F)

lemma *frameResChainEqvt*[*eqvt*]:
  fixes *perm* :: *name prm*
  and *lst* :: *name list*
  and *F* :: $'a$::*fs-name frame*

  shows *perm* $\cdot$ ((⦇ν*xvec*⦈)*F*) = (⦇ν*(perm* $\cdot$ *xvec*)⦈)(*perm* $\cdot$ *F*)
by(*induct-tac xvec*, *auto*)

lemma *frameResChainFresh*:
  fixes *x* :: *name*
  and *xvec* :: *name list*
  and *F* :: $'a$::*fs-name frame*

  shows *x* ♯ (⦇ν*xvec*⦈)*F* = (*x* ∈ *set xvec* ∨ *x* ♯ *F*)
by (*induct xvec*) (*simp-all add*: *abs-fresh*)

lemma *frameResChainFreshSet*:
  fixes *Xs* :: *name set*
  and *xvec* :: *name list*
  and *F* :: $'a$::*fs-name frame*

  shows *Xs* ♯* (⦇ν*xvec*⦈)*F* = (∀ *x*∈*Xs*. *x* ∈ *set xvec* ∨ *x* ♯ *F*)
by (*simp add*: *fresh-star-def frameResChainFresh*)

lemma *frameChainAlpha*:

fixes p :: *name prm*
and *xvec* :: *name list*
and F :: *'a::fs-name frame*

assumes *xvecFreshF*: $(p \cdot xvec) \sharp* F$
and S: *set* $p \subseteq$ *set xvec* $\times$ *set* $(p \cdot xvec)$

shows $(\!|\nu* xvec|\!)F = (\!|\nu*(p \cdot xvec)|\!)(p \cdot F)$
proof −
note *pt-name-inst at-name-inst S*
moreover have *set xvec* $\sharp*$ $((\!|\nu* xvec|\!)F)$
by (*simp add*: *frameResChainFreshSet*)
moreover from *xvecFreshF* **have** *set* $(p \cdot xvec) \sharp* ((\!|\nu* xvec|\!)F)$
by (*simp add*: *frameResChainFreshSet*) (*simp add*: *fresh-star-def*)
ultimately have $(\!|\nu* xvec|\!)F = p \cdot ((\!|\nu* xvec|\!)F)$
by (*rule-tac pt-freshs-freshs* [*symmetric*])
then show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *frameChainAlpha'*:
fixes p :: *name prm*
and A_P :: *name list*
and Ψ_P :: *'a::fs-name*

assumes $(p \cdot A_P) \sharp* \Psi_P$
and S: *set* $p \subseteq$ *set* $A_P \times$ *set* $(p \cdot A_P)$

shows $\langle A_P, \Psi_P\rangle = \langle(p \cdot A_P), p \cdot \Psi_P\rangle$
using *assms*
by(*subst frameChainAlpha*) (*auto simp add*: *fresh-star-def*)

lemma *alphaFrameRes*:
fixes x :: *name*
and F :: *'a::fs-name frame*
and y :: *name*

assumes $y \sharp F$

shows $(\!|\nu x|\!)F = (\!|\nu y|\!)([(x, y)] \cdot F)$
proof(*cases* $x = y$)
assume $x{=}y$
thus *?thesis* **by** *simp*
next
assume $x \neq y$
with ‹$y \sharp F$› **show** *?thesis*
by(*perm-simp add*: *frame.inject alpha calc-atm fresh-left*)
qed

lemma *frameChainAppend*:

```
  fixes xvec :: name list
  and   yvec :: name list
  and   F    :: 'a::fs-name frame

  shows (|ν*(xvec@yvec)|)F = (|ν*xvec|)((|ν*yvec|)F)
by(induct xvec) auto

lemma frameChainEqLength:
  fixes xvec :: name list
  and   Ψ    :: 'a::fs-name
  and   yvec :: name list
  and   Ψ'   :: 'a::fs-name

  assumes ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩

  shows length xvec = length yvec
proof −
  obtain n where n = length xvec by auto
  with assms show ?thesis
  proof(induct n arbitrary: xvec yvec Ψ Ψ')
    case(0 xvec yvec Ψ Ψ')
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› have yvec = []
      by(case-tac yvec) auto
    ultimately show ?case by simp
  next
    case(Suc n xvec yvec Ψ Ψ')
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› ‹xvec = x # xvec'›
    obtain y yvec' where ⟨(x#xvec'), Ψ⟩ = ⟨(y#yvec'), Ψ'⟩
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence EQ: (|νx|)(|ν*xvec'|)(FAssert Ψ) = (|νy|)(|ν*yvec'|)(FAssert Ψ')
      by simp
    have IH: ⋀xvec yvec Ψ Ψ'. ⟦⟨xvec, (Ψ::'a)⟩ = ⟨yvec, (Ψ'::'a)⟩; n = length xvec⟧
⟹ length xvec = length yvec
      by fact
    show ?case
    proof(case-tac x = y)
      assume x = y
      with EQ have ⟨xvec', Ψ⟩ = ⟨yvec', Ψ'⟩
        by(simp add: alpha frame.inject)
      with IH ‹length xvec' = n› have length xvec' = length yvec'
        by blast
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
      show ?case by simp
    next
```

```
      assume x ≠ y
      with EQ have ⟨xvec', Ψ⟩ = [(x, y)] • ⟨yvec', Ψ'⟩
        by(simp add: alpha frame.inject)
      hence ⟨xvec', Ψ⟩ = ⟨([(x, y)] • yvec'), ([(x, y)] • Ψ')⟩
        by(simp add: eqvts)
      with IH ‹length xvec' = n› have length xvec' = length ([(x, y)] • yvec')
        by blast
      hence length xvec' = length yvec'
        by simp
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
      show ?case by simp
    qed
  qed
qed

lemma frameEqFresh:
  fixes F :: ('a::fs-name) frame
  and   G :: 'a frame
  and   x :: name
  and   y :: name

  assumes (|νx|)F = (|νy|)G
  and     x ♯ F

  shows y ♯ G
using assms
by(auto simp add: frame.inject alpha fresh-left calc-atm)

lemma frameEqSupp:
  fixes F :: ('a::fs-name) frame
  and   G :: 'a frame
  and   x :: name
  and   y :: name

  assumes (|νx|)F = (|νy|)G
  and     x ∈ supp F

  shows y ∈ supp G
using assms
apply(auto simp add: frame.inject alpha fresh-left calc-atm)
apply(drule-tac pi=[(x, y)] in pt-set-bij2[OF pt-name-inst, OF at-name-inst])
by(simp add: eqvts calc-atm)

lemma frameChainEqSuppEmpty[dest]:
  fixes xvec :: name list
  and   Ψ    :: 'a::fs-name
  and   yvec :: name list
  and   Ψ'   :: 'a::fs-name
```

```
  assumes ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩
  and     supp Ψ = ({}::name set)

  shows Ψ = Ψ'
proof −
  obtain n where n = length xvec by auto
  with assms show ?thesis
  proof(induct n arbitrary: xvec yvec Ψ Ψ')
    case(0 xvec yvec Ψ Ψ')
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› have yvec = []
      by(case-tac yvec) auto
    ultimately show ?case  using ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩›
      by(simp add: frame.inject)
  next
    case(Suc n xvec yvec Ψ Ψ')
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› ‹xvec = x # xvec'›
    obtain y yvec' where ⟨(x#xvec'), Ψ⟩ = ⟨(y#yvec'), Ψ'⟩
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence EQ: (|νx|)(|ν*xvec'|)(FAssert Ψ) = (|νy|)(|ν*yvec'|)(FAssert Ψ')
      by simp
     have IH: ⋀xvec yvec Ψ Ψ'. [[⟨xvec, (Ψ::'a)⟩ = ⟨yvec, (Ψ'::'a)⟩; supp Ψ =
({}::name set); n = length xvec]] ⟹ Ψ = Ψ'
      by fact
    show ?case
    proof(case-tac x = y)
      assume x = y
      with EQ have ⟨xvec', Ψ⟩ = ⟨yvec', Ψ'⟩
        by(simp add: alpha frame.inject)
      with IH ‹length xvec' = n› ‹supp Ψ = {}› show ?case
        by simp
    next
      assume x ≠ y
      with EQ have ⟨xvec', Ψ⟩ = [(x, y)] • ⟨yvec', Ψ'⟩
        by(simp add: alpha frame.inject)
      hence ⟨xvec', Ψ⟩ = ⟨([(x, y)] • yvec'), ([(x, y)] • Ψ')⟩
        by(simp add: eqvts)
      with IH ‹length xvec' = n› ‹supp Ψ = {}› have Ψ = [(x, y)] • Ψ'
        by(simp add: eqvts)
      moreover with ‹supp Ψ = {}› have supp([(x, y)] • Ψ') = ({}::name set)
        by simp
      hence x ♯ ([(x, y)] • Ψ') and y ♯ ([(x, y)] • Ψ')
        by(simp add: fresh-def)+
      with ‹x ≠ y› have x ♯ Ψ' and y ♯ Ψ'
        by(simp add: fresh-left calc-atm)+
```

```
    ultimately show ?case by simp
   qed
  qed
qed

lemma frameChainEq:
  fixes xvec :: name list
  and   Ψ    :: 'a::fs-name
  and   yvec :: name list
  and   Ψ'   :: 'a::fs-name

  assumes ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩
  and     xvec ♯* yvec

  obtains p where (set p) ⊆ (set xvec) × set (yvec) and distinctPerm p and Ψ'
= p • Ψ
proof −
  assume ⋀p. ⟦set p ⊆ set xvec × set yvec; distinctPerm p; Ψ' = p • Ψ⟧ ⟹ thesis
  moreover obtain n where n = length xvec by auto
  with assms have ∃ p. (set p) ⊆ (set xvec) × set (yvec) ∧ distinctPerm p ∧  Ψ'
= p • Ψ
  proof(induct n arbitrary: xvec yvec Ψ Ψ')
    case(0 xvec yvec Ψ Ψ')
    have Eq: ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩ by fact
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with Eq have yvec = []
      by(case-tac yvec) auto
    ultimately show ?case using Eq
      by(simp add: frame.inject)
  next
    case(Suc n xvec yvec Ψ Ψ')
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› ‹xvec = x # xvec'›
    obtain y yvec' where ⟨(x#xvec'), Ψ⟩ = ⟨(y#yvec'), Ψ'⟩
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence EQ: (|νx|)(|ν∗xvec'|)(FAssert Ψ) = (|νy|)(|ν∗yvec'|)(FAssert Ψ')
      by simp
    from ‹xvec = x#xvec'› ‹yvec=y#yvec'› ‹xvec ♯* yvec›
    have x ≠ y and xvec' ♯* yvec' and x ♯ yvec' and y ♯ xvec'
      by auto
    have IH: ⋀xvec yvec Ψ Ψ'. ⟦⟨xvec, (Ψ::'a)⟩ = ⟨yvec, (Ψ'::'a)⟩; xvec ♯* yvec; n
= length xvec⟧ ⟹
                                ∃ p. (set p) ⊆ (set xvec) × (set yvec) ∧ distinctPerm p
∧  Ψ' = p • Ψ
      by fact
```

from EQ ‹$x \neq y$› **have** EQ': $\langle xvec', \Psi\rangle = ([(x, y)] \cdot \langle yvec', \Psi'\rangle)$
and $xFresh\Psi'$: $x \sharp (\!|\nu * yvec'|\!)(FAssert\ \Psi')$
by(*simp add*: *frame.inject alpha*)+

show *?case*
proof(*case-tac* $x \sharp \langle xvec', \Psi\rangle$)
assume $x \sharp \langle xvec', \Psi\rangle$
with EQ **have** $y \sharp \langle yvec', \Psi'\rangle$
by(*rule frameEqFresh*)
with $xFresh\Psi'$ EQ' **have** $\langle xvec', \Psi\rangle = \langle yvec', \Psi'\rangle$
by(*simp*)
with ‹$xvec' \sharp* yvec'$› ‹*length* $xvec' = n$› *IH*
obtain p **where** S: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm* p **and** $\Psi' = p \cdot \Psi$
by *blast*
from S **have** $(set\ p) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
with ‹$xvec = x\#xvec'$› ‹$yvec = y\#yvec'$› ‹*distinctPerm* p› ‹$\Psi' = p \cdot \Psi$›
show *?case* **by** *blast*
next
assume $\neg(x \sharp (\!|\nu * xvec'|\!)(FAssert\ \Psi))$
hence $xSupp\Psi$: $x \in supp(\langle xvec', \Psi\rangle)$
by(*simp add*: *fresh-def*)
with EQ **have** $y \in supp\ (\langle yvec', \Psi'\rangle)$
by(*rule frameEqSupp*)
hence $y \sharp yvec'$
by(*induct yvec'*) (*auto simp add*: *frame.supp abs-supp*)
with ‹$x \sharp yvec'$› EQ' **have** $\langle xvec', \Psi\rangle = \langle yvec', ([(x, y)] \cdot \Psi')\rangle$
by(*simp add*: *eqvts*)
with ‹$xvec' \sharp* yvec'$› ‹*length* $xvec' = n$› *IH*
obtain p **where** S: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm* p **and** $([(x, y)] \cdot \Psi') = p \cdot \Psi$
by *blast*

from $xSupp\Psi$ **have** $x \sharp xvec'$
by(*induct xvec'*) (*auto simp add*: *frame.supp abs-supp*)
with ‹$x \sharp yvec'$› ‹$y \sharp xvec'$› ‹$y \sharp yvec'$› S **have** $x \sharp p$ **and** $y \sharp p$
apply(*induct p*)
by(*auto simp add*: *name-list-supp*) (*auto simp add*: *fresh-def*)
from S **have** $(set\ ((x, y)\#p)) \subseteq (set(x\#xvec')) \times (set(y\#yvec'))$
by *force*
moreover from ‹$x \neq y$› ‹$x \sharp p$› ‹$y \sharp p$› S ‹*distinctPerm* p›
have *distinctPerm*$((x,y)\#p)$ **by** *simp*
moreover from ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $y\#(p \cdot xvec') = ((x, y)\#p) \cdot (x\#xvec')$
by(*simp add*: *eqvts calc-atm freshChainSimps*)
moreover from ‹$([(x, y)] \cdot \Psi') = p \cdot \Psi$›
have $([(x, y)] \cdot [(x, y)] \cdot \Psi') = [(x, y)] \cdot p \cdot \Psi$
by(*simp add*: *pt-bij*)
hence $\Psi' = ((x, y)\#p) \cdot \Psi$ **by** *simp*

```
    ultimately show ?case using ‹xvec=x#xvec'› ‹yvec=y#yvec'›
      by blast
  qed
 qed
 ultimately show ?thesis by blast
qed

lemma frameChainEq':
 fixes xvec :: name list
 and   Ψ    :: 'a::fs-name
 and   yvec :: name list
 and   Ψ'   :: 'a::fs-name

 assumes ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩
 and     xvec ♯* yvec
 and     distinct xvec
 and     distinct yvec

 obtains p where (set p) ⊆ (set xvec) × set (p · xvec) and distinctPerm p and
yvec = p · xvec and Ψ' = p · Ψ
proof −
 assume ⋀p. ⟦set p ⊆ set xvec × set (p · xvec); distinctPerm p; yvec = p · xvec;
Ψ' = p · Ψ⟧ ⟹ thesis
 moreover obtain n where n = length xvec by auto
 with assms have ∃ p. (set p) ⊆ (set xvec) × set (yvec) ∧ distinctPerm p ∧  yvec
= p · xvec ∧ Ψ' = p · Ψ
 proof(induct n arbitrary: xvec yvec Ψ Ψ')
  case(0 xvec yvec Ψ Ψ')
  have Eq: ⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩ by fact
  from ‹0 = length xvec› have xvec = [] by auto
  moreover with Eq have yvec = []
   by(case-tac yvec) auto
  ultimately show ?case using Eq
   by(simp add: frame.inject)
 next
  case(Suc n xvec yvec Ψ Ψ')
  from ‹Suc n = length xvec›
  obtain x xvec' where xvec = x#xvec' and length xvec' = n
   by(case-tac xvec) auto
  from ‹⟨xvec, Ψ⟩ = ⟨yvec, Ψ'⟩› ‹xvec = x # xvec'›
  obtain y yvec' where ⟨(x#xvec'), Ψ⟩ = ⟨(y#yvec'), Ψ'⟩
   and yvec = y#yvec'
   by(case-tac yvec) auto
  hence EQ: (|νx|)(|ν*xvec'|)(FAssert Ψ) = (|νy|)(|ν*yvec'|)(FAssert Ψ')
   by simp
  from ‹xvec = x#xvec'› ‹yvec=y#yvec'› ‹xvec ♯* yvec›
  have x ≠ y and xvec' ♯* yvec' and x ♯ yvec' and y ♯ xvec'
   by auto
  from ‹distinct xvec› ‹distinct yvec› ‹xvec=x#xvec'› ‹yvec=y#yvec'› have x ♯
```

xvec' **and** $y \sharp yvec'$ **and** *distinct xvec'* **and** *distinct yvec'*
by *simp+*
have *IH*: $\bigwedge xvec\ yvec\ \Psi\ \Psi'.\ [\![\langle xvec,\ (\Psi::'a)\rangle = \langle yvec,\ (\Psi'::'a)\rangle;\ xvec\ \sharp*\ yvec;\ distinct\ xvec;\ distinct\ yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p.\ (set\ p) \subseteq (set\ xvec) \times (set\ yvec) \wedge distinctPerm\ p \wedge\ \ yvec = p \cdot xvec \wedge \Psi' = p \cdot \Psi$
by *fact*
from *EQ* ‹$x \neq y$› ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› **have** $\langle xvec',\ \Psi\rangle = \langle yvec',\ ([(x,\ y)] \cdot \Psi')\rangle$
by(*simp add*: *frame.inject alpha eqvts*)
with ‹*xvec'* $\sharp*$ *yvec'*› ‹*distinct xvec'*› ‹*distinct yvec'*› ‹*length xvec'* = *n*› *IH*
obtain *p* **where** *S*: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm p* **and** $yvec' = p \cdot xvec'$ **and** $[(x,\ y)] \cdot \Psi' = p \cdot \Psi$
by *metis*
from *S* **have** $set((x,\ y)\#p) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover from ‹$x \sharp xvec'$› ‹$x \sharp yvec'$› ‹$y \sharp xvec'$› ‹$y \sharp yvec'$› *S* **have** $x \sharp p$ **and** $y \sharp p$
apply(*induct p*)
by(*auto simp add*: *name-list-supp*) (*auto simp add*: *fresh-def*)

with *S* ‹*distinctPerm p*› ‹$x \neq y$› **have** $distinctPerm((x,\ y)\#p)$ **by** *auto*
moreover from ‹$yvec' = p \cdot xvec'$› ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $(y\#yvec') = ((x,\ y)\#p) \cdot (x\#xvec')$
by(*simp add*: *freshChainSimps calc-atm*)
moreover from ‹$([(x,\ y)] \cdot \Psi') = p \cdot \Psi$›
have $([(x,\ y)] \cdot [(x,\ y)] \cdot \Psi') = [(x,\ y)] \cdot p \cdot \Psi$
by(*simp add*: *pt-bij*)
hence $\Psi' = ((x,\ y)\#p) \cdot \Psi$
by *simp*
ultimately show *?case* **using** ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*›
by *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *frameEq*[*simp*]:
fixes A_F :: *name list*
and Ψ :: *'a*::*fs-name*
and Ψ' :: *'a*

shows $\langle A_F,\ \Psi\rangle = \langle \varepsilon,\ \Psi'\rangle = (A_F = [] \wedge \Psi = \Psi')$
and $\langle \varepsilon,\ \Psi'\rangle = \langle A_F,\ \Psi\rangle\ = (A_F = [] \wedge \Psi = \Psi')$
proof −
{
assume $\langle A_F,\ \Psi\rangle = \langle \varepsilon,\ \Psi'\rangle$
hence *A*: $\langle A_F,\ \Psi\rangle = \langle [],\ \Psi'\rangle$ **by** *simp*
hence *length* A_F = *length* ([]::*name list*)
by(*rule frameChainEqLength*)
with *A* **have** $A_F = []$ **and** $\Psi = \Psi'$ **by**(*auto simp add*: *frame.inject*)
}

thus $\langle A_F, \Psi\rangle = \langle\varepsilon, \Psi'\rangle = (A_F = [] \wedge \Psi = \Psi')$
and $\langle\varepsilon, \Psi'\rangle = \langle A_F, \Psi\rangle = (A_F = [] \wedge \Psi = \Psi')$
by *auto*
qed

lemma *distinctFrame*:
fixes A_F :: *name list*
and Ψ_F :: $'a$::*fs-name*
and C :: $'b$::*fs-name*

assumes $A_F \sharp* C$

obtains A_F' **where** $\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F\rangle$ **and** *distinct* A_F' **and** $A_F' \sharp* C$
proof −
assume $\bigwedge A_F'. [\![\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F\rangle; \textit{distinct } A_F'; A_F' \sharp* C]\!] \Longrightarrow \textit{thesis}$
moreover from *assms* **have** $\exists A_F'. \langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F\rangle \wedge \textit{distinct } A_F' \wedge A_F' \sharp* C$
proof(*induct* A_F)
case *Nil*
thus *?case* **by** *simp*
next
case(*Cons a* A_F)
then obtain A_F' **where** *Eq*: $\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F\rangle$ **and** *distinct* A_F' **and** $A_F' \sharp* C$ **by** *force*
from $\langle(a\#A_F) \sharp* C\rangle$ **have** $a \sharp C$ **and** $A_F \sharp* C$ **by** *simp*+
show *?case*
proof(*case-tac* $a \sharp \langle A_F', \Psi_F\rangle$)
assume $a \sharp \langle A_F', \Psi_F\rangle$
obtain b::*name* **where** $b \sharp A_F'$ **and** $b \sharp \Psi_F$ **and** $b \sharp C$ **by**(*generate-fresh name*, *auto*)
have $\langle(a\#A_F), \Psi_F\rangle = \langle(b\#A_F'), \Psi_F\rangle$
proof −
from *Eq* **have** $\langle(a\#A_F), \Psi_F\rangle = \langle(a\#A_F'), \Psi_F\rangle$ **by**(*simp add*: *frame.inject*)
moreover from $\langle b \sharp \Psi_F\rangle$ **have** $\ldots = (\!|\nu b|\!)([(a, b)] \cdot (\!|\nu* A_F'|\!)(\textit{FAssert } \Psi_F))$
by(*force intro*: *alphaFrameRes simp add*: *frameResChainFresh*)
ultimately show *?thesis* **using** $\langle a \sharp \langle A_F', \Psi_F\rangle\rangle$ $\langle b \sharp \Psi_F\rangle$
by(*simp add*: *frameResChainFresh*)
qed
moreover from $\langle \textit{distinct } A_F'\rangle$ $\langle b \sharp A_F'\rangle$ **have** *distinct*$(b\#A_F')$ **by** *simp*
moreover from $\langle A_F' \sharp* C\rangle$ $\langle b \sharp C\rangle$ **have** $(b\#A_F') \sharp* C$ **by** *simp*+
ultimately show *?case* **by** *blast*
next
from *Eq* **have** $\langle(a\#A_F), \Psi_F\rangle = \langle(a\#A_F'), \Psi_F\rangle$ **by**(*simp add*: *frame.inject*)
moreover assume $\neg(a \sharp \langle A_F', \Psi_F\rangle)$
hence $a \sharp A_F'$ **apply**(*simp add*: *fresh-def*)
by(*induct* A_F') (*auto simp add*: *supp-list-nil supp-list-cons supp-atm frame.supp abs-supp*)
with $\langle \textit{distinct } A_F'\rangle$ **have** *distinct*$(a\#A_F')$ **by** *simp*
moreover from $\langle A_F' \sharp* C\rangle$ $\langle a \sharp C\rangle$ **have** $(a\#A_F') \sharp* C$ **by** *simp*+

```
    ultimately show ?case by blast
  qed
 qed
 ultimately show ?thesis using ‹A_F ♯* C›
  by blast
qed

lemma freshFrame:
 fixes F :: ('a::fs-name) frame
 and   C :: 'b ::fs-name

 obtains A_F Ψ_F where F = ⟨A_F, Ψ_F⟩ and distinct A_F and A_F ♯* C
proof −
 assume ⋀A_F Ψ_F. [[F = ⟨A_F, Ψ_F⟩; distinct A_F; A_F ♯* C]] ⟹ thesis
 moreover have ∃ A_F Ψ_F. F = ⟨A_F, Ψ_F⟩ ∧ A_F ♯* C
 proof(nominal-induct F avoiding: C rule: frame.strong-induct)
  case(FAssert Ψ_F)
  have FAssert Ψ_F = ⟨[], Ψ_F⟩ by simp
  moreover have ([]::name list) ♯* C by simp
  ultimately show ?case by force
 next
  case(FRes a F)
  from ‹⋀C. ∃ A_F Ψ_F. F = ⟨A_F, Ψ_F⟩ ∧ A_F ♯* C›
  obtain A_F Ψ_F  where F = ⟨A_F, Ψ_F⟩ and A_F ♯* C
   by blast
  with ‹a ♯ C› have (⦇νa⦈)F = (⦇ν*(a#A_F)⦈)(FAssert Ψ_F) and (a#A_F) ♯* C
   by simp+
  thus ?case by blast
 qed
 ultimately show ?thesis
  by(auto, rule-tac distinctFrame) auto
qed

locale assertionAux =
 fixes SCompose :: 'b::fs-name ⇒ 'b ⇒ 'b   (infixr ⊗ 80)
 and   SImp     :: 'b ⇒ 'c::fs-name ⇒ bool (- ⊢ - [70, 70] 70)
 and   SBottom  :: 'b                      (⊥ 90)
 and   SChanEq  :: 'a::fs-name ⇒ 'a ⇒ 'c   (- ↔ - [80, 80] 80)

 assumes statEqvt[eqvt]:   ⋀p::name prm. p ∙ (Ψ ⊢ Φ) = (p ∙ Ψ) ⊢ (p ∙ Φ)
 and     statEqvt'[eqvt]:  ⋀p::name prm. p ∙ (Ψ ⊗ Ψ') = (p ∙ Ψ) ⊗ (p ∙ Ψ')
 and     statEqvt''[eqvt]: ⋀p::name prm. p ∙ (M ↔ N) = (p ∙ M) ↔ (p ∙ N)
 and     permBottom[eqvt]: ⋀p::name prm. (p ∙ SBottom) = SBottom

begin

lemma statClosed:
 fixes Ψ :: 'b
 and   φ :: 'c
```

and $p :: name\ prm$

assumes $\Psi \vdash \varphi$

shows $(p \cdot \Psi) \vdash (p \cdot \varphi)$
using *assms statEqvt*
by(*simp add*: *perm-bool*)

lemma *compSupp*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$

shows $(supp(\Psi \otimes \Psi')::name\ set) \subseteq ((supp\ \Psi) \cup (supp\ \Psi'))$
proof(*auto simp add*: *eqvts supp-def*)
fix $x::name$
let $?P = \lambda y.\ ([(x,\ y)] \cdot \Psi) \otimes [(x,\ y)] \cdot \Psi' \neq \Psi \otimes \Psi'$
let $?Q = \lambda y\ \Psi.\ ([(x,\ y)] \cdot \Psi) \neq \Psi$
assume *finite* $\{y.\ ?Q\ y\ \Psi'\}$
moreover assume *finite* $\{y.\ ?Q\ y\ \Psi\}$ **and** *infinite* $\{y.\ ?P(y)\}$
hence *infinite*$(\{y.\ ?P(y)\} - \{y.\ ?Q\ y\ \Psi\})$ **by**(*rule Diff-infinite-finite*)
ultimately have *infinite*$((\{y.\ ?P(y)\} - \{y.\ ?Q\ y\ \Psi\}) - \{y.\ ?Q\ y\ \Psi'\})$ **by**(*rule Diff-infinite-finite*)
hence *infinite*$(\{y.\ ?P(y) \wedge \neg(?Q\ y\ \Psi) \wedge \neg\ (?Q\ y\ \Psi')\})$ **by**(*simp add*: *set-diff-eq*)
moreover have $\{y.\ ?P(y) \wedge \neg(?Q\ y\ \Psi) \wedge \neg\ (?Q\ y\ \Psi')\} = \{\}$ **by** *auto*
ultimately have *infinite* $\{\}$ **by**(*drule-tac Infinite-cong*) *auto*
thus *False* **by** *simp*
qed

lemma *chanEqSupp*:
fixes $M :: 'a$
and $N :: 'a$

shows $(supp(M \leftrightarrow N)::name\ set) \subseteq ((supp\ M) \cup (supp\ N))$
proof(*auto simp add*: *eqvts supp-def*)
fix $x::name$
let $?P = \lambda y.\ ([(x,\ y)] \cdot M) \leftrightarrow [(x,\ y)] \cdot N \neq M \leftrightarrow N$
let $?Q = \lambda y\ M.\ ([(x,\ y)] \cdot M) \neq M$
assume *finite* $\{y.\ ?Q\ y\ N\}$
moreover assume *finite* $\{y.\ ?Q\ y\ M\}$ **and** *infinite* $\{y.\ ?P(y)\}$
hence *infinite*$(\{y.\ ?P(y)\} - \{y.\ ?Q\ y\ M\})$ **by**(*rule Diff-infinite-finite*)
ultimately have *infinite*$((\{y.\ ?P(y)\} - \{y.\ ?Q\ y\ M\}) - \{y.\ ?Q\ y\ N\})$ **by**(*rule Diff-infinite-finite*)
hence *infinite*$(\{y.\ ?P(y) \wedge \neg(?Q\ y\ M) \wedge \neg\ (?Q\ y\ N)\})$ **by**(*simp add*: *set-diff-eq*)
moreover have $\{y.\ ?P(y) \wedge \neg(?Q\ y\ M) \wedge \neg\ (?Q\ y\ N)\} = \{\}$ **by** *auto*
ultimately have *infinite* $\{\}$ **by**(*drule-tac Infinite-cong*) *auto*
thus *False* **by** *simp*
qed

lemma *freshComp*[*intro*]:

fixes x :: *name*
and Ψ :: $'b$
and Ψ' :: $'b$

assumes $x \sharp \Psi$
and $x \sharp \Psi'$

shows $x \sharp \Psi \otimes \Psi'$
using *assms compSupp*
by(*auto simp add*: *fresh-def*)

lemma *freshCompChain*[*intro*]:
fixes *xvec* :: *name list*
and *Xs* :: *name set*
and Ψ :: $'b$
and Ψ' :: $'b$

shows $[\![xvec \sharp* \Psi; xvec \sharp* \Psi']\!] \Longrightarrow xvec \sharp* (\Psi \otimes \Psi')$
and $[\![Xs \sharp* \Psi; Xs \sharp* \Psi']\!] \Longrightarrow Xs \sharp* (\Psi \otimes \Psi')$
by(*auto simp add*: *fresh-star-def*)

lemma *freshChanEq*[*intro*]:
fixes x :: *name*
and M :: $'a$
and N :: $'a$

assumes $x \sharp M$
and $x \sharp N$

shows $x \sharp M \leftrightarrow N$
using *assms chanEqSupp*
by(*auto simp add*: *fresh-def*)

lemma *freshChanEqChain*[*intro*]:
fixes *xvec* :: *name list*
and *Xs* :: *name set*
and M :: $'a$
and N :: $'a$

shows $[\![xvec \sharp* M; xvec \sharp* N]\!] \Longrightarrow xvec \sharp* (M \leftrightarrow N)$
and $[\![Xs \sharp* M; Xs \sharp* N]\!] \Longrightarrow Xs \sharp* (M \leftrightarrow N)$
by(*auto simp add*: *fresh-star-def*)

lemma *suppBottom*[*simp*]:
shows ((*supp SBottom*)::*name set*) = {}
by(*auto simp add*: *supp-def permBottom*)

lemma *freshBottom*[*simp*]:
fixes x :: *name*

shows $x \sharp \bot$
by(*simp add*: *fresh-def*)

lemma *freshBottoChain*[*simp*]:
fixes *xvec* :: *name list*
and *Xs* :: *name set*

shows $xvec \sharp* (\bot)$
and $Xs \sharp* (\bot)$
by(*auto simp add*: *fresh-star-def*)

lemma *chanEqClosed*:
fixes $\Psi :: 'b$
and $M :: 'a$
and $N :: 'a$
and $p :: name\ prm$

assumes $\Psi \vdash M \leftrightarrow N$

shows $(p \cdot \Psi) \vdash (p \cdot M) \leftrightarrow (p \cdot N)$
proof −
from ‹$\Psi \vdash M \leftrightarrow N$› **have** $(p \cdot \Psi) \vdash p \cdot (M \leftrightarrow N)$
by(*rule statClosed*)
thus *?thesis* **by**(*simp add*: *eqvts*)
qed

definition
AssertionStatImp :: $'b \Rightarrow 'b \Rightarrow bool$ (**infix** $\hookrightarrow$ *70*)
where $(\Psi \hookrightarrow \Psi') \equiv (\forall \Phi.\ \Psi \vdash \Phi \longrightarrow \Psi' \vdash \Phi)$

definition
AssertionStatEq :: $'b \Rightarrow 'b \Rightarrow bool$ (**infix** $\simeq$ *70*)
where $(\Psi \simeq \Psi') \equiv \Psi \hookrightarrow \Psi' \wedge \Psi' \hookrightarrow \Psi$

lemma *statImpEnt*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $\Phi :: 'c$

assumes $\Psi \hookrightarrow \Psi'$
and $\Psi \vdash \Phi$

shows $\Psi' \vdash \Phi$
using *assms*
by(*simp add*: *AssertionStatImp-def*)

lemma *statEqEnt*:
fixes $\Psi :: 'b$

and $\Psi' :: 'b$
and $\Phi :: 'c$

assumes $\Psi \simeq \Psi'$
and $\Psi \vdash \Phi$

shows $\Psi' \vdash \Phi$
using *assms*
by(*auto simp add*: *AssertionStatEq-def intro*: *statImpEnt*)

lemma *AssertionStatImpClosed*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $p :: name\ prm$

assumes $\Psi \hookrightarrow \Psi'$

shows $(p \cdot \Psi) \hookrightarrow (p \cdot \Psi')$
proof(*auto simp add*: *AssertionStatImp-def*)
fix φ
assume $(p \cdot \Psi) \vdash \varphi$
hence $\Psi \vdash rev\ p \cdot \varphi$ **by**(*drule-tac p=rev p* **in** *statClosed*) *auto*
with $\langle \Psi \hookrightarrow \Psi' \rangle$ **have** $\Psi' \vdash rev\ p \cdot \varphi$ **by**(*simp add*: *AssertionStatImp-def*)
thus $(p \cdot \Psi') \vdash \varphi$ **by**(*drule-tac p=p* **in** *statClosed*) *auto*
qed

lemma *AssertionStatEqClosed*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $p :: name\ prm$

assumes $\Psi \simeq \Psi'$

shows $(p \cdot \Psi) \simeq (p \cdot \Psi')$
using *assms*
by(*auto simp add*: *AssertionStatEq-def intro*: *AssertionStatImpClosed*)

lemma *AssertionStatImpEqvt*[*eqvt*]:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $p :: name\ prm$

shows $(p \cdot (\Psi \hookrightarrow \Psi')) = ((p \cdot \Psi) \hookrightarrow (p \cdot \Psi'))$
by(*simp add*: *AssertionStatImp-def eqvts*)

lemma *AssertionStatEqEqvt*[*eqvt*]:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $p :: name\ prm$

shows $(p \cdot (\Psi \simeq \Psi')) = ((p \cdot \Psi) \simeq (p \cdot \Psi'))$
by(*simp add*: *AssertionStatEq-def eqvts*)

lemma *AssertionStatImpRefl*[*simp*]:
fixes $\Psi :: 'b$

shows $\Psi \hookrightarrow \Psi$
by(*simp add*: *AssertionStatImp-def*)

lemma *AssertionStatEqRefl*[*simp*]:
fixes $\Psi :: 'b$

shows $\Psi \simeq \Psi$
by(*simp add*: *AssertionStatEq-def*)

lemma *AssertionStatEqSym*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$

assumes $\Psi \simeq \Psi'$

shows $\Psi' \simeq \Psi$
using *assms*
by(*auto simp add*: *AssertionStatEq-def*)

lemma *AssertionStatImpTrans*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $\Psi'' :: 'b$

assumes $\Psi \hookrightarrow \Psi'$
and $\Psi' \hookrightarrow \Psi''$

shows $\Psi \hookrightarrow \Psi''$
using *assms*
by(*simp add*: *AssertionStatImp-def*)

lemma *AssertionStatEqTrans*:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$
and $\Psi'' :: 'b$

assumes $\Psi \simeq \Psi'$
and $\Psi' \simeq \Psi''$

shows $\Psi \simeq \Psi''$
using *assms*
by(*auto simp add*: *AssertionStatEq-def intro*: *AssertionStatImpTrans*)

definition
FrameImp :: *'b::fs-name frame* ⇒ *'c* ⇒ *bool* (**infixl** $\vdash_F$ *70*)
where $(F \vdash_F \Phi) = (\exists A_F\ \Psi_F.\ F = \langle A_F, \Psi_F\rangle \wedge A_F \sharp* \Phi \wedge (\Psi_F \vdash \Phi))$

lemma *frameImpI*:
fixes F :: *'b frame*
and φ :: *'c*
and A_F :: *name list*
and Ψ_F :: *'b*

assumes $F = \langle A_F, \Psi_F\rangle$
and $A_F \sharp* \varphi$
and $\Psi_F \vdash \varphi$

shows $F \vdash_F \varphi$
using *assms*
by(*force simp add*: *FrameImp-def*)

lemma *frameImpAlphaEnt*:
fixes A_F :: *name list*
and Ψ_F :: *'b*
and A_F' :: *name list*
and Ψ_F' :: *'b*
and φ :: *'c*

assumes $\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F'\rangle$
and $A_F \sharp* \varphi$
and $A_F' \sharp* \varphi$
and $\Psi_F' \vdash \varphi$

shows $\Psi_F \vdash \varphi$
proof −
from ‹$\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F'\rangle$›
obtain *n* **where** $n = length\ A_F$ **by** *blast*
moreover from ‹$\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F'\rangle$›
have $length\ A_F = length\ A_F'$
by(*rule frameChainEqLength*)
ultimately show *?thesis* **using** *assms*
proof(*induct n arbitrary*: $A_F\ A_F'\ \Psi_F'$ *rule*: *nat.induct*)
case(*zero* $A_F\ A_F'\ \Psi_F'$)
thus *?case* **by**(*auto simp add*: *frame.inject*)
next
case(*Suc n* $A_F\ A_F'\ \Psi_F'$)
from ‹*Suc n* = *length* A_F›
obtain *x xs* **where** $A_F = x\#xs$ **and** $n = length\ xs$
by(*case-tac* A_F) *auto*
from ‹$\langle A_F, \Psi_F\rangle = \langle A_F', \Psi_F'\rangle$› ‹$A_F = x\ \#\ xs$›
obtain *y ys* **where** $\langle(x\#xs), \Psi_F\rangle = \langle(y\#ys), \Psi_F'\rangle$ **and** $A_F' = y\#ys$

by(*case-tac* A_F') *auto*
hence *EQ*: $(\!|\nu x|\!)(\!|\nu{*}xs|\!)(FAssert\ \Psi_F) = (\!|\nu y|\!)(\!|\nu{*}ys|\!)(FAssert\ \Psi_F')$
by *simp*
from ‹$A_F = x \# xs$› ‹$A_F' = y \# ys$› ‹*length* A_F = *length* A_F'› ‹$A_F \sharp* \varphi$› ‹$A_F' \sharp* \varphi$›
have *length xs* = *length ys* **and** $xs \sharp* \varphi$ **and** $ys \sharp* \varphi$ **and** $x \sharp \varphi$ **and** $y \sharp \varphi$
by *auto*

have *IH*: $\bigwedge xs\ ys\ \Psi_F'.\ [\![n = length\ xs;\ length\ xs = length\ ys;\ \langle xs, \Psi_F\rangle = \langle ys, (\Psi_F'::'b)\rangle;\ xs \sharp* \varphi;\ ys \sharp* \varphi;\ \Psi_F' \vdash \varphi]\!] \Longrightarrow \Psi_F \vdash \varphi$
by *fact*
show *?case*
proof(*case-tac* $x = y$)
assume $x = y$
with *EQ* **have** $\langle xs, \Psi_F\rangle = \langle ys, \Psi_F'\rangle$ **by**(*simp add*: *alpha frame.inject*)
with *IH* ‹$n = length\ xs$› ‹*length xs* = *length ys*› ‹$xs \sharp* \varphi$› ‹$ys \sharp* \varphi$› ‹$\Psi_F' \vdash \varphi$›
show *?case* **by** *blast*
next
assume $x \neq y$
with *EQ* **have** $\langle xs, \Psi_F\rangle = [(x, y)] \cdot \langle ys, \Psi_F'\rangle$ **by**(*simp add*: *alpha frame.inject*)
hence $\langle xs, \Psi_F\rangle = \langle([(x, y)] \cdot ys), ([(x, y)] \cdot \Psi_F')\rangle$ **by**(*simp add*: *eqvts*)
moreover from ‹*length xs* = *length ys*› **have** *length xs* = *length*$([(x, y)] \cdot ys)$
by *auto*
moreover from ‹$ys \sharp* \varphi$› **have** $([(x, y)] \cdot ys) \sharp* ([(x, y)] \cdot \varphi)$
by(*simp add*: *fresh-star-bij*)
with ‹$x \sharp \varphi$› ‹$y \sharp \varphi$› **have** $([(x, y)] \cdot ys) \sharp* \varphi$
by *simp*
moreover with ‹$\Psi_F' \vdash \varphi$› **have** $([(x, y)] \cdot \Psi_F') \vdash ([(x, y)] \cdot \varphi)$
by(*simp add*: *statClosed*)
with ‹$x \sharp \varphi$› ‹$y \sharp \varphi$› **have** $([(x, y)] \cdot \Psi_F') \vdash \varphi$
by *simp*
ultimately show *?case* **using** *IH* ‹$n = length\ xs$› ‹$xs \sharp* \varphi$›
by *blast*
qed
qed
qed

lemma *frameImpEAux*:
fixes F :: $'b$ *frame*
and Φ :: $'c$

assumes $F \vdash_F \Phi$
and $F = \langle A_F, \Psi_F\rangle$
and $A_F \sharp* \Phi$

shows $\Psi_F \vdash \Phi$
using *assms*
by(*auto simp add*: *FrameImp-def dest*: *frameImpAlphaEnt*)

lemma *frameImpE*:
  fixes F :: *'b frame*
  and Φ :: *'c*

  assumes $\langle A_F, \Psi_F\rangle \vdash_F \Phi$
  and $A_F \sharp* \Phi$

  shows $\Psi_F \vdash \Phi$
using *assms*
by(*auto elim*: *frameImpEAux*)

lemma *frameImpClosed*:
  fixes F :: *'b frame*
  and Φ :: *'c*
  and p :: *name prm*

  assumes $F \vdash_F \Phi$

  shows $(p \cdot F) \vdash_F (p \cdot \Phi)$
using *assms*
by(*force simp add*: *FrameImp-def eqvts pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*]
        intro: *statClosed*)

lemma *frameImpEqvt*[*eqvt*]:
  fixes F :: *'b frame*
  and Φ :: *'c*
  and p :: *name prm*

  shows $(p \cdot (F \vdash_F \Phi)) = (p \cdot F) \vdash_F (p \cdot \Phi)$
proof −
  have $F \vdash_F \Phi \Longrightarrow (p \cdot F) \vdash_F (p \cdot \Phi)$
    by(*rule frameImpClosed*)
  moreover have $(p \cdot F) \vdash_F (p \cdot \Phi) \Longrightarrow F \vdash_F \Phi$
    by(*drule-tac* $p = rev\ p$ **in** *frameImpClosed*) *simp*
  ultimately show *?thesis*
    by(*auto simp add*: *perm-bool*)
qed

lemma *frameImpEmpty*[*simp*]:
  fixes Ψ :: *'b*
  and φ :: *'c*

  shows $\langle \varepsilon, \Psi\rangle \vdash_F \varphi = \Psi \vdash \varphi$
by(*auto simp add*: *FrameImp-def*)

definition
  FrameStatImp :: *'b frame* ⇒ *'b frame*⇒ *bool* (**infix** $\hookrightarrow_F$ *70*)
  where $(F \hookrightarrow_F G) \equiv (\forall \varphi.\ F \vdash_F \varphi \longrightarrow G \vdash_F \varphi)$

definition
FrameStatEq :: *'b frame* ⇒ *'b frame*⇒ *bool* (**infix** $\simeq_F$ *70*)
where $(F \simeq_F G) \equiv F \hookrightarrow_F G \wedge G \hookrightarrow_F F$

lemma *FrameStatImpClosed*:
fixes F :: *'b frame*
and G :: *'b frame*
and p :: *name prm*

assumes $F \hookrightarrow_F G$

shows $(p \cdot F) \hookrightarrow_F (p \cdot G)$
proof(*auto simp add*: *FrameStatImp-def*)
fix φ
assume $(p \cdot F) \vdash_F \varphi$
hence $F \vdash_F rev\ p \cdot \varphi$ **by**(*drule-tac p=rev p* **in** *frameImpClosed*) *auto*
with ‹$F \hookrightarrow_F G$› **have** $G \vdash_F rev\ p \cdot \varphi$ **by**(*simp add*: *FrameStatImp-def*)
thus $(p \cdot G) \vdash_F \varphi$ **by**(*drule-tac p=p* **in** *frameImpClosed*) *auto*
qed

lemma *FrameStatEqClosed*:
fixes F :: *'b frame*
and G :: *'b frame*
and p :: *name prm*

assumes $F \simeq_F G$

shows $(p \cdot F) \simeq_F (p \cdot G)$
using *assms*
by(*auto simp add*: *FrameStatEq-def intro*: *FrameStatImpClosed*)

lemma *FrameStatImpEqvt*[*eqvt*]:
fixes F :: *'b frame*
and G :: *'b frame*
and p :: *name prm*

shows $(p \cdot (F \hookrightarrow_F G)) = ((p \cdot F) \hookrightarrow_F (p \cdot G))$
by(*simp add*: *FrameStatImp-def eqvts*)

lemma *FrameStatEqEqvt*[*eqvt*]:
fixes F :: *'b frame*
and G :: *'b frame*
and p :: *name prm*

shows $(p \cdot (F \simeq_F G)) = ((p \cdot F) \simeq_F (p \cdot G))$
by(*simp add*: *FrameStatEq-def eqvts*)

lemma *FrameStatImpRefl*[*simp*]:

fixes $F :: 'b\ frame$

shows $F \hookrightarrow_F F$
by(*simp add*: *FrameStatImp-def*)

lemma *FrameStatEqRefl*[*simp*]:
fixes $F :: 'b\ frame$

shows $F \simeq_F F$
by(*simp add*: *FrameStatEq-def*)

lemma *FrameStatEqSym*:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$

assumes $F \simeq_F G$

shows $G \simeq_F F$
using *assms*
by(*auto simp add*: *FrameStatEq-def*)

lemma *FrameStatImpTrans*:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$
and $H :: 'b\ frame$

assumes $F \hookrightarrow_F G$
and $G \hookrightarrow_F H$

shows $F \hookrightarrow_F H$
using *assms*
by(*simp add*: *FrameStatImp-def*)

lemma *FrameStatEqTrans*:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$
and $H :: 'b\ frame$

assumes $F \simeq_F G$
and $G \simeq_F H$

shows $F \simeq_F H$
using *assms*
by(*auto simp add*: *FrameStatEq-def intro*: *FrameStatImpTrans*)

lemma *fsCompose*[*simp*]: *finite*((*supp SCompose*)::*name set*)
by(*simp add*: *supp-def perm-fun-def eqvts*)

nominal-primrec

```
  insertAssertion :: 'b frame ⇒ 'b ⇒ 'b frame
where
  insertAssertion (FAssert Ψ) Ψ' = FAssert (Ψ' ⊗ Ψ)
| x ♯ Ψ' ⟹ insertAssertion ((⌊νx⌋)F) Ψ' = (⌊νx⌋)(insertAssertion F Ψ')
apply(finite-guess add: fsCompose)+
apply(rule TrueI)+
apply(simp add: abs-fresh)
apply(rule supports-fresh[of supp Ψ'])
apply(force simp add: perm-fun-def eqvts fresh-def[symmetric] supports-def)
apply(simp add: fs-name1)
apply(simp add: fresh-def[symmetric])
apply(fresh-guess)+
done

lemma insertAssertionEqvt[eqvt]:
  fixes p :: name prm
  and   F :: 'b frame
  and   Ψ :: 'b

  shows p • (insertAssertion F Ψ) = insertAssertion (p • F) (p • Ψ)
by(nominal-induct F avoiding: p Ψ rule: frame.strong-induct)
  (auto simp add: at-prm-fresh[OF at-name-inst]
                  pt-fresh-perm-app[OF pt-name-inst, OF at-name-inst] eqvts)


nominal-primrec
  mergeFrame :: 'b frame ⇒ 'b frame ⇒ 'b frame
where
  mergeFrame (FAssert Ψ) G = insertAssertion G Ψ
| x ♯ G ⟹ mergeFrame ((⌊νx⌋)F) G = (⌊νx⌋)(mergeFrame F G)
apply(finite-guess add: fsCompose)+
apply(rule TrueI)+
apply(simp add: abs-fresh)
apply(simp add: fs-name1)
apply(rule supports-fresh[of supp G])
apply(force simp add: perm-fun-def eqvts fresh-def[symmetric] supports-def)
apply(simp add: fs-name1)
apply(simp add: fresh-def[symmetric])
apply(fresh-guess)+
done

notation mergeFrame (infixr ⊗F 80)

abbreviation
  frameBottomJudge (⊥F) where ⊥F ≡ (FAssert SBottom)

lemma mergeFrameEqvt[eqvt]:
  fixes p :: name prm
  and   F :: 'b frame
```

and $G :: 'b\ frame$

shows $p \cdot (mergeFrame\ F\ G) = mergeFrame\ (p \cdot F)\ (p \cdot G)$
by(*nominal-induct F avoiding*: *p G rule*: *frame.strong-induct*)
(*auto simp add*: *at-prm-fresh*[*OF at-name-inst*]
pt-fresh-perm-app[*OF pt-name-inst*, *OF at-name-inst*] *eqvts*)

nominal-primrec
$extractFrame :: ('a, 'b, 'c)\ psi \Rightarrow 'b\ frame$
and $extractFrame' :: ('a, 'b, 'c)\ input \Rightarrow 'b\ frame$
and $extractFrame'' :: ('a, 'b, 'c)\ psiCase \Rightarrow 'b\ frame$

where
$extractFrame\ (\mathbf{0}) = \langle \varepsilon, \bot \rangle$
$| extractFrame\ (M(\!|I) = \langle \varepsilon, \bot \rangle$
$| extractFrame\ (M\langle N \rangle.P) = \langle \varepsilon, \bot \rangle$
$| extractFrame\ (Case\ C) = \langle \varepsilon, \bot \rangle$
$| extractFrame\ (P \parallel Q) = (extractFrame\ P) \otimes_F (extractFrame\ Q)$
$| extractFrame\ ((\{\!|\Psi|\!\}::('a, 'b, 'c)\ psi)) = \langle \varepsilon, \Psi \rangle$

$| extractFrame\ ((|\nu x|)P) = (|\nu x|)(extractFrame\ P)$
$| extractFrame\ (!P) = \langle \varepsilon, \bot \rangle$

$| extractFrame'\ ((Trm\ M\ P)::('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ input) = \langle \varepsilon, \bot \rangle$

$| extractFrame'\ (Bind\ x\ I) = \langle \varepsilon, \bot \rangle$

$| extractFrame''\ (\bot_c::('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ psiCase) = \langle \varepsilon, \bot \rangle$
$| extractFrame''\ (\Box\Phi \Rightarrow P\ C) = \langle \varepsilon, \bot \rangle$
apply(*finite-guess add*: *fsCompose*)+
apply(*rule TrueI*)+
apply(*simp add*: *abs-fresh*)+
apply(*fresh-guess add*: *freshBottom*)+
apply(*rule supports-fresh*[*of* {}])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess add*: *freshBottom*)+
apply(*rule supports-fresh*[*of* {}])
apply(*force simp add*: *perm-fun-def eqvts fresh-def*[*symmetric*] *supports-def*)
apply(*simp add*: *fs-name1*)
apply(*simp add*: *fresh-def*[*symmetric*])
apply(*fresh-guess add*: *freshBottom*)+
done

lemmas *extractFrameSimps* = *extractFrame-extractFrame'-extractFrame''.simps*

lemma *extractFrameEqvt*[*eqvt*]:
fixes $p :: name\ prm$

and $P :: ('a, 'b, 'c)\ psi$
and $I :: ('a, 'b, 'c)\ input$
and $C :: ('a, 'b, 'c)\ psiCase$

shows $p \cdot (extractFrame\ P) = extractFrame\ (p \cdot P)$
and $p \cdot (extractFrame'\ I) = extractFrame'\ (p \cdot I)$
and $p \cdot (extractFrame''\ C) = extractFrame''\ (p \cdot C)$
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *p rule*: *psi-input-psiCase.strong-inducts*)
(*auto simp add*: *at-prm-fresh*[*OF at-name-inst*] *eqvts permBottom*
pt-fresh-perm-app[*OF pt-name-inst*, *OF at-name-inst*])

lemma *insertAssertionFresh*[*intro*]:
fixes $F :: 'b\ frame$
and $\Psi :: 'b$
and $x :: name$

assumes $x \sharp F$
and $x \sharp \Psi$

shows $x \sharp (insertAssertion\ F\ \Psi)$
using *assms*
by(*nominal-induct F avoiding*: *x* Ψ *rule*: *frame.strong-induct*)
(*auto simp add*: *abs-fresh*)

lemma *insertAssertionFreshChain*[*intro*]:
fixes $F :: 'b\ frame$
and $\Psi :: 'b$
and $xvec :: name\ list$
and $Xs :: name\ set$

shows $[\![xvec \sharp* F;\ xvec \sharp* \Psi]\!] \Longrightarrow xvec \sharp* (insertAssertion\ F\ \Psi)$
and $[\![Xs \sharp* F;\ Xs \sharp* \Psi]\!] \Longrightarrow Xs \sharp* (insertAssertion\ F\ \Psi)$
by(*auto simp add*: *fresh-star-def*)

lemma *mergeFrameFresh*[*intro*]:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$
and $x :: name$

shows $[\![x \sharp F;\ x \sharp G]\!] \Longrightarrow x \sharp (mergeFrame\ F\ G)$
by(*nominal-induct F avoiding*: *x G rule*: *frame.strong-induct*)
(*auto simp add*: *abs-fresh*)

lemma *mergeFrameFreshChain*[*intro*]:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$
and $xvec :: name\ list$
and $Xs :: name\ set$

shows $[\![xvec \sharp* F; xvec \sharp* G]\!] \Longrightarrow xvec \sharp* (mergeFrame\ F\ G)$
and $[\![Xs \sharp* F; Xs \sharp* G]\!] \Longrightarrow Xs \sharp* (mergeFrame\ F\ G)$
by(*auto simp add*: *fresh-star-def*)

lemma *extractFrameFresh*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $I :: ('a, 'b, 'c)\ input$
and $C :: ('a, 'b, 'c)\ psiCase$
and $x :: name$

shows $x \sharp P \Longrightarrow x \sharp extractFrame\ P$
and $x \sharp I \Longrightarrow x \sharp extractFrame'\ I$
and $x \sharp C \Longrightarrow x \sharp extractFrame''\ C$
by(*nominal-induct P* **and** *I* **and** *C avoiding*: *x rule*: *psi-input-psiCase.strong-inducts*)
(*auto simp add*: *abs-fresh*)

lemma *extractFrameFreshChain*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $I :: ('a, 'b, 'c)\ input$
and $C :: ('a, 'b, 'c)\ psiCase$
and $xvec :: name\ list$
and $Xs :: name\ set$

shows $xvec \sharp* P \Longrightarrow xvec \sharp* extractFrame\ P$
and $xvec \sharp* I \Longrightarrow xvec \sharp* extractFrame'\ I$
and $xvec \sharp* C \Longrightarrow xvec \sharp* extractFrame''\ C$
and $Xs \sharp* P \Longrightarrow Xs \sharp* extractFrame\ P$
and $Xs \sharp* I \Longrightarrow Xs \sharp* extractFrame'\ I$
and $Xs \sharp* C \Longrightarrow Xs \sharp* extractFrame''\ C$
by(*auto simp add*: *fresh-star-def intro*: *extractFrameFresh*)

lemma *guardedFrameSupp*[*simp*]:
fixes $P :: ('a, 'b, 'c)\ psi$
and $I :: ('a, 'b, 'c)\ input$
and $C :: ('a, 'b, 'c)\ psiCase$
and $x :: name$

shows $guarded\ P \Longrightarrow x \sharp (extractFrame\ P)$
and $guarded'\ I \Longrightarrow x \sharp (extractFrame'\ I)$
and $guarded''\ C \Longrightarrow x \sharp (extractFrame''\ C)$
by(*nominal-induct P* **and** *I* **and** *C arbitrary*: *x rule*: *psi-input-psiCase.strong-inducts*)
(*auto simp add*: *frameResChainFresh abs-fresh*)

lemma *frameResChainFresh'*:
fixes $xvec :: name\ list$
and $yvec :: name\ list$
and $F :: 'b\ frame$

shows $(xvec \sharp* (\langle\!|\nu * yvec|\!\rangle F)) = (\forall x \in set\ xvec.\ x \in set\ yvec \vee x \sharp F)$
by(*simp add*: *frameResChainFresh fresh-star-def*)

lemma *frameChainFresh*[*simp*]:
fixes $xvec :: name\ list$
and $\Psi :: 'b$
and $Xs :: name\ set$

shows $xvec \sharp* (FAssert\ \Psi) = xvec \sharp* \Psi$
and $Xs \sharp* (FAssert\ \Psi) = Xs \sharp* \Psi$
by(*simp add*: *fresh-star-def*)+

lemma *frameResChainFresh''*[*simp*]:
fixes $xvec :: name\ list$
and $yvec :: name\ list$
and $F :: 'b\ frame$

assumes $xvec \sharp* yvec$

shows $xvec \sharp* (\langle\!|\nu * yvec|\!\rangle F) = xvec \sharp* F$

using *assms*
by(*simp-all add*: *frameResChainFresh'*)
(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)

lemma *frameResChainFresh'''*[*simp*]:
fixes $x :: name$
and $xvec :: name\ list$
and $F :: 'b\ frame$

assumes $x \sharp xvec$

shows $x \sharp (\langle\!|\nu * xvec|\!\rangle F) = x \sharp F$
using *assms*
by(*induct xvec*) (*auto simp add*: *abs-fresh*)

lemma *FFreshBottom*[*simp*]:
fixes $xvec :: name\ list$
and $Xs :: name\ set$

shows $xvec \sharp* (\bot_F)$
and $Xs \sharp* (\bot_F)$
by(*auto simp add*: *fresh-star-def*)

lemma *SFreshBottom*[*simp*]:
fixes $xvec :: name\ list$
and $Xs :: name\ set$

shows $xvec \sharp* (SBottom)$

and $Xs \sharp* (SBottom)$
by(*auto simp add*: *fresh-star-def*)

lemma *freshFrameDest*[*dest*]:
fixes A_F :: *name list*
and Ψ_F :: *'b*
and *xvec* :: *name list*

assumes $xvec \sharp* (\langle A_F, \Psi_F\rangle)$

shows $xvec \sharp* A_F \Longrightarrow xvec \sharp* \Psi_F$
and $A_F \sharp* xvec \Longrightarrow xvec \sharp* \Psi_F$
proof −
from *assms* **have** $(set\ xvec) \sharp* (\langle A_F, \Psi_F\rangle)$
by(*simp add*: *fresh-star-def*)
moreover assume $xvec \sharp* A_F$
ultimately show $xvec \sharp* \Psi_F$
by(*simp add*: *frameResChainFreshSet*) (*force simp add*: *fresh-def name-list-supp fresh-star-def*)
next
from *assms* **have** $(set\ xvec) \sharp* (\langle A_F, \Psi_F\rangle)$
by(*simp add*: *fresh-star-def*)
moreover assume $A_F \sharp* xvec$
ultimately show $xvec \sharp* \Psi_F$
by(*simp add*: *frameResChainFreshSet*) (*force simp add*: *fresh-def name-list-supp fresh-star-def*)
qed

lemma *insertAssertionSimps*[*simp*]:
fixes A_F :: *name list*
and Ψ_F :: *'b*
and Ψ :: *'b*

assumes $A_F \sharp* \Psi$

shows $insertAssertion\ (\langle A_F, \Psi_F\rangle)\ \Psi = \langle A_F, \Psi \otimes \Psi_F\rangle$
using *assms*
by(*induct* A_F *arbitrary*: *F*) *auto*

lemma *mergeFrameSimps*[*simp*]:
fixes A_F :: *name list*
and Ψ_F :: *'b*
and Ψ :: *'b*

assumes $A_F \sharp* \Psi$

shows $(\langle A_F, \Psi_F\rangle) \otimes_F \langle \varepsilon, \Psi\rangle = \langle A_F, \Psi_F \otimes \Psi\rangle$
using *assms*
by(*induct* A_F *arbitrary*: *F*) *auto*

lemma *mergeFrames*[*simp*]:
  fixes A_F :: *name list*
  and Ψ_F :: *'b*
  and A_G :: *name list*
  and Ψ_G :: *'b*

  assumes $A_F \sharp* A_G$
  and $A_F \sharp* \Psi_G$
  and $A_G \sharp* \Psi_F$

  shows $(\langle A_F, \Psi_F\rangle) \otimes_F (\langle A_G, \Psi_G\rangle) = (\langle (A_F @ A_G), \Psi_F \otimes \Psi_G\rangle)$
using *assms*
by(*induct* A_F) *auto*

lemma *frameImpResFreshLeft*:
  fixes F :: *'b frame*
  and x :: *name*

  assumes $x \sharp F$

  shows $(\!|\nu x|\!)F \hookrightarrow_F F$
proof(*auto simp add*: *FrameStatImp-def*)
  fix φ::*'c*
  obtain A_F Ψ_F **where** *Feq*: $F = \langle A_F, \Psi_F\rangle$ **and** $A_F \sharp* (x, \varphi)$
    by(*rule freshFrame*)
  from ‹$A_F \sharp* (x, \varphi)$› **have** $x \sharp A_F$ **and** $A_F \sharp* \varphi$ **by** *simp+*
  obtain y **where** $y \sharp \varphi$ **and** $y \sharp F$ **and** $x \neq y$
    by(*generate-fresh name*, *auto*)

  assume $(\!|\nu x|\!)F \vdash_F \varphi$
  with ‹$y \sharp F$› **have** $(\!|\nu y|\!)([(x, y)] \cdot F) \vdash_F \varphi$ **by**(*simp add*: *alphaFrameRes*)
  with ‹$x \sharp F$› ‹$y \sharp F$› **have** $(\!|\nu y|\!)F \vdash_F \varphi$ **by** *simp*
  with *Feq* **have** $\langle (y\#A_F), \Psi_F\rangle \vdash_F \varphi$ **by** *simp*
  with *Feq* ‹$A_F \sharp* \varphi$› ‹$y \sharp \varphi$› **show** $F \vdash_F \varphi$
    by(*force intro*: *frameImpI dest*: *frameImpE simp del*: *frameResChain.simps*)
qed

lemma *frameImpResFreshRight*:
  fixes F :: *'b frame*
  and x :: *name*

  assumes $x \sharp F$

  shows $F \hookrightarrow_F (\!|\nu x|\!)F$
proof(*auto simp add*: *FrameStatImp-def*)
  fix φ::*'c*
  obtain A_F Ψ_F **where** *Feq*: $F = \langle A_F, \Psi_F\rangle$ **and** $A_F \sharp* (x, \varphi)$
    by(*rule freshFrame*)

from ‹A_F ♯* (x, φ)› **have** x ♯ A_F **and** A_F ♯* φ **by** *simp+*
obtain y **where** y ♯ φ **and** y ♯ F **and** $x \neq y$
by(*generate-fresh name*, *auto*)

assume $F \vdash_F \varphi$
with *Feq* ‹A_F ♯* φ› ‹y ♯ φ› **have** $\langle(y\#A_F), \Psi_F\rangle \vdash_F \varphi$
by(*force intro*: *frameImpI dest*: *frameImpE simp del*: *frameResChain.simps*)
moreover with ‹y ♯ F› ‹x ♯ F› *Feq* **show** $(\!|\nu x|\!)F \vdash_F \varphi$
by(*subst alphaFrameRes*) *auto*
qed

lemma *frameResFresh*:
fixes $F :: 'b$ *frame*
and $x ::$ *name*

assumes x ♯ F

shows $(\!|\nu x|\!)F \simeq_F F$
using *assms*
by(*auto simp add*: *FrameStatEq-def intro*: *frameImpResFreshLeft frameImpResFreshRight*)

lemma *frameImpResPres*:
fixes $F :: 'b$ *frame*
and $G :: 'b$ *frame*
and $x ::$ *name*

assumes $F \hookrightarrow_F G$

shows $(\!|\nu x|\!)F \hookrightarrow_F (\!|\nu x|\!)G$
proof(*auto simp add*: *FrameStatImp-def*)
fix $\varphi::'c$
obtain A_F Ψ_F **where** *Feq*: $F = \langle A_F, \Psi_F\rangle$ **and** A_F ♯* (x, φ)
by(*rule freshFrame*)
from ‹A_F ♯* (x, φ)› **have** x ♯ A_F **and** A_F ♯* φ **by** *simp+*
obtain y **where** y ♯ A_F **and** y ♯ F **and** y ♯ G
and $x \neq y$ **and** y ♯ φ
by(*generate-fresh name*, *auto*)
assume $(\!|\nu x|\!)F \vdash_F \varphi$
with ‹y ♯ F› **have** $(\!|\nu y|\!)([(x, y)] \cdot F) \vdash_F \varphi$ **by**(*simp add*: *alphaFrameRes*)
with *Feq* ‹x ♯ A_F› ‹y ♯ A_F› **have** $\langle(y\#A_F), [(x, y)] \cdot \Psi_F\rangle \vdash_F \varphi$ **by**(*simp add*: *eqvts*)
with ‹y ♯ φ› ‹A_F ♯* φ› **have** $\langle A_F, [(x, y)] \cdot \Psi_F\rangle \vdash_F \varphi$
by(*force intro*: *frameImpI dest*: *frameImpE simp del*: *frameResChain.simps*)
hence $([(x, y)] \cdot \langle A_F, [(x, y)] \cdot \Psi_F\rangle) \vdash_F ([(x, y)] \cdot \varphi)$
by(*rule frameImpClosed*)
with ‹x ♯ A_F› ‹y ♯ A_F› *Feq* **have** $F \vdash_F [(x, y)] \cdot \varphi$
by(*simp add*: *eqvts*)
with ‹$F \hookrightarrow_F G$› **have** $G \vdash_F [(x, y)] \cdot \varphi$ **by**(*simp add*: *FrameStatImp-def*)

obtain A_G Ψ_G **where** *Geq*: $G = \langle A_G, \Psi_G\rangle$ **and** $A_G \sharp* (x, y, \varphi)$
by(*rule freshFrame*)
from ‹$A_G \sharp* (x, y, \varphi)$› **have** $x \sharp A_G$ **and** $y \sharp A_G$ **and** $A_G \sharp* \varphi$ **by** *simp+*
from ‹$G \vdash_F [(x, y)] \cdot \varphi$› **have** $([(x, y)] \cdot G) \vdash_F [(x, y)] \cdot [(x, y)] \cdot \varphi$
by(*rule frameImpClosed*)
with *Geq* ‹$x \sharp A_G$› ‹$y \sharp A_G$› **have** $\langle A_G, [(x, y)] \cdot \Psi_G\rangle \vdash_F \varphi$ **by**(*simp add*: *eqvts*)
with ‹$y \sharp \varphi$› ‹$A_G \sharp* \varphi$› **have** $\langle (y\#A_G), [(x, y)] \cdot \Psi_G\rangle \vdash_F \varphi$
by(*force intro*: *frameImpI dest*: *frameImpE simp del*: *frameResChain.simps*)
with ‹$y \sharp G$› ‹$x \sharp A_G$› ‹$y \sharp A_G$› *Geq* **show** $(\!|\nu x|\!) G \vdash_F \varphi$
by(*subst alphaFrameRes*) (*fastforce simp add*: *eqvts*)+
qed

lemma *frameResPres*:
fixes $F :: 'b$ *frame*
and $G :: 'b$ *frame*
and $x ::$ *name*

assumes $F \simeq_F G$

shows $(\!|\nu x|\!) F \simeq_F (\!|\nu x|\!) G$
using *assms*
by(*auto simp add*: *FrameStatEq-def intro*: *frameImpResPres*)

lemma *frameImpResComm*:
fixes $x ::$ *name*
and $y ::$ *name*
and $F :: 'b$ *frame*

shows $(\!|\nu x|\!)((\!|\nu y|\!) F) \hookrightarrow_F (\!|\nu y|\!)((\!|\nu x|\!) F)$
proof(*case-tac* $x = y$)
assume $x = y$
thus *?thesis* **by** *simp*
next
assume $x \neq y$
show *?thesis*
proof(*auto simp add*: *FrameStatImp-def*)
fix $\varphi::'c$
obtain A_F Ψ_F **where** *Feq*: $F = \langle A_F, \Psi_F\rangle$ **and** $A_F \sharp* (x, y, \varphi)$
by(*rule freshFrame*)
then have $x \sharp A_F$ **and** $y \sharp A_F$ **and** $A_F \sharp* \varphi$ **by** *simp+*

obtain $x'::$*name* **where** $x' \neq x$ **and** $x' \neq y$ **and** $x' \sharp F$ **and** $x' \sharp \varphi$ **and** $x' \sharp A_F$
by(*generate-fresh name*) *auto*
obtain $y'::$*name* **where** $y' \neq x$ **and** $y' \neq y$ **and** $y' \neq x'$ **and** $y' \sharp F$ **and** $y' \sharp \varphi$ **and** $y' \sharp A_F$
by(*generate-fresh name*) *auto*

from ‹$y' \sharp F$› **have** $(\!|\nu x|\!)((\!|\nu y|\!)F) = (\!|\nu x|\!)((\!|\nu y'|\!)([(y, y')] \cdot F))$
by(*simp add*: *alphaFrameRes*)
moreover from ‹$x' \sharp F$› ‹$x' \neq y$› ‹$y' \neq x'$› **have** $\ldots = (\!|\nu x'|\!)([(x, x')] \cdot ((\!|\nu y'|\!)([(y, y')] \cdot F)))$
by(*rule-tac alphaFrameRes*) (*simp add*: *abs-fresh fresh-left*)
moreover with ‹$y' \neq x'$› ‹$y' \neq x$› **have** $\ldots = (\!|\nu x'|\!)((\!|\nu y'|\!)([(x, x')] \cdot [(y, y')] \cdot F))$
by(*simp add*: *eqvts calc-atm*)
ultimately have *A*: $(\!|\nu x|\!)((\!|\nu y|\!)F) = (\!|\nu x'|\!)((\!|\nu y'|\!)((\!|\nu * A_F|\!)(FAssert([(x, x')] \cdot [(y, y')] \cdot \Psi_F))))$
using *Feq* ‹$x \sharp A_F$› ‹$x' \sharp A_F$› ‹$y \sharp A_F$› ‹$y' \sharp A_F$›
by(*simp add*: *eqvts*)

from ‹$x' \sharp F$› **have** $(\!|\nu y|\!)((\!|\nu x|\!)F) = (\!|\nu y|\!)((\!|\nu x'|\!)([(x, x')] \cdot F))$
by(*simp add*: *alphaFrameRes*)
moreover from ‹$y' \sharp F$› ‹$y' \neq x$› ‹$y' \neq x'$› **have** $\ldots = (\!|\nu y'|\!)([(y, y')] \cdot ((\!|\nu x'|\!)([(x, x')] \cdot F)))$
by(*rule-tac alphaFrameRes*) (*simp add*: *abs-fresh fresh-left*)
moreover with ‹$y' \neq x'$› ‹$x' \neq y$› **have** $\ldots = (\!|\nu y'|\!)((\!|\nu x'|\!)([(y, y')] \cdot [(x, x')] \cdot F))$
by(*simp add*: *eqvts calc-atm*)
moreover with ‹$x' \neq x$› ‹$x' \neq y$› ‹$y' \neq x$› ‹$y' \neq y$› ‹$y' \neq x'$› ‹$x \neq y$›
have $\ldots = (\!|\nu y'|\!)((\!|\nu x'|\!)([(x, x')] \cdot [(y, y')] \cdot F))$
apply(*simp add*: *eqvts*)
by(*subst perm-compose*) (*simp add*: *calc-atm*)
ultimately have *B*: $(\!|\nu y|\!)((\!|\nu x|\!)F) = (\!|\nu y'|\!)((\!|\nu x'|\!)((\!|\nu * A_F|\!)(FAssert([(x, x')] \cdot [(y, y')] \cdot \Psi_F))))$
using *Feq* ‹$x \sharp A_F$› ‹$x' \sharp A_F$› ‹$y \sharp A_F$› ‹$y' \sharp A_F$›
by(*simp add*: *eqvts*)

from ‹$x' \sharp \varphi$› ‹$y' \sharp \varphi$› ‹$A_F \sharp * \varphi$›
have $\langle (x' \# y' \# A_F), [(x, x')] \cdot [(y, y')] \cdot \Psi_F \rangle \vdash_F \varphi = \langle (y' \# x' \# A_F), [(x, x')] \cdot [(y, y')] \cdot \Psi_F \rangle \vdash_F \varphi$
by(*force dest*: *frameImpE intro*: *frameImpI simp del*: *frameResChain.simps*)
with *A B* **have** $((\!|\nu x|\!)((\!|\nu y|\!)F)) \vdash_F \varphi = ((\!|\nu y|\!)((\!|\nu x|\!)F)) \vdash_F \varphi$
by *simp*
moreover assume $((\!|\nu x|\!)((\!|\nu y|\!)F)) \vdash_F \varphi$
ultimately show $((\!|\nu y|\!)((\!|\nu x|\!)F)) \vdash_F \varphi$ **by** *simp*
qed
qed

lemma *frameResComm*:
fixes x :: *name*
and y :: *name*
and F :: *'b frame*

shows $(\!|\nu x|\!)((\!|\nu y|\!)F) \simeq_F (\!|\nu y|\!)((\!|\nu x|\!)F)$
by(*auto simp add*: *FrameStatEq-def intro*: *frameImpResComm*)

lemma *frameImpResCommLeft'*:
fixes x :: *name*
and *xvec* :: *name list*
and F :: *'b frame*

shows $(\!|\nu x|\!)((\!|\nu{*}xvec|\!)F) \hookrightarrow_F (\!|\nu{*}xvec|\!)((\!|\nu x|\!)F)$
by(*induct xvec*) (*auto intro*: *frameImpResComm FrameStatImpTrans frameImpResPres*)

lemma *frameImpResCommRight'*:
fixes x :: *name*
and *xvec* :: *name list*
and F :: *'b frame*

shows $(\!|\nu{*}xvec|\!)((\!|\nu x|\!)F) \hookrightarrow_F (\!|\nu x|\!)((\!|\nu{*}xvec|\!)F)$
by(*induct xvec*) (*auto intro*: *frameImpResComm FrameStatImpTrans frameImpResPres*)

lemma *frameResComm'*:
fixes x :: *name*
and *xvec* :: *name list*
and F :: *'b frame*

shows $(\!|\nu x|\!)((\!|\nu{*}xvec|\!)F) \simeq_F (\!|\nu{*}xvec|\!)((\!|\nu x|\!)F)$
by(*induct xvec*) (*auto intro*: *frameResComm FrameStatEqTrans frameResPres*)

lemma *frameImpChainComm*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and F :: *'b frame*

shows $(\!|\nu{*}xvec|\!)((\!|\nu{*}yvec|\!)F) \hookrightarrow_F (\!|\nu{*}yvec|\!)((\!|\nu{*}xvec|\!)F)$
by(*induct xvec*) (*auto intro*: *frameImpResCommLeft' FrameStatImpTrans frameImpResPres*)

lemma *frameResChainComm*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and F :: *'b frame*

shows $(\!|\nu{*}xvec|\!)((\!|\nu{*}yvec|\!)F) \simeq_F (\!|\nu{*}yvec|\!)((\!|\nu{*}xvec|\!)F)$
by(*induct xvec*) (*auto intro*: *frameResComm' FrameStatEqTrans frameResPres*)

lemma *frameImpNilStatEq*[*simp*]:
fixes Ψ :: *'b*
and Ψ' :: *'b*

shows $(\langle\varepsilon, \Psi\rangle \hookrightarrow_F \langle\varepsilon, \Psi'\rangle) = (\Psi \hookrightarrow \Psi')$
by(*simp add*: *FrameStatImp-def AssertionStatImp-def FrameImp-def*)

lemma *frameNilStatEq*[*simp*]:
  fixes Ψ :: $'b$
  and Ψ' :: $'b$

  shows $(\langle\varepsilon, \Psi\rangle \simeq_F \langle\varepsilon, \Psi'\rangle) = (\Psi \simeq \Psi')$
by(*simp add*: *FrameStatEq-def AssertionStatEq-def FrameImp-def*)

lemma *extractFrameChainStatImp*:
  fixes *xvec* :: *name list*
  and *P* :: $('a, 'b, 'c)$ *psi*

  shows *extractFrame*$((\!|\nu{*}xvec|\!)P) \hookrightarrow_F (\!|\nu{*}xvec|\!)$(*extractFrame P*)
by(*induct xvec*) (*auto intro*: *frameImpResPres*)

lemma *extractFrameChainStatEq*:
  fixes *xvec* :: *name list*
  and *P* :: $('a, 'b, 'c)$ *psi*

  shows *extractFrame*$((\!|\nu{*}xvec|\!)P) \simeq_F (\!|\nu{*}xvec|\!)$(*extractFrame P*)
by(*induct xvec*) (*auto intro*: *frameResPres*)

lemma *insertAssertionExtractFrameFreshImp*:
  fixes *xvec* :: *name list*
  and Ψ :: $'b$
  and *P* :: $('a, 'b, 'c)$ *psi*

  assumes *xvec* $\sharp*$ Ψ

  shows *insertAssertion*(*extractFrame*$((\!|\nu{*}xvec|\!)P)$) $\Psi \hookrightarrow_F (\!|\nu{*}xvec|\!)$(*insertAssertion* (*extractFrame P*) Ψ)
using *assms*
by(*induct xvec*) (*auto intro*: *frameImpResPres*)

lemma *insertAssertionExtractFrameFresh*:
  fixes *xvec* :: *name list*
  and Ψ :: $'b$
  and *P* :: $('a, 'b, 'c)$ *psi*

  assumes *xvec* $\sharp*$ Ψ

  shows *insertAssertion*(*extractFrame*$((\!|\nu{*}xvec|\!)P)$) $\Psi \simeq_F (\!|\nu{*}xvec|\!)$(*insertAssertion* (*extractFrame P*) Ψ)
using *assms*
by(*induct xvec*) (*auto intro*: *frameResPres*)

lemma *frameImpResChainPres*:
  fixes *F* :: $'b$ *frame*

```
  and   G    :: 'b frame
  and   xvec :: name list

  assumes F ↪F G

  shows (|ν*xvec|)F ↪F (|ν*xvec|)G
using assms
by(induct xvec) (auto intro: frameImpResPres)

lemma frameResChainPres:
  fixes F    :: 'b frame
  and   G    :: 'b frame
  and   xvec :: name list

  assumes F ≃F G

  shows (|ν*xvec|)F ≃F (|ν*xvec|)G
using assms
by(induct xvec) (auto intro: frameResPres)

lemma insertAssertionE:
  fixes F  :: ('b::fs-name) frame
  and   Ψ  :: 'b
  and   Ψ' :: 'b
  and   A_F :: name list

  assumes insertAssertion F Ψ = ⟨A_F, Ψ'⟩
  and     A_F ♯* F
  and     A_F ♯* Ψ
  and     distinct A_F

  obtains Ψ_F where F = ⟨A_F, Ψ_F⟩ and Ψ' = Ψ ⊗ Ψ_F
proof -
  assume A: ⋀Ψ_F. [[F = ⟨A_F, Ψ_F⟩; Ψ' = Ψ ⊗ Ψ_F]] ⟹ thesis
  from assms have ∃Ψ_F. F = ⟨A_F, Ψ_F⟩ ∧ Ψ' = Ψ ⊗ Ψ_F
  proof(nominal-induct F avoiding: Ψ A_F Ψ' rule: frame.strong-induct)
    case(FAssert Ψ A_F Ψ')
    thus ?case by auto
  next
    case(FRes x F Ψ A_F Ψ')
    from ‹insertAssertion ((|νx|)F) Ψ = ⟨A_F, Ψ'⟩› ‹x ♯ Ψ›
    obtain y A_F' where A_F = y#A_F' by(induct A_F) auto
    with ‹insertAssertion ((|νx|)F) Ψ = ⟨A_F, Ψ'⟩› ‹x ♯ Ψ› ‹x ♯ A_F›
    have A: insertAssertion F Ψ = ⟨([(x, y)] • A_F'), [(x, y)] • Ψ'⟩
      by(simp add: frame.inject alpha eqvts)
    from ‹A_F = y#A_F'› ‹A_F ♯* Ψ› have y ♯ Ψ and A_F' ♯* Ψ by simp+
    from ‹distinct A_F› ‹A_F = y#A_F'› have y ♯ A_F' and distinct A_F' by auto
    from ‹A_F ♯* ((|νx|)F)› ‹x ♯ A_F› ‹A_F = y#A_F'› have y ♯ F and A_F' ♯* F
and x ♯ A_F'
```

apply −
apply(*auto simp add*: *abs-fresh*)
apply(*hypsubst-thin*)
apply(*subst fresh-star-def*)
apply(*erule rev-mp*)
apply(*subst fresh-star-def*)
apply(*clarify*)
apply(*erule-tac x*=*xa* **in** *ballE*)
apply(*simp add*: *abs-fresh*)
apply *auto*
by(*simp add*: *fresh-def name-list-supp*)
with ‹$x \sharp A_F{}'$› ‹$y \sharp A_F{}'$› **have** $([(x, y)] \cdot A_F{}') \sharp* F$ **by** *simp*
from ‹$A_F{}' \sharp* \Psi$› **have** $([(x, y)] \cdot A_F{}') \sharp* ([(x, y)] \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $([(x, y)] \cdot A_F{}') \sharp* \Psi$ **by** *simp*
with ‹$\bigwedge \Psi\ A_F\ \Psi'.\ [\![$*insertAssertion* $F\ \Psi = \langle A_F, \Psi'\rangle$; $A_F \sharp* F$; $A_F \sharp* \Psi$; *distinct* $A_F]\!] \Longrightarrow \exists \Psi_F.\ F = \langle A_F, \Psi_F\rangle \wedge \Psi' = \Psi \otimes \Psi_F$› A
‹$([(x, y)] \cdot A_F{}') \sharp* F$› ‹*distinct* $A_F{}'$› ‹$x \sharp A_F{}'$› ‹$y \sharp A_F{}'$›
obtain Ψ_F **where** *Feq*: $F = \langle A_F{}', \Psi_F\rangle$ **and** Ψeq: $([(x, y)] \cdot \Psi') = \Psi \otimes \Psi_F$
by *force*

from *Feq* **have** $(\!|\nu x|\!)F = \langle (x\#A_F{}'), \Psi_F\rangle$ **by**(*simp add*: *frame.inject*)
hence $([(x, y)] \cdot (\!|\nu x|\!)F) = [(x, y)] \cdot \langle (x\#A_F{}'), \Psi_F\rangle$ **by** *simp*
hence $(\!|\nu x|\!)F = \langle A_F, [(x, y)] \cdot \Psi_F\rangle$ **using** ‹$y \sharp F$› ‹$A_F = y\#A_F{}'$› ‹$x \sharp A_F$› ‹$y \sharp A_F{}'$›
by(*simp add*: *eqvts calc-atm alphaFrameRes*)

moreover from Ψeq **have** $[(x, y)] \cdot ([(x, y)] \cdot \Psi') = [(x, y)] \cdot (\Psi \otimes \Psi_F)$
by *simp*
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi' = \Psi \otimes ([(x, y)] \cdot \Psi_F)$ **by**(*simp add*: *eqvts*)
ultimately show *?case*
by *blast*
qed
with *A* **show** *?thesis*
by *blast*
qed

lemma *mergeFrameE*:
fixes F :: *'b frame*
and G :: *'b frame*
and A_{FG} :: *name list*
and Ψ_{FG} :: *'b*

assumes *mergeFrame* $F\ G = \langle A_{FG}, \Psi_{FG}\rangle$
and *distinct* A_{FG}
and $A_{FG} \sharp* F$
and $A_{FG} \sharp* G$

obtains $A_F\ \Psi_F\ A_G\ \Psi_G$ **where** $A_{FG} = A_F @ A_G$ **and** $\Psi_{FG} = \Psi_F \otimes \Psi_G$ **and** F

$= \langle A_F, \Psi_F \rangle$ **and** $G = \langle A_G, \Psi_G \rangle$ **and** $A_F \sharp* \Psi_G$ **and** $A_G \sharp* \Psi_F$
proof −
assume *A*: $\bigwedge A_F\ A_G\ \Psi_F\ \Psi_G.\ [\![A_{FG} = A_F@A_G;\ \Psi_{FG} = \Psi_F \otimes \Psi_G;\ F = \langle A_F, \Psi_F \rangle;\ G = \langle A_G, \Psi_G \rangle;\ A_F \sharp* \Psi_G;\ A_G \sharp* \Psi_F]\!] \Longrightarrow$ *thesis*
from *assms* **have** $\exists A_F\ \Psi_F\ A_G\ \Psi_G.\ A_{FG} = A_F@A_G \wedge \Psi_{FG} = \Psi_F \otimes \Psi_G \wedge F = \langle A_F, \Psi_F \rangle \wedge G = \langle A_G, \Psi_G \rangle \wedge A_F \sharp* \Psi_G \wedge A_G \sharp* \Psi_F$
proof(*nominal-induct F avoiding*: *G* A_{FG} Ψ_{FG} *rule*: *frame.strong-induct*)
case(*FAssert* Ψ *G* A_{FG} Ψ_{FG})
thus *?case*
apply *auto*
apply(*rule-tac x*=[] **in** *exI*)
by(*drule-tac insertAssertionE*) *auto*
next
case(*FRes x F G* A_{FG} Ψ_{FG})
from ‹*mergeFrame* $((\!|\nu x|\!)F)$ $G = \langle A_{FG}, \Psi_{FG} \rangle$› ‹$x \sharp G$›
obtain *y* A_{FG}' **where** $A_{FG} = y\#A_{FG}'$ **by**(*induct* A_{FG}) *auto*
with ‹$A_{FG} \sharp* ((\!|\nu x|\!)F)$› ‹$x \sharp A_{FG}$› **have** $A_{FG}' \sharp* F$ **and** $x \sharp A_{FG}'$
by(*auto simp add*: *supp-list-cons fresh-star-def fresh-def name-list-supp abs-supp frame.supp*)
from ‹$A_{FG} = y\#A_{FG}'$› ‹$A_{FG} \sharp* G$› **have** $y \sharp G$ **and** $A_{FG}' \sharp* G$ **by** *simp+*
from ‹$A_{FG} = y\#A_{FG}'$› ‹$A_{FG} \sharp* ((\!|\nu x|\!)F)$› ‹$x \sharp A_{FG}$› **have** $y \sharp F$ **and** $A_{FG}' \sharp* F$
apply(*auto simp add*: *abs-fresh frameResChainFreshSet*)
apply(*hypsubst-thin*)
by(*induct* A_{FG}') (*auto simp add*: *abs-fresh*)
from ‹*distinct* A_{FG}› ‹$A_{FG} = y\#A_{FG}'$› **have** $y \sharp A_{FG}'$ **and** *distinct* A_{FG}' **by** *auto*

with ‹$A_{FG} = y\#A_{FG}'$› ‹*mergeFrame* $((\!|\nu x|\!)F)$ $G = \langle A_{FG}, \Psi_{FG} \rangle$› ‹$x \sharp G$› ‹$x \sharp A_{FG}$› ‹$y \sharp A_{FG}'$›
have *mergeFrame F G* $= \langle A_{FG}', [(x, y)] \cdot \Psi_{FG} \rangle$
by(*simp add*: *frame.inject alpha eqvts*)
with ‹*distinct* A_{FG}'› ‹$A_{FG}' \sharp* F$› ‹$A_{FG}' \sharp* G$›
‹$\bigwedge G\ A_{FG}\ \Psi_{FG}.\ [\![$*mergeFrame F G* $= \langle A_{FG}, \Psi_{FG} \rangle$; *distinct* A_{FG}; $A_{FG} \sharp* F$; $A_{FG} \sharp* G]\!] \Longrightarrow \exists A_F\ \Psi_F\ A_G\ \Psi_G.\ A_{FG} = A_F@A_G \wedge \Psi_{FG} = \Psi_F \otimes \Psi_G \wedge F = \langle A_F, \Psi_F \rangle \wedge G = \langle A_G, \Psi_G \rangle \wedge A_F \sharp* \Psi_G \wedge A_G \sharp* \Psi_F$›
obtain $A_F\ \Psi_F\ A_G\ \Psi_G$ **where** $A_{FG}' = A_F@A_G$ **and** $([(x, y)] \cdot \Psi_{FG}) = \Psi_F \otimes \Psi_G$ **and** *FrF*: $F = \langle A_F, \Psi_F \rangle$ **and** *FrG*: $G = \langle A_G, \Psi_G \rangle$ **and** $A_F \sharp* \Psi_G$ **and** $A_G \sharp* \Psi_F$
by *metis*

from ‹$A_{FG}' = A_F@A_G$› ‹$A_{FG} = y\#A_{FG}'$› **have** $A_{FG} = (y\#A_F)@A_G$ **by** *simp*
moreover from ‹$A_{FG}' = A_F@A_G$› ‹$y \sharp A_{FG}'$› ‹$x \sharp A_{FG}'$› **have** $x \sharp A_F$ **and** $y \sharp A_F$ **and** $x \sharp A_G$ **and** $y \sharp A_G$ **by** *simp+*
with ‹$y \sharp G$› ‹$x \sharp G$› ‹$x \sharp A_{FG}$› *FrG* **have** $y \sharp \Psi_G$ **and** $x \sharp \Psi_G$
by *auto*
from ‹$([(x, y)] \cdot \Psi_{FG}) = \Psi_F \otimes \Psi_G$› **have** $([(x, y)] \cdot [(x, y)] \cdot \Psi_{FG}) = [(x, y)] \cdot (\Psi_F \otimes \Psi_G)$

by *simp*
with ‹$x \sharp \Psi_G$› ‹$y \sharp \Psi_G$› **have** $\Psi_{FG} = ([(x, y)] \cdot \Psi_F) \otimes \Psi_G$ **by**(*simp add*: *eqvts*)
moreover from *FrF* **have** $([(x, y)] \cdot F) = [(x, y)] \cdot \langle A_F, \Psi_F\rangle$ **by** *simp*
with ‹$x \sharp A_F$› ‹$y \sharp A_F$› **have** $([(x, y)] \cdot F) = \langle A_F, [(x, y)] \cdot \Psi_F\rangle$ **by**(*simp add*: *eqvts*)
hence $(\!|\nu y|\!)([(x, y)] \cdot F) = \langle (y\#A_F), [(x, y)] \cdot \Psi_F\rangle$ **by**(*simp add*: *frame.inject*)
with ‹$y \sharp F$› **have** $(\!|\nu x|\!)F = \langle (y\#A_F), [(x, y)] \cdot \Psi_F\rangle$ **by**(*simp add*: *alphaFrameRes*)
moreover with ‹$A_G \sharp* \Psi_F$› **have** $([(x, y)] \cdot A_G) \sharp* ([(x, y)] \cdot \Psi_F)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp A_G$› ‹$y \sharp A_G$› **have** $A_G \sharp* ([(x, y)] \cdot \Psi_F)$ **by** *simp*
moreover from ‹$A_F \sharp* \Psi_G$› ‹$y \sharp \Psi_G$› **have** $(y\#A_F) \sharp* \Psi_G$ **by** *simp*
ultimately show *?case* **using** *FrG*
by *blast*
qed
with *A* **show** *?thesis* **by** *blast*
qed

lemma *mergeFrameRes1*[*simp*]:
fixes A_F :: *name list*
and Ψ_F :: $'b$
and x :: *name*
and A_G :: *name list*
and Ψ_G :: $'b$

assumes $A_F \sharp* \Psi_G$
and $A_F \sharp* A_G$
and $x \sharp A_F$
and $x \sharp \Psi_F$
and $A_G \sharp* \Psi_F$

shows $(\langle A_F, \Psi_F\rangle) \otimes_F (\!|\nu x|\!)(\langle A_G, \Psi_G\rangle)) = (\langle (A_F@x\#A_G), \Psi_F \otimes \Psi_G\rangle)$
using *assms*
apply(*fold frameResChain.simps*)
by(*rule mergeFrames*) *auto*

lemma *mergeFrameRes2*[*simp*]:
fixes A_F :: *name list*
and Ψ_F :: $'b$
and x :: *name*
and A_G :: *name list*
and Ψ_G :: $'b$

assumes $A_F \sharp* \Psi_G$
and $A_G \sharp* A_F$
and $x \sharp A_F$
and $x \sharp \Psi_F$
and $A_G \sharp* \Psi_F$

shows $(\langle A_F, \Psi_F\rangle) \otimes_F (\langle\!|\nu x|\!\rangle(\langle A_G, \Psi_G\rangle)) = (\langle(A_F @ x \# A_G), \Psi_F \otimes \Psi_G\rangle)$
using *assms*
apply(*fold frameResChain.simps*)
by(*rule mergeFrames*) *auto*

lemma *insertAssertionResChain*[*simp*]:
fixes *xvec* :: *name list*
and F :: $'b$ *frame*
and Ψ :: $'b$

assumes *xvec* $\sharp*$ Ψ

shows *insertAssertion* $(\langle\!|\nu*xvec|\!\rangle F)$ Ψ = $\langle\!|\nu*xvec|\!\rangle$(*insertAssertion* F Ψ)
using *assms*
by(*induct xvec*) *auto*

lemma *extractFrameResChain*[*simp*]:
fixes *xvec* :: *name list*
and P :: ($'a$, $'b$, $'c$) *psi*

shows *extractFrame*($\langle\!|\nu*xvec|\!\rangle P$) = $\langle\!|\nu*xvec|\!\rangle$(*extractFrame* P)
by(*induct xvec*) *auto*

lemma *frameResFreshChain*:
fixes *xvec* :: *name list*
and F :: $'b$ *frame*

assumes *xvec* $\sharp*$ F

shows $\langle\!|\nu*xvec|\!\rangle F \simeq_F F$
using *assms*
proof(*induct xvec*)
case *Nil*
thus *?case* **by** *simp*
next
case(*Cons x xvec*)
thus *?case*
by *auto* (*metis frameResPres frameResFresh FrameStatEqTrans*)
qed

end

locale *assertion* = *assertionAux SCompose SImp SBottom SChanEq*
for *SCompose* :: $'b$::*fs-name* ⇒ $'b$ ⇒ $'b$
and *SImp* :: $'b$ ⇒ $'c$::*fs-name* ⇒ *bool*
and *SBottom* :: $'b$
and *SChanEq* :: $'a$::*fs-name* ⇒ $'a$ ⇒ $'c$ +

assumes *chanEqSym*: *SImp* Ψ (*SChanEq M N*) ⟹ *SImp* Ψ (*SChanEq N M*)

and *chanEqTrans*: $[\![SImp\ \Psi\ (SChanEq\ M\ N);\ SImp\ \Psi\ (SChanEq\ N\ L)]\!] \Longrightarrow SImp\ \Psi\ (SChanEq\ M\ L)$
and *Composition*: $assertionAux.AssertionStatEq\ SImp\ \Psi\ \Psi' \Longrightarrow assertionAux.AssertionStatEq\ SImp\ (SCompose\ \Psi\ \Psi'')\ (SCompose\ \Psi'\ \Psi'')$
and *Identity*: $assertionAux.AssertionStatEq\ SImp\ (SCompose\ \Psi\ SBottom)\ \Psi$
and *Associativity*: $assertionAux.AssertionStatEq\ SImp\ (SCompose\ (SCompose\ \Psi\ \Psi')\ \Psi'')\ (SCompose\ \Psi\ (SCompose\ \Psi'\ \Psi''))$
and *Commutativity*: $assertionAux.AssertionStatEq\ SImp\ (SCompose\ \Psi\ \Psi')\ (SCompose\ \Psi'\ \Psi)$

begin

notation *SCompose* (**infixr** $\otimes$ *90*)
notation *SImp* (- $\vdash$ - [*85*, *85*] *85*)
notation *SChanEq* (- $\leftrightarrow$ - [*90*, *90*] *90*)
notation *SBottom* ($\perp$ *90*)

lemma *compositionSym*:
fixes Ψ :: *'b*
and Ψ' :: *'b*
and Ψ'' :: *'b*

assumes $\Psi \simeq \Psi'$

shows $\Psi'' \otimes \Psi \simeq \Psi'' \otimes \Psi'$
proof −
have $\Psi'' \otimes \Psi \simeq \Psi \otimes \Psi''$ **by**(*rule Commutativity*)
moreover from *assms* **have** $\Psi \otimes \Psi'' \simeq \Psi' \otimes \Psi''$ **by**(*rule Composition*)
moreover have $\Psi' \otimes \Psi'' \simeq \Psi'' \otimes \Psi'$ **by**(*rule Commutativity*)
ultimately show *?thesis* **by**(*blast intro*: *AssertionStatEqTrans*)
qed

lemma *Composition'*:
fixes Ψ :: *'b*
and Ψ' :: *'b*
and Ψ'' :: *'b*
and Ψ''' :: *'b*

assumes $\Psi \simeq \Psi'$
and $\Psi'' \simeq \Psi'''$

shows $\Psi \otimes \Psi'' \simeq \Psi' \otimes \Psi'''$
using *assms*
by(*metis Composition Commutativity AssertionStatEqTrans*)

lemma *composition'*:
fixes Ψ :: *'b*

and Ψ' :: $'b$
and Ψ'' :: $'b$
and Ψ''' :: $'b$

assumes $\Psi \simeq \Psi'$

shows $(\Psi \otimes \Psi'') \otimes \Psi''' \simeq (\Psi' \otimes \Psi'') \otimes \Psi'''$
proof −
have $(\Psi \otimes \Psi'') \otimes \Psi''' \simeq \Psi \otimes (\Psi'' \otimes \Psi''')$
by(*rule Associativity*)
moreover from *assms* **have** $\Psi \otimes (\Psi'' \otimes \Psi''') \simeq \Psi' \otimes (\Psi'' \otimes \Psi''')$
by(*rule Composition*)
moreover have $\Psi' \otimes (\Psi'' \otimes \Psi''') \simeq (\Psi' \otimes \Psi'') \otimes \Psi'''$
by(*rule Associativity*[*THEN AssertionStatEqSym*])
ultimately show *?thesis* **by**(*blast dest*: *AssertionStatEqTrans*)
qed

lemma *associativitySym*:
fixes Ψ :: $'b$
and Ψ' :: $'b$
and Ψ'' :: $'b$

shows $(\Psi \otimes \Psi') \otimes \Psi'' \simeq (\Psi \otimes \Psi'') \otimes \Psi'$
proof −
have $(\Psi \otimes \Psi') \otimes \Psi'' \simeq \Psi \otimes (\Psi' \otimes \Psi'')$
by(*rule Associativity*)
moreover have $\Psi \otimes (\Psi' \otimes \Psi'') \simeq \Psi \otimes (\Psi'' \otimes \Psi')$
by(*rule compositionSym*[*OF Commutativity*])
moreover have $\Psi \otimes (\Psi'' \otimes \Psi') \simeq (\Psi \otimes \Psi'') \otimes \Psi'$
by(*rule AssertionStatEqSym*[*OF Associativity*])
ultimately show *?thesis*
by(*blast dest*: *AssertionStatEqTrans*)
qed

lemma *frameIntAssociativity*:
fixes A_F :: *name list*
and Ψ :: $'b$
and Ψ' :: $'b$
and Ψ'' :: $'b$

shows $\langle A_F, (\Psi \otimes \Psi') \otimes \Psi'' \rangle \simeq_F \langle A_F, \Psi \otimes (\Psi' \otimes \Psi'') \rangle$
by(*induct* A_F) (*auto intro*: *Associativity frameResPres*)

lemma *frameIntCommutativity*:
fixes A_F :: *name list*
and Ψ :: $'b$
and Ψ' :: $'b$

shows $\langle A_F, \Psi \otimes \Psi' \rangle \simeq_F \langle A_F, \Psi' \otimes \Psi \rangle$

by(*induct* A_F) (*auto intro*: *Commutativity frameResPres*)

lemma *frameIntIdentity*:
fixes A_F :: *name list*
and Ψ_F :: $'b$

shows $\langle A_F, \Psi_F \otimes SBottom\rangle \simeq_F \langle A_F, \Psi_F\rangle$
by(*induct* A_F) (*auto intro*: *Identity frameResPres*)

lemma *frameIntComposition*:
fixes Ψ :: $'b$
and Ψ' :: $'b$
and A_F :: *name list*
and Ψ_F :: $'b$

assumes $\Psi \simeq \Psi'$

shows $\langle A_F, \Psi \otimes \Psi_F\rangle \simeq_F \langle A_F, \Psi' \otimes \Psi_F\rangle$
using *assms*
by(*induct* A_F) (*auto intro*: *Composition frameResPres*)

lemma *frameIntCompositionSym*:
fixes Ψ :: $'b$
and Ψ' :: $'b$
and A_F :: *name list*
and Ψ_F :: $'b$

assumes $\Psi \simeq \Psi'$

shows $\langle A_F, \Psi_F \otimes \Psi\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi'\rangle$
using *assms*
by(*induct* A_F) (*auto intro*: *compositionSym frameResPres*)

lemma *frameCommutativity*:
fixes F :: $'b$ *frame*
and G :: $'b$ *frame*

shows $F \otimes_F G \simeq_F G \otimes_F F$
proof −
obtain A_F Ψ_F **where** $F = \langle A_F, \Psi_F\rangle$ **and** $A_F \sharp* G$
by(*rule freshFrame*)
moreover obtain A_G Ψ_G **where** $G = \langle A_G, \Psi_G\rangle$ **and** $A_G \sharp* \Psi_F$ **and** $A_G \sharp* A_F$
by(*rule-tac* C=(A_F, Ψ_F) **in** *freshFrame*) *auto*
moreover from ‹$A_F \sharp* G$› ‹$G = \langle A_G, \Psi_G\rangle$› ‹$A_G \sharp* A_F$› **have** $A_F \sharp* \Psi_G$
by *auto*
ultimately show *?thesis*
by *auto* (*metis FrameStatEqTrans frameChainAppend frameResChainComm frameIntCommutativity*)
qed

lemma *frameScopeExt*:
fixes x :: *name*
and F :: $'b$ *frame*
and G :: $'b$ *frame*

assumes $x \sharp F$

shows $(\!|\nu x|\!)(F \otimes_F G) \simeq_F F \otimes_F ((\!|\nu x|\!) G)$
proof −
have $(\!|\nu x|\!)(F \otimes_F G) \simeq_F (\!|\nu x|\!)(G \otimes_F F)$
by(*metis frameResPres frameCommutativity*)
with ‹$x \sharp F$› **have** $(\!|\nu x|\!)(F \otimes_F G) \simeq_F ((\!|\nu x|\!) G) \otimes_F F$
by *simp*
moreover have $((\!|\nu x|\!) G) \otimes_F F \simeq_F F \otimes_F ((\!|\nu x|\!) G)$
by(*rule frameCommutativity*)
ultimately show *?thesis* **by**(*rule FrameStatEqTrans*)
qed

lemma *insertDoubleAssertionStatEq*:
fixes F :: $'b$ *frame*
and Ψ :: $'b$
and Ψ' :: $'b$

shows *insertAssertion*(*insertAssertion* F Ψ) $\Psi' \simeq_F$ (*insertAssertion* F) ($\Psi \otimes \Psi'$)
proof −
obtain A_F Ψ_F **where** $F = \langle A_F, \Psi_F \rangle$ **and** $A_F \sharp* \Psi$ **and** $A_F \sharp* \Psi'$ **and** $A_F \sharp* (\Psi \otimes \Psi')$
by(*rule-tac* C=(Ψ, Ψ') **in** *freshFrame*) *auto*
thus *?thesis*
by *auto* (*metis frameIntComposition Commutativity frameIntAssociativity FrameStatEqTrans FrameStatEqSym*)
qed

lemma *guardedStatEq*:
fixes P :: ($'a$, $'b$, $'c$) *psi*
and I :: ($'a$, $'b$, $'c$) *input*
and C :: ($'a$, $'b$, $'c$) *psiCase*
and A_P :: *name list*
and Ψ_P :: $'b$

shows $[\![$*guarded* P; *extractFrame* $P = \langle A_P, \Psi_P \rangle]\!] \Longrightarrow \Psi_P \simeq \bot \wedge$ *supp* $\Psi_P =$ ({}::*name set*)
and $[\![$*guarded'* I; *extractFrame'* $I = \langle A_P, \Psi_P \rangle]\!] \Longrightarrow \Psi_P \simeq \bot \wedge$ *supp* $\Psi_P =$ ({}::*name set*)
and $[\![$*guarded''* C; *extractFrame''* $C = \langle A_P, \Psi_P \rangle]\!] \Longrightarrow \Psi_P \simeq \bot \wedge$ *supp* $\Psi_P =$ ({}::*name set*)
proof(*nominal-induct* P **and** I **and** C *arbitrary*: A_P Ψ_P *rule*: *psi-input-psiCase.strong-inducts*)

case(*PsiNil* A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Output* M N P A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Input* M *In* A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Case psiCase* A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Par* P Q A_{PQ} Ψ_{PQ})
from ‹*guarded*($P \parallel Q$)› **have** *guarded* P **and** *guarded* Q **by** *simp+*
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* Q$ **by**(*rule freshFrame*)
obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* \Psi_P$
by(*rule-tac* C=(A_P, Ψ_P) **in** *freshFrame*) *auto*

from ‹$\bigwedge A_P\ \Psi_P.\ [\![$*guarded* P; *extractFrame* $P = \langle A_P, \Psi_P\rangle$$]\!] \Longrightarrow \Psi_P \simeq \bot \wedge$ (*supp* Ψ_P = ({}::*name set*))› ‹*guarded* P› *FrP*
have $\Psi_P \simeq \bot$ **and** *supp* Ψ_P = ({}::*name set*) **by** *simp+*
from ‹$\bigwedge A_Q\ \Psi_Q.\ [\![$*guarded* Q; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$$]\!] \Longrightarrow \Psi_Q \simeq \bot \wedge$ (*supp* Ψ_Q = ({}::*name set*))› ‹*guarded* Q› *FrQ*
have $\Psi_Q \simeq \bot$ **and** *supp* Ψ_Q = ({}::*name set*) **by** *simp+*

from ‹$A_P \sharp* Q$› *FrQ* ‹$A_Q \sharp* A_P$› **have** $A_P \sharp* \Psi_Q$ **by**(*drule-tac extractFrame-FreshChain*) *auto*
with ‹$A_Q \sharp* A_P$› ‹$A_Q \sharp* \Psi_P$› *FrP FrQ* ‹*extractFrame*($P \parallel Q$) = $\langle A_{PQ}, \Psi_{PQ}\rangle$› **have** $\langle(A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$
by *auto*
with ‹*supp* Ψ_P = {}› ‹*supp* Ψ_Q = {}› *compSupp* **have** $\Psi_{PQ} = \Psi_P \otimes \Psi_Q$
by *blast*
moreover from ‹$\Psi_P \simeq \bot$› ‹$\Psi_Q \simeq \bot$› **have** $\Psi_P \otimes \Psi_Q \simeq \bot$
by(*metis Composition Identity Associativity Commutativity AssertionStatEq-Trans*)
ultimately show *?case* **using** ‹*supp* Ψ_P = {}› ‹*supp* Ψ_Q = {}› *compSupp*
by *blast*
next
case(*Res* x P A_{xP} Ψ_{xP})
from ‹*guarded*($(\!|\nu x|\!)P$)› **have** *guarded* P **by** *simp*
moreover obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by**(*rule freshFrame*)
moreover note ‹$\bigwedge A_P\ \Psi_P.\ [\![$*guarded* P; *extractFrame* $P = \langle A_P, \Psi_P\rangle$$]\!] \Longrightarrow \Psi_P \simeq \bot \wedge$ (*supp* Ψ_P = ({}::*name set*))›
ultimately have $\Psi_P \simeq \bot$ **and** *supp* Ψ_P = ({}::*name set*) **by** *auto*
from *FrP* ‹*extractFrame*($(\!|\nu x|\!)P$) = $\langle A_{xP}, \Psi_{xP}\rangle$› **have** $\langle(x\#A_P), \Psi_P\rangle = \langle A_{xP}, \Psi_{xP}\rangle$ **by** *simp*

with ‹*supp* $\Psi_P = \{\}$› **have** $\Psi_P = \Psi_{xP}$ **by**(*auto simp del*: *frameResChain.simps*)
with ‹$\Psi_P \simeq \bot$› ‹*supp* $\Psi_P = \{\}$› **show** *?case*
by *simp*
next
case(*Assert* Ψ A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Bang P* A_P Ψ_P)
thus *?case* **by** *simp*
next
case(*Trm M P*)
thus *?case* **by** *simp*
next
case(*Bind x I*)
thus *?case* **by** *simp*
next
case *EmptyCase*
thus *?case* **by** *simp*
next
case(*Cond* φ *P psiCase*)
thus *?case* **by** *simp*
qed

end

end

theory *Semantics*
imports *Frame*
begin

nominal-datatype ($'a$, $'b$, $'c$) *boundOutput* =
BOut $'a$::*fs-name* ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi* (- $\prec''$ - [*110*, *110*] *110*)
| *BStep* «*name*» ($'a$, $'b$, $'c$) *boundOutput* (⦇ν-⦈- [*110*, *110*] *110*)

primrec *BOresChain* :: *name list* ⇒ ($'a$::*fs-name*, $'b$::*fs-name*, $'c$::*fs-name*) *boundOutput* ⇒
($'a$, $'b$, $'c$) *boundOutput* **where**
Base: *BOresChain* [] *B* = *B*
| *Step*: *BOresChain* (*x*#*xs*) *B* = ⦇νx⦈(*BOresChain xs B*)

abbreviation
BOresChainJudge (⦇ν∗-⦈- [*80*, *80*] *80*) **where** ⦇ν∗*xvec*⦈*B* ≡ *BOresChain xvec B*

lemma *BOresChainEqvt*[*eqvt*]:
fixes *perm* :: *name prm*
and *lst* :: *name list*
and *B* :: ($'a$::*fs-name*, $'b$::*fs-name*, $'c$::*fs-name*) *boundOutput*

shows $perm \cdot ((\!|\nu{*}xvec|\!)B) = (\!|\nu{*}(perm \cdot xvec)|\!)(perm \cdot B)$
by(*induct-tac xvec*, *auto*)

lemma *BOresChainSimps*[*simp*]:
fixes $xvec :: name\ list$
and $N :: 'a{::}fs\text{-}name$
and $P :: ('a, 'b{::}fs\text{-}name, 'c{::}fs\text{-}name)\ psi$
and $N' :: 'a$
and $P' :: ('a, 'b, 'c)\ psi$
and $B :: ('a, 'b, 'c)\ boundOutput$
and $B' :: ('a, 'b, 'c)\ boundOutput$

shows $((\!|\nu{*}xvec|\!)N \prec' P = N' \prec' P') = (xvec = [] \wedge N = N' \wedge P = P')$
and $(N' \prec' P' = (\!|\nu{*}xvec|\!)N \prec' P) = (xvec = [] \wedge N = N' \wedge P = P')$
and $(N' \prec' P' = N \prec' P) = (N = N' \wedge P = P')$
and $((\!|\nu{*}xvec|\!)B = (\!|\nu{*}xvec|\!)B') = (B = B')$
by(*induct xvec*) (*auto simp add*: *boundOutput.inject alpha*)

lemma *outputFresh*[*simp*]:
fixes $Xs :: name\ set$
and $xvec :: name\ list$
and $N :: 'a{::}fs\text{-}name$
and $P :: ('a, 'b{::}fs\text{-}name, 'c{::}fs\text{-}name)\ psi$

shows $(Xs \sharp* (N \prec' P)) = ((Xs \sharp* N) \wedge (Xs \sharp* P))$
and $(xvec \sharp* (N \prec' P)) = ((xvec \sharp* N) \wedge (xvec \sharp* P))$
by(*auto simp add*: *fresh-star-def*)

lemma *boundOutputFresh*:
fixes $x :: name$
and $xvec :: name\ list$
and $B :: ('a{::}fs\text{-}name, 'b{::}fs\text{-}name, 'c{::}fs\text{-}name)\ boundOutput$

shows $(x \sharp ((\!|\nu{*}xvec|\!)B)) = (x \in set\ xvec \vee x \sharp B)$
by (*induct xvec*) (*simp-all add*: *abs-fresh*)

lemma *boundOutputFreshSet*:
fixes $Xs :: name\ set$
and $xvec :: name\ list$
and $B :: ('a{::}fs\text{-}name, 'b{::}fs\text{-}name, 'c{::}fs\text{-}name)\ boundOutput$
and $yvec :: name\ list$
and $x :: name$

shows $Xs \sharp* ((\!|\nu{*}xvec|\!)B) = (\forall x{\in}Xs.\ x \in set\ xvec \vee x \sharp B)$
and $yvec \sharp* ((\!|\nu{*}xvec|\!)B) = (\forall x{\in}(set\ yvec).\ x \in set\ xvec \vee x \sharp B)$
and $Xs \sharp* ((\!|\nu x|\!)B) = Xs \sharp* [x].B$
and $xvec \sharp* ((\!|\nu x|\!)B) = xvec \sharp* [x].B$
by(*simp add*: *fresh-star-def boundOutputFresh*)+

lemma *BOresChainSupp*:
fixes *xvec* :: *name list*
and *B* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *boundOutput*

shows (*supp*(⦇*ν*∗*xvec*⦈*B*)::*name set*) = (*supp B*) − (*supp xvec*)
by(*induct xvec*)
(*auto simp add*: *boundOutput.supp supp-list-nil supp-list-cons abs-supp supp-atm*)

lemma *boundOutputFreshSimps*[*simp*]:
fixes *Xs* :: *name set*
and *xvec* :: *name list*
and *B* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *boundOutput*
and *yvec* :: *name list*
and *x* :: *name*

shows *Xs* ♯∗ *xvec* ⟹ (*Xs* ♯∗ (⦇*ν*∗*xvec*⦈*B*)) = (*Xs* ♯∗ *B*)
and *yvec* ♯∗ *xvec* ⟹ *yvec* ♯∗ (⦇*ν*∗*xvec*⦈*B*) = *yvec* ♯∗ *B*
and *xvec* ♯∗ (⦇*ν*∗*xvec*⦈*B*)
and *x* ♯ *xvec* ⟹ *x* ♯ ⦇*ν*∗*xvec*⦈*B* = *x* ♯ *B*
apply(*simp add*: *boundOutputFreshSet*) **apply**(*force simp add*: *fresh-star-def name-list-supp fresh-def*)
apply(*simp add*: *boundOutputFreshSet*) **apply**(*force simp add*: *fresh-star-def name-list-supp fresh-def*)
apply(*simp add*: *boundOutputFreshSet*)
by(*simp add*: *BOresChainSupp fresh-def*)

lemma *boundOutputChainAlpha*:
fixes *p* :: *name prm*
and *xvec* :: *name list*
and *B* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *boundOutput*
and *yvec* :: *name list*

assumes *xvecFreshB*: (*p* ∙ *xvec*) ♯∗ *B*
and *S*: *set p* ⊆ *set xvec* × *set* (*p* ∙ *xvec*)
and (*set xvec*) ⊆ (*set yvec*)

shows (⦇*ν*∗*yvec*⦈*B*) = (⦇*ν*∗(*p* ∙ *yvec*)⦈(*p* ∙ *B*))
proof −
note *pt-name-inst at-name-inst S*
moreover from ‹(*set xvec*) ⊆ (*set yvec*)› **have** *set xvec* ♯∗ (⦇*ν*∗*yvec*⦈*B*)
by(*force simp add*: *boundOutputFreshSet*)
moreover from *xvecFreshB* ‹(*set xvec*) ⊆ (*set yvec*)› **have** *set* (*p* ∙ *xvec*) ♯∗ (⦇*ν*∗*yvec*⦈*B*)
by (*simp add*: *boundOutputFreshSet*) (*simp add*: *fresh-star-def*)
ultimately have (⦇*ν*∗*yvec*⦈*B*) = *p* ∙ (⦇*ν*∗*yvec*⦈*B*)
by (*rule-tac pt-freshs-freshs* [*symmetric*])
then show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *boundOutputChainAlpha'*:
  fixes p :: *name prm*
  and *xvec* :: *name list*
  and B :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
  and *yvec* :: *name list*
  and *zvec* :: *name list*

  assumes *xvecFreshB*: *xvec* $\sharp*$ B
  and S: *set p* $\subseteq$ *set xvec* $\times$ *set yvec*
  and *yvec* $\sharp*$ $(\langle\!|\nu*zvec|\!\rangle B)$

  shows $(\langle\!|\nu*zvec|\!\rangle B) = (\langle\!|\nu*(p \cdot zvec)|\!\rangle(p \cdot B))$
proof −
  note *pt-name-inst at-name-inst S* ‹*yvec* $\sharp*$ $(\langle\!|\nu*zvec|\!\rangle B)$›
  moreover from *xvecFreshB* **have** *set* (*xvec*) $\sharp*$ $(\langle\!|\nu*zvec|\!\rangle B)$
    by (*simp add*: *boundOutputFreshSet*) (*simp add*: *fresh-star-def*)
  ultimately have $(\langle\!|\nu*zvec|\!\rangle B) = p \cdot (\langle\!|\nu*zvec|\!\rangle B)$
    by (*rule-tac pt-freshs-freshs* [*symmetric*]) *auto*
  then show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *boundOutputChainAlpha''*:
  fixes p :: *name prm*
  and *xvec* :: *name list*
  and M :: '*a*::*fs-name*
  and P :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
  and *yvec* :: *name list*

  assumes $(p \cdot xvec) \sharp* M$
  and $(p \cdot xvec) \sharp* P$
  and *set p* $\subseteq$ *set xvec* $\times$ *set* $(p \cdot xvec)$
  and $(set\ xvec) \subseteq (set\ yvec)$

  shows $(\langle\!|\nu*yvec|\!\rangle M \prec' P) = (\langle\!|\nu*(p \cdot yvec)|\!\rangle(p \cdot M) \prec' (p \cdot P))$
using *assms*
by(*subst boundOutputChainAlpha*) *auto*

lemma *boundOutputChainSwap*:
  fixes x :: *name*
  and y :: *name*
  and N :: '*a*::*fs-name*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
  and *xvec* :: *name list*

  assumes $y \sharp N$
  and $y \sharp P$
  and $x \in (set\ xvec)$

  shows $\langle\!|\nu*xvec|\!\rangle N \prec' P = \langle\!|\nu*([(x,\ y)] \cdot xvec)|\!\rangle([(x\ ,y)] \cdot N) \prec' ([(x,\ y)] \cdot P)$

proof(*case-tac* $x{=}y$)
  assume $x{=}y$
  thus *?thesis* **by** *simp*
next
  assume $x \neq y$
  with *assms* **show** *?thesis*
    by(*rule-tac xvec*=[x] **in** *boundOutputChainAlpha''*) (*auto simp add*: *calc-atm*)
qed

lemma *alphaBoundOutput*:
  fixes x :: *name*
  and y :: *name*
  and B :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*

  assumes $y \sharp B$

  shows $(\!|\nu x|\!)B = (\!|\nu y|\!)([(x, y)] \cdot B)$
using *assms*
by(*auto simp add*: *boundOutput.inject alpha fresh-left calc-atm*)

lemma *boundOutputEqFresh*:
  fixes B :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
  and C :: ('*a*, '*b*, '*c*) *boundOutput*
  and x :: *name*
  and y :: *name*

  assumes $(\!|\nu x|\!)B = (\!|\nu y|\!)C$
  and $x \sharp B$

  shows $y \sharp C$
using *assms*
by(*auto simp add*: *boundOutput.inject alpha fresh-left calc-atm*)

lemma *boundOutputEqSupp*:
  fixes B :: ('*a*::*fs-name*, '*b*::*fs-name*, '*c*::*fs-name*) *boundOutput*
  and C :: ('*a*, '*b*, '*c*) *boundOutput*
  and x :: *name*
  and y :: *name*

  assumes $(\!|\nu x|\!)B = (\!|\nu y|\!)C$
  and $x \in supp\ B$

  shows $y \in supp\ C$
using *assms*
apply(*auto simp add*: *boundOutput.inject alpha fresh-left calc-atm*)
apply(*drule-tac pi*=[(x, y)] **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
by(*simp add*: *eqvts calc-atm*)

lemma *boundOutputChainEq*:

fixes *xvec* :: *name list*
and *B* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *boundOutput*
and *yvec* :: *name list*
and *B'* :: (*'a*, *'b*, *'c*) *boundOutput*

assumes $(\!|\nu * xvec|\!)B = (\!|\nu * yvec|\!)B'$
and *xvec* $\sharp*$ *yvec*
and *length xvec* = *length yvec*

shows $\exists p.\ (set\ p) \subseteq (set\ xvec) \times set\ (yvec) \wedge distinctPerm\ p \wedge\ B = p \cdot B' \wedge (set\ (map\ fst\ p)) \subseteq (supp\ B) \wedge xvec \sharp* B' \wedge yvec \sharp* B$
proof −
obtain *n* **where** *n* = *length xvec* **by** *auto*
with *assms* **show** *?thesis*
proof(*induct n arbitrary*: *xvec yvec B B'*)
case(*0 xvec yvec B B'*)
have *Eq*: $(\!|\nu * xvec|\!)B = (\!|\nu * yvec|\!)B'$ **by** *fact*
from ‹*0* = *length xvec*› **have** *xvec* = [] **by** *auto*
moreover with ‹*length xvec* = *length yvec*› **have** *yvec* = []
by(*case-tac yvec*) *auto*
ultimately show *?case* **using** *Eq*
by(*simp add*: *boundOutput.inject*)
next
case(*Suc n xvec yvec B B'*)
from ‹*Suc n* = *length xvec*›
obtain *x xvec'* **where** *xvec* = *x*#*xvec'* **and** *length xvec'* = *n*
by(*case-tac xvec*) *auto*
from ‹$(\!|\nu * xvec|\!)B = (\!|\nu * yvec|\!)B'$› ‹*xvec* = *x* # *xvec'*› ‹*length xvec* = *length yvec*›
obtain *y yvec'* **where** $(\!|\nu * (x\#xvec')|\!)B = (\!|\nu * (y\#yvec')|\!)B'$
and *yvec* = *y*#*yvec'* **and** *length xvec'* = *length yvec'*
by(*case-tac yvec*) *auto*
hence *EQ*: $(\!|\nu x|\!)((\!|\nu * xvec'|\!)B) = (\!|\nu y|\!)((\!|\nu * yvec'|\!)B')$
by *simp*
from ‹*xvec* = *x*#*xvec'*› ‹*yvec*=*y*#*yvec'*› ‹*xvec* $\sharp*$ *yvec*›
have $x \neq y$ **and** *xvec'* $\sharp*$ *yvec'* **and** *x* $\sharp$ *yvec'* **and** *y* $\sharp$ *xvec'*
by *auto*
have *IH*: $\bigwedge xvec\ yvec\ B\ B'.\ [\![(\!|\nu * xvec|\!)(B::('a::fs\text{-}name, 'b::fs\text{-}name, 'c::fs\text{-}name)\ boundOutput) = (\!|\nu * yvec|\!)B';\ xvec \sharp* yvec;\ length\ xvec = length\ yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p.\ (set\ p) \subseteq (set\ xvec) \times (set\ yvec) \wedge distinctPerm\ p \wedge\ B = p \cdot B' \wedge set(map\ fst\ p) \subseteq supp\ B \wedge xvec \sharp* B' \wedge yvec \sharp* B$
by *fact*
from *EQ* ‹*x* $\neq$ *y*› **have** *EQ'*: $(\!|\nu * xvec'|\!)B = ([(x, y)] \cdot ((\!|\nu * yvec'|\!)B'))$
and *xFreshB'*: *x* $\sharp$ $((\!|\nu * yvec'|\!)B')$
and *yFreshB*: *y* $\sharp$ $((\!|\nu * xvec'|\!)B)$
by(*metis boundOutput.inject alpha*)+
from *xFreshB'* ‹*x* $\sharp$ *yvec'*› **have** *x* $\sharp$ *B'*
by(*auto simp add*: *boundOutputFresh*) (*simp add*: *fresh-def name-list-supp*)+
from *yFreshB* ‹*y* $\sharp$ *xvec'*› **have** *y* $\sharp$ *B*
by(*auto simp add*: *boundOutputFresh*) (*simp add*: *fresh-def name-list-supp*)+

show *?case*
proof(*case-tac* $x \sharp (\!|\nu{*}xvec'|\!)B$)
assume *xFreshB*: $x \sharp (\!|\nu{*}xvec'|\!)B$
with *EQ* **have** *yFreshB'*: $y \sharp (\!|\nu{*}yvec'|\!)B'$
by(*rule boundOutputEqFresh*)
with *xFreshB' EQ'* **have** $(\!|\nu{*}xvec'|\!)B = (\!|\nu{*}yvec'|\!)B'$
by(*simp*)
with ‹$xvec' \sharp* yvec'$› ‹*length xvec' = length yvec'*› ‹*length xvec' = n*› *IH*
obtain p **where** S: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm p* **and** $B = p \cdot B'$
and $set(map\ fst\ p) \subseteq supp\ B$ **and** $xvec' \sharp* B'$ **and** $yvec' \sharp* B$
by *blast*
from S **have** $(set\ p) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover note ‹$xvec = x\#xvec'$› ‹$yvec=y\#yvec'$› ‹*distinctPerm p*› ‹$B = p \cdot B'$›
‹$xvec' \sharp* B'$› ‹$x \sharp B'$› ‹$x \sharp B'$› ‹$yvec' \sharp* B$› ‹$y \sharp B$› ‹$set(map\ fst\ p) \subseteq supp\ B$›

ultimately show *?case* **by** *auto*
next
assume $\neg(x \sharp (\!|\nu{*}xvec'|\!)B)$
hence *xSuppB*: $x \in supp((\!|\nu{*}xvec'|\!)B)$
by(*simp add*: *fresh-def*)
with *EQ* **have** *ySuppB'*: $y \in supp\ ((\!|\nu{*}yvec'|\!)B')$
by(*rule boundOutputEqSupp*)
hence $y \sharp yvec'$
by(*induct yvec'*) (*auto simp add*: *boundOutput.supp abs-supp*)
with ‹$x \sharp yvec'$› *EQ'* **have** $(\!|\nu{*}xvec'|\!)B = (\!|\nu{*}yvec'|\!)([(x,\ y)] \cdot B')$
by(*simp add*: *eqvts*)
with ‹$xvec' \sharp* yvec'$› ‹*length xvec' = length yvec'*› ‹*length xvec' = n*› *IH*
obtain p **where** S: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm p* **and** $B = p \cdot [(x,\ y)] \cdot B'$
and $set(map\ fst\ p) \subseteq supp\ B$ **and** $xvec' \sharp* ([(x,\ y)] \cdot B')$ **and** $yvec' \sharp* B$
by *blast*

from *xSuppB* **have** $x \sharp xvec'$
by(*induct xvec'*) (*auto simp add*: *boundOutput.supp abs-supp*)
with ‹$x \sharp yvec'$› ‹$y \sharp xvec'$› ‹$y \sharp yvec'$› S **have** $x \sharp p$ **and** $y \sharp p$
apply(*induct p*)
by(*auto simp add*: *name-list-supp*) (*auto simp add*: *fresh-def*)
from S **have** $(set\ ((x,\ y)\#p)) \subseteq (set(x\#xvec')) \times (set(y\#yvec'))$
by *force*
moreover from ‹$x \neq y$› ‹$x \sharp p$› ‹$y \sharp p$› S ‹*distinctPerm p*› **have** *distinctPerm*$((x,y)\#p)$ **by** *simp*
moreover from ‹$B = p \cdot [(x,\ y)] \cdot B'$› ‹$x \sharp p$› ‹$y \sharp p$› **have** $B = [(x,\ y)] \cdot p \cdot B'$
by(*subst perm-compose*) *simp*
hence $B = ((x,\ y)\#p) \cdot B'$ **by** *simp*

moreover from ‹$xvec' \sharp* ([(x,\ y)] \cdot B')$› **have** $([(x,\ y)] \cdot xvec') \sharp* ([(x,\ y)] \cdot [(x,\ y)] \cdot B')$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› ‹$x \sharp B'$› **have** $(x\#xvec') \sharp* B'$ **by** *simp*
moreover from ‹$y \sharp B$› ‹$yvec' \sharp* B$› **have** $(y\#yvec') \sharp* B$ **by** *simp*
moreover from ‹$set(map\ fst\ p) \subseteq supp\ B$› *xSuppB* ‹$x \sharp xvec'$›
have $set(map\ fst\ ((x,\ y)\#p)) \subseteq supp\ B$
by(*simp add*: *BOresChainSupp*)
ultimately show *?case* **using** ‹$xvec=x\#xvec'$› ‹$yvec=y\#yvec'$›
by *metis*
qed
qed
qed

lemma *boundOutputChainEqLength*:
fixes *xvec* :: *name list*
and M :: $'a$::*fs-name*
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*
and *yvec* :: *name list*
and N :: $'a$::*fs-name*
and Q :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

assumes $(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$

shows *length xvec* = *length yvec*
proof −
obtain *n* **where** $n = length\ xvec$ **by** *auto*
with *assms* **show** *?thesis*
proof(*induct n arbitrary*: *xvec yvec M P N Q*)
case(*0 xvec yvec M P N Q*)
from ‹$0 = length\ xvec$› **have** $xvec = []$ **by** *auto*
moreover with ‹$(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$› **have** $yvec = []$
by(*case-tac yvec*) *auto*
ultimately show *?case* **by** *simp*
next
case(*Suc n xvec yvec M P N Q*)
from ‹$Suc\ n = length\ xvec$›
obtain *x xvec'* **where** $xvec = x\#xvec'$ **and** $length\ xvec' = n$
by(*case-tac xvec*) *auto*
from ‹$(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$› ‹$xvec = x\ \#\ xvec'$›
obtain *y yvec'* **where** $(\!|\nu*(x\#xvec')|\!)M \prec' P = (\!|\nu*(y\#yvec')|\!)N \prec' Q$
and $yvec = y\#yvec'$
by(*case-tac yvec*) *auto*
hence *EQ*: $(\!|\nu x|\!)((\!|\nu*xvec'|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu*yvec'|\!)N \prec' Q)$
by *simp*
have *IH*: $\bigwedge xvec\ yvec\ M\ P\ N\ Q.\ [\![(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' (Q::('a,\ 'b,\ 'c)\ psi);\ n = length\ xvec]\!] \Longrightarrow length\ xvec = length\ yvec$
by *fact*
show *?case*

```
    proof(case-tac x = y)
      assume x = y
      with EQ have (⦇ν*xvec'⦈)M ≺' P = (⦇ν*yvec'⦈)N ≺' Q
        by(simp add: alpha boundOutput.inject)
      with IH ‹length xvec' = n› have length xvec' = length yvec'
        by blast
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
      show ?case by simp
    next
      assume x ≠ y
      with EQ have (⦇ν*xvec'⦈)M ≺' P = [(x, y)] ∙ (⦇ν*yvec'⦈)N ≺' Q
        by(simp add: alpha boundOutput.inject)
      hence (⦇ν*xvec'⦈)M ≺' P = (⦇ν*([(x, y)] ∙ yvec')⦈)([(x, y)] ∙ N) ≺' ([(x, y)] ∙ Q)
        by(simp add: eqvts)
      with IH ‹length xvec' = n› have length xvec' = length ([(x, y)] ∙ yvec')
        by blast
      hence length xvec' = length yvec'
        by simp
      with ‹xvec = x#xvec'› ‹yvec=y#yvec'›
      show ?case by simp
    qed
  qed
qed

lemma boundOutputChainEq':
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a::fs-name, 'b::fs-name, 'c::fs-name) psi

  assumes (⦇ν*xvec⦈)M ≺' P = (⦇ν*yvec⦈)N ≺' Q
  and     xvec ♯* yvec

  shows ∃p. (set p) ⊆ (set xvec) × set (yvec) ∧ distinctPerm p ∧  M = p ∙ N ∧
P = p ∙ Q ∧ xvec ♯* N ∧ xvec ♯* Q ∧ yvec ♯* M ∧ yvec ♯* P
using assms
apply(frule-tac boundOutputChainEqLength)
apply(drule-tac boundOutputChainEq)
by(auto simp add: boundOutput.inject)

lemma boundOutputChainEq'':
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a::fs-name, 'b::fs-name, 'c::fs-name) psi
```

assumes $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' Q$
and $xvec \sharp* yvec$
and $distinct\ xvec$
and $distinct\ yvec$

obtains p **where** $(set\ p) \subseteq (set\ xvec) \times set\ (p \cdot xvec)$ **and** $distinctPerm\ p$ **and** $yvec = p \cdot xvec$ **and** $N = p \cdot M$ **and** $Q = p \cdot P$ **and** $xvec \sharp* N$ **and** $xvec \sharp* Q$ **and** $(p \cdot xvec) \sharp* M$ **and** $(p \cdot xvec) \sharp* P$
proof −

assume $\bigwedge p.\ [\![set\ p \subseteq set\ xvec \times set\ (p \cdot xvec);\ distinctPerm\ p;\ yvec = p \cdot xvec;\ N = p \cdot M;\ Q = p \cdot P;\ xvec \sharp* N;\ xvec \sharp* Q;\ (p \cdot xvec) \sharp* M;\ (p \cdot xvec) \sharp* P]\!] \Longrightarrow thesis$

moreover obtain n **where** $n = length\ xvec$ **by** $auto$
with $assms$ **have** $\exists p.\ (set\ p) \subseteq (set\ xvec) \times set\ (yvec) \wedge distinctPerm\ p \wedge\ yvec = p \cdot xvec \wedge N = p \cdot M \wedge Q = p \cdot P \wedge xvec \sharp* N \wedge xvec \sharp* Q \wedge (p \cdot xvec) \sharp* M \wedge (p \cdot xvec) \sharp* P$
proof($induct\ n\ arbitrary$: $xvec\ yvec\ M\ P\ N\ Q$)
case($0\ xvec\ yvec\ M\ P\ N\ Q$)
have Eq: $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' Q$ **by** $fact$
from ‹$0 = length\ xvec$› **have** $xvec = []$ **by** $auto$
moreover with Eq **have** $yvec = []$
by($case\text{-}tac\ yvec$) $auto$
ultimately show $?case$ **using** Eq
by($simp\ add$: $boundOutput.inject$)
next
case($Suc\ n\ xvec\ yvec\ M\ P\ N\ Q$)
from ‹$Suc\ n = length\ xvec$›
obtain $x\ xvec'$ **where** $xvec = x\#xvec'$ **and** $length\ xvec' = n$
by($case\text{-}tac\ xvec$) $auto$
from ‹$(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' Q$› ‹$xvec = x\ \#\ xvec'$›
obtain $y\ yvec'$ **where** $(\!|\nu{*}(x\#xvec')|\!)M \prec' P = (\!|\nu{*}(y\#yvec')|\!)N \prec' Q$
and $yvec = y\#yvec'$
by($case\text{-}tac\ yvec$) $auto$
hence EQ: $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' Q)$
by $simp$
from ‹$xvec = x\#xvec'$› ‹$yvec{=}y\#yvec'$› ‹$xvec \sharp* yvec$›
have $x \neq y$ **and** $xvec' \sharp* yvec'$ **and** $x \sharp yvec'$ **and** $y \sharp xvec'$
by $auto$
from ‹$distinct\ xvec$› ‹$distinct\ yvec$› ‹$xvec{=}x\#xvec'$› ‹$yvec{=}y\#yvec'$› **have** $x \sharp xvec'$ **and** $y \sharp yvec'$ **and** $distinct\ xvec'$ **and** $distinct\ yvec'$
by $simp{+}$
have IH: $\bigwedge xvec\ yvec\ M\ P\ N\ Q.\ [\![(\!|\nu{*}xvec|\!)(M::'a) \prec' (P::('a,\ 'b,\ 'c)\ psi) = (\!|\nu{*}yvec|\!)N \prec' Q;\ xvec \sharp* yvec;\ distinct\ xvec;\ distinct\ yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p.\ (set\ p) \subseteq (set\ xvec) \times (set\ yvec) \wedge distinctPerm\ p \wedge\ yvec = p \cdot xvec \wedge N = p \cdot M \wedge Q = p \cdot P \wedge xvec \sharp* N \wedge xvec \sharp* Q \wedge (p \cdot xvec) \sharp* M \wedge (p \cdot xvec) \sharp* P$
by $fact$

from EQ ‹$x \neq y$› ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› ‹$y \sharp xvec'$› ‹$x \sharp xvec'$› **have** $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}yvec'|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$ **and** $x \sharp N$ **and** $x \sharp Q$ **and** $y \sharp M$ **and** $y \sharp P$
apply −
apply(*simp add*: *boundOutput.inject alpha eqvts*)
apply(*simp add*: *boundOutput.inject alpha eqvts*)
apply(*simp add*: *boundOutput.inject alpha eqvts*)
by(*simp add*: *boundOutput.inject alpha' eqvts*)+
with ‹$xvec' \sharp* yvec'$› ‹*distinct xvec'*› ‹*distinct yvec'*› ‹*length xvec'* = *n*› *IH*
obtain p **where** S: $(set\ p) \subseteq (set\ xvec') \times (set\ yvec')$ **and** *distinctPerm p* **and** $yvec' = p \cdot xvec'$ **and** $([(x, y)] \cdot N) = p \cdot M$ **and** $([(x, y)] \cdot Q) = p \cdot P$ **and** $xvec' \sharp* ([(x, y)] \cdot N)$ **and** $xvec' \sharp* ([(x, y)] \cdot Q)$ **and** $yvec' \sharp* M$ **and** $yvec' \sharp* P$
by *metis*
from S **have** $set((x, y)\#p) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover from ‹$x \sharp xvec'$› ‹$x \sharp yvec'$› ‹$y \sharp xvec'$› ‹$y \sharp yvec'$› S **have** $x \sharp p$ **and** $y \sharp p$
apply(*induct p*)
by(*auto simp add*: *fresh-prod name-list-supp*) (*auto simp add*: *fresh-def*)

with S ‹*distinctPerm p*› ‹$x \neq y$› **have** $distinctPerm((x, y)\#p)$ **by** *auto*
moreover from ‹$yvec' = p \cdot xvec'$› ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $(y\#yvec') = ((x, y)\#p) \cdot (x\#xvec')$
by(*simp add*: *eqvts calc-atm perm-compose freshChainSimps*)
moreover from ‹$([(x, y)] \cdot N) = p \cdot M$›
have $([(x, y)] \cdot [(x, y)] \cdot N) = [(x, y)] \cdot p \cdot M$
by(*simp add*: *pt-bij*)
hence $N = ((x, y)\#p) \cdot M$ **by** *simp*
moreover from ‹$([(x, y)] \cdot Q) = p \cdot P$›
have $([(x, y)] \cdot [(x, y)] \cdot Q) = [(x, y)] \cdot p \cdot P$
by(*simp add*: *pt-bij*)
hence $Q = ((x, y)\#p) \cdot P$ **by** *simp*
moreover from ‹$xvec' \sharp* ([(x, y)] \cdot N)$› **have** $([(x, y)] \cdot xvec') \sharp* ([(x, y)] \cdot [(x, y)] \cdot N)$
by(*subst fresh-star-bij*)
with ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $xvec' \sharp* N$ **by** *simp*
with ‹$x \sharp N$› **have** $(x\#xvec') \sharp* N$ **by** *simp*
moreover from ‹$xvec' \sharp* ([(x, y)] \cdot Q)$› **have** $([(x, y)] \cdot xvec') \sharp* ([(x, y)] \cdot [(x, y)] \cdot Q)$
by(*subst fresh-star-bij*)
with ‹$x \sharp xvec'$› ‹$y \sharp xvec'$› **have** $xvec' \sharp* Q$ **by** *simp*
with ‹$x \sharp Q$› **have** $(x\#xvec') \sharp* Q$ **by** *simp*
moreover from ‹$y \sharp M$› ‹$yvec' \sharp* M$› **have** $(y\#yvec') \sharp* M$ **by** *simp*
moreover from ‹$y \sharp P$› ‹$yvec' \sharp* P$› **have** $(y\#yvec') \sharp* P$ **by** *simp*
ultimately show *?case* **using** ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*›
by *metis*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *boundOutputEqSupp′*:
fixes x :: *name*
and *xvec* :: *name list*
and M :: *'a::fs-name*
and P :: *('a, 'b::fs-name, 'c::fs-name) psi*
and y :: *name*
and *yvec* :: *name list*
and N :: *'a*
and Q :: *('a, 'b, 'c) psi*

assumes *Eq*: $(\!|\nu x|\!)((\!|\nu * xvec|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu * yvec|\!)N \prec' Q)$
and $x \neq y$
and $x \sharp yvec$
and $x \sharp xvec$
and $y \sharp xvec$
and $y \sharp yvec$
and $xvec \sharp* yvec$
and $x \in supp\ M$

shows $y \in supp\ N$
proof −
from *Eq* ‹$x \neq y$› ‹$x \sharp yvec$› ‹$y \sharp yvec$› **have** $(\!|\nu * xvec|\!)M \prec' P = (\!|\nu * yvec|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
by(*simp add*: *boundOutput.inject alpha eqvts*)
then obtain p **where** *S*: *set p* ⊆ *set xvec* × *set yvec* **and** $M = p \cdot [(x, y)] \cdot N$ **and** *distinctPerm p* **using** ‹*xvec* ♯* *yvec*›
by(*blast dest*: *boundOutputChainEq′*)
with ‹$x \in supp\ M$› **have** $x \in supp(p \cdot [(x, y)] \cdot N)$ **by** *simp*
hence $(p \cdot x) \in p \cdot supp(p \cdot [(x, y)] \cdot N)$
by(*simp add*: *pt-set-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp xvec$› ‹$x \sharp yvec$› *S* ‹*distinctPerm p*› **have** $x \in supp([(x, y)] \cdot N)$
by(*simp add*: *eqvts*)
hence $([(x, y)] \cdot x) \in ([(x, y)] \cdot (supp([(x, y)] \cdot N)))$
by(*simp add*: *pt-set-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \neq y$› **show** *?thesis* **by**(*simp add*: *calc-atm eqvts*)
qed

lemma *boundOutputChainOpenIH*:
fixes *xvec* :: *name list*
and x :: *name*
and B :: *('a::fs-name, 'b::fs-name, 'c::fs-name) boundOutput*
and *yvec* :: *name list*
and y :: *name*
and B' :: *('a, 'b, 'c) boundOutput*

assumes *Eq*: $(\!|\nu * xvec|\!)((\!|\nu x|\!)B) = (\!|\nu * yvec|\!)((\!|\nu y|\!)B')$
and *L*: *length xvec* = *length yvec*
and *xFreshB′*: $x \sharp B'$
and *xFreshxvec*: $x \sharp xvec$

and *xFreshyvec*: $x \sharp yvec$

shows $(\!|\nu * xvec|\!)B = (\!|\nu * yvec|\!)([(x,\ y)] \cdot B')$
using *assms*
proof(*induct n==length xvec arbitrary*: *xvec yvec y B' rule*: *nat.induct*)
case(*zero xvec yvec y B'*)
have $0 = length\ xvec$ **and** $length\ xvec = length\ yvec$ **by** *fact+*
moreover have $(\!|\nu * xvec|\!)(\!|\nu x|\!)B = (\!|\nu * yvec|\!)(\!|\nu y|\!)B'$ **by** *fact*
ultimately show *?case* **by**(*auto simp add*: *boundOutput.inject alpha*)
next
case(*Suc n xvec yvec y B'*)
have *L*: $length\ xvec = length\ yvec$ **and** $Suc\ n = length\ xvec$ **by** *fact+*
then obtain $x'\ xvec'\ y'\ yvec'$ **where** *xEq*: $xvec = x'\#xvec'$ **and** *yEq*: $yvec = y'\#yvec'$
and *L'*: $length\ xvec' = length\ yvec'$
by(*cases xvec*, *auto*, *cases yvec*, *auto*)
have *xFreshB'*: $x \sharp B'$ **by** *fact*
have $x \sharp xvec$ **and** $x \sharp yvec$ **by** *fact+*
with *xEq yEq* **have** *xineqx'*: $x \neq x'$ **and** *xFreshxvec'*: $x \sharp xvec'$
and *xineqy'*: $x \neq y'$ **and** *xFreshyvec'*: $x \sharp yvec'$
by *simp+*
have $(\!|\nu * xvec|\!)(\!|\nu x|\!)B = (\!|\nu * yvec|\!)(\!|\nu y|\!)B'$ **by** *fact*
with *xEq yEq* **have** *Eq*: $(\!|\nu x'|\!)((\!|\nu * xvec'|\!)(\!|\nu x|\!)B) = (\!|\nu y'|\!)((\!|\nu * yvec'|\!)(\!|\nu y|\!)B')$ **by** *simp*
have $Suc\ n = length\ xvec$ **by** *fact*
with *xEq* **have** *L''*: $n = length\ xvec'$ **by** *simp*
have $(\!|\nu x'|\!)((\!|\nu * xvec'|\!)B) = (\!|\nu y'|\!)((\!|\nu * yvec'|\!)([(x,\ y)] \cdot B'))$
proof(*case-tac x'=y'*)
assume *x'eqy'*: $x' = y'$
with *Eq* **have** $(\!|\nu * xvec'|\!)(\!|\nu x|\!)B = (\!|\nu * yvec'|\!)(\!|\nu y|\!)B'$ **by**(*simp add*: *boundOutput.inject alpha*)
hence $(\!|\nu * xvec'|\!)B = (\!|\nu * yvec'|\!)([(x,\ y)] \cdot B')$ **using** *L' xFreshB' xFreshxvec' xFreshyvec' L''*
by(*rule-tac Suc*)
with *x'eqy'* **show** *?thesis* **by**(*simp add*: *boundOutput.inject alpha*)
next
assume *x'ineqy'*: $x' \neq y'$
with *Eq* **have** *Eq'*: $(\!|\nu * xvec'|\!)(\!|\nu x|\!)B = (\!|\nu * ([(x',\ y')] \cdot yvec')|\!)(\!|\nu([(x',\ y')] \cdot y)|\!)([(x',\ y')] \cdot B')$
and $x'FreshB'$: $x' \sharp (\!|\nu * yvec'|\!)(\!|\nu y|\!)B'$
by(*simp add*: *boundOutput.inject alpha eqvts*)+
from *L'* **have** $length\ xvec' = length\ ([(x',\ y')] \cdot yvec')$ **by** *simp*
moreover from *xineqx' xineqy' xFreshB'* **have** $x \sharp [(x',\ y')] \cdot B'$ **by**(*simp add*: *fresh-left calc-atm*)
moreover from *xineqx' xineqy' xFreshyvec'* **have** $x \sharp [(x',\ y')] \cdot yvec'$ **by**(*simp add*: *fresh-left calc-atm*)
ultimately have $(\!|\nu * xvec'|\!)B = (\!|\nu * ([(x',\ y')] \cdot yvec')|\!)([(x,\ ([(x',\ y')] \cdot y))] \cdot [(x',\ y')] \cdot B')$ **using** *Eq' xFreshxvec' L''*
by(*rule-tac Suc*)

```
    moreover from x'FreshB' have x' ♯ (|ν*yvec'|)([(x, y)] • B')
    proof(case-tac x' ♯ yvec')
      assume x' ♯ yvec'
      with x'FreshB' have x'FreshB': x' ♯ (|νy|)B'
        by(simp add: fresh-def BOresChainSupp)
      show ?thesis
      proof(case-tac x'=y)
        assume x'eqy: x' = y
        show ?thesis
        proof(case-tac x=y)
          assume x=y
        with xFreshB' x'eqy show ?thesis by(simp add: BOresChainSupp fresh-def)
        next
          assume x ≠ y
          with ‹x ♯ B'› have y ♯ [(x, y)] • B' by(simp add: fresh-left calc-atm)
          with x'eqy show ?thesis by(simp add: BOresChainSupp fresh-def)
        qed
      next
        assume x'ineqy: x' ≠ y
        with x'FreshB' have x' ♯ B' by(simp add: abs-fresh)
       with xineqx' x'ineqy have x' ♯ ([(x, y)] • B') by(simp add: fresh-left calc-atm)
        thus ?thesis by(simp add: BOresChainSupp fresh-def)
      qed
    next
      assume ¬x' ♯ yvec'
      thus ?thesis by(simp add: BOresChainSupp fresh-def)
    qed
    ultimately show ?thesis using x'ineqy' xineqx' xineqy'
      apply(simp add: boundOutput.inject alpha eqvts)
      apply(subst perm-compose[of [(x', y')]])
      by(simp add: calc-atm)
  qed
  with xEq yEq show ?case by simp
qed

lemma boundOutputPar1Dest:
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a, 'b, 'c) psi
  and   R    :: ('a, 'b, 'c) psi

  assumes (|ν*xvec|)M ≺' P = (|ν*yvec|)N ≺' (Q ∥ R)
  and     xvec ♯* R
  and     yvec ♯* R

  obtains T where P = T ∥ R and (|ν*xvec|)M ≺' T = (|ν*yvec|)N ≺' Q
```

proof −
assume $\bigwedge T.\ [\![P = T \parallel R;\ (\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' Q]\!] \Longrightarrow thesis$
moreover obtain n **where** $n = length\ xvec$ **by** *auto*
with *assms* **have** $\exists T.\ P = T \parallel R \wedge (\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' Q$
proof(*induct n arbitrary*: *xvec yvec M N P Q R*)
case(*0 xvec yvec M N P Q R*)
have *Eq*: $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$ **by** *fact*
from ‹*0* = *length xvec*› **have** $xvec = []$ **by** *auto*
moreover with *Eq* **have** $yvec = []$
by(*case-tac yvec*) *auto*
ultimately show *?case* **using** *Eq*
by(*simp add*: *boundOutput.inject*)
next
case(*Suc n xvec yvec M N P Q R*)
from ‹*Suc n* = *length xvec*›
obtain $x\ xvec'$ **where** $xvec = x\#xvec'$ **and** $length\ xvec' = n$
by(*case-tac xvec*) *auto*
from ‹$(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$› ‹$xvec = x\ \#\ xvec'$›
obtain $y\ yvec'$ **where** $(\!|\nu{*}(x\#xvec')|\!)M \prec' P = (\!|\nu{*}(y\#yvec')|\!)N \prec' (Q \parallel R)$
and $yvec = y\#yvec'$
by(*case-tac yvec*) *auto*
hence *EQ*: $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R))$
by *simp*
from ‹$xvec \sharp* R$› ‹$yvec \sharp* R$› ‹$xvec = x\#xvec'$› ‹$yvec = y\#yvec'$›
have $x \sharp R$ **and** $xvec' \sharp* R$ **and** $y \sharp R$ **and** $yvec' \sharp* R$ **by** *auto*
show *?case*
proof(*case-tac x = y*)
assume $x = y$
with *EQ* **have** $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R)$
by(*simp add*: *boundOutput.inject alpha*)
with ‹$xvec' \sharp* R$› ‹$yvec' \sharp* R$› ‹*length xvec'* = *n*›
obtain T **where** $P = T \parallel R$ **and** $(\!|\nu{*}xvec'|\!)M \prec' T = (\!|\nu{*}yvec'|\!)N \prec' Q$
by(*drule-tac Suc*) *auto*
with ‹$xvec{=}x\#xvec'$› ‹$yvec{=}y\#yvec'$› ‹$x{=}y$› **show** *?case*
by(*force simp add*: *boundOutput.inject alpha*)
next
assume $x \neq y$
with *EQ* ‹$x \sharp R$› ‹$y \sharp R$›
have $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}([(x,\ y)] \cdot yvec')|\!)([(x,\ y)] \cdot N) \prec' (([(x,\ y)] \cdot Q) \parallel R)$
and *xFreshQR*: $x \sharp (\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R)$
by(*simp add*: *boundOutput.inject alpha eqvts*)+
moreover from ‹$yvec' \sharp* R$› **have** $([(x,\ y)] \cdot yvec') \sharp* ([(x,\ y)] \cdot R)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp R$› ‹$y \sharp R$› **have** $([(x,\ y)] \cdot yvec') \sharp* R$ **by** *simp*
moreover note ‹$xvec' \sharp* R$› ‹*length xvec'* = *n*›
ultimately obtain T **where** $P = T \parallel R$ **and** *A*: $(\!|\nu{*}xvec'|\!)M \prec' T = (\!|\nu{*}([(x,\ y)] \cdot yvec')|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)$
by(*drule-tac Suc*) *auto*

from A **have** $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' T) = (\!|\nu x|\!)((\!|\nu{*}([(x,\ y)] \cdot yvec')|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q))$
by(*simp add*: *boundOutput.inject alpha*)
moreover from *xFreshQR* **have** $x \sharp (\!|\nu{*}yvec'|\!)N \prec' Q$
by(*force simp add*: *boundOutputFresh*)
ultimately show *?thesis* **using** ‹$P = T \parallel R$› ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*› *xFreshQR*
by(*force simp add*: *alphaBoundOutput name-swap eqvts*)
qed
qed
ultimately show *?thesis*
by *blast*
qed

lemma *boundOutputPar1Dest'*:
fixes *xvec* :: *name list*
and M :: *'a*::*fs-name*
and P :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
and *yvec* :: *name list*
and N :: *'a*
and Q :: (*'a*, *'b*, *'c*) *psi*
and R :: (*'a*, *'b*, *'c*) *psi*

assumes $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$
and *xvec* $\sharp*$ *yvec*

obtains $T\ p$ **where** *set* $p \subseteq$ *set xvec* $\times$ *set yvec* **and** $P = T \parallel (p \cdot R)$ **and** $(\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' Q$
proof −
assume $\bigwedge p\ T.\ [\![$*set* $p \subseteq$ *set xvec* $\times$ *set yvec*; $P = T \parallel (p \cdot R)$; $(\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' Q]\!] \Longrightarrow$ *thesis*
moreover obtain *n* **where** *n* = *length xvec* **by** *auto*
with *assms* **have** $\exists p\ T.$ *set* $p \subseteq$ *set xvec* $\times$ *set yvec* $\wedge\ P = T \parallel (p \cdot R) \wedge (\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' Q$
proof(*induct n arbitrary*: *xvec yvec M N P Q R*)
case(*0 xvec yvec M N P Q R*)
have *Eq*: $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$ **by** *fact*
from ‹*0* = *length xvec*› **have** *xvec* = [] **by** *auto*
moreover with *Eq* **have** *yvec* = []
by(*case-tac yvec*) *auto*
ultimately show *?case* **using** *Eq*
by(*simp add*: *boundOutput.inject*)
next
case(*Suc n xvec yvec M N P Q R*)
from ‹*Suc n* = *length xvec*›
obtain *x xvec'* **where** *xvec* = *x*#*xvec'* **and** *length xvec'* = *n*
by(*case-tac xvec*) *auto*
from ‹$(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R)$› ‹*xvec* = *x* # *xvec'*›

obtain y $yvec'$ **where** $(\!|\nu*(x\#xvec')|\!)M \prec' P = (\!|\nu*(y\#yvec')|\!)N \prec' (Q \parallel R)$
and $yvec = y\#yvec'$
by(*case-tac yvec*) *auto*
hence *Eq*: $(\!|\nu x|\!)((\!|\nu*xvec'|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu*yvec'|\!)N \prec' (Q \parallel R))$
by *simp*
from ‹$xvec = x\#xvec'$› ‹$yvec{=}y\#yvec'$› ‹$xvec \sharp* yvec$› **have** $x \neq y$ **and** $x \sharp yvec'$ **and** $y \sharp xvec'$ **and** $xvec' \sharp* yvec'$
by *auto*
from *Eq* ‹$x \neq y$› **have** *Eq'*: $(\!|\nu*xvec'|\!)M \prec' P = [(x, y)] \cdot (\!|\nu*yvec'|\!)N \prec' (Q \parallel R)$
and *xFreshQR*: $x \sharp (\!|\nu*yvec'|\!)N \prec' (Q \parallel R)$
by(*simp add*: *boundOutput.inject alpha*)+
have *IH*: $\bigwedge xvec\ yvec\ M\ N\ P\ Q\ R.\ [\![(\!|\nu*xvec|\!)M \prec' (P::('a, 'b, 'c)\ psi) = (\!|\nu*yvec|\!)N \prec' (Q \parallel R);\ xvec \sharp* yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p\ T.\ set\ p \subseteq set\ xvec \times set\ yvec \wedge P = T \parallel (p \cdot R) \wedge (\!|\nu*xvec|\!)M \prec' T = (\!|\nu*yvec|\!)N \prec' Q$
by *fact*
show *?case*
proof(*case-tac* $x \sharp (\!|\nu*xvec'|\!)M \prec' P$)
assume $x \sharp (\!|\nu*xvec'|\!)M \prec' P$
with *Eq* **have** *yFreshQR*: $y \sharp (\!|\nu*yvec'|\!)N \prec' (Q \parallel R)$
by(*rule boundOutputEqFresh*)
with *Eq' xFreshQR* **have** $(\!|\nu*xvec'|\!)M \prec' P = (\!|\nu*yvec'|\!)N \prec' (Q \parallel R)$
by *simp*
with ‹$xvec' \sharp* yvec'$› ‹$length\ xvec' = n$›
obtain p T **where** *S*: $set\ p \subseteq set\ xvec' \times set\ yvec'$ **and** $P = T \parallel (p \cdot R)$ **and** *A*: $(\!|\nu*xvec'|\!)M \prec' T = (\!|\nu*yvec'|\!)N \prec' Q$
by(*drule-tac IH*) *auto*
from *yFreshQR xFreshQR* **have** *yFreshQ*: $y \sharp (\!|\nu*yvec'|\!)N \prec' Q$ **and** *xFreshQ*: $x \sharp (\!|\nu*yvec'|\!)N \prec' Q$
by(*force simp add*: *BOresChainSupp fresh-def boundOutput.supp psi.supp*)+
hence $(\!|\nu x|\!)((\!|\nu*yvec'|\!)N \prec' Q) = (\!|\nu y|\!)((\!|\nu*yvec'|\!)N \prec' Q)$ **by** (*subst alphaBoundOutput*) *simp*+
with *A* **have** $(\!|\nu x|\!)((\!|\nu*xvec'|\!)M \prec' T) = (\!|\nu y|\!)((\!|\nu*yvec'|\!)N \prec' Q)$ **by** *simp*
with ‹$xvec{=}x\#xvec'$› ‹$yvec{=}y\#yvec'$› *S* ‹$P = T \parallel (p \cdot R)$› **show** *?case*
by *auto*
next
assume $\neg(x \sharp (\!|\nu*xvec'|\!)M \prec' P)$
hence $x \in supp((\!|\nu*xvec'|\!)M \prec' P)$ **by**(*simp add*: *fresh-def*)
with *Eq* **have** $y \in supp((\!|\nu*yvec'|\!)N \prec' (Q \parallel R))$
by(*rule boundOutputEqSupp*)
hence $y \sharp yvec'$ **by**(*simp add*: *BOresChainSupp fresh-def*)
with *Eq'* ‹$x \sharp yvec'$› **have** $(\!|\nu*xvec'|\!)M \prec' P = (\!|\nu*yvec'|\!)([(x, y)] \cdot N) \prec' (([(x, y)] \cdot Q) \parallel ([(x, y)] \cdot R))$
by(*simp add*: *eqvts*)
moreover note ‹$xvec' \sharp* yvec'$› ‹$length\ xvec' = n$›
ultimately obtain p T **where** *S*: $set\ p \subseteq set\ xvec' \times set\ yvec'$ **and** $P = T \parallel (p \cdot [(x, y)] \cdot R)$ **and** *A*: $(\!|\nu*xvec'|\!)M \prec' T = (\!|\nu*yvec'|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
by(*drule-tac IH*) *auto*

from S **have** $set(p@[(x,\ y)]) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover from ‹$P = T \parallel (p \cdot [(x,\ y)] \cdot R)$› **have** $P = T \parallel ((p\ @\ [(x,\ y)]) \cdot R)$
by(*simp add*: *pt2*[*OF pt-name-inst*])
moreover from *xFreshQR* **have** *xFreshQ*: $x \sharp (\!|\nu * yvec'|\!) N \prec' Q$
by(*force simp add*: *BOresChainSupp fresh-def boundOutput.supp psi.supp*)+
with ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› ‹$x \neq y$› **have** $y \sharp (\!|\nu * yvec'|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)$
by(*simp add*: *fresh-left calc-atm*)
with ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› **have** $(\!|\nu x|\!)((\!|\nu * yvec'|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)) = (\!|\nu y|\!)((\!|\nu * yvec'|\!)N \prec' Q)$
by(*subst alphaBoundOutput*) (*assumption* | *simp add*: *eqvts*)+
with *A* **have** $(\!|\nu x|\!)((\!|\nu * xvec'|\!)M \prec' T) = (\!|\nu y|\!)((\!|\nu * yvec'|\!)N \prec' Q)$ **by** *simp*
ultimately show *?thesis* **using** ‹$xvec = x\#xvec'$› ‹$yvec = y\#yvec'$›
by(*rule-tac* $x=p@[(x,\ y)]$ **in** *exI*) *force*
qed
qed
ultimately show *?thesis*
by *blast*
qed

lemma *boundOutputPar2Dest*:
fixes *xvec* :: *name list*
and M :: $'a$::*fs-name*
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*
and *yvec* :: *name list*
and N :: $'a$
and Q :: ($'a$, $'b$, $'c$) *psi*
and R :: ($'a$, $'b$, $'c$) *psi*

assumes $(\!|\nu * xvec|\!) M \prec' P = (\!|\nu * yvec|\!) N \prec' (Q \parallel R)$
and *xvec* $\sharp*$ Q
and *yvec* $\sharp*$ Q

obtains T **where** $P = Q \parallel T$ **and** $(\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' R$
proof −
assume $\bigwedge T.\ [\![P = Q \parallel T;\ (\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' R]\!] \Longrightarrow$ *thesis*
moreover obtain n **where** $n = length\ xvec$ **by** *auto*
with *assms* **have** $\exists T.\ P = Q \parallel T \wedge (\!|\nu * xvec|\!) M \prec' T = (\!|\nu * yvec|\!) N \prec' R$
proof(*induct n arbitrary*: *xvec yvec M N P Q R*)
case(*0 xvec yvec M N P Q R*)
have *Eq*: $(\!|\nu * xvec|\!) M \prec' P = (\!|\nu * yvec|\!) N \prec' (Q \parallel R)$ **by** *fact*
from ‹$0 = length\ xvec$› **have** $xvec = []$ **by** *auto*
moreover with *Eq* **have** $yvec = []$
by(*case-tac yvec*) *auto*
ultimately show *?case* **using** *Eq*
by(*simp add*: *boundOutput.inject*)
next

case(*Suc n xvec yvec M N P Q R*)
from ‹*Suc n = length xvec*›
obtain *x xvec′* **where** *xvec = x#xvec′* **and** *length xvec′ = n*
by(*case-tac xvec*) *auto*
from ‹(|*ν***xvec*|)*M* ≺′ *P* = (|*ν***yvec*|)*N* ≺′ (*Q* ∥ *R*)› ‹*xvec = x # xvec′*›
obtain *y yvec′* **where** (|*ν**(*x*#*xvec′*)|)*M* ≺′ *P* = (|*ν**(*y*#*yvec′*)|)*N* ≺′ (*Q* ∥ *R*)
and *yvec = y#yvec′*
by(*case-tac yvec*) *auto*
hence *EQ*: (|*νx*|)((|*ν***xvec′*|)*M* ≺′ *P*) = (|*νy*|)((|*ν***yvec′*|)*N* ≺′ (*Q* ∥ *R*))
by *simp*
from ‹*xvec* ♯* *Q*› ‹*yvec* ♯* *Q*› ‹*xvec = x#xvec′*› ‹*yvec = y#yvec′*›
have *x* ♯ *Q* **and** *xvec′* ♯* *Q* **and** *y* ♯ *Q* **and** *yvec′* ♯* *Q* **by** *auto*
have *IH*: ⋀*xvec yvec M N P Q R*. ⟦(|*ν***xvec*|)*M* ≺′ (*P*::('*a*, '*b*, '*c*) *psi*) = (|*ν***yvec*|)*N* ≺′ (*Q* ∥ *R*); *xvec* ♯* *Q*; *yvec* ♯* *Q*; *n = length xvec*⟧ ⟹ ∃ *T*. *P = Q* ∥ *T* ∧ (|*ν***xvec*|)*M* ≺′ *T* = (|*ν***yvec*|)*N* ≺′ *R*
by *fact*
show *?case*
proof(*case-tac x = y*)
assume *x = y*
with *EQ* **have** (|*ν***xvec′*|)*M* ≺′ *P* = (|*ν***yvec′*|)*N* ≺′ (*Q* ∥ *R*)
by(*simp add*: *boundOutput.inject alpha*)
with ‹*xvec′* ♯* *Q*› ‹*yvec′* ♯* *Q*› ‹*length xvec′ = n*›
obtain *T* **where** *P = Q* ∥ *T* **and** (|*ν***xvec′*|)*M* ≺′ *T* = (|*ν***yvec′*|)*N* ≺′ *R*
by(*drule-tac IH*) *auto*
with ‹*xvec=x#xvec′*› ‹*yvec=y#yvec′*› ‹*x=y*› **show** *?case*
by(*force simp add*: *boundOutput.inject alpha*)
next
assume *x ≠ y*
with *EQ* ‹*x* ♯ *Q*› ‹*y* ♯ *Q*›
have (|*ν***xvec′*|)*M* ≺′ *P* = (|*ν**([(*x*, *y*)] ∙ *yvec′*)|)([(*x*, *y*)] ∙ *N*) ≺′ (*Q* ∥ ([(*x*, *y*)] ∙ *R*))
and *xFreshQR*: *x* ♯ (|*ν***yvec′*|)*N* ≺′ (*Q* ∥ *R*)
by(*simp add*: *boundOutput.inject alpha eqvts*)+
moreover from ‹*yvec′* ♯* *Q*› **have** ([(*x*, *y*)] ∙ *yvec′*) ♯* ([(*x*, *y*)] ∙ *Q*)
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*x* ♯ *Q*› ‹*y* ♯ *Q*› **have** ([(*x*, *y*)] ∙ *yvec′*) ♯* *Q* **by** *simp*
moreover note ‹*xvec′* ♯* *Q*› ‹*length xvec′ = n*›
ultimately obtain *T* **where** *P = Q* ∥ *T* **and** *A*: (|*ν***xvec′*|)*M* ≺′ *T* = (|*ν**([(*x*, *y*)] ∙ *yvec′*)|)([(*x*, *y*)] ∙ *N*) ≺′ ([(*x*, *y*)] ∙ *R*)
by(*drule-tac IH*) *auto*

from *A* **have** (|*νx*|)((|*ν***xvec′*|)*M* ≺′ *T*) = (|*νx*|)((|*ν**([(*x*, *y*)] ∙ *yvec′*)|)([(*x*, *y*)] ∙ *N*) ≺′ ([(*x*, *y*)] ∙ *R*))
by(*simp add*: *boundOutput.inject alpha*)
moreover from *xFreshQR* **have** *x* ♯ (|*ν***yvec′*|)*N* ≺′ *R*
by(*force simp add*: *boundOutputFresh*)
ultimately show *?thesis* **using** ‹*P = Q* ∥ *T*› ‹*xvec=x#xvec′*› ‹*yvec=y#yvec′*› *xFreshQR*
by(*force simp add*: *alphaBoundOutput name-swap eqvts*)

```
    qed
  qed
  ultimately show ?thesis
    by blast
qed

lemma boundOutputPar2Dest':
  fixes xvec :: name list
  and   M    :: 'a::fs-name
  and   P    :: ('a, 'b::fs-name, 'c::fs-name) psi
  and   yvec :: name list
  and   N    :: 'a
  and   Q    :: ('a, 'b, 'c) psi
  and   R    :: ('a, 'b, 'c) psi

  assumes (|ν*xvec|)M ≺' P = (|ν*yvec|)N ≺' (Q ∥ R)
  and     xvec ♯* yvec

  obtains T p where set p ⊆ set xvec × set yvec and P = (p · Q) ∥ T and
(|ν*xvec|)M ≺' T = (|ν*yvec|)N ≺' R
proof −
  assume ⋀p T. [[set p ⊆ set xvec × set yvec; P = (p · Q) ∥ T; (|ν*xvec|)M ≺' T
= (|ν*yvec|)N ≺' R]] ⟹ thesis
  moreover obtain n where n = length xvec by auto
  with assms have ∃p T. set p ⊆ set xvec × set yvec ∧ P = (p · Q) ∥ T ∧
(|ν*xvec|)M ≺' T = (|ν*yvec|)N ≺' R
  proof(induct n arbitrary: xvec yvec M N P Q R)
    case(0 xvec yvec M N P Q R)
    have Eq: (|ν*xvec|)M ≺' P = (|ν*yvec|)N ≺' (Q ∥ R) by fact
    from ‹0 = length xvec› have xvec = [] by auto
    moreover with Eq have yvec = []
      by(case-tac yvec) auto
    ultimately show ?case using Eq
      by(simp add: boundOutput.inject)
  next
    case(Suc n xvec yvec M N P Q R)
    from ‹Suc n = length xvec›
    obtain x xvec' where xvec = x#xvec' and length xvec' = n
      by(case-tac xvec) auto
    from ‹(|ν*xvec|)M ≺' P = (|ν*yvec|)N ≺' (Q ∥ R)› ‹xvec = x # xvec'›
    obtain y yvec' where (|ν*(x#xvec')|)M ≺' P = (|ν*(y#yvec')|)N ≺' (Q ∥ R)
      and yvec = y#yvec'
      by(case-tac yvec) auto
    hence Eq: (|νx|)((|ν*xvec'|)M ≺' P) = (|νy|)((|ν*yvec'|)N ≺' (Q ∥ R))
      by simp
     from ‹xvec = x#xvec'› ‹yvec=y#yvec'› ‹xvec ♯* yvec› have x ≠ y and x ♯
yvec' and y ♯ xvec' and xvec' ♯* yvec'
      by auto
    from Eq ‹x ≠ y› have Eq': (|ν*xvec'|)M ≺' P = [(x, y)] · (|ν*yvec'|)N ≺' (Q ∥
```

R)
and *xFreshQR*: $x \sharp (\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R)$
by(*simp add*: *boundOutput.inject alpha*)+
have *IH*: $\bigwedge xvec\ yvec\ M\ N\ P\ Q\ R.\ [\![(\!|\nu{*}xvec|\!)M \prec' (P::('a, 'b, 'c)\ psi) = (\!|\nu{*}yvec|\!)N \prec' (Q \parallel R);\ xvec \sharp* yvec;\ n = length\ xvec]\!] \Longrightarrow \exists p\ T.\ set\ p \subseteq set\ xvec \times set\ yvec \wedge P = (p \cdot Q) \parallel T \wedge (\!|\nu{*}xvec|\!)M \prec' T = (\!|\nu{*}yvec|\!)N \prec' R$
by *fact*
show *?case*
proof(*case-tac* $x \sharp (\!|\nu{*}xvec'|\!)M \prec' P$)
assume $x \sharp (\!|\nu{*}xvec'|\!)M \prec' P$
with *Eq* **have** *yFreshQR*: $y \sharp (\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R)$
by(*rule boundOutputEqFresh*)
with *Eq'* *xFreshQR* **have** $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R)$
by *simp*
with ‹$xvec' \sharp* yvec'$› ‹$length\ xvec' = n$›
obtain $p\ T$ **where** S: $set\ p \subseteq set\ xvec' \times set\ yvec'$ **and** $P = (p \cdot Q) \parallel T$ **and** A: $(\!|\nu{*}xvec'|\!)M \prec' T = (\!|\nu{*}yvec'|\!)N \prec' R$
by(*drule-tac IH*) *auto*
from *yFreshQR xFreshQR* **have** *yFreshR*: $y \sharp (\!|\nu{*}yvec'|\!)N \prec' R$ **and** *xFreshQ*: $x \sharp (\!|\nu{*}yvec'|\!)N \prec' R$
by(*force simp add*: *BOresChainSupp fresh-def boundOutput.supp psi.supp*)+
hence $(\!|\nu x|\!)((\!|\nu{*}yvec'|\!)N \prec' R) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' R)$ **by** (*subst alphaBoundOutput*) *simp*+
with *A* **have** $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' T) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' R)$ **by** *simp*
with ‹$xvec{=}x\#xvec'$› ‹$yvec{=}y\#yvec'$› S ‹$P = (p \cdot Q) \parallel T$› **show** *?case*
by *auto*
next
assume $\neg(x \sharp (\!|\nu{*}xvec'|\!)M \prec' P)$
hence $x \in supp((\!|\nu{*}xvec'|\!)M \prec' P)$ **by**(*simp add*: *fresh-def*)
with *Eq* **have** $y \in supp((\!|\nu{*}yvec'|\!)N \prec' (Q \parallel R))$
by(*rule boundOutputEqSupp*)
hence $y \sharp yvec'$ **by**(*simp add*: *BOresChainSupp fresh-def*)
with *Eq'* ‹$x \sharp yvec'$› **have** $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}yvec'|\!)([(x, y)] \cdot N) \prec' (([(x, y)] \cdot Q) \parallel ([(x, y)] \cdot R))$
by(*simp add*: *eqvts*)
moreover note ‹$xvec' \sharp* yvec'$› ‹$length\ xvec' = n$›
ultimately obtain $p\ T$ **where** S: $set\ p \subseteq set\ xvec' \times set\ yvec'$ **and** $P = (p \cdot [(x, y)] \cdot Q) \parallel T$ **and** A: $(\!|\nu{*}xvec'|\!)M \prec' T = (\!|\nu{*}yvec'|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot R)$
by(*drule-tac IH*) *auto*

from S **have** $set(p@[(x, y)]) \subseteq set(x\#xvec') \times set(y\#yvec')$ **by** *auto*
moreover from ‹$P = (p \cdot [(x, y)] \cdot Q) \parallel T$› **have** $P = ((p\ @\ [(x, y)]) \cdot Q) \parallel T$
by(*simp add*: *pt2*[*OF pt-name-inst*])
moreover from *xFreshQR* **have** *xFreshR*: $x \sharp (\!|\nu{*}yvec'|\!)N \prec' R$
by(*force simp add*: *BOresChainSupp fresh-def boundOutput.supp psi.supp*)+
with ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› ‹$x \neq y$› **have** $y \sharp (\!|\nu{*}yvec'|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot R)$

by(*simp add*: *fresh-left calc-atm*)
with ‹$x \sharp yvec'$› ‹$y \sharp yvec'$› **have** $(\!|\nu x|\!)((\!|\nu{*}yvec'|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot R)) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' R)$
by(*subst alphaBoundOutput*) (*assumption* | *simp add*: *eqvts*)+
with *A* **have** $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' T) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' R)$ **by** *simp*
ultimately show *?thesis* **using** ‹*xvec*=*x*#*xvec'*› ‹*yvec*=*y*#*yvec'*›
by(*rule-tac x*=*p*@[(*x*, *y*)] **in** *exI*) *force*
qed
qed
ultimately show *?thesis*
by *blast*
qed

lemma *boundOutputApp*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *B* :: (*'a*::*fs-name*, *'b*::*fs-name*, *'c*::*fs-name*) *boundOutput*

shows $(\!|\nu{*}(xvec@yvec)|\!)B = (\!|\nu{*}xvec|\!)((\!|\nu{*}yvec|\!)B)$
by(*induct xvec*) *auto*

lemma *openInjectAuxAuxAux*:
fixes *x* :: *name*
and *xvec* :: *name list*

shows $\exists y\ yvec.\ x \# xvec = yvec @ [y] \wedge length\ xvec = length\ yvec$
apply(*induct xvec arbitrary*: *x*)
apply *auto*
apply(*subgoal-tac* $\exists y\ yvec.\ a \# xvec = yvec @ [y] \wedge length\ xvec = length\ yvec$)
apply(*clarify*)
apply(*rule-tac x*=*y* **in** *exI*)
by *auto*

lemma *openInjectAuxAux*:
fixes *xvec1* :: *name list*
and *xvec2* :: *name list*
and *yvec* :: *name list*

assumes *length*(*xvec1*@*xvec2*) = *length yvec*

shows $\exists yvec1\ yvec2.\ yvec = yvec1@yvec2 \wedge length\ xvec1 = length\ yvec1 \wedge length\ xvec2 = length\ yvec2$
using *assms*
apply(*induct yvec arbitrary*: *xvec1*)
apply *simp*
apply *simp*
apply(*case-tac xvec1*)
apply *simp*
apply *simp*

apply(*subgoal-tac* ∃ *yvec1 yvec2*.
yvec = *yvec1* @ *yvec2* ∧ *length list* = *length yvec1* ∧ *length xvec2* = *length yvec2*)
apply(*clarify*)
apply(*rule-tac x*=*a*#*yvec1* **in** *exI*)
apply(*rule-tac x*=*yvec2* **in** *exI*)
by *auto*

lemma *openInjectAux*:
fixes *xvec1* :: *name list*
and *x* :: *name*
and *xvec2* :: *name list*
and *yvec* :: *name list*

assumes *length*(*xvec1*@*x*#*xvec2*) = *length yvec*

shows ∃ *yvec1 y yvec2*. *yvec* = *yvec1*@*y*#*yvec2* ∧ *length xvec1* = *length yvec1* ∧ *length xvec2* = *length yvec2*
using *assms*
apply(*case-tac yvec*)
apply *simp*
apply *simp*
apply(*subgoal-tac* ∃(*yvec1*::*name list*) (*yvec2*::*name list*). *yvec1*@*yvec2* = *list* ∧ *length xvec1* = *length yvec1* ∧ *length xvec2* = *length yvec2*)
apply(*clarify*)
apply *hypsubst-thin*
apply *simp*
apply(*subgoal-tac* ∃ *y* (*yvec*::*name list*). *a* # *yvec1* = *yvec* @ [*y*] ∧ *length yvec1* = *length yvec*)
apply(*clarify*)
apply(*rule-tac x*=*yvec* **in** *exI*)
apply(*rule-tac x*=*y* **in** *exI*)
apply *simp*
apply(*rule-tac x*=*yvec2* **in** *exI*)
apply *simp*
apply(*rule openInjectAuxAuxAux*)
apply(*insert openInjectAuxAux*)
apply *simp*
by *blast*

lemma *boundOutputOpenDest*:
fixes *yvec* :: *name list*
and *M* :: *'a*::*fs-name*
and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
and *xvec1* :: *name list*
and *x* :: *name*
and *xvec2* :: *name list*
and *N* :: *'a*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

assumes *Eq*: $(\!|\nu*(xvec1@x\#xvec2)|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$
and $x \sharp xvec1$
and $x \sharp yvec$
and $x \sharp N$
and $x \sharp Q$
and *distinct yvec*

obtains *yvec1 y yvec2* **where** *yvec=yvec1@y#yvec2* **and** *length xvec1 = length yvec1* **and** *length xvec2 = length yvec2*
and $(\!|\nu*(xvec1@xvec2)|\!)M \prec' P = (\!|\nu*(yvec1@yvec2)|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
proof −
assume *Ass*: $\bigwedge yvec1\ y\ yvec2.$
$[\![yvec = yvec1\ @\ y\ \#\ yvec2;\ length\ xvec1 = length\ yvec1;\ length\ xvec2 = length\ yvec2;$
$(\!|\nu*(xvec1\ @\ xvec2)|\!)M \prec' P = (\!|\nu*(yvec1\ @\ yvec2)|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)]\!]$
$\Longrightarrow$ *thesis*
from *Eq* **have** *length*$(xvec1@x\#xvec2)$ = *length yvec* **by**(*rule boundOutputChainEqLength*)
then obtain *yvec1 y yvec2* **where** *A*: *yvec = yvec1@y#yvec2* **and** *length xvec1 = length yvec1*
and *length xvec2 = length yvec2*
by(*metis openInjectAux sym*)

from ‹*distinct yvec*› *A* **have** $y \sharp yvec2$ **by** *simp*
from *A* ‹$x \sharp yvec$› **have** $x \sharp yvec2$ **and** $x \sharp yvec1$ **by** *simp+*
with *Eq* ‹*length xvec1 = length yvec1*› ‹$x \sharp N$› ‹$x \sharp Q$› ‹$y \sharp yvec2$› ‹$x \sharp xvec1$› *A*
have $(\!|\nu*(xvec1@xvec2)|\!)M \prec' P = (\!|\nu*(yvec1@yvec2)|\!)([(x, y)] \cdot N) \prec' ([(x, y)] \cdot Q)$
by(*force dest*: *boundOutputChainOpenIH simp add*: *boundOutputApp BOresChain-Supp fresh-def boundOutput.supp eqvts*)
with ‹*length xvec1 = length yvec1*› ‹*length xvec2 = length yvec2*› *A Ass* **show** *?thesis*
by *blast*
qed

lemma *boundOutputOpenDest'*:
fixes *yvec* :: *name list*
and *M* :: *'a::fs-name*
and *P* :: (*'a*, *'b::fs-name*, *'c::fs-name*) *psi*
and *xvec1* :: *name list*
and *x* :: *name*
and *xvec2* :: *name list*
and *N* :: *'a*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

assumes *Eq*: $(\!|\nu*(xvec1@x\#xvec2)|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' Q$

and $x \sharp xvec1$
and $x \sharp yvec$
and $x \sharp N$
and $x \sharp Q$

obtains *yvec1 y yvec2* **where** *yvec=yvec1@y#yvec2* **and** *length xvec1 = length yvec1* **and** *length xvec2 = length yvec2*
and $(\!|\nu*(xvec1@xvec2)|\!)M \prec' P = (\!|\nu*(yvec1@[(x,\ y)] \cdot yvec2)|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)$
proof −
assume *Ass*: $\bigwedge yvec1\ y\ yvec2.$
$[\![yvec = yvec1\ @\ y\ \#\ yvec2;\ length\ xvec1 = length\ yvec1;\ length\ xvec2 = length\ yvec2;$
$(\!|\nu*(xvec1\ @\ xvec2)|\!)M \prec' P = (\!|\nu*(yvec1\ @\ ([(x,\ y)] \cdot yvec2))|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)]\!]$
$\Longrightarrow$ *thesis*
from *Eq* **have** *length*(*xvec1@x#xvec2*) = *length yvec* **by**(*rule boundOutputChainEqLength*)
then obtain *yvec1 y yvec2* **where** *A*: *yvec = yvec1@y#yvec2* **and** *length xvec1 = length yvec1*
and *length xvec2 = length yvec2*
by(*metis openInjectAux sym*)

from *A* ‹*x* ♯ *yvec*› **have** *x* ♯ *yvec2* **and** *x* ♯ *yvec1* **by** *simp+*
with *Eq* ‹*length xvec1 = length yvec1*› ‹*x* ♯ *N*› ‹*x* ♯ *Q*› ‹*x* ♯ *xvec1*› *A*
have $(\!|\nu*(xvec1@xvec2)|\!)M \prec' P = (\!|\nu*(yvec1@([(x,\ y)] \cdot yvec2))|\!)([(x,\ y)] \cdot N) \prec' ([(x,\ y)] \cdot Q)$
by(*force dest*: *boundOutputChainOpenIH simp add*: *boundOutputApp BOresChainSupp fresh-def boundOutput.supp eqvts*)
with ‹*length xvec1 = length yvec1*› ‹*length xvec2 = length yvec2*› *A Ass* **show** *?thesis*
by *blast*
qed

lemma *boundOutputScopeDest*:
fixes *xvec* :: *name list*
and *M* :: *'a*::*fs-name*
and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*
and *yvec* :: *name list*
and *N* :: *'a*
and *x* :: *name*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

assumes $(\!|\nu*xvec|\!)M \prec' P = (\!|\nu*yvec|\!)N \prec' (\!|\nu z|\!)Q$
and $z \sharp xvec$
and $z \sharp yvec$

obtains *R* **where** $P = (\!|\nu z|\!)R$ **and** $(\!|\nu*xvec|\!)M \prec' R = (\!|\nu*yvec|\!)N \prec' Q$
proof −

assume $\bigwedge R.\ [\![P = (\!|\nu z|\!)R;\ (\!|\nu{*}xvec|\!)M \prec' R = (\!|\nu{*}yvec|\!)N \prec' Q]\!] \Longrightarrow thesis$
moreover obtain n **where** $n = length\ xvec$ **by** *auto*
with *assms* **have** $\exists R.\ P = (\!|\nu z|\!)R \wedge (\!|\nu{*}xvec|\!)M \prec' R = (\!|\nu{*}yvec|\!)N \prec' Q$
proof(*induct n arbitrary*: *xvec yvec M N P Q z*)
case(*0 xvec yvec M N P Q z*)
have *Eq*: $(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' (\!|\nu z|\!)Q$ **by** *fact*
from ‹*0* = *length xvec*› **have** $xvec = []$ **by** *auto*
moreover with *Eq* **have** $yvec = []$
by(*case-tac yvec*) *auto*
ultimately show *?case* **using** *Eq*
by(*simp add*: *boundOutput.inject*)
next
case(*Suc n xvec yvec M N P Q z*)
from ‹*Suc n* = *length xvec*›
obtain $x\ xvec'$ **where** $xvec = x\#xvec'$ **and** $length\ xvec' = n$
by(*case-tac xvec*) *auto*
from ‹$(\!|\nu{*}xvec|\!)M \prec' P = (\!|\nu{*}yvec|\!)N \prec' ((\!|\nu z|\!)Q)$› ‹$xvec = x\ \#\ xvec'$›
obtain $y\ yvec'$ **where** $(\!|\nu{*}(x\#xvec')|\!)M \prec' P = (\!|\nu{*}(y\#yvec')|\!)N \prec' (\!|\nu z|\!)Q$
and $yvec = y\#yvec'$
by(*case-tac yvec*) *auto*
hence *EQ*: $(\!|\nu x|\!)((\!|\nu{*}xvec'|\!)M \prec' P) = (\!|\nu y|\!)((\!|\nu{*}yvec'|\!)N \prec' (\!|\nu z|\!)Q)$
by *simp*
from ‹$z \sharp xvec$› ‹$z \sharp yvec$› ‹$xvec = x\#xvec'$› ‹$yvec = y\#yvec'$›
have $z \neq x$ **and** $z \neq y$ **and** $z \sharp xvec'$ **and** $z \sharp yvec'$
by *simp+*
have *IH*: $\bigwedge xvec\ yvec\ M\ N\ P\ Q\ z.\ [\![(\!|\nu{*}xvec|\!)M \prec' (P::('a,\ 'b,\ 'c)\ psi) = (\!|\nu{*}yvec|\!)N \prec' (\!|\nu z|\!)Q;\ z \sharp xvec;\ z \sharp yvec;\ n = length\ xvec]\!] \Longrightarrow \exists R.\ P = (\!|\nu z|\!)R \wedge (\!|\nu{*}xvec|\!)M \prec' R = (\!|\nu{*}yvec|\!)N \prec' Q$
by *fact*
show *?case*
proof(*case-tac x = y*)
assume $x = y$
with *EQ* **have** $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}yvec'|\!)N \prec' (\!|\nu z|\!)Q$
by(*simp add*: *boundOutput.inject alpha*)
with ‹$z \sharp xvec'$› ‹$z \sharp yvec'$› ‹*length xvec'* = *n*›
obtain R **where** $P = (\!|\nu z|\!)R$ **and** $(\!|\nu{*}xvec'|\!)M \prec' R = (\!|\nu{*}yvec'|\!)N \prec' Q$
by(*drule-tac IH*) *auto*
with ‹$xvec{=}x\#xvec'$› ‹$yvec{=}y\#yvec'$› ‹$x{=}y$› **show** *?case*
by(*force simp add*: *boundOutput.inject alpha*)
next
assume $x \neq y$
with *EQ* ‹$z \neq x$› ‹$z \neq y$›
have $(\!|\nu{*}xvec'|\!)M \prec' P = (\!|\nu{*}([(x,\ y)] \cdot yvec')|\!)([(x,\ y)] \cdot N) \prec' (\!|\nu z|\!)([(x,\ y)] \cdot Q)$
and *xFreshzQ*: $x \sharp (\!|\nu{*}yvec'|\!)N \prec' (\!|\nu z|\!)Q$
by(*simp add*: *boundOutput.inject alpha eqvts*)+
moreover from ‹$z \neq x$› ‹$z \neq y$› ‹$z \sharp yvec'$› ‹$x \neq y$› **have** $z \sharp ([(x,\ y)] \cdot yvec')$
by(*simp add*: *fresh-left calc-atm*)
moreover note ‹$z \sharp xvec'$› ‹*length xvec'* = *n*›

```
    ultimately obtain R where P = (|νz|)R and A: (|ν*xvec'|)M ≺' R = (|ν*([(x, y)] • yvec')|)([(x, y)] • N) ≺' ([(x, y)] • Q)
      by(drule-tac IH) auto

    from A have (|νx|)((|ν*xvec'|)M ≺' R) = (|νx|)((|ν*([(x, y)] • yvec')|)([(x, y)] • N) ≺' ([(x, y)] • Q))
      by(simp add: boundOutput.inject alpha)
    moreover from xFreshzQ ‹z ≠ x› have x ♯ (|ν*yvec'|)N ≺' Q
      by(simp add: boundOutputFresh abs-fresh)
    ultimately show ?thesis using ‹P = (|νz|)R› ‹xvec=x#xvec'› ‹yvec=y#yvec'› xFreshzQ
      by(force simp add: alphaBoundOutput name-swap eqvts)
   qed
  qed
  ultimately show ?thesis
   by blast
qed

nominal-datatype ('a, 'b, 'c) residual =
  RIn 'a::fs-name 'a ('a, 'b::fs-name, 'c::fs-name) psi
| ROut 'a ('a, 'b, 'c) boundOutput
| RTau ('a, 'b, 'c) psi

nominal-datatype 'a action = In 'a::fs-name 'a      (-(|-|) [90, 90] 90)
                | Out 'a::fs-name name list 'a (-(|ν*-|)⟨-⟩ [90, 90, 90] 90)
                | Tau                          (τ 90)

nominal-primrec bn :: ('a::fs-name) action ⇒ name list
  where
  bn (M(|N|)) = []
| bn (M(|ν*xvec|)⟨N⟩) = xvec
| bn (τ) = []
by(rule TrueI)+

lemma bnEqvt[eqvt]:
  fixes p :: name prm
  and   α :: ('a::fs-name) action

  shows (p • bn α) = bn(p • α)
by(nominal-induct α rule: action.strong-induct) auto

nominal-primrec create-residual :: ('a::fs-name) action ⇒ ('a, 'b::fs-name, 'c::fs-name) psi ⇒ ('a, 'b, 'c) residual (- ≺ - [80, 80] 80)
where
  (M(|N|)) ≺ P = RIn M N P
| M(|ν*xvec|)⟨N⟩ ≺ P = ROut M ((|ν*xvec|)(N ≺' P))
| τ ≺ P = (RTau P)
by(rule TrueI)+
```

nominal-primrec *subject* :: ('*a*::*fs-name*) *action* ⇒ '*a option*
  where
  subject (*M*⟦*N*⟧) = *Some M*
| *subject* (*M*⟦*ν*∗*xvec*⟧⟨*N*⟩) = *Some M*
| *subject* (*τ*) = *None*
by(*rule TrueI*)+

nominal-primrec *object* :: ('*a*::*fs-name*) *action* ⇒ '*a option*
  where
  object (*M*⟦*N*⟧) = *Some N*
| *object* (*M*⟦*ν*∗*xvec*⟧⟨*N*⟩) = *Some N*
| *object* (*τ*) = *None*
by(*rule TrueI*)+

lemma *optionFreshChain*[*simp*]:
  fixes *xvec* :: *name list*
  and *X* :: *name set*

  shows *xvec* ♯∗ (*Some x*) = *xvec* ♯∗ *x*
  and *X* ♯∗ (*Some x*) = *X* ♯∗ *x*
  and *xvec* ♯∗ *None*
  and *X* ♯∗ *None*
by(*auto simp add*: *fresh-star-def fresh-some fresh-none*)

lemmas [*simp*] = *fresh-some fresh-none*

lemma *actionFresh*[*simp*]:
  fixes *x* :: *name*
  and *α* :: ('*a*::*fs-name*) *action*

  shows (*x* ♯ *α*) = (*x* ♯ (*subject α*) ∧ *x* ♯ (*bn α*) ∧ *x* ♯ (*object α*))
by(*nominal-induct α rule*: *action.strong-induct*) *auto*

lemma *actionFreshChain*[*simp*]:
  fixes *X* :: *name set*
  and *α* :: ('*a*::*fs-name*) *action*
  and *xvec* :: *name list*

  shows (*X* ♯∗ *α*) = (*X* ♯∗ (*subject α*) ∧ *X* ♯∗ (*bn α*) ∧ *X* ♯∗ (*object α*))
  and (*xvec* ♯∗ *α*) = (*xvec* ♯∗ (*subject α*) ∧ *xvec* ♯∗ (*bn α*) ∧ *xvec* ♯∗ (*object α*))
by(*auto simp add*: *fresh-star-def*)

lemma *subjectEqvt*[*eqvt*]:
  fixes *p* :: *name prm*
  and *α* :: ('*a*::*fs-name*) *action*

  shows (*p* ∙ *subject α*) = *subject*(*p* ∙ *α*)
by(*nominal-induct α rule*: *action.strong-induct*) *auto*

lemma *okjectEqvt*[*eqvt*]:
  fixes p :: *name prm*
  and α :: ('*a*::*fs-name*) *action*

  shows $(p \cdot object\ \alpha) = object(p \cdot \alpha)$
by(*nominal-induct* α *rule*: *action.strong-induct*) *auto*

lemma *create-residualEqvt*[*eqvt*]:
  fixes p :: *name prm*
  and α :: ('*a*::*fs-name*) *action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

  shows $(p \cdot (\alpha \prec P)) = (p \cdot \alpha) \prec (p \cdot P)$
by(*nominal-induct* α *rule*: *action.strong-induct*)
  (*auto simp add*: *eqvts*)

lemma *residualFresh*:
  fixes x :: *name*
  and α :: '*a*::*fs-name action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

  shows $(x \sharp (\alpha \prec P)) = (x \sharp (subject\ \alpha) \wedge (x \in (set(bn(\alpha))) \vee (x \sharp object(\alpha) \wedge x \sharp P)))$
by(*nominal-induct* α *rule*: *action.strong-induct*)
  (*auto simp add*: *fresh-some fresh-none boundOutputFresh*)

lemma *residualFresh2*[*simp*]:
  fixes x :: *name*
  and α :: ('*a*::*fs-name*) *action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

  assumes $x \sharp \alpha$
  and $x \sharp P$

  shows $x \sharp \alpha \prec P$
using *assms*
by(*nominal-induct* α *rule*: *action.strong-induct*) *auto*

lemma *residualFreshChain2*[*simp*]:
  fixes *xvec* :: *name list*
  and X :: *name set*
  and α :: ('*a*::*fs-name*) *action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

  shows $[\![xvec \sharp* \alpha;\ xvec \sharp* P]\!] \Longrightarrow xvec \sharp* (\alpha \prec P)$
  and $[\![X \sharp* \alpha;\ X \sharp* P]\!] \Longrightarrow X \sharp* (\alpha \prec P)$
by(*auto simp add*: *fresh-star-def*)

lemma *residualFreshSimp*[*simp*]:

fixes x :: *name*
and M :: $'a$::*fs-name*
and N :: $'a$
and P :: ($'a$, $'b$::*fs-name*, $'c$::*fs-name*) *psi*

shows $x \sharp (M(\!|N|\!) \prec P) = (x \sharp M \wedge x \sharp N \wedge x \sharp P)$
and $x \sharp (M(\!|\nu * xvec|\!)\langle N\rangle \prec P) = (x \sharp M \wedge x \sharp ((\!|\nu * xvec|\!)(N \prec' P)))$
and $x \sharp (\tau \prec P) = (x \sharp P)$
by(*auto simp add*: *residualFresh*)

lemma *residualInject'*:

shows $(\alpha \prec P = RIn\ M\ N\ Q) = (P = Q \wedge \alpha = M(\!|N|\!))$
and $(\alpha \prec P = ROut\ M\ B) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu * xvec|\!)\langle N\rangle \wedge B = (\!|\nu * xvec|\!)(N \prec' P))$
and $(\alpha \prec P = RTau\ Q) = (\alpha = \tau \wedge P = Q)$
and $(RIn\ M\ N\ Q = \alpha \prec P) = (P = Q \wedge \alpha = M(\!|N|\!))$
and $(ROut\ M\ B = \alpha \prec P) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu * xvec|\!)\langle N\rangle \wedge B = (\!|\nu * xvec|\!)(N \prec' P))$
and $(RTau\ Q = \alpha \prec P) = (\alpha = \tau \wedge P = Q)$
proof −
show $(\alpha \prec P = RIn\ M\ N\ Q) = (P = Q \wedge \alpha = M(\!|N|\!))$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
next
show $(\alpha \prec P = ROut\ M\ B) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu * xvec|\!)\langle N\rangle \wedge B = (\!|\nu * xvec|\!)(N \prec' P))$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
next
show $(\alpha \prec P = RTau\ Q) = (\alpha = \tau \wedge P = Q)$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
next
show $(RIn\ M\ N\ Q = \alpha \prec P) = (P = Q \wedge \alpha = M(\!|N|\!))$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
next
show $(ROut\ M\ B = \alpha \prec P) = (\exists\, xvec\ N.\ \alpha = M(\!|\nu * xvec|\!)\langle N\rangle \wedge B = (\!|\nu * xvec|\!)(N \prec' P))$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
next
show $(RTau\ Q = \alpha \prec P) = (\alpha = \tau \wedge P = Q)$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residual.inject action.inject*)
qed

lemma *residualFreshChainSimp*[*simp*]:
  fixes *xvec* :: *name list*
  and *X* :: *name set*
  and *M* :: *'a*::*fs-name*
  and *N* :: *'a*
  and *yvec* :: *name list*
  and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

  shows *xvec* ♯* (*M*⟨⟨*N*⟩⟩ ≺ *P*) = (*xvec* ♯* *M* ∧ *xvec* ♯* *N* ∧ *xvec* ♯* *P*)
  and *xvec* ♯* (*M*⟨⟨*ν*∗*yvec*⟩⟩⟨*N*⟩ ≺ *P*) = (*xvec* ♯* *M* ∧ *xvec* ♯* ((⟨⟨*ν*∗*yvec*⟩⟩(*N* ≺′ *P*)))
  and *xvec* ♯* (*τ* ≺ *P*) = (*xvec* ♯* *P*)
  and *X* ♯* (*M*⟨⟨*N*⟩⟩ ≺ *P*) = (*X* ♯* *M* ∧ *X* ♯* *N* ∧ *X* ♯* *P*)
  and *X* ♯* (*M*⟨⟨*ν*∗*yvec*⟩⟩⟨*N*⟩ ≺ *P*) = (*X* ♯* *M* ∧ *X* ♯* (⟨⟨*ν*∗*yvec*⟩⟩(*N* ≺′ *P*)))
  and *X* ♯* (*τ* ≺ *P*) = (*X* ♯* *P*)
by(*auto simp add*: *fresh-star-def*)

lemma *residualFreshChainSimp2*[*simp*]:
  fixes *xvec* :: *name list*
  and *X* :: *name set*
  and *M* :: *'a*::*fs-name*
  and *N* :: *'a*
  and *yvec* :: *name list*
  and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

  shows *xvec* ♯* (*RIn M N P*) = (*xvec* ♯* *M* ∧ *xvec* ♯* *N* ∧ *xvec* ♯* *P*)
  and *xvec* ♯* (*ROut M B*) = (*xvec* ♯* *M* ∧ *xvec* ♯* *B*)
  and *xvec* ♯* (*RTau P*) = (*xvec* ♯* *P*)
  and *X* ♯* (*RIn M N P*) = (*X* ♯* *M* ∧ *X* ♯* *N* ∧ *X* ♯* *P*)
  and *X* ♯* (*ROut M B*) = (*X* ♯* *M* ∧ *X* ♯* *B*)
  and *X* ♯* (*RTau P*) = (*X* ♯* *P*)
by(*auto simp add*: *fresh-star-def*)

lemma *freshResidual3*[*dest*]:
  fixes *x* :: *name*
  and *α* :: (*'a*::*fs-name*) *action*
  and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

  assumes *x* ♯ *bn α*
  and *x* ♯ *α* ≺ *P*

  shows *x* ♯ *α* **and** *x* ♯ *P*
using *assms*
by(*nominal-induct rule*: *action.strong-induct*) *auto*

lemma *freshResidualChain3*[*dest*]:
  fixes *xvec* :: *name list*
  and *α* :: (*'a*::*fs-name*) *action*
  and *P* :: (*'a*, *'b*::*fs-name*, *'c*::*fs-name*) *psi*

assumes *xvec* $\sharp*$ $(\alpha \prec P)$
and *xvec* $\sharp*$ *bn* α

shows *xvec* $\sharp*$ α **and** *xvec* $\sharp*$ P
using *assms*
by(*nominal-induct rule*: *action.strong-induct*) *auto*

lemma *freshResidual4*[*dest*]:
fixes x :: *name*
and α :: ('*a*::*fs-name*) *action*
and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

assumes $x \sharp \alpha \prec P$

shows $x \sharp$ *subject* α
using *assms*
by(*nominal-induct rule*: *action.strong-induct*) *auto*

lemma *freshResidualChain4*[*dest*]:
fixes *xvec* :: *name list*
and α :: ('*a*::*fs-name*) *action*
and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*

assumes *xvec* $\sharp*$ $(\alpha \prec P)$

shows *xvec* $\sharp*$ *subject* α
using *assms*
by(*nominal-induct rule*: *action.strong-induct*) *auto*

lemma *alphaOutputResidual*:
fixes M :: '*a*::*fs-name*
and *xvec* :: *name list*
and N :: '*a*
and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
and p :: *name prm*

assumes $(p \cdot xvec) \sharp* N$
and $(p \cdot xvec) \sharp* P$
and *set* $p \subseteq$ *set xvec* $\times$ *set*$(p \cdot xvec)$
and *set xvec* $\subseteq$ *set yvec*

shows $M\langle\!|\nu*yvec|\!\rangle\langle N\rangle \prec P = M\langle\!|\nu*(p \cdot yvec)|\!\rangle\langle(p \cdot N)\rangle \prec (p \cdot P)$
using *assms*
by(*simp add*: *boundOutputChainAlpha''*)

lemmas[*simp del*] = *create-residual.simps*

lemma *residualInject''*:

assumes $bn\ \alpha = bn\ \beta$

shows $(\alpha \prec P = \beta \prec Q) = (\alpha = \beta \wedge P = Q)$
using *assms*
apply(*nominal-induct* α *rule*: *action.strong-induct*)
apply(*auto simp add*: *residual.inject create-residual.simps residualInject' action.inject boundOutput.inject*)
by(*rule-tac* x=$bn\ \beta$ **in** *exI*) *auto*

lemmas *residualInject* = *residual.inject create-residual.simps residualInject' residualInject''*

lemma *bnFreshResidual*[*simp*]:
fixes α :: ('*a*::*fs-name*) *action*

shows $(bn\ \alpha)\ \sharp*\ (\alpha \prec P) = bn\ \alpha\ \sharp*\ (subject\ \alpha)$
by(*nominal-induct* α *rule*: *action.strong-induct*)
(*auto simp add*: *residualFresh fresh-some fresh-star-def*)

lemma *actionCases*[*case-names cInput cOutput cTau*]:
fixes α :: ('*a*::*fs-name*) *action*

assumes $\bigwedge M\ N.\ \alpha = M(\!|N|\!) \Longrightarrow Prop$
and $\bigwedge M\ xvec\ N.\ \alpha = M(\!|\nu * xvec|\!)\langle N\rangle \Longrightarrow Prop$
and $\alpha = \tau \Longrightarrow Prop$

shows *Prop*
using *assms*
by(*nominal-induct* α *rule*: *action.strong-induct*) *auto*

lemma *actionPar1Dest*:
fixes α :: ('*a*::*fs-name*) *action*
and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
and β :: ('*a*::*fs-name*) *action*
and Q :: ('*a*, '*b*, '*c*) *psi*
and R :: ('*a*, '*b*, '*c*) *psi*

assumes $\alpha \prec P = \beta \prec (Q \parallel R)$
and $bn\ \alpha\ \sharp*\ bn\ \beta$

obtains $T\ p$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn\ \beta)$ **and** $P = T \parallel (p \cdot R)$ **and** $\alpha \prec T = \beta \prec Q$
using *assms*
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*drule-tac boundOutputPar1Dest'*) *auto*

lemma *actionPar2Dest*:

fixes α :: ('a::*fs-name*) *action*
and P :: ('a, 'b::*fs-name*, 'c::*fs-name*) *psi*
and β :: ('a::*fs-name*) *action*
and Q :: ('a, 'b, 'c) *psi*
and R :: ('a, 'b, 'c) *psi*

assumes $\alpha \prec P = \beta \prec (Q \parallel R)$
and *bn* $\alpha \sharp* bn\ \beta$

obtains $T\ p$ **where** *set* $p \subseteq set(bn\ \alpha) \times set(bn\ \beta)$ **and** $P = (p \cdot Q) \parallel T$ **and** $\alpha \prec T = \beta \prec R$
using *assms*
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*drule-tac boundOutputPar2Dest'*) *auto*

lemma *actionScopeDest*:
fixes α :: ('a::*fs-name*) *action*
and P :: ('a, 'b::*fs-name*, 'c::*fs-name*) *psi*
fixes β :: ('a::*fs-name*) *action*
and x :: *name*
and Q :: ('a, 'b, 'c) *psi*

assumes $\alpha \prec P = \beta \prec (\!|\nu x|\!) Q$
and $x \sharp bn\ \alpha$
and $x \sharp bn\ \beta$

obtains R **where** $P = (\!|\nu x|\!) R$ **and** $\alpha \prec R = \beta \prec Q$
using *assms*
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*drule-tac boundOutputScopeDest*) *auto*

abbreviation
outputJudge (-⟨-⟩ [*110*, *110*] *110*) **where** $M\langle N\rangle \equiv M(\!|\nu*([])|\!)\langle N\rangle$

declare [[*unify-trace-bound=100*]]

locale *env* = *substPsi substTerm substAssert substCond* +
assertion SCompose' SImp' SBottom' SChanEq'
for *substTerm* :: ('a::*fs-name*) ⇒ *name list* ⇒ 'a::*fs-name list* ⇒ 'a
and *substAssert* :: ('b::*fs-name*) ⇒ *name list* ⇒ 'a::*fs-name list* ⇒ 'b
and *substCond* :: ('c::*fs-name*) ⇒ *name list* ⇒ 'a::*fs-name list* ⇒ 'c
and *SCompose'* :: 'b ⇒ 'b ⇒ 'b
and *SImp'* :: 'b ⇒ 'c ⇒ *bool*
and *SBottom'* :: 'b
and *SChanEq'* :: 'a ⇒ 'a ⇒ 'c
begin
notation *SCompose'* (**infixr** ⊗ *90*)

notation *SImp′* (- ⊢ - [*85*, *85*] *85*)
notation *FrameImp* (- ⊢$_F$ - [*85*, *85*] *85*)
abbreviation
FBottomJudge (⊥$_F$ *90*) **where** ⊥$_F$ ≡ (*FAssert SBottom′*)
notation *SChanEq′* (- ↔ - [*90*, *90*] *90*)
notation *substTerm* (-[-::=-] [*100*, *100*, *100*] *100*)
notation *subs* (-[-::=-] [*100*, *100*, *100*] *100*)
notation *AssertionStatEq* (- ≃ - [*80*, *80*] *80*)
notation *FrameStatEq* (- ≃$_F$ - [*80*, *80*] *80*)
notation *SBottom′* (**1** *190*)
abbreviation *insertAssertion′* (*insertAssertion*) **where** *insertAssertion′* ≡ *assertionAux.insertAssertion* (⊗)

inductive *semantics* :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *residual* ⇒ *bool*
(- ▷ - ⟼ - [*50*, *50*, *50*] *50*)
where
cInput: ⟦Ψ ⊢ *M* ↔ *K*; *distinct xvec*; *set xvec* ⊆ *supp N*; *xvec* ♯* *Tvec*;
length xvec = *length Tvec*;
xvec ♯* Ψ; *xvec* ♯* *M*; *xvec* ♯* *K*⟧ ⟹ Ψ ▷ *M*(⦇λ*xvec *N*⦈).*P* ⟼ *K*(⦇(*N*[*xvec*::=*Tvec*])⦈) ≺ *P*[*xvec*::=*Tvec*]
| *Output*: ⟦Ψ ⊢ *M* ↔ *K*⟧ ⟹ Ψ ▷ *M*⟨*N*⟩.*P* ⟼ *K*⟨*N*⟩ ≺ *P*
| *Case*: ⟦Ψ ▷ *P* ⟼ *Rs*; (φ, *P*) *mem Cs*; Ψ ⊢ φ; *guarded P*⟧ ⟹ Ψ ▷ *Cases Cs* ⟼ *Rs*

| *cPar1*: ⟦(Ψ ⊗ Ψ$_Q$) ▷ *P* ⟼α ≺ *P′*; *extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩; *distinct A*$_Q$;
A$_Q$ ♯* *P*; *A*$_Q$ ♯* *Q*; *A*$_Q$ ♯* Ψ; *A*$_Q$ ♯* α; *A*$_Q$ ♯* *P′*; *distinct*(*bn* α);
bn α ♯* Ψ; *bn* α ♯* Ψ$_Q$; *bn* α ♯* *Q*; *bn* α ♯* *P*; *bn* α ♯* (*subject* α)⟧ ⟹
Ψ ▷ *P* ∥ *Q* ⟼α ≺ (*P′* ∥ *Q*)
| *cPar2*: ⟦(Ψ ⊗ Ψ$_P$) ▷ *Q* ⟼α ≺ *Q′*; *extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩; *distinct A*$_P$;
A$_P$ ♯* *P*; *A*$_P$ ♯* *Q*; *A*$_P$ ♯* Ψ; *A*$_P$ ♯* α; *A*$_P$ ♯* *Q′*; *distinct*(*bn* α);
bn α ♯* Ψ; *bn* α ♯* Ψ$_P$; *bn* α ♯* *P*; *bn* α ♯* *Q*; *bn* α ♯* (*subject* α)⟧ ⟹
Ψ ▷ *P* ∥ *Q* ⟼α ≺ (*P* ∥ *Q′*)
| *cComm1*: ⟦Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼ *M*⦇*N*⦈ ≺ *P′*; *extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩; *distinct A*$_P$;
Ψ ⊗ Ψ$_P$ ▷ *Q* ⟼ *K*⦇ν*xvec⦈⟨*N*⟩ ≺ *Q′*; *extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩; *distinct A*$_Q$;
Ψ ⊗ Ψ$_P$ ⊗ Ψ$_Q$ ⊢ *M* ↔ *K*;
A$_P$ ♯* Ψ; *A*$_P$ ♯* Ψ$_Q$; *A*$_P$ ♯* *P*; *A*$_P$ ♯* *M*; *A*$_P$ ♯* *N*; *A*$_P$ ♯* *P′*;
A$_P$ ♯* *Q*; *A*$_P$ ♯* *Q′*; *A*$_P$ ♯* *A*$_Q$; *A*$_P$ ♯* *xvec*;
A$_Q$ ♯* Ψ; *A*$_Q$ ♯* Ψ$_P$; *A*$_Q$ ♯* *P*; *A*$_Q$ ♯* *N*; *A*$_Q$ ♯* *P′*;
A$_Q$ ♯* *Q*; *A*$_Q$ ♯* *K*; *A*$_Q$ ♯* *Q′*; *A*$_Q$ ♯* *xvec*; *distinct xvec*;
xvec ♯* Ψ; *xvec* ♯* Ψ$_P$; *xvec* ♯* Ψ$_Q$; *xvec* ♯* *P*; *xvec* ♯* *M*;
xvec ♯* *Q*; *xvec* ♯* *K*⟧ ⟹
Ψ ▷ *P* ∥ *Q* ⟼ τ ≺ ⦇ν*xvec⦈(*P′* ∥ *Q′*)
| *cComm2*: ⟦Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼ *M*⦇ν*xvec⦈⟨*N*⟩ ≺ *P′*; *extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩; *distinct A*$_P$;
Ψ ⊗ Ψ$_P$ ▷ *Q* ⟼ *K*⦇*N*⦈ ≺ *Q′*; *extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩; *distinct A*$_Q$;
Ψ ⊗ Ψ$_P$ ⊗ Ψ$_Q$ ⊢ *M* ↔ *K*;

$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$;
$A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$; *distinct xvec*;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* Q$; $xvec \sharp* K]\!] \Longrightarrow$
$\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$

| *cOpen*: $[\![\Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $x \sharp xvec$; $x \sharp yvec$; $x \sharp M$; $x \sharp \Psi$;
distinct xvec; *distinct yvec*;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* yvec$; $yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M]\!] \Longrightarrow$
$\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu * (xvec @ x \# yvec)|\!)\langle N\rangle \prec P'$

| *cScope*: $[\![\Psi \rhd P \longmapsto \alpha \prec P'$; $x \sharp \Psi$; $x \sharp \alpha$; $bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp* (subject\ \alpha)$; $distinct(bn\ \alpha)]\!] \Longrightarrow \Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec ((\!|\nu x|\!)P')$

| *Bang*: $[\![\Psi \rhd P \parallel\ !P \longmapsto Rs$; *guarded* $P]\!] \Longrightarrow \Psi \rhd\ !P \longmapsto Rs$

abbreviation
semanticsBottomJudge (- $\longmapsto$ - [50, 50] 50) **where** $P \longmapsto Rs \equiv \mathbf{1} \rhd P \longmapsto Rs$

equivariance *env.semantics*

nominal-inductive2 *env.semantics*
avoids *cInput*: *set xvec*
| *cPar1*: *set* $A_Q \cup$ *set*$(bn\ \alpha)$
| *cPar2*: *set* $A_P \cup$ *set*$(bn\ \alpha)$
| *cComm1*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cComm2*: *set* $A_P \cup$ *set* $A_Q \cup$ *set xvec*
| *cOpen*: $\{x\} \cup$ *set xvec* $\cup$ *set yvec*
| *cScope*: $\{x\} \cup$ *set*$(bn\ \alpha)$
apply(*auto intro*: *substTerm.subst4Chain subst4Chain simp add*: *abs-fresh residualFresh*)
apply(*force simp add*: *fresh-star-def abs-fresh*)
apply(*simp add*: *boundOutputFresh*)
apply(*simp add*: *boundOutputFreshSet*)
apply(*simp add*: *boundOutputFreshSet*)
by(*simp add*: *fresh-star-def abs-fresh*)

lemma *nilTrans*[*dest*]:
fixes Ψ :: *'b*
and *Rs* :: (*'a*, *'b*, *'c*) *residual*
and *M* :: *'a*
and *xvec* :: *name list*
and *N* :: *'a*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *K* :: *'a*
and *yvec* :: *name list*
and *N'* :: *'a*
and *P'* :: (*'a*, *'b*, *'c*) *psi*

and $CsP :: ('c \times ('a, 'b, 'c)\ psi)\ list$
and $\Psi' :: 'b$

shows $\Psi \rhd \mathbf{0} \longmapsto Rs \Longrightarrow False$
and $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto K(\!|\nu * yvec|\!)\langle N'\rangle \prec P' \Longrightarrow False$
and $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \tau \prec P' \Longrightarrow False$
and $\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|N'|\!) \prec P' \Longrightarrow False$
and $\Psi \rhd M\langle N\rangle.P \longmapsto \tau \prec P' \Longrightarrow False$
and $\Psi \rhd \{\!|\Psi'|\!\} \longmapsto Rs \Longrightarrow False$
apply(*cases rule*: *semantics.cases*) **apply** *auto*
apply(*cases rule*: *semantics.cases*) **apply**(*auto simp add*: *residualInject*)
apply(*cases rule*: *semantics.cases*) **apply**(*auto simp add*: *residualInject*)
apply(*cases rule*: *semantics.cases*) **apply**(*auto simp add*: *residualInject*)
apply(*cases rule*: *semantics.cases*) **apply**(*auto simp add*: *residualInject*)
by(*cases rule*: *semantics.cases*) (*auto simp add*: *residualInject*)

lemma *residualEq*:
fixes $\alpha :: 'a\ action$
and $P :: ('a, 'b, 'c)\ psi$
and $\beta :: 'a\ action$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\alpha \prec P = \beta \prec Q$
and $bn\ \alpha\ \sharp*\ (bn\ \beta)$
and $distinct(bn\ \alpha)$
and $distinct(bn\ \beta)$
and $bn\ \alpha\ \sharp*\ (\alpha \prec P)$
and $bn\ \beta\ \sharp*\ (\beta \prec Q)$

obtains p **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $distinctPerm\ p$ **and** $\beta = p \cdot \alpha$ **and** $Q = p \cdot P$ **and** $bn\ \alpha\ \sharp*\ \beta$ **and** $bn\ \alpha\ \sharp*\ Q$ **and** $bn(p \cdot \alpha)\ \sharp*\ \alpha$ **and** $bn(p \cdot \alpha)\ \sharp*\ P$
using *assms*
proof(*nominal-induct* α *rule*: *action.strong-induct*)
case(*In M N*)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*Out M xvec N*)
thus *?case*
by(*auto simp add*: *residualInject*)
(*drule-tac boundOutputChainEq''*, *auto*)
next
case *Tau*
thus *?case* **by**(*simp add*: *residualInject*)
qed

lemma *semanticsInduct*[*consumes 3*, *case-names cAlpha cInput cOutput cCase cPar1 cPar2 cComm1 cComm2 cOpen cScope cBang*]:
fixes $\Psi :: 'b$

and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*
and *Prop* :: $'d$::*fs-name* $\Rightarrow$ $'b$ $\Rightarrow$ $('a, 'b, 'c)$ *psi* $\Rightarrow$
$'a$ *action* $\Rightarrow$ $('a, 'b, 'c)$ *psi* $\Rightarrow$ *bool*
and C :: $'d$::*fs-name*

assumes $\Psi \rhd P \longmapsto \alpha \prec P'$
and *bn* $\alpha \sharp* (subject\ \alpha)$
and *distinct*(*bn* α)
and *rAlpha*: $\bigwedge \Psi\ P\ \alpha\ P'\ p\ C.\ [\![bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* (subject\ \alpha);$
$bn\ \alpha \sharp* C;\ bn\ \alpha \sharp* (bn(p \cdot \alpha));$
$set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha));\ distinctPerm\ p;$
$(bn(p \cdot \alpha)) \sharp* \alpha;\ (bn(p \cdot \alpha)) \sharp* P';\ Prop\ C\ \Psi\ P\ \alpha\ P'$
$]\!] \Longrightarrow$
$Prop\ C\ \Psi\ P\ (p \cdot \alpha)\ (p \cdot P')$
and *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(K(\!|(N[xvec::=Tvec])|\!))\ (P[xvec::=Tvec])$
and *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)$
$(K\langle N\rangle)\ P$
and *rCase*: $\bigwedge \Psi\ P\ \alpha\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto \alpha \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ \alpha\ P';$
$(\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ \alpha\ P'$
and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct$
$A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ \alpha\ P';$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ A_Q \sharp* C;$
$distinct(bn\ \alpha);\ bn\ \alpha \sharp* Q;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* \Psi_Q;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C]\!]$
$\Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P' \parallel Q)$
and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct$
$A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ \alpha\ Q';$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* C;$
$distinct(bn\ \alpha);\ bn\ \alpha \sharp* Q;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* \Psi_P;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C]\!]$
$\Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P \parallel Q')$
and *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|N|\!))$
$P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$

$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$; $\bigwedge C$. *Prop* C $(\Psi \otimes \Psi_P)$ Q $(K(\!|\nu{*}xvec|\!)\langle N\rangle)$ Q';
extractFrame $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp{*} \Psi$; $A_P \sharp{*} \Psi_Q$; $A_P \sharp{*} P$; $A_P \sharp{*} M$; $A_P \sharp{*} N$; $A_P \sharp{*} P'$;
$A_P \sharp{*} Q$; $A_P \sharp{*} Q'$; $A_P \sharp{*} A_Q$; $A_P \sharp{*} xvec$; $A_Q \sharp{*} \Psi$; $A_Q \sharp{*} \Psi_P$;
$A_Q \sharp{*} P$; $A_Q \sharp{*} N$; $A_Q \sharp{*} P'$; $A_Q \sharp{*} Q$; $A_Q \sharp{*} K$; $A_Q \sharp{*} Q'$; *distinct xvec*;
$A_Q \sharp{*} xvec$; $xvec \sharp{*} \Psi$; $xvec \sharp{*} \Psi_P$; $xvec \sharp{*} \Psi_Q$; $xvec \sharp{*} P$; $xvec \sharp{*} M$;
$xvec \sharp{*} Q$; $xvec \sharp{*} K$; $A_P \sharp{*} C$; $A_Q \sharp{*} C$; $xvec \sharp{*} C$⟧ ⟹
Prop C Ψ $(P \parallel Q)$ (τ) $((\!|\nu{*}xvec|\!)(P' \parallel Q'))$

and *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C$.
⟦$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$; $\bigwedge C$. *Prop* C $(\Psi \otimes \Psi_Q)$ P $(M(\!|\nu{*}xvec|\!)\langle N\rangle)$ P';
extractFrame $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; $\bigwedge C$. *Prop* C $(\Psi \otimes \Psi_P)$ Q $(K(\!|N|\!))$ Q';
extractFrame $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp{*} \Psi$; $A_P \sharp{*} \Psi_Q$; $A_P \sharp{*} P$; $A_P \sharp{*} M$; $A_P \sharp{*} N$; $A_P \sharp{*} P'$;
$A_P \sharp{*} Q$; $A_P \sharp{*} Q'$; $A_P \sharp{*} A_Q$; $A_P \sharp{*} xvec$; $A_Q \sharp{*} \Psi$; $A_Q \sharp{*} \Psi_P$;
$A_Q \sharp{*} P$; $A_Q \sharp{*} N$; $A_Q \sharp{*} P'$; $A_Q \sharp{*} Q$; $A_Q \sharp{*} K$; $A_Q \sharp{*} Q'$; *distinct xvec*;
$A_Q \sharp{*} xvec$; $xvec \sharp{*} \Psi$; $xvec \sharp{*} \Psi_P$; $xvec \sharp{*} \Psi_Q$; $xvec \sharp{*} P$; $xvec \sharp{*} M$;
$xvec \sharp{*} Q$; $xvec \sharp{*} K$; $A_P \sharp{*} C$; $A_Q \sharp{*} C$; $xvec \sharp{*} C$⟧ ⟹
Prop C Ψ $(P \parallel Q)$ (τ) $((\!|\nu{*}xvec|\!)(P' \parallel Q'))$

and *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C$.
⟦$\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P'$; $x \in supp\ N$; $\bigwedge C$. *Prop* C Ψ P $(M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle)$ P';
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp yvec$; $xvec \sharp{*} \Psi$; $xvec \sharp{*} P$; $xvec \sharp{*} M$; *distinct xvec*; *distinct yvec*;
$yvec \sharp{*} \Psi$; $yvec \sharp{*} P$; $yvec \sharp{*} M$; $yvec \sharp{*} C$; $x \sharp C$; $xvec \sharp{*} C$⟧ ⟹
Prop C Ψ $((\!|\nu x|\!)P)$ $(M(\!|\nu{*}(xvec@x\#yvec)|\!)\langle N\rangle)$ P'

and *rScope*: $\bigwedge \Psi\ P\ \alpha\ P'\ x\ C$.
⟦$\Psi \rhd P \longmapsto \alpha \prec P'$; $\bigwedge C$. *Prop* C Ψ P α P';
$x \sharp \Psi$; $x \sharp \alpha$; $bn\ \alpha \sharp{*} \Psi$;
$bn\ \alpha \sharp{*} P$; $bn\ \alpha \sharp{*} (subject\ \alpha)$; $x \sharp C$; $bn\ \alpha \sharp{*} C$; *distinct*$(bn\ \alpha)$⟧ ⟹
Prop C Ψ $((\!|\nu x|\!)P)$ α $((\!|\nu x|\!)P')$

and *rBang*: $\bigwedge \Psi\ P\ \alpha\ P'\ C$.
⟦$\Psi \rhd P \parallel !P \longmapsto \alpha \prec P'$; *guarded* P; $\bigwedge C$. *Prop* C Ψ $(P \parallel !P)$ α P'⟧ ⟹
Prop C Ψ $(!P)$ α P'

shows *Prop* C Ψ P α P'
using ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹$bn\ \alpha \sharp{*} (subject\ \alpha)$› ‹*distinct*$(bn\ \alpha)$›
proof(*nominal-induct x3*==$\alpha \prec P'$ *avoiding*: α C *arbitrary*: P' *rule*: *seman-*

tics.strong-induct)
case(*cInput* Ψ M K *xvec* N *Tvec* P α C P')
thus *?case* **by**(*force intro*: *rInput simp add*: *residualInject*)
next
case(*Output* Ψ M K N P α C P')
thus *?case* **by**(*force intro*: *rOutput simp add*: *residualInject*)
next
case(*Case* Ψ P *Rs* φ *Cs* α C)
thus *?case* **by**(*auto intro*: *rCase*)
next
case(*cPar1* Ψ Ψ_Q P α P' Q A_Q α' C P'')
note ‹$\alpha \prec (P' \parallel Q) = \alpha' \prec P''$›
moreover from ‹*bn* $\alpha \sharp* \alpha'$› **have** *bn* $\alpha \sharp* (bn\ \alpha')$ **by** *auto*
moreover note ‹*distinct* (*bn* α)› ‹*distinct*(*bn* α')›
moreover from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha' \sharp*$ *subject* α'›
have *bn* $\alpha \sharp* (\alpha \prec P' \parallel Q)$ **and** *bn* $\alpha' \sharp* (\alpha' \prec P'')$ **by** *simp+*
ultimately obtain p **where** S: (*set* p) $\subseteq$ (*set*(*bn* α)) $\times$ (*set*(*bn*($p \cdot \alpha$))) **and** *distinctPerm* p
and αEq: $\alpha' = p \cdot \alpha$ **and** $P'eq$: $P'' = p \cdot (P' \parallel Q)$ **and** (*bn*($p \cdot \alpha$)) $\sharp* \alpha$
and (*bn*($p \cdot \alpha$)) $\sharp* (P' \parallel Q)$
by(*rule residualEq*)

note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹*distinct* A_Q›
moreover from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinct*(*bn* α)›
have $\bigwedge C$. *Prop* C ($\Psi \otimes \Psi_Q$) P α P' **by**(*rule-tac cPar1*) *auto*
moreover note ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* P'$› ‹$A_Q \sharp* C$›
‹*bn* $\alpha \sharp* Q$› ‹*distinct*(*bn* α)› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* \Psi_Q$› ‹*bn* $\alpha \sharp* P$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha \sharp* C$›
ultimately have *Prop* C Ψ ($P \parallel Q$) α ($P' \parallel Q$)
by(*rule-tac rPar1*)

with ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$› ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha \sharp* C$› ‹*bn* $\alpha \sharp*$ *bn* α'› S ‹*distinctPerm* p› ‹*bn*($p \cdot \alpha$) $\sharp* \alpha$› ‹*bn*($p \cdot \alpha$) $\sharp* (P' \parallel Q)$› ‹$A_Q \sharp* C$›
have *Prop* C Ψ ($P \parallel Q$) ($p \cdot \alpha$) ($p \cdot (P' \parallel Q)$)
by(*rule-tac rAlpha*) *auto*
with αEq $P'eq$ ‹*distinctPerm* p› **show** *?case* **by** *simp*
next
case(*cPar2* Ψ Ψ_P Q α Q' P A_P α' C Q'')
note ‹$\alpha \prec (P \parallel Q') = \alpha' \prec Q''$›
moreover from ‹*bn* $\alpha \sharp* \alpha'$› **have** *bn* $\alpha \sharp* (bn\ \alpha')$ **by** *auto*
moreover note ‹*distinct* (*bn* α)› ‹*distinct*(*bn* α')›
moreover from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha' \sharp*$ *subject* α'›
have *bn* $\alpha \sharp* (\alpha \prec P \parallel Q')$ **and** *bn* $\alpha' \sharp* (\alpha' \prec Q'')$ **by** *simp+*
ultimately obtain p **where** S: (*set* p) $\subseteq$ (*set*(*bn* α)) $\times$ (*set*(*bn*($p \cdot \alpha$))) **and** *distinctPerm* p
and αEq: $\alpha' = p \cdot \alpha$ **and** $Q'eq$: $Q'' = p \cdot (P \parallel Q')$ **and** (*bn*($p \cdot \alpha$)) $\sharp* \alpha$

and $(bn(p \cdot \alpha)) \sharp* (P \parallel Q')$
by(*rule residualEq*)

note ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹*distinct* A_P›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹*distinct*$(bn\ \alpha)$›
have $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ \alpha\ Q'$ **by**(*rule-tac cPar2*) *auto*

moreover note ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* Q'$› ‹$A_P \sharp* C$›
‹$bn\ \alpha \sharp* Q$› ‹*distinct*$(bn\ \alpha)$› ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$›
ultimately have $Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P \parallel Q')$
by(*rule-tac rPar2*)
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$bn\ \alpha \sharp* (bn\ \alpha')$› S ‹*distinctPerm p*› ‹$bn(p \cdot \alpha) \sharp* \alpha$› ‹$bn(p \cdot \alpha) \sharp* (P \parallel Q')$›
have $Prop\ C\ \Psi\ (P \parallel Q)\ (p \cdot \alpha)\ (p \cdot (P \parallel Q'))$
by(*rule-tac rAlpha*) *auto*
with *αEq Q'eq* ‹*distinctPerm p*› **show** *?case* **by** *simp*
next
case(*cComm1* $\Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ \alpha\ C\ P''$)
hence $Prop\ C\ \Psi\ (P \parallel Q)\ (\tau)\ (([\![\nu* xvec]\!])(P' \parallel Q'))$
by(*rule-tac rComm1*) (*assumption* | *simp*)+
thus *?case* **using** ‹$\tau \prec ([\![\nu* xvec]\!])(P' \parallel Q') = \alpha \prec P''$›
by(*simp add*: *residualInject*)
next
case(*cComm2* $\Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ \alpha\ C\ P''$)
hence $Prop\ C\ \Psi\ (P \parallel Q)\ (\tau)\ (([\![\nu* xvec]\!])(P' \parallel Q'))$
by(*rule-tac rComm2*) (*assumption* | *simp*)+
thus *?case* **using** ‹$\tau \prec ([\![\nu* xvec]\!])(P' \parallel Q') = \alpha \prec P''$›
by(*simp add*: *residualInject*)
next
case(*cOpen* $\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ \alpha\ C\ P''$)
note ‹$M([\![\nu*(xvec@x\#yvec)]\!])\langle N \rangle \prec P' = \alpha \prec P''$›
moreover from ‹$xvec \sharp* \alpha$› ‹$x \sharp \alpha$› ‹$yvec \sharp* \alpha$› **have** $(xvec@x\#yvec) \sharp* (bn\ \alpha)$
by *auto*
moreover from ‹$xvec \sharp* yvec$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹*distinct xvec*› ‹*distinct yvec*›
have *distinct*$(xvec@x\#yvec)$
by(*auto simp add*: *fresh-star-def*) (*simp add*: *fresh-def name-list-supp*)
moreover note ‹*distinct*$(bn\ \alpha)$›
moreover from ‹$xvec \sharp* M$› ‹$x \sharp M$› ‹$yvec \sharp* M$› **have** $(xvec@x\#yvec) \sharp* M$
by *auto*
hence $(xvec@x\#yvec) \sharp* (M([\![\nu*(xvec@x\#yvec)]\!])\langle N \rangle \prec P')$ **by** *auto*
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› **have** $bn\ \alpha \sharp* (\alpha \prec P'')$ **by** *simp*
ultimately obtain p **where** S: $(set\ p) \subseteq (set(xvec@x\#yvec)) \times (set(p \cdot (xvec@x\#yvec)))$
and *distinctPerm p*
and *αeq*: $\alpha = (p \cdot M)([\![\nu*(p \cdot (xvec@x\#yvec))]\!])\langle (p \cdot N) \rangle$ **and** *P'eq*: $P'' = (p \cdot P')$
and A: $(xvec@x\#yvec) \sharp* ((p \cdot M)([\![\nu*(p \cdot (xvec@x\#yvec))]\!])\langle (p \cdot N) \rangle)$
and B: $(p \cdot (xvec@x\#yvec)) \sharp* (M([\![\nu*(xvec@x\#yvec)]\!])\langle N \rangle)$

```
          and C: (p • (xvec@x#yvec)) ♯* P'
    by(rule-tac residualEq) (assumption | simp)+

  note ‹Ψ ▷ P ⟼M⦇νx⦈⟨N⟩ ≺ P'› ‹x ∈ (supp N)›

  moreover {
    fix C
    from ‹xvec ♯* M› ‹yvec ♯* M› have (xvec@yvec) ♯* M by simp
   moreover from ‹distinct xvec› ‹distinct yvec› ‹xvec ♯* yvec› have distinct(xvec@yvec)
      by auto (simp add: fresh-star-def name-list-supp fresh-def)
    ultimately have Prop C Ψ P (M⦇νx⦈⟨N⟩) P' by(rule-tac cOpen)
auto
  }

  moreover note ‹x ♯ Ψ› ‹x ♯ M› ‹x ♯ xvec› ‹x ♯ yvec› ‹xvec ♯* Ψ› ‹xvec ♯* P›
‹xvec ♯* M›
                ‹yvec ♯* Ψ› ‹yvec ♯* P› ‹yvec ♯* M› ‹yvec ♯* C› ‹x ♯ C› ‹xvec ♯* C›
‹distinct xvec› ‹distinct yvec›
  ultimately have Prop C Ψ (⦇νx⦈P) (M⦇νx⦈⟨N⟩) P'
    by(rule-tac rOpen)

  with ‹xvec ♯* Ψ› ‹yvec ♯* Ψ› ‹xvec ♯* P› ‹yvec ♯* P› ‹xvec ♯* M› ‹yvec ♯* M›
       ‹yvec ♯* C›  S ‹distinctPerm p› ‹x ♯ C› ‹xvec ♯* C›
       ‹x ♯ Ψ› ‹x ♯ M› ‹x ♯ xvec› ‹x ♯ yvec› A B C
  have Prop C Ψ (⦇νx⦈P) (p • (M⦇νx⦈⟨N⟩)) (p • P')
    apply(rule-tac α=M⦇νx⦈⟨N⟩ in rAlpha)
    apply(assumption | simp)+
    apply(fastforce simp add: fresh-star-def abs-fresh)
    by(assumption | simp)+
  with αeq P'eq show ?case by simp
next
  case(cScope Ψ P α P' x α' C P'')
  note ‹α ≺ (⦇νx⦈P') = α' ≺ P''›
  moreover from ‹bn α ♯* α'› have bn α ♯* (bn α') by auto
  moreover note ‹distinct (bn α)› ‹distinct(bn α')›
  moreover from ‹bn α ♯* subject α› ‹bn α' ♯* subject α'›
  have bn α ♯* (α ≺ ⦇νx⦈P') and bn α' ♯* (α' ≺ P'') by simp+
  ultimately obtain p where S: (set p) ⊆ (set(bn α)) × (set(bn(p • α))) and
distinctPerm p
                       and αEq: α' = p • α and P'eq: P'' = p • (⦇νx⦈P') and (bn(p
• α)) ♯* α
                       and (bn(p • α)) ♯* (⦇νx⦈P')
    by(rule residualEq)

  note ‹Ψ ▷ P ⟼α ≺ P'›
  moreover from ‹bn α ♯* subject α› ‹distinct(bn α)›
  have ⋀C. Prop C Ψ P α P' by(rule-tac cScope) auto

  moreover note ‹x ♯ Ψ› ‹x ♯ α› ‹bn α ♯* Ψ› ‹bn α ♯* P› ‹bn α ♯* subject α›
```

$\langle x \sharp C\rangle$ $\langle bn\ \alpha \sharp* C\rangle$ $\langle distinct(bn\ \alpha)\rangle$
ultimately have *Prop C* Ψ $((\!|\nu x|\!)P)$ α $((\!|\nu x|\!)P')$
by(*rule rScope*)
with $\langle bn\ \alpha \sharp* \Psi\rangle$ $\langle bn\ \alpha \sharp* P\rangle$ $\langle x \sharp \alpha\rangle$ $\langle bn\ \alpha \sharp* subject\ \alpha\rangle$ $\langle bn\ \alpha \sharp* C\rangle$ $\langle bn\ \alpha \sharp* (bn\ \alpha')\rangle$ *S* $\langle distinctPerm\ p\rangle$ $\langle bn(p \cdot \alpha) \sharp* \alpha\rangle$ $\langle bn(p \cdot \alpha) \sharp* ((\!|\nu x|\!)P')\rangle$
have *Prop C* Ψ $((\!|\nu x|\!)P)$ $(p \cdot \alpha)$ $(p \cdot ((\!|\nu x|\!)P'))$
by(*rule-tac rAlpha*) *simp+*
with *αEq P'eq* $\langle distinctPerm\ p\rangle$ **show** *?case* **by** *simp*
next
case(*Bang* Ψ *P Rs* α *C*)
thus *?case* **by**(*rule-tac rBang*) *auto*
qed

lemma *outputInduct*[*consumes 1*, *case-names cOutput cCase cPar1 cPar2 cOpen cScope cBang*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *M* :: *'a*
and *B* :: (*'a*, *'b*, *'c*) *boundOutput*
and *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
'a ⇒ (*'a*, *'b*, *'c*) *boundOutput* ⇒ *bool*
and *C* :: *'d*::*fs-name*

assumes $\Psi \rhd P \longmapsto ROut\ M\ B$
and *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)\ K\ (N \prec' P)$
and *rCase*: $\bigwedge \Psi\ P\ M\ B\ \varphi\ Cs\ C.$
$[\![\Psi \rhd P \longmapsto (ROut\ M\ B);\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ B;\ (\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ M\ B$
and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ \ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ ((\!|\nu * xvec|\!)N \prec' P');$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;$
$A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* C;\ xvec \sharp* Q;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!)N \prec' (P' \parallel Q))$
and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ \ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu * xvec|\!)N \prec' Q');$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;$
$A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* C;\ xvec \sharp* P;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* Q;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu * xvec|\!)N \prec' (P \parallel Q'))$
and *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu * (xvec @ yvec)|\!)N \prec' P');$

$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp yvec$; $xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* yvec$; $yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $yvec \sharp* C$; $x \sharp C$;
$xvec \sharp* C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu*(xvec@x\#yvec)|\!)N \prec' P')$

and *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$\llbracket \Psi \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'; \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu*xvec|\!)N \prec' P')$;
$x \sharp \Psi$; $x \sharp M$; $x \sharp xvec$; $x \sharp N$; $xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$;
$x \sharp C$; $xvec \sharp* C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu*xvec|\!)N \prec' (\!|\nu x|\!)P')$

and *rBang*: $\bigwedge \Psi\ P\ M\ B\ C.$
$\llbracket \Psi \rhd P \parallel\ !P \longmapsto (ROut\ M\ B);\ guarded\ P; \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ M\ B\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ B$

shows $Prop\ C\ \Psi\ P\ M\ B$
using ‹$\Psi \rhd P \longmapsto (ROut\ M\ B)$›
proof(*nominal-induct* $\Psi\ P\ Rs==(ROut\ M\ B)$ *avoiding*: C *arbitrary*: B *rule*: *semantics.strong-induct*)
case(*cInput* $\Psi\ M\ K\ xvec\ N\ Tvec\ P\ C$)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*Output* $\Psi\ M\ K\ N\ P\ C$)
thus *?case* **by**(*force simp add*: *residualInject intro*: *rOutput*)
next
case(*Case* $\Psi\ P\ Rs\ \varphi\ Cs\ C$)
thus *?case* **by**(*force intro*: *rCase*)
next
case(*cPar1* $\Psi\ \Psi_Q\ P\ \alpha\ P'\ Q\ A_Q\ C$)
thus *?case* **by**(*force intro*: *rPar1 simp add*: *residualInject*)
next
case(*cPar2* $\Psi\ \Psi_P\ Q\ \alpha\ Q'\ P\ A_P\ C$)
thus *?case* **by**(*force intro*: *rPar2 simp add*: *residualInject*)
next
case *cComm1*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cComm2*
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cOpen* $\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C\ B$)
thus *?case* **by**(*force intro*: *rOpen simp add*: *residualInject*)
next
case(*cScope* $\Psi\ P\ M\ \alpha\ P'\ x\ C$)
thus *?case* **by**(*force intro*: *rScope simp add*: *residualInject*)
next
case(*Bang* $\Psi\ P\ Rs\ C$)
thus *?case* **by**(*force intro*: *rBang*)
qed

lemma *boundOutputBindObject*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and M :: $'a$
  and *yvec* :: *name list*
  and N :: $'a$
  and P' :: $('a, 'b, 'c)$ *psi*
  and y :: *name*

  assumes $\Psi \rhd P \longmapsto \alpha \prec P'$
  and *bn* $\alpha \sharp* subject\ \alpha$
  and *distinct*(*bn* α)
  and $y \in set(bn\ \alpha)$

  shows $y \in supp(object\ \alpha)$
using *assms*
proof(*nominal-induct avoiding*: P' *arbitrary*: y *rule*: *semanticsInduct*)
  case(*cAlpha* Ψ P α P' p P'' y)
  from ‹$y \in set(bn(p \cdot \alpha))$› **have** $(p \cdot y) \in (p \cdot set(bn(p \cdot \alpha)))$
    by(*rule pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
  hence $(p \cdot y) \in set(bn\ \alpha)$ **using** ‹*distinctPerm p*›
    by(*simp add*: *eqvts*)
  hence $(p \cdot y) \in supp(object\ \alpha)$ **by**(*rule cAlpha*)
  hence $(p \cdot p \cdot y) \in (p \cdot supp(object\ \alpha))$
    by(*rule pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
  thus *?case* **using** ‹*distinctPerm p*›
    by(*simp add*: *eqvts*)
next
  case *cInput*
  thus *?case* **by**(*simp add*: *supp-list-nil*)
next
  case *cOutput*
  thus *?case* **by**(*simp add*: *supp-list-nil*)
next
  case *cCase*
  thus *?case* **by** *simp*
next
  case *cPar1*
  thus *?case* **by** *simp*
next
  case *cPar2*
  thus *?case* **by** *simp*
next
  case *cComm1*
  thus *?case* **by**(*simp add*: *supp-list-nil*)
next
  case *cComm2*
  thus *?case* **by**(*simp add*: *supp-list-nil*)
next

case *cOpen*
thus *?case* **by**(*auto simp add*: *supp-list-cons supp-list-append supp-atm supp-some*)
next
case *cScope*
thus *?case* **by** *simp*
next
case *cBang*
thus *?case* **by** *simp*
qed

lemma *alphaBoundOutputChain'*:
fixes *yvec* :: *name list*
and *xvec* :: *name list*
and *B* :: (*'a*, *'b*, *'c*) *boundOutput*

assumes *length xvec* = *length yvec*
and *yvec* ♯* *B*
and *yvec* ♯* *xvec*
and *distinct yvec*

shows (|*ν***xvec*|)*B* = (|*ν***yvec*|)([*xvec yvec*] •$_v$ *B*)
using *assms*
proof(*induct rule*: *composePermInduct*)
case *cBase*
show *?case* **by** *simp*
next
case(*cStep x xvec y yvec*)
thus *?case*
apply *auto*
by(*subst alphaBoundOutput*[*of y*]) (*auto simp add*: *eqvts*)
qed

lemma *alphaBoundOutputChain''*:
fixes *yvec* :: *name list*
and *xvec* :: *name list*
and *N* :: *'a*
and *P* :: (*'a*, *'b*, *'c*) *psi*

assumes *length xvec* = *length yvec*
and *yvec* ♯* *N*
and *yvec* ♯* *P*
and *yvec* ♯* *xvec*
and *distinct yvec*

shows (|*ν***xvec*|)(*N* ≺' *P*) = (|*ν***yvec*|)(([*xvec yvec*] •$_v$ *N*) ≺' ([*xvec yvec*] •$_v$ *P*))
proof −
from *assms* **have** (|*ν***xvec*|)(*N* ≺' *P*) = (|*ν***yvec*|)([*xvec yvec*] •$_v$ (*N* ≺' *P*))
by(*simp add*: *alphaBoundOutputChain'*)
thus *?thesis* **by** *simp*

qed

lemma *alphaDistinct*:
fixes *xvec* :: *name list*
and *N* :: *'a*
and *P* :: *('a, 'b, 'c) psi*
and *yvec* :: *name list*
and *M* :: *'a*
and *Q* :: *('a, 'b, 'c) psi*

assumes $\alpha \prec P = \beta \prec Q$
and *distinct*(*bn* α)
and $\bigwedge x.\ x \in set(bn\ \alpha) \Longrightarrow x \in supp(object\ \alpha)$
and *bn* α $\sharp*$ *bn* β
and *bn* α $\sharp*$ (*object* β)
and *bn* α $\sharp*$ *Q*

shows *distinct*(*bn* β)
using *assms*
proof(*rule-tac actionCases*[**where** α=α], *auto simp add*: *residualInject supp-some*)
fix *xvec M yvec N*
assume *Eq*: $(\!|\nu*xvec|\!)N \prec' P = (\!|\nu*yvec|\!)M \prec' Q$
assume *distinct xvec* **and** *xvec* $\sharp*$ *M* **and** *xvec* $\sharp*$ *yvec* **and** *xvec* $\sharp*$ *Q*
assume *Mem*: $\bigwedge x.\ x \in set\ xvec \Longrightarrow x \in (supp\ N)$
show *distinct yvec*
proof −
from *Eq* **have** *length xvec* = *length yvec*
by(*rule boundOutputChainEqLength*)
with *Eq* ‹*distinct xvec*› ‹*xvec* $\sharp*$ *yvec*› ‹*xvec* $\sharp*$ *M*› ‹*xvec* $\sharp*$ *Q*› *Mem* **show** *?thesis*
proof(*induct n*==*length xvec arbitrary*: *xvec yvec M Q rule*: *nat.induct*)
case(*zero xvec yvec M Q*)
thus *?case* **by** *simp*
next
case(*Suc n xvec yvec M Q*)
have *L*: *length xvec* = *length yvec* **and** *Suc n* = *length xvec* **by** *fact+*
then obtain *x xvec' y yvec'* **where** *xEq*: *xvec* = *x#xvec'* **and** *yEq*: *yvec* = *y#yvec'*
and *L'*: *length xvec'* = *length yvec'*
by(*cases xvec*, *auto*, *cases yvec*, *auto*)
have *xvecFreshyvec*: *xvec* $\sharp*$ *yvec* **and** *xvecDist*: *distinct xvec* **by** *fact+*
with *xEq yEq* **have** *xineqy*: $x \neq y$ **and** *xvec'Freshyvec'*: *xvec'* $\sharp*$ *yvec'*
and *xvec'Dist*: *distinct xvec'* **and** *xFreshxvec'*: *x* $\sharp$ *xvec'*
and *xFreshyvec'*: *x* $\sharp$ *yvec'* **and** *yFreshxvec'*: *y* $\sharp$ *xvec'*
by *auto*
have *Eq*: $(\!|\nu*xvec|\!)N \prec' P = (\!|\nu*yvec|\!)M \prec' Q$ **by** *fact*
with *xEq yEq xineqy* **have** *Eq'*: $(\!|\nu*xvec'|\!)N \prec' P = (\!|\nu*([(x, y)] \cdot yvec')|\!)([(x, y)] \cdot M) \prec' ([(x, y)] \cdot Q)$
by(*simp add*: *boundOutput.inject alpha eqvts*)
moreover have *Mem*:$\bigwedge x.\ x \in set\ xvec \Longrightarrow x \in supp\ N$ **by** *fact*

with *xEq* **have** $\bigwedge x.\ x \in set\ xvec' \Longrightarrow x \in supp\ N$ **by** *simp*
moreover have $xvec \sharp* M$ **by** *fact*
with *xEq xFreshxvec' yFreshxvec'* **have** $xvec' \sharp* ([(x,\ y)] \cdot M)$ **by** *simp*
moreover have *xvecFreshQ*: $xvec \sharp* Q$ **by** *fact*
with *xEq xFreshxvec' yFreshxvec'* **have** $xvec' \sharp* ([(x,\ y)] \cdot Q)$ **by** *simp*
moreover have $Suc\ n = length\ xvec$ **by** *fact*
with *xEq* **have** $n = length\ xvec'$ **by** *simp*
moreover from *xvec'Freshyvec' xFreshxvec' yFreshxvec'* **have** $xvec' \sharp* ([(x,\ y)] \cdot yvec')$
by *simp*
moreover from *L'* **have** $length\ xvec' = length([(x,\ y)] \cdot yvec')$ **by** *simp*
ultimately have $distinct([(x,\ y)] \cdot yvec')$ **using** *xvec'Dist*
by(*rule-tac Suc*) (*assumption* | *simp*)+
hence *distinct yvec'* **by** *simp*
from *Mem xEq* **have** *xSuppN*: $x \in supp\ N$ **by** *simp*
from *L* ‹*distinct xvec*› ‹*xvec* $\sharp*$ *yvec*› ‹*xvec* $\sharp*$ *M*› ‹*xvec* $\sharp*$ *Q*›
have $(\!|\nu * yvec|\!)M \prec' Q = (\!|\nu * xvec|\!)([yvec\ xvec] \cdot_v M) \prec' ([yvec\ xvec] \cdot_v Q)$
by(*simp add*: *alphaBoundOutputChain''*)
with *Eq* **have** $N = [yvec\ xvec] \cdot_v M$ **by** *simp*
with *xEq yEq* **have** $N = [(y,\ x)] \cdot [yvec'\ xvec'] \cdot_v M$
by *simp*
with *xSuppN* **have** *ySuppM*: $y \in supp([yvec'\ xvec'] \cdot_v M)$
by(*drule-tac pi*=$[(x,\ y)]$ **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
(*simp add*: *calc-atm eqvts name-swap*)
have $y \sharp yvec'$
proof(*simp add*: *fresh-def*, *rule notI*)
assume $y \in supp\ yvec'$
hence *y mem yvec'*
by(*induct yvec'*) (*auto simp add*: *supp-list-nil supp-list-cons supp-atm*)
moreover from ‹*xvec* $\sharp*$ *M*› *xEq xFreshxvec'* **have** $xvec' \sharp* M$ **by** *simp*
ultimately have $y \sharp [yvec'\ xvec'] \cdot_v M$ **using** *L' xvec'Freshyvec' xvec'Dist*
by(*force intro*: *freshChainPerm*)
with *ySuppM* **show** *False* **by**(*simp add*: *fresh-def*)
qed
with ‹*distinct yvec'*› *yEq* **show** *?case* **by** *simp*
qed
qed
qed

lemma *boundOutputDistinct*:
fixes Ψ :: *'b*
and *P* :: *('a, 'b, 'c) psi*
and α :: *'a action*
and *P'* :: *('a, 'b, 'c) psi*

assumes $\Psi \rhd P \longmapsto \alpha \prec P'$

shows $distinct(bn\ \alpha)$
using *assms*

proof(*nominal-induct* Ψ P *x3*==$\alpha \prec P'$ *avoiding*: α P' *rule*: *semantics.strong-induct*)
case *cInput*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *Output*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *Case*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cPar1*
thus *?case* **by**(*force intro*: *alphaDistinct boundOutputBindObject*)
next
case *cPar2*
thus *?case* **by**(*force intro*: *alphaDistinct boundOutputBindObject*)
next
case *cComm1*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cComm2*
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cOpen* Ψ P M *xvec yvec* N P' x α P'')
note ‹$M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle \prec P' = \alpha \prec P''$›
moreover from ‹*xvec* $\sharp*$ *yvec*› ‹$x \sharp$ *xvec*› ‹$x \sharp$ *yvec*› ‹*distinct xvec*› ‹*distinct yvec*›
have *distinct*(*bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$))
by *auto* (*simp add*: *fresh-star-def fresh-def name-list-supp*)
moreover {
fix *y*
from ‹$\Psi \rhd P \longmapsto M\langle\!|\nu*(xvec@yvec)|\!\rangle\langle N\rangle \prec P'$› ‹$x \in$ *supp* N› ‹$x \sharp$ *xvec*› ‹$x \sharp$ *yvec*› ‹$x \sharp$ M› ‹$x \sharp \Psi$› ‹*distinct xvec*› ‹*distinct yvec*› ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ P› ‹*xvec* $\sharp*$ M› ‹*xvec* $\sharp*$ *yvec*› ‹*yvec* $\sharp*$ Ψ› ‹*yvec* $\sharp*$ P› ‹*yvec* $\sharp*$ M›
have $\Psi \rhd (\!|\nu x|\!)P \longmapsto M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle \prec P'$ **by**(*rule semantics.cOpen*)
moreover moreover from ‹*xvec* $\sharp*$ M› ‹$x \sharp$ M› ‹*yvec* $\sharp*$ M›
have *bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$) $\sharp*$ (*subject*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$))
by *simp*
moreover note ‹*distinct*(*bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$))›
moreover assume $y \in$ *set*(*bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$))

ultimately have $y \in$ *supp*(*object*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$))
by(*rule-tac boundOutputBindObject*)
}
moreover from ‹*xvec* $\sharp*$ α› ‹$x \sharp \alpha$› ‹*yvec* $\sharp*$ α›
have *bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$) $\sharp*$ *bn* α **and** *bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$) $\sharp*$ *object* α **by** *simp+*
moreover from ‹*xvec* $\sharp*$ P''› ‹$x \sharp P''$› ‹*yvec* $\sharp*$ P''›
have *bn*($M\langle\!|\nu*(xvec@x\#yvec)|\!\rangle\langle N\rangle$) $\sharp*$ P'' **by** *simp*
ultimately show *?case* **by**(*rule alphaDistinct*)
next

case *cScope*
thus *?case*
by(*rule-tac alphaDistinct*, *auto*) (*rule-tac boundOutputBindObject*, *auto*)
next
case *Bang*
thus *?case* **by** *simp*
qed

lemma *inputDistinct*:
fixes Ψ :: *'b*
and *M* :: *'a*
and *xvec* :: *name list*
and *N* :: *'a*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rs* :: (*'a*, *'b*, *'c*) *residual*

assumes $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto Rs$

shows *distinct xvec*
using *assms*
by(*nominal-induct* Ψ $P{=}{=}M(\!|\lambda * xvec\ N|\!).P$ *Rs avoiding*: *xvec N P rule*: *semantics.strong-induct*)
(*auto simp add*: *psi.inject intro*: *alphaInputDistinct*)

lemma *outputInduct'*[*consumes 2*, *case-names cAlpha cOutput cCase cPar1 cPar2 cOpen cScope cBang*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *M* :: *'a*
and *yvec* :: *name list*
and *N* :: *'a*
and *P'* :: (*'a*, *'b*, *'c*) *psi*
and *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
'a ⇒ *name list* ⇒ *'a* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool*
and *C* :: *'d*::*fs-name*

assumes $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *xvec* $\sharp*$ *M*
and *rAlpha*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ p\ C.$ $[\![xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* C;\ xvec \sharp* (p \cdot xvec);$
set p ⊆ *set xvec* × *set*(*p* · *xvec*); *distinctPerm p*;
(*p* · *xvec*) $\sharp*$ *N*; (*p* · *xvec*) $\sharp*$ *P'*; *Prop C* Ψ *P M xvec N P'*$]\!] \Longrightarrow$
Prop C Ψ *P M* (*p* · *xvec*) (*p* · *N*) (*p* · *P'*)
and *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ [\![\Psi \vdash M \leftrightarrow K]\!] \Longrightarrow$ *Prop C* Ψ ($M\langle N\rangle.P$) *K* ([]) *N P*
and *rCase*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ \varphi\ Cs\ C.$ $[\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';$ $\bigwedge C.$ *Prop C* Ψ *P M xvec N P'*; (*φ*, *P*) *mem Cs*; $\Psi \vdash \varphi$; *guarded P*$]\!] \Longrightarrow$
Prop C Ψ (*Cases Cs*) *M xvec N P'*

and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ \ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ xvec\ N\ P';$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;$
$A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* C;\ xvec \sharp* Q;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P' \parallel Q)$
and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ xvec\ N\ \ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ \ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ xvec\ N\ Q';$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;$
$A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* C;\ xvec \sharp* Q;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ xvec\ N\ (P \parallel Q')$
and *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ (xvec@yvec)\ N\ P';$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp yvec;\ xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;$
$yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ yvec \sharp* C;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ (xvec@x\#yvec)\ N\ P'$
and *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ xvec\ N\ P';$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ xvec \sharp* \Psi;$
$xvec \sharp* P;\ xvec \sharp* M;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ xvec\ N\ ((\!|\nu x|\!)P')$
and *rBang*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ M\ xvec\ N\ P']\!] \Longrightarrow$
$Prop\ C\ \Psi\ (!P)\ M\ xvec\ N\ P'$
shows $Prop\ C\ \Psi\ P\ M\ xvec\ N\ P'$
proof −
note ‹$\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$›
moreover from ‹$xvec \sharp* M$› **have** $bn(M(\!|\nu{*}xvec|\!)\langle N\rangle) \sharp* subject(M(\!|\nu{*}xvec|\!)\langle N\rangle)$ **by** *simp*
moreover from ‹$\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$› **have** $distinct(bn(M(\!|\nu{*}xvec|\!)\langle N\rangle))$
by(*rule boundOutputDistinct*)
ultimately show *?thesis*
proof(*nominal-induct* $\Psi\ P\ \alpha{==}M(\!|\nu{*}xvec|\!)\langle N\rangle\ P'$ *avoiding*: *C arbitrary*: *M xvec N rule*: *semanticsInduct*)
case(*cAlpha* $\Psi\ P\ \alpha\ P'\ p\ C\ M\ xvec\ N$)
from ‹$(p \cdot \alpha) = M(\!|\nu{*}xvec|\!)\langle N\rangle$› **have** $(p \cdot p \cdot \alpha) = p \cdot (M(\!|\nu{*}xvec|\!)\langle N\rangle)$
by(*simp add*: *fresh-bij*)
with ‹*distinctPerm p*› **have** *A*: $\alpha = (p \cdot M)(\!|\nu{*}(p \cdot xvec)|\!)\langle(p \cdot N)\rangle$
by(*simp add*: *eqvts*)
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$ › ‹$bn\ \alpha \sharp* C$› ‹$bn\ \alpha \sharp* bn(p \cdot \alpha)$› ‹*distinctPerm p*›
have $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* (p \cdot M)$ **and** $(p$

$\cdot$ *xvec*) $\sharp*$ *C* **and** $(p \cdot xvec) \sharp* (p \cdot p \cdot xvec)$
by *auto*
moreover from *A* ‹*set p* $\subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$))› ‹*distinctPerm p*›
have *S*: *set p* $\subseteq$ *set*($p \cdot xvec$) $\times$ *set*($p \cdot p \cdot xvec$) **by** *simp*
moreover note ‹*distinctPerm p*›
moreover from *A* ‹*bn*($p \cdot \alpha$) $\sharp*$ α› ‹*bn*($p \cdot \alpha$) $\sharp*$ *P*′›
have $(p \cdot p \cdot xvec) \sharp* (p \cdot N)$ **and** $(p \cdot p \cdot xvec) \sharp* P'$ **by** *simp*+
moreover from *A* **have** *Prop C* Ψ *P* $(p \cdot M)$ $(p \cdot xvec)$ $(p \cdot N)$ *P*′
by(*rule cAlpha*)
ultimately have *Prop C* Ψ *P* $(p \cdot M)$ $(p \cdot p \cdot xvec)$ $(p \cdot p \cdot N)$ $(p \cdot P')$
by(*rule rAlpha*)
moreover from *A* ‹*bn* α $\sharp*$ *subject* α› **have** $(p \cdot xvec) \sharp* (p \cdot M)$ **by** *simp*
hence *xvec* $\sharp*$ *M* **by**(*simp add*: *fresh-star-bij*)
from *A* ‹*bn*($p \cdot \alpha$) $\sharp*$ α› ‹*distinctPerm p*› **have** *xvec* $\sharp*$ $(p \cdot M)$ **by** *simp*
hence $(p \cdot xvec) \sharp* (p \cdot p \cdot M)$ **by**(*simp add*: *fresh-star-bij*)
with ‹*distinctPerm p*› **have** $(p \cdot xvec) \sharp* M$ **by** *simp*
with ‹*xvec* $\sharp*$ *M*› *S* ‹*distinctPerm p*› **have** $(p \cdot M) = M$ **by** *simp*
ultimately show *?case* **using** *S* ‹*distinctPerm p*› **by** *simp*
next
case *cInput*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cOutput*
thus *?case* **by**(*force dest*: *rOutput simp add*: *action.inject*)
next
case *cCase*
thus *?case* **by**(*force intro*: *rCase*)
next
case *cPar1*
thus *?case* **by**(*force intro*: *rPar1*)
next
case *cPar2*
thus *?case* **by**(*force intro*: *rPar2*)
next
case *cComm1*
thus *?case* **by**(*simp add*: *action.inject*)
next
case *cComm2*
thus *?case* **by**(*simp add*: *action.inject*)
next
case *cOpen*
thus *?case* **by**(*fastforce intro*: *rOpen simp add*: *action.inject*)
next
case *cScope*
thus *?case* **by**(*fastforce intro*: *rScope*)
next
case *cBang*
thus *?case* **by**(*fastforce intro*: *rBang*)
qed

qed

lemma *inputInduct*[*consumes 1*, *case-names cInput cCase cPar1 cPar2 cScope cBang*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and N :: *'a*
and P' :: (*'a*, *'b*, *'c*) *psi*
and *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *'a* ⇒ *'a* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
and *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$K\ (N[xvec::=Tvec])\ (P[xvec::=Tvec])$
and *rCase*: $\bigwedge \Psi\ P\ M\ N\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P';\ (\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ M\ N\ P'$
and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P';\ distinct\ A_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;$
$A_Q \sharp* P';\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q)$
and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ N\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q';\ distinct\ A_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;$
$A_P \sharp* Q';\ A_P \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P \parallel Q')$
and *rScope*: $\bigwedge \Psi\ P\ M\ N\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P';\ x \sharp \Psi;\ x \sharp M;\ x \sharp N;\ x \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (\langle\!|\nu x|\!\rangle P)\ M\ N\ (\langle\!|\nu x|\!\rangle P')$
and *rBang*: $\bigwedge \Psi\ P\ M\ N\ P'\ C.$
$[\![\Psi \rhd P \parallel\ !P \longmapsto M(\!|N|\!) \prec P';\ guarded\ P;\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ M\ N\ P']\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ N\ P'$
shows *Prop C* Ψ *P M N P'*
using *Trans*
proof(*nominal-induct* Ψ *P Rs*==$M(\!|N|\!) \prec P'$ *avoiding*: *C arbitrary*: *P' rule*: *semantics.strong-induct*)
case(*cInput* Ψ *M K xvec N Tvec P C*)

thus *?case*
by(*force intro*: *rInput simp add*: *residualInject action.inject*)
next
case(*Output* Ψ *M K N P C*)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*Case* Ψ *P Rs* φ *CS C*)
thus *?case* **by**(*force intro*: *rCase*)
next
case(*cPar1* Ψ Ψ_Q *P* α *P'* *Q* A_Q *C P''*)
thus *?case* **by**(*force intro*: *rPar1 simp add*: *residualInject*)
next
case(*cPar2* Ψ Ψ*P Q* α *Q' xvec P C Q''*)
thus *?case* **by**(*force intro*: *rPar2 simp add*: *residualInject*)
next
case(*cComm1* Ψ Ψ*Q P M N P' xvec* Ψ*P Q K zvec Q' yvec C PQ*)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cComm2* Ψ Ψ*Q P M zvec N P' xvec* Ψ*P Q K yvec Q' C PQ*)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cOpen* Ψ *P M xvec N P' x yvec C P''*)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cScope* Ψ *P* α *P' x C P''*)
thus *?case* **by**(*force intro*: *rScope simp add*: *residualInject*)
next
case(*Bang* Ψ *P Rs C*)
thus *?case* **by**(*force intro*: *rBang*)
qed

lemma *tauInduct*[*consumes 1*, *case-names cCase cPar1 cPar2 cComm1 cComm2 cScope cBang*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rs* :: (*'a*, *'b*, *'c*) *residual*
and *Prop* :: *'d*::*fs-name* $\Rightarrow$ *'b* $\Rightarrow$ (*'a*, *'b*, *'c*) *psi* $\Rightarrow$
(*'a*, *'b*, *'c*) *psi* $\Rightarrow$ *bool*
and *C* :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto \tau \prec P'$
and *rCase*: $\bigwedge \Psi\ P\ P'\ \varphi\ Cs\ C.\ [\![\Psi \rhd P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ P\ P';\ (\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$
Prop C Ψ (*Cases Cs*) *P'*
and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ P'\ A_Q\ Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \tau \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ P';$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;$

$A_Q \sharp* P'$; $A_Q \sharp* C]] \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ (P' \parallel Q)$

and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ Q'\ A_P\ P\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \tau \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ Q'$;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$;
$A_P \sharp* Q'$; $A_P \sharp* C]] \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ (P \parallel Q')$

and *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C]] \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ ((\!|\nu * xvec|\!)(P' \parallel Q'))$

and *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C]] \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ ((\!|\nu * xvec|\!)(P' \parallel Q'))$

and *rScope*: $\bigwedge \Psi\ P\ P'\ x\ C.$
$[\![\Psi \rhd P \longmapsto \tau \prec P'$; $\bigwedge C.$ *Prop* $C\ \Psi\ P\ P'$; $x \sharp \Psi$; $x \sharp C]] \Longrightarrow$
Prop $C\ \Psi\ ((\!|\nu x|\!)P)\ ((\!|\nu x|\!)P')$

and *rBang*: $\bigwedge \Psi\ P\ P'\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto \tau \prec P'$; *guarded* P; $\bigwedge C.$ *Prop* $C\ \Psi\ (P \parallel !P)\ P']\!] \Longrightarrow$ *Prop* $C\ \Psi\ (!P)\ P'$

shows *Prop* $C\ \Psi\ P\ P'$
using *Trans*
proof(*nominal-induct* $\Psi\ P\ Rs{==}\tau \prec P'$ *avoiding*: C *arbitrary*: P' *rule*: *semantics.strong-induct*)
case(*cInput* $M\ K\ xvec\ N\ Tvec\ P\ C$)
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*Output* $\Psi\ M\ K\ N\ P\ C$)

thus *?case* **by**(*simp add*: *residualInject*)
next
case(*Case* Ψ P *Rs* φ *Cs* C)
thus *?case* **by**(*force intro*: *rCase simp add*: *residualInject*)
next
case(*cPar1* Ψ Ψ_Q P α P' A_Q Q C P'')
thus *?case* **by**(*force intro*: *rPar1 simp add*: *residualInject*)
next
case(*cPar2* Ψ Ψ_P Q α Q' A_P P C Q'')
thus *?case* **by**(*force intro*: *rPar2 simp add*: *residualInject*)
next
case(*cComm1* Ψ Ψ_Q P M N P' A_P Ψ_P Q K *xvec* Q' A_Q C PQ)
thus *?case* **by**(*force intro*: *rComm1 simp add*: *residualInject*)
next
case(*cComm2* Ψ Ψ_Q P M *xvec* N P' A_P ΨP Q' A_Q C PQ)
thus *?case* **by**(*force intro*: *rComm2 simp add*: *residualInject*)
next
case(*cOpen* Ψ P M *xvec* N P' x *yvec* C P'')
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cScope* Ψ P α P' x C P'')
thus *?case* **by**(*force intro*: *rScope simp add*: *residualInject*)
next
case(*Bang* Ψ P *Rs* C)
thus *?case* **by**(*force intro*: *rBang simp add*: *residualInject*)
qed

lemma *semanticsFrameInduct*[*consumes 3*, *case-names cAlpha cInput cOutput cCase cPar1 cPar2 cComm1 cComm2 cOpen cScope cBang*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rs* :: (*'a*, *'b*, *'c*) *residual*
and A_P :: *name list*
and Ψ_P :: *'b*
and *Prop* :: *'d*::*fs-name* $\Rightarrow$ *'b* $\Rightarrow$ (*'a*, *'b*, *'c*) *psi* $\Rightarrow$
(*'a*, *'b*, *'c*) *residual* $\Rightarrow$ *name list* $\Rightarrow$ *'b* $\Rightarrow$ *bool*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto Rs$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and *rAlpha*: $\bigwedge \Psi\ P\ A_P\ \Psi_P\ p\ Rs\ C$. $[\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* Rs;\ A_P \sharp* C;$
set $p \subseteq$ *set* $A_P \times$ *set*$(p \cdot A_P)$; *distinctPerm* p;
Prop $C\ \Psi\ P\ Rs\ A_P\ \Psi_P]\!] \Longrightarrow$ *Prop* $C\ \Psi\ P\ Rs\ (p \cdot A_P)\ (p \cdot \Psi_P)$
and *rInput*: $\bigwedge \Psi\ M\ K$ *xvec* N *Tvec* $P\ C$.
$[\![\Psi \vdash M \leftrightarrow K$; *distinct xvec*; *set xvec* $\subseteq$ *supp* N;
length xvec = *length Tvec*; *xvec* $\sharp* \Psi$;

$xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$(K(\!|(N[xvec::=Tvec])|\!) \prec (P[xvec::=Tvec]))\ ([])\ (\mathbf{1})$

and *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C.\ \Psi \vdash M \leftrightarrow K \Longrightarrow Prop\ C\ \Psi\ (M\langle N\rangle.P)\ (K\langle N\rangle \prec P)\ ([])\ (\mathbf{1})$

and *rCase*: $\bigwedge \Psi\ P\ Rs\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto Rs;\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ Rs\ A_P\ \Psi_P;$
$(\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};$
$(supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Rs;\ A_P \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (Cases\ Cs)\ Rs\ ([])\ (\mathbf{1})$

and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (\alpha \prec P')\ A_P\ \Psi_P;\ distinct(bn\ \alpha);$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* \Psi_P;\ bn\ \alpha \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ bn\ \alpha \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P' \parallel Q))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (\alpha \prec Q')\ A_Q\ \Psi_Q;\ distinct(bn\ \alpha);$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* \alpha;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* \alpha;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* \Psi_P;\ bn\ \alpha \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ bn\ \alpha \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ ((M(\!|N|\!)) \prec P')\ A_P\ \Psi_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|\nu* xvec|\!)\langle N\rangle \prec Q')\ A_Q\ \Psi_Q;\ distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;$

$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$⟧ ⟹
Prop $C\ \Psi\ (P \parallel Q)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C$.
⟦$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|\nu * xvec|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\bigwedge C$. *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|N|\!) \prec Q')\ A_Q\ \Psi_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$⟧ ⟹
Prop $C\ \Psi\ (P \parallel Q)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C$.
⟦$\Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* $C\ \Psi\ P\ (M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P')\ A_P\ \Psi_P$; $x \in supp\ N$; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$; $xvec \sharp* yvec$; *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$; $yvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C$⟧ ⟹
Prop $C\ \Psi\ ((\!|\nu x|\!)P)\ (M(\!|\nu * (xvec @ x\#yvec)|\!)\langle N\rangle \prec P')\ (x\#A_P)\ \Psi_P$

and *rScope*: $\bigwedge \Psi\ P\ \alpha\ P'\ x\ A_P\ \Psi_P\ C$.
⟦$\Psi \rhd P \longmapsto \alpha \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* $C\ \Psi\ P\ (\alpha \prec P')\ A_P\ \Psi_P$;
$x \sharp \Psi$; $x \sharp \alpha$; $x \sharp A_P$; $A_P \sharp* \Psi$; $A_P \sharp* P$;
$A_P \sharp* \alpha$; $A_P \sharp* P'$; *distinct*$(bn\ \alpha)$;
$bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp* subject\ \alpha$; $bn\ \alpha \sharp* \Psi_P$;
$A_P \sharp* C$; $x \sharp C$; $bn\ \alpha \sharp* C$⟧ ⟹
Prop $C\ \Psi\ ((\!|\nu x|\!)P)\ (\alpha \prec ((\!|\nu x|\!)P'))\ (x\#A_P)\ \Psi_P$

and *rBang*: $\bigwedge \Psi\ P\ Rs\ A_P\ \Psi_P\ C$.
⟦$\Psi \rhd P \parallel\ !P \longmapsto Rs$; *guarded* P; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* $C\ \Psi\ (P \parallel\ !P)\ Rs\ A_P\ (\Psi_P \otimes \mathbf{1})$; $\Psi_P \simeq \mathbf{1}$; $supp\ \Psi_P = (\{\}::name\ set)$;
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* Rs$; $A_P \sharp* C$⟧ ⟹ *Prop* $C\ \Psi\ (!P)\ Rs\ ([])\ (\mathbf{1})$

shows *Prop* $C\ \Psi\ P\ Rs\ A_P\ \Psi_P$
using *Trans FrP* ‹*distinct* A_P›
proof(*nominal-induct avoiding*: $A_P\ \Psi_P\ C$ *rule*: *semantics.strong-induct*)
case(*cInput* $\Psi\ M\ K\ xvec\ N\ Tvec\ P\ A_P\ \Psi_P\ C$)
from ‹*extractFrame* $(M(\!|\lambda * xvec\ N|\!).P) = \langle A_P, \Psi_P\rangle$›

have $A_P = []$ **and** $\Psi_P = \mathbf{1}$
by *auto*
with ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp* N› ‹*length xvec* = *length Tvec*›
‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ M› ‹*xvec* $\sharp*$ K› ‹*xvec* $\sharp*$ C›
show *?case* **by**(*blast intro*: *rInput*)
next
case(*Output* Ψ M K N P A_P Ψ_P)
from ‹*extractFrame* $(M\langle N\rangle.P) = \langle A_P, \Psi_P\rangle$›
have $A_P = []$ **and** $\Psi_P = \mathbf{1}$
by *auto*
with ‹$\Psi \vdash M \leftrightarrow K$› **show** *?case*
by(*blast intro*: *rOutput*)
next
case(*Case* Ψ P *Rs* φ *Cs* A_{cP} Ψ_{cP} C)
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P
and $A_P \sharp* (\Psi, P, Rs, C)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* P$ **and** $A_P \sharp* Rs$ **and** $A_P \sharp* C$
by *simp+*
note ‹$\Psi \rhd P \longmapsto Rs$› *FrP* ‹*distinct* A_P›
moreover from *FrP* ‹*distinct* A_P› ‹$\bigwedge A_P\ \Psi_P\ C.\ [\![$*extractFrame* $P = \langle A_P, \Psi_P\rangle$*; distinct* $A_P]\!] \Longrightarrow$ *Prop* C Ψ P *Rs* A_P Ψ_P›
have $\bigwedge C.$ *Prop* C Ψ P *Rs* A_P Ψ_P **by** *simp*
moreover note ‹(φ, P) *mem Cs*› ‹$\Psi \vdash \varphi$› ‹*guarded* P›
moreover from ‹*guarded* P› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::name\ set)$
by(*metis guardedStatEq*)+
moreover note ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Rs$› ‹$A_P \sharp* C$›
ultimately have *Prop* C Ψ (*Cases Cs*) *Rs* ([]) (**1**)
by(*rule rCase*)
thus *?case* **using** ‹*extractFrame*(*Cases Cs*) = $\langle A_{cP}, \Psi_{cP}\rangle$› **by** *simp*
next
case(*cPar1* Ψ Ψ_Q P α P' Q A_Q A_{PQ} Ψ_{PQ} C)
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P
$A_P \sharp* (P, Q, \Psi, \alpha, P', A_Q, A_{PQ}, C, \Psi_Q)$
by(*rule freshFrame*)
hence $A_P \sharp* P$ **and** $A_P \sharp* Q$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* \alpha$ **and** $A_P \sharp* P'$
and $A_P \sharp* A_Q$ **and** $A_P \sharp* A_{PQ}$ **and** $A_P \sharp* C$ **and** $A_P \sharp* \Psi_Q$
by *simp+*

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*

from ‹$A_Q \sharp* P$› ‹$A_P \sharp* A_Q$› *FrP* **have** $A_Q \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \alpha$› *FrP* **have** *bn* $\alpha \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹*extractFrame*$(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› *FrP FrQ* ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$›

‹A_Q ♯* Ψ_P›
have $\langle(A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$
by *simp*
moreover from ‹*distinct* A_P› ‹*distinct* A_Q› ‹A_P ♯* A_Q› **have** *distinct*($A_P@A_Q$)
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain p **where** S: *set* $p \subseteq$ *set*($A_P@A_Q$) × *set*(($p \cdot A_P$)@($p \cdot A_Q$))
and *distinctPerm* p
and Ψeq: $\Psi_{PQ} = p \cdot (\Psi_P \otimes \Psi_Q)$ **and** *Aeq*: $A_{PQ} = (p \cdot A_P)@(p \cdot A_Q)$
using ‹A_P ♯* A_{PQ}› ‹A_Q ♯* A_{PQ}› ‹*distinct* A_{PQ}›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› *FrP* ‹*distinct* A_P› *FrQ* ‹*distinct* A_Q›

moreover from *FrP* ‹*distinct* A_P› ‹$\bigwedge A_P\ \Psi_P\ C.\ [\![$*extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* $A_P]\!] \Longrightarrow$ *Prop* C ($\Psi \otimes \Psi_Q$) P ($\alpha \prec P'$) A_P Ψ_P›
have $\bigwedge C$. *Prop* C ($\Psi \otimes \Psi_Q$) P ($\alpha \prec P'$) A_P Ψ_P **by** *simp*

moreover note ‹A_P ♯* P› ‹A_P ♯* Q› ‹A_P ♯* Ψ› ‹A_P ♯* α› ‹A_P ♯* P'› ‹A_P ♯* A_Q› ‹A_P ♯* Ψ_Q›
‹A_Q ♯* P› ‹A_Q ♯* Q› ‹A_Q ♯* Ψ› ‹A_Q ♯* α› ‹A_Q ♯* P'› ‹A_Q ♯* Ψ_P› ‹*distinct*(*bn* α)›
‹*bn* α ♯* Ψ› ‹*bn* α ♯* P› ‹*bn* α ♯* Q› ‹*bn* α ♯* *subject* α› ‹*bn* α ♯* Ψ_P› ‹*bn* α ♯* Ψ_Q›
‹A_P ♯* C› ‹A_Q ♯* C› ‹*bn* α ♯* C›
ultimately have *Prop* C Ψ ($P \parallel Q$) ($\alpha \prec (P' \parallel Q)$) ($A_P@A_Q$) ($\Psi_P \otimes \Psi_Q$)
by(*rule-tac rPar1*)
with ‹A_P ♯* Ψ› ‹A_P ♯* P› ‹A_P ♯* Q› ‹A_P ♯* α› ‹A_P ♯* P'› ‹A_P ♯* A_{PQ}› ‹A_P ♯* C›
‹A_Q ♯* Ψ› ‹A_Q ♯* P› ‹A_Q ♯* Q› ‹A_Q ♯* α› ‹A_Q ♯* P'› ‹A_Q ♯* A_{PQ}› ‹A_Q ♯* C›
S ‹*distinctPerm* p› *Aeq*
have *Prop* C Ψ ($P \parallel Q$) ($\alpha \prec (P' \parallel Q)$) ($p \cdot (A_P@A_Q)$) ($p \cdot (\Psi_P \otimes \Psi_Q)$)
by(*rule-tac rAlpha*) (*assumption* | *simp add*: *eqvts*)+
with Ψeq *Aeq* **show** *?case* **by**(*simp add*: *eqvts*)
next
case(*cPar2* Ψ Ψ_P Q α Q' P A_P A_{PQ} Ψ_{PQ} C)
obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** *distinct* A_Q
A_Q ♯* (P, Q, Ψ, α, Q', A_P, A_{PQ}, C, Ψ_P)
by(*rule freshFrame*)
hence A_Q ♯* P **and** A_Q ♯* Q **and** A_Q ♯* Ψ **and** A_Q ♯* α **and** A_Q ♯* Q'
and A_Q ♯* A_P **and** A_Q ♯* A_{PQ} **and** A_Q ♯* C **and** A_Q ♯* Ψ_P
by *simp*+

from ‹A_Q ♯* A_P› **have** A_P ♯* A_Q **by** *simp*
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*

from ‹A_P ♯* Q› ‹A_Q ♯* A_P› *FrQ* **have** A_P ♯* Ψ_Q
by(*force dest*: *extractFrameFreshChain*)

from ‹$bn\ \alpha \sharp* Q$› ‹$A_Q \sharp* \alpha$› FrQ **have** $bn\ \alpha \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)

from ‹$extractFrame(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› $FrP\ FrQ$ ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have $\langle(A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$
by *simp*
moreover from ‹$distinct\ A_P$› ‹$distinct\ A_Q$› ‹$A_P \sharp* A_Q$› **have** $distinct(A_P@A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain p **where** S: $(set\ p \subseteq (set(A_P@A_Q)) \times (set\ A_{PQ}))$ **and** *distinctPerm p*
and Ψeq: $\Psi_{PQ} = p \cdot (\Psi_P \otimes \Psi_Q)$ **and** Aeq: $A_{PQ} = ((p \cdot A_P)@(p \cdot A_Q))$
using ‹$A_P \sharp* A_{PQ}$› ‹$A_Q \sharp* A_{PQ}$› ‹$distinct\ A_{PQ}$›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

note ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› FrP ‹$distinct\ A_P$› FrQ ‹$distinct\ A_Q$›

moreover from FrQ ‹$distinct\ A_Q$› ‹$\bigwedge A_Q\ \Psi_Q\ C.\ [\![extractFrame\ Q = \langle A_Q, \Psi_Q\rangle; distinct\ A_Q]\!] \Longrightarrow Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (\alpha \prec Q')\ A_Q\ \Psi_Q$›
have $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (\alpha \prec Q')\ A_Q\ \Psi_Q$ **by** *simp*

moreover note ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* Q'$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$›
‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* Q'$› ‹$A_Q \sharp* \Psi_P$› ‹$distinct(bn\ \alpha)$›
‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \Psi_Q$›
‹$A_P \sharp* C$› ‹$A_Q \sharp* C$› ‹$bn\ \alpha \sharp* C$›
ultimately have $Prop\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P \parallel Q'))\ (A_P@A_Q)\ (\Psi_P \otimes \Psi_Q)$
by(*rule-tac rPar2*)

with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* Q'$› ‹$A_P \sharp* A_{PQ}$› ‹$A_P \sharp* C$›
‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* Q'$› ‹$A_Q \sharp* A_{PQ}$› ‹$A_Q \sharp* C$›
S ‹$distinctPerm\ p$› Aeq
have $Prop\ C\ \Psi\ (P \parallel Q)\ (\alpha \prec (P \parallel Q'))\ (p \cdot (A_P@A_Q))\ (p \cdot (\Psi_P \otimes \Psi_Q))$
by(*rule-tac rAlpha*) (*assumption* | *simp add*: *eqvts*)+
with $\Psi eq\ Aeq$ **show** *?case* **by**(*simp add*: *eqvts*)
next
case(*cComm1* $\Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ A_{PQ}\ \Psi_{PQ}\ C$)
from ‹$distinct\ A_P$› ‹$distinct\ A_Q$› ‹$A_P \sharp* A_Q$› **have** $distinct(A_P@A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
from *cComm1* **have** $Prop\ C\ \Psi\ (P \parallel Q)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P@A_Q)\ (\Psi_P \otimes \Psi_Q)$
by(*rule-tac rComm1*)
moreover from ‹$extractFrame(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› ‹$extractFrame\ P = \langle A_P, \Psi_P\rangle$› ‹$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle$›

$\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$
have $\langle(A_P@A_Q), (\Psi_P \otimes \Psi_Q)\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$
by *simp*
with $\langle A_P \sharp* A_{PQ}\rangle$ $\langle A_Q \sharp* A_{PQ}\rangle$ $\langle distinct(A_P@A_Q)\rangle$ $\langle distinct\ A_{PQ}\rangle$
obtain p **where** S: $(set\ p \subseteq (set(A_P@A_Q)) \times (set\ A_{PQ}))$ **and** *distinctPerm* p **and** Ψeq: $\Psi_{PQ} = p \cdot (\Psi_P \otimes \Psi_Q)$ **and** *Aeq*: $A_{PQ} = p \cdot (A_P@A_Q)$
by(*rule-tac frameChainEq′*) (*assumption* | *simp*)+
moreover note $\langle A_P \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_P \sharp* xvec\rangle$
$\langle A_Q \sharp* xvec\rangle$ $\langle A_P \sharp* P'\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_P \sharp* Q'\rangle$ $\langle A_Q \sharp* Q'\rangle$ $\langle A_P \sharp* A_{PQ}\rangle$ $\langle A_Q \sharp* A_{PQ}\rangle$
$\langle A_P \sharp* C\rangle$ $\langle A_Q \sharp* C\rangle$
ultimately have *Prop* C Ψ $(P \parallel Q)$ $(\tau \prec (\!|\nu*xvec|\!)(P' \parallel Q'))$ $(p \cdot (A_P@A_Q))$ $(p \cdot (\Psi_P \otimes \Psi_Q))$
by(*rule-tac rAlpha*) *auto*
with Ψeq *Aeq* **show** *?case* **by** *simp*
next
case(*cComm2* Ψ Ψ_Q P M *xvec* N P' A_P Ψ_P Q K Q' A_Q A_{PQ} Ψ_{PQ} C)
from $\langle distinct\ A_P\rangle$ $\langle distinct\ A_Q\rangle$ $\langle A_P \sharp* A_Q\rangle$ **have** *distinct*$(A_P@A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
from *cComm2* **have** *Prop* C Ψ $(P \parallel Q)$ $(\tau \prec (\!|\nu*xvec|\!)(P' \parallel Q'))$ $(A_P@A_Q)$ $(\Psi_P \otimes \Psi_Q)$
by(*rule-tac rComm2*)
moreover from $\langle extractFrame(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle\rangle$ $\langle extractFrame\ P = \langle A_P, \Psi_P\rangle\rangle$ $\langle extractFrame\ Q = \langle A_Q, \Psi_Q\rangle\rangle$
$\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$
have $\langle(A_P@A_Q), (\Psi_P \otimes \Psi_Q)\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$
by *simp*
with $\langle A_P \sharp* A_{PQ}\rangle$ $\langle A_Q \sharp* A_{PQ}\rangle$ $\langle distinct(A_P@A_Q)\rangle$ $\langle distinct\ A_{PQ}\rangle$
obtain p **where** S: $(set\ p \subseteq (set(A_P@A_Q)) \times (set\ A_{PQ}))$ **and** *distinctPerm* p **and** Ψeq: $\Psi_{PQ} = p \cdot (\Psi_P \otimes \Psi_Q)$ **and** *Aeq*: $A_{PQ} = p \cdot (A_P@A_Q)$
by(*rule-tac frameChainEq′*) (*assumption* | *simp*)+
moreover note $\langle A_P \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_P \sharp* xvec\rangle$
$\langle A_Q \sharp* xvec\rangle$ $\langle A_P \sharp* P'\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_P \sharp* Q'\rangle$ $\langle A_Q \sharp* Q'\rangle$ $\langle A_P \sharp* A_{PQ}\rangle$ $\langle A_Q \sharp* A_{PQ}\rangle$
$\langle A_P \sharp* C\rangle$ $\langle A_Q \sharp* C\rangle$
ultimately have *Prop* C Ψ $(P \parallel Q)$ $(\tau \prec (\!|\nu*xvec|\!)(P' \parallel Q'))$ $(p \cdot (A_P@A_Q))$ $(p \cdot (\Psi_P \otimes \Psi_Q))$
by(*rule-tac rAlpha*) *auto*
with Ψeq *Aeq* **show** *?case* **by** *simp*
next
case(*cOpen* Ψ P M *xvec* *yvec* N P' x A_{xP} Ψ_{xP} C)
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P **and** $A_P \sharp* (\Psi, P, M, xvec, yvec, N, P', A_{xP}, \Psi_{xP}, C, x)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* P$ **and** $A_P \sharp* M$ **and** $A_P \sharp* xvec$**and** $A_P \sharp* yvec$ **and** $A_P \sharp* N$ **and** $A_P \sharp* P'$
and $A_P \sharp* A_{xP}$ **and** $A_P \sharp* \Psi_{xP}$ **and** $A_P \sharp* C$ **and** $x \sharp A_P$

by *simp+*

from ‹*xvec* ♯* *P*› ‹*A*$_P$ ♯* *xvec*› *FrP* **have** *xvec* ♯* Ψ$_P$
by(*force dest*: *extractFrameFreshChain*)
from ‹*yvec* ♯* *P*› ‹*A*$_P$ ♯* *yvec*› *FrP* **have** *yvec* ♯* Ψ$_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹*extractFrame*(⦇ν*x*⦈*P*) = ⟨*A*$_{xP}$, Ψ$_{xP}$⟩› *FrP*
have ⟨(*x*#*A*$_P$), Ψ$_P$⟩ = ⟨*A*$_{xP}$, Ψ$_{xP}$⟩
by *simp*
moreover from ‹*x* ♯ *A*$_P$› ‹*distinct A*$_P$› **have** *distinct*(*x*#*A*$_P$) **by** *simp*
ultimately obtain *p* **where** *S*: *set p* ⊆ *set* (*x*#*A*$_P$) × *set* (*p* ∙ (*x*#*A*$_P$)) **and** *distinctPerm p*
and Ψ*eq*: Ψ$_{xP}$ = *p* ∙ Ψ$_P$ **and** *Aeq*: *A*$_{xP}$ = (*p* ∙ *x*)#(*p* ∙ *A*$_P$)
using ‹*A*$_P$ ♯* *A*$_{xP}$›‹*x* ♯ *A*$_{xP}$› ‹*distinct A*$_{xP}$›
by(*rule-tac frameChainEq′*) (*assumption* | *simp add*: *eqvts*)+

note ‹Ψ ▷ *P* ⟼*M*⦇ν*(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P′*› *FrP* ‹*distinct A*$_P$›
moreover from *FrP* ‹*distinct A*$_P$› ‹⋀*A*$_P$ Ψ$_P$ *C*. ⟦*extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩; *distinct A*$_P$⟧ ⟹ *Prop C* Ψ *P* (*M*⦇ν*(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P′*) *A*$_P$ Ψ$_P$›
have ⋀*C*. *Prop C* Ψ *P* (*M*⦇ν*(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P′*) *A*$_P$ Ψ$_P$ **by** *simp*
moreover note ‹*x* ♯ Ψ› ‹*x* ♯ *M*› ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*x* ∈ *supp N*› ‹*x* ♯ *A*$_P$› ‹*A*$_P$ ♯* Ψ› ‹*A*$_P$ ♯* Ψ› ‹*A*$_P$ ♯* *P*› ‹*A*$_P$ ♯* *M*› ‹*A*$_P$ ♯* *xvec*› ‹*A*$_P$ ♯* *yvec*› ‹*A*$_P$ ♯* *N*› ‹*A*$_P$ ♯* *P′*›
‹*xvec* ♯* Ψ› ‹*xvec* ♯* *P*› ‹*xvec* ♯* *M*› ‹*xvec* ♯* Ψ$_P$› ‹*yvec* ♯* Ψ› ‹*yvec* ♯* *P*› ‹*yvec* ♯* *M*› ‹*yvec* ♯* Ψ$_P$›
‹*A*$_P$ ♯* *C*› ‹*x* ♯ *C*› ‹*xvec* ♯* *C*› ‹*yvec* ♯* *C*› ‹*xvec* ♯* *yvec*› ‹*distinct xvec*› ‹*distinct yvec*›
ultimately have *Prop C* Ψ (⦇ν*x*⦈*P*) (*M*⦇ν*(*xvec*@*x*#*yvec*)⦈⟨*N*⟩ ≺ *P′*) (*x*#*A*$_P$) Ψ$_P$
by(*rule-tac rOpen*)

with ‹*A*$_P$ ♯* Ψ› ‹*A*$_P$ ♯* *P*› ‹*A*$_P$ ♯* *M*› ‹*A*$_P$ ♯* *xvec*› ‹*A*$_P$ ♯* *yvec*› ‹*A*$_P$ ♯* *N*› ‹*A*$_P$ ♯* *P′*› ‹*A*$_P$ ♯* *A*$_{xP}$› ‹*A*$_P$ ♯* *C*› ‹*x* ♯ *A*$_{xP}$› ‹*A*$_P$ ♯* *A*$_{xP}$› ‹*x* ♯ *A*$_P$›
‹*x* ♯ Ψ› ‹*x* ♯ *M*› ‹*x* ♯ *C*› ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› *Aeq*
S ‹*distinctPerm p*›
have *Prop C* Ψ (⦇ν*x*⦈*P*) (*M*⦇ν*(*xvec*@*x*#*yvec*)⦈⟨*N*⟩ ≺ *P′*) (*p* ∙ (*x*#*A*$_P$)) (*p* ∙ Ψ$_P$)
by(*rule-tac A*$_P$=*x*#*A*$_P$ **in** *rAlpha*) (*assumption* | *simp add*: *abs-fresh fresh-star-def boundOutputFresh*)+
with Ψ*eq Aeq* **show** *?case* **by**(*simp add*: *eqvts*)
next
case(*cScope* Ψ *P* α *P′ x A*$_{xP}$ Ψ$_{xP}$ *C*)
obtain *A*$_P$ Ψ$_P$ **where** *FrP*: *extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩ **and** *distinct A*$_P$
and *A*$_P$ ♯* (Ψ, *P*, α, *P′*, *A*$_{xP}$, Ψ$_{xP}$, *C*, *x*)
by(*rule freshFrame*)
hence *A*$_P$ ♯* Ψ **and** *A*$_P$ ♯* *P* **and** *A*$_P$ ♯* α **and** *A*$_P$ ♯* *P′*
and *A*$_P$ ♯* *A*$_{xP}$ **and** *A*$_P$ ♯* Ψ$_{xP}$ **and** *A*$_P$ ♯* *C* **and** *x* ♯ *A*$_P$
by *simp+*

from ‹$bn\ \alpha \sharp* P$› ‹$A_P \sharp* \alpha$› *FrP* **have** $bn\ \alpha \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹$extractFrame((\!|\nu x|\!)P) = \langle A_{xP}, \Psi_{xP}\rangle$› *FrP*
have $\langle(x\#A_P), \Psi_P\rangle = \langle A_{xP}, \Psi_{xP}\rangle$
by *simp*
moreover from ‹$x \sharp A_P$› ‹*distinct* A_P› **have** *distinct*$(x\#A_P)$ **by** *simp*
ultimately obtain p **where** S: *set* $p \subseteq$ *set* $(x\#A_P) \times$ *set* $(p \cdot (x\#A_P))$ **and** *distinctPerm* p
and Ψeq: $\Psi_{xP} = p \cdot \Psi_P$ **and** Aeq: $A_{xP} = (p \cdot x)\#(p \cdot A_P)$
using ‹$A_P \sharp* A_{xP}$›‹$x \sharp A_{xP}$› ‹*distinct* A_{xP}›
by(*rule-tac frameChainEq′*) (*assumption* | *simp add*: *eqvts*)+

note ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› *FrP* ‹*distinct* A_P›
moreover from *FrP* ‹*distinct* A_P› ‹$\bigwedge A_P\ \Psi_P\ C.\ [\![extractFrame\ P = \langle A_P, \Psi_P\rangle; distinct\ A_P]\!] \Longrightarrow Prop\ C\ \Psi\ P\ (\alpha \prec P')\ A_P\ \Psi_P$›
have $\bigwedge C.\ Prop\ C\ \Psi\ P\ (\alpha \prec P')\ A_P\ \Psi_P$ **by** *simp*
moreover note ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp A_P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* P'$› ‹*distinct*$(bn\ \alpha)$›
‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* \Psi_P$› ‹$A_P \sharp* C$› ‹$x \sharp C$› ‹$bn\ \alpha \sharp* C$›
ultimately have $Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\alpha \prec ((\!|\nu x|\!)P'))\ (x\#A_P)\ \Psi_P$
by(*rule-tac rScope*)

with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* P'$› ‹$A_P \sharp* A_{xP}$› ‹$A_P \sharp* C$› ‹$x \sharp A_{xP}$› ‹$A_P \sharp* A_{xP}$› ‹$x \sharp A_P$›
‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp C$› *Aeq*
S ‹*distinctPerm* p›
have $Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ (\alpha \prec ((\!|\nu x|\!)P'))\ (p \cdot (x\#A_P))\ (p \cdot \Psi_P)$
by(*rule-tac* $A_P{=}x\#A_P$ **in** *rAlpha*) (*assumption* | *simp add*: *abs-fresh fresh-star-def*)+
with Ψeq *Aeq* **show** *?case* **by**(*simp add*: *eqvts*)
next
case(*Bang* Ψ P *Rs* A_{bP} Ψ_{bP} C)

obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P
and $A_P \sharp* (\Psi, P, Rs, C)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* P$ **and** $A_P \sharp* Rs$ **and** $A_P \sharp* C$
by *simp*+

note ‹$\Psi \rhd P \parallel !P \longmapsto Rs$› ‹*guarded* P› *FrP* ‹*distinct* A_P›
moreover from *FrP* **have** *extractFrame* $(P \parallel !P) = \langle A_P, \Psi_P \otimes \mathbf{1}\rangle$
by *simp*
with ‹*distinct* A_P› ‹$\bigwedge A_P\ \Psi_P\ C.\ [\![extractFrame\ (P \parallel !P) = \langle A_P, \Psi_P\rangle; distinct\ A_P]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ Rs\ A_P\ \Psi_P$›
have $\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ Rs\ A_P\ (\Psi_P \otimes \mathbf{1})$ **by** *simp*
moreover from ‹*guarded* P› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::name\ set)$
by(*metis guardedStatEq*)+

moreover note ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Rs$› ‹$A_P \sharp* C$›
ultimately have *Prop* C Ψ ($!P$) Rs ([]) (**1**)
by(*rule rBang*)
thus *?case* **using** ‹*extractFrame*($!P$) = $\langle A_{bP}, \Psi_{bP}\rangle$› **by** *simp*
qed

lemma *semanticsFrameInduct'*[*consumes 5*, *case-names cAlpha cFrameAlpha cInput cOutput cCase cPar1 cPar2 cComm1 cComm2 cOpen cScope cBang*]:
fixes Ψ :: $'b$
and P :: ($'a$, $'b$, $'c$) *psi*
and Rs :: ($'a$, $'b$, $'c$) *residual*
and A_P :: *name list*
and Ψ_P :: $'b$
and *Prop* :: $'d$::*fs-name* ⇒ $'b$ ⇒ ($'a$, $'b$, $'c$) *psi* ⇒ $'a$ *action* ⇒
($'a$, $'b$, $'c$) *psi* ⇒ *name list* ⇒ $'b$ ⇒ *bool*
and C :: $'d$::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *distinct* A_P
and *bn* $\alpha \sharp* subject\ \alpha$
and *distinct*(*bn* α)
and *rAlpha*: $\bigwedge \Psi\ P\ \alpha\ P'\ p\ A_P\ \Psi_P\ C$. ⟦*bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp* subject\ \alpha$; *bn* $\alpha \sharp* \Psi_P$;
bn $\alpha \sharp* C$; *bn* $\alpha \sharp* (p \cdot \alpha)$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* \alpha$; $A_P \sharp* P'$; $A_P \sharp* C$;
set $p \subseteq$ *set*(*bn* α) × *set*(*bn*($p \cdot \alpha$)); *distinctPerm* p;
bn($p \cdot \alpha$) $\sharp* \alpha$; (*bn*($p \cdot \alpha$)) $\sharp* P'$; *Prop* C Ψ P α P' A_P Ψ_P⟧ ⟹
Prop C Ψ P ($p \cdot \alpha$) ($p \cdot P'$) A_P Ψ_P
and *rFrameAlpha*: $\bigwedge \Psi\ P\ A_P\ \Psi_P\ p\ \alpha\ P'\ C$. ⟦$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* (p \cdot A_P)$; $A_P \sharp* \alpha$; $A_P \sharp* P'$; $A_P \sharp* C$;
set $p \subseteq$ *set* A_P × *set*($p \cdot A_P$); *distinctPerm* p; $A_P \sharp* subject\ \alpha$;
Prop C Ψ P α P' A_P Ψ_P⟧ ⟹ *Prop* C Ψ P α P' ($p \cdot A_P$) ($p \cdot \Psi_P$)
and *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C$.
⟦$\Psi \vdash M \leftrightarrow K$; *distinct xvec*; *set xvec* ⊆ *supp* N;
length xvec = *length Tvec*; *xvec* $\sharp* \Psi$;
xvec $\sharp* M$; *xvec* $\sharp* K$; *xvec* $\sharp* C$⟧ ⟹
Prop C Ψ (M⦇$\lambda* xvec\ N$⦈.P)
(K⦇(N[*xvec*::=*Tvec*])⦈) (P[*xvec*::=*Tvec*]) ([]) (**1**)
and *rOutput*: $\bigwedge \Psi\ M\ K\ N\ P\ C$. $\Psi \vdash M \leftrightarrow K$ ⟹ *Prop* C Ψ ($M\langle N\rangle$.P) ($K\langle N\rangle$) P ([]) (**1**)
and *rCase*: $\bigwedge \Psi\ P\ \alpha\ P'\ \varphi\ Cs\ A_P\ \Psi_P\ C$. ⟦$\Psi \rhd P \longmapsto \alpha \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P; $\bigwedge C$. *Prop* C Ψ P α P' A_P Ψ_P;
(φ, P) *mem Cs*; $\Psi \vdash \varphi$; *guarded* P; $\Psi_P \simeq$ **1**; (*supp* Ψ_P) = ({}::*name set*);

$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* \alpha$; $A_P \sharp* P'$; $A_P \sharp* C\rrbracket \Longrightarrow$ *Prop* $C\ \Psi\ (Cases\ Cs)\ \alpha\ P'\ ([])$ (**1**)

and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ \alpha\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* A_P; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* A_Q; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ \alpha\ P'\ A_P\ \Psi_P$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* \alpha$; $A_P \sharp* P'$; $A_P \sharp* A_Q$; $A_P \sharp* \Psi_Q$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* \alpha$; $A_Q \sharp* P'$; $A_Q \sharp* \Psi_P$; $bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp* Q$; $bn\ \alpha \sharp*$ *subject* α; $bn\ \alpha \sharp* \Psi_P$; $bn\ \alpha \sharp* \Psi_Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $bn\ \alpha \sharp* C\rrbracket \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ \alpha\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$\llbracket \Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* A_P; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* A_Q; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ \alpha\ Q'\ A_Q\ \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* \alpha$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* \Psi_Q$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* \alpha$; $A_Q \sharp* Q'$; $A_Q \sharp* \Psi_P$; $bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp* Q$; $bn\ \alpha \sharp*$ *subject* α; $bn\ \alpha \sharp* \Psi_P$; $bn\ \alpha \sharp* \Psi_Q$; $A_P \sharp* C$; $A_Q \sharp* C$; $bn\ \alpha \sharp* C\rrbracket \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ \alpha\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* A_P; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|N|\!))\ P'\ A_P\ \Psi_P$; $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* A_Q; $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct* $xvec$; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|\nu * xvec|\!)\langle N \rangle)\ Q'\ A_Q\ \Psi_Q$; $A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C\rrbracket \Longrightarrow$
Prop $C\ \Psi\ (P \parallel Q)\ (\tau)\ ((\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P \rangle$; *distinct* A_P; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_Q)\ P\ (M(\!|\nu * xvec|\!)\langle N \rangle)\ P'\ A_P\ \Psi_P$; $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$; *distinct* A_Q; $\bigwedge C.$ *Prop* $C\ (\Psi \otimes \Psi_P)\ Q\ (K(\!|N|\!))\ Q'\ A_Q\ \Psi_Q$; $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct* $xvec$;

$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$;
$A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$;
$xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$$\rrbracket \Longrightarrow$
Prop C Ψ $(P \parallel Q)$ (τ) $((\!|\nu* xvec|\!)(P' \parallel Q'))$ $(A_P @ A_Q)$ $(\Psi_P \otimes \Psi_Q)$

and *rOpen*: $\bigwedge \Psi$ P M *xvec yvec* N P' x A_P Ψ_P y C.
$\llbracket \Psi \rhd P \longmapsto M(\!|\nu*(xvec @ yvec)|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* C Ψ P $(M(\!|\nu*(xvec @ yvec)|\!)\langle N\rangle)$ P' A_P Ψ_P; $x \in$ *supp* N; $x \sharp \Psi$; $x \sharp M$;
$x \sharp A_P$; $x \sharp xvec$; $x \sharp yvec$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$;
$A_P \sharp* xvec$; $A_P \sharp* yvec$; $xvec \sharp* yvec$; *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* \Psi_P$;
$yvec \sharp* \Psi$; $yvec \sharp* P$; $yvec \sharp* M$; $A_P \sharp* C$; $x \sharp C$; $xvec \sharp* C$; $yvec \sharp* C$;
$y \neq x$; $y \sharp \Psi$; $y \sharp P$; $y \sharp M$; $y \sharp xvec$; $y \sharp yvec$; $y \sharp N$; $y \sharp P'$; $y \sharp A_P$; $y \sharp \Psi_P$; $y \sharp C$$\rrbracket \Longrightarrow$
Prop C Ψ $((\!|\nu x|\!)P)$ $(M(\!|\nu*(xvec @ y\#yvec)|\!)\langle([(x,\ y)] \cdot N)\rangle)$ $([(x,\ y)] \cdot P')$ $(x\#A_P)$ Ψ_P

and *rScope*: $\bigwedge \Psi$ P α P' x A_P Ψ_P C.
$\llbracket \Psi \rhd P \longmapsto \alpha \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* C Ψ P α P' A_P Ψ_P;
$x \sharp \Psi$; $x \sharp \alpha$; $x \sharp A_P$; $A_P \sharp* \Psi$; $A_P \sharp* P$;
$A_P \sharp* \alpha$; $A_P \sharp* P'$;
$bn\ \alpha \sharp* \Psi$; $bn\ \alpha \sharp* P$; $bn\ \alpha \sharp*$ *subject* α; $bn\ \alpha \sharp* \Psi_P$;
$A_P \sharp* C$; $x \sharp C$; $bn\ \alpha \sharp* C$$\rrbracket \Longrightarrow$
Prop C Ψ $((\!|\nu x|\!)P)$ α $((\!|\nu x|\!)P')$ $(x\#A_P)$ Ψ_P

and *rBang*: $\bigwedge \Psi$ P α P' A_P Ψ_P C.
$\llbracket \Psi \rhd P \parallel !P \longmapsto \alpha \prec P'$; *guarded* P; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\bigwedge C$. *Prop* C Ψ $(P \parallel !P)$ α P' A_P $(\Psi_P \otimes \mathbf{1})$; $\Psi_P \simeq \mathbf{1}$; *supp* Ψ_P = ({}::*name set*);
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* \alpha$; $A_P \sharp* P'$; $A_P \sharp* C$$\rrbracket \Longrightarrow$ *Prop* C Ψ $(!P)$ α P' $([])$ $(\mathbf{1})$

shows *Prop* C Ψ P α P' A_P Ψ_P
using *Trans FrP* ‹*distinct* A_P› ‹$bn\ \alpha \sharp*$ *subject* α› ‹*distinct*($bn\ \alpha$)›
proof(*nominal-induct* Ψ P *Rs*==$\alpha \prec P'$ A_P Ψ_P *avoiding*: C α P' *rule*: *semanticsFrameInduct*)
case *cAlpha*
thus *?case* **using** *rFrameAlpha*
by *auto*
next
case *cInput*
thus *?case* **using** *rInput*
by(*auto simp add*: *residualInject*)
next

case *cOutput*
thus *?case* **using** *rOutput*
by(*auto simp add*: *residualInject*)
next
case *cCase*
thus *?case* **using** *rCase*
by(*auto simp add*: *residualInject*)
next
case(*cPar1* Ψ Ψ_Q P α P' A_Q Q A_P Ψ_P C α' P'')
note ‹$\alpha \prec (P' \parallel Q) = \alpha' \prec P''$›
moreover from ‹$bn\ \alpha \sharp* \alpha'$› **have** $bn\ \alpha \sharp* (bn\ \alpha')$ **by** *auto*
moreover note ‹*distinct* $(bn\ \alpha)$› ‹*distinct*$(bn\ \alpha')$›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha' \sharp* subject\ \alpha'$›
have $bn\ \alpha \sharp* (\alpha \prec P' \parallel Q)$ **and** $bn\ \alpha' \sharp* (\alpha' \prec P'')$ **by** *simp+*
ultimately obtain p **where** S: $(set\ p) \subseteq (set(bn\ \alpha)) \times (set(bn(p \cdot \alpha)))$ **and** *distinctPerm p*
and αEq: $\alpha' = p \cdot \alpha$ **and** $P'eq$: $P'' = p \cdot (P' \parallel Q)$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$
and $(bn(p \cdot \alpha)) \sharp* (P' \parallel Q)$
by(*rule residualEq*)

note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹*distinct* A_Q›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹*distinct*$(bn\ \alpha)$› ‹$A_P \sharp* \alpha$›
have $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ \alpha\ P'\ A_P\ \Psi_P$ **by**(*rule-tac cPar1*) *auto*

moreover note ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* P'$› ‹$A_Q \sharp* C$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* P'$› ‹$A_P \sharp* C$›
‹$bn\ \alpha \sharp* Q$› ‹*distinct*$(bn\ \alpha)$› ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* \Psi_Q$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$›
‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹*distinct* A_P› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \Psi_P$›
ultimately have $Prop\ C\ \Psi\ (P \parallel Q)\ \alpha\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$
by(*rule-tac rPar1*)
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$bn\ \alpha \sharp* (bn\ \alpha')$› S ‹*distinctPerm* p› ‹$bn(p \cdot \alpha) \sharp* \alpha$› ‹$bn(p \cdot \alpha) \sharp* (P' \parallel Q)$› ‹$bn\ \alpha \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \Psi_Q$› ‹$A_P \sharp* \alpha$› ‹$A_Q \sharp* \alpha$› ‹$A_P \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› αEq ‹$bn\ \alpha \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \alpha'$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_P \sharp* P'$› ‹$A_Q \sharp* P'$› ‹$A_P \sharp* C$› ‹$A_Q \sharp* C$›
have $Prop\ C\ \Psi\ (P \parallel Q)\ (p \cdot \alpha)\ (p \cdot (P' \parallel Q))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$
by(*rule-tac rAlpha*) *auto*
with αEq $P'eq$ ‹*distinctPerm* p› **show** *?case* **by** *simp*
next
case(*cPar2* Ψ Ψ_P Q α Q' A_P P A_Q Ψ_Q C α' Q'')
note ‹$\alpha \prec (P \parallel Q') = \alpha' \prec Q''$›
moreover from ‹$bn\ \alpha \sharp* \alpha'$› **have** $bn\ \alpha \sharp* (bn\ \alpha')$ **by** *auto*
moreover note ‹*distinct* $(bn\ \alpha)$› ‹*distinct*$(bn\ \alpha')$›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha' \sharp* subject\ \alpha'$›
have $bn\ \alpha \sharp* (\alpha \prec P \parallel Q')$ **and** $bn\ \alpha' \sharp* (\alpha' \prec Q'')$ **by** *simp+*
ultimately obtain p **where** S: $(set\ p) \subseteq (set(bn\ \alpha)) \times (set(bn(p \cdot \alpha)))$ **and**

distinctPerm p
and *αEq*: $\alpha' = p \cdot \alpha$ **and** *Q'eq*: $Q'' = p \cdot (P \parallel Q')$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$
and $(bn(p \cdot \alpha)) \sharp* (P \parallel Q')$
by(*rule residualEq*)

note ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹*extractFrame P* = $\langle A_P, \Psi_P \rangle$› ‹*distinct* A_P›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹*distinct*$(bn\ \alpha)$› ‹$A_Q \sharp* \alpha$›
have $\bigwedge C$. *Prop C* $(\Psi \otimes \Psi_P)$ *Q* α *Q'* A_Q Ψ_Q **by**(*rule-tac cPar2*) *auto*

moreover note ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* Q'$› ‹$A_Q \sharp* C$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* Q'$› ‹$A_P \sharp* C$›
‹$bn\ \alpha \sharp* Q$› ‹*distinct*$(bn\ \alpha)$› ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* \Psi_Q$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$›
‹*extractFrame Q* = $\langle A_Q, \Psi_Q \rangle$› ‹*distinct* A_Q› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \Psi_P$›
ultimately have *Prop C* Ψ $(P \parallel Q)$ α $(P \parallel Q')$ $(A_P @ A_Q)$ $(\Psi_P \otimes \Psi_Q)$
by(*rule-tac rPar2*) *auto*
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$bn\ \alpha \sharp* (bn\ \alpha')$› *S* ‹*distinctPerm p*› ‹$bn(p \cdot \alpha) \sharp* \alpha$› ‹$bn(p \cdot \alpha) \sharp* (P \parallel Q')$› ‹$bn\ \alpha \sharp* \Psi_P$› ‹$bn\ \alpha \sharp* \Psi_Q$› ‹$A_P \sharp* \alpha$› ‹$A_Q \sharp* \alpha$› ‹$A_P \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› *αEq* ‹$bn\ \alpha \sharp* \alpha'$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_P \sharp* Q'$› ‹$A_Q \sharp* Q'$› ‹$A_P \sharp* C$› ‹$A_Q \sharp* C$›
have *Prop C* Ψ $(P \parallel Q)$ $(p \cdot \alpha)$ $(p \cdot (P \parallel Q'))$ $(A_P @ A_Q)$ $(\Psi_P \otimes \Psi_Q)$
by(*rule-tac rAlpha*) *auto*
with *αEq Q'eq* ‹*distinctPerm p*› **show** *?case* **by** *simp*
next
case(*cComm1* Ψ Ψ_Q *P M N P'* A_P Ψ_P *Q K xvec Q'* A_Q *C* α *P''*)
thus *?case* **using** *rComm1*
apply(*auto*)
apply(*drule-tac x*=$M\langle\!|N|\!\rangle$ **in** *meta-spec*)
apply(*drule-tac x*=$K(\!|\nu * xvec|\!)\langle N \rangle$ **in** *meta-spec*)
apply(*drule-tac x*=*P'* **in** *meta-spec*)
apply(*drule-tac x*=*Q'* **in** *meta-spec*)
apply *auto*
apply(*drule-tac x*=Ψ **in** *meta-spec*)
apply(*drule-tac x*=Ψ_Q **in** *meta-spec*)
apply(*drule-tac x*=*P* **in** *meta-spec*)
apply(*drule-tac x*=*M* **in** *meta-spec*)
apply(*drule-tac x*=*N* **in** *meta-spec*)
apply(*drule-tac x*=*P'* **in** *meta-spec*)
apply(*drule-tac x*=A_P **in** *meta-spec*)
apply(*drule-tac x*=Ψ_P **in** *meta-spec*)
apply(*drule-tac x*=*Q* **in** *meta-spec*)
apply(*drule-tac x*=*K* **in** *meta-spec*)
apply(*drule-tac x*=*xvec* **in** *meta-spec*)
apply(*drule-tac x*=*Q'* **in** *meta-spec*)
apply(*drule-tac x*=A_Q **in** *meta-spec*)
apply *auto*

apply(*subgoal-tac* $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (K\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle)\ Q'\ A_Q\ \Psi_Q$)
apply(*subgoal-tac* $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (M\langle\!|N|\!\rangle)\ P'\ A_P\ \Psi_P$)
by(*auto simp add*: *residualInject*)
next
case(*cComm2* $\Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C\ \alpha\ Q''$)
thus *?case* **using** *rComm2*
apply(*drule-tac* $x{=}M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle$ **in** *meta-spec*)
apply(*drule-tac* $x{=}K\langle\!|N|\!\rangle$ **in** *meta-spec*)
apply(*drule-tac* $x{=}P'$ **in** *meta-spec*)
apply(*drule-tac* $x{=}Q'$ **in** *meta-spec*)
apply *auto*
apply(*drule-tac* $x{=}\Psi$ **in** *meta-spec*)
apply(*drule-tac* $x{=}\Psi_Q$ **in** *meta-spec*)
apply(*drule-tac* $x{=}P$ **in** *meta-spec*)
apply(*drule-tac* $x{=}M$ **in** *meta-spec*)
apply(*drule-tac* $x{=}xvec$ **in** *meta-spec*)
apply(*drule-tac* $x{=}N$ **in** *meta-spec*)
apply(*drule-tac* $x{=}P'$ **in** *meta-spec*)
apply(*drule-tac* $x{=}A_P$ **in** *meta-spec*)
apply(*drule-tac* $x{=}\Psi_P$ **in** *meta-spec*)
apply(*drule-tac* $x{=}Q$ **in** *meta-spec*)
apply(*drule-tac* $x{=}K$ **in** *meta-spec*)
apply(*drule-tac* $x{=}Q'$ **in** *meta-spec*)
apply(*drule-tac* $x{=}A_Q$ **in** *meta-spec*)
apply *auto*
apply(*subgoal-tac* $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ (M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle)\ P'\ A_P\ \Psi_P$)
apply(*subgoal-tac* $\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ (K\langle\!|N|\!\rangle)\ Q'\ A_Q\ \Psi_Q$)
by(*auto simp add*: *residualInject*)
next
case(*cOpen* $\Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C\ \alpha\ P''$)
note $\langle M\langle\!|\nu{*}(xvec@x\#yvec)|\!\rangle\langle N\rangle \prec P' = \alpha \prec P''\rangle$
moreover from $\langle xvec\ \sharp{*}\ \alpha\rangle\ \langle x\ \sharp\ \alpha\rangle\ \langle yvec\ \sharp{*}\ \alpha\rangle$ **have** $(xvec@x\#yvec)\ \sharp{*}\ (bn\ \alpha)$
by *auto*
moreover from $\langle xvec\ \sharp{*}\ yvec\rangle\ \langle x\ \sharp\ xvec\rangle\ \langle x\ \sharp\ yvec\rangle\ \langle distinct\ xvec\rangle\ \langle distinct\ yvec\rangle$
have *distinct*$(xvec@x\#yvec)$
by(*auto simp add*: *fresh-star-def*) (*simp add*: *fresh-def name-list-supp*)
moreover note $\langle distinct(bn\ \alpha)\rangle$
moreover from $\langle xvec\ \sharp{*}\ M\rangle\ \langle x\ \sharp\ M\rangle\ \langle yvec\ \sharp{*}\ M\rangle$ **have** $(xvec@x\#yvec)\ \sharp{*}\ M$
by *auto*
hence $(xvec@x\#yvec)\ \sharp{*}\ (M\langle\!|\nu{*}(xvec@x\#yvec)|\!\rangle\langle N\rangle \prec P')$ **by** *auto*
moreover from $\langle bn\ \alpha\ \sharp{*}\ subject\ \alpha\rangle$ **have** $bn\ \alpha\ \sharp{*}\ (\alpha \prec P'')$ **by** *simp*
ultimately obtain p **where** S: $(set\ p) \subseteq (set(xvec@x\#yvec)) \times (set(p \cdot (xvec@x\#yvec)))$
and *distinctPerm p*
and αeq: $\alpha = (p \cdot M)\langle\!|\nu{*}(p \cdot (xvec@x\#yvec))|\!\rangle\langle(p \cdot N)\rangle$ **and** $P'eq$: P''
$= (p \cdot P')$
and A: $(xvec@x\#yvec)\ \sharp{*}\ ((p \cdot M)\langle\!|\nu{*}(p \cdot (xvec@x\#yvec))|\!\rangle\langle(p \cdot N)\rangle)$
and B: $(p \cdot (xvec@x\#yvec))\ \sharp{*}\ (M\langle\!|\nu{*}(xvec@x\#yvec)|\!\rangle\langle N\rangle)$
and C: $(p \cdot (xvec@x\#yvec))\ \sharp{*}\ P'$
by(*rule-tac residualEq*) (*assumption* | *simp*)+

note ‹$\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$› ‹$x \in (supp\ N)$›

moreover {
fix C
from ‹$xvec \sharp* M$› ‹$yvec \sharp* M$› **have** $(xvec@yvec) \sharp* M$ **by** *simp*
moreover from ‹*distinct xvec*› ‹*distinct yvec*› ‹$xvec \sharp* yvec$› **have** *distinct*$(xvec@yvec)$
by *auto* (*simp add*: *fresh-star-def name-list-supp fresh-def*)
ultimately have *Prop* $C\ \Psi\ P\ (M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle)\ P'\ A_P\ \Psi_P$ **using** ‹$A_P \sharp* xvec$› ‹$A_P \sharp* yvec$› ‹$A_P \sharp* M$› ‹$A_P \sharp* N$›
by(*rule-tac cOpen*) *auto*
}
moreover obtain y::*name* **where** $y \sharp \Psi$ **and** $y \neq x$ **and** $y \sharp P$ **and** $y \sharp xvec$ **and** $y \sharp yvec$ **and** $y \sharp \alpha$ **and** $y \sharp P'$ **and** $y \sharp A_P$ **and** $y \sharp \Psi_P$ **and** $y \sharp M$ **and** $y \sharp N$ **and** $y \sharp C$ **and** $y \sharp p$
by(*generate-fresh name*) *auto*
moreover note ‹$x \sharp \Psi$› ‹$x \sharp M$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* P$› ‹$xvec \sharp* M$›
‹$yvec \sharp* \Psi$› ‹$yvec \sharp* P$› ‹$yvec \sharp* M$› ‹$yvec \sharp* C$› ‹$x \sharp C$› ‹$xvec \sharp* C$› ‹*distinct xvec*› ‹*distinct yvec*›
‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› ‹*distinct* A_P› ‹$x \sharp A_P$› ‹$xvec \sharp* yvec$› ‹$xvec \sharp* \Psi_P$›
‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* xvec$› ‹$A_P \sharp* yvec$› ‹$A_P \sharp* N$› ‹$A_P \sharp* P'$› ‹$A_P \sharp* C$›
ultimately have *Prop* $C\ \Psi\ ((\!|\nu x|\!)P)\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle([(x,\ y)] \cdot N)\rangle)\ ([(x,\ y)] \cdot P')\ (x\#A_P)\ \Psi_P$
by(*rule-tac rOpen*)
moreover have $(([(x,\ y)] \cdot p) \cdot [(x,\ y)] \cdot M) = [(x,\ y)] \cdot p \cdot M$
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹$y \sharp M$› ‹$x \sharp \alpha$› αeq ‹$y \sharp p$› ‹$x \sharp M$› **have** *D*: $(([(x,\ y)] \cdot p) \cdot M) = p \cdot M$
by(*auto simp add*: *eqvts freshChainSimps*)
moreover have $(([(x,\ y)] \cdot p) \cdot [(x,\ y)] \cdot xvec) = [(x,\ y)] \cdot p \cdot xvec$
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹$y \sharp xvec$› ‹$x \sharp \alpha$› αeq ‹$y \sharp p$› ‹$x \sharp xvec$› **have** *E*: $(([(x,\ y)] \cdot p) \cdot xvec) = p \cdot xvec$
by(*auto simp add*: *eqvts freshChainSimps*)
moreover have $(([(x,\ y)] \cdot p) \cdot [(x,\ y)] \cdot yvec) = [(x,\ y)] \cdot p \cdot yvec$
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹$y \sharp yvec$› ‹$x \sharp \alpha$› αeq ‹$y \sharp p$› ‹$x \sharp yvec$› **have** *F*: $(([(x,\ y)] \cdot p) \cdot yvec) = p \cdot yvec$
by(*auto simp add*: *eqvts freshChainSimps*)
moreover have $(([(x,\ y)] \cdot p) \cdot [(x,\ y)] \cdot x) = [(x,\ y)] \cdot p \cdot x$
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹$y \neq x$› ‹$y \sharp p$› **have** *G*: $(([(x,\ y)] \cdot p) \cdot y) = p \cdot x$
apply(*simp add*: *freshChainSimps calc-atm*)
apply(*subgoal-tac* $y \neq p \cdot x$)
apply(*clarsimp*)
using *A* αeq
apply(*simp add*: *eqvts*)

apply(*subst fresh-atm*[*symmetric*])
apply(*simp only*: *freshChainSimps*)
by *simp*
moreover have (([(*x*, *y*)] ∙ *p*) ∙ [(*x*, *y*)] ∙ *N*) = [(*x*, *y*)] ∙ *p* ∙ *N*
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹*y* ♯ *N*› ‹*x* ♯ α› ‹*y* ♯ *p*› α*eq* **have** *H*: (([(*x*, *y*)] ∙ *p*) ∙ [(*x*, *y*)] ∙ *N*) = *p* ∙ *N*
by(*auto simp add*: *eqvts freshChainSimps*)
moreover have (([(*x*, *y*)] ∙ *p*) ∙ [(*x*, *y*)] ∙ *P*′) = [(*x*, *y*)] ∙ *p* ∙ *P*′
by(*subst perm-compose*[*symmetric*]) *simp*
with ‹*y* ♯ *P*′› ‹*x* ♯ *P*′′› ‹*y* ♯ *p*› *P*′*eq* **have** *I*: (([(*x*, *y*)] ∙ *p*) ∙ [(*x*, *y*)] ∙ *P*′) = *p* ∙ *P*′
by(*auto simp add*: *eqvts freshChainSimps*)
from ‹*y* ♯ *p*› ‹*y* ≠ *x*› **have** *y* ≠ *p* ∙ *x*
apply(*subst fresh-atm*[*symmetric*])
apply(*simp only*: *freshChainSimps*)
by *simp*
moreover from *S* **have** ([(*x*, *y*)] ∙ *set p*) ⊆ [(*x*, *y*)] ∙ (*set*(*xvec*@*x*#*yvec*) × *set*(*p* ∙ (*xvec*@*x*#*yvec*)))
by(*simp*)
with ‹*y* ≠ *p* ∙ *x*› ‹(([(*x*, *y*)] ∙ *p*) ∙ *y*) = *p* ∙ *x*› ‹*x* ♯ *xvec*› ‹*y* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*y* ♯ *yvec*› ‹*y* ♯ *p*› ‹*x* ♯ α› α*eq* **have**
set([(*x*, *y*)] ∙ *p*) ⊆ *set*(*xvec*@*y*#*yvec*) × *set*(([(*x*, *y*)] ∙ *p*) ∙ (*xvec*@*y*#*yvec*))
by(*simp add*: *eqvts calc-atm perm-compose*)
moreover note ‹*xvec* ♯* Ψ› ‹*yvec* ♯* Ψ› ‹*xvec* ♯* *P*› ‹*yvec* ♯* *P*› ‹*xvec* ♯* *M*› ‹*yvec* ♯* *M*›
‹*yvec* ♯* *C*› *S* ‹*distinctPerm p*› ‹*x* ♯ *C*› ‹*xvec* ♯* *C*› ‹*xvec* ♯* Ψ*P*› ‹*yvec* ♯* Ψ*P*› ‹*x* ♯ Ψ›
‹*A*P ♯* *xvec*› ‹*x* ♯ *A*P*› ‹*A*P ♯* *yvec*› ‹*A*P ♯* *M*› ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*x* ♯ *M*› ‹*x* ♯ *A*P*› ‹*A*P ♯* *N*›
A B C α*eq* ‹*A*P ♯* α› ‹*y* ♯ Ψ› ‹*y* ≠ *x*› ‹*y* ♯ *P*› ‹*y* ♯ *M*› ‹*y* ♯ Ψ*P*› ‹*y* ♯ *C*› ‹*xvec* ♯* α› ‹*x* ♯ α› ‹*yvec* ♯* α› ‹*y* ♯ α› ‹*A*P ♯* *P*› ‹*A*P ♯* Ψ› ‹*y* ♯ *A*P*› ‹*y* ♯ *N*› ‹*A*P ♯* *P*′› ‹*y* ♯ *P*′› ‹*A*P ♯* *C*› *P*′*eq*
ultimately have *Prop C* Ψ (⦇ν*x*⦈*P*) (([(*x*, *y*)] ∙ *p*) ∙ (*M*⦇ν*(*xvec*@*y*#*yvec*)⦈⟨([(*x*, *y*)] ∙ *N*)⟩)) (([(*x*, *y*)] ∙ *p*) ∙ [(*x*, *y*)] ∙ *P*′) (*x*#*A*P*) Ψ*P*
apply(*rule-tac* α=*M*⦇ν*(*xvec*@*y*#*yvec*)⦈⟨([(*x*, *y*)] ∙ *N*)⟩ **in** *rAlpha*)
apply(*assumption* | *simp*)+
apply(*simp add*: *eqvts*)
apply(*assumption* | *simp add*: *abs-fresh*)+
apply(*simp add*: *fresh-left calc-atm*)
apply(*assumption* | *simp*)+
apply(*simp add*: *fresh-left calc-atm*)
apply(*assumption* | *simp*)+
by(*simp add*: *eqvts fresh-left*)+
with α*eq* *P*′*eq* *D E F G H I* **show** *?case*
by(*simp add*: *eqvts*)
next
case(*cScope* Ψ *P* α *P*′ *x* *A*P* Ψ*P* *C* α′ *P*′′)
note ‹α ≺ (⦇ν*x*⦈*P*′) = α′ ≺ *P*′′›
moreover from ‹*bn* α ♯* α′› **have** *bn* α ♯* (*bn* α′) **by** *auto*

moreover note ‹*distinct* (*bn* α)› ‹*distinct*(*bn* α')›
moreover from ‹*bn* α $\sharp*$ *subject* α› ‹*bn* α' $\sharp*$ *subject* α'›
have *bn* α $\sharp*$ ($\alpha \prec (\!|\nu x|\!) P'$) **and** *bn* α' $\sharp*$ ($\alpha' \prec P''$) **by** *simp*+
ultimately obtain p **where** S: (*set* p) $\subseteq$ (*set*(*bn* α)) $\times$ (*set*(*bn*($p \cdot \alpha$))) **and** *distinctPerm* p
and αEq: $\alpha' = p \cdot \alpha$ **and** $P'eq$: $P'' = p \cdot ((\!|\nu x|\!) P')$ **and** (*bn*($p \cdot \alpha$)) $\sharp*$ α
and (*bn*($p \cdot \alpha$)) $\sharp*$ ($(\!|\nu x|\!) P'$)
by(*rule residualEq*)

note ‹$\Psi \rhd P \longmapsto \alpha \prec P'$›
moreover from ‹*bn* α $\sharp*$ *subject* α› ‹*distinct*(*bn* α)›
have $\bigwedge C$. *Prop* C Ψ P α P' A_P Ψ_P **by**(*rule-tac cScope*) *auto*

moreover note ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P› ‹*bn* α $\sharp*$ *subject* α› ‹*bn* α $\sharp*$ Ψ_P›
‹$x \sharp C$› ‹*bn* α $\sharp*$ C› ‹*distinct*(*bn* α)› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$›
‹*distinct* A_P› ‹$x \sharp A_P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* P'$› ‹$A_P \sharp* C$›
ultimately have *Prop* C Ψ ($(\!|\nu x|\!) P$) α ($(\!|\nu x|\!) P'$) ($x \# A_P$) Ψ_P
by(*rule-tac rScope*)
with ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P› ‹$x \sharp \alpha$› ‹*bn* α $\sharp*$ *subject* α› ‹*bn* α $\sharp*$ C› ‹*bn* α $\sharp*$ (*bn* α')› S ‹*distinctPerm* p› ‹*bn*($p \cdot \alpha$) $\sharp*$ α› ‹*bn*($p \cdot \alpha$) $\sharp*$ ($(\!|\nu x|\!) P'$)› ‹$A_P \sharp* \alpha$› ‹$A_P \sharp* \alpha'$› αEq ‹$x \sharp \alpha'$› ‹*bn* α $\sharp*$ Ψ_P› ‹*bn* α $\sharp*$ α'› ‹$x \sharp \Psi$› ‹$A_P \sharp* \Psi$› ‹$x \sharp A_P$› ‹$A_P \sharp* P$› ‹$A_P \sharp* P'$› ‹$x \sharp C$› ‹$A_P \sharp* C$›
have *Prop* C Ψ ($(\!|\nu x|\!) P$) ($p \cdot \alpha$) ($p \cdot ((\!|\nu x|\!) P')$) ($x \# A_P$) Ψ_P
by(*rule-tac rAlpha*) (*simp add*: *abs-fresh*)+
with αEq $P'eq$ ‹*distinctPerm* p› **show** *?case* **by** *simp*
next
case(*cBang* Ψ P *Rs* A_P Ψ_P C α)
thus *?case* **by**(*rule-tac rBang*) *auto*
qed

lemma *inputFrameInduct*[*consumes 3*, *case-names cAlpha cInput cCase cPar1 cPar2 cScope cBang*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and N :: *'a*
and P' :: (*'a*, *'b*, *'c*) *psi*
and *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
'a ⇒ *'a* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *name list* ⇒ *'b* ⇒ *bool*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and *rAlpha*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ p\ C$. $[\![A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* M; A_P \sharp* N; A_P \sharp* P'; A_P \sharp* (p \cdot A_P); A_P \sharp* C;$

$set\ p \subseteq set\ A_P \times set(p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P]\!] \Longrightarrow Prop\ C\ \Psi$
$P\ M\ N\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$

and *rInput*: $\bigwedge \Psi\ M\ K\ xvec\ N\ Tvec\ P\ C.$
$[\![\Psi \vdash M \leftrightarrow K;\ distinct\ xvec;\ set\ xvec \subseteq supp\ N;$
$length\ xvec = length\ Tvec;\ xvec \sharp* \Psi;$
$xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (M(\!|\lambda* xvec\ N|\!).P)$
$K\ (N[xvec ::= Tvec])\ (P[xvec ::= Tvec])\ ([])\ (\mathbf{1})$

and *rCase*: $\bigwedge \Psi\ P\ M\ N\ P'\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$(\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \Psi_P \simeq \mathbf{1};$
$(supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ M\ N\ P'\ ([])\ (\mathbf{1})$

and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ M\ N\ P'\ A_P\ \Psi_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ M\ N\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ N\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ N\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rScope*: $\bigwedge \Psi\ P\ M\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P;\ x \sharp \Psi;\ x \sharp M;\ x \sharp N;$
$x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* C;\ x \sharp C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ N\ ((\!|\nu x|\!)P')\ (x \# A_P)\ \Psi_P$

and *rBang*: $\bigwedge \Psi\ P\ M\ N\ P'\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto M(\!|N|\!) \prec P';\ guarded\ P;\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ N\ P'\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow$

Prop C Ψ (!*P*) *M N P'* ([]) (**1**)
shows *Prop C* Ψ *P M N P' A*$_P$ Ψ_P
using *assms*
by(*nominal-induct* Ψ *P Rs*==*M*(⇩*N*) ≺ *P' A*$_P$ Ψ_P *avoiding*: *C arbitrary*: *P' rule*: *semanticsFrameInduct*)
(*auto simp add*: *residualInject*)

lemma *outputFrameInduct*[*consumes 3*, *case-names cAlpha cOutput cCase cPar1 cPar2 cOpen cScope cBang*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *M* :: *'a*
and *B* :: (*'a*, *'b*, *'c*) *boundOutput*
and *A*$_P$ :: *name list*
and Ψ_P :: *'b*
and *Prop* :: *'d*::*fs-name* ⇒ *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
'a ⇒ (*'a*, *'b*, *'c*) *boundOutput* ⇒ *name list* ⇒ *'b* ⇒ *bool*
and *C* :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto ROut\ M\ B$
and *FrP*: *extractFrame P* = ⟨*A*$_P$, Ψ_P⟩
and *distinct A*$_P$
and *rAlpha*: ⋀Ψ *P M A*$_P$ Ψ_P *p B C*. ⟦*A*$_P$ ♯* Ψ; *A*$_P$ ♯* *P*; *A*$_P$ ♯* *M*; *A*$_P$ ♯* (*p* ∙ *A*$_P$); *A*$_P$ ♯* *B*; *A*$_P$ ♯* *C*;
set p ⊆ *set A*$_P$ × *set*(*p* ∙ *A*$_P$); *distinctPerm p*;
Prop C Ψ *P M B A*$_P$ Ψ_P⟧ ⟹ *Prop C* Ψ *P M B* (*p* ∙ *A*$_P$) (*p* ∙ Ψ_P)
and *rOutput*: ⋀Ψ *M K N P C*. $\Psi \vdash M \leftrightarrow K$ ⟹ *Prop C* Ψ (*M*⟨*N*⟩.*P*) *K* (*N* ≺' *P*) ([]) (**1**)
and *rCase*: ⋀Ψ *P M B* φ *Cs A*$_P$ Ψ_P *C*. ⟦$\Psi \rhd P \longmapsto (ROut\ M\ B)$; *extractFrame P* = ⟨*A*$_P$, Ψ_P⟩; *distinct A*$_P$; ⋀*C*. *Prop C* Ψ *P M B A*$_P$ Ψ_P;
(φ, *P*) *mem Cs*; $\Psi \vdash \varphi$; *guarded P*; $\Psi_P \simeq \mathbf{1}$; (*supp* Ψ_P) = ({}::*name set*);
A$_P$ ♯* Ψ; *A*$_P$ ♯* *P*; *A*$_P$ ♯* *M*; *A*$_P$ ♯* *B*; *A*$_P$ ♯* *C*⟧ ⟹ *Prop C* Ψ (*Cases Cs*) *M B* ([]) (**1**)
and *rPar1*: ⋀Ψ Ψ_Q *P M xvec N P' A*$_Q$ *Q A*$_P$ Ψ_P *C*.
⟦$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$;
extractFrame P = ⟨*A*$_P$, Ψ_P⟩; *distinct A*$_P$;
extractFrame Q = ⟨*A*$_Q$, Ψ_Q⟩; *distinct A*$_Q$;
⋀*C*. *Prop C* ($\Psi \otimes \Psi_Q$) *P M* (⦇ν*xvec⦈*N* ≺' *P'*) *A*$_P$ Ψ_P;
A$_P$ ♯* *P*; *A*$_P$ ♯* *Q*; *A*$_P$ ♯* Ψ; *A*$_P$ ♯* *M*; *A*$_P$ ♯* *xvec*; *A*$_P$ ♯* *N*; *A*$_P$ ♯* *P'*; *A*$_P$ ♯* *A*$_Q$; *A*$_P$ ♯* Ψ_Q;
A$_Q$ ♯* *P*; *A*$_Q$ ♯* *Q*; *A*$_Q$ ♯* Ψ; *A*$_Q$ ♯* *M*; *A*$_Q$ ♯* *xvec*; *A*$_Q$ ♯* *N*; *A*$_Q$ ♯* *P'*; *A*$_Q$ ♯* Ψ_P;
xvec ♯* Ψ; *xvec* ♯* *P*; *xvec* ♯* *Q*; *xvec* ♯* *M*; *xvec* ♯* Ψ_P; *xvec* ♯* Ψ_Q;
A$_P$ ♯* *C*; *A*$_Q$ ♯* *C*; *xvec* ♯* *C*⟧ ⟹
Prop C Ψ (*P* ∥ *Q*) *M* (⦇ν*xvec⦈*N* ≺' (*P'* ∥ *Q*)) (*A*$_P$@*A*$_Q$) ($\Psi_P \otimes \Psi_Q$)
and *rPar2*: ⋀Ψ Ψ_P *Q M xvec N Q' A*$_P$ *P A*$_Q$ Ψ_Q *C*.

$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ M\ ((\!|\nu{*}xvec|\!)N \prec' Q')\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* Q;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;$
$A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ M\ ((\!|\nu{*}xvec|\!)N \prec' (P \parallel Q'))\ (A_P@A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rOpen*: $\bigwedge \Psi\ P\ M\ xvec\ yvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu{*}(xvec@yvec)|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu{*}(xvec@yvec)|\!)N \prec' P')\ A_P\ \Psi_P;\ x \in supp\ N;\ x \sharp \Psi;\ x \sharp M;$
$x \sharp A_P;\ x \sharp xvec;\ x \sharp yvec;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* xvec;\ A_P \sharp* yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;$
$yvec \sharp* \Psi;\ yvec \sharp* P;\ yvec \sharp* M;\ A_P \sharp* C;\ x \sharp C;\ xvec \sharp* C;\ yvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu{*}(xvec@x\#yvec)|\!)N \prec' P')\ (x\#A_P)\ \Psi_P$

and *rScope*: $\bigwedge \Psi\ P\ M\ xvec\ N\ P'\ x\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ P\ M\ ((\!|\nu{*}xvec|\!)N \prec' P')\ A_P\ \Psi_P;$
$x \sharp \Psi;\ x \sharp M;\ x \sharp xvec;\ x \sharp N;\ x \sharp A_P;\ A_P \sharp* \Psi;\ A_P \sharp* P;$
$A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* xvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* \Psi_P;$
$A_P \sharp* C;\ x \sharp C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ M\ ((\!|\nu{*}xvec|\!)N \prec' ((\!|\nu x|\!)P'))\ (x\#A_P)\ \Psi_P$

and *rBang*: $\bigwedge \Psi\ P\ M\ B\ A_P\ \Psi_P\ C.$
$[\![\Psi \rhd P \parallel !P \longmapsto ROut\ M\ B;\ guarded\ P;\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B\ A_P\ (\Psi_P \otimes \mathbf{1});\ \Psi_P \simeq \mathbf{1};\ supp\ \Psi_P = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ B\ ([])\ (\mathbf{1})$

shows $Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P$

proof −

{

fix B

assume $\Psi \rhd P \longmapsto ROut\ M\ B$

hence $Prop\ C\ \Psi\ P\ M\ B\ A_P\ \Psi_P$ **using** $FrP\ \langle distinct\ A_P\rangle$

proof(*nominal-induct* $\Psi\ P\ Rs==ROut\ M\ B\ A_P\ \Psi_P$ *avoiding*: C *arbitrary*: B *rule*: *semanticsFrameInduct*)

case *cAlpha*

```
    thus ?case by(fastforce intro: rAlpha)
  next
    case cInput
    thus ?case by(simp add: residualInject)
  next
    case cOutput
    thus ?case by(force intro: rOutput simp add: residualInject)
  next
    case cCase
    thus ?case by(force intro: rCase simp add: residualInject)
  next
    case cPar1
    thus ?case
      by(fastforce intro: rPar1 simp add: residualInject)
  next
    case cPar2
    thus ?case
      by(fastforce intro: rPar2 simp add: residualInject)
  next
    case cComm1
    thus ?case by(simp add: residualInject)
  next
    case cComm2
    thus ?case by(simp add: residualInject)
  next
    case cOpen
    thus ?case by(fastforce intro: rOpen simp add: residualInject)
  next
    case cScope
    thus ?case by(force intro: rScope simp add: residualInject)
  next
    case cBang
    thus ?case by(force intro: rBang simp add: residualInject)
  qed
 }
 with Trans show ?thesis by(simp add: residualInject)
qed

lemma tauFrameInduct[consumes 3, case-names cAlpha cCase cPar1 cPar2 cComm1
cComm2 cScope cBang]:
 fixes Ψ    :: 'b
 and   P    :: ('a, 'b, 'c) psi
 and   P'   :: ('a, 'b, 'c) psi
 and   Prop :: 'd::fs-name ⇒ 'b ⇒ ('a, 'b, 'c) psi ⇒
               ('a, 'b, 'c) psi ⇒ name list ⇒ 'b ⇒ bool
 and   C    :: 'd::fs-name

 assumes Trans: Ψ ▷ P ⟼τ ≺ P'
 and     FrP: extractFrame P = ⟨A_P, Ψ_P⟩
```

and *distinct* A_P

and *rAlpha*: $\bigwedge \Psi\ P\ P'\ A_P\ \Psi_P\ p\ C.\ [\![A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* P';\ A_P \sharp* (p \cdot A_P);\ A_P \sharp* C;$
$set\ p \subseteq set\ A_P \times set\ (p \cdot A_P);\ distinctPerm\ p;$
$Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P]\!] \Longrightarrow Prop\ C\ \Psi\ P\ P'\ (p \cdot A_P)\ (p \cdot \Psi_P)$

and *rCase*: $\bigwedge \Psi\ P\ P'\ \varphi\ Cs\ A_P\ \Psi_P\ C.\ [\![\Psi \rhd P \longmapsto \tau \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;\ \bigwedge C.\ Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P;$
$(\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P;\ \ \Psi_P \simeq \mathbf{1};\ (supp\ \Psi_P) = (\{\}::name\ set);$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* P';\ A_P \sharp* C]\!] \Longrightarrow Prop\ C\ \Psi\ (Cases\ Cs)\ P'\ ([])\ (\mathbf{1})$

and *rPar1*: $\bigwedge \Psi\ \Psi_Q\ P\ P'\ A_Q\ Q\ A_P\ \Psi_P\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \tau \prec P';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_Q)\ P\ P'\ A_P\ \Psi_P;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* P';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* P';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (P' \parallel Q)\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rPar2*: $\bigwedge \Psi\ \Psi_P\ Q\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ C.$
$[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \tau \prec Q';$
$extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\bigwedge C.\ Prop\ C\ (\Psi \otimes \Psi_P)\ Q\ Q'\ A_Q\ \Psi_Q;$
$A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* \Psi;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* \Psi_Q;$
$A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* \Psi;\ A_Q \sharp* Q';\ A_Q \sharp* \Psi_P;$
$A_P \sharp* C;\ A_Q \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ (P \parallel Q')\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm1*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';$
$A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* K;\ A_Q \sharp* Q';$
$A_Q \sharp* xvec;\ xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;$
$xvec \sharp* Q;\ xvec \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ ((\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rComm2*: $\bigwedge \Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ distinct\ xvec;$

$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* K$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$; $xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* Q$; $xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel Q)\ ((\!|\nu * xvec|\!)(P' \parallel Q'))\ (A_P @ A_Q)\ (\Psi_P \otimes \Psi_Q)$

and *rScope*: $\bigwedge \Psi\ P\ P'\ x\ A_P\ \Psi_P\ C.$
$\llbracket \Psi \rhd P \longmapsto \tau \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$; $\bigwedge C.\ Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P$; $x \sharp \Psi$; $x \sharp A_P$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* P'$; $A_P \sharp* C$; $x \sharp C\rrbracket \Longrightarrow$
$Prop\ C\ \Psi\ ((\!|\nu x|\!)P)\ ((\!|\nu x|\!)P')\ (x \# A_P)\ \Psi_P$

and *rBang*: $\bigwedge \Psi\ P\ P'\ A_P\ \Psi_P\ C.$
$\llbracket \Psi \rhd P \parallel !P \longmapsto \tau \prec P'$; $guarded\ P$; $extractFrame\ P = \langle A_P, \Psi_P \rangle$; $distinct\ A_P$; $\bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ P'\ A_P\ (\Psi_P \otimes \mathbf{1})$; $\Psi_P \simeq \mathbf{1}$; $supp\ \Psi_P = (\{\}::name\ set)$; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* P'$; $A_P \sharp* C\rrbracket \Longrightarrow Prop\ C\ \Psi\ (!P)\ P'\ ([])\ (\mathbf{1})$

shows $Prop\ C\ \Psi\ P\ P'\ A_P\ \Psi_P$
using *Trans FrP* ‹*distinct* A_P›
proof(*nominal-induct* $\Psi\ P\ Rs == \tau \prec P'\ A_P\ \Psi_P$ *avoiding*: C *arbitrary*: P' *rule*: *semanticsFrameInduct*)
case *cAlpha*
thus *?case* **by**(*force intro*: *rAlpha simp add*: *residualInject*)
next
case *cInput*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cOutput*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cCase*
thus *?case* **by**(*force intro*: *rCase simp add*: *residualInject*)
next
case *cPar1*
thus *?case* **by**(*force intro*: *rPar1 simp add*: *residualInject*)
next
case *cPar2*
thus *?case* **by**(*force intro*: *rPar2 simp add*: *residualInject*)
next
case *cComm1*
thus *?case* **by**(*force intro*: *rComm1 simp add*: *residualInject*)
next
case *cComm2*
thus *?case* **by**(*force intro*: *rComm2 simp add*: *residualInject*)
next
case *cOpen*

```
  thus ?case by(simp add: residualInject)
next
  case cScope
  thus ?case by(force intro: rScope simp add: residualInject)
next
  case cBang
  thus ?case by(force intro: rBang simp add: residualInject)
qed

lemma inputFreshDerivative:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
  and   N  :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   x  :: name

  assumes Ψ ▷ P ⟼M⦇N⦈ ≺ P'
  and     x ♯ P
  and     x ♯ N

  shows x ♯ P'
proof -
  have bn(M⦇N⦈) ♯* subject(M⦇N⦈) and distinct(bn(M⦇N⦈)) by simp+
  with ‹Ψ ▷ P ⟼M⦇N⦈ ≺ P'› show ?thesis using ‹x ♯ P› ‹x ♯ N›
  proof(nominal-induct Ψ P α==M⦇N⦈ P' avoiding: x rule: semanticsInduct)
    case(cAlpha Ψ P α P' p x)
    thus ?case by simp
  next
    case(cInput Ψ M' K xvec N' Tvec P x)
      from ‹K⦇(N'[xvec::=Tvec])⦈ = M⦇N⦈› have M = K and NeqN': N =
N'[xvec::=Tvec] by(simp add: action.inject)+
    note ‹length xvec = length Tvec› ‹distinct xvec› then
    moreover have x ♯ Tvec using ‹set xvec ⊆ supp N'› ‹x ♯ N› NeqN'
      by(blast intro: substTerm.subst3)
    moreover from ‹xvec ♯* x› ‹x ♯ M'⦇λ*xvec N'⦈.P›
   have x ♯ P by(simp add: inputChainFresh) (simp add: name-list-supp fresh-def)
    ultimately show ?case using ‹xvec ♯* x› by auto
  next
    case(cOutput Ψ M  K N P x)
    thus ?case by simp
  next
    case(cCase Ψ P P' φ Cs x)
    thus ?case by(induct Cs, auto)
  next
    case(cPar1 Ψ Ψ_Q P P' xvec Q x)
    thus ?case by simp
  next
    case(cPar2 Ψ Ψ_P Q Q' xvec P x)
```

```
    thus ?case by simp
  next
    case(cComm1 Ψ Ψ_Q P M N P' A_P Ψ_P Q K xvec Q' A_Q x)
    thus ?case by simp
  next
    case(cComm2 Ψ Ψ_Q P M xwec N P' A_P Ψ_P Q K Q' A_Q x)
    thus ?case by simp
  next
    case(cOpen Ψ P M xvec yvec N P' x y)
    thus ?case by simp
  next
    case(cScope Ψ P P' x y)
    thus ?case by(simp add: abs-fresh)
  next
    case(cBang Ψ P P' x)
    thus ?case by simp
  qed
qed

lemma inputFreshChainDerivative:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   M    :: 'a
  and   N    :: 'a
  and   P'   :: ('a, 'b, 'c) psi
  and   xvec :: name list

  assumes Ψ ▷ P ⟼M⦇N⦈ ≺ P'
  and     xvec ♯* P
  and     xvec ♯* N

  shows xvec ♯* P'
using assms
by(induct xvec)
  (auto intro: inputFreshDerivative)

lemma outputFreshDerivative:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   M    :: 'a
  and   xvec :: name list
  and   N    :: 'a
  and   P'   :: ('a, 'b, 'c) psi
  and   x    :: name

  assumes Ψ ▷ P ⟼M⦇ν*xvec⦈⟨N⟩ ≺ P'
  and     xvec ♯* M
  and     distinct xvec
  and     x ♯ P
```

and $x \sharp xvec$

shows $x \sharp N$
and $x \sharp P'$
proof −
note ‹$\Psi \rhd P \longmapsto M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle \prec P'$›
moreover from ‹$xvec \sharp* M$› **have** $bn(M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle) \sharp* subject(M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle)$ **by** *simp*
moreover from ‹*distinct xvec*› **have** $distinct(bn(M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle))$ **by** *simp*
ultimately show $x \sharp N$ **using** ‹$x \sharp P$› ‹$x \sharp xvec$›
proof(*nominal-induct* Ψ P $\alpha{==}M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle$ P' *avoiding*: *x arbitrary*: *M xvec N rule*: *semanticsInduct*)
case(*cAlpha* Ψ P α P' p x M *xvec* N)
have *S*: *set* $p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **by** *fact*
from ‹$(p \cdot \alpha) = M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle$› **have** $(p \cdot p \cdot \alpha) = p \cdot (M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle)$ **by**(*simp add*: *fresh-star-bij*)
with ‹*distinctPerm p*› **have** $\alpha = (p \cdot M)\langle\!|\nu{*}(p \cdot xvec)|\!\rangle\langle(p \cdot N)\rangle$ **by** *simp*
moreover from ‹$(p \cdot \alpha) = M\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle$› ‹$x \sharp xvec$› **have** $x \sharp (bn(p \cdot \alpha))$ **by** *simp*
with ‹$(bn\ \alpha) \sharp* x$› ‹$x \sharp xvec$› *S* **have** $x \sharp (p \cdot xvec)$
by(*drule-tac pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=*p* **and** *x*=*xvec*]) *simp*
ultimately have $x \sharp (p \cdot N)$ **using** ‹$x \sharp P$› **by**(*rule-tac cAlpha*)
hence $(p \cdot x) \sharp (p \cdot p \cdot N)$ **by**(*simp add*: *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› ‹$bn(\alpha) \sharp* x$› ‹$x \sharp (bn(p \cdot \alpha))$›*S* **show** *?case* **by** *simp*
next
case *cInput*
thus *?case* **by** *simp*
next
case *cOutput*
thus *?case* **by**(*simp add*: *action.inject*)
next
case *cCase*
thus *?case*
by(*rule-tac cCase*) (*auto dest*: *memFresh*)
next
case *cPar1*
thus *?case* **by** *simp*
next
case *cPar2*
thus *?case* **by** *simp*
next
case *cComm1*
thus *?case* **by** *simp*
next
case *cComm2*
thus *?case* **by** *simp*
next

case(*cOpen* Ψ P M *xvec* *yvec* N P' x y M' *zvec* N')
from ‹$M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle = M'(\!|\nu*zvec|\!)\langle N'\rangle$› **have** *zvec* = *xvec@x#yvec* **and** $N = N'$
by(*simp add*: *action.inject*)+
from ‹$y \sharp (\!|\nu x|\!)P$› ‹$x \sharp y$› **have** $y \sharp P$ **by**(*simp add*: *abs-fresh*)
moreover from ‹$y \sharp$ *zvec*› ‹*zvec* = *xvec@x#yvec*›**have** $y \sharp$ (*xvec@yvec*)
by *simp*
ultimately have $y \sharp N$ **by**(*rule-tac cOpen*) *auto*
with ‹$N = N'$› **show** *?case* **by** *simp*
next
case *cScope*
thus *?case* **by**(*auto simp add*: *abs-fresh*)
next
case *cBang*
thus *?case* **by** *simp*
qed
next
note ‹$\Psi \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$›
moreover from ‹*xvec* $\sharp*$ M› **have** *bn*($M(\!|\nu*xvec|\!)\langle N\rangle$) $\sharp*$ *subject*($M(\!|\nu*xvec|\!)\langle N\rangle$) **by** *simp*
moreover from ‹*distinct xvec*› **have** *distinct*(*bn*($M(\!|\nu*xvec|\!)\langle N\rangle$)) **by** *simp*
ultimately show $x \sharp P'$ **using** ‹$x \sharp P$› ‹$x \sharp$ *xvec*›
proof(*nominal-induct* Ψ P α==$M(\!|\nu*xvec|\!)\langle N\rangle$ P' *avoiding*: *x arbitrary*: *M xvec N rule*: *semanticsInduct*)
case(*cAlpha* Ψ P α P' p x M *xvec* N)
have *S*: *set* $p \subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$)) **by** *fact*
from ‹$(p \cdot \alpha) = M(\!|\nu*xvec|\!)\langle N\rangle$› **have** $(p \cdot p \cdot \alpha) = p \cdot (M(\!|\nu*xvec|\!)\langle N\rangle)$ **by**(*simp add*: *fresh-star-bij*)
with ‹*distinctPerm p*› **have** $\alpha = (p \cdot M)(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle$ **by** *simp*
moreover from ‹$(p \cdot \alpha) = M(\!|\nu*xvec|\!)\langle N\rangle$› ‹$x \sharp$ *xvec*› **have** $x \sharp$ (*bn*($p \cdot \alpha$)) **by** *simp*
with ‹(*bn* α) $\sharp*$ x› ‹$x \sharp$ *xvec*› *S* **have** $x \sharp (p \cdot xvec)$
by(*drule-tac pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=*p* **and** *x*=*xvec*]) *simp*
ultimately have $x \sharp P'$ **using** ‹$x \sharp P$› **by**(*rule-tac cAlpha*)
hence $(p \cdot x) \sharp (p \cdot P')$ **by**(*simp add*: *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› ‹*bn*(α) $\sharp*$ x› ‹$x \sharp$ (*bn*($p \cdot \alpha$))›*S* **show** *?case* **by** *simp*
next
case *cInput*
thus *?case* **by** *simp*
next
case *cOutput*
thus *?case* **by**(*simp add*: *action.inject*)
next
case *cCase*
thus *?case* **by**(*fastforce simp add*: *action.inject dest*: *memFresh*)
next
case *cPar1*

```
    thus ?case by simp
  next
    case cPar2
    thus ?case by simp
  next
    case cComm1
    thus ?case by simp
  next
    case cComm2
    thus ?case by simp
  next
    case(cOpen Ψ P M xvec yvec N P' x y M' zvec N')
   from ‹M⟨⌊ν∗(xvec@x#yvec)⌋⟩⟨N⟩ = M'⟨⌊ν∗zvec⌋⟩⟨N'⟩› have zvec = xvec@x#yvec

      by(simp add: action.inject)
    from ‹y ♯ ⟨⌊νx⌋⟩P› ‹x ♯ y›  have y ♯ P by(simp add: abs-fresh)
    moreover from ‹y ♯ zvec› ‹zvec = xvec@x#yvec›have y ♯ (xvec@yvec)
      by simp
    ultimately show y ♯ P' by(rule-tac cOpen) auto
  next
    case cScope
    thus ?case by(auto simp add: abs-fresh)
  next
    case cBang
    thus ?case by simp
  qed
qed

lemma outputFreshChainDerivative:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   M    :: 'a
  and   xvec :: name list
  and   N    :: 'a
  and   P'   :: ('a, 'b, 'c) psi
  and   yvec :: name list

  assumes Ψ ▷ P ⟼M⟨⌊ν∗xvec⌋⟩⟨N⟩ ≺ P'
  and     xvec ♯∗ M
  and     distinct xvec
  and     yvec ♯∗ P
  and     yvec ♯∗ xvec

  shows yvec ♯∗ N
  and   yvec ♯∗ P'
using assms
by(induct yvec) (auto intro: outputFreshDerivative)

lemma tauFreshDerivative:
```

```
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   P' :: ('a, 'b, 'c) psi
  and   x  :: name

  assumes Ψ ▷ P ⟼τ ≺ P'
  and     x ♯ P

  shows x ♯ P'
proof −
  have bn(τ) ♯* subject(τ) and distinct(bn(τ)) by simp+
  with ‹Ψ ▷ P ⟼τ ≺ P'› show ?thesis using ‹x ♯ P›
  proof(nominal-induct Ψ P α==(τ::('a action)) P' avoiding: x rule: semantic-
sInduct)
    case cAlpha
    thus ?case by simp
  next
    case cInput
    thus ?case by simp
  next
    case cOutput
    thus ?case by simp
  next
    case cCase
    thus ?case by(auto dest: memFresh)
  next
    case cPar1
    thus ?case by simp
  next
    case cPar2
    thus ?case by simp
  next
    case cComm1
    thus ?case
    by(fastforce dest: inputFreshDerivative outputFreshDerivative simp add: resChain-
Fresh)
  next
    case cComm2
    thus ?case
    by(fastforce dest: inputFreshDerivative outputFreshDerivative simp add: resChain-
Fresh)
  next
    case cOpen
    thus ?case by simp
  next
    case cScope
    thus ?case by(simp add: abs-fresh)
  next
    case cBang
```

```
    thus ?case by simp
  qed
qed

lemma tauFreshChainDerivative:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   M    :: 'a
  and   N    :: 'a
  and   P'   :: ('a, 'b, 'c) psi
  and   xvec :: name list

  assumes Ψ ▷ P ⟼τ ≺ P'
  and     xvec ♯* P

  shows xvec ♯* P'
using assms
by(induct xvec) (auto intro: tauFreshDerivative)

lemma freeFreshDerivative:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   α  :: 'a action
  and   P' :: ('a, 'b, 'c) psi
  and   x  :: name

  assumes Ψ ▷ P ⟼α ≺ P'
  and     bn α ♯* subject α
  and     distinct(bn α)
  and     x ♯ α
  and     x ♯ P

  shows   x ♯ P'
using assms
by(rule-tac actionCases[where α=α])
  (auto intro: inputFreshDerivative tauFreshDerivative outputFreshDerivative)

lemma freeFreshChainDerivative:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   α    :: 'a action
  and   P'   :: ('a, 'b, 'c) psi
  and   xvec :: name list

  assumes Ψ ▷ P ⟼α ≺ P'
  and     bn α ♯* subject α
  and     distinct(bn α)
  and     xvec ♯* P
  and     xvec ♯* α
```

shows *xvec* $\sharp*$ *P'*
using *assms*
by(*auto intro*: *freeFreshDerivative simp add*: *fresh-star-def*)

lemma *Input*:
fixes Ψ :: *'b*
and *M* :: *'a*
and *K* :: *'a*
and *xvec* :: *name list*
and *N* :: *'a*
and *Tvec* :: *'a list*

assumes $\Psi \vdash M \leftrightarrow K$
and *distinct xvec*
and *set xvec* $\subseteq$ *supp N*
and *length xvec* = *length Tvec*

shows $\Psi \rhd M(\!|\lambda* xvec\ N|\!).P \longmapsto K(\!|N[xvec::=Tvec]|\!) \prec P[xvec::=Tvec]$
proof −
obtain *p* **where** *xvecFreshPsi*: ((*p*::*name prm*) · (*xvec*::*name list*)) $\sharp*$ Ψ
and *xvecFreshM*: (*p* · *xvec*) $\sharp*$ *M*
and *xvecFreshN*: (*p* · *xvec*) $\sharp*$ *N*
and *xvecFreshK*: (*p* · *xvec*) $\sharp*$ *K*
and *xvecFreshTvec*: (*p* · *xvec*) $\sharp*$ *Tvec*
and *xvecFreshP*: (*p* · *xvec*) $\sharp*$ *P*
and *S*: (*set p*) $\subseteq$ (*set xvec*) × (*set*(*p* · *xvec*))
and *dp*: *distinctPerm p*
by(*rule-tac xvec*=*xvec* **and** *c*=(Ψ, *M*, *K*, *N*, *P*, *Tvec*) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)
note ‹$\Psi \vdash M \leftrightarrow K$›
moreover from ‹*distinct xvec*› **have** *distinct*(*p* · *xvec*)
by *simp*
moreover from ‹(*set xvec*) $\subseteq$ (*supp N*)› **have** (*p* · (*set xvec*)) $\subseteq$ (*p* · (*supp N*))
by *simp*
hence *set*(*p* · *xvec*) $\subseteq$ *supp*(*p* · *N*)
by(*simp add*: *eqvts*)
moreover from ‹*length xvec* = *length Tvec*› **have** *length*(*p* · *xvec*) = *length Tvec*
by *simp*
ultimately have $\Psi \rhd M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P) \longmapsto K(\!|(p \cdot N)[(p \cdot xvec)::=Tvec]|\!) \prec (p \cdot P)[(p \cdot xvec)::=Tvec]$
using *xvecFreshPsi xvecFreshM xvecFreshK xvecFreshTvec*
by(*rule-tac cInput*)
thus *?thesis* **using** *xvecFreshN xvecFreshP S* ‹*length xvec* = *length Tvec*› *dp*
by(*auto simp add*: *inputChainAlpha' substTerm.renaming renaming*)
qed

lemma *residualAlpha*:
fixes *p* :: *name prm*

```
  and   α :: 'a action
  and   P :: ('a, 'b, 'c) psi

  assumes bn(p · α) ♯* object  α
  and     bn(p · α) ♯* P
  and     bn α ♯* subject α
  and     bn(p · α) ♯* subject α
  and     set p ⊆ set(bn α) × set(bn(p · α))

  shows α ≺ P = (p · α) ≺ (p · P)
using assms
apply(rule-tac α=α in actionCases)
apply(simp only: eqvts bn.simps)
apply simp
apply(simp add: boundOutputChainAlpha'' residualInject)
by simp

lemma Par1:
  fixes Ψ   :: 'b
  and   Ψ_Q  :: 'b
  and   P   :: ('a, 'b, 'c) psi
  and   α   :: 'a action
  and   P'  :: ('a, 'b, 'c) psi
  and   A_Q  :: name list
  and   Q   :: ('a, 'b, 'c) psi

  assumes Trans: Ψ ⊗ Ψ_Q ▷ P ⟼α ≺ P'
  and     extractFrame Q = ⟨A_Q, Ψ_Q⟩
  and     bn α ♯* Q
  and     A_Q ♯* Ψ
  and     A_Q ♯* P
  and     A_Q ♯* α

  shows Ψ ▷ P ∥ Q ⟼α ≺ (P' ∥ Q)
proof −
  {
    fix Ψ   :: 'b
    and Ψ_Q  :: 'b
    and P   :: ('a, 'b, 'c) psi
    and α   :: 'a action
    and P'  :: ('a, 'b, 'c) psi
    and A_Q  :: name list
    and Q   :: ('a, 'b, 'c) psi

    assume Ψ ⊗ Ψ_Q ▷ P ⟼α ≺ P'
    and    extractFrame Q = ⟨A_Q, Ψ_Q⟩
    and    bn α ♯* Q
    and    bn α ♯* subject α
    and    A_Q ♯* Ψ
```

and $A_Q \sharp* P$
and $A_Q \sharp* \alpha$
and *distinct* A_Q

have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$
proof −
from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› **have** *distinct*(*bn* α) **by**(*rule boundOutputDistinct*)
obtain *q*::*name prm* **where** $bn(q \cdot \alpha) \sharp* \Psi$ **and** $bn(q \cdot \alpha) \sharp* P$ **and** $bn(q \cdot \alpha) \sharp* Q$ **and** $bn(q \cdot \alpha) \sharp* \alpha$
and $bn(q \cdot \alpha) \sharp* A_Q$ **and** $bn(q \cdot \alpha) \sharp* P'$ **and** $bn(q \cdot \alpha) \sharp* \Psi_Q$
and *Sq*: (*set q*) ⊆ (*set* (*bn* α)) × (*set*($bn(q \cdot \alpha)$))
by(*rule-tac xvec*=*bn* α **and** *c*=(Ψ, *P*, *Q*, α, A_Q, Ψ_Q, P') **in** *name-list-avoiding*) (*auto simp add*: *eqvts*)
obtain *p*::*name prm* **where** $(p \cdot A_Q) \sharp* \Psi$ **and** $(p \cdot A_Q) \sharp* P$ **and** $(p \cdot A_Q) \sharp* Q$ **and** $(p \cdot A_Q) \sharp* \alpha$
and $(p \cdot A_Q) \sharp* \alpha$ **and** $(p \cdot A_Q) \sharp* (q \cdot \alpha)$ **and** $(p \cdot A_Q) \sharp* P'$
and $(p \cdot A_Q) \sharp* (q \cdot P')$ **and** $(p \cdot A_Q) \sharp* \Psi_Q$ **and** *Sp*: (*set p*) ⊆ (*set* A_Q) × (*set*($p \cdot A_Q$))
by(*rule-tac xvec*=A_Q **and** *c*=(Ψ, *P*, *Q*, α, *bn* α, $q \cdot \alpha$, P', $(q \cdot P')$, Ψ_Q) **in** *name-list-avoiding*) *auto*
from ‹*distinct*(*bn* α)› **have** *distinct*($bn(q \cdot \alpha)$)
by(*rule-tac* α=α **in** *actionCases*) (*auto simp add*: *eqvts*)
from ‹$A_Q \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* A_Q$› *Sq* **have** $A_Q \sharp* (q \cdot \alpha)$
apply(*rule-tac* α=α **in** *actionCases*)
apply(*simp only*: *bn.simps eqvts*, *simp*)
apply(*simp add*: *freshChainSimps*)
by *simp*
from ‹*bn* $\alpha \sharp*$ *subject* α› **have** $(q \cdot (bn\ \alpha)) \sharp* (q \cdot (subject\ \alpha))$
by(*simp add*: *fresh-star-bij*)
hence $bn(q \cdot \alpha) \sharp*$ *subject*$(q \cdot \alpha)$ **by**(*simp add*: *eqvts*)
from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› ‹$bn(q \cdot \alpha) \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* P'$› ‹*bn* $\alpha \sharp*$ (*subject* α)› *Sq*
have *Trans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto (q \cdot \alpha) \prec (q \cdot P')$
by(*force simp add*: *residualAlpha*)
hence $A_Q \sharp* (q \cdot P')$ **using** ‹$bn(q \cdot \alpha) \sharp*$ *subject*$(q \cdot \alpha)$› ‹*distinct*($bn(q \cdot \alpha)$)› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* (q \cdot \alpha)$›
by(*auto intro*: *freeFreshChainDerivative*)
from *Trans* **have** $(p \cdot (\Psi \otimes \Psi_Q)) \rhd (p \cdot P) \longmapsto p \cdot ((q \cdot \alpha) \prec (q \cdot P'))$
by(*rule semantics.eqvt*)
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* (q \cdot \alpha)$› ‹$(p \cdot A_Q) \sharp* (q \cdot \alpha)$› ‹$A_Q \sharp* (q \cdot P')$›
‹$(p \cdot A_Q) \sharp* \Psi$› ‹$(p \cdot A_Q) \sharp* P$› ‹$(p \cdot A_Q) \sharp* (q \cdot P')$› *Sp*
have $\Psi \otimes (p \cdot \Psi_Q) \rhd P \longmapsto (q \cdot \alpha) \prec (q \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹$(p \cdot A_Q) \sharp* \Psi_Q$› *Sp* **have** *extractFrame* $Q = \langle (p \cdot A_Q), (p \cdot \Psi_Q) \rangle$
by(*simp add*: *frameChainAlpha′ eqvts*)

moreover from ‹$(bn(q \cdot \alpha))$ $\sharp*$ Ψ_Q› ‹$(bn(q \cdot \alpha))$ $\sharp*$ A_Q› ‹$(p \cdot A_Q)$ $\sharp*$ $(q \cdot \alpha)$› *Sp*
have $(bn(q \cdot \alpha))$ $\sharp*$ $(p \cdot \Psi_Q)$
by(*simp add*: *freshAlphaPerm*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(p \cdot A_Q)$ **by** *simp*
ultimately have $\Psi \rhd P \parallel Q \longmapsto(q \cdot \alpha) \prec ((q \cdot P') \parallel Q)$
using ‹$(p \cdot A_Q)$ $\sharp*$ P› ‹$(p \cdot A_Q)$ $\sharp*$ Q› ‹$(p \cdot A_Q)$ $\sharp*$ Ψ› ‹$(p \cdot A_Q)$ $\sharp*$ $(q \cdot \alpha)$›
‹$(p \cdot A_Q)$ $\sharp*$ $(q \cdot P')$› ‹$(bn(q \cdot \alpha))$ $\sharp*$ Ψ› ‹$(bn(q \cdot \alpha))$ $\sharp*$ Q› ‹$(bn(q \cdot \alpha))$ $\sharp*$ P›
‹$(bn(q \cdot \alpha))$ $\sharp*$ (*subject* $(q \cdot \alpha)$)› ‹*distinct*$(bn(q \cdot \alpha))$›
by(*rule-tac cPar1*)

thus *?thesis* **using** ‹$bn(q \cdot \alpha)$ $\sharp*$ α› ‹$bn(q \cdot \alpha)$ $\sharp*$ P'› ‹bn α $\sharp*$ *subject* α› ‹$bn(q \cdot \alpha)$ $\sharp*$ Q› ‹bn α $\sharp*$ Q› *Sq*
by(*force simp add*: *residualAlpha*)
qed
}
note *Goal* = *this*
from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q$ $\sharp*$ Ψ› ‹A_Q $\sharp*$ P› ‹A_Q $\sharp*$ α›
obtain $A_Q{}'$ **where** *FrQ*: *extractFrame* $Q = \langle A_Q{}', \Psi_Q\rangle$ **and** *distinct* $A_Q{}'$ **and** $A_Q{}'$ $\sharp*$ Ψ **and** $A_Q{}'$ $\sharp*$ P **and** $A_Q{}'$ $\sharp*$ α
by(*rule-tac* C=(Ψ, P, α) **in** *distinctFrame*) *auto*
show *?thesis*
proof(*induct rule*: *actionCases*[**where** α=α])
case(*cInput* M N)
from *Trans FrQ* ‹$A_Q{}'$ $\sharp*$ Ψ› ‹$A_Q{}'$ $\sharp*$ P› ‹$A_Q{}'$ $\sharp*$ α› ‹*distinct* $A_Q{}'$› ‹bn α $\sharp*$ Q›
show *?case* **using** ‹$\alpha = M(\!|N|\!)$› **by**(*force intro*: *Goal*)
next
case *cTau*
from *Trans FrQ* ‹$A_Q{}'$ $\sharp*$ Ψ› ‹$A_Q{}'$ $\sharp*$ P› ‹$A_Q{}'$ $\sharp*$ α› ‹*distinct* $A_Q{}'$› ‹bn α $\sharp*$ Q›
show *?case* **using** ‹$\alpha = \tau$› **by**(*force intro*: *Goal*)
next
case(*cOutput* M *xvec* N)
from ‹$\alpha = M(\!|\nu*xvec|\!)\langle N\rangle$› ‹$A_Q{}'$ $\sharp*$ α› ‹bn α $\sharp*$ Q› **have** *xvec* $\sharp*$ $A_Q{}'$ **and** *xvec* $\sharp*$ Q
by *simp+*
obtain p **where** $(p \cdot xvec)$ $\sharp*$ N **and** $(p \cdot xvec)$ $\sharp*$ P' **and** $(p \cdot xvec)$ $\sharp*$ Q
and $(p \cdot xvec)$ $\sharp*$ M **and** $(p \cdot xvec)$ $\sharp*$ $A_Q{}'$
and S: *set* $p \subseteq$ *set xvec* $\times$ *set*$(p \cdot xvec)$
by(*rule-tac xvec*=*xvec* **and** c=(N, P', Q, M, $A_Q{}'$) **in** *name-list-avoiding*) *auto*
from *Trans* ‹α=$M(\!|\nu*xvec|\!)\langle N\rangle$› **have** $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$ **by** *simp*
with ‹$(p \cdot xvec)$ $\sharp*$ N› ‹$(p \cdot xvec)$ $\sharp*$ P'› S
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot P')$
by(*simp add*: *boundOutputChainAlpha''* *create-residual.simps*)
moreover from ‹*xvec* $\sharp*$ $A_Q{}'$› ‹$(p \cdot xvec)$ $\sharp*$ $A_Q{}'$› ‹$A_Q{}'$ $\sharp*$ α› S
have $A_Q{}'$ $\sharp*$ $(p \cdot \alpha)$ **by**(*simp add*: *freshChainSimps del*: *actionFreshChain*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot P') \parallel Q$
using *FrQ* ‹$A_Q{}'$ $\sharp*$ Ψ› ‹$A_Q{}'$ $\sharp*$ P› ‹*distinct* $A_Q{}'$› ‹$(p \cdot xvec)$ $\sharp*$ Q› ‹$A_Q{}'$ $\sharp*$ α›

‹(p • xvec) ♯* M› ‹α = M⦇ν∗xvec⦈⟨N⟩›
by(*force intro*: *Goal*)
with ‹(p • xvec) ♯* N› ‹(p • xvec) ♯* P'› ‹(p • xvec) ♯* Q› ‹xvec ♯* Q› S ‹α = M⦇ν∗xvec⦈⟨N⟩›
show *?case*
by(*simp add*: *boundOutputChainAlpha''* *eqvts* *create-residual.simps*)
qed
qed

lemma *Par2*:
fixes Ψ :: 'b
and Ψ_P :: 'b
and Q :: ('a, 'b, 'c) *psi*
and α :: 'a *action*
and Q' :: ('a, 'b, 'c) *psi*
and A_P :: *name list*
and P :: ('a, 'b, 'c) *psi*

assumes *Trans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *bn* α ♯* P
and A_P ♯* Ψ
and A_P ♯* Q
and A_P ♯* α

shows $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$
proof −
{
fix Ψ :: 'b
and Ψ_P :: 'b
and Q :: ('a, 'b, 'c) *psi*
and α :: 'a *action*
and Q' :: ('a, 'b, 'c) *psi*
and A_P :: *name list*
and P :: ('a, 'b, 'c) *psi*

assume $\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *bn* α ♯* P
and *bn* α ♯* *subject* α
and A_P ♯* Ψ
and A_P ♯* Q
and A_P ♯* α
and *distinct* A_P

have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$
proof −
from ‹Ψ ⊗ Ψ_P ▷ Q ⟼α ≺ Q'› **have** *distinct*(*bn* α) **by**(*rule* *boundOutput-Distinct*)

obtain q::*name prm* **where** $bn(q \cdot \alpha) \sharp* \Psi$ **and** $bn(q \cdot \alpha) \sharp* P$ **and** $bn(q \cdot \alpha) \sharp* Q$ **and** $bn(q \cdot \alpha) \sharp* \alpha$
and $bn(q \cdot \alpha) \sharp* A_P$ **and** $bn(q \cdot \alpha) \sharp* Q'$ **and** $bn(q \cdot \alpha) \sharp* \Psi_P$
and *Sq*: $(set\ q) \subseteq (set\ (bn\ \alpha)) \times (set(bn(q \cdot \alpha)))$
by(*rule-tac xvec*=$bn\ \alpha$ **and** c=(Ψ, P, Q, α, A_P, Ψ_P, Q') **in** *name-list-avoiding*) (*auto simp add*: *eqvts*)
obtain p::*name prm* **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$ **and** $(p \cdot A_P) \sharp* \alpha$
and $(p \cdot A_P) \sharp* \alpha$ **and** $(p \cdot A_P) \sharp* (q \cdot \alpha)$ **and** $(p \cdot A_P) \sharp* Q'$
and $(p \cdot A_P) \sharp* (q \cdot Q')$ **and** $(p \cdot A_P) \sharp* \Psi_P$
and *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$
by(*rule-tac xvec*=A_P **and** c=(Ψ, P, Q, α, $q \cdot \alpha$, Q', $(q \cdot Q')$, Ψ_P) **in** *name-list-avoiding*) *auto*
from ‹*distinct*($bn\ \alpha$)› **have** *distinct*($bn(q \cdot \alpha)$)
by(*rule-tac* α=α **in** *actionCases*) (*auto simp add*: *eqvts*)
from ‹$A_P \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* A_P$› *Sq* **have** $A_P \sharp* (q \cdot \alpha)$
apply(*rule-tac* α=α **in** *actionCases*)
apply(*simp only*: *bn.simps eqvts*, *simp*)
apply(*simp add*: *freshChainSimps*)
by *simp*
from ‹$bn\ \alpha \sharp* subject\ \alpha$› **have** $(q \cdot (bn\ \alpha)) \sharp* (q \cdot (subject\ \alpha))$
by(*simp add*: *fresh-star-bij*)
hence $bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$ **by**(*simp add*: *eqvts*)
from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹$bn(q \cdot \alpha) \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* Q'$› ‹$bn\ \alpha \sharp* (subject\ \alpha)$› *Sq*
have *Trans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto (q \cdot \alpha) \prec (q \cdot Q')$
by(*force simp add*: *residualAlpha*)
hence $A_P \sharp* (q \cdot Q')$ **using** ‹$bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$› ‹*distinct*($bn(q \cdot \alpha)$)› ‹$A_P \sharp* Q$› ‹$A_P \sharp* (q \cdot \alpha)$›
by(*auto intro*: *freeFreshChainDerivative*)
from *Trans* **have** $(p \cdot (\Psi \otimes \Psi_P)) \rhd (p \cdot Q) \longmapsto p \cdot ((q \cdot \alpha) \prec (q \cdot Q'))$
by(*rule semantics.eqvt*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* (q \cdot \alpha)$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$› ‹$A_P \sharp* (q \cdot Q')$›
‹$(p \cdot A_P) \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(p \cdot A_P) \sharp* (q \cdot Q')$› *Sp*
have $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (q \cdot \alpha) \prec (q \cdot Q')$ **by**(*simp add*: *eqvts*)
moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹$(p \cdot A_P) \sharp* \Psi_P$› *Sp* **have** *extractFrame* $P = \langle (p \cdot A_P), (p \cdot \Psi_P) \rangle$
by(*simp add*: *frameChainAlpha'* *eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* \Psi_P$› ‹$(bn(q \cdot \alpha)) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$› *Sp*
have $(bn(q \cdot \alpha)) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshAlphaPerm*)
moreover from ‹*distinct* A_P› **have** *distinct*$(p \cdot A_P)$ **by** *simp*
ultimately have $\Psi \rhd P \parallel Q \longmapsto (q \cdot \alpha) \prec (P \parallel (q \cdot Q'))$
using ‹$(p \cdot A_P) \sharp* P$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$›
‹$(p \cdot A_P) \sharp* (q \cdot Q')$› ‹$(bn(q \cdot \alpha)) \sharp* \Psi$› ‹$(bn(q \cdot \alpha)) \sharp* Q$› ‹$(bn(q \cdot$

$\alpha)) \sharp* P$›
‹$(bn(q \cdot \alpha)) \sharp* (subject\ (q \cdot \alpha))$› ‹$distinct(bn(q \cdot \alpha))$›
by(*rule-tac cPar2*)

thus *?thesis* **using** ‹$bn(q \cdot \alpha) \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* Q'$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn(q \cdot \alpha) \sharp* P$› ‹$bn\ \alpha \sharp* P$› *Sq*
by(*force simp add*: *residualAlpha*)
qed
}
note *Goal* = *this*
from ‹$extractFrame\ P = \langle A_P, \Psi_P\rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \alpha$›
obtain $A_P{}'$ **where** *FrP*: $extractFrame\ P = \langle A_P{}', \Psi_P\rangle$ **and** $distinct\ A_P{}'$ **and** $A_P{}' \sharp* \Psi$ **and** $A_P{}' \sharp* Q$ **and** $A_P{}' \sharp* \alpha$
by(*rule-tac* $C=(\Psi, Q, \alpha)$ **in** *distinctFrame*) *auto*
show *?thesis*
proof(*induct rule*: *actionCases*[**where** $\alpha=\alpha$])
case(*cInput M N*)
from *Trans FrP* ‹$A_P{}' \sharp* \Psi$› ‹$A_P{}' \sharp* Q$› ‹$A_P{}' \sharp* \alpha$› ‹$distinct\ A_P{}'$› ‹$bn\ \alpha \sharp* P$›
show *?case* **using** ‹$\alpha = M(\!|N|\!)$› **by**(*force intro*: *Goal*)
next
case *cTau*
from *Trans FrP* ‹$A_P{}' \sharp* \Psi$› ‹$A_P{}' \sharp* Q$› ‹$A_P{}' \sharp* \alpha$› ‹$distinct\ A_P{}'$› ‹$bn\ \alpha \sharp* P$›
show *?case* **using** ‹$\alpha = \tau$› **by**(*force intro*: *Goal*)
next
case(*cOutput M xvec N*)
from ‹$\alpha = M(\!|\nu*xvec|\!)\langle N\rangle$› ‹$A_P{}' \sharp* \alpha$› ‹$bn\ \alpha \sharp* P$› **have** $xvec \sharp* A_P{}'$ **and** $xvec \sharp* P$
by *simp+*
obtain *p* **where** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* Q'$ **and** $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* M$ **and** $(p \cdot xvec) \sharp* A_P{}'$
and *S*: $set\ p \subseteq set\ xvec \times set(p \cdot xvec)$
by(*rule-tac xvec=xvec* **and** $c=(N, Q', P, M, A_P{}')$ **in** *name-list-avoiding*) *auto*
from *Trans* ‹$\alpha=M(\!|\nu*xvec|\!)\langle N\rangle$› **have** $\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$ **by** *simp*
with ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* Q'$› *S*
have $\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot Q')$
by(*simp add*: *boundOutputChainAlpha''* *create-residual.simps*)
moreover from ‹$xvec \sharp* A_P{}'$› ‹$(p \cdot xvec) \sharp* A_P{}'$› ‹$A_P{}' \sharp* \alpha$› *S*
have $A_P{}' \sharp* (p \cdot \alpha)$ **by**(*simp add*: *freshChainSimps del*: *actionFreshChain*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec P \parallel (p \cdot Q')$
using *FrP* ‹$A_P{}' \sharp* \Psi$› ‹$A_P{}' \sharp* Q$› ‹$distinct\ A_P{}'$› ‹$(p \cdot xvec) \sharp* P$› ‹$A_P{}' \sharp* \alpha$›
‹$(p \cdot xvec) \sharp* M$› ‹$\alpha = M(\!|\nu*xvec|\!)\langle N\rangle$›
by(*force intro*: *Goal*)
with ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* Q'$› ‹$(p \cdot xvec) \sharp* P$› ‹$xvec \sharp* P$› *S* ‹$\alpha = M(\!|\nu*xvec|\!)\langle N\rangle$›
show *?case*
by(*simp add*: *boundOutputChainAlpha''* *eqvts create-residual.simps*)
qed
qed

lemma *Open*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and *yvec* :: *name list*
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and x :: *name*

assumes *Trans*: $\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$
and $x \in supp\ N$
and $x \sharp \Psi$
and $x \sharp M$
and $x \sharp xvec$
and $x \sharp yvec$

shows $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
proof −
from *Trans* **have** *distinct*(*xvec*@*yvec*) **by**(*force dest*: *boundOutputDistinct*)
hence *xvec* $\sharp*$ *yvec* **by**(*induct xvec*) *auto*

obtain p **where** $(p \cdot yvec) \sharp* \Psi$ **and** $(p \cdot yvec) \sharp* P$ **and** $(p \cdot yvec) \sharp* M$
and $(p \cdot yvec) \sharp* yvec$ **and** $(p \cdot yvec) \sharp* N$ **and** $(p \cdot yvec) \sharp* P'$
and $x \sharp (p \cdot yvec)$ **and** $(p \cdot yvec) \sharp* xvec$
and *Sp*: $(set\ p) \subseteq (set\ yvec) \times (set(p \cdot yvec))$
by(*rule-tac xvec*=*yvec* **and** c=(Ψ, P, M, *xvec*, *yvec*, N, P', x) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)
obtain q **where** $(q \cdot xvec) \sharp* \Psi$ **and** $(q \cdot xvec) \sharp* P$ **and** $(q \cdot xvec) \sharp* M$
and $(q \cdot xvec) \sharp* xvec$ **and** $(q \cdot xvec) \sharp* N$ **and** $(q \cdot xvec) \sharp* P'$
and $x \sharp (q \cdot xvec)$ **and** $(q \cdot xvec) \sharp* yvec$
and $(q \cdot xvec) \sharp* p$ **and** $(q \cdot xvec) \sharp* (p \cdot yvec)$
and *Sq*: $(set\ q) \subseteq (set\ xvec) \times (set(q \cdot xvec))$
by(*rule-tac xvec*=*xvec* **and** c=(Ψ, P, M, *xvec*, *yvec*, $p \cdot yvec$, N, P', x, p) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

note $\langle\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'\rangle$
moreover from $\langle(p \cdot yvec) \sharp* N\rangle$ $\langle(q \cdot xvec) \sharp* N\rangle$ $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$ *Sp Sq*
have $((p@q) \cdot (xvec\ @\ yvec)) \sharp* N$ **apply**(*simp only*: *eqvts*) **apply**(*simp only*: *pt2*[*OF pt-name-inst*])
by *simp*
moreover from $\langle(p \cdot yvec) \sharp* P'\rangle$ $\langle(q \cdot xvec) \sharp* P'\rangle$ $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$ *Sp Sq*
have $((p@q) \cdot (xvec\ @\ yvec)) \sharp* P'$ **by**(*simp del*: *freshAlphaPerm add*: *eqvts pt2*[*OF pt-name-inst*])
moreover from *Sp Sq* $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot$

yvec)› ‹(*p* · *yvec*) ♯* *xvec*›
have *Spq*: *set*(*p*@*q*) ⊆ *set*(*xvec*@*yvec*) × *set*((*p*@*q*) · (*xvec*@*yvec*))
by(*simp add*: *pt2*[*OF pt-name-inst*] *eqvts*) *blast*
ultimately have Ψ ▷ *P* ⟼*M*⦇*ν**((*p*@*q*) · (*xvec*@*yvec*))⦈⟨((*p*@*q*) · *N*)⟩ ≺ ((*p*@*q*) · *P*′)
apply(*simp add*: *create-residual.simps*)
by(*erule-tac rev-mp*) (*subst boundOutputChainAlpha*, *auto*)

with *Sp Sq* ‹*xvec* ♯* *yvec*› ‹(*q* · *xvec*) ♯* *yvec*› ‹(*q* · *xvec*) ♯* (*p* · *yvec*)› ‹(*p* · *yvec*) ♯* *xvec*›
have Ψ ▷ *P* ⟼*M*⦇*ν**((*q* · *xvec*)@(*p* · *yvec*))⦈⟨((*p*@*q*) · *N*)⟩ ≺ ((*p*@*q*) · *P*′)
by(*simp add*: *eqvts pt2*[*OF pt-name-inst*] *del*: *freshAlphaPerm*)
moreover from ‹*x* ∈ *supp N*› **have** ((*p*@*q*) · *x*) ∈ (*p*@*q*) · (*supp N*)
by(*simp add*: *pt-set-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*x* ♯ (*q* · *xvec*)› ‹*x* ♯ (*p* · *yvec*)› *Sp Sq*
have *x* ∈ *supp*((*p*@*q*)· *N*) **by**(*simp add*: *eqvts pt2*[*OF pt-name-inst*])
moreover from ‹*distinct*(*xvec*@*yvec*)› **have** *distinct*(*q* · *xvec*) **and** *distinct*(*p* · *yvec*)
by *auto*
moreover note ‹*x* ♯ (*q* · *xvec*)› ‹*x* ♯ (*p* · *yvec*)› ‹*x* ♯ *M*› ‹*x* ♯ Ψ›
‹(*q* · *xvec*) ♯* Ψ› ‹(*q* · *xvec*) ♯* *P*› ‹(*q* · *xvec*) ♯* *M*› ‹(*q* · *xvec*) ♯* (*p* · *yvec*)›
‹(*p* · *yvec*) ♯* Ψ› ‹(*p* · *yvec*) ♯* *P*› ‹(*p* · *yvec*) ♯* *M*› ‹*distinct*(*q* · *xvec*)›
ultimately have Ψ ▷ ⦇*νx*⦈*P* ⟼*M*⦇*ν**((*q* · *xvec*)@*x*#(*p* · *yvec*))⦈⟨((*p*@*q*) · *N*)⟩ ≺ ((*p*@*q*) · *P*′)
by(*rule-tac cOpen*)
with ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*x* ♯ (*q* · *xvec*)› ‹*x* ♯ (*p* · *yvec*)›
‹*xvec* ♯* *yvec*› ‹(*q* · *xvec*) ♯* *yvec*› ‹(*q* · *xvec*) ♯* (*p* · *yvec*)› ‹(*p* · *yvec*) ♯* *xvec*› *Sp Sq*
have Ψ ▷ ⦇*νx*⦈*P* ⟼*M*⦇*ν**((*p*@*q*) · (*xvec*@*x*#*yvec*))⦈⟨((*p*@*q*) · *N*)⟩ ≺ ((*p*@*q*) · *P*′)
by(*simp add*: *eqvts pt2*[*OF pt-name-inst*] *del*: *freshAlphaPerm*)
thus *?thesis* **using** ‹((*p*@*q*) · (*xvec* @ *yvec*)) ♯* *N*› ‹((*p*@*q*) · (*xvec* @ *yvec*)) ♯* *P*′› *Spq*
apply(*simp add*: *create-residual.simps*)
by(*erule-tac rev-mp*) (*subst boundOutputChainAlpha*, *auto*)
qed

lemma *Scope*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and *P*′ :: (*'a*, *'b*, *'c*) *psi*
and *x* :: *name*

assumes Ψ ▷ *P* ⟼α ≺ *P*′
and *x* ♯ Ψ
and *x* ♯ α

```
  shows Ψ ▷ (⦇νx⦈)P ⟼α ≺ (⦇νx⦈)P'
proof −
  {
    fix Ψ P M xvec N P' x

    assume Ψ ▷ P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ P'
    and    (x::name) ♯ Ψ
    and    x ♯ M
    and    x ♯ xvec
    and    x ♯ N

    obtain p::name prm where (p • xvec) ♯* Ψ and (p • xvec) ♯* P and (p • xvec)
♯* M and (p • xvec) ♯* xvec
                         and (p • xvec) ♯* N and (p • xvec) ♯* P' and x ♯ (p • xvec)
                         and S: (set p) ⊆ (set xvec) × (set(p • xvec))
      by(rule-tac xvec=xvec and c=(Ψ, P, M, xvec, N, P', x) in name-list-avoiding)
         (auto simp add: eqvts fresh-star-prod)
    from ‹Ψ ▷ P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ P'› ‹(p • xvec) ♯* N› ‹(p • xvec) ♯* P'› S
    have Ψ ▷ P ⟼M(⦇ν*(p • xvec)⦈)⟨(p • N)⟩ ≺ (p • P')
      by(simp add: boundOutputChainAlpha'' create-residual.simps)
    moreover hence distinct(p • xvec) by(force dest: boundOutputDistinct)
    moreover note ‹x ♯ Ψ› ‹x ♯ M› ‹x ♯ (p • xvec)›
    moreover from ‹x ♯ xvec› ‹x ♯ p • xvec› ‹x ♯ N› S have x ♯ (p • N)
      by(simp add: fresh-left del: freshAlphaSwap)
     ultimately have Ψ ▷ (⦇νx⦈)P ⟼M(⦇ν*(p • xvec)⦈)⟨(p • N)⟩ ≺ (⦇νx⦈)(p • P')
using ‹(p • xvec) ♯* Ψ› ‹(p • xvec) ♯* P› ‹(p • xvec) ♯* M›
      by(rule-tac cScope) auto
    moreover from ‹x ♯ xvec› ‹x ♯ p • xvec› S have p • x = x by simp
     ultimately have Ψ ▷ (⦇νx⦈)P ⟼M(⦇ν*(p • xvec)⦈)⟨(p • N)⟩ ≺ (p • ((⦇νx⦈)P'))
by simp
    moreover from ‹(p • xvec) ♯* P'› ‹x ♯ xvec› ‹x ♯ (p • xvec)› have (p • xvec)
♯* (⦇νx⦈)P'
      by(simp add: abs-fresh-star)
     ultimately haveΨ ▷ (⦇νx⦈)P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ (⦇νx⦈)P' using ‹(p • xvec)
♯* N› S
      by(simp add: boundOutputChainAlpha'' create-residual.simps)
  }
  note Goal = this
  show ?thesis
  proof(induct rule: actionCases[where α=α])
    case(cInput M N)
    with assms show ?case by(force intro: cScope)
  next
    case(cOutput M xvec N)
    with assms show ?case by(force intro: Goal)
  next
    case cTau
    with assms show ?case by(force intro: cScope)
  qed
```

qed

lemma *inputSwapFrameSubject*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $N :: 'a$
and $P' :: ('a, 'b, 'c)\ psi$
and $x :: name$
and $y :: name$

assumes $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
and $x \sharp P$
and $y \sharp P$

shows $([(x, y)] \cdot \Psi) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$
using *assms*
proof(*nominal-induct avoiding*: *x y rule*: *inputInduct*)
case(*cInput* Ψ *M K xvec N Tvec P x y*)
from ‹$x \sharp M(\!|\lambda*xvec\ N|\!).P$› **have** $x \sharp M$ **by** *simp*
from ‹$y \sharp M(\!|\lambda*xvec\ N|\!).P$› **have** $y \sharp M$ **by** *simp*
from ‹$\Psi \vdash M \leftrightarrow K$› **have** $([(x, y)] \cdot \Psi) \vdash ([(x, y)] \cdot M) \leftrightarrow ([(x, y)] \cdot K)$
by(*rule chanEqClosed*)
with ‹$x \sharp M$› ‹$y \sharp M$› **have** $([(x, y)] \cdot \Psi) \vdash M \leftrightarrow ([(x, y)] \cdot K)$
by(*simp*)
thus *?case* **using** ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec* = *length Tvec*›
by(*rule Input*)
next
case(*cCase* Ψ *P M N P'* φ *Cs x y*)
from ‹$x \sharp$ *Cases Cs*› ‹$y \sharp$ *Cases Cs*› ‹(φ, P) *mem Cs*› **have** $x \sharp \varphi$ **and** $x \sharp P$ **and** $y \sharp \varphi$ **and** $y \sharp P$
by(*auto dest*: *memFresh*)
from ‹$x \sharp P$› ‹$y \sharp P$› **have** $([(x ,y)] \cdot \Psi) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$ **by**(*rule cCase*)
moreover note ‹(φ, P) *mem Cs*›
moreover from ‹$\Psi \vdash \varphi$› **have** $([(x, y)] \cdot \Psi) \vdash ([(x, y)] \cdot \varphi)$ **by**(*rule statClosed*)
with ‹$x \sharp \varphi$› ‹$y \sharp \varphi$› **have** $([(x, y)] \cdot \Psi) \vdash \varphi$ **by** *simp*
ultimately show *?case* **using** ‹*guarded P*› **by**(*rule Case*)
next
case(*cPar1* Ψ Ψ_Q *P M N P'* A_Q *Q x y*)
from ‹$x \sharp P \parallel Q$› **have** $x \sharp P$ **and** $x \sharp Q$ **by** *simp+*
from ‹$y \sharp P \parallel Q$› **have** $y \sharp P$ **and** $y \sharp Q$ **by** *simp+*
from ‹$x \sharp P$› ‹$y \sharp P$› ‹$\bigwedge x\ y.\ [\![x \sharp P; y \sharp P]\!] \Longrightarrow ([(x, y)] \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$›
have $([(x, y)] \cdot \Psi) \otimes ([(x, y)] \cdot \Psi_Q) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|N|\!) \prec P'$
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame Q* = $\langle A_Q, \Psi_Q\rangle$› **have** $([(x, y)] \cdot (extractFrame\ Q)) = ([(x, y)] \cdot \langle A_Q, \Psi_Q\rangle)$

by *simp*
with ‹A_Q ♯* x› ‹x ♯ Q› ‹A_Q ♯* y› ‹y ♯ Q› **have** $\langle A_Q, ([(x, y)] \cdot \Psi_Q)\rangle$ = *extractFrame* Q
by(*simp add*: *eqvts*)
moreover from ‹A_Q ♯* Ψ› **have** $([(x, y)] \cdot A_Q)$ ♯* $([(x, y)] \cdot \Psi)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_Q ♯* x› ‹A_Q ♯* y› **have** A_Q ♯* $([(x, y)] \cdot \Psi)$ **by** *simp*
moreover from ‹A_Q ♯* M› **have** $([(x, y)] \cdot A_Q)$ ♯* $([(x, y)] \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_Q ♯* x› ‹A_Q ♯* y› **have** A_Q ♯* $([(x, y)] \cdot M)$ **by** *simp*
ultimately show *?case* **using** ‹A_Q ♯* P› ‹A_Q ♯* N›
by(*rule-tac Par1*) *auto*
next
case(*cPar2* Ψ Ψ_P Q M N Q' A_P P x y)
from ‹x ♯ $P \parallel Q$› **have** x ♯ P **and** x ♯ Q **by** *simp*+
from ‹y ♯ $P \parallel Q$› **have** y ♯ P **and** y ♯ Q **by** *simp*+
from ‹x ♯ Q› ‹y ♯ Q› ‹$\bigwedge x\ y.\ [\![x \sharp Q; y \sharp Q]\!] \Longrightarrow ([(x, y)] \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto([(x, y)] \cdot M)(\!|N|\!) \prec Q'$›
have $([(x, y)] \cdot \Psi) \otimes ([(x, y)] \cdot \Psi_P) \rhd Q \longmapsto([(x, y)] \cdot M)(\!|N|\!) \prec Q'$
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› **have** $([(x, y)] \cdot$ (*extractFrame* $P)) = ([(x, y)] \cdot \langle A_P, \Psi_P\rangle)$
by *simp*
with ‹A_P ♯* x› ‹x ♯ P› ‹A_P ♯* y› ‹y ♯ P› **have** $\langle A_P, ([(x, y)] \cdot \Psi_P)\rangle$ = *extract-Frame* P
by(*simp add*: *eqvts*)
moreover from ‹A_P ♯* Ψ› **have** $([(x, y)] \cdot A_P)$ ♯* $([(x, y)] \cdot \Psi)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_P ♯* x› ‹A_P ♯* y› **have** A_P ♯* $([(x, y)] \cdot \Psi)$ **by** *simp*
moreover from ‹A_P ♯* M› **have** $([(x, y)] \cdot A_P)$ ♯* $([(x, y)] \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_P ♯* x› ‹A_P ♯* y› **have** A_P ♯* $([(x, y)] \cdot M)$ **by** *simp*
ultimately show *?case* **using** ‹A_P ♯* Q› ‹A_P ♯* N›
by(*rule-tac Par2*) *auto*
next
case(*cScope* Ψ P M N P' z x y)
from ‹x ♯ $(\!|\nu z|\!)P$› ‹z ♯ x› **have** x ♯ P **by**(*simp add*: *abs-fresh*)
from ‹y ♯ $(\!|\nu z|\!)P$› ‹z ♯ y› **have** y ♯ P **by**(*simp add*: *abs-fresh*)
from ‹x ♯ P› ‹y ♯ P› ‹$\bigwedge x\ y.\ [\![x \sharp P; y \sharp P]\!] \Longrightarrow ([(x, y)] \cdot \Psi) \rhd P \longmapsto([(x, y)] \cdot M)(\!|N|\!) \prec P'$›
have $([(x, y)] \cdot \Psi) \rhd P \longmapsto([(x, y)] \cdot M)(\!|N|\!) \prec P'$ **by** *simp*
moreover with ‹z ♯ Ψ› **have** $([(x, y)] \cdot z)$ ♯ $[(x, y)] \cdot \Psi$
by(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹z ♯ x› ‹z ♯ y› **have** z ♯ $[(x, y)] \cdot \Psi$ **by** *simp*
moreover with ‹z ♯ M› **have** $([(x, y)] \cdot z)$ ♯ $[(x, y)] \cdot M$
by(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹z ♯ x› ‹z ♯ y› **have** z ♯ $[(x, y)] \cdot M$ **by** *simp*
ultimately show *?case* **using** ‹z ♯ N›

```
    by(rule-tac Scope) (assumption | simp)+
next
  case(cBang Ψ P M N P' x y)
  thus ?case by(force intro: Bang)
qed

lemma inputPermFrameSubject:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
  and   N  :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   p  :: name prm
  and   Xs :: name set
  and   Ys :: name set

  assumes Ψ ▷ P ⟼M⦇N⦈ ≺ P'
  and     S: set p ⊆ Xs × Ys
  and     Xs ♯* P
  and     Ys ♯* P

  shows (p • Ψ) ▷ P ⟼ (p • M)⦇N⦈ ≺ P'
using S
proof(induct p)
  case Nil
  from ‹Ψ ▷ P ⟼M⦇N⦈ ≺ P'›
  show ?case by simp
next
  case(Cons a p)
  from ‹set(a#p) ⊆ Xs × Ys› have set p ⊆ Xs × Ys by auto
  with ‹set p ⊆ Xs × Ys ⟹ (p • Ψ) ▷ P ⟼ (p • M)⦇N⦈ ≺ P'›
  have Trans: (p • Ψ) ▷ P ⟼ (p • M)⦇N⦈ ≺ P' by simp
  from ‹set(a#p) ⊆ Xs × Ys› show ?case
  proof(cases a, clarsimp)
    fix a b
    assume a ∈ Xs and b ∈ Ys
    with ‹Xs ♯* P› ‹Ys ♯* P›
    have a ♯ P and b ♯ P
      by(auto simp add: fresh-star-def)
    with Trans show ([(a, b)] • p • Ψ) ▷ P ⟼ ([(a, b)] • p • M)⦇N⦈ ≺ P'
      by(rule inputSwapFrameSubject)
  qed
qed

lemma inputSwapSubject:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
  and   N  :: 'a
```

```
  and   P' :: ('a, 'b, 'c) psi
  and   x  :: name
  and   y  :: name

  assumes Ψ ▷ P ⟼M⟮N⟯ ≺ P'
  and     x ♯ P
  and     y ♯ P
  and     x ♯ Ψ
  and     y ♯ Ψ

  shows Ψ ▷ P ⟼ ([(x, y)] • M)⟮N⟯ ≺ P'
proof −
  from ‹Ψ ▷ P ⟼M⟮N⟯ ≺ P'› ‹x ♯ P› ‹y ♯ P›
  have ([(x, y)] • Ψ) ▷ P ⟼([(x, y)] • M)⟮N⟯ ≺ P'
    by(rule inputSwapFrameSubject)
  with ‹x ♯ Ψ› ‹y ♯ Ψ› show ?thesis
    by simp
qed

lemma inputPermSubject:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
  and   N  :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   p  :: name prm
  and   Xs :: name set
  and   Ys :: name set

  assumes Ψ ▷ P ⟼M⟮N⟯ ≺ P'
  and     S: set p ⊆ Xs × Ys
  and     Xs ♯* P
  and     Ys ♯* P
  and     Xs ♯* Ψ
  and     Ys ♯* Ψ

  shows Ψ ▷ P ⟼ (p • M)⟮N⟯ ≺ P'
proof −
  from ‹Ψ ▷ P ⟼M⟮N⟯ ≺ P'› S ‹Xs ♯* P› ‹Ys ♯* P›
  have (p • Ψ) ▷ P ⟼(p • M)⟮N⟯ ≺ P'
    by(rule inputPermFrameSubject)
  with ‹Xs ♯* Ψ› ‹Ys ♯* Ψ› S show ?thesis
    by simp
qed

lemma inputSwapFrame:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
```

```
  and   N :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   x :: name
  and   y :: name

  assumes Ψ ▷ P ⟼M⦇N⦈ ≺ P'
  and     x ♯ P
  and     y ♯ P
  and     x ♯ M
  and     y ♯ M

  shows ([(x, y)] • Ψ) ▷ P ⟼ M⦇N⦈ ≺ P'
proof −
  from ‹Ψ ▷ P ⟼M⦇N⦈ ≺ P'› ‹x ♯ P› ‹y ♯ P›
  have ([(x, y)] • Ψ) ▷ P ⟼([(x, y)] • M)⦇N⦈ ≺ P'
    by(rule inputSwapFrameSubject)
  with ‹x ♯ M› ‹y ♯ M› show ?thesis
    by simp
qed

lemma inputPermFrame:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   M :: 'a
  and   N :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   p :: name prm
  and   Xs :: name set
  and   Ys :: name set

  assumes Ψ ▷ P ⟼M⦇N⦈ ≺ P'
  and     S: set p ⊆ Xs × Ys
  and     Xs ♯* P
  and     Ys ♯* P
  and     Xs ♯* M
  and     Ys ♯* M

  shows (p • Ψ) ▷ P ⟼ M⦇N⦈ ≺ P'
proof −
  from ‹Ψ ▷ P ⟼M⦇N⦈ ≺ P'› S ‹Xs ♯* P› ‹Ys ♯* P›
  have (p • Ψ) ▷ P ⟼(p • M)⦇N⦈ ≺ P'
    by(rule inputPermFrameSubject)
  with ‹Xs ♯* M› ‹Ys ♯* M› S show ?thesis
    by simp
qed

lemma inputAlpha:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
```

and M :: $'a$
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and p :: *name prm*
and *xvec* :: *name list*

assumes $\Psi \rhd P \longmapsto M\langle\!|N|\!\rangle \prec P'$
and *set p* $\subseteq$ (*set xvec*) $\times$ (*set* ($p \cdot$ *xvec*))
and *distinctPerm p*
and *xvec* $\sharp*$ P
and ($p \cdot$ *xvec*) $\sharp*$ P

shows $\Psi \rhd P \longmapsto M\langle\!|(p \cdot N)|\!\rangle \prec (p \cdot P')$
proof −
from ‹$\Psi \rhd P \longmapsto M\langle\!|N|\!\rangle \prec P'$› ‹*set p* $\subseteq$ (*set xvec*) $\times$ (*set* ($p \cdot$ *xvec*))› ‹*xvec* $\sharp*$ P› ‹($p \cdot$ *xvec*) $\sharp*$ P›
have ($p \cdot \Psi$) $\rhd$ $P \longmapsto (p \cdot M)\langle\!|N|\!\rangle \prec P'$ **by**(*rule-tac inputPermFrameSubject*) *auto*
hence ($p \cdot p \cdot \Psi$) $\rhd$ ($p \cdot P$) $\longmapsto (p \cdot ((p \cdot M)\langle\!|N|\!\rangle \prec P'))$ **by**(*rule eqvts*)
with ‹*distinctPerm p*› ‹*xvec* $\sharp*$ P› ‹($p \cdot$ *xvec*) $\sharp*$ P› ‹*set p* $\subseteq$ (*set xvec*) $\times$ (*set* ($p \cdot$ *xvec*))›
show *?thesis* **by**(*simp add*: *eqvts*)
qed

lemma *frameFresh*[*dest*]:
fixes x :: *name*
and A_F :: *name list*
and Ψ_F :: $'b$

assumes $x \sharp A_F$
and $x \sharp \langle A_F, \Psi_F\rangle$

shows $x \sharp \Psi_F$
using *assms*
by(*simp add*: *frameResChainFresh*) (*simp add*: *fresh-def name-list-supp*)

lemma *outputSwapFrameSubject*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and x :: *name*
and y :: *name*

assumes $\Psi \rhd P \longmapsto M\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$
and *xvec* $\sharp*$ M
and $x \sharp P$
and $y \sharp P$

shows $([(x, y)] \cdot \Psi) \rhd P \longmapsto([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
using *assms*
proof(*nominal-induct avoiding*: *x y rule*: *outputInduct'*)
case *cAlpha*
thus *?case* **by**(*simp add*: *create-residual.simps boundOutputChainAlpha''*)
next
case(*cOutput* Ψ *M K N P x y*)
from ‹$x \sharp M\langle N\rangle.P$› **have** $x \sharp M$ **by** *simp*
from ‹$y \sharp M\langle N\rangle.P$› **have** $y \sharp M$ **by** *simp*
from ‹$\Psi \vdash M \leftrightarrow K$› **have** $([(x, y)] \cdot \Psi) \vdash ([(x, y)] \cdot M) \leftrightarrow ([(x, y)] \cdot K)$
by(*rule chanEqClosed*)
with ‹$x \sharp M$› ‹$y \sharp M$› **have** $([(x, y)] \cdot \Psi) \vdash M \leftrightarrow ([(x, y)] \cdot K)$
by(*simp*)
thus *?case* **by**(*rule Output*)
next
case(*cCase* Ψ *P M xvec N P'* φ *Cs x y*)
from ‹$x \sharp$ *Cases Cs*› ‹$y \sharp$ *Cases Cs*› ‹(φ, P) *mem Cs*› **have** $x \sharp \varphi$ **and** $x \sharp P$ **and** $y \sharp \varphi$ **and** $y \sharp P$
by(*auto dest*: *memFresh*)
from ‹$x \sharp P$› ‹$y \sharp P$› **have** $([(x ,y)] \cdot \Psi) \rhd P \longmapsto([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by**(*rule cCase*)
moreover note ‹(φ, P) *mem Cs*›
moreover from ‹$\Psi \vdash \varphi$› **have** $([(x, y)] \cdot \Psi) \vdash ([(x, y)] \cdot \varphi)$ **by**(*rule statClosed*)
with ‹$x \sharp \varphi$› ‹$y \sharp \varphi$› **have** $([(x, y)] \cdot \Psi) \vdash \varphi$ **by** *simp*
ultimately show *?case* **using** ‹*guarded P*› **by**(*rule Case*)
next
case(*cPar1* Ψ Ψ_Q *P M xvec N P'* A_Q *Q x y*)
from ‹$x \sharp P \parallel Q$› **have** $x \sharp P$ **and** $x \sharp Q$ **by** *simp+*
from ‹$y \sharp P \parallel Q$› **have** $y \sharp P$ **and** $y \sharp Q$ **by** *simp+*
from ‹$x \sharp P$› ‹$y \sharp P$› ‹$\bigwedge x\ y.\ [\![x \sharp P; y \sharp P]\!] \Longrightarrow ([(x, y)] \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$›
have $([(x, y)] \cdot \Psi) \otimes ([(x, y)] \cdot \Psi_Q) \rhd P \longmapsto([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame Q* = $\langle A_Q, \Psi_Q\rangle$› **have** $([(x, y)] \cdot \langle A_Q, \Psi_Q\rangle) = ([(x, y)] \cdot (\mathit{extractFrame}\ Q))$
by *simp*
with ‹A_Q $\sharp*$ *x*› ‹$x \sharp Q$› ‹A_Q $\sharp*$ *y*› ‹$y \sharp Q$› **have** $\langle A_Q, ([(x, y)] \cdot \Psi_Q)\rangle$ = *extractFrame Q*
by(*simp add*: *eqvts*)
moreover from ‹A_Q $\sharp*$ Ψ› **have** $([(x, y)] \cdot A_Q)$ $\sharp*$ $([(x, y)] \cdot \Psi)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_Q $\sharp*$ *x*› ‹A_Q $\sharp*$ *y*› **have** A_Q $\sharp*$ $([(x, y)] \cdot \Psi)$ **by** *simp*
moreover from ‹A_Q $\sharp*$ *M*› **have** $([(x, y)] \cdot A_Q)$ $\sharp*$ $([(x, y)] \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_Q $\sharp*$ *x*› ‹A_Q $\sharp*$ *y*› **have** A_Q $\sharp*$ $([(x, y)] \cdot M)$ **by** *simp*
ultimately show *?case* **using** ‹A_Q $\sharp*$ *P*› ‹A_Q $\sharp*$ *N*› ‹*xvec* $\sharp*$ *Q*› ‹A_Q $\sharp*$ *xvec*›
by(*rule-tac Par1*) *auto*

next
case(*cPar2* Ψ Ψ_P Q M *xvec* N Q' A_P P x y)
from ‹$x \sharp P \parallel Q$› **have** $x \sharp P$ **and** $x \sharp Q$ **by** *simp*+
from ‹$y \sharp P \parallel Q$› **have** $y \sharp P$ **and** $y \sharp Q$ **by** *simp*+
from ‹$x \sharp Q$› ‹$y \sharp Q$› ‹$\bigwedge x\ y.\ [\![x \sharp Q;\ y \sharp Q]\!] \Longrightarrow ([(x,\ y)] \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$›
have $([(x,\ y)] \cdot \Psi) \otimes ([(x,\ y)] \cdot \Psi_P) \rhd Q \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $P = \langle A_P,\ \Psi_P\rangle$› **have** $([(x,\ y)] \cdot \langle A_P,\ \Psi_P\rangle) = ([(x,\ y)] \cdot (\mathit{extractFrame}\ P))$
by *simp*
with ‹$A_P \sharp* x$› ‹$x \sharp P$› ‹$A_P \sharp* y$› ‹$y \sharp P$› **have** $\langle A_P,\ ([(x,\ y)] \cdot \Psi_P)\rangle = \mathit{extractFrame}\ P$
by(*simp add*: *eqvts*)
moreover from ‹$A_P \sharp* \Psi$› **have** $([(x,\ y)] \cdot A_P) \sharp* ([(x,\ y)] \cdot \Psi)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* x$› ‹$A_P \sharp* y$› **have** $A_P \sharp* ([(x,\ y)] \cdot \Psi)$ **by** *simp*
moreover from ‹$A_P \sharp* M$› **have** $([(x,\ y)] \cdot A_P) \sharp* ([(x,\ y)] \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* x$› ‹$A_P \sharp* y$› **have** $A_P \sharp* ([(x,\ y)] \cdot M)$ **by** *simp*
ultimately show *?case* **using** ‹$A_P \sharp* Q$› ‹$A_P \sharp* N$› ‹$\mathit{xvec} \sharp* P$› ‹$A_P \sharp* \mathit{xvec}$›
by(*rule-tac Par2*) *auto*
next
case(*cOpen* Ψ P M *xvec yvec* N P' z x y)
from ‹$x \sharp (\!|\nu z|\!)P$› ‹$z \sharp x$› **have** $x \sharp P$ **by**(*simp add*: *abs-fresh*)
from ‹$y \sharp (\!|\nu z|\!)P$› ‹$z \sharp y$› **have** $y \sharp P$ **by**(*simp add*: *abs-fresh*)
from ‹$x \sharp P$› ‹$y \sharp P$› ‹$\bigwedge x\ y.\ [\![x \sharp P;\ y \sharp P]\!] \Longrightarrow ([(x,\ y)] \cdot \Psi) \rhd P \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * (\mathit{xvec}@\mathit{yvec})|\!)\langle N\rangle \prec P'$›
have $([(x,\ y)] \cdot \Psi) \rhd P \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * (\mathit{xvec}@\mathit{yvec})|\!)\langle N\rangle \prec P'$ **by** *simp*
moreover with ‹$z \sharp \Psi$› **have** $([(x,\ y)] \cdot z) \sharp [(x,\ y)] \cdot \Psi$
by(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$z \sharp x$› ‹$z \sharp y$› **have** $z \sharp [(x,\ y)] \cdot \Psi$ **by** *simp*
moreover with ‹$z \sharp M$› **have** $([(x,\ y)] \cdot z) \sharp [(x,\ y)] \cdot M$
by(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$z \sharp x$› ‹$z \sharp y$› **have** $z \sharp [(x,\ y)] \cdot M$ **by** *simp*
ultimately show *?case* **using** ‹$z \in \mathit{supp}\ N$› ‹$z \sharp \mathit{xvec}$› ‹$z \sharp \mathit{yvec}$›
by(*rule-tac Open*) (*assumption* | *simp*)+
next
case(*cScope* Ψ P M *xvec* N P' z x y)
from ‹$x \sharp (\!|\nu z|\!)P$› ‹$z \sharp x$› **have** $x \sharp P$ **by**(*simp add*: *abs-fresh*)
from ‹$y \sharp (\!|\nu z|\!)P$› ‹$z \sharp y$› **have** $y \sharp P$ **by**(*simp add*: *abs-fresh*)
from ‹$x \sharp P$› ‹$y \sharp P$› ‹$\bigwedge x\ y.\ [\![x \sharp P;\ y \sharp P]\!] \Longrightarrow ([(x,\ y)] \cdot \Psi) \rhd P \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$›
have $([(x,\ y)] \cdot \Psi) \rhd P \longmapsto ([(x,\ y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by** *simp*
moreover with ‹$z \sharp \Psi$› **have** $([(x,\ y)] \cdot z) \sharp [(x,\ y)] \cdot \Psi$
by(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$z \sharp x$› ‹$z \sharp y$› **have** $z \sharp [(x,\ y)] \cdot \Psi$ **by** *simp*
moreover with ‹$z \sharp M$› **have** $([(x,\ y)] \cdot z) \sharp [(x,\ y)] \cdot M$

```
    by(simp add: pt-fresh-bij[OF pt-name-inst, OF at-name-inst])
  with ‹z ♯ x› ‹z ♯ y› have z ♯ [(x, y)] • M by simp
  ultimately show ?case using ‹z ♯ N› ‹z ♯ xvec›
    by(rule-tac Scope) (assumption | simp)+
next
  case(cBang Ψ P M B x y)
  thus ?case by(force intro: Bang)
qed

lemma outputPermFrameSubject:
  fixes Ψ    :: 'b
  and   P    :: ('a, 'b, 'c) psi
  and   M    :: 'a
  and   xvec :: name list
  and   N    :: 'a
  and   P'   :: ('a, 'b, 'c) psi
  and   p    :: name prm
  and   yvec :: name list
  and   zvec :: name list

  assumes Ψ ▷ P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ P'
  and     S: set p ⊆ set yvec × set zvec
  and     yvec ♯* P
  and     zvec ♯* P

  shows (p • Ψ) ▷ P ⟼(p • M)(⦇ν*xvec⦈)⟨N⟩ ≺ P'
proof −
  {
    fix xvec N P' Xs YS
    assume Ψ ▷ P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ P' and xvec ♯* M and xvec ♯* yvec and
xvec ♯* zvec
    have (p • Ψ) ▷ P ⟼(p • M)(⦇ν*xvec⦈)⟨N⟩ ≺ P' using S
    proof(induct p)
      case Nil
      from ‹Ψ ▷ P ⟼M(⦇ν*xvec⦈)⟨N⟩ ≺ P'›
      show ?case by simp
    next
      case(Cons a p)
      from ‹set(a#p) ⊆ set yvec × set zvec› have set p ⊆ set yvec × set zvec by
auto
      then have Trans: (p • Ψ) ▷ P ⟼(p • M)(⦇ν*xvec⦈)⟨N⟩ ≺ P' by(rule Cons)
      from ‹set(a#p) ⊆ set yvec × set zvec› show ?case
      proof(cases a, clarsimp)
        fix x y
        note Trans
        moreover from ‹xvec ♯* yvec› ‹xvec ♯* zvec› ‹set p ⊆ set yvec × set zvec›
‹xvec ♯* M› have xvec ♯* (p • M)
          by(simp add: freshChainSimps)
        moreover assume x ∈ set yvec and y ∈ set zvec
```

with ‹*yvec* ♯* *P*› ‹*zvec* ♯* *P*› **have** *x* ♯ *P* **and** *y* ♯ *P*
by(*auto simp add*: *fresh-star-def*)
ultimately show ([(*x*, *y*)] ∙ *p* ∙ Ψ) ▷ *P* ⟼([(*x*, *y*)] ∙ *p* ∙ *M*)⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′
by(*rule outputSwapFrameSubject*)
qed
qed
}
note *Goal* = *this*
obtain *q*::*name prm* **where** (*q* ∙ *xvec*) ♯* *yvec* **and** (*q* ∙ *xvec*) ♯* *zvec* **and** (*q* ∙ *xvec*) ♯* *xvec*
and (*q* ∙ *xvec*) ♯* *N* **and** (*q* ∙ *xvec*) ♯* *P*′ **and** (*q* ∙ *xvec*) ♯* *M*
and *Sq*: (*set q*) ⊆ (*set xvec*) × (*set*(*q* ∙ *xvec*))
by(*rule-tac xvec*=*xvec* **and** *c*=(*P*, *xvec*, *yvec*, *zvec*, *N*, *M*, *P*′) **in** *name-list-avoiding*) *auto*
with ‹Ψ ▷ *P* ⟼*M*⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′› **have** Ψ ▷ *P* ⟼*M*⦇*ν*∗(*q* ∙ *xvec*)⦈⟨(*q* ∙ *N*)⟩ ≺ (*q* ∙ *P*′)
by(*simp add*: *boundOutputChainAlpha*′′ *residualInject*)
hence (*p* ∙ Ψ) ▷ *P* ⟼(*p* ∙ *M*)⦇*ν*∗(*q* ∙ *xvec*)⦈⟨(*q* ∙ *N*)⟩ ≺ (*q* ∙ *P*′)
using ‹(*q* ∙ *xvec*) ♯* *M*› ‹(*q* ∙ *xvec*) ♯* *yvec*› ‹(*q* ∙ *xvec*) ♯* *zvec*›
by(*rule Goal*)
with ‹(*q* ∙ *xvec*) ♯* *N*› ‹(*q* ∙ *xvec*) ♯* *P*′› *Sq* **show** *?thesis*
by(*simp add*: *boundOutputChainAlpha*′′ *residualInject*)
qed

lemma *outputSwapSubject*:
fixes Ψ :: ′*b*
and *P* :: (′*a*, ′*b*, ′*c*) *psi*
and *M* :: ′*a*
and *B* :: (′*a*, ′*b*, ′*c*) *boundOutput*
and *x* :: *name*
and *y* :: *name*

assumes Ψ ▷ *P* ⟼*M*⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′
and *xvec* ♯* *M*
and *x* ♯ *P*
and *y* ♯ *P*
and *x* ♯ Ψ
and *y* ♯ Ψ

shows Ψ ▷ *P* ⟼([(*x*, *y*)] ∙ *M*)⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′
proof −
from ‹Ψ ▷ *P* ⟼*M*⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′› ‹*xvec* ♯* *M*› ‹*x* ♯ *P*› ‹*y* ♯ *P*›
have ([(*x*, *y*)] ∙ Ψ) ▷ *P* ⟼([(*x*, *y*)] ∙ *M*)⦇*ν*∗*xvec*⦈⟨*N*⟩ ≺ *P*′
by(*rule outputSwapFrameSubject*)
with ‹*x* ♯ Ψ› ‹*y* ♯ Ψ› **show** *?thesis*
by *simp*
qed

lemma *outputPermSubject*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and B :: $('a, 'b, 'c)$ *boundOutput*
and p :: *name prm*
and *yvec* :: *name list*
and *zvec* :: *name list*

assumes $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and S: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
and *yvec* $\sharp*$ P
and *zvec* $\sharp*$ P
and *yvec* $\sharp*$ Ψ
and *zvec* $\sharp*$ Ψ

shows $\Psi \rhd P \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
proof −
from *assms* **have** $(p \cdot \Psi) \rhd P \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule-tac outputPermFrameSubject*)
with S ‹*yvec* $\sharp*$ Ψ› ‹*zvec* $\sharp*$ Ψ› **show** *?thesis*
by *simp*
qed

lemma *outputSwapFrame*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and B :: $('a, 'b, 'c)$ *boundOutput*
and x :: *name*
and y :: *name*

assumes $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *xvec* $\sharp*$ M
and $x \sharp P$
and $y \sharp P$
and $x \sharp M$
and $y \sharp M$

shows $([(x, y)] \cdot \Psi) \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
proof −
from ‹$\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$› ‹*xvec* $\sharp*$ M› ‹$x \sharp P$› ‹$y \sharp P$›
have $([(x, y)] \cdot \Psi) \rhd P \longmapsto ([(x, y)] \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule outputSwapFrameSubject*)
with ‹$x \sharp M$› ‹$y \sharp M$› **show** *?thesis*
by *simp*
qed

lemma *outputPermFrame*:

fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and B :: $('a, 'b, 'c)$ *boundOutput*
and p :: *name prm*
and *yvec* :: *name list*
and *zvec* :: *name list*

assumes $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and S: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
and *yvec* $\sharp*$ P
and *zvec* $\sharp*$ P
and *yvec* $\sharp*$ M
and *zvec* $\sharp*$ M

shows $(p \cdot \Psi) \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
proof −
from *assms* **have** $(p \cdot \Psi) \rhd P \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule-tac outputPermFrameSubject*)
with S ‹*yvec* $\sharp*$ M› ‹*zvec* $\sharp*$ M› **show** *?thesis*
by *simp*
qed

lemma *Comm1*:
fixes Ψ :: $'b$
and Ψ_Q :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and K :: $'a$
and *xvec* :: *name list*
and Q' :: $('a, 'b, 'c)$ *psi*
and A_Q :: *name list*

assumes $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$
and *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_P \sharp* \Psi$
and $A_P \sharp* P$
and $A_P \sharp* Q$
and $A_P \sharp* M$
and $A_P \sharp* A_Q$
and $A_Q \sharp* \Psi$

and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* K$
and $xvec \sharp* P$

shows $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
proof −
{
fix Ψ :: $'b$
and Ψ_Q :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and K :: $'a$
and $xvec$ :: *name list*
and Q' :: $('a, 'b, 'c)$ *psi*
and A_Q :: *name list*

assume $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$
and *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
and *distinct* A_Q
and $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_P \sharp* \Psi$
and $A_P \sharp* P$
and $A_P \sharp* Q$
and $A_P \sharp* M$
and $A_P \sharp* A_Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* K$
and $xvec \sharp* P$

have $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
proof −

obtain r::*name prm* **where** $(r \cdot xvec) \sharp* \Psi$ **and** $(r \cdot xvec) \sharp* P$ **and** $(r \cdot xvec) \sharp* Q$ **and** $(r \cdot xvec) \sharp* M$
and $(r \cdot xvec) \sharp* K$ **and** $(r \cdot xvec) \sharp* N$ **and** $(r \cdot xvec) \sharp* A_P$ **and** $(r \cdot xvec) \sharp* A_Q$
and $(r \cdot xvec) \sharp* P'$ **and** $(r \cdot xvec) \sharp* Q'$ **and** $(r \cdot xvec) \sharp* \Psi_P$ **and** $(r \cdot xvec) \sharp* \Psi_Q$

and *Sr*: $(set\ r) \subseteq (set\ xvec) \times (set(r \cdot xvec))$ **and** $distinctPerm\ r$

by(*rule-tac* $xvec{=}xvec$ **and** $c{=}(\Psi, P, Q, M, K, N, A_P, A_Q, \Psi_P, \Psi_Q, P', Q')$ **in** *name-list-avoiding*)

(*auto simp add*: *eqvts fresh-star-prod*)

obtain $q{::}name\ prm$ **where** $(q \cdot A_Q) \sharp* \Psi$ **and** $(q \cdot A_Q) \sharp* P$ **and** $(q \cdot A_Q) \sharp* Q$ **and** $(q \cdot A_Q) \sharp* K$

and $(q \cdot A_Q) \sharp* (r \cdot N)$ **and** $(q \cdot A_Q) \sharp* (r \cdot xvec)$ **and** $(q \cdot A_Q) \sharp* (r \cdot Q')$

and $(q \cdot A_Q) \sharp* (r \cdot P')$ **and** $(q \cdot A_Q) \sharp* \Psi_P$ **and** $(q \cdot A_Q) \sharp* A_P$ **and** $(q \cdot A_Q) \sharp* \Psi_Q$

and *Sq*: $set\ q \subseteq set\ A_Q \times set(q \cdot A_Q)$

by(*rule-tac* $xvec{=}A_Q$ **and** $c{=}(\Psi, P, Q, K, r \cdot N, r \cdot xvec, \Psi_Q, A_P, \Psi_P, r \cdot Q', r \cdot P')$ **in** *name-list-avoiding*)

(*auto simp add*: *eqvts fresh-star-prod*)

obtain $p{::}name\ prm$ **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$ **and** $(p \cdot A_P) \sharp* M$

and $(p \cdot A_P) \sharp* (r \cdot N)$ **and** $(p \cdot A_P) \sharp* (r \cdot xvec)$ **and** $(p \cdot A_P) \sharp* (r \cdot Q')$

and $(p \cdot A_P) \sharp* (r \cdot P')$ **and** $(p \cdot A_P) \sharp* \Psi_P$ **and** $(p \cdot A_P) \sharp* \Psi_Q$ **and** $(p \cdot A_P) \sharp* A_Q$

and $(p \cdot A_P) \sharp* (q \cdot A_Q)$ **and** *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$

by(*rule-tac* $xvec{=}A_P$ **and** $c{=}(\Psi, P, Q, M, r \cdot N, r \cdot xvec, A_Q, q \cdot A_Q, \Psi_Q, \Psi_P, r \cdot Q', r \cdot P')$ **in** *name-list-avoiding*)

(*auto simp add*: *eqvts fresh-star-prod*)

have *FrP*: $extractFrame\ P = \langle A_P, \Psi_P \rangle$ **by** *fact*

have *FrQ*: $extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*

from $\langle A_P \sharp* Q \rangle$ *FrQ* $\langle A_P \sharp* A_Q \rangle$ **have** $A_P \sharp* \Psi_Q$

by(*drule-tac extractFrameFreshChain*) *auto*

from $\langle A_Q \sharp* P \rangle$ *FrP* $\langle A_P \sharp* A_Q \rangle$ **have** $A_Q \sharp* \Psi_P$

by(*drule-tac extractFrameFreshChain*) *auto*

from $\langle (r \cdot xvec) \sharp* A_P \rangle$ $\langle (p \cdot A_P) \sharp* (r \cdot xvec) \rangle$ $\langle (r \cdot xvec) \sharp* A_P \rangle$ *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot A_P)$

by(*simp add*: *freshChainSimps*)

from $\langle \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P' \rangle$ *Sr* $\langle distinctPerm\ r \rangle$ $\langle xvec \sharp* P \rangle$ $\langle (r \cdot xvec) \sharp* P \rangle$

have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|(r \cdot N)|\!) \prec (r \cdot P')$

by(*rule inputAlpha*)

hence $(q \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto (q \cdot M)(\!|(r \cdot N)|\!) \prec (r \cdot P')$ **using** *Sq* $\langle A_Q \sharp* P \rangle$ $\langle (q \cdot A_Q) \sharp* P \rangle$

by(*rule-tac inputPermFrameSubject*) (*assumption* | *simp*)+

hence *PTrans*: $\Psi \otimes (q \cdot \Psi_Q) \rhd P \longmapsto (q \cdot M)(\!|(r \cdot N)|\!) \prec (r \cdot P')$ **using** *Sq* $\langle A_Q \sharp* \Psi \rangle$ $\langle (q \cdot A_Q) \sharp* \Psi \rangle$

by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› *Sp* ‹$(p \cdot A_P) \sharp* \Psi_P$›
have *FrP*: *extractFrame* $P = \langle(p \cdot A_P), (p \cdot \Psi_P)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_P› **have** *distinct*$(p \cdot A_P)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q'$› *Sr* ‹$(r \cdot xvec) \sharp* N$› ‹$(r \cdot xvec) \sharp* Q'$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot Q')$
by(*simp add*: *boundOutputChainAlpha''* *create-residual.simps*)
hence $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto (p \cdot K)(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* Q$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(r \cdot xvec) \sharp* K$› ‹$(r \cdot xvec) \sharp* A_P$› ‹$(r \cdot xvec) \sharp* (p \cdot A_P)$›
by(*rule-tac outputPermFrameSubject*) (*assumption* | *auto*)
hence *QTrans*: $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (p \cdot K)(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$›
by(*simp add*: *eqvts*)
moreover hence *distinct*$(r \cdot xvec)$ **by**(*force dest*: *boundOutputDistinct*)
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› *Sq* ‹$(q \cdot A_Q) \sharp* \Psi_Q$›
have *FrQ*: *extractFrame* $Q = \langle(q \cdot A_Q), (q \cdot \Psi_Q)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(q \cdot A_Q)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(p \cdot q \cdot (\Psi \otimes \Psi_P \otimes \Psi_Q)) \vdash (p \cdot q \cdot M) \leftrightarrow (p \cdot q \cdot K)$
by(*rule-tac chanEqClosed*)+
with ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$(q \cdot A_Q) \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$(q \cdot A_Q) \sharp* \Psi_P$›
‹$A_P \sharp* \Psi_Q$› ‹$(p \cdot A_P) \sharp* \Psi_Q$› ‹$A_P \sharp* M$› ‹$(p \cdot A_P) \sharp* M$› ‹$(q \cdot A_Q) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$›
‹$A_Q \sharp* K$› ‹$(q \cdot A_Q) \sharp* K$› ‹$A_P \sharp* A_Q$› ‹$(p \cdot A_P) \sharp* A_Q$› *Sp Sq*
have $\Psi \otimes (p \cdot \Psi_P) \otimes (q \cdot \Psi_Q) \vdash (q \cdot M) \leftrightarrow (p \cdot K)$ **by**(*simp add*: *eqvts freshChainSimps*)
moreover note ‹$(p \cdot A_P) \sharp* \Psi$›
moreover from ‹$(p \cdot A_P) \sharp* A_Q$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$› ‹$(p \cdot A_P) \sharp* \Psi_Q$› *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(p \cdot A_P) \sharp* P$›
moreover from ‹$(p \cdot A_P) \sharp* A_Q$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$› ‹$(p \cdot A_P) \sharp* M$› *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(p \cdot A_P) \sharp* (r \cdot N)$› ‹$(p \cdot A_P) \sharp* (r \cdot P')$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(p \cdot A_P) \sharp* (r \cdot Q')$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$›
‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(q \cdot A_Q) \sharp* \Psi$›
moreover from ‹$(q \cdot A_Q) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* A_Q$› ‹$(q \cdot A_Q) \sharp* \Psi_P$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$› *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(q \cdot A_Q) \sharp* P$› ‹$(q \cdot A_Q) \sharp* (r \cdot N)$›‹$(q \cdot A_Q) \sharp* (r \cdot P')$› ‹$(q \cdot A_Q) \sharp* Q$›
moreover from ‹$(q \cdot A_Q) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* A_Q$› ‹$(q \cdot A_Q) \sharp* K$› ‹$(p \cdot$

A_P) $\sharp*$ $(q \cdot A_Q)$› *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(q \cdot A_Q) \sharp* (r \cdot Q')$› ‹$(q \cdot A_Q) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* \Psi$›
moreover from ‹$(r \cdot xvec) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* \Psi_P$› *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover from ‹$(r \cdot xvec) \sharp* A_Q$› ‹$(q \cdot A_Q) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* \Psi_Q$› *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(r \cdot xvec) \sharp* P$›
moreover from ‹$(r \cdot xvec) \sharp* A_Q$› ‹$(q \cdot A_Q) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* M$› *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(r \cdot xvec) \sharp* Q$›
moreover from ‹$(r \cdot xvec) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* K$› *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu*(r \cdot xvec)|\!)((r \cdot P') \parallel (r \cdot Q'))$
by(*rule-tac cComm1*)
with ‹$(r \cdot xvec) \sharp* P'$› ‹$(r \cdot xvec) \sharp* Q'$› *Sr*
show *?thesis*
by(*subst resChainAlpha*) *auto*
qed
}
note *Goal* = *this*
note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$› ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$› ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$›
moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* A_Q$›
obtain A_P' **where** *extractFrame* $P = \langle A_P', \Psi_P\rangle$ **and** *distinct* A_P' **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* P$ **and** $A_P' \sharp* Q$ **and** $A_P' \sharp* M$ **and** $A_P' \sharp* A_Q$
by(*rule-tac* C=(Ψ, P, Q, M, A_Q) **in** *distinctFrame*) *auto*
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹$A_P' \sharp* A_Q$›
obtain A_Q' **where** *extractFrame* $Q = \langle A_Q', \Psi_Q\rangle$ **and** *distinct* A_Q' **and** $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* P$ **and** $A_Q' \sharp* Q$ **and** $A_Q' \sharp* K$ **and** $A_P' \sharp* A_Q'$
apply(*rule-tac* C=(Ψ, P, Q, K, A_P') **in** *distinctFrame*) **by** *auto*
ultimately show *?thesis* **using** ‹*xvec* $\sharp* P$›
by(*rule-tac Goal*)
qed

lemma *Comm2*:
fixes Ψ :: *'b*
and Ψ_Q :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*

and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and K :: $'a$
and Q' :: $('a, 'b, 'c)$ *psi*
and A_Q :: *name list*

assumes $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
and *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_P \sharp* \Psi$
and $A_P \sharp* P$
and $A_P \sharp* Q$
and $A_P \sharp* M$
and $A_P \sharp* A_Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* K$
and $xvec \sharp* Q$

shows $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
proof −
{
fix Ψ :: $'b$
and Ψ_Q :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and K :: $'a$
and Q' :: $('a, 'b, 'c)$ *psi*
and A_Q :: *name list*

assume $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *distinct* A_P
and $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
and *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *distinct* A_Q
and $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_P \sharp* \Psi$

and $A_P \sharp* P$
and $A_P \sharp* Q$
and $A_P \sharp* M$
and $A_P \sharp* A_Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* K$
and $xvec \sharp* Q$

have $\Psi \rhd P \parallel Q \longmapsto\tau \prec (\!|\nu* xvec|\!)(P' \parallel Q')$
proof −

obtain r::*name prm* **where** $(r \cdot xvec) \sharp* \Psi$ **and** $(r \cdot xvec) \sharp* P$ **and** $(r \cdot xvec) \sharp* Q$ **and** $(r \cdot xvec) \sharp* M$
and $(r \cdot xvec) \sharp* K$ **and** $(r \cdot xvec) \sharp* N$ **and** $(r \cdot xvec) \sharp* A_P$ **and** $(r \cdot xvec) \sharp* A_Q$
and $(r \cdot xvec) \sharp* P'$ **and** $(r \cdot xvec) \sharp* Q'$ **and** $(r \cdot xvec) \sharp* \Psi_P$ **and** $(r \cdot xvec) \sharp* \Psi_Q$
and *Sr*: $(set\ r) \subseteq (set\ xvec) \times (set(r \cdot xvec))$ **and** *distinctPerm r*
by(*rule-tac xvec=xvec* **and** c=(Ψ, P, Q, M, K, N, A_P, A_Q, Ψ_P, Ψ_Q, P', Q') **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

obtain q::*name prm* **where** $(q \cdot A_Q) \sharp* \Psi$ **and** $(q \cdot A_Q) \sharp* P$ **and** $(q \cdot A_Q) \sharp* Q$ **and** $(q \cdot A_Q) \sharp* K$
and $(q \cdot A_Q) \sharp* (r \cdot N)$ **and** $(q \cdot A_Q) \sharp* (r \cdot xvec)$ **and** $(q \cdot A_Q) \sharp* (r \cdot Q')$
and $(q \cdot A_Q) \sharp* (r \cdot P')$ **and** $(q \cdot A_Q) \sharp* \Psi_P$ **and** $(q \cdot A_Q) \sharp* A_P$ **and** $(q \cdot A_Q) \sharp* \Psi_Q$
and *Sq*: $set\ q \subseteq set\ A_Q \times set(q \cdot A_Q)$
by(*rule-tac xvec*=A_Q **and** c=(Ψ, P, Q, K, $r \cdot N$, $r \cdot xvec$, Ψ_Q, A_P, Ψ_P, $r \cdot Q'$, $r \cdot P'$) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

obtain p::*name prm* **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$ **and** $(p \cdot A_P) \sharp* M$
and $(p \cdot A_P) \sharp* (r \cdot N)$ **and** $(p \cdot A_P) \sharp* (r \cdot xvec)$ **and** $(p \cdot A_P) \sharp* (r \cdot Q')$
and $(p \cdot A_P) \sharp* (r \cdot P')$ **and** $(p \cdot A_P) \sharp* \Psi_P$ **and** $(p \cdot A_P) \sharp* \Psi_Q$ **and** $(p \cdot A_P) \sharp* A_Q$
and $(p \cdot A_P) \sharp* (q \cdot A_Q)$ **and** *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$
by(*rule-tac xvec*=A_P **and** c=(Ψ, P, Q, M, $r \cdot N$, $r \cdot xvec$, A_Q, $q \cdot A_Q$, Ψ_Q, Ψ_P, $r \cdot Q'$, $r \cdot P'$) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **by** *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*

from ‹A_P ♯* Q› FrQ ‹A_P ♯* A_Q› **have** A_P ♯* Ψ_Q
by(*drule-tac extractFrameFreshChain*) *auto*
from ‹A_Q ♯* P› FrP ‹A_P ♯* A_Q› **have** A_Q ♯* Ψ_P
by(*drule-tac extractFrameFreshChain*) *auto*

from ‹$(r \cdot xvec)$ ♯* A_Q› ‹$(q \cdot A_Q)$ ♯* $(r \cdot xvec)$› ‹$(r \cdot xvec)$ ♯* A_Q› Sq **have** $(r \cdot xvec)$ ♯* $(q \cdot A_Q)$
by(*simp add*: *freshChainSimps*)

from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$› Sr ‹$(r \cdot xvec)$ ♯* N› ‹$(r \cdot xvec)$ ♯* P'›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N)\rangle \prec (r \cdot P')$
by(*simp add*: *boundOutputChainAlpha''* *create-residual.simps*)
hence $(q \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto (q \cdot M)(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N)\rangle \prec (r \cdot P')$ **using** Sq ‹A_Q ♯* P› ‹$(q \cdot A_Q)$ ♯* P› ‹$(r \cdot xvec)$ ♯* M› ‹$(r \cdot xvec)$ ♯* A_Q› ‹$(r \cdot xvec)$ ♯* $(q \cdot A_Q)$›
by(*rule-tac outputPermFrameSubject*) (*assumption* | *auto*)
hence *PTrans*: $\Psi \otimes (q \cdot \Psi_Q) \rhd P \longmapsto (q \cdot M)(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N)\rangle \prec (r \cdot P')$ **using** Sq ‹A_Q ♯* Ψ› ‹$(q \cdot A_Q)$ ♯* Ψ›
by(*simp add*: *eqvts*)
moreover hence *distinct*$(r \cdot xvec)$ **by**(*force dest*: *boundOutputDistinct*)

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› Sp ‹$(p \cdot A_P)$ ♯* Ψ_P›
have *FrP*: *extractFrame* $P = \langle (p \cdot A_P), (p \cdot \Psi_P)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_P› **have** *distinct*$(p \cdot A_P)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$› Sr ‹*distinctPerm* r› ‹$xvec$ ♯* Q› ‹$(r \cdot xvec)$ ♯* Q›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|(r \cdot N)|\!) \prec (r \cdot Q')$
by(*rule inputAlpha*)
hence $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto (p \cdot K)(\!|(r \cdot N)|\!) \prec (r \cdot Q')$ **using** Sp ‹A_P ♯* Q› ‹$(p \cdot A_P)$ ♯* Q›
by(*rule-tac inputPermFrameSubject*) (*assumption* | *simp*)+
hence *QTrans*: $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (p \cdot K)(\!|(r \cdot N)|\!) \prec (r \cdot Q')$ **using** Sp ‹A_P ♯* Ψ› ‹$(p \cdot A_P)$ ♯* Ψ›
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› Sq ‹$(q \cdot A_Q)$ ♯* Ψ_Q›
have *FrQ*: *extractFrame* $Q = \langle (q \cdot A_Q), (q \cdot \Psi_Q)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(q \cdot A_Q)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(p \cdot q \cdot (\Psi \otimes \Psi_P \otimes \Psi_Q)) \vdash (p \cdot q \cdot M) \leftrightarrow (p \cdot q \cdot K)$
by(*rule-tac chanEqClosed*)+
with ‹A_P ♯* Ψ› ‹$(p \cdot A_P)$ ♯* Ψ› ‹A_Q ♯* Ψ› ‹$(q \cdot A_Q)$ ♯* Ψ› ‹A_Q ♯* Ψ_P› ‹$(q \cdot A_Q)$ ♯* Ψ_P›
‹A_P ♯* Ψ_Q› ‹$(p \cdot A_P)$ ♯* Ψ_Q› ‹A_P ♯* M› ‹$(p \cdot A_P)$ ♯* M› ‹$(q \cdot A_Q)$ ♯*

$A_P\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$
$\langle A_Q \sharp* K\rangle$ $\langle(q \cdot A_Q) \sharp* K\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle(p \cdot A_P) \sharp* A_Q\rangle$ *Sp Sq*
have $\Psi \otimes (p \cdot \Psi_P) \otimes (q \cdot \Psi_Q) \vdash (q \cdot M) \leftrightarrow (p \cdot K)$
by(*simp add*: *eqvts freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* \Psi\rangle$
moreover from $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ $\langle(p \cdot A_P) \sharp* \Psi_Q\rangle$ *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* P\rangle$
moreover from $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ $\langle(p \cdot A_P) \sharp* M\rangle$ *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* (r \cdot N)\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot P')\rangle$ $\langle(p \cdot A_P) \sharp* Q\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot Q')\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$
$\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(q \cdot A_Q) \sharp* \Psi\rangle$
moreover from $\langle(q \cdot A_Q) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* \Psi_P\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(q \cdot A_Q) \sharp* P\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot N)\rangle$$\langle(q \cdot A_Q) \sharp* (r \cdot P')\rangle$ $\langle(q \cdot A_Q) \sharp* Q\rangle$
moreover from $\langle(q \cdot A_Q) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* K\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(q \cdot A_Q) \sharp* (r \cdot Q')\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi\rangle$
moreover from $\langle(r \cdot xvec) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi_P\rangle$ *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(r \cdot xvec) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi_Q\rangle$ *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(r \cdot xvec) \sharp* P\rangle$
moreover from $\langle(r \cdot xvec) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* M\rangle$ *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(r \cdot xvec) \sharp* Q\rangle$
moreover from $\langle(r \cdot xvec) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* K\rangle$ *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu*(r \cdot xvec)|\!)((r \cdot P') \parallel (r \cdot Q'))$
by(*rule-tac cComm2*)
with $\langle(r \cdot xvec) \sharp* P'\rangle$ $\langle(r \cdot xvec) \sharp* Q'\rangle$ *Sr*
show *?thesis*
by(*subst resChainAlpha*) *auto*
qed
}
note *Goal* = *this*
note $\langle\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'\rangle$ $\langle\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'\rangle$ $\langle\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K\rangle$

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* A_Q$›
obtain A_P' **where** *extractFrame* $P = \langle A_P', \Psi_P\rangle$ **and** *distinct* A_P' **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* P$ **and** $A_P' \sharp* Q$ **and** $A_P' \sharp* M$ **and** $A_P' \sharp* A_Q$
by(*rule-tac* C=(Ψ, P, Q, M, A_Q) **in** *distinctFrame*) *auto*
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹$A_P' \sharp* A_Q$›
obtain A_Q' **where** *extractFrame* $Q = \langle A_Q', \Psi_Q\rangle$ **and** *distinct* A_Q' **and** $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* P$ **and** $A_Q' \sharp* Q$ **and** $A_Q' \sharp* K$ **and** $A_P' \sharp* A_Q'$
by(*rule-tac* C=(Ψ, P, Q, K, A_P') **in** *distinctFrame*) *auto*
ultimately show *?thesis* **using** ‹*xvec* $\sharp* Q$›
by(*rule-tac Goal*)
qed

lemma *semanticsCasesAux*[*consumes 1*, *case-names cInput cOutput cCase cPar1 cPar2 cComm1 cComm2 cOpen cScope cBang*]:
fixes Ψ :: *'b*
and *cP* :: (*'a*, *'b*, *'c*) *psi*
and *cRs* :: (*'a*, *'b*, *'c*) *residual*
and *C* :: *'d*::*fs-name*
and *x* :: *name*

assumes $\Psi \rhd cP \longmapsto cRs$
and *rInput*: $\bigwedge M\ K\ xvec\ N\ Tvec\ P.\ [\![cP = M(\!|\lambda* xvec\ N|\!).P;\ cRs = K(\!|(N[xvec::=Tvec])|\!) \prec P[xvec::=Tvec];$
$\Psi \vdash M \leftrightarrow K$; *distinct xvec*; *set xvec* $\subseteq$ *supp N*; *length xvec*=*length Tvec*;
xvec $\sharp*$ *Tvec*; *xvec* $\sharp*$ Ψ; *xvec* $\sharp*$ *M*; *xvec* $\sharp*$ *K*; *xvec* $\sharp*$ *C*$]\!] \Longrightarrow$ *Prop*
and *rOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = K\langle N\rangle \prec P;\ \Psi \vdash M \leftrightarrow K]\!] \Longrightarrow$ *Prop*
and *rCase*: $\bigwedge Cs\ P\ \varphi.\ [\![cP = Cases\ Cs;\ \Psi \rhd P \longmapsto cRs;\ (\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$ *Prop*

and *rPar1*: $\bigwedge \Psi_Q\ P\ \alpha\ P'\ Q\ A_Q.\ [\![cP = P \parallel Q;\ cRs = \alpha \prec (P' \parallel Q);$
$(\Psi \otimes \Psi_Q) \rhd P \longmapsto (\alpha \prec P');$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \Psi$; $A_Q \sharp* \alpha$; $A_Q \sharp* C$; $A_Q \sharp* P'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* \Psi_Q$;
bn $\alpha \sharp* Q$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp*$ *subject* α; *bn* $\alpha \sharp* C$; *distinct*(*bn* α)$]\!] \Longrightarrow$
Prop
and *rPar2*: $\bigwedge \Psi_P\ Q\ \alpha\ Q'\ P\ A_P.\ [\![cP = P \parallel Q;\ cRs = \alpha \prec (P \parallel Q');$
$(\Psi \otimes \Psi_P) \rhd Q \longmapsto \alpha \prec Q';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \Psi$; $A_P \sharp* \alpha$; $A_P \sharp* C$;
$A_P \sharp* Q'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* \Psi_P$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp* Q$; *bn* $\alpha \sharp*$ *subject* α; *bn* $\alpha \sharp* C$; *distinct*(*bn* α)$]\!] \Longrightarrow$ *Prop*

and *rComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu * xvec|\!) P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;$
$distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q,$
$\Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp*$
$M;\ A_P \sharp* N;$
$A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;$
$A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_Q$
$\sharp* xvec;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp*$
$Q;$
$xvec \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C;\ distinct\ xvec]\!] \Longrightarrow Prop$
and *rComm2*: $\bigwedge \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu * xvec|\!) P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P,$
$\Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;$
$distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp*$
$M;\ A_P \sharp* N;$
$A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* xvec;\ A_Q \sharp* \Psi;$
$A_Q \sharp* \Psi_P;$
$A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* Q';\ A_Q$
$\sharp* xvec;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* \Psi_Q;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp*$
$Q;$
$xvec \sharp* K;\ A_P \sharp* C;\ A_Q \sharp* C;\ xvec \sharp* C;\ distinct\ xvec]\!] \Longrightarrow Prop$
and *rOpen*: $\bigwedge P\ M\ xvec\ yvec\ N\ P'\ x.$
$[\![cP = (\!|\nu x|\!) P;\ cRs = M(\!|\nu * (xvec @ x \# yvec)|\!)\langle N\rangle \prec P';$
$\Psi \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P';\ x \in supp\ N;\ x \sharp xvec;\ x \sharp$
$yvec;\ x \sharp M;\ x \sharp \Psi;\ distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* yvec;\ yvec \sharp* \Psi;\ yvec \sharp* P;$
$yvec \sharp* M;\ xvec \sharp* C;\ x \sharp C;\ yvec \sharp* C]\!] \Longrightarrow$
$Prop$
and *rScope*: $\bigwedge P\ \alpha\ P'\ x.\ [\![cP = (\!|\nu x|\!) P;\ cRs = \alpha \prec (\!|\nu x|\!) P';$
$\Psi \rhd P \longmapsto \alpha \prec P';\ x \sharp \Psi;\ x \sharp \alpha;\ x \sharp C;\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha$
$\sharp* P;\ bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C;\ distinct(bn\ \alpha)]\!] \Longrightarrow Prop$
and *rBang*: $\bigwedge P.\ [\![cP = !P;\ \Psi \rhd P \parallel !P \longmapsto cRs;\ guarded\ P]\!] \Longrightarrow Prop$
shows *Prop*
using ‹$\Psi \rhd cP \longmapsto cRs$›
proof(*cases rule*: *semantics.cases*)
case(*cInput M K xvec N Tvec P*)
obtain *p*::*name prm* **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* M$ **and** $(p \cdot xvec)$
$\sharp* N$ **and** $(p \cdot xvec) \sharp* K$
and $(p \cdot xvec) \sharp* Tvec$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* C$
and *S*: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm*

p
by(*rule-tac xvec=xvec* **and** *c=*(Ψ, *M*, *K*, *N*, *P*, *C*, *Tvec*) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)
from ‹$cP = M\langle\!\langle\lambda*xvec\ N\rangle\!\rangle.P$› ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* P$› *S*
have $cP = M\langle\!\langle\lambda*(p \cdot xvec)\ (p \cdot N)\rangle\!\rangle.(p \cdot P)$
by(*simp add*: *inputChainAlpha′*)
moreover from ‹$cRs = K\langle\!\langle(N[xvec::=Tvec])\rangle\!\rangle \prec P[xvec::=Tvec]$› ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* P$› *S* ‹*length xvec = length Tvec*› ‹*distinctPerm p*›
have $cRs = K\langle\!\langle((p \cdot N)[(p \cdot xvec)::=Tvec])\rangle\!\rangle \prec (p \cdot P)[(p \cdot xvec)::=Tvec]$
by(*auto simp add*: *substTerm.renaming renaming residualInject*)

moreover note ‹$\Psi \vdash M \leftrightarrow K$›
moreover from ‹*distinct xvec*› **have** *distinct*$(p \cdot xvec)$
by *simp*
moreover from ‹$(set\ xvec) \subseteq (supp\ N)$› **have** $(p \cdot (set\ xvec)) \subseteq (p \cdot (supp\ N))$
by *simp*
hence $set(p \cdot xvec) \subseteq supp(p \cdot N)$
by(*simp add*: *eqvts*)
moreover from ‹*length xvec = length Tvec*› **have** *length*$(p \cdot xvec) =$ *length Tvec*
by *simp*
ultimately show *?thesis* **using** ‹$(p \cdot xvec) \sharp* Tvec$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* M$› ‹$(p \cdot xvec) \sharp* K$›
‹$(p \cdot xvec) \sharp* C$›
by(*rule rInput*)
next
case(*Output M K N P*)
thus *?thesis* **by**(*rule rOutput*)
next
case(*Case P* φ *Cs*)
thus *?thesis* **by**(*rule rCase*)
next
case(*cPar1* Ψ_Q *P* α *P′ Q* A_Q)
obtain *q*::*name prm* **where** $(bn(q \cdot \alpha)) \sharp* \Psi$ **and** $(bn(q \cdot \alpha)) \sharp* P$ **and** $(bn(q \cdot \alpha)) \sharp* Q$
and $(bn(q \cdot \alpha)) \sharp* \alpha$ **and** $(bn(q \cdot \alpha)) \sharp* A_Q$ **and** $(bn(q \cdot \alpha)) \sharp* P'$ **and** $(bn(q \cdot \alpha)) \sharp* \Psi_Q$
and *distinctPerm q*
and $(bn(q \cdot \alpha)) \sharp* C$ **and** *Sq*: $(set\ q) \subseteq set(bn\ \alpha) \times (set(bn(q \cdot \alpha)))$
by(*rule-tac xvec=bn* α **and** *c=*(Ψ, *P*, *Q*, α, A_Q, Ψ_Q, *P′*, *C*) **in** *name-list-avoiding*) (*auto simp add*: *eqvts*)
obtain *p*::*name prm* **where** $(p \cdot A_Q) \sharp* \Psi$ **and** $(p \cdot A_Q) \sharp* P$ **and** $(p \cdot A_Q) \sharp* Q$
and $(p \cdot A_Q) \sharp* \alpha$ **and** $(p \cdot A_Q) \sharp* (q \cdot \alpha)$ **and** $(p \cdot A_Q) \sharp* P'$
and $(p \cdot A_Q) \sharp* (q \cdot P')$ **and** $(p \cdot A_Q) \sharp* \Psi_Q$ **and** $(p \cdot A_Q) \sharp* C$
and *Sp*: $(set\ p) \subseteq (set\ A_Q) \times (set(p \cdot A_Q))$ **and** *distinctPerm p*

by(*rule-tac xvec*=A_Q **and** *c*=(Ψ, P, Q, α, $q \cdot \alpha$, P', $(q \cdot P')$, Ψ_Q, C) **in** *name-list-avoiding*) *auto*
from ‹$A_Q \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* A_Q$› *Sq* ‹*distinctPerm q*› **have** $A_Q \sharp* (q \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, *of* - - *q*]) (*simp add*: *eqvts*)
from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinctPerm q*› **have** $bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, *of* - - *q*]) (*simp add*: *eqvts*)
from ‹*distinct*(*bn* α)› ‹*distinctPerm q*› **have** *distinct*($bn(q \cdot \alpha)$)
by(*subst distinctClosed*[*symmetric*, *of* - *q*]) (*simp add*: *eqvts*)
note ‹$cP = P \parallel Q$›

moreover from ‹$cRs = \alpha \prec (P' \parallel Q)$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$(bn(q \cdot \alpha)) \sharp* \alpha$› ‹$(bn(q \cdot \alpha)) \sharp* P'$› ‹$(bn(q \cdot \alpha)) \sharp* Q$› ‹*bn* $\alpha \sharp* Q$› *Sq*
have $cRs = (q \cdot \alpha) \prec (q \cdot P') \parallel Q$
by(*force simp add*: *residualAlpha*)
moreover from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$(bn(q \cdot \alpha)) \sharp* \alpha$› ‹$(bn(q \cdot \alpha)) \sharp* P'$› *Sq*
have *Trans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto (q \cdot \alpha) \prec (q \cdot P')$
by(*force simp add*: *residualAlpha*)
hence $A_Q \sharp* (q \cdot P')$ **using** ‹$bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$› ‹*distinct*($bn(q \cdot \alpha)$)› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* (q \cdot \alpha)$›
by(*drule-tac freeFreshChainDerivative*) *auto*

from *Trans* **have** $(p \cdot (\Psi \otimes \Psi_Q)) \rhd (p \cdot P) \longmapsto p \cdot ((q \cdot \alpha) \prec (q \cdot P'))$
by(*rule semantics.eqvt*)
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* (q \cdot \alpha)$› ‹$A_Q \sharp* (q \cdot P')$› ‹$(bn(q \cdot \alpha)) \sharp* A_Q$› ‹$(p \cdot A_Q) \sharp* (q \cdot \alpha)$›
‹$(p \cdot A_Q) \sharp* \Psi$› ‹$(p \cdot A_Q) \sharp* P$› ‹$(p \cdot A_Q) \sharp* (q \cdot P')$› *Sp*
have $\Psi \otimes (p \cdot \Psi_Q) \rhd P \longmapsto (q \cdot \alpha) \prec (q \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹$(p \cdot A_Q) \sharp* \Psi_Q$› *Sp* **have** *extractFrame* $Q = \langle (p \cdot A_Q), (p \cdot \Psi_Q) \rangle$
by(*simp add*: *frameChainAlpha' eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* \Psi_Q$› ‹$(bn(q \cdot \alpha)) \sharp* A_Q$› ‹$(p \cdot A_Q) \sharp* (q \cdot \alpha)$› *Sp* **have** $(bn(q \cdot \alpha)) \sharp* (p \cdot \Psi_Q)$
by(*simp add*: *freshAlphaPerm*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(p \cdot A_Q)$ **by** *simp*
ultimately show *?thesis*
using ‹$(p \cdot A_Q) \sharp* P$› ‹$(p \cdot A_Q) \sharp* Q$› ‹$(p \cdot A_Q) \sharp* \Psi$› ‹$(p \cdot A_Q) \sharp* (q \cdot \alpha)$›
‹$(p \cdot A_Q) \sharp* (q \cdot P')$› ‹$(bn(q \cdot \alpha)) \sharp* \Psi$› ‹$(bn(q \cdot \alpha)) \sharp* Q$› ‹$(bn(q \cdot \alpha)) \sharp* P$›
‹$(bn(q \cdot \alpha)) \sharp* C$› ‹$(p \cdot A_Q) \sharp* C$› ‹*bn* $(q \cdot \alpha) \sharp*$ *subject* $(q \cdot \alpha)$› ‹*distinct*($bn(q \cdot \alpha)$)›
by(*rule-tac rPar1*)
next
case(*cPar2* Ψ_P Q α Q' P A_P)
obtain *q*::*name prm* **where** $(bn(q \cdot \alpha)) \sharp* \Psi$ **and** $(bn(q \cdot \alpha)) \sharp* P$ **and** $(bn(q \cdot \alpha)) \sharp* Q$
and $(bn(q \cdot \alpha)) \sharp* \alpha$ **and** $(bn(q \cdot \alpha)) \sharp* A_P$ **and** $(bn(q \cdot \alpha)) \sharp* Q'$ **and** $(bn(q \cdot \alpha)) \sharp* \Psi_P$
and *distinctPerm q*

and $(bn(q \cdot \alpha)) \sharp* C$ **and** *Sq*: $(set\ q) \subseteq set(bn\ \alpha) \times (set(bn(q \cdot \alpha)))$
by(*rule-tac xvec*=*bn* α **and** *c*=(Ψ, P, Q, α, A_P, Ψ_P, Q', C) **in** *name-list-avoiding*) (*auto simp add*: *eqvts*)
obtain *p*::*name prm* **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$
and $(p \cdot A_P) \sharp* \alpha$ **and** $(p \cdot A_P) \sharp* (q \cdot \alpha)$ **and** $(p \cdot A_P) \sharp* Q'$
and $(p \cdot A_P) \sharp* (q \cdot Q')$ **and** $(p \cdot A_P) \sharp* \Psi_P$ **and** $(p \cdot A_P) \sharp* C$
and *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$ **and** *distinctPerm p*
by(*rule-tac xvec*=A_P **and** *c*=(Ψ, P, Q, α, $q \cdot \alpha$, Q', $(q \cdot Q')$, Ψ_P, C) **in** *name-list-avoiding*) *auto*
from ‹$A_P \sharp* \alpha$› ‹$bn(q \cdot \alpha) \sharp* A_P$› *Sq* ‹*distinctPerm q*› **have** $A_P \sharp* (q \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, *of* - - *q*]) (*simp add*: *eqvts*)
from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinctPerm q*› **have** $bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, *of* - - *q*]) (*simp add*: *eqvts*)
from ‹*distinct*(*bn* α)› ‹*distinctPerm q*› **have** $distinct(bn(q \cdot \alpha))$
by(*subst distinctClosed*[*symmetric*, *of* - *q*]) (*simp add*: *eqvts*)
note ‹$cP = P \parallel Q$›

moreover from ‹$cRs = \alpha \prec (P \parallel Q')$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$(bn(q \cdot \alpha)) \sharp* \alpha$› ‹$(bn(q \cdot \alpha)) \sharp* Q'$› ‹$(bn(q \cdot \alpha)) \sharp* P$› ‹*bn* $\alpha \sharp* P$› *Sq*
have $cRs = (q \cdot \alpha) \prec P \parallel (q \cdot Q')$
by(*force simp add*: *residualAlpha*)
moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$(bn(q \cdot \alpha)) \sharp* \alpha$› ‹$(bn(q \cdot \alpha)) \sharp* Q'$› *Sq*
have *Trans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto (q \cdot \alpha) \prec (q \cdot Q')$
by(*force simp add*: *residualAlpha*)
hence $A_P \sharp* (q \cdot Q')$ **using** ‹$bn(q \cdot \alpha) \sharp* subject(q \cdot \alpha)$› ‹$distinct(bn(q \cdot \alpha))$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* (q \cdot \alpha)$›
by(*drule-tac freeFreshChainDerivative*) *auto*

from *Trans* **have** $(p \cdot (\Psi \otimes \Psi_P)) \rhd (p \cdot Q) \longmapsto p \cdot ((q \cdot \alpha) \prec (q \cdot Q'))$
by(*rule semantics.eqvt*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* (q \cdot \alpha)$› ‹$A_P \sharp* (q \cdot Q')$› ‹$(bn(q \cdot \alpha)) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$›
‹$(p \cdot A_P) \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(p \cdot A_P) \sharp* (q \cdot Q')$› *Sp*
have $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (q \cdot \alpha) \prec (q \cdot Q')$ **by**(*simp add*: *eqvts*)
moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹$(p \cdot A_P) \sharp* \Psi_P$› *Sp* **have** *extractFrame* $P = \langle (p \cdot A_P), (p \cdot \Psi_P) \rangle$
by(*simp add*: *frameChainAlpha'* *eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* \Psi_P$› ‹$(bn(q \cdot \alpha)) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$› *Sp* **have** $(bn(q \cdot \alpha)) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshAlphaPerm*)
moreover from ‹*distinct* A_P› **have** $distinct(p \cdot A_P)$ **by** *simp*
ultimately show *?thesis*
using ‹$(p \cdot A_P) \sharp* P$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* (q \cdot \alpha)$›
‹$(p \cdot A_P) \sharp* (q \cdot Q')$› ‹$(bn(q \cdot \alpha)) \sharp* \Psi$› ‹$(bn(q \cdot \alpha)) \sharp* Q$› ‹$(bn(q \cdot \alpha)) \sharp* P$›

$\langle(bn(q \cdot \alpha)) \sharp* C\rangle$ $\langle(p \cdot A_P) \sharp* C\rangle$ $\langle bn\ (q \cdot \alpha) \sharp* subject\ (q \cdot \alpha)\rangle$ $\langle distinct(bn(q \cdot \alpha))\rangle$
by(*rule-tac rPar2*)
next
case(*cComm1* Ψ_Q P M N P' A_P Ψ_P Q K *xvec* Q' A_Q)
obtain r::*name prm* **where** $(r \cdot xvec) \sharp* \Psi$ **and** $(r \cdot xvec) \sharp* P$ **and** $(r \cdot xvec) \sharp* Q$ **and** $(r \cdot xvec) \sharp* M$
and $(r \cdot xvec) \sharp* K$ **and** $(r \cdot xvec) \sharp* N$ **and** $(r \cdot xvec) \sharp* A_P$ **and** $(r \cdot xvec) \sharp* A_Q$
and $(r \cdot xvec) \sharp* P'$ **and** $(r \cdot xvec) \sharp* Q'$ **and** $(r \cdot xvec) \sharp* \Psi_P$ **and** $(r \cdot xvec) \sharp* \Psi_Q$
and $(r \cdot xvec) \sharp* C$ **and** *Sr*: $(set\ r) \subseteq (set\ xvec) \times (set(r \cdot xvec))$ **and** *distinctPerm r*
by(*rule-tac xvec*=*xvec* **and** c=(Ψ, P, Q, M, K, N, A_P, A_Q, Ψ_P, Ψ_Q, P', Q', C) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts*)

obtain q::*name prm* **where** $(q \cdot A_Q) \sharp* \Psi$ **and** $(q \cdot A_Q) \sharp* P$ **and** $(q \cdot A_Q) \sharp* Q$ **and** $(q \cdot A_Q) \sharp* K$
and $(q \cdot A_Q) \sharp* N$ **and** $(q \cdot A_Q) \sharp* xvec$ **and** $(q \cdot A_Q) \sharp* Q'$ **and** $(q \cdot A_Q) \sharp* P'$
and $(q \cdot A_Q) \sharp* \Psi_P$ **and** $(q \cdot A_Q) \sharp* A_P$ **and** $(q \cdot A_Q) \sharp* \Psi_Q$ **and** $(q \cdot A_Q) \sharp* (r \cdot xvec)$
and $(q \cdot A_Q) \sharp* C$ **and** *Sq*: $(set\ q) \subseteq (set\ A_Q) \times (set(q \cdot A_Q))$
by(*rule-tac xvec*=A_Q **and** c=(Ψ, P, Q, K, N, *xvec*, $r \cdot xvec$, Ψ_Q, A_P, Ψ_P, Q', P', C) **in** *name-list-avoiding*) *clarsimp*

obtain p::*name prm* **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$ **and** $(p \cdot A_P) \sharp* M$
and $(p \cdot A_P) \sharp* N$ **and** $(p \cdot A_P) \sharp* xvec$ **and** $(p \cdot A_P) \sharp* Q'$ **and** $(p \cdot A_P) \sharp* A_Q$
and $(p \cdot A_P) \sharp* P'$ **and** $(p \cdot A_P) \sharp* \Psi_P$ **and** $(p \cdot A_P) \sharp* \Psi_Q$ **and** $(p \cdot A_P) \sharp* (q \cdot A_Q)$
and $(p \cdot A_P) \sharp* C$ **and** $(p \cdot A_P) \sharp* (r \cdot xvec)$ **and** *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$
by(*rule-tac xvec*=A_P **and** c=(Ψ, P, Q, M, N, *xvec*, $r \cdot xvec$, A_Q, $q \cdot A_Q$, Ψ_Q, Ψ_P, Q', P', C) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*

from $\langle A_P \sharp* Q\rangle$ *FrQ* $\langle A_P \sharp* A_Q\rangle$ **have** $A_P \sharp* \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
from $\langle A_Q \sharp* P\rangle$ *FrP* $\langle A_P \sharp* A_Q\rangle$ **have** $A_Q \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*
note $\langle cP = P \parallel Q\rangle$
moreover from $\langle(r \cdot xvec) \sharp* P'\rangle$ $\langle(r \cdot xvec) \sharp* Q'\rangle$ **have** $(r \cdot xvec) \sharp* (P' \parallel Q')$

by *simp*
with ‹$cRs = \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$› ‹$(r \cdot xvec) \sharp* N$› *Sr*
have $cRs = \tau \prec (\!|\nu * (r \cdot xvec)|\!)(r \cdot (P' \parallel Q'))$ **by**(*simp add*: *resChainAlpha residualInject*)
hence $cRs = \tau \prec (\!|\nu * (r \cdot xvec)|\!)((r \cdot P') \parallel (r \cdot Q'))$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$› *Sr* ‹*distinctPerm r*› ‹$xvec \sharp* P$› ‹$(r \cdot xvec) \sharp* P$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|(r \cdot N)|\!) \prec (r \cdot P')$
by(*rule inputAlpha*)
hence $(q \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto (q \cdot M)(\!|(r \cdot N)|\!) \prec (r \cdot P')$ **using** *Sq* ‹$A_Q \sharp* P$› ‹$(q \cdot A_Q) \sharp* P$›
by(*rule-tac inputPermFrameSubject*) (*assumption* | *simp*)+
hence *PTrans*: $\Psi \otimes (q \cdot \Psi_Q) \rhd P \longmapsto (q \cdot M)(\!|(r \cdot N)|\!) \prec (r \cdot P')$ **using** *Sq* ‹$A_Q \sharp* \Psi$› ‹$(q \cdot A_Q) \sharp* \Psi$›
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› *Sp* ‹$(p \cdot A_P) \sharp* \Psi_P$›
have *FrP*: *extractFrame* $P = \langle (p \cdot A_P), (p \cdot \Psi_P) \rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_P› **have** *distinct*$(p \cdot A_P)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$› *Sr* ‹$(r \cdot xvec) \sharp* N$› ‹$(r \cdot xvec) \sharp* Q'$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N) \rangle \prec (r \cdot Q')$
by(*simp add*: *boundOutputChainAlpha''* *residualInject*)
hence $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto (p \cdot K)(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N) \rangle \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* Q$› ‹$(p \cdot A_P) \sharp* Q$› ‹$(r \cdot xvec) \sharp* K$›‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* A_P$›
by(*rule-tac outputPermFrameSubject*) (*assumption* | *auto*)

hence *QTrans*: $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (p \cdot K)(\!|\nu * (r \cdot xvec)|\!)\langle (r \cdot N) \rangle \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$›
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› *Sq* ‹$(q \cdot A_Q) \sharp* \Psi_Q$›
have *FrQ*: *extractFrame* $Q = \langle (q \cdot A_Q), (q \cdot \Psi_Q) \rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(q \cdot A_Q)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(p \cdot q \cdot (\Psi \otimes \Psi_P \otimes \Psi_Q)) \vdash (p \cdot q \cdot M) \leftrightarrow (p \cdot q \cdot K)$
by(*rule-tac chanEqClosed*)+
with ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$(q \cdot A_Q) \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$(q \cdot A_Q) \sharp* \Psi_P$›
‹$A_P \sharp* \Psi_Q$› ‹$(p \cdot A_P) \sharp* \Psi_Q$› ‹$A_P \sharp* M$› ‹$(p \cdot A_P) \sharp* M$› ‹$(q \cdot A_Q) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (q \cdot A_Q)$›
‹$A_Q \sharp* K$› ‹$(q \cdot A_Q) \sharp* K$› ‹$A_P \sharp* A_Q$› ‹$(p \cdot A_P) \sharp* A_Q$› *Sp Sq*
have $\Psi \otimes (p \cdot \Psi_P) \otimes (q \cdot \Psi_Q) \vdash (q \cdot M) \leftrightarrow (p \cdot K)$

by(*simp add*: *eqvts freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* \Psi\rangle$
moreover from $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ $\langle(p \cdot A_P) \sharp* \Psi_Q\rangle$ *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* P\rangle$
moreover from $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ $\langle(p \cdot A_P) \sharp* M\rangle$ *Sq* **have** $(p \cdot A_P) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(p \cdot A_P) \sharp* xvec\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(p \cdot A_P) \sharp* N\rangle$ *Sr* **have** $(p \cdot A_P) \sharp* (r \cdot N)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(p \cdot A_P) \sharp* xvec\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(p \cdot A_P) \sharp* P'\rangle$ *Sr* **have** $(p \cdot A_P) \sharp* (r \cdot P')$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* Q\rangle$
moreover from $\langle(p \cdot A_P) \sharp* xvec\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(p \cdot A_P) \sharp* Q'\rangle$ *Sr* **have** $(p \cdot A_P) \sharp* (r \cdot Q')$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(q \cdot A_Q) \sharp* \Psi\rangle$
moreover from $\langle(q \cdot A_Q) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* \Psi_P\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(q \cdot A_Q) \sharp* P\rangle$
moreover from $\langle(q \cdot A_Q) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* K\rangle$ $\langle(p \cdot A_P) \sharp* (q \cdot A_Q)\rangle$ *Sp Sq* **have** $(q \cdot A_Q) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(q \cdot A_Q) \sharp* xvec\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(q \cdot A_Q) \sharp* N\rangle$ *Sr* **have** $(q \cdot A_Q) \sharp* (r \cdot N)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(q \cdot A_Q) \sharp* xvec\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(q \cdot A_Q) \sharp* P'\rangle$ *Sr* **have** $(q \cdot A_Q) \sharp* (r \cdot P')$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(q \cdot A_Q) \sharp* Q\rangle$
moreover from $\langle(q \cdot A_Q) \sharp* xvec\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(q \cdot A_Q) \sharp* Q'\rangle$ *Sr* **have** $(q \cdot A_Q) \sharp* (r \cdot Q')$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi\rangle$
moreover from $\langle(r \cdot xvec) \sharp* A_P\rangle$ $\langle(p \cdot A_P) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi_P\rangle$ *Sp* **have** $(r \cdot xvec) \sharp* (p \cdot \Psi_P)$
by(*simp add*: *freshChainSimps*)
moreover from $\langle(r \cdot xvec) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* \Psi_Q\rangle$ *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot \Psi_Q)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(r \cdot xvec) \sharp* P\rangle$
moreover from $\langle(r \cdot xvec) \sharp* A_Q\rangle$ $\langle(q \cdot A_Q) \sharp* (r \cdot xvec)\rangle$ $\langle(r \cdot xvec) \sharp* M\rangle$ *Sq* **have** $(r \cdot xvec) \sharp* (q \cdot M)$
by(*simp add*: *freshChainSimps*)
moreover note $\langle(r \cdot xvec) \sharp* Q\rangle$

moreover from ‹$(r \cdot xvec) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* K$› *Sp*
have $(r \cdot xvec) \sharp* (p \cdot K)$
by(*simp add*: *freshChainSimps*)
moreover note ‹$(p \cdot A_P) \sharp* C$› ‹$(q \cdot A_Q) \sharp* C$› ‹$(r \cdot xvec) \sharp* C$›
moreover from ‹*distinct xvec*› **have** *distinct*$(r \cdot xvec)$ **by** *simp*
ultimately show *?thesis* **by**(*rule rComm1*)
next
case(*cComm2* Ψ_Q P M *xvec* N P' A_P Ψ_P Q K Q' A_Q)
obtain r::*name prm* **where** $(r \cdot xvec) \sharp* \Psi$ **and** $(r \cdot xvec) \sharp* P$ **and** $(r \cdot xvec) \sharp* Q$ **and** $(r \cdot xvec) \sharp* M$
and $(r \cdot xvec) \sharp* K$ **and** $(r \cdot xvec) \sharp* N$ **and** $(r \cdot xvec) \sharp* A_P$ **and** $(r \cdot xvec) \sharp* A_Q$
and $(r \cdot xvec) \sharp* P'$ **and** $(r \cdot xvec) \sharp* Q'$ **and** $(r \cdot xvec) \sharp* \Psi_P$ **and** $(r \cdot xvec) \sharp* \Psi_Q$
and $(r \cdot xvec) \sharp* C$ **and** *Sr*: $(set\ r) \subseteq (set\ xvec) \times (set(r \cdot xvec))$ **and** *distinctPerm r*
by(*rule-tac xvec*=*xvec* **and** *c*=(Ψ, P, Q, M, K, N, A_P, A_Q, Ψ_P, Ψ_Q, P', Q', C) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts*)

obtain q::*name prm* **where** $(q \cdot A_Q) \sharp* \Psi$ **and** $(q \cdot A_Q) \sharp* P$ **and** $(q \cdot A_Q) \sharp* Q$ **and** $(q \cdot A_Q) \sharp* K$
and $(q \cdot A_Q) \sharp* N$ **and** $(q \cdot A_Q) \sharp* xvec$ **and** $(q \cdot A_Q) \sharp* Q'$ **and** $(q \cdot A_Q) \sharp* P'$
and $(q \cdot A_Q) \sharp* \Psi_P$ **and** $(q \cdot A_Q) \sharp* A_P$ **and** $(q \cdot A_Q) \sharp* \Psi_Q$ **and** $(q \cdot A_Q) \sharp* (r \cdot xvec)$
and $(q \cdot A_Q) \sharp* C$ **and** *Sq*: $(set\ q) \subseteq (set\ A_Q) \times (set(q \cdot A_Q))$
by(*rule-tac xvec*=A_Q **and** *c*=(Ψ, P, Q, K, N, *xvec*, $r \cdot xvec$, Ψ_Q, A_P, Ψ_P, Q', P', C) **in** *name-list-avoiding*) *clarsimp*

obtain p::*name prm* **where** $(p \cdot A_P) \sharp* \Psi$ **and** $(p \cdot A_P) \sharp* P$ **and** $(p \cdot A_P) \sharp* Q$ **and** $(p \cdot A_P) \sharp* M$
and $(p \cdot A_P) \sharp* N$ **and** $(p \cdot A_P) \sharp* xvec$ **and** $(p \cdot A_P) \sharp* Q'$ **and** $(p \cdot A_P) \sharp* A_Q$
and $(p \cdot A_P) \sharp* P'$ **and** $(p \cdot A_P) \sharp* \Psi_P$ **and** $(p \cdot A_P) \sharp* \Psi_Q$ **and** $(p \cdot A_P) \sharp* (q \cdot A_Q)$
and $(p \cdot A_P) \sharp* C$ **and** $(p \cdot A_P) \sharp* (r \cdot xvec)$ **and** *Sp*: $(set\ p) \subseteq (set\ A_P) \times (set(p \cdot A_P))$
by(*rule-tac xvec*=A_P **and** *c*=(Ψ, P, Q, M, N, *xvec*, $r \cdot xvec$, A_Q, $q \cdot A_Q$, Ψ_Q, Ψ_P, Q', P', C) **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)

have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **by** *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*

from ‹$A_P \sharp* Q$› *FrQ* ‹$A_P \sharp* A_Q$› **have** $A_P \sharp* \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
from ‹$A_Q \sharp* P$› *FrP* ‹$A_P \sharp* A_Q$› **have** $A_Q \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*

note ‹$cP = P \parallel Q$›
moreover from ‹$(r \cdot xvec) \sharp* P'$› ‹$(r \cdot xvec) \sharp* Q'$› **have** $(r \cdot xvec) \sharp* (P' \parallel Q')$
by *simp*
with ‹$cRs = \tau \prec (\!|\nu* xvec|\!)(P' \parallel Q')$› ‹$(r \cdot xvec) \sharp* N$› *Sr*
have $cRs = \tau \prec (\!|\nu*(r \cdot xvec)|\!)(r \cdot (P' \parallel Q'))$ **by**(*simp add*: *resChainAlpha residualInject*)
hence $cRs = \tau \prec (\!|\nu*(r \cdot xvec)|\!)((r \cdot P') \parallel (r \cdot Q'))$
by *simp*

moreover from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'$› *Sr* ‹$(r \cdot xvec) \sharp* N$› ‹$(r \cdot xvec) \sharp* P'$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot P')$ **by**(*simp add*: *boundOutputChainAlpha''* *residualInject*)
hence $(q \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto (q \cdot M)(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot P')$ **using** *Sq* ‹$A_Q \sharp* P$› ‹$(q \cdot A_Q) \sharp* P$› ‹$(r \cdot xvec) \sharp* M$› ‹$(r \cdot xvec) \sharp* A_Q$› ‹$(q \cdot A_Q) \sharp* (r \cdot xvec)$›
by(*rule-tac outputPermFrameSubject*) (*assumption* | *auto*)
hence *PTrans*: $\Psi \otimes (q \cdot \Psi_Q) \rhd P \longmapsto (q \cdot M)(\!|\nu*(r \cdot xvec)|\!)\langle(r \cdot N)\rangle \prec (r \cdot P')$ **using** *Sq* ‹$A_Q \sharp* \Psi$› ‹$(q \cdot A_Q) \sharp* \Psi$›
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› *Sp* ‹$(p \cdot A_P) \sharp* \Psi_P$›
have *FrP*: *extractFrame* $P = \langle(p \cdot A_P), (p \cdot \Psi_P)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_P› **have** *distinct*$(p \cdot A_P)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$› *Sr* ‹*distinctPerm* r› ‹$xvec \sharp* Q$› ‹$(r \cdot xvec) \sharp* Q$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|(r \cdot N)|\!) \prec (r \cdot Q')$ **by**(*rule inputAlpha*)
hence $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto (p \cdot K)(\!|(r \cdot N)|\!) \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* Q$› ‹$(p \cdot A_P) \sharp* Q$›
by(*rule-tac inputPermFrameSubject*) (*assumption* | *simp*)+
hence *QTrans*: $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto (p \cdot K)(\!|(r \cdot N)|\!) \prec (r \cdot Q')$ **using** *Sp* ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$›
by(*simp add*: *eqvts*)

moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› *Sq* ‹$(q \cdot A_Q) \sharp* \Psi_Q$›
have *FrQ*: *extractFrame* $Q = \langle(q \cdot A_Q), (q \cdot \Psi_Q)\rangle$
by(*simp add*: *frameChainAlpha*)
moreover from ‹*distinct* A_Q› **have** *distinct*$(q \cdot A_Q)$ **by** *simp*

moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(p \cdot q \cdot (\Psi \otimes \Psi_P \otimes \Psi_Q)) \vdash (p \cdot q \cdot M) \leftrightarrow (p \cdot q \cdot K)$
by(*rule-tac chanEqClosed*)+
with ‹$A_P \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$(q \cdot A_Q) \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$(q \cdot A_Q) \sharp* \Psi_P$›
‹$A_P \sharp* \Psi_Q$› ‹$(p \cdot A_P) \sharp* \Psi_Q$› ‹$A_P \sharp* M$› ‹$(p \cdot A_P) \sharp* M$› ‹$(q \cdot A_Q) \sharp* A_P$›

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‹(p · A_P) ♯* (q · A_Q)›
      ‹A_Q ♯* K› ‹(q · A_Q) ♯* K› ‹A_P ♯* A_Q› ‹(p · A_P) ♯* A_Q›  Sp Sq
  have Ψ ⊗ (p · Ψ_P) ⊗ (q · Ψ_Q) ⊢ (q · M) ↔ (p · K)
    by(simp add: eqvts freshChainSimps)
  moreover note ‹(p · A_P) ♯* Ψ›
  moreover from ‹(p · A_P) ♯* A_Q› ‹(p · A_P) ♯* (q · A_Q)› ‹(p · A_P) ♯* Ψ_Q› Sq
have (p · A_P) ♯* (q · Ψ_Q)
    by(simp add: freshChainSimps)
  moreover note ‹(p · A_P) ♯* P›
  moreover from ‹(p · A_P) ♯* A_Q› ‹(p · A_P) ♯* (q · A_Q)› ‹(p · A_P) ♯* M› Sq
have (p · A_P) ♯* (q · M)
    by(simp add: freshChainSimps)
  moreover from ‹(p · A_P) ♯* xvec› ‹(p · A_P) ♯* (r · xvec)› ‹(p · A_P) ♯* N› Sr
have (p · A_P) ♯* (r · N)
    by(simp add: freshChainSimps)
  moreover from ‹(p · A_P) ♯* xvec› ‹(p · A_P) ♯* (r · xvec)› ‹(p · A_P) ♯* P'› Sr
have (p · A_P) ♯* (r · P')
    by(simp add: freshChainSimps)
  moreover note  ‹(p · A_P) ♯* Q›
  moreover from ‹(p · A_P) ♯* xvec› ‹(p · A_P) ♯* (r · xvec)› ‹(p · A_P) ♯* Q'› Sr
have (p · A_P) ♯* (r · Q')
    by(simp add: freshChainSimps)
  moreover note ‹(p · A_P) ♯* (q · A_Q)› ‹(p · A_P) ♯* (r · xvec)› ‹(q · A_Q) ♯* Ψ›
  moreover from ‹(q · A_Q) ♯* A_P› ‹(p · A_P) ♯* A_Q› ‹(q · A_Q) ♯* Ψ_P› ‹(p · A_P)
♯* (q · A_Q)› Sp Sq have (q · A_Q) ♯* (p · Ψ_P)
    by(simp add: freshChainSimps)
  moreover note ‹(q · A_Q) ♯* P›
  moreover from ‹(q · A_Q) ♯* A_P› ‹(p · A_P) ♯* A_Q› ‹(q · A_Q) ♯* K› ‹(p · A_P)
♯* (q · A_Q)› Sp Sq have (q · A_Q) ♯* (p · K)
    by(simp add: freshChainSimps)
  moreover from ‹(q · A_Q) ♯* xvec› ‹(q · A_Q) ♯* (r · xvec)› ‹(q · A_Q) ♯* N› Sr
have (q · A_Q) ♯* (r · N)
    by(simp add: freshChainSimps)
  moreover from ‹(q · A_Q) ♯* xvec› ‹(q · A_Q) ♯* (r · xvec)› ‹(q · A_Q) ♯* P'› Sr
have (q · A_Q) ♯* (r · P')
    by(simp add: freshChainSimps)
  moreover note ‹(q · A_Q) ♯* Q›
  moreover from ‹(q · A_Q) ♯* xvec› ‹(q · A_Q) ♯* (r · xvec)› ‹(q · A_Q) ♯* Q'› Sr
have (q · A_Q) ♯* (r · Q')
    by(simp add: freshChainSimps)
  moreover note ‹(q · A_Q) ♯* (r · xvec)› ‹(r · xvec) ♯* Ψ›
  moreover from ‹(r · xvec) ♯* A_P› ‹(p · A_P) ♯* (r · xvec)› ‹(r · xvec) ♯* Ψ_P›
Sp have (r · xvec) ♯* (p · Ψ_P)
    by(simp add: freshChainSimps)
  moreover from ‹(r · xvec) ♯* A_Q› ‹(q · A_Q) ♯* (r · xvec)› ‹(r · xvec) ♯* Ψ_Q›
Sq have (r · xvec) ♯* (q · Ψ_Q)
    by(simp add: freshChainSimps)
  moreover note ‹(r · xvec) ♯* P›
  moreover from ‹(r · xvec) ♯* A_Q› ‹(q · A_Q) ♯* (r · xvec)› ‹(r · xvec) ♯* M› Sq
```

have $(r \cdot xvec) \sharp* (q \cdot M)$
  by(*simp add*: *freshChainSimps*)
 moreover note ‹$(r \cdot xvec) \sharp* Q$›
 moreover from ‹$(r \cdot xvec) \sharp* A_P$› ‹$(p \cdot A_P) \sharp* (r \cdot xvec)$› ‹$(r \cdot xvec) \sharp* K$› *Sp*
have $(r \cdot xvec) \sharp* (p \cdot K)$
  by(*simp add*: *freshChainSimps*)
 moreover note ‹$(p \cdot A_P) \sharp* C$› ‹$(q \cdot A_Q) \sharp* C$› ‹$(r \cdot xvec) \sharp* C$›
 moreover from ‹*distinct xvec*› **have** *distinct*$(r \cdot xvec)$ **by** *simp*
 ultimately show *?thesis* **by**(*rule rComm2*)
next
 case(*cOpen P M xvec yvec N P' x*)
  from ‹$\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$› **have** *distinct*$(xvec@yvec)$ **by**(*force dest*: *boundOutputDistinct*)
 hence $xvec \sharp* yvec$ **by**(*induct xvec*) *auto*
 obtain p **where** $(p \cdot yvec) \sharp* \Psi$ **and** $(p \cdot yvec) \sharp* P$ **and** $(p \cdot yvec) \sharp* M$
          and $(p \cdot yvec) \sharp* yvec$ **and** $(p \cdot yvec) \sharp* N$ **and** $(p \cdot yvec) \sharp* P'$
          and $x \sharp (p \cdot yvec)$ **and** $(p \cdot yvec) \sharp* xvec$
          and $(p \cdot yvec) \sharp* C$ **and** *Sp*: $(set\ p) \subseteq (set\ yvec) \times (set(p \cdot yvec))$
  by(*rule-tac xvec=yvec* **and** *c*=(Ψ, P, M, *xvec*, *yvec*, N, P', x, C) **in** *name-list-avoiding*)
    (*auto simp add*: *eqvts fresh-star-prod*)
 obtain q **where** $(q \cdot xvec) \sharp* \Psi$ **and** $(q \cdot xvec) \sharp* P$ **and** $(q \cdot xvec) \sharp* M$
          and $(q \cdot xvec) \sharp* xvec$ **and** $(q \cdot xvec) \sharp* N$ **and** $(q \cdot xvec) \sharp* P'$
          and $x \sharp (q \cdot xvec)$ **and** $(q \cdot xvec) \sharp* yvec$
          and $(q \cdot xvec) \sharp* p$ **and** $(q \cdot xvec) \sharp* (p \cdot yvec)$
          and $(q \cdot xvec) \sharp* C$ **and** *Sq*: $(set\ q) \subseteq (set\ xvec) \times (set(q \cdot xvec))$
   by(*rule-tac xvec=xvec* **and** *c*=(Ψ, P, M, *xvec*, *yvec*, $p \cdot yvec$, N, P', x, p, C) **in** *name-list-avoiding*)
    (*auto simp add*: *eqvts fresh-star-prod*)
 obtain y::*name* **where** $y \sharp P$ **and** $y \sharp C$ **and** $y \sharp xvec$ **and** $y \sharp yvec$ **and** $y \neq x$ **and** $y \sharp N$
            and $y \sharp (q \cdot xvec)$ **and** $y \sharp (p \cdot yvec)$ **and** $y \sharp M$ **and** $y \sharp \Psi$ **and** $y \sharp P'$
   by(*generate-fresh name*) (*auto simp add*: *freshChainSimps*)

 from ‹$cP = (\!|\nu x|\!)P$› ‹$y \sharp P$› **have** $cP = (\!|\nu y|\!)([(x, y)] \cdot P)$ **by**(*simp add*: *alphaRes*)
 moreover have $cRs = M(\!|\nu*((q \cdot xvec)@y\#(p \cdot yvec))|\!)\langle((q@(x, y)\#p) \cdot N)\rangle \prec ((q@(x, y)\#p) \cdot P')$
 proof −
   note ‹$cRs = M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$›
   moreover have $(\!|\nu*(xvec@x\#yvec)|\!)N \prec' P' = (\!|\nu*xvec|\!)((\!|\nu x|\!)((\!|\nu*yvec|\!)N \prec' P'))$ **by**(*simp add*: *boundOutputApp*)
   moreover from ‹$(p \cdot yvec) \sharp* N$› ‹$(p \cdot yvec) \sharp* P'$› *Sp* **have** ... $= (\!|\nu*xvec|\!)((\!|\nu x|\!)((\!|\nu*(p \cdot yvec)|\!)(p \cdot N) \prec' (p \cdot P')))$
     by(*simp add*: *boundOutputChainAlpha''*)
   moreover with ‹$y \sharp N$› ‹$y \sharp P'$› ‹$y \sharp (p \cdot yvec)$› ‹$y \sharp yvec$› ‹$x \sharp yvec$› ‹$x \sharp (p \cdot yvec)$› *Sp*
    have ... $= (\!|\nu*xvec|\!)((\!|\nu y|\!)((\!|\nu*(p \cdot yvec)|\!)(([[(x, y)] \cdot p \cdot N) \prec' ([[(x, y)] \cdot p \cdot P'))))$
    by(*subst alphaBoundOutput*[**where** y=y]) (*simp add*: *freshChainSimps eqvts*)+

moreover hence ... = $(\!|\nu*xvec|\!)((\!|\nu y|\!)((\!|\nu*(p \cdot yvec)|\!)((((x,\ y)\#p) \cdot N) \prec' (((x,\ y)\#p) \cdot P')))$

by *simp*

moreover from $\langle(q \cdot xvec) \sharp* N\rangle$ $\langle(q \cdot xvec) \sharp* P'\rangle$ $\langle xvec \sharp* yvec\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle y \sharp xvec\rangle$ $\langle y \sharp (q \cdot xvec)\rangle$ $\langle x \sharp xvec\rangle$ $\langle x \sharp (q \cdot xvec)\rangle$ *Sp Sq*

have ... = $(\!|\nu*(q \cdot xvec)|\!)((\!|\nu y|\!)((\!|\nu*(p \cdot yvec)|\!)((q \cdot ((x,\ y)\#p) \cdot N) \prec' (q \cdot ((x,\ y)\#p) \cdot P'))))$

apply(*subst boundOutputChainAlpha*[**where** $p{=}q$ **and** *xvec*=*xvec* **and** *yvec*=*xvec*])

defer

apply *assumption*

apply *simp*

apply(*simp add*: *eqvts*)

apply(*simp add*: *eqvts*)

apply(*simp add*: *boundOutputFreshSet*(*4*))

apply(*rule conjI*)

apply(*simp add*: *freshChainSimps*)

apply(*simp add*: *freshChainSimps*)

done

moreover hence ... = $(\!|\nu*(q \cdot xvec@y\#(p \cdot yvec))|\!)((q@(x,\ y)\#p) \cdot N) \prec' ((q@(x,\ y)\#p) \cdot P')$

by(*simp only*: *pt2*[*OF pt-name-inst*] *boundOutputApp BOresChain.simps*)

ultimately show *?thesis*

by(*simp only*: *residualInject*)

qed

moreover have $\Psi \rhd ([(x,\ y)] \cdot P) \longmapsto M(\!|\nu*((q \cdot xvec)@(p \cdot yvec))|\!)\langle((q@(x,\ y)\#p) \cdot N)\rangle \prec ((q@(x,\ y)\#p) \cdot P')$

proof −

note$\langle \Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'\rangle$

moreover from $\langle(p \cdot yvec) \sharp* N\rangle$ $\langle(q \cdot xvec) \sharp* N\rangle$ $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$ *Sp Sq*

have $((q@p) \cdot (xvec\ @\ yvec)) \sharp* N$ **apply**(*simp only*: *eqvts*) **apply**(*simp only*: *pt2*[*OF pt-name-inst*])

by *simp*

moreover from $\langle(p \cdot yvec) \sharp* P'\rangle$ $\langle(q \cdot xvec) \sharp* P'\rangle$ $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$ *Sp Sq*

have $((q@p) \cdot (xvec\ @\ yvec)) \sharp* P'$ **by**(*simp del*: *freshAlphaPerm add*: *eqvts pt2*[*OF pt-name-inst*])

moreover from *Sp Sq* $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$

have *Spq*: *set*$(q@p) \subseteq$ *set*$(xvec@yvec) \times$ *set*$((q@p) \cdot (xvec@yvec))$

by(*simp add*: *pt2*[*OF pt-name-inst*] *eqvts*) *blast*

ultimately have $\Psi \rhd P \longmapsto M(\!|\nu*((q@p) \cdot (xvec@yvec))|\!)\langle((q@p) \cdot N)\rangle \prec ((q@p) \cdot P')$

apply(*simp only*: *residualInject*)

by(*erule-tac rev-mp*) (*subst boundOutputChainAlpha*, *auto*)

with *Sp Sq* $\langle xvec \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* yvec\rangle$ $\langle(q \cdot xvec) \sharp* (p \cdot yvec)\rangle$ $\langle(p \cdot yvec) \sharp* xvec\rangle$

have $\Psi \rhd P \longmapsto M(\!|\nu*((q \cdot xvec)@(p \cdot yvec))|\!)\langle((q@p) \cdot N)\rangle \prec ((q@p) \cdot P')$

by(*simp add*: *eqvts pt2*[*OF pt-name-inst*] *del*: *freshAlphaPerm*)
hence $([(x,\ y)] \cdot \Psi) \rhd ([(x,\ y)] \cdot P) \longmapsto [(x,\ y)] \cdot (M\langle\!|\nu*((q \cdot xvec)@(p \cdot yvec))|\!\rangle\langle((q@p) \cdot N)\rangle \prec ((q@p) \cdot P'))$
by(*rule semantics.eqvt*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› ‹$x \sharp M$› ‹$y \sharp M$› ‹$x \sharp xvec$› ‹$y \sharp xvec$› ‹$x \sharp (q \cdot xvec)$› ‹$y \sharp (q \cdot xvec)$ ›‹$x \sharp yvec$› ‹$y \sharp yvec$› ‹$x \sharp (p \cdot yvec)$› ‹$y \sharp (p \cdot yvec)$› *Sp Sq*
show *?thesis*
apply(*simp add*: *eqvts pt2*[*OF pt-name-inst*])
by(*subst perm-compose*[*of q*], *simp*)+
qed
moreover from ‹$x \in supp\ N$› **have** $((q@(x,\ y)\#p) \cdot x) \in ((q@(x,\ y)\#p) \cdot (supp\ N))$
by(*simp add*: *pt-set-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$x \sharp (q \cdot xvec)$› ‹$x \sharp (p \cdot yvec)$› ‹$y \sharp xvec$› ‹$y \sharp (q \cdot xvec)$› *Sp Sq*
have $y \in supp((q@(x,\ y)\#p) \cdot N)$ **by**(*simp add*: *pt2*[*OF pt-name-inst*] *calc-atm eqvts*)
moreover from ‹*distinct xvec*› **have** $distinct(q \cdot xvec)$ **by** *simp*
moreover from ‹*distinct yvec*› **have** $distinct(p \cdot yvec)$ **by** *simp*
moreover note ‹$x \sharp (q \cdot xvec)$› ‹$x \sharp (p \cdot yvec)$› ‹$x \sharp M$› ‹$x \sharp \Psi$›
‹$(q \cdot xvec) \sharp* \Psi$› ‹$(q \cdot xvec) \sharp* P$› ‹$(q \cdot xvec) \sharp* M$› ‹$(q \cdot xvec) \sharp* (p \cdot yvec)$›
‹$(p \cdot yvec) \sharp* \Psi$› ‹$(p \cdot yvec) \sharp* P$› ‹$(p \cdot yvec) \sharp* M$› ‹$y \sharp (q \cdot xvec)$› ‹$y \sharp (p \cdot yvec)$› ‹$y \sharp M$› ‹$y \sharp C$› ‹$y \sharp \Psi$›
‹$(p \cdot yvec) \sharp* C$› ‹$(q \cdot xvec) \sharp* C$›
ultimately show *Prop* **by**(*rule-tac rOpen*) (*assumption* | *simp*)+
next
case(*cScope P* α *P' x*)
obtain *p*::*name prm* **where** $(bn(p \cdot \alpha)) \sharp* \Psi$ **and** $(bn(p \cdot \alpha)) \sharp* P$
and $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$ **and** $x \sharp bn(p \cdot \alpha)$
and *distinctPerm p*
and $(bn(p \cdot \alpha)) \sharp* C$ **and** *Sp*: $(set\ p) \subseteq set(bn\ \alpha) \times (set(bn(p \cdot \alpha)))$
by(*rule-tac xvec*=*bn* α **and** *c*=(Ψ, *P*, α, *x*, *P'*, *C*) **in** *name-list-avoiding*) (*auto simp add*: *eqvts*)
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp (p \cdot P')$ **and** $y \sharp (p \cdot \alpha)$ **and** $y \sharp C$
by(*generate-fresh name*) (*auto simp add*: *freshChainSimps simp del*: *actionFresh*)
from ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinctPerm p*› **have** $bn(p \cdot \alpha) \sharp* subject(p \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, *of - - p*]) (*simp add*: *eqvts*)
from ‹$distinct(bn\ \alpha)$› ‹*distinctPerm p*› **have** $distinct(bn(p \cdot \alpha))$
by(*subst distinctClosed*[*symmetric*, *of - p*]) (*simp add*: *eqvts*)
from ‹$x \sharp \alpha$› ‹$x \sharp (bn(p \cdot \alpha))$› ‹*distinctPerm p*› *Sp* **have** $x \sharp (p \cdot \alpha)$
by(*subst fresh-bij*[*symmetric*, *of - - p*]) (*simp add*: *eqvts freshChainSimps*)

moreover from ‹$cP = (\!|\nu x|\!)P$› ‹$y \sharp P$› **have** $cP = (\!|\nu y|\!)([(x,\ y)] \cdot P)$ **by**(*simp add*: *alphaRes*)
moreover from ‹$cRs = \alpha \prec (\!|\nu x|\!)P'$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$(bn(p \cdot \alpha)) \sharp* \alpha$› ‹*x*

$\sharp\ bn(p \cdot \alpha)\rangle\ \langle(bn(p \cdot \alpha))\ \sharp*\ P'\rangle\ \langle x\ \sharp\ \alpha\rangle\ Sp$
have $cRs = (p \cdot \alpha) \prec (\!|\nu x|\!)(p \cdot P')$
by(*force simp add*: *residualAlpha*)
with $\langle y\ \sharp\ (p \cdot P')\rangle$ **have** $cRs = (p \cdot \alpha) \prec (\!|\nu y|\!)([(x,\ y)] \cdot p \cdot P')$
by(*simp add*: *alphaRes*)
moreover from $\langle \Psi \rhd P \longmapsto \alpha \prec P'\rangle\ \langle bn\ \alpha\ \sharp*\ subject\ \alpha\rangle\ \langle(bn(p \cdot \alpha))\ \sharp*\ \alpha\rangle$
$\langle(bn(p \cdot \alpha))\ \sharp*\ P'\rangle\ Sp$
have $\Psi \rhd P \longmapsto (p \cdot \alpha) \prec (p \cdot P')$ **by**(*force simp add*: *residualAlpha*)
hence$([(x,\ y)] \cdot \Psi) \rhd ([(x,\ y)] \cdot P) \longmapsto [(x,\ y)] \cdot ((p \cdot \alpha) \prec (p \cdot P'))$
by(*rule eqvts*)
with $\langle x\ \sharp\ \Psi\rangle\ \langle y\ \sharp\ \Psi\rangle\ \langle y\ \sharp\ (p \cdot \alpha)\rangle\ \langle x\ \sharp\ (p \cdot \alpha)\rangle\ Sp\ \langle distinctPerm\ p\rangle$
have $\Psi \rhd ([(x,\ y)] \cdot P) \longmapsto (p \cdot \alpha) \prec ([(x,\ y)] \cdot p \cdot P')$
by(*simp add*: *eqvts*)
moreover from $\langle bn(p \cdot \alpha)\ \sharp*\ P\rangle\ \langle y\ \sharp\ (p \cdot \alpha)\rangle\ \langle y\ \sharp\ P\rangle$ **have** $bn(p \cdot \alpha)\ \sharp*\ ([(x,$
$y)] \cdot P)$
by(*auto simp add*: *fresh-star-def fresh-left calc-atm*) (*simp add*: *fresh-def name-list-supp*)
moreover from $\langle distinct(bn\ \alpha)\rangle$ **have** $distinct(p \cdot bn\ \alpha)$ **by** *simp*
hence $distinct(bn(p \cdot \alpha))$ **by**(*simp add*: *eqvts*)
ultimately show *?thesis*
using $\langle y\ \sharp\ \Psi\rangle\ \langle y\ \sharp\ (p \cdot \alpha)\rangle\ \langle y\ \sharp\ C\rangle\ \langle bn(p \cdot \alpha)\ \sharp*\ \Psi\rangle\ \langle bn(p \cdot \alpha)\ \sharp*\ subject(p \cdot$
$\alpha)\rangle\ \langle bn(p \cdot \alpha)\ \sharp*\ C\rangle$
by(*rule-tac rScope*)
next
case(*Bang P*)
thus *?thesis* **by**(*rule-tac rBang*)
qed

nominal-primrec
$inputLength :: ('a::fs\text{-}name,\ 'b::fs\text{-}name,\ 'c::fs\text{-}name)\ psi \Rightarrow nat$
and $inputLength' :: ('a::fs\text{-}name,\ 'b::fs\text{-}name,\ 'c::fs\text{-}name)\ input \Rightarrow nat$
and $inputLength'' :: ('a::fs\text{-}name,\ 'b::fs\text{-}name,\ 'c::fs\text{-}name)\ psiCase \Rightarrow nat$

where
$inputLength\ (\mathbf{0}) = 0$
$|\ inputLength\ (M\langle N\rangle.P) = 0$
$|\ inputLength\ (M(\!|I) = inputLength'\ I$
$|\ inputLength\ (Case\ C) = 0$
$|\ inputLength\ (P \parallel Q) = 0$
$|\ inputLength\ (\nu x)P) = 0$
$|\ inputLength\ (\{\!|\Psi|\!\}) = 0$
$|\ inputLength\ (!P) = 0$

$|\ inputLength'\ (Trm\ M\ P) = 0$
$|\ inputLength'\ (\nu\ y\ I) = 1 + (inputLength'\ I)$

$|\ inputLength''\ (\bot_c) = 0$
$|\ inputLength''\ (\Box\Phi \Rightarrow P\ C) = 0$
apply(*finite-guess*)+
apply(*rule TrueI*)+

by(*fresh-guess add*: *fresh-nat*)+

nominal-primrec *boundOutputLength* :: ('*a*, '*b*, '*c*) *boundOutput* ⇒ *nat*
where
  boundOutputLength (*BOut M P*) = *0*
| *boundOutputLength* (*BStep x B*) = (*boundOutputLength B*) + *1*
apply(*finite-guess*)+
apply(*rule TrueI*)+
by(*fresh-guess add*: *fresh-nat*)+

nominal-primrec *residualLength* :: ('*a*, '*b*, '*c*) *residual* ⇒ *nat*
where
  residualLength (*RIn M N P*) = *0*
| *residualLength* (*ROut M B*) = *boundOutputLength B*
| *residualLength* (*RTau P*) = *0*
by(*rule TrueI*)+

lemma *inputLengthProc*[*simp*]:
  shows *inputLength*(*M*⟪λ∗*xvec N*⟫.*P*) = *length xvec*
by(*induct xvec*) *auto*

lemma *boundOutputLengthSimp*[*simp*]:
  shows *residualLength*(*M*⟪*ν*∗*xvec*⟫⟨*N*⟩ ≺ *P*) = *length xvec*
by(*induct xvec*) (*auto simp add*: *residualInject*)

lemma *boundOuputLengthSimp2*[*simp*]:
  shows *residualLength*(*α* ≺ *P*) = *length*(*bn α*)
by(*nominal-induct α rule*: *action.strong-induct*, *auto*) (*auto simp add*: *residualInject*)

lemmas [*simp del*] = *inputLength-inputLength'-inputLength''.simps residualLength.simps boundOutputLength.simps*

lemma *constructPerm*:
  fixes *xvec* :: *name list*
  and *yvec* :: *name list*

  assumes *length xvec* = *length yvec*
  and *xvec* ♯∗ *yvec*
  and *distinct xvec*
  and *distinct yvec*

  obtains *p* **where** *set p* ⊆ *set xvec* × *set*(*p* ∙ *xvec*) **and** *distinctPerm p* **and** *yvec* = *p* ∙ *xvec*
proof −
  assume ⋀*p*. ⟦*set p* ⊆ *set xvec* × *set* (*p* ∙ *xvec*); *distinctPerm p*; *yvec* = *p* ∙ *xvec*⟧ ⟹ *thesis*
  moreover obtain *n* **where** *n* = *length xvec* **by** *auto*
  with *assms* **have** ∃ *p*. (*set p*) ⊆ (*set xvec*) × *set* (*yvec*) ∧ *distinctPerm p* ∧ *yvec*

$= p \cdot xvec$
proof(*induct n arbitrary*: *xvec yvec*)
case(*0 xvec yvec*)
thus *?case* **by** *simp*
next
case(*Suc n xvec yvec*)
from ‹*Suc n = length xvec*›
obtain *x xvec'* **where** *xvec = x#xvec'* **and** *length xvec' = n*
by(*case-tac xvec*) *auto*
from ‹*length xvec = length yvec*› ‹*xvec = x # xvec'*›
obtain *y yvec'* **where** *length xvec' = length yvec'* **and** *yvec = y#yvec'*
by(*case-tac yvec*) *auto*
from ‹*xvec = x#xvec'*› ‹*yvec=y#yvec'*› ‹*xvec ♯* yvec*›
have *x ≠ y* **and** *xvec' ♯* yvec'* **and** *x ♯ yvec'* **and** *y ♯ xvec'*
by(*auto simp add*: *fresh-list-cons*)
from ‹*distinct xvec*› ‹*distinct yvec*› ‹*xvec=x#xvec'*› ‹*yvec=y#yvec'*› **have** *x ♯ xvec'* **and** *y ♯ yvec'* **and** *distinct xvec'* **and** *distinct yvec'*
by *simp+*
from ‹*Suc n = length xvec*› ‹*xvec=x#xvec'*› **have** *n = length xvec'* **by** *simp*
with ‹*length xvec' = length yvec'*› ‹*xvec' ♯* yvec'*› ‹*distinct xvec'*› ‹*distinct yvec'*›
obtain *p* **where** *S*: *set p ⊆ set xvec' × set yvec'* **and** *distinctPerm p* **and** *yvec' = p · xvec'*
by(*drule-tac Suc*) *auto*
from *S* **have** *set((x, y)#p) ⊆ set(x#xvec') × set(y#yvec')* **by** *auto*
moreover from ‹*x ♯ xvec'*› ‹*x ♯ yvec'*› ‹*y ♯ xvec'*› ‹*y ♯ yvec'*› *S* **have** *x ♯ p* **and** *y ♯ p*
apply(*induct p*)
by(*auto simp add*: *fresh-list-nil fresh-list-cons fresh-prod name-list-supp*) (*auto simp add*: *fresh-def*)

with *S* ‹*distinctPerm p*› ‹*x ≠ y*› **have** *distinctPerm((x, y)#p)* **by** *auto*
moreover from ‹*yvec' = p · xvec'*› ‹*x ♯ p*› ‹*y ♯ p*› ‹*x ♯ xvec'*› ‹*y ♯ xvec'*› **have** *(y#yvec') = ((x, y)#p) · (x#xvec')*
by(*simp add*: *calc-atm freshChainSimps*)
ultimately show *?case* **using** ‹*xvec=x#xvec'*› ‹*yvec=y#yvec'*›
by *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *distinctApend*[*simp*]:
fixes *xvec* :: *name list*
and *yvec* :: *name list*

shows *(set xvec ∩ set yvec = {}) = xvec ♯* yvec*
by(*auto simp add*: *fresh-star-def name-list-supp fresh-def*)

lemma *lengthAux*:
fixes *xvec* :: *name list*

and y :: *name*
and *yvec* :: *name list*

assumes *length xvec* = *length*(*y*#*yvec*)

obtains *z zvec* **where** *xvec* = *z*#*zvec* **and** *length zvec* = *length yvec*
using *assms*
by(*induct xvec arbitrary*: *yvec y*) *auto*

lemma *lengthAux2*:
fixes *xvec* :: *name list*
and *yvec* :: *name list*
and *zvec* :: *name list*

assumes *length xvec* = *length*(*yvec*@*y*#*zvec*)

obtains *xvec1 x xvec2* **where** *xvec*=*xvec1*@*x*#*xvec2* **and** *length xvec1* = *length yvec* **and** *length xvec2* = *length zvec*
proof −
assume ⋀*xvec1 x xvec2*.
⟦*xvec* = *xvec1* @ *x* # *xvec2*; *length xvec1* = *length yvec*;
length xvec2 = *length zvec*⟧
⟹ *thesis*
moreover from *assms* **have** ∃ *xvec1 x xvec2*. *xvec*=*xvec1*@*x*#*xvec2* ∧ *length xvec1* = *length yvec* ∧ *length xvec2* = *length zvec*
apply(*rule-tac x*=*take* (*length yvec*) *xvec* **in** *exI*)
apply(*rule-tac x*=*hd*(*drop* (*length yvec*) *xvec*) **in** *exI*)
apply(*rule-tac x*=*tl*(*drop* (*length yvec*) *xvec*) **in** *exI*)
by *auto*
ultimately show *?thesis* **by** *blast*
qed

lemma *semanticsCases*[*consumes 11*, *case-names cInput cCase cPar1 cPar2 cComm1 cComm2 cScope cBang*]:
fixes Ψ :: *'b*
and *cP* :: (*'a*, *'b*, *'c*) *psi*
and *cRs* :: (*'a*, *'b*, *'c*) *residual*
and *C* :: *'d*::*fs-name*
and *x1* :: *name*
and *x2* :: *name*
and *xvec1* :: *name list*
and *xvec2* :: *name list*
and *xvec3* :: *name list*
and *xvec4* :: *name list*
and *xvec5* :: *name list*

assumes Ψ ▷ *cP* ⟼*cRs*
and *length xvec1* = *inputLength cP* **and** *distinct xvec1*
and *length xvec2* = *residualLength cRs* **and** *distinct xvec2*

and *length xvec3 = residualLength cRs* **and** *distinct xvec3*
and *length xvec4 = residualLength cRs* **and** *distinct xvec4*
and *length xvec5 = residualLength cRs* **and** *distinct xvec5*
and *rInput*: $\bigwedge M\ K\ N\ Tvec\ P.\ ([\![xvec1\ \sharp*\ \Psi;\ xvec1\ \sharp*\ cP;\ xvec1\ \sharp*\ cRs]\!] \Longrightarrow cP = M(\!|\lambda*xvec1\ N|\!).P \wedge\ \ cRs = K(\!|(N[xvec1::=Tvec])|\!) \prec P[xvec1::=Tvec] \wedge \Psi \vdash M \leftrightarrow K \wedge distinct\ xvec1 \wedge set\ xvec1 \subseteq supp\ N \wedge length\ xvec1{=}length\ Tvec \wedge xvec1\ \sharp*\ Tvec \wedge xvec1\ \sharp*\ \Psi \wedge xvec1\ \sharp*\ M \wedge xvec1\ \sharp*\ K) \Longrightarrow Prop$
and *rOutput*: $\bigwedge M\ K\ N\ P.\ [\![cP = M\langle N\rangle.P;\ cRs = K\langle N\rangle \prec P;\ \Psi \vdash M \leftrightarrow K]\!] \Longrightarrow Prop$
and *rCase*: $\bigwedge Cs\ P\ \varphi.\ [\![cP = Cases\ Cs;\ \Psi \rhd P \longmapsto cRs;\ (\varphi,\ P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow Prop$
and *rPar1*: $\bigwedge \Psi_Q\ P\ \alpha\ P'\ Q\ A_Q.\ ([\![xvec2\ \sharp*\ \Psi;\ xvec2\ \sharp*\ cP;\ xvec2\ \sharp*\ cRs]\!] \Longrightarrow cP = P \parallel Q \wedge cRs = \alpha \prec (P' \parallel Q) \wedge xvec2 = bn\ \alpha \wedge \Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P' \wedge extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle \wedge distinct\ A_Q \wedge A_Q\ \sharp*\ P \wedge A_Q\ \sharp*\ Q \wedge A_Q\ \sharp*\ \Psi \wedge A_Q\ \sharp*\ \alpha \wedge A_Q\ \sharp*\ P' \wedge A_Q\ \sharp*\ C) \Longrightarrow Prop$
and *rPar2*: $\bigwedge \Psi_P\ Q\ \alpha\ Q'\ P\ A_P.\ ([\![xvec3\ \sharp*\ \Psi;\ xvec3\ \sharp*\ cP;\ xvec3\ \sharp*\ cRs]\!] \Longrightarrow cP = P \parallel Q \wedge cRs = \alpha \prec (P \parallel Q') \wedge xvec3 = bn\ \alpha \wedge \Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q' \wedge extractFrame\ P = \langle A_P,\ \Psi_P\rangle \wedge distinct\ A_P \wedge A_P\ \sharp*\ P \wedge A_P\ \sharp*\ Q \wedge A_P\ \sharp*\ \Psi \wedge A_P\ \sharp*\ \alpha \wedge A_P\ \sharp*\ Q' \wedge A_P\ \sharp*\ C) \Longrightarrow Prop$
and *rComm1*: $\bigwedge \Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu*xvec|\!)P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_P\ \sharp*\ \Psi;\ A_P\ \sharp*\ \Psi_Q;\ A_P\ \sharp*\ P;\ A_P\ \sharp*\ M;\ A_P\ \sharp*\ N;$
$A_P\ \sharp*\ P';\ A_P\ \sharp*\ Q;\ A_P\ \sharp*\ Q';\ A_P\ \sharp*\ A_Q;\ A_P\ \sharp*\ xvec;\ A_Q\ \sharp*\ \Psi;\ A_Q\ \sharp*\ \Psi_P;$
$A_Q\ \sharp*\ P;\ A_Q\ \sharp*\ K;\ A_Q\ \sharp*\ N;\ A_Q\ \sharp*\ P';\ A_Q\ \sharp*\ Q;\ A_Q\ \sharp*\ Q';\ A_Q\ \sharp*\ xvec;$
$xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ \Psi_P;\ xvec\ \sharp*\ \Psi_Q;\ xvec\ \sharp*\ P;\ xvec\ \sharp*\ M;\ xvec\ \sharp*\ Q;$
$xvec\ \sharp*\ K;\ A_P\ \sharp*\ C;\ A_Q\ \sharp*\ C;\ xvec\ \sharp*\ C;\ distinct\ xvec]\!] \Longrightarrow Prop$
and *rComm2*: $\bigwedge \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q.$
$[\![cP = P \parallel Q;\ cRs = \tau \prec (\!|\nu*xvec|\!)P' \parallel Q';$
$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P';\ extractFrame\ P = \langle A_P,\ \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';\ extractFrame\ Q = \langle A_Q,\ \Psi_Q\rangle;$

distinct A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; $A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$;
$A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* xvec$; $A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$;
$A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* Q'$; $A_Q \sharp* xvec$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* \Psi_Q$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* Q$;
$xvec \sharp* K$; $A_P \sharp* C$; $A_Q \sharp* C$; $xvec \sharp* C$; *distinct xvec*$\rrbracket \Longrightarrow$ *Prop*
and *rOpen*: $\bigwedge P\ M\ xvec\ y\ yvec\ N\ P'$.
($\llbracket xvec4 \sharp* \Psi$; $xvec4 \sharp* cP$; $xvec4 \sharp* cRs$; $x1 \sharp \Psi$; $x1 \sharp cP$; $x1 \sharp cRs$; $x1 \sharp xvec4\rrbracket \Longrightarrow$
$cP = (\!|\nu x1|\!)P \wedge cRs = M(\!|\nu*(xvec@x1\#yvec)|\!)\langle N\rangle \prec P' \wedge xvec4{=}xvec@y\#yvec \wedge$
$\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P' \wedge x1 \in supp\ N \wedge x1 \sharp xvec \wedge x1 \sharp yvec \wedge$
distinct xvec $\wedge$ *distinct yvec* $\wedge$ $xvec \sharp* \Psi \wedge xvec \sharp* P \wedge xvec \sharp* M \wedge xvec \sharp* yvec \wedge$
$yvec \sharp* \Psi \wedge yvec \sharp* P \wedge yvec \sharp* M) \Longrightarrow$ *Prop*
and *rScope*: $\bigwedge P\ \alpha\ P'$. ($\llbracket xvec5 \sharp* \Psi$; $xvec5 \sharp* cP$; $xvec5 \sharp* cRs$; $x2 \sharp \Psi$; $x2 \sharp cP$; $x2 \sharp cRs$; $x2 \sharp xvec5\rrbracket \Longrightarrow$
$cP = (\!|\nu x2|\!)P \wedge cRs = \alpha \prec (\!|\nu x2|\!)P' \wedge\ xvec5 = bn\ \alpha \wedge$
$\Psi \rhd P \longmapsto \alpha \prec P' \wedge x2 \sharp \Psi \wedge x2 \sharp \alpha \wedge bn\ \alpha \sharp* subject\ \alpha \wedge distinct(bn\ \alpha)) \Longrightarrow$ *Prop*
and *rBang*: $\bigwedge P$. $\llbracket cP = !P$; $\Psi \rhd P \parallel !P \longmapsto cRs$; *guarded* $P\rrbracket \Longrightarrow$ *Prop*
shows *Prop*
using ‹$\Psi \rhd cP \longmapsto cRs$›
proof(*cases rule*: *semanticsCasesAux*[**where** C=($xvec1$, $xvec2$, $xvec3$, $xvec4$, $xvec5$, $x1$, $x2$, cP, cRs, C)])
case(*cInput M K xvec N Tvec P*)
have *B*: $cP = M(\!|\lambda* xvec\ N|\!).P$ **and** *C*: $cRs = K(\!|(N[xvec::=Tvec])|\!) \prec (P[xvec::=Tvec])$
by *fact+*
from ‹$xvec \sharp* (xvec1, xvec2, xvec3, xvec4, xvec5, x1, x2, cP, cRs, C)$› **have** $xvec \sharp* xvec1$ **by** *simp*

from ‹*length xvec1* = *inputLength cP*› *B* **have** *length xvec1* = *length xvec*
by *simp*
then obtain p **where** *S*: *set* $p \subseteq$ *set xvec* $\times$ *set*($p \cdot xvec$) **and** *distinctPerm p* **and** $xvec1 = p \cdot xvec$
using ‹*xvec* $\sharp*$ *xvec1*› ‹*distinct xvec*› ‹*distinct xvec1*›
by(*rule-tac constructPerm*[**where** *xvec*=*xvec* **and** *yvec*=*xvec1*]) *auto*
show *?thesis*
proof(*rule rInput*[**where** *M*=*M* **and** *K*=*K* **and** $N = p \cdot N$ **and** *Tvec*=*Tvec* **and** P=$p \cdot P$], *goal-cases*)
case *1*
from *B* ‹*xvec* $\sharp*$ *xvec1*› ‹*xvec1* $\sharp*$ *cP*› **have** $xvec1 \sharp* N$ **and** $xvec1 \sharp* P$
by(*auto simp add*: *fresh-star-def inputChainFresh name-list-supp*) (*auto simp*

add: *fresh-def*)

from ‹$cP = M(\!|\lambda{*}xvec\ N|\!).P$› S ‹$xvec1\ \sharp{*}\ N$› ‹$xvec1\ \sharp{*}\ P$› ‹$xvec1 = p \cdot xvec$›
have $cP = M(\!|\lambda{*}xvec1\ (p \cdot N)|\!).(p \cdot P)$
apply *simp*
by(*subst inputChainAlpha*) *auto*
moreover from ‹$cRs = K(\!|(N[xvec::=Tvec])|\!) \prec P[xvec::=Tvec]$› S ‹$xvec1\ \sharp{*}\ N$› ‹$xvec1\ \sharp{*}\ P$› ‹$xvec1 = p \cdot xvec$› ‹*length xvec = length Tvec*› ‹*distinctPerm p*›
have $cRs = K(\!|((p \cdot N)[xvec1::=Tvec])|\!) \prec (p \cdot P)[xvec1::=Tvec]$
by(*simp add*: *renaming substTerm.renaming*)
moreover note ‹$\Psi \vdash M \leftrightarrow K$›
moreover from ‹*distinct xvec*› ‹$xvec1 = p \cdot xvec$› **have** *distinct xvec1* **by** *simp*
moreover from ‹*set xvec* $\subseteq$ *supp N*› **have** $(p \cdot set\ xvec) \subseteq (p \cdot (supp\ N))$
by(*simp add*: *eqvts*)
with ‹$xvec1 = p \cdot xvec$› **have** $set\ xvec1 \subseteq supp(p \cdot N)$ **by**(*simp add*: *eqvts*)
moreover from ‹*length xvec = length Tvec*› ‹$xvec1 = p \cdot xvec$› **have** *length xvec1 = length Tvec*
by *simp*

moreover from ‹$xvec1\ \sharp{*}\ cRs$› C ‹*length xvec = length Tvec*› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*›
have $(set\ xvec1)\ \sharp{*}\ Tvec$
by(*rule-tac substTerm.subst3Chain*[**where** $T{=}N$]) *auto*
hence $xvec1\ \sharp{*}\ Tvec$ **by** *simp*
moreover from ‹$xvec\ \sharp{*}\ Tvec$› **have** $(p \cdot xvec)\ \sharp{*}\ (p \cdot Tvec)$ **by**(*simp add*: *fresh-star-bij*)
with S ‹$xvec\ \sharp{*}\ Tvec$› ‹$xvec1\ \sharp{*}\ Tvec$› ‹$xvec1 = p \cdot xvec$› **have** $xvec1\ \sharp{*}\ Tvec$ **by** *simp*
moreover from ‹$xvec\ \sharp{*}\ M$› **have** $(p \cdot xvec)\ \sharp{*}\ (p \cdot M)$ **by**(*simp add*: *fresh-star-bij*)
with S ‹$xvec\ \sharp{*}\ M$› ‹$xvec1\ \sharp{*}\ cP$› B ‹$xvec1 = p \cdot xvec$› **have** $xvec1\ \sharp{*}\ M$ **by** *simp*
moreover from ‹$xvec\ \sharp{*}\ K$› **have** $(p \cdot xvec)\ \sharp{*}\ (p \cdot K)$ **by**(*simp add*: *fresh-star-bij*)
with S ‹$xvec\ \sharp{*}\ K$› ‹$xvec1\ \sharp{*}\ cRs$› C ‹$xvec1 = p \cdot xvec$› **have** $xvec1\ \sharp{*}\ K$ **by** *simp*
ultimately show *?case* **using** ‹$xvec1\ \sharp{*}\ \Psi$› **by** *blast*
qed
next
case(*cOutput M K N P*)
thus *?thesis* **by**(*rule rOutput*)
next
case(*cCase Cs P* φ)
thus *?thesis* **by**(*rule rCase*)
next
case(*cPar1* Ψ_Q P α P' Q A_Q)
have *B*: $cP = P \parallel Q$ **and** *C*: $cRs = \alpha \prec P' \parallel Q$
by *fact+*
from ‹*bn* $\alpha\ \sharp{*}$ (*xvec1*, *xvec2*, *xvec3*, *xvec4*, *xvec5*, *x1*, *x2*, *cP*, *cRs*, *C*)› **have** *bn*

α $\sharp*$ *xvec2* **by** *simp*
from ‹A_Q $\sharp*$ (*xvec1*, *xvec2*, *xvec3*, *xvec4*, *xvec5*, *x1*, *x2*, *cP*, *cRs*, *C*)› **have** A_Q $\sharp*$ *xvec2* **and** A_Q $\sharp*$ *C* **by** *simp+*

from ‹*length xvec2* = *residualLength cRs*› *C* **have** *length xvec2* = *length*(*bn* α)
by *simp*
then obtain *p* **where** *S*: *set p* $\subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$)) **and** *distinctPerm p* **and** *xvec2*= *bn*($p \cdot \alpha$)
using ‹*bn* α $\sharp*$ *xvec2*› ‹*distinct*(*bn* α)› ‹*distinct xvec2*›
by(*rule-tac constructPerm*[**where** *xvec*=*bn* α **and** *yvec*=*xvec2*]) (*auto simp add*: *eqvts*)
show *?thesis*
proof(*rule rPar1*[**where** *P*=*P* **and** *Q*=*Q* **and** α=$p \cdot \alpha$ **and** *P'*=$p \cdot P'$ **and** A_Q=A_Q **and** Ψ_Q=Ψ_Q], *goal-cases*)
case *1*
note ‹*cP* = *P* $\parallel$ *Q*›
moreover from *B C S* ‹*bn* α $\sharp*$ *xvec2*› ‹*xvec2* $\sharp*$ *cRs*› ‹*xvec2* = *bn*($p \cdot \alpha$)› ‹*bn* α $\sharp*$ *subject* α› ‹*xvec2* $\sharp*$ *cP*› ‹*bn* α $\sharp*$ *Q*›
have *cRs* = ($p \cdot \alpha$) $\prec$ ($p \cdot P'$) $\parallel$ *Q*
apply *auto*
by(*subst residualAlpha*[**where** *p*=*p*]) *auto*
moreover note ‹*xvec2* = *bn*($p \cdot \alpha$)›
moreover from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› *S B C S* ‹*bn* α $\sharp*$ *xvec2*› ‹*xvec2* $\sharp*$ *cRs*› ‹*xvec2* = *bn*($p \cdot \alpha$)› ‹*bn* α $\sharp*$ *subject* α› ‹*xvec2* $\sharp*$ *cP*›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto (p \cdot \alpha) \prec (p \cdot P')$
by(*subst residualAlpha*[*symmetric*]) *auto*
moreover note ‹*extractFrame Q* = $\langle A_Q, \Psi_Q \rangle$› ‹*distinct* A_Q› ‹A_Q $\sharp*$ *P*› ‹A_Q $\sharp*$ *Q*› ‹A_Q $\sharp*$ Ψ› ‹A_Q $\sharp*$ α›
moreover from ‹A_Q $\sharp*$ α› ‹A_Q $\sharp*$ *xvec2*› *S* ‹*xvec2* = *bn*($p \cdot \alpha$)› ‹*distinctPerm p*› **have** A_Q $\sharp*$ ($p \cdot \alpha$)
by(*subst fresh-star-bij*[*symmetric*, **where** *pi*=*p*]) *simp*
moreover from ‹A_Q $\sharp*$ *P'*› ‹A_Q $\sharp*$ α› ‹A_Q $\sharp*$ *xvec2*› *S* ‹*xvec2* = *bn*($p \cdot \alpha$)› ‹*distinctPerm p*› **have** A_Q $\sharp*$ ($p \cdot P'$)
by(*subst fresh-star-bij*[*symmetric*, **where** *pi*=*p*]) *simp*
moreover note ‹A_Q $\sharp*$ *C*›
ultimately show *?case* **by** *blast*
qed
next
case(*cPar2* Ψ_P *Q* α *Q'* *P* A_P)
have *B*: *cP* = *P* $\parallel$ *Q* **and** *C*: *cRs* = $\alpha \prec P \parallel Q'$
by *fact+*
from ‹*bn* α $\sharp*$ (*xvec1*, *xvec2*, *xvec3*, *xvec4*, *xvec5*, *x1*, *x2*, *cP*, *cRs*, *C*)› **have** *bn* α $\sharp*$ *xvec3* **by** *simp*
from ‹A_P $\sharp*$ (*xvec1*, *xvec2*, *xvec3*, *xvec4*, *xvec5*, *x1*, *x2*, *cP*, *cRs*, *C*)› **have** A_P $\sharp*$ *xvec3* **and** A_P $\sharp*$ *C* **by** *simp+*

from ‹*length xvec3* = *residualLength cRs*› *C* **have** *length xvec3* = *length*(*bn* α)
by *simp*
then obtain *p* **where** *S*: *set p* $\subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$)) **and** *distinctPerm*

p **and** *xvec3* = $bn(p \cdot \alpha)$
using ‹$bn\ \alpha\ \sharp*\ xvec3$› ‹$distinct(bn\ \alpha)$› ‹$distinct\ xvec3$›
by(*rule-tac constructPerm*[**where** *xvec*=$bn\ \alpha$ **and** *yvec*=*xvec3*]) (*auto simp add*: *eqvts*)
show *?thesis*
proof(*rule rPar2*[**where** P=P **and** Q=Q **and** α=$p \cdot \alpha$ **and** Q'=$p \cdot Q'$ **and** A_P=A_P **and** Ψ_P=Ψ_P], *goal-cases*)
case *1*
note ‹$cP = P \parallel Q$›
moreover from $B\ C\ S$ ‹$bn\ \alpha\ \sharp*\ xvec3$› ‹$xvec3\ \sharp*\ cRs$› ‹$xvec3 = bn(p \cdot \alpha)$› ‹$bn\ \alpha\ \sharp*\ subject\ \alpha$› ‹$xvec3\ \sharp*\ cP$› ‹$bn\ \alpha\ \sharp*\ P$›
have $cRs = (p \cdot \alpha) \prec P \parallel (p \cdot Q')$
apply *auto*
by(*subst residualAlpha*[**where** *p*=*p*]) *auto*
moreover note ‹$xvec3 = bn(p \cdot \alpha)$›
moreover from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› $S\ B\ C\ S$ ‹$bn\ \alpha\ \sharp*\ xvec3$› ‹$xvec3\ \sharp*\ cRs$› ‹$xvec3 = bn(p \cdot \alpha)$› ‹$bn\ \alpha\ \sharp*\ subject\ \alpha$› ‹$xvec3\ \sharp*\ cP$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto (p \cdot \alpha) \prec (p \cdot Q')$
by(*subst residualAlpha*[*symmetric*]) *auto*
moreover note ‹$extractFrame\ P = \langle A_P, \Psi_P \rangle$› ‹$distinct\ A_P$› ‹$A_P\ \sharp*\ P$› ‹$A_P\ \sharp*\ Q$› ‹$A_P\ \sharp*\ \Psi$› ‹$A_P\ \sharp*\ \alpha$›
moreover from ‹$A_P\ \sharp*\ \alpha$› ‹$A_P\ \sharp*\ xvec3$› S ‹$xvec3 = bn(p \cdot \alpha)$› ‹$distinctPerm\ p$› **have** $A_P\ \sharp*\ (p \cdot \alpha)$
by(*subst fresh-star-bij*[*symmetric*, **where** *pi*=*p*]) *simp*
moreover from ‹$A_P\ \sharp*\ Q'$› ‹$A_P\ \sharp*\ \alpha$› ‹$A_P\ \sharp*\ xvec3$› S ‹$xvec3 = bn(p \cdot \alpha)$› ‹$distinctPerm\ p$› **have** $A_P\ \sharp*\ (p \cdot Q')$
by(*subst fresh-star-bij*[*symmetric*, **where** *pi*=*p*]) *simp*
moreover note ‹$A_P\ \sharp*\ C$›
ultimately show *?case* **by** *blast*
qed
next
case(*cComm1* $\Psi_Q\ P\ M\ N\ P'\ A_P\ \Psi_P\ Q\ K\ xvec\ Q'\ A_Q$)
thus *?thesis* **by**(*rule-tac rComm1*[**where** *P*=*P* **and** *Q*=*Q*]) (*assumption* | *simp*)+
next
case(*cComm2* $\Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q$)
thus *?thesis* **by**(*rule-tac rComm2*[**where** *P*=*P* **and** *Q*=*Q*]) (*assumption* | *simp*)+
next
case(*cOpen P M xvec yvec N P' x*)
have *B*: $cP = (\!|\nu x|\!)P$ **and** *C*: $cRs = M(\!|\nu*(xvec@x\#yvec)|\!)\langle N \rangle \prec P'$
by *fact*+
from ‹$xvec\ \sharp*\ (xvec1,\ xvec2,\ xvec3,\ xvec4,\ xvec5,\ x1,\ x2,\ cP,\ cRs,\ C)$› **have** $xvec\ \sharp*\ xvec4$ **and** $xvec\ \sharp*\ cP$ **and** $xvec\ \sharp*\ cRs$ **and** $x1\ \sharp\ xvec$ **by** *simp*+
from ‹$x\ \sharp\ (xvec1,\ xvec2,\ xvec3,\ xvec4,\ xvec5,\ x1,\ x2,\ cP,\ cRs,\ C)$› **have** $x\ \sharp\ xvec4$ **and** $x\ \sharp\ cP$ **and** $x\ \sharp\ cRs$ **and** $x \neq x1$ **by** *simp*+
from ‹$yvec\ \sharp*\ (xvec1,\ xvec2,\ xvec3,\ xvec4,\ xvec5,\ x1,\ x2,\ cP,\ cRs,\ C)$› **have** $yvec\ \sharp*\ xvec4$ **and** $yvec\ \sharp*\ cP$ **and** $yvec\ \sharp*\ cRs$ **and** $x1\ \sharp\ yvec$ **by** *simp*+

from ‹$xvec\ \sharp*\ cRs$› ‹$x\ \sharp\ cRs$› ‹$yvec\ \sharp*\ cRs$› C **have** $(xvec@x\#yvec)\ \sharp*\ M$ **by** *simp*

from ‹*xvec* ♯* Ψ› ‹*x* ♯ Ψ› ‹*yvec* ♯* Ψ› **have** (*xvec*@*x*#*yvec*) ♯* Ψ **by** *simp*
from ‹*length xvec4* = *residualLength cRs*› *C* **obtain** *xvec′ y yvec′* **where** *D*: *xvec4* = *xvec′*@*y*#*yvec′* **and** *length xvec′* = *length xvec* **and** *length yvec′* = *length yvec*
by(*rule-tac lengthAux2*) *auto*
with ‹*distinct xvec*› ‹*distinct yvec*› ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*xvec* ♯* *yvec*› ‹*xvec* ♯* *xvec4*› ‹*yvec* ♯* *xvec4*› ‹*x* ♯ *xvec4*› ‹*distinct xvec4*›
have *distinct xvec′* **and** *distinct yvec′* **and** *xvec′* ♯* *yvec′* **and** *x* ≠ *y* **and** *y* ♯ *xvec′* **and** *y* ♯ *yvec′*
and *x* ♯ *xvec′* **and** *x* ♯ *yvec′* **and** *y* ♯ *xvec* **and** *y* ♯ *yvec* **and** *xvec* ♯* *xvec′* **and** *yvec* ♯* *yvec′*
by *auto*
from ‹*length xvec′* = *length xvec*› ‹*xvec* ♯* *xvec′*› ‹*distinct xvec*› ‹*distinct xvec′*›
obtain *p* **where** *Sp*: *set p* ⊆ *set xvec* × *set*(*p* ∙ *xvec*) **and** *distinctPerm p* **and** *E*: *xvec′* = *p* ∙ *xvec*
by(*metis constructPerm*)
from ‹*length yvec′* = *length yvec*› ‹*yvec* ♯* *yvec′*› ‹*distinct yvec*› ‹*distinct yvec′*›
obtain *q* **where** *Sq*: *set q* ⊆ *set yvec* × *set*(*q* ∙ *yvec*) **and** *distinctPerm q* **and** *F*: *yvec′* = *q* ∙ *yvec*
by(*metis constructPerm*)

show *?thesis*
proof(*rule rOpen*[**where** *P*=([(*x*, *x1*)] ∙ *P*) **and** *xvec*=*p* ∙ *xvec* **and** *y*=*y* **and** *yvec*=*q* ∙ *yvec* **and** *N*=(*p*@(*x1*, *x*)#*q*) ∙ *N* **and** *P′*=(*p*@(*x1*, *x*)#*q*) ∙ *P′* **and** *M*=*M*], *goal-cases*)
case *1*
from ‹*xvec* ♯* *xvec4*› ‹*x* ♯ *xvec4*› ‹*x1* ♯ *xvec4*› ‹*yvec* ♯* *xvec4*› *D E F*
have *x* ≠ *y* **and** *x1* ≠ *y* **and** *x1* ♯ *p* ∙ *xvec* **and** *x1* ♯ *q* ∙ *yvec* **by** *simp+*
from ‹*xvec4* ♯* *cRs*› ‹*x1* ♯ *cRs*› *C* **have** *xvec4* ♯* *M* **and** *x1* ♯ *M* **by** *simp+*
from ‹*cP* = (⦇*νx*⦈)*P*› ‹*x* ♯ *cP*› ‹*x* ≠ *x1*› **have** ([(*x*, *x1*)] ∙ *cP*) = [(*x*, *x1*)] ∙ (⦇*νx*⦈)*P*
by *simp*
with ‹*x* ♯ *cP*› ‹*x1* ♯ *cP*› **have** *cP* = (⦇*νx1*⦈)([(*x*, *x1*)] ∙ *P*) **by**(*simp add*: *eqvts calc-atm*)
moreover from *C* **have** ((*p*@(*x1*, *x*)#*q*) ∙ *cRs*) = (*p*@(*x1*, *x*)#*q*) ∙ (*M*⦇*νx*∗(*xvec*@*x*#*yvec*)⦈⟨*N*⟩ ≺ *P′*) **by**(*simp add*: *fresh-star-bij*)
with *Sp Sq* ‹*xvec4* ♯* *cRs*› *D E F* ‹*xvec* ♯* *cRs*› ‹*x* ♯ *cRs*› ‹*yvec* ♯* *cRs*› ‹*xvec4* ♯* *M*› ‹(*xvec*@*x*#*yvec*) ♯* *M*› ‹*xvec* ♯* *xvec4*› ‹*x* ♯ *xvec4*› ‹*yvec* ♯* *xvec4*› ‹*xvec* ♯* *yvec*› ‹*x* ♯ *xvec*› ‹*x* ♯ *yvec*› ‹*y* ♯ *xvec′*› ‹*y* ♯ *yvec′*› ‹*xvec′* ♯* *yvec′*› ‹*x1* ♯ *xvec*› ‹*x1* ♯ *yvec*› ‹*x1* ≠ *y*› ‹*x1* ♯ *xvec4*› ‹*x1* ♯ *cRs*› ‹*x1* ♯ *cRs*› ‹*x* ≠ *x1*› ‹*x1* ♯ *M*›
have *cRs* = *M*⦇*νx*∗((*p* ∙ *xvec*)@*x1*#(*q* ∙ *yvec*))⦈⟨((*p*@(*x1*, *x*)#*q*) ∙ *N*)⟩ ≺ ((*p*@(*x1*, *x*)#*q*) ∙ *P′*)
by(*simp add*: *eqvts pt2*[*OF pt-name-inst*] *calc-atm*)
moreover from *D E F* **have** *xvec4* = (*p* ∙ *xvec*)@*y*#(*q* ∙ *yvec*) **by** *simp*
moreover from ‹Ψ ▷ *P* ⟼*M*⦇*νx*∗(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P′*› **have** ((*p*@(*x1*, *x*)#*q*) ∙ Ψ) ▷ ((*p*@(*x1*, *x*)#*q*) ∙ *P*) ⟼((*p*@(*x1*, *x*)#*q*) ∙ (*M*⦇*νx*∗(*xvec*@*yvec*)⦈⟨*N*⟩ ≺ *P′*))
by(*intro eqvts*)
with *Sp Sq B C D E F* ‹*xvec4* ♯* Ψ› ‹(*xvec*@*x*#*yvec*) ♯* Ψ› ‹*xvec4* ♯* *cRs*›

‹$x \sharp xvec4$› C D ‹$x \sharp cRs$› ‹$yvec \sharp* cRs$› ‹$xvec4 \sharp* M$› ‹$(xvec@x\#yvec) \sharp* M$› ‹$x \sharp M$› ‹$x1 \sharp cRs$› ‹$x \neq x1$› ‹$x1 \sharp xvec$› ‹$x1 \sharp yvec$› ‹$xvec \sharp* xvec4$› ‹$yvec \sharp* xvec4$› ‹$x1 \sharp xvec4$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$x1 \sharp \Psi$› ‹$xvec4 \sharp* cP$› ‹$xvec \sharp* P$› ‹$yvec \sharp* P$› ‹$xvec' \sharp* yvec'$› ‹$x1 \sharp xvec4$› ‹$xvec4 \sharp* cP$› ‹$yvec \sharp* xvec4$› ‹$xvec \sharp* xvec4$› ‹$x \neq x1$› ‹$xvec \sharp* yvec$›

have $\Psi \rhd ([(x, x1)] \cdot P) \longmapsto M\langle\!|\nu*((p \cdot xvec)@(q \cdot yvec))|\!\rangle\langle((p@(x1, x)\#q) \cdot N)\rangle \prec ((p@(x1, x)\#q) \cdot P')$

by(*simp add*: *eqvts pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*] *pt2*[*OF pt-name-inst*] *name-swap*)

moreover from ‹$x \in supp\ N$› **have** $((p@(x1, x)\#q) \cdot x) \in ((p@(x1, x)\#q) \cdot supp\ N)$

by(*simp add*: *pt-set-bij*[*OF pt-name-inst*, *OF at-name-inst*])

hence $x1 \in supp((p@(x1, x)\#q) \cdot N)$

using ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$x1 \sharp xvec$› ‹$x1 \sharp yvec$› ‹$x \sharp xvec4$› ‹$x1 \sharp xvec4$› ‹$xvec \sharp* xvec4$› ‹$yvec \sharp* xvec4$› ‹$xvec' \sharp* yvec'$› D E F Sp Sq ‹$x \neq x1$›

by(*simp add*: *eqvts pt2*[*OF pt-name-inst*] *calc-atm*)

moreover from ‹$x1 \sharp xvec4$› D E F **have** $x1 \sharp (p \cdot xvec)$ **and** $x1 \sharp (q \cdot yvec)$ **by** *simp+*

moreover from ‹*distinct xvec'*› ‹*distinct yvec'*› E F **have** $distinct(p \cdot xvec)$ **and** $distinct(q \cdot yvec)$ **by** *simp+*

moreover from ‹$xvec' \sharp* yvec'$› E F **have** $(p \cdot xvec) \sharp* (q \cdot yvec)$ **by** *auto*

moreover from ‹$xvec \sharp* \Psi$› **have** $(p \cdot xvec) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

with Sp D E ‹$xvec4 \sharp* \Psi$› ‹$xvec \sharp* \Psi$› **have** $(p \cdot xvec) \sharp* \Psi$ **by**(*simp add*: *eqvts*)

moreover from ‹$yvec \sharp* \Psi$› **have** $(p \cdot yvec) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

with Sq D F ‹$xvec4 \sharp* \Psi$› ‹$yvec \sharp* \Psi$› **have** $(q \cdot yvec) \sharp* \Psi$ **by**(*simp add*: *eqvts*)

moreover from ‹$xvec4 \sharp* cP$› ‹$x \sharp xvec4$› ‹$x1 \sharp xvec4$› B D E F **have** $(p \cdot xvec) \sharp* ([(x, x1)] \cdot P)$ **and** $(q \cdot yvec) \sharp* ([(x, x1)] \cdot P)$

by *simp+*

moreover from ‹$xvec4 \sharp* M$› C D E F **have** $(p \cdot xvec) \sharp* M$ **and** $(q \cdot yvec) \sharp* M$ **by** *simp+*

ultimately show *?case*

by *blast*

qed

next

case(*cScope P* α *P' x*)

have *B*: $cP = \langle\!|\nu x|\!\rangle P$ **and** *C*: $cRs = \alpha \prec \langle\!|\nu x|\!\rangle P'$

by *fact+*

from ‹$bn\ \alpha \sharp* (xvec1, xvec2, xvec3, xvec4, xvec5, x1, x2, cP, cRs, C)$› **have** $bn\ \alpha \sharp* xvec5$ **and** $x2 \sharp bn\ \alpha$ **by** *simp+*

from ‹$x \sharp (xvec1, xvec2, xvec3, xvec4, xvec5, x1, x2, cP, cRs, C)$› **have** $x \sharp xvec5$ **and** $x \neq x2$ **and** $x \sharp cRs$ **by** *simp+*

from ‹$length\ xvec5 = residualLength\ cRs$› C **have** $length\ xvec5 = length(bn\ \alpha)$

by *simp*

then obtain p **where** S: *set* $p \subseteq$ *set*$(bn\ \alpha) \times$ *set*$(bn(p \cdot \alpha))$ **and** *distinctPerm* p **and** $xvec5 = bn(p \cdot \alpha)$
using ‹$bn\ \alpha\ \sharp*\ xvec5$› ‹*distinct*$(bn\ \alpha)$› ‹*distinct* $xvec5$›
by(*rule-tac constructPerm*[**where** $xvec=bn\ \alpha$ **and** $yvec=xvec5$]) (*auto simp add*: *eqvts*)
show *?thesis*
proof(*rule rScope*[**where** $P=[(x, x2)] \cdot P$ **and** $\alpha=[(x, x2)] \cdot p \cdot \alpha$ **and** $P'=[(x, x2)] \cdot p \cdot P'$], *goal-cases*)
case *1*
from ‹$x2\ \sharp\ cRs$› C ‹$x2\ \sharp\ bn\ \alpha$› ‹$x \neq x2$› **have** $x2\ \sharp\ \alpha$ **and** $x2\ \sharp\ P'$ **by**(*auto simp add*: *abs-fresh*)
from ‹$cP = (\!|\nu x|\!)P$› ‹$x2\ \sharp\ cP$› ‹$x \neq x2$› **have** $cP = (\!|\nu x2|\!)([(x, x2)] \cdot P)$
by(*simp add*: *alphaRes abs-fresh*)
moreover from $B\ C\ S$ ‹$bn\ \alpha\ \sharp*\ xvec5$› ‹$xvec5\ \sharp*\ cRs$› ‹$xvec5 = bn(p \cdot \alpha)$› ‹$bn\ \alpha\ \sharp*$ *subject* α› ‹$xvec5\ \sharp*\ cP$› ‹$x\ \sharp\ \alpha$› ‹$x\ \sharp\ xvec5$›
have $cRs = (p \cdot \alpha) \prec (\!|\nu x|\!)(p \cdot P')$
apply *auto*
by(*subst residualAlpha*[**where** $p=p$] *alphaRes*) (*auto simp del*: *actionFresh*)
hence $([(x, x2)] \cdot cRs) = [(x, x2)] \cdot ((p \cdot \alpha) \prec (\!|\nu x|\!)(p \cdot P'))$
by *simp*
with ‹$x2\ \sharp\ cRs$› ‹$x\ \sharp\ cRs$› **have** $cRs = ([(x, x2)] \cdot p \cdot \alpha) \prec (\!|\nu x2|\!)([(x, x2)] \cdot p \cdot P')$
by(*simp add*: *eqvts calc-atm*)
moreover from ‹$xvec5 = bn(p \cdot \alpha)$› **have** $([(x, x2)] \cdot xvec5) = ([(x, x2)] \cdot bn(p \cdot \alpha))$
by *simp*
with ‹$x\ \sharp\ xvec5$› ‹$x2\ \sharp\ xvec5$› **have** $xvec5 = bn([(x, x2)] \cdot p \cdot \alpha)$
by(*simp add*: *eqvts*)
moreover from ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› $S\ B\ C\ S$ ‹$bn\ \alpha\ \sharp*\ xvec5$› ‹$xvec5\ \sharp*\ cRs$› ‹$xvec5 = bn(p \cdot \alpha)$› ‹$bn\ \alpha\ \sharp*$ *subject* α› ‹$xvec5\ \sharp*\ cP$› ‹$x\ \sharp\ xvec5$›
have $\Psi \rhd P \longmapsto (p \cdot \alpha) \prec (p \cdot P')$
by(*subst residualAlpha*[*symmetric*]) *auto*
hence $([(x, x2)] \cdot \Psi) \rhd ([(x, x2)] \cdot P) \longmapsto ([(x, x2)] \cdot ((p \cdot \alpha) \prec (p \cdot P')))$
by(*rule eqvt*)
with ‹$x\ \sharp\ \Psi$› ‹$x2\ \sharp\ \Psi$› **have** $\Psi \rhd ([(x, x2)] \cdot P) \longmapsto ([(x, x2)] \cdot p \cdot \alpha) \prec ([(x, x2)] \cdot p \cdot P')$
by(*simp add*: *eqvts*)
moreover note ‹$x2\ \sharp\ \Psi$›
moreover from ‹$x\ \sharp\ \alpha$› ‹$x2\ \sharp\ \alpha$› ‹$x\ \sharp\ xvec5$› ‹$x2\ \sharp\ xvec5$› S ‹$x \neq x2$› ‹$xvec5 = bn(p \cdot \alpha)$› **have** $x2\ \sharp\ [(x, x2)] \cdot p \cdot \alpha$
apply(*subgoal-tac* $x\ \sharp\ p \wedge x2\ \sharp\ p$)
apply(*simp add*: *perm-compose freshChainSimps del*: *actionFresh*)
by(*auto dest*: *freshAlphaSwap*)
moreover from ‹$bn\ \alpha\ \sharp*$ *subject* α› **have** $([(x, x2)] \cdot p \cdot (bn\ \alpha))\ \sharp*\ ([(x, x2)] \cdot p \cdot (subject\ \alpha))$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
hence $bn([(x, x2)] \cdot p \cdot \alpha)\ \sharp*\ subject([(x, x2)] \cdot p \cdot \alpha)$
by(*simp add*: *eqvts*)
moreover from ‹*distinct*$(bn\ \alpha)$› **have** *distinct*$([(x, x2)] \cdot p \cdot (bn\ \alpha))$ **by** *simp*

hence *distinct*(*bn*([(*x*, *x2*)] · *p* · α)) **by**(*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
qed
next
case(*cBang P*)
thus *?thesis* **by**(*rule-tac rBang*) *auto*
qed

lemma *parCases*[*consumes 5*, *case-names cPar1 cPar2 cComm1 cComm2*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and *T* :: (*'a*, *'b*, *'c*) *psi*
and *C* :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto \alpha \prec T$
and *bn* α $\sharp*$ Ψ
and *bn* α $\sharp*$ *P*
and *bn* α $\sharp*$ *Q*
and *bn* α $\sharp*$ *subject* α
and *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q$. $[\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$A_Q \sharp* \Psi$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* \alpha$; $A_Q \sharp* P'$; $A_Q \sharp* C]\!] \Longrightarrow$ *Prop* α $(P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P$. $[\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* \alpha$; $A_P \sharp* Q'$; $A_P \sharp* C]\!] \Longrightarrow$ *Prop* α $(P \parallel Q')$
and *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C]\!] \Longrightarrow$
Prop (τ) $((\!|\nu* xvec|\!)(P' \parallel Q'))$
and *rComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q$.
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q';$ *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp*$

Q; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C]\!] \Longrightarrow$
$Prop\ (\tau)\ ((\!|\nu*xvec|\!)(P' \parallel Q'))$

shows $Prop\ \alpha\ T$
proof −
from *Trans* **have** $distinct(bn\ \alpha)$ **by**(*auto dest*: *boundOutputDistinct*)
have $length(bn\ \alpha) = residualLength(\alpha \prec T)$ **by** *simp*
note *Trans*
moreover have $length\ [] = inputLength(P \parallel Q)$ **and** $distinct\ []$
by(*auto simp add*: *inputLength-inputLength'-inputLength''.simps*)
moreover note ‹$length(bn\ \alpha) = residualLength(\alpha \prec T)$› ‹$distinct(bn\ \alpha)$›
moreover note ‹$length(bn\ \alpha) = residualLength(\alpha \prec T)$› ‹$distinct(bn\ \alpha)$›
moreover note ‹$length(bn\ \alpha) = residualLength(\alpha \prec T)$› ‹$distinct(bn\ \alpha)$›
moreover note ‹$length(bn\ \alpha) = residualLength(\alpha \prec T)$› ‹$distinct(bn\ \alpha)$›
moreover obtain x::*name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp Q$ **and** $x \sharp \alpha$ **and** $x \sharp T$
by(*generate-fresh name*) *auto*
ultimately show *?thesis* **using** ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$›
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - $C\ x\ x$])
apply(*auto simp add*: *psi.inject*)
apply(*force simp add*: *residualInject residualInject' intro*: *rPar1*)
apply(*force simp add*: *residualInject residualInject' intro*: *rPar2*)
apply(*fastforce simp add*: *residualInject residualInject' intro*: *rComm1*)
by(*fastforce simp add*: *residualInject residualInject' intro*: *rComm2*)
qed

lemma *parInputCases*[*consumes 1*, *case-names cPar1 cPar2*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $N :: 'a$
and $R :: ('a, 'b, 'c)\ psi$
and $C :: 'd$::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|N|\!) \prec R$
and *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* N;\ A_Q \sharp* C]\!] \Longrightarrow Prop\ (P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* C]\!] \Longrightarrow Prop\ (P \parallel Q')$
shows $Prop\ R$
proof −

from *Trans* **obtain** α **where** $\Psi \rhd P \parallel Q \longmapsto \alpha \prec R$ **and** $bn\ \alpha \sharp* \Psi$ **and** $bn\ \alpha \sharp* P$ **and** $bn\ \alpha \sharp* Q$ **and** $bn\ \alpha \sharp* subject\ \alpha$ **and** $\alpha = M(\!|N|\!)$ **by** *auto*
thus *?thesis* **using** *rPar1 rPar2*
by(*induct rule*: *parCases*) (*auto simp add*: *residualInject*)
qed

lemma *parOutputCases*[*consumes 5*, *case-names cPar1 cPar2*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $xvec :: name\ list$
and $N :: 'a$
and $R :: ('a, 'b, 'c)\ psi$
and $C :: 'd::fs\text{-}name$

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec R$
and $xvec \sharp* \Psi$
and $xvec \sharp* P$
and $xvec \sharp* Q$
and $xvec \sharp* M$
and *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* xvec;\ A_Q \sharp* N;\ A_Q \sharp* C;\ A_Q \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* M;\ A_P \sharp* xvec;\ A_P \sharp* N;\ A_P \sharp* C;\ A_P \sharp* xvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (P \parallel Q')$
shows *Prop R*
proof −
from *Trans* **have** *distinct xvec* **by**(*auto dest*: *boundOutputDistinct*)
obtain α **where** $\alpha = M(\!|\nu* xvec|\!)\langle N\rangle$ **by** *simp*
with *Trans* ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* P$› ‹$xvec \sharp* Q$› ‹$xvec \sharp* M$›
have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec R$ **and** $bn\ \alpha \sharp* \Psi$ **and** $bn\ \alpha \sharp* P$ **and** $bn\ \alpha \sharp* Q$ $bn\ \alpha \sharp* subject\ \alpha$
by *simp*+
thus *?thesis* **using** ‹$\alpha = M(\!|\nu* xvec|\!)\langle N\rangle$› *rPar1 rPar2* ‹*distinct xvec*›
by(*induct rule*: *parCases*[**where** $C = (xvec, C)$]) (*auto simp add*: *residualInject*)
qed

lemma *theEqvt*[*eqvt-force*]:
fixes $p :: name\ prm$
and $\alpha :: 'a\ action$

assumes $\alpha \neq \tau$

shows $(p \cdot the(subject\ \alpha)) = the(p \cdot (subject\ \alpha))$
using *assms*

by(*induct rule*: *actionCases*[**where** α=α]) *auto*

lemma *theSubjectFresh*[*simp*]:
  fixes α :: $'a$ *action*
  and x :: *name*

  assumes $\alpha \neq \tau$

  shows $x \sharp$ *the*(*subject* α) = $x \sharp$ *subject* α
using *assms*
by(*cases rule*: *actionCases*) *auto*

lemma *theSubjectFreshChain*[*simp*]:
  fixes α :: $'a$ *action*
  and *xvec* :: *name list*

  assumes $\alpha \neq \tau$

  shows *xvec* $\sharp*$ *the*(*subject* α) = *xvec* $\sharp*$ *subject* α
using *assms*
by(*cases rule*: *actionCases*) *auto*

lemma *obtainPrefix*:
  fixes Ψ :: $'b$
  and P :: ($'a$, $'b$, $'c$) *psi*
  and α :: $'a$ *action*
  and P' :: ($'a$, $'b$, $'c$) *psi*
  and A_P :: *name list*
  and Ψ_P :: $'b$
  and B :: *name list*

  assumes $\Psi \rhd P \longmapsto \alpha \prec P'$
  and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
  and *distinct* A_P
  and *bn* $\alpha \sharp*$ *subject* α
  and *distinct*(*bn* α)
  and $\alpha \neq \tau$
  and $B \sharp* P$
  and $A_P \sharp* \Psi$
  and $A_P \sharp* B$
  and $A_P \sharp* P$
  and $A_P \sharp*$ *subject* α

  obtains M **where** $\Psi \otimes \Psi_P \vdash$ *the*(*subject* α) $\leftrightarrow M$ **and** $B \sharp* M$
using *assms*
proof(*nominal-induct avoiding*: B *arbitrary*: *thesis rule*: *semanticsFrameInduct'*)
  case(*cAlpha* Ψ P α P' p A_P Ψ_P B)
  then obtain M **where** *subjEq*: $\Psi \otimes \Psi_P \vdash$ *the*(*subject* α) $\leftrightarrow M$ **and** $B \sharp* M$
    by(*rule-tac cAlpha*) *auto*

from ‹*set p* ⊆ *set*(*bn α*) × *set*(*bn*(*p* ∙ *α*))› ‹*bn α* ♯* *subject α*› ‹*bn*(*p* ∙ *α*) ♯* *α*› *subjEq*
have Ψ ⊗ Ψ$_P$ ⊢ *the*(*subject*(*p* ∙ *α*)) ↔ *M*
by(*simp add*: *subjectEqvt*[*symmetric*])
thus *?case* **using** *cAlpha* ‹*B* ♯* *M*›
by *auto*
next
case(*cFrameAlpha* Ψ *P* *A*$_P$ Ψ$_P$ *p* *α* *P*′ *B*)
then obtain *M* **where** *subjEq*: Ψ ⊗ Ψ$_P$ ⊢ *the*(*subject α*) ↔ *M* **and** *B* ♯* *M*
by(*rule-tac cFrameAlpha*) *auto*
have *S*: *set p* ⊆ *set A*$_P$ × *set* (*p* ∙ *A*$_P$) **by** *fact*
from *subjEq* **have** (*p* ∙ (Ψ ⊗ Ψ$_P$)) ⊢ (*p* ∙ *the*(*subject α*)) ↔ (*p* ∙ *M*)
by(*rule chanEqClosed*)
with ‹*A*$_P$ ♯* Ψ› ‹(*p* ∙ *A*$_P$) ♯* Ψ› ‹(*p* ∙ *A*$_P$) ♯* *subject α*› *S* ‹*α* ≠ *τ*› ‹*A*$_P$ ♯* *α*›
have Ψ ⊗ (*p* ∙ Ψ$_P$) ⊢ *the*(*subject α*) ↔ (*p* ∙ *M*)
by(*simp add*: *eqvts del*: *subjectEqvt*)
moreover from ‹*B* ♯* *M*› **have** (*p* ∙ *B*) ♯* (*p* ∙ *M*)
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*A*$_P$ ♯* *B*› ‹(*p* ∙ *A*$_P$) ♯* *B*› *S* **have** *B* ♯* (*p* ∙ *M*) **by**(*simp add*: *eqvts*)
ultimately show *?case* **by**(*rule cFrameAlpha*)
next
case(*cInput* Ψ *M K xvec N Tvec P B*)
from ‹Ψ ⊢ *M* ↔ *K*› **have** Ψ ⊗ **1** ⊢ *M* ↔ *K*
by(*blast intro*: *statEqEnt AssertionStatEqSym*[*OF Identity*])
hence Ψ ⊗ **1** ⊢ *K* ↔ *M* **by**(*rule chanEqSym*)
moreover from ‹*B* ♯* (*M*⦇λ∗*xvec N*⦈.*P*)› **have** *B* ♯* *M* **by** *simp*
ultimately show *?case* **by**(*rule-tac cInput*) *auto*
next
case(*cOutput* Ψ *M K N P B*)
from ‹Ψ ⊢ *M* ↔ *K*› **have** Ψ ⊗ **1** ⊢ *M* ↔ *K*
by(*blast intro*: *statEqEnt AssertionStatEqSym*[*OF Identity*])
hence Ψ ⊗ **1** ⊢ *K* ↔ *M*
by(*rule chanEqSym*)
moreover from ‹*B* ♯* (*M*⟨*N*⟩.*P*)› **have** *B* ♯* *M* **by** *simp*
ultimately show *?case* **by**(*rule-tac cOutput*) *auto*
next
case(*cCase* Ψ *P* *α* *P*′ *φ* *Cs* *A*$_P$ Ψ$_P$ *B*)
then obtain *M* **where** Ψ ⊗ Ψ$_P$ ⊢ *the*(*subject α*) ↔ *M* **and** *B* ♯* *M*
by(*rule-tac cCase*) (*auto dest*: *memFreshChain*)
with ‹Ψ$_P$ ≃ **1**› **show** *?case* **by**(*blast intro*: *cCase statEqEnt compositionSym Identity*)
next
case(*cPar1* Ψ Ψ$_Q$ *P* *α* *P*′ *A*$_Q$ *Q* *A*$_P$ Ψ$_P$ *B*)
then obtain *M* **where** (Ψ ⊗ Ψ$_Q$) ⊗ Ψ$_P$ ⊢ *the*(*subject α*) ↔ *M* **and** *B* ♯* *M*
apply(*rule-tac cPar1*) **by** *assumption auto*
thus *?case*
by(*metis cPar1 statEqEnt Associativity Commutativity AssertionStatEqTrans Composition*)
next

```
  case(cPar2 Ψ Ψ_P Q α Q' A_P P A_Q Ψ_Q B)
  then obtain M where (Ψ ⊗ Ψ_P) ⊗ Ψ_Q ⊢ the(subject α) ↔ M and B ♯* M
    by(rule-tac cPar2) auto
  thus ?case by(metis cPar2 statEqEnt Associativity)
next
  case cComm1
  thus ?case by simp
next
  case cComm2
  thus ?case by simp
next
  case(cOpen Ψ P M xvec yvec N P' x A_P Ψ_P B)
  then obtain K where Ψ ⊗ Ψ_P ⊢ M ↔ K and B ♯* K
    apply(rule-tac cOpen) by force auto
  thus ?case by(fastforce intro: cOpen)
next
  case(cScope Ψ P α P' x A_P Ψ_P B)
  then obtain M where Ψ ⊗ Ψ_P ⊢ the(subject α) ↔ M and B ♯* M
    by(rule-tac cScope) auto
  thus ?case by(fastforce intro: cScope)
next
  case(cBang Ψ P α P' A_P Ψ_P B)
  then obtain K where Ψ ⊗ Ψ_P ⊗ 1 ⊢ the(subject α) ↔ K and B ♯* K
    by(rule-tac cBang) auto
  with ‹Ψ_P ≃ 1› show ?case by(metis cBang statEqEnt compositionSym Identity)
qed

lemma inputRenameSubject:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   M  :: 'a
  and   N  :: 'a
  and   P' :: ('a, 'b, 'c) psi
  and   A_P :: name list
  and   Ψ_P :: 'b

  assumes Ψ ▷ P ⟼M(⦇N⦈) ≺ P'
  and     extractFrame P = ⟨A_P, Ψ_P⟩
  and     distinct A_P
  and     Ψ ⊗ Ψ_P ⊢ M ↔ K
  and     A_P ♯* Ψ
  and     A_P ♯* P
  and     A_P ♯* M
  and     A_P ♯* K

  shows Ψ ▷ P ⟼K(⦇N⦈) ≺ P'
using assms
proof(nominal-induct avoiding: K rule: inputFrameInduct)
  case(cAlpha Ψ P M N P' A_P Ψ_P  p K)
```

have S: *set* $p \subseteq$ *set* $A_P \times$ *set* $(p \cdot A_P)$ **by** *fact*
from ‹$\Psi \otimes (p \cdot \Psi_P) \vdash M \leftrightarrow K$› **have** $(p \cdot (\Psi \otimes (p \cdot \Psi_P))) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$
by(*rule chanEqClosed*)
with S ‹*distinctPerm* p› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$› ‹$(p \cdot A_P) \sharp* \Psi$› ‹$(p \cdot A_P) \sharp* M$› ‹$(p \cdot A_P) \sharp* K$›
have $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$ **by**(*simp add*: *eqvts*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$[\![\Psi \otimes \Psi_P \vdash M \leftrightarrow K; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* M; A_P \sharp* K]\!] \Longrightarrow \Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$›
show *?case* **by** *blast*
next
case(*cInput* Ψ M K *xvec* N *Tvec* P K')
from ‹$\Psi \otimes \mathbf{1} \vdash K \leftrightarrow K'$› **have** $\Psi \vdash K \leftrightarrow K'$
by(*blast intro*: *statEqEnt Identity*)
with ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \vdash M \leftrightarrow K'$
by(*rule chanEqTrans*)
thus *?case* **using** ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp* N› ‹*length xvec* = *length Tvec*›
by(*rule Input*)
next
case(*cCase* Ψ P M N P' φ *Cs* A_P Ψ_P K)
from ‹$\Psi \otimes \mathbf{1} \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity compositionSym AssertionStatEqSym*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$\bigwedge K.\ [\![\Psi \otimes \Psi_P \vdash M \leftrightarrow K; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* M; A_P \sharp* K]\!] \Longrightarrow \Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$›
have $\Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$ **by** *force*
thus *?case* **using** ‹(φ, P) *mem Cs*› ‹$\Psi \vdash \varphi$› ‹*guarded* P› **by**(*rule Case*)
next
case(*cPar1* Ψ Ψ_Q P M N P' A_Q Q A_P Ψ_P K)
from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K$
by(*metis statEqEnt Associativity Composition AssertionStatEqTrans Commutativity*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_Q$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$\bigwedge K.\ [\![(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K; A_P \sharp* (\Psi \otimes \Psi_Q); A_P \sharp* P; A_P \sharp* M; A_P \sharp* K]\!] \Longrightarrow \Psi \otimes \Psi_Q \rhd P \longmapsto K(\!|N|\!) \prec P'$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto K(\!|N|\!) \prec P'$ **by** *force*
thus *?case* **using** ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* K$› ‹$A_Q \sharp* N$›
by(*rule-tac Par1*) *auto*
next
case(*cPar2* Ψ Ψ_P Q M N Q' A_P P A_Q Ψ_Q K)
from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash M \leftrightarrow K$
by(*rule statEqEnt*[*OF AssertionStatEqSym*[*OF Associativity*]])
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* M$› ‹$A_Q \sharp* K$›
‹$\bigwedge K.\ [\![(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash M \leftrightarrow K; A_Q \sharp* (\Psi \otimes \Psi_P); A_Q \sharp* Q; A_Q \sharp* M; A_Q \sharp* K]\!] \Longrightarrow \Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$ **by** *force*
thus *?case* **using** ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp*$

$K\rangle$ $\langle A_P \sharp* N\rangle$
  by(*rule-tac Par2*) *auto*
next
  case(*cScope* Ψ P M N P' x A_P Ψ_P)
  hence $\Psi \rhd P \longmapsto K(\!|N|\!) \prec P'$ **by** *force*
  with $\langle x \sharp \Psi\rangle$ $\langle x \sharp K\rangle$ $\langle x \sharp N\rangle$ **show** *?case*
    by(*rule-tac Scope*) *auto*
next
  case(*cBang* Ψ P M N P' A_P Ψ_P K)
  from $\langle \Psi \otimes \mathbf{1} \vdash M \leftrightarrow K\rangle$ $\langle \Psi_P \simeq \mathbf{1}\rangle$ **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K$
    by(*blast intro*: *statEqEnt Identity compositionSym AssertionStatEqSym*)
  with $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K\rangle$
    $\langle \bigwedge K.\ [\![\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K;\ A_P \sharp* \Psi;\ A_P \sharp* (P \parallel !P);\ A_P \sharp* M;\ A_P \sharp* K]\!] \Longrightarrow \Psi \rhd P \parallel !P \longmapsto K(\!|N|\!) \prec P'\rangle$
  have $\Psi \rhd P \parallel !P \longmapsto K(\!|N|\!) \prec P'$ **by** *force*
  thus *?case* **using** $\langle guarded\ P\rangle$ **by**(*rule Bang*)
qed

lemma *outputRenameSubject*:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*
  and M :: *'a*
  and *xvec* :: *name list*
  and N :: *'a*
  and P' :: (*'a*, *'b*, *'c*) *psi*
  and A_P :: *name list*
  and Ψ_P :: *'b*

  assumes $\Psi \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$
  and *extractFrame* $P = \langle A_P, \Psi_P\rangle$
  and *distinct* A_P
  and $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$
  and $A_P \sharp* \Psi$
  and $A_P \sharp* P$
  and $A_P \sharp* M$
  and $A_P \sharp* K$

  shows $\Psi \rhd P \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec P'$
using *assms*
apply(*simp add*: *residualInject*)
proof(*nominal-induct avoiding*: *K rule*: *outputFrameInduct*)
  case(*cAlpha* Ψ P M A_P Ψ_P p B K)
  have *S*: *set* $p \subseteq$ *set* $A_P \times$ *set*$(p \cdot A_P)$ **by** *fact*
  from $\langle \Psi \otimes (p \cdot \Psi_P) \vdash M \leftrightarrow K\rangle$ **have** $(p \cdot (\Psi \otimes (p \cdot \Psi_P))) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$
    by(*rule chanEqClosed*)
  with *S* $\langle$*distinctPerm p*$\rangle$ $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K\rangle$ $\langle (p \cdot A_P) \sharp* \Psi\rangle$ $\langle (p \cdot A_P) \sharp* M\rangle$ $\langle (p \cdot A_P) \sharp* K\rangle$
  have $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$ **by**(*simp add*: *eqvts*)

with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
show *?case* **by**(*blast intro*: *cAlpha*)
next
case(*cOutput* Ψ M K N P K')
from ‹$\Psi \otimes \mathbf{1} \vdash K \leftrightarrow K'$› **have** $\Psi \vdash K \leftrightarrow K'$
by(*blast intro*: *statEqEnt Identity*)
with ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \vdash M \leftrightarrow K'$
by(*rule chanEqTrans*)
thus *?case* **using** *Output* **by**(*force simp add*: *residualInject*)
next
case(*cCase* Ψ P M B φ Cs A_P Ψ_P K)
from ‹$\Psi \otimes \mathbf{1} \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity compositionSym AssertionStatEqSym*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$\bigwedge K.\ [\![\Psi \otimes \Psi_P \vdash M \leftrightarrow K;\ A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* K]\!] \Longrightarrow \Psi \rhd P \longmapsto (ROut\ K\ B)$›
have $\Psi \rhd P \longmapsto ROut\ K\ B$ **by** *force*
thus *?case* **using** ‹(φ, P) *mem* Cs› ‹$\Psi \vdash \varphi$› ‹*guarded* P› **by**(*rule Case*)
next
case(*cPar1* Ψ Ψ_Q P M *xvec* N P' A_Q Q A_P Ψ_P K)
from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K$
by(*metis statEqEnt Associativity Composition AssertionStatEqTrans Commutativity*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_Q$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$\bigwedge K.\ [\![(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K;\ A_P \sharp* (\Psi \otimes \Psi_Q);\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* K]\!] \Longrightarrow \Psi \otimes \Psi_Q \rhd P \longmapsto (ROut\ K\ ((\!|\nu* xvec|\!) N \prec' P'))$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec P'$ **by**(*force simp add*: *residualInject*)
thus *?case* **using** ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹*xvec* $\sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* K$› ‹$A_Q \sharp* xvec$› ‹$A_Q \sharp* N$› *Par1*[**where** $\alpha{=}K(\!|\nu* xvec|\!)\langle N\rangle$]
by(*auto simp add*: *residualInject*)
next
case(*cPar2* Ψ Ψ_P Q M *xvec* N Q' A_P P A_Q Ψ_Q K)
from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash M \leftrightarrow K$
by(*rule statEqEnt*[*OF AssertionStatEqSym*[*OF Associativity*]])
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* M$› ‹$A_Q \sharp* K$›
‹$\bigwedge K.\ [\![(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash M \leftrightarrow K;\ A_Q \sharp* (\Psi \otimes \Psi_P);\ A_Q \sharp* Q;\ A_Q \sharp* M;\ A_Q \sharp* K]\!] \Longrightarrow \Psi \otimes \Psi_P \rhd Q \longmapsto ROut\ K\ ((\!|\nu* xvec|\!) N \prec' Q')$›
have $\Psi \otimes \Psi_P \rhd Q \longmapsto ROut\ K\ ((\!|\nu* xvec|\!) N \prec' Q')$ **by** *force*
thus *?case* **using** ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› ‹*xvec* $\sharp* P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* K$› ‹$A_P \sharp* xvec$› ‹$A_P \sharp* N$› *Par2*[**where** $\alpha{=}K(\!|\nu* xvec|\!)\langle N\rangle$]
by(*auto simp add*: *residualInject*)
next
case(*cOpen* Ψ P M *xvec* *yvec* N P' x A_P Ψ_P)
hence $\Psi \rhd P \longmapsto K(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$ **by**(*force simp add*: *residualInject*)
with ‹$x \in supp\ N$› ‹$x \sharp \Psi$› ‹$x \sharp K$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› *Open* **show** *?case*
by(*auto simp add*: *residualInject*)
next
case(*cScope* Ψ P M *xvec* N P' x A_P Ψ_P)
hence $\Psi \rhd P \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec P'$ **by**(*force simp add*: *residualInject*)

with ‹$x \sharp \Psi$› ‹$x \sharp K$› ‹$x \sharp xvec$› ‹$x \sharp N$› *Scope*[**where** $\alpha{=}K(\!|\nu{*}xvec|\!)\langle N\rangle$] **show** *?case*
by(*auto simp add*: *residualInject*)
next
case(*cBang* Ψ P M B A_P Ψ_P K)
from ‹$\Psi \otimes \mathbf{1} \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity compositionSym AssertionStatEqSym*)
with ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K$›
‹$\bigwedge K.\ [\![\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K;\ A_P \sharp* \Psi;\ A_P \sharp* (P \parallel !P);\ A_P \sharp* M;\ A_P \sharp* K]\!] \Longrightarrow \Psi \rhd P \parallel !P \longmapsto ROut\ K\ B$›
have $\Psi \rhd P \parallel !P \longmapsto ROut\ K\ B$ **by** *force*
thus *?case* **using** ‹*guarded P*› **by**(*rule Bang*)
qed

lemma *parCasesSubject*[*consumes 7*, *case-names cPar1 cPar2 cComm1 cComm2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and R :: (*'a*, *'b*, *'c*) *psi*
and C :: *'d*::*fs-name*
and *yvec* :: *name list*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto \alpha \prec R$
and *bn* $\alpha \sharp* \Psi$
and *bn* $\alpha \sharp* P$
and *bn* $\alpha \sharp* Q$
and *bn* $\alpha \sharp* subject\ \alpha$
and *yvec* $\sharp* P$
and *yvec* $\sharp* Q$
and *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* \alpha;\ A_Q \sharp* C]\!] \Longrightarrow Prop\ \alpha\ (P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* Q;\ A_P \sharp* \alpha;\ A_P \sharp* C]\!] \Longrightarrow Prop\ \alpha\ (P \parallel Q')$
and *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;$
$distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ yvec \sharp* M;\ yvec \sharp* K;\ distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* xvec;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp* Q;\ A_Q \sharp* xvec;\ A_Q \sharp* Q';\ A_Q \sharp* C;$
$xvec \sharp* \Psi;\ xvec \sharp* \Psi_P;\ xvec \sharp* P;\ xvec \sharp* M;\ xvec \sharp* K;\ xvec \sharp* Q;\ xvec \sharp* \Psi_Q;\ xvec \sharp* C]\!] \Longrightarrow$
$Prop\ (\tau)\ ((\!|\nu{*}xvec|\!)(P' \parallel Q'))$
and *rComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q.$

$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; $yvec \sharp* M$; $yvec \sharp* K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C]\!] \Longrightarrow$
Prop (τ) $((\!|\nu * xvec|\!)(P' \parallel Q'))$

shows *Prop* α R
using *Trans* ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$›
proof(*induct rule*: *parCases*[**where** C=(C, *yvec*)])
case(*cPar1* $P'\ A_Q\ \Psi_Q$)
thus *?case* **by**(*rule-tac rPar1*) *auto*
next
case(*cPar2* $Q'\ A_P\ \Psi_P$)
thus *?case* **by**(*rule-tac rPar2*) *auto*
next
case(*cComm1* $\Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q$)
from ‹$A_P \sharp* (C, yvec)$› ‹$A_Q \sharp* (C, yvec)$› ‹$xvec \sharp* (C, yvec)$›
have $A_P \sharp* C$ **and** $A_Q \sharp* C$ **and** $xvec \sharp* C$ **and** $A_P \sharp* yvec$ **and** $A_Q \sharp* yvec$ **and** $xvec \sharp* yvec$
by *simp+*

have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *MeqK*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$ **by** *fact+*

from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$› *FrP* ‹*distinct* A_P› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$yvec \sharp* P$› ‹$A_P \sharp* \Psi$›
‹$A_P \sharp* A_Q$› ‹$A_P \sharp* yvec$› ‹$A_P \sharp* xvec$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$xvec \sharp* P$› ‹$A_P \sharp* \Psi_Q$›
obtain M' **where** *MeqM'*: $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $xvec \sharp* M'$ **and** $yvec \sharp* M'$ **and** $A_Q \sharp* M'$
by(*rule-tac* B=$xvec@yvec@A_Q$ **in** *obtainPrefix*) (*assumption* | *force*)+
from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$› *FrQ* ‹*distinct* A_Q› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$yvec \sharp* Q$› ‹$A_Q \sharp* \Psi$›
‹$A_P \sharp* A_Q$› ‹$A_Q \sharp* yvec$› ‹$A_Q \sharp* xvec$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹$xvec \sharp* Q$› ‹$A_Q \sharp* \Psi_P$› ‹$xvec \sharp* K$› ‹*distinct xvec*›
obtain K' **where** *KeqK'*: $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow K'$ **and** $xvec \sharp* K'$ **and** $yvec \sharp* K'$ **and** $A_P \sharp* K'$
by(*rule-tac* B=$xvec@yvec@A_P$ **in** *obtainPrefix*) (*assumption* | *force* | *metis freshChainSym*)+

from *MeqK KeqK'* **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans*)
with ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$› *FrP* ‹*distinct* A_P›

have $\Psi \otimes \Psi_Q \rhd P \longmapsto K'\langle\!|N|\!\rangle \prec P'$ **using** $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K'\rangle$
by(*rule-tac inputRenameSubject*) (*assumption* | *force*)+
moreover note *FrP* $\langle$*distinct* $A_P\rangle$
moreover from *MeqK MeqM'* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)
with $\langle\Psi \otimes \Psi_P \rhd Q \longmapsto K\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$
have $\Psi \otimes \Psi_P \rhd Q \longmapsto M'\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec Q'$ **using** $\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle A_Q \sharp* M'\rangle$
by(*rule-tac outputRenameSubject*) (*assumption* | *force*)+
moreover note *FrQ* $\langle$*distinct* $A_Q\rangle$
moreover from *MeqM' KeqK' MeqK* **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)
moreover note $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* K'\rangle$ $\langle A_P \sharp* N\rangle$ $\langle A_P \sharp* P'\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_P \sharp* xvec\rangle$ $\langle A_P \sharp* Q'\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* C\rangle$
$\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* M'\rangle$ $\langle A_Q \sharp* N\rangle$ $\langle A_Q \sharp* Q'\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_Q \sharp* C\rangle$
$\langle xvec \sharp* \Psi\rangle$ $\langle xvec \sharp* \Psi_P\rangle$ $\langle xvec \sharp* P\rangle$ $\langle xvec \sharp* M'\rangle$ $\langle xvec \sharp* K'\rangle$ $\langle xvec \sharp* Q\rangle$ $\langle xvec \sharp* \Psi_Q\rangle$ $\langle xvec \sharp* C\rangle$ $\langle yvec \sharp* M'\rangle$ $\langle yvec \sharp* K'\rangle$ $\langle$*distinct xvec*$\rangle$
ultimately show *?case*
by(*rule-tac rComm1*)
next
case(*cComm2* Ψ_Q M *xvec* N P' A_P Ψ_P K Q' A_Q)
from $\langle A_P \sharp* (C, yvec)\rangle$ $\langle A_Q \sharp* (C, yvec)\rangle$ $\langle xvec \sharp* (C, yvec)\rangle$
have $A_P \sharp* C$ **and** $A_Q \sharp* C$ **and** $xvec \sharp* C$ **and** $A_P \sharp* yvec$ **and** $A_Q \sharp* yvec$ **and** $xvec \sharp* yvec$
by *simp*+

have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** *MeqK*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$ **by** *fact*+

from $\langle\Psi \otimes \Psi_Q \rhd P \longmapsto M\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle yvec \sharp* P\rangle$ $\langle A_P \sharp* \Psi\rangle$
$\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* yvec\rangle$ $\langle A_P \sharp* xvec\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle xvec \sharp* P\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle xvec \sharp* M\rangle$ $\langle$*distinct xvec*$\rangle$
obtain M' **where** *MeqM'*: $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $xvec \sharp* M'$ **and** $yvec \sharp* M'$ **and** $A_Q \sharp* M'$
by(*rule-tac* B=*xvec*@*yvec*@A_Q **in** *obtainPrefix*) (*assumption* | *force*)+
from $\langle\Psi \otimes \Psi_P \rhd Q \longmapsto K\langle\!|N|\!\rangle \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle yvec \sharp* Q\rangle$ $\langle A_Q \sharp* \Psi\rangle$
$\langle A_P \sharp* A_Q\rangle$ $\langle A_Q \sharp* yvec\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle xvec \sharp* Q\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$
obtain K' **where** *KeqK'*: $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow K'$ **and** $xvec \sharp* K'$ **and** $yvec \sharp* K'$ **and** $A_P \sharp* K'$
by(*rule-tac* B=*xvec*@*yvec*@A_P **in** *obtainPrefix*) (*assumption* | *force* | *metis freshChainSym*)+

from *MeqK KeqK′* **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans*)
with $\langle \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$
have $\Psi \otimes \Psi_Q \rhd P \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **using** $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K'\rangle$
by(*rule-tac outputRenameSubject*) (*assumption* | *force*)+
moreover note *FrP* $\langle$*distinct* $A_P\rangle$
moreover from *MeqK MeqM′* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)
with $\langle \Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$
have $\Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q'$ **using** $\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle A_Q \sharp* M'\rangle$
by(*rule-tac inputRenameSubject*) (*assumption* | *force*)+
moreover note *FrQ* $\langle$*distinct* $A_Q\rangle$
moreover from *MeqM′ KeqK′ MeqK* **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)
moreover note $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* K'\rangle$ $\langle A_P \sharp* N\rangle$ $\langle A_P \sharp* P'\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_P \sharp* xvec\rangle$ $\langle A_P \sharp* Q'\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* C\rangle$
$\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* M'\rangle$ $\langle A_Q \sharp* N\rangle$ $\langle A_Q \sharp* Q'\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_Q \sharp* C\rangle$
$\langle xvec \sharp* \Psi\rangle$ $\langle xvec \sharp* \Psi_P\rangle$ $\langle xvec \sharp* P\rangle$ $\langle xvec \sharp* M'\rangle$ $\langle xvec \sharp* K'\rangle$ $\langle xvec \sharp* Q\rangle$ $\langle xvec \sharp* \Psi_Q\rangle$ $\langle xvec \sharp* C\rangle$ $\langle yvec \sharp* M'\rangle$ $\langle yvec \sharp* K'\rangle$ $\langle$*distinct xvec*$\rangle$
ultimately show *?case*
by(*rule-tac rComm2*)
qed

lemma *inputCases*[*consumes 1*, *case-names cInput*]:
fixes Ψ :: *'b*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes *Trans*: $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P'$
and *rInput*: $\bigwedge K\ Tvec.\ [\![\Psi \vdash M \leftrightarrow K;\ set\ xvec \subseteq supp\ N;\ length\ xvec = length\ Tvec;\ distinct\ xvec]\!] \Longrightarrow Prop\ (K(\!|N[xvec::=Tvec]|\!))\ (P[xvec::=Tvec])$

shows *Prop* α P'
proof −
{
fix *xvec N P*
assume *Trans*: $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P'$
and $xvec \sharp* \Psi$ **and** $xvec \sharp* M$ **and** $xvec \sharp* \alpha$ **and** $xvec \sharp* P'$ **and** *distinct xvec*
and *rInput*: $\bigwedge K\ Tvec.\ [\![\Psi \vdash M \leftrightarrow K;\ set\ xvec \subseteq supp\ N;\ length\ xvec = length$

Tvec; *distinct xvec*⟧ ⟹ *Prop* (*K*(⦇*N*[*xvec*::=*Tvec*]⦈) (*P*[*xvec*::=*Tvec*])

from *Trans* **have** *bn* α = []
apply −
by(*ind-cases* $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P'$) (*auto simp add*: *residualInject*)
from *Trans* **have** *distinct*(*bn* α) **by**(*auto dest*: *boundOutputDistinct*)
have *length*(*bn* α) = *residualLength*($\alpha \prec P'$) **by** *simp*
note *Trans*
moreover have *length xvec* = *inputLength*($M(\!|\lambda * xvec\ N|\!).P$) **by** *auto*
moreover note ‹*distinct xvec*›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover obtain *x*::*name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp M$ **and** $x \sharp xvec$ **and** $x \sharp \alpha$ **and** $x \sharp P'$ **and** $x \sharp N$
by(*generate-fresh name*) *auto*
ultimately have *Prop* α P' **using** ‹*bn* α = []› ‹*xvec* $\sharp*$ Ψ›‹*xvec* $\sharp*$ *M*› ‹*xvec* $\sharp*$ α› ‹*xvec* $\sharp*$ P'›
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - *C x*])
apply(*force simp add*: *residualInject psi.inject rInput*)
by(*fastforce simp add*: *residualInject psi.inject inputChainFresh*)+
}
note *Goal* = *this*
moreover obtain *p* :: *name prm* **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* M$ **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* \alpha$ **and** $(p \cdot xvec) \sharp* P'$ **and** *S*: *set p* ⊆ *set xvec* × *set*($p \cdot xvec$)
and *distinctPerm p*
by(*rule-tac xvec*=*xvec* **and** *c*=(Ψ, *M*, *N*, *P*, α, P') **in** *name-list-avoiding*) *auto*
from *Trans* ‹($p \cdot xvec$) $\sharp*$ *N*› ‹($p \cdot xvec$) $\sharp*$ *P*› *S* **have** $\Psi \rhd M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P) \longmapsto \alpha \prec P'$
by(*simp add*: *inputChainAlpha'*)
moreover {
fix *K Tvec*
assume $\Psi \vdash M \leftrightarrow K$
moreover assume *set*($p \cdot xvec$) ⊆ *supp*($p \cdot N$)
hence ($p \cdot$ *set*($p \cdot xvec$)) ⊆ ($p \cdot$ *supp*($p \cdot N$)) **by** *simp*
with ‹*distinctPerm p*› **have** *set xvec* ⊆ *supp N* **by**(*simp add*: *eqvts*)
moreover assume *length*($p \cdot xvec$) = *length*(*Tvec*::'*a list*)
hence *length xvec* = *length Tvec* **by** *simp*
moreover assume *distinct xvec*
ultimately have *Prop* (*K*(⦇*N*[*xvec*::=*Tvec*]⦈) (*P*[*xvec*::=*Tvec*])
by(*rule rInput*)
with ‹*length xvec* = *length Tvec*› *S* ‹*distinctPerm p*› ‹($p \cdot xvec$) $\sharp*$ *N*› ‹($p \cdot xvec$) $\sharp*$ *P*›
have *Prop* (*K*(⦇($p \cdot N$)[($p \cdot xvec$)::=*Tvec*]⦈) (($p \cdot P$)[($p \cdot xvec$)::=*Tvec*])
by(*simp add*: *renaming substTerm.renaming*)

}
moreover from *Trans* **have** *distinct xvec* **by**(*rule inputDistinct*)
hence *distinct*($p \cdot xvec$) **by** *simp*
ultimately show *?thesis* **using** ‹($p \cdot xvec$) $\sharp*$ Ψ› ‹($p \cdot xvec$) $\sharp*$ M› ‹($p \cdot xvec$) $\sharp*$ α› ‹($p \cdot xvec$) $\sharp*$ P'› ‹*distinct xvec*›
by(*rule-tac Goal*) *assumption+*
qed

lemma *outputCases*[*consumes 1*, *case-names cOutput*]:
fixes Ψ :: $'b$
and M :: $'a$
and N :: $'a$
and P :: ($'a$, $'b$, $'c$) *psi*
and α :: $'a$ *action*
and P' :: ($'a$, $'b$, $'c$) *psi*

assumes $\Psi \rhd M\langle N\rangle.P \longmapsto \alpha \prec P'$
and $\bigwedge K.\ \Psi \vdash M \leftrightarrow K \Longrightarrow Prop\ (K\langle N\rangle)\ P$

shows *Prop* α P'
using *assms*
by(*cases rule*: *semantics.cases*) (*auto simp add*: *residualInject psi.inject*)

lemma *caseCases*[*consumes 1*, *case-names cCase*]:
fixes Ψ :: $'b$
and *Cs* :: ($'c \times$ ($'a$, $'b$, $'c$) *psi*) *list*
and α :: $'a$ *action*
and P' :: ($'a$, $'b$, $'c$) *psi*

assumes *Trans*: $\Psi \rhd$ (*Cases Cs*) $\longmapsto$ *Rs*
and *rCase*: $\bigwedge\varphi\ P.\ [\![\Psi \rhd P \longmapsto Rs;\ (\varphi, P)\ mem\ Cs;\ \Psi \vdash \varphi;\ guarded\ P]\!] \Longrightarrow$ *Prop*

shows *Prop*
using *assms*
by(*cases rule*: *semantics.cases*) (*auto simp add*: *residualInject psi.inject*)

lemma *resCases*[*consumes 7*, *case-names cOpen cRes*]:
fixes Ψ :: $'b$
and x :: *name*
and P :: ($'a$, $'b$, $'c$) *psi*
and α :: $'a$ *action*
and P' :: ($'a$, $'b$, $'c$) *psi*
and C :: $'d$::*fs-name*

assumes *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec P'$
and $x \sharp \Psi$
and $x \sharp \alpha$
and $x \sharp P'$

and $bn\ \alpha \sharp* \Psi$
and $bn\ \alpha \sharp* P$
and $bn\ \alpha \sharp* subject\ \alpha$
and $rOpen$: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec ([(x,\ y)] \cdot P');\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$ $distinct\ xvec;\ distinct\ yvec;$
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$ $yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
and $rScope$: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \alpha \prec P']\!] \Longrightarrow Prop\ \alpha\ ((\!|\nu x|\!)P')$

shows $Prop\ \alpha\ P'$
proof −
from *Trans* **have** $distinct(bn\ \alpha)$ **by**(*auto dest*: *boundOutputDistinct*)
have $length(bn\ \alpha) = residualLength(\alpha \prec P')$ **by** *simp*
note *Trans*
moreover have $length\ [] = inputLength((\!|\nu x|\!)P)$ **and** $distinct\ []$
by(*auto simp add*: *inputLength-inputLength'-inputLength''.simps*)
moreover note $\langle length(bn\ \alpha) = residualLength(\alpha \prec P')\rangle\ \langle distinct(bn\ \alpha)\rangle$
moreover note $\langle length(bn\ \alpha) = residualLength(\alpha \prec P')\rangle\ \langle distinct(bn\ \alpha)\rangle$
moreover note $\langle length(bn\ \alpha) = residualLength(\alpha \prec P')\rangle\ \langle distinct(bn\ \alpha)\rangle$
moreover note $\langle length(bn\ \alpha) = residualLength(\alpha \prec P')\rangle\ \langle distinct(bn\ \alpha)\rangle$
ultimately show *?thesis* **using** $\langle bn\ \alpha \sharp* \Psi\rangle\ \langle bn\ \alpha \sharp* P\rangle\ \langle bn\ \alpha \sharp* subject\ \alpha\rangle\ \langle x \sharp \Psi\rangle\ \langle x \sharp \alpha\rangle\ \langle x \sharp P'\rangle\ \langle distinct(bn\ \alpha)\rangle$
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - - - *x x*])
apply(*auto simp add*: *psi.inject alpha abs-fresh residualInject boundOutputApp boundOutput.inject eqvts*)
apply(*subgoal-tac* $y \in supp\ Na$)
apply(*rule-tac rOpen*)
apply(*auto simp add*: *residualInject boundOutputApp*)
apply(*drule-tac pi*=[(*x*, *y*)] **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *calc-atm eqvts*)
by(*rule rScope*)
qed

lemma *resCases'*[*consumes 7*, *case-names cOpen cRes*]:
fixes Ψ :: *'b*
and x :: *name*
and P :: *('a, 'b, 'c) psi*
and α :: *'a action*
and P' :: *('a, 'b, 'c) psi*
and C :: *'d::fs-name*

assumes *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec P'$
and $x \sharp \Psi$
and $x \sharp \alpha$
and $x \sharp P'$

and $bn\ \alpha \sharp* \Psi$
and $bn\ \alpha \sharp* P$
and $bn\ \alpha \sharp* subject\ \alpha$
and $rOpen$: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P';\ y \in supp\ N;$
$x \sharp N;\ x \sharp P';\ x \neq y;\ y \sharp xvec;\ y \sharp yvec;\ y \sharp M;$ *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi;\ y \sharp \Psi;\ yvec \sharp* \Psi;\ xvec \sharp* P;\ y \sharp P;$ $yvec \sharp* P;\ xvec \sharp* M;\ y \sharp M;$
$yvec \sharp* M;\ xvec \sharp* yvec]\!] \Longrightarrow$
$Prop\ (M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle)\ P'$
and $rScope$: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \alpha \prec P']\!] \Longrightarrow Prop\ \alpha\ ((\!|\nu x|\!)P')$

shows $Prop\ \alpha\ P'$
proof −
from *Trans* **have** *distinct*(*bn* α) **by**(*auto dest*: *boundOutputDistinct*)
have *length*(*bn* α) = *residualLength*($\alpha \prec P'$) **by** *simp*
note *Trans*
moreover have *length* [] = *inputLength*($(\!|\nu x|\!)P$) **and** *distinct* []
by(*auto simp add*: *inputLength-inputLength'-inputLength''.simps*)
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
ultimately show *?thesis* **using** ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$› ‹*bn* $\alpha \sharp* subject\ \alpha$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp P'$› ‹*distinct*(*bn* α)›
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - - - $x\ x$])
apply(*auto simp add*: *psi.inject alpha abs-fresh residualInject boundOutputApp boundOutput.inject eqvts*)
apply(*subgoal-tac* $y \in supp\ Na$)
apply(*rule-tac rOpen*)
apply(*auto simp add*: *residualInject boundOutputApp*)
apply(*drule-tac pi*=[(*x*, *y*)] **in** *semantics.eqvt*)
apply(*simp add*: *calc-atm eqvts*)
apply(*drule-tac pi*=[(*x*, *y*)] **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *calc-atm eqvts*)
by(*rule rScope*)
qed

abbreviation
statImpJudge (- ↪ - [*80*, *80*] *80*)
where $\Psi \hookrightarrow \Psi' \equiv AssertionStatImp\ \Psi\ \Psi'$

lemma *statEqTransition*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rs* :: (*'a*, *'b*, *'c*) *residual*
and Ψ' :: *'b*

```
  assumes Ψ ▷ P ⟼ Rs
  and     Ψ ≃ Ψ'

  shows Ψ' ▷ P ⟼ Rs
using assms
proof(nominal-induct avoiding: Ψ' rule: semantics.strong-induct)
  case(cInput Ψ M K xvec N Tvec P Ψ')
  from ‹Ψ ≃ Ψ'› ‹Ψ ⊢ M ↔ K› have Ψ' ⊢ M ↔ K
    by(simp add: AssertionStatImp-def AssertionStatEq-def)
  thus ?case using ‹distinct xvec› ‹set xvec ⊆ supp N› ‹length xvec = length Tvec›
    by(rule Input)
next
  case(Output Ψ M K N P Ψ')
  from ‹Ψ ≃ Ψ'› ‹Ψ ⊢ M ↔ K› have Ψ' ⊢ M ↔ K
    by(simp add: AssertionStatImp-def AssertionStatEq-def)
  thus ?case by(rule semantics.Output)
next
  case(Case Ψ P Rs φ Cs Ψ')
  then have Ψ' ▷ P ⟼ Rs by(rule-tac Case)
  moreover note ‹(φ, P) mem Cs›
  moreover from ‹Ψ ≃ Ψ'› ‹Ψ ⊢ φ› have Ψ' ⊢ φ
    by(simp add: AssertionStatImp-def AssertionStatEq-def)
  ultimately show ?case using ‹guarded P› by(rule semantics.Case)
next
  case(cPar1 Ψ ΨQ P α P' xvec Q Ψ')
  thus ?case
    by(rule-tac Par1) (auto intro: Composition)
next
  case(cPar2 Ψ ΨP Q α Q' xvec P Ψ')
  thus ?case
    by(rule-tac Par2) (auto intro: Composition)
next
  case(cComm1 Ψ ΨQ P M N P' xvec ΨP Q K zvec Q' yvec Ψ')
  thus ?case
    by(clarsimp, rule-tac Comm1) (blast intro: Composition statEqEnt)+
next
  case(cComm2 Ψ ΨQ P M zvec N P' xvec ΨP Q K Q' yvec Ψ')
  thus ?case
    by(clarsimp, rule-tac Comm2) (blast intro: Composition statEqEnt)+
next
  case(cOpen Ψ P M xvec N P' x yvec Ψ')
  thus ?case by(force intro: Open)
next
  case(cScope Ψ P α P' x Ψ')
  thus ?case by(force intro: Scope)
next
  case(Bang Ψ P Rs Ψ')
  thus ?case by(force intro: semantics.Bang)
qed
```

lemma *actionPar1Dest'*:
  fixes α :: ('*a*::*fs-name*) *action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
  and β :: ('*a*::*fs-name*) *action*
  and Q :: ('*a*, '*b*, '*c*) *psi*
  and R :: ('*a*, '*b*, '*c*) *psi*

  assumes $\alpha \prec P = \beta \prec (Q \parallel R)$
  and *bn* $\alpha \sharp* R$
  and *bn* $\beta \sharp* R$

  obtains T **where** $P = T \parallel R$ **and** $\alpha \prec T = \beta \prec Q$
using *assms*
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*drule-tac boundOutputPar1Dest*) *auto*

lemma *actionPar2Dest'*:
  fixes α :: ('*a*::*fs-name*) *action*
  and P :: ('*a*, '*b*::*fs-name*, '*c*::*fs-name*) *psi*
  and β :: ('*a*::*fs-name*) *action*
  and Q :: ('*a*, '*b*, '*c*) *psi*
  and R :: ('*a*, '*b*, '*c*) *psi*

  assumes $\alpha \prec P = \beta \prec (Q \parallel R)$
  and *bn* $\alpha \sharp* Q$
  and *bn* $\beta \sharp* Q$

  obtains T **where** $P = Q \parallel T$ **and** $\alpha \prec T = \beta \prec R$
using *assms*
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*drule-tac boundOutputPar2Dest*) *auto*

lemma *expandNonTauFrame*:
  fixes Ψ :: '*b*
  and P :: ('*a*, '*b*, '*c*) *psi*
  and α :: '*a action*
  and P' :: ('*a*, '*b*, '*c*) *psi*
  and A_P :: *name list*
  and Ψ_P :: '*b*
  and C :: '*d*::*fs-name*
  and C' :: '*e*::*fs-name*

  assumes *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
  and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
  and *distinct* A_P
  and *bn* $\alpha \sharp* subject\ \alpha$

and $A_P \sharp* P$
and $A_P \sharp* \alpha$
and $A_P \sharp* C$
and $A_P \sharp* C'$
and $bn\ \alpha \sharp* P$
and $bn\ \alpha \sharp* C'$
and $\alpha \neq \tau$

obtains $p\ \Psi'\ A_P'\ \Psi_P'$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinctPerm p* **and**
extractFrame $P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$ **and**
$A_P' \sharp* C$ **and** $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$ **and** *distinct* A_P'
proof −
assume *A*: $\bigwedge p\ \Psi'\ \Psi_P'\ A_P'$.
$[\![set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha)); (p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$; *distinctPerm p*;
extractFrame $P' = \langle A_P', \Psi_P' \rangle$; $A_P' \sharp* P'$; $A_P' \sharp* \alpha$; $A_P' \sharp* (p \cdot \alpha)$;
$A_P' \sharp* C$; $(bn(p \cdot \alpha)) \sharp* C'$; $(bn(p \cdot \alpha)) \sharp* \alpha$; $(bn(p \cdot \alpha)) \sharp* P'$; *distinct* $A_P'$$]\!]$
$\Longrightarrow$ *thesis*

from *Trans* **have** *distinct*$(bn\ \alpha)$ **by**(*auto dest*: *boundOutputDistinct*)

with *Trans* ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$A_P \sharp* P$› ‹$A_P \sharp* \alpha$› **have** $A_P \sharp* P'$
by(*drule-tac freeFreshChainDerivative*) *auto*
{
fix X :: *name list*
and Y :: *'b list*
and Z :: *('a, 'b, 'c) psi list*

assume $bn\ \alpha \sharp* X$ **and** $bn\ \alpha \sharp* Y$ **and** $bn\ \alpha \sharp* Z$ **and** $A_P \sharp* X$ **and** $A_P \sharp* Y$ **and** $A_P \sharp* Z$

with *assms* **obtain** $p\ \Psi'\ A_P'\ \Psi_P'$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinctPerm p*
and *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$
and $A_P' \sharp* C$ **and** $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$
and $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$ **and** *distinct* A_P'
and $(bn(p \cdot \alpha)) \sharp* X$ **and** $(bn(p \cdot \alpha)) \sharp* Y$ **and** $(bn(p \cdot \alpha)) \sharp* Z$
using ‹$A_P \sharp* P'$› ‹*distinct*$(bn\ \alpha)$›
proof(*nominal-induct* $\Psi\ P\ Rs == \alpha \prec P'\ A_P\ \Psi_P$ *avoiding*: $C\ C'\ \alpha\ P'\ X\ Y\ Z$ *arbitrary*: *thesis rule*: *semanticsFrameInduct*)
case(*cAlpha* $\Psi\ P\ A_P\ \Psi_P\ p\ C\ C'\ \alpha\ P'\ X\ Y\ Z$)

then obtain $q\ \Psi'\ A_P'\ \Psi_P'$ **where** *Sq*: $set\ q \subseteq set(bn\ \alpha) \times set(bn(q \cdot \alpha))$ **and** *PeqP'*: $(q \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinctPerm q*
and *FrP'*: $extractFrame\ P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (q \cdot \alpha)$
and $A_P' \sharp* C$ **and** $(bn(q \cdot \alpha)) \sharp* C'$ **and** $(bn(q \cdot \alpha)) \sharp* \alpha$ **and** $(bn(q \cdot \alpha)) \sharp* P'$
and $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$ **and** *distinct* A_P'
and $(bn(q \cdot \alpha)) \sharp* X$ **and** $(bn(q \cdot \alpha)) \sharp* Y$ **and** $(bn(q \cdot \alpha)) \sharp* Z$
by *metis*

have *Sp*: $set\ p \subseteq set\ A_P \times set\ (p \cdot A_P)$ **by** *fact*

from *Sq* **have** $(p \cdot set\ q) \subseteq p \cdot (set(bn\ \alpha) \times set(bn(q \cdot \alpha)))$
by(*simp add*: *subsetClosed*)
hence $set(p \cdot q) \subseteq set(bn(p \cdot \alpha)) \times set(p \cdot bn(q \cdot \alpha))$
by(*simp add*: *eqvts*)
with $\langle A_P \sharp* \alpha \rangle$ $\langle (p \cdot A_P) \sharp* \alpha \rangle$ *Sp* **have** $set(p \cdot q) \subseteq set(bn\ \alpha) \times set(bn((p \cdot q) \cdot \alpha))$
by(*simp add*: *perm-compose bnEqvt*[*symmetric*])
moreover from *PeqP'* **have** $(p \cdot (q \cdot \Psi_P) \otimes \Psi') \simeq (p \cdot \Psi_P')$
by(*simp add*: *AssertionStatEqClosed*)
hence $((p \cdot q) \cdot p \cdot \Psi_P) \otimes (p \cdot \Psi') \simeq (p \cdot \Psi_P')$
apply(*subst perm-compose*[*symmetric*])
by(*simp add*: *eqvts*)
moreover from $\langle distinctPerm\ q \rangle$ **have** $distinctPerm\ (p \cdot q)$
by *simp*
moreover from $\langle (bn(q \cdot \alpha)) \sharp* C' \rangle$ **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot C')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* \alpha \rangle$ $\langle (p \cdot A_P) \sharp* \alpha \rangle$ $\langle A_P \sharp* C' \rangle$ $\langle (p \cdot A_P) \sharp* C' \rangle$ *Sp* **have** $bn((p \cdot q) \cdot \alpha) \sharp* C'$
by(*simp add*: *perm-compose bnEqvt*[*symmetric*])
moreover from *FrP'* **have** $(p \cdot extractFrame\ P') = p \cdot \langle A_P', \Psi_P' \rangle$ **by** *simp*
with $\langle A_P \sharp* P' \rangle$ $\langle (p \cdot A_P) \sharp* P' \rangle$ *Sp* **have** $extractFrame\ P' = \langle p \cdot A_P', p \cdot \Psi_P' \rangle$
by(*simp add*: *eqvts*)
moreover from $\langle A_P' \sharp* P' \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot P')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* P' \rangle$ $\langle (p \cdot A_P) \sharp* P' \rangle$ *Sp* **have** $(p \cdot A_P') \sharp* P'$ **by** *simp*
moreover from $\langle A_P' \sharp* \alpha \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* \alpha \rangle$ $\langle (p \cdot A_P) \sharp* \alpha \rangle$ *Sp* **have** $(p \cdot A_P') \sharp* \alpha$ **by** *simp*
moreover from $\langle A_P' \sharp* C \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot C)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* C \rangle$ $\langle (p \cdot A_P) \sharp* C \rangle$ *Sp* **have** $(p \cdot A_P') \sharp* C$ **by** *simp*
moreover from $\langle (bn(q \cdot \alpha)) \sharp* \alpha \rangle$ **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* \alpha \rangle$ $\langle (p \cdot A_P) \sharp* \alpha \rangle$ $\langle A_P \sharp* \alpha \rangle$ $\langle (p \cdot A_P) \sharp* \alpha \rangle$ *Sp* **have** $bn((p \cdot$

$q) \cdot \alpha) \sharp* \alpha$
by(*simp add*: *perm-compose eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* P'$› **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot P')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* \alpha$› ‹$(p \cdot A_P) \sharp* \alpha$› ‹$A_P \sharp* P'$› ‹$(p \cdot A_P) \sharp* P'$› *Sp* **have** $bn((p$
$\cdot q) \cdot \alpha) \sharp* P'$
by(*simp add*: *perm-compose eqvts*)
moreover from ‹$A_P' \sharp* (q \cdot \alpha)$› **have** $(p \cdot A_P') \sharp* (p \cdot q \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with *Sp* ‹$A_P \sharp* \alpha$› ‹$(p \cdot A_P) \sharp* \alpha$› **have** $(p \cdot A_P') \sharp* ((p \cdot q) \cdot \alpha)$
by(*simp add*: *perm-compose*)
moreover from ‹$A_P' \sharp* X$› **have** $(p \cdot A_P') \sharp* (p \cdot X)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* X$› ‹$(p \cdot A_P) \sharp* X$› *Sp* **have** $(p \cdot A_P') \sharp* X$ **by** *simp*
moreover from ‹$A_P' \sharp* Y$› **have** $(p \cdot A_P') \sharp* (p \cdot Y)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* Y$› ‹$(p \cdot A_P) \sharp* Y$› *Sp* **have** $(p \cdot A_P') \sharp* Y$ **by** *simp*
moreover from ‹$A_P' \sharp* Z$› **have** $(p \cdot A_P') \sharp* (p \cdot Z)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* Z$› ‹$(p \cdot A_P) \sharp* Z$› *Sp* **have** $(p \cdot A_P') \sharp* Z$ **by** *simp*
moreover from ‹$(bn(q \cdot \alpha)) \sharp* X$› **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot X)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* \alpha$› ‹$(p \cdot A_P) \sharp* \alpha$› ‹$A_P \sharp* X$› ‹$(p \cdot A_P) \sharp* X$› *Sp* **have** $bn((p \cdot$
$q) \cdot \alpha) \sharp* X$
by(*simp add*: *perm-compose eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* Y$› **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot Y)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* \alpha$› ‹$(p \cdot A_P) \sharp* \alpha$› ‹$A_P \sharp* Y$› ‹$(p \cdot A_P) \sharp* Y$› *Sp* **have** $bn((p \cdot$
$q) \cdot \alpha) \sharp* Y$
by(*simp add*: *perm-compose eqvts*)
moreover from ‹$(bn(q \cdot \alpha)) \sharp* Z$› **have** $(p \cdot bn(q \cdot \alpha)) \sharp* (p \cdot Z)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_P \sharp* \alpha$› ‹$(p \cdot A_P) \sharp* \alpha$› ‹$A_P \sharp* Z$› ‹$(p \cdot A_P) \sharp* Z$› *Sp* **have** $bn((p \cdot$
$q) \cdot \alpha) \sharp* Z$
by(*simp add*: *perm-compose eqvts*)
moreover from ‹*distinct* A_P'› **have** $distinct(p \cdot A_P')$ **by** *simp*
ultimately show *?case*
by(*rule-tac cAlpha*)
next
case(*cInput* Ψ M K *xvec* N *Tvec* P C C' α P' X Y Z)
moreover obtain A_P Ψ_P **where** $extractFrame(P[xvec::=Tvec]) = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and $A_P \sharp* (C, P[xvec::=Tvec], \alpha, P', X, Y, Z, N)$
by(*rule freshFrame*)
moreover have $\mathbf{1} \otimes \Psi_P \simeq \Psi_P$
by(*blast intro*: *Identity Commutativity AssertionStatEqTrans*)
ultimately show *?case*
by(*rule-tac cInput*) (*assumption* | *simp add*: *residualInject*)+
next

case(*cOutput* Ψ M K N P C C' α P' X Y Z)
moreover obtain A_P Ψ_P **where** *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P
and $A_P \sharp* (C, C', P, \alpha, N, P', X, Y, Z)$
by(*rule freshFrame*)
moreover have $\mathbf{1} \otimes \Psi_P \simeq \Psi_P$
by(*blast intro*: *Identity Commutativity AssertionStatEqTrans*)
ultimately show *?case* **by**(*simp add*: *residualInject*)
next
case(*cCase* Ψ P φ *Cs* A_P Ψ_P C C' α P' X Y Z)
moreover from ‹*bn* $\alpha \sharp* (Cases\ Cs)$› ‹$(\varphi, P)$ *mem Cs*› **have** *bn* $\alpha \sharp* P$ **by**(*auto dest*: *memFreshChain*)
ultimately obtain p Ψ' A_P' Ψ_P' **where** S: *set* $p \subseteq$ *set*(*bn* α) $\times$ *set*(*bn*$(p \cdot \alpha)$)
and *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P'\rangle$
and *PeqP'*: $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinct* A_P'
and $A_P' \sharp* C$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$
and $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$
and *distinctPerm* p **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$
and $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* X$ **and** $(bn(p \cdot \alpha)) \sharp* Y$ **and** $(bn(p \cdot \alpha)) \sharp* Z$
by(*rule-tac cCase*) (*assumption* | *simp* (*no-asm-use*))+
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $(p \cdot \Psi_P) \simeq (p \cdot \mathbf{1})$
by(*simp add*: *AssertionStatEqClosed*)
hence $(p \cdot \Psi_P) \simeq \mathbf{1}$ **by**(*simp add*: *permBottom*)
with *PeqP'* **have** $(\mathbf{1} \otimes \Psi') \simeq \Psi_P'$
by(*metis Identity AssertionStatEqTrans composition' Commutativity Associativity AssertionStatEqSym*)
ultimately show *?case* **using** *cCase* ‹*bn* $\alpha \sharp* P$›
by(*rule-tac cCase*(*20*)) (*assumption* | *simp*)+
next
case(*cPar1* Ψ Ψ_Q P α P' A_Q Q A_P Ψ_P C C' α' PQ' X Y Z)
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
by *fact*+

note ‹*bn* $\alpha' \sharp* subject\ \alpha'$›
moreover from ‹*bn* $\alpha' \sharp* (P \parallel Q)$› **have** *bn* $\alpha' \sharp* P$ **and** *bn* $\alpha' \sharp* Q$ **by** *simp*+
moreover with *FrP FrQ* ‹$A_P \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› **have** *bn* $\alpha' \sharp* \Psi_P$ **and** *bn* $\alpha' \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+

moreover from ‹*bn* $\alpha' \sharp* X$› ‹$A_Q \sharp* \alpha'$› **have** *bn* $\alpha' \sharp* (X @ A_Q)$ **by** *simp*
moreover from ‹*bn* $\alpha' \sharp* Y$› ‹*bn* $\alpha' \sharp* \Psi_Q$› **have** *bn* $\alpha' \sharp* (\Psi_Q \# Y)$ **by** *simp*
moreover from ‹*bn* $\alpha' \sharp* Z$› ‹*bn* $\alpha' \sharp* Q$› **have** *bn* $\alpha' \sharp* (Q \# Z)$ **by** *simp*
moreover from ‹$A_P \sharp* X$› ‹$A_P \sharp* A_Q$› **have** $A_P \sharp* (X @ A_Q)$ **by** *simp*

moreover from ‹$A_P \sharp* Y$› ‹$A_P \sharp* \Psi_Q$› **have** $A_P \sharp* (\Psi_Q \# Y)$ **by** *force*
moreover from ‹$A_P \sharp* Z$› ‹$A_P \sharp* Q$› **have** $A_P \sharp* (Q \# Z)$ **by** *simp*
moreover from ‹$\alpha \prec (P' \parallel Q) = \alpha' \prec PQ'$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha' \sharp* Q$› ‹$bn\ \alpha \sharp* \alpha'$›
obtain P'' **where** A: $\alpha \prec P' = \alpha' \prec P''$ **and** $PQ' = P'' \parallel Q$
by(*metis actionPar1Dest'*)
moreover from ‹$A_P \sharp* PQ'$› ‹$PQ' = P'' \parallel Q$› **have** $A_P \sharp* P''$ **by** *simp*
ultimately obtain $p\ \Psi'\ A_P'\ \Psi_P'$ **where** S: *set* $p \subseteq$ *set*$(bn\ \alpha') \times$ *set* $(bn(p \cdot \alpha'))$ **and** *PeqP'*: $((p \cdot \Psi_P) \otimes \Psi') \simeq \Psi_P'$
and *distinctPerm* p **and** $(bn(p \cdot \alpha')) \sharp* C'$ **and** *FrP'*: *extractFrame* $P'' = \langle A_P', \Psi_P' \rangle$
and $A_P' \sharp* P''$ **and** $A_P' \sharp* \alpha'\ A_P' \sharp* (p \cdot \alpha')$ **and** $A_P' \sharp* C$
and $(bn(p \cdot \alpha')) \sharp* \alpha'$ **and** $(bn(p \cdot \alpha')) \sharp* P''$ **and** *distinct* A_P'
and $A_P' \sharp* (X \mathbin{@} A_Q)$ **and** $A_P' \sharp* (\Psi_Q \# Y)$
and $A_P' \sharp* (Q \# Z)$ **and** $(bn(p \cdot \alpha')) \sharp* (X \mathbin{@} A_Q)$ **and** $(bn(p \cdot \alpha')) \sharp* (\Psi_Q \# Y)$
and $(bn(p \cdot \alpha')) \sharp* (Q \# Z)$ **using** *cPar1*
by(*rule-tac cPar1*)

then have $A_P' \sharp* Q$ **and** $A_P' \sharp* Z$ **and** $A_P' \sharp* A_Q$ **and** $A_P' \sharp* X$ **and** $A_P' \sharp* \Psi_Q$ **and** $A_P' \sharp* Y$
and $(bn(p \cdot \alpha')) \sharp* A_Q$ **and** $(bn(p \cdot \alpha')) \sharp* X$ **and** $(bn(p \cdot \alpha')) \sharp* Y$ **and** $(bn(p \cdot \alpha')) \sharp* Z$ **and** $(bn(p \cdot \alpha')) \sharp* \Psi_Q$
and $(bn(p \cdot \alpha')) \sharp* Q$
by(*simp del*: *freshChainSimps*)+

from ‹$A_Q \sharp* PQ'$› ‹$PQ' = P'' \parallel Q$› ‹$A_P' \sharp* A_Q$› *FrP'* **have** $A_Q \sharp* \Psi_P'$
by(*force dest*: *extractFrameFreshChain*)
note S
moreover from *PeqP'* **have** $((p \cdot (\Psi_P \otimes \Psi_Q)) \otimes \Psi') \simeq \Psi_P' \otimes (p \cdot \Psi_Q)$
by(*simp add*: *eqvts*) (*metis Composition Associativity AssertionStatEqTrans AssertionStatEqSym Commutativity*)
with ‹$(bn(p \cdot \alpha')) \sharp* \Psi_Q$› ‹$bn\ \alpha' \sharp* \Psi_Q$› S **have** $((p \cdot (\Psi_P \otimes \Psi_Q)) \otimes \Psi') \simeq \Psi_P' \otimes \Psi_Q$
by *simp*
moreover from ‹$PQ' = P'' \parallel Q$› ‹$A_P' \sharp* A_Q$› ‹$A_P' \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P'$› ‹$A_Q \sharp* PQ'$› *FrP' FrQ* **have** *extractFrame* $PQ' = \langle (A_P' @ A_Q), \Psi_P' \otimes \Psi_Q \rangle$
by *simp*
moreover note ‹*distinctPerm* p› ‹$(bn(p \cdot \alpha')) \sharp* C'$›
moreover from ‹$A_P' \sharp* P''$› ‹$A_P' \sharp* Q$› ‹$PQ' = P'' \parallel Q$› **have** $A_P' \sharp* PQ'$ **by** *simp*
moreover note ‹$A_Q \sharp* PQ'$› ‹$A_P' \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› ‹$A_P' \sharp* C$› ‹$A_Q \sharp* C$› ‹$(bn(p \cdot \alpha')) \sharp* \alpha'$›
moreover from ‹$bn\ \alpha' \sharp* Q$› **have** $(bn(p \cdot \alpha')) \sharp* (p \cdot Q)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *bnEqvt*[*symmetric*])
with ‹$bn\ \alpha \sharp* Q$› ‹$(bn(p \cdot \alpha')) \sharp* Q$› S **have** $(bn(p \cdot \alpha')) \sharp* Q$ **by** *simp*
with ‹$(bn(p \cdot \alpha')) \sharp* P''$› ‹$PQ' = P'' \parallel Q$› **have** $(bn(p \cdot \alpha')) \sharp* PQ'$ **by** *simp*

moreover from ‹$A_P{}' \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› **have** $(A_P{}'@A_Q) \sharp* \alpha'$ **by** *simp*
moreover from ‹$A_P{}' \sharp* C$› ‹$A_Q \sharp* C$› **have** $(A_P{}'@A_Q) \sharp* C$ **by** *simp*
moreover from ‹$A_P{}' \sharp* X$› ‹$A_Q \sharp* X$› **have** $(A_P{}'@A_Q) \sharp* X$ **by** *simp*
moreover from ‹$A_P{}' \sharp* Y$› ‹$A_Q \sharp* Y$› **have** $(A_P{}'@A_Q) \sharp* Y$ **by** *simp*
moreover from ‹$A_P{}' \sharp* Z$› ‹$A_Q \sharp* Z$› **have** $(A_P{}'@A_Q) \sharp* Z$ **by** *simp*
moreover from ‹$A_P{}' \sharp* PQ'$› ‹$A_Q \sharp* PQ'$› **have** $(A_P{}'@A_Q) \sharp* PQ'$ **by** *simp*
moreover from ‹$A_P{}' \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› **have** $(A_P{}'@A_Q) \sharp* \alpha'$ **by** *simp*
moreover from ‹$A_Q \sharp* \alpha'$› **have** $(p \cdot A_Q) \sharp* (p \cdot \alpha')$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *bnEqvt*[*symmetric*])
with S ‹$A_Q \sharp* \alpha'$› ‹$bn(p \cdot \alpha') \sharp* A_Q$› **have** $A_Q \sharp* (p \cdot \alpha')$
by *simp*
with ‹$A_P{}' \sharp* (p \cdot \alpha')$› ‹$A_Q \sharp* \alpha'$› ‹$bn(p \cdot \alpha') \sharp* A_Q$› S **have** $(A_P{}'@A_Q) \sharp* (p \cdot \alpha')$
by *simp*
moreover from ‹$A_P{}' \sharp* A_Q$› ‹*distinct* $A_P{}'$› ‹*distinct* A_Q› **have** *distinct*$(A_P{}'@A_Q)$ **by** *auto*
moreover note ‹$(bn(p \cdot \alpha')) \sharp* X$› ‹$(bn(p \cdot \alpha')) \sharp* Y$› ‹$(bn(p \cdot \alpha')) \sharp* Z$›
ultimately show *?case* **using** *cPar1*
by *metis*
next
case(*cPar2* Ψ Ψ_P Q α Q' A_P P A_Q Ψ_Q C C' α' PQ' X Y Z)
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
by *fact+*

note ‹$bn\ \alpha' \sharp* subject\ \alpha'$›
moreover from ‹$bn\ \alpha' \sharp* (P \parallel Q)$› **have** $bn\ \alpha' \sharp* Q$ **and** $bn\ \alpha' \sharp* P$ **by** *simp+*
moreover with *FrP FrQ* ‹$A_P \sharp* \alpha'$› ‹$A_Q \sharp* \alpha'$› **have** $bn\ \alpha' \sharp* \Psi_P$ **and** $bn\ \alpha' \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+

moreover from ‹$bn\ \alpha' \sharp* X$› ‹$A_P \sharp* \alpha'$› **have** $bn\ \alpha' \sharp* (X@A_P)$ **by** *simp*
moreover from ‹$bn\ \alpha' \sharp* Y$› ‹$bn\ \alpha' \sharp* \Psi_P$› **have** $bn\ \alpha' \sharp* (\Psi_P \# Y)$ **by** *simp*
moreover from ‹$bn\ \alpha' \sharp* Z$› ‹$bn\ \alpha' \sharp* P$› **have** $bn\ \alpha' \sharp* (P \# Z)$ **by** *simp*
moreover from ‹$A_Q \sharp* X$› ‹$A_P \sharp* A_Q$› **have** $A_Q \sharp* (X@A_P)$ **by** *simp*
moreover from ‹$A_Q \sharp* Y$› ‹$A_Q \sharp* \Psi_P$› **have** $A_Q \sharp* (\Psi_P \# Y)$ **by** *force*
moreover from ‹$A_Q \sharp* Z$› ‹$A_Q \sharp* P$› **have** $A_Q \sharp* (P \# Z)$ **by** *simp*
moreover from ‹$\alpha \prec (P \parallel Q') = \alpha' \prec PQ'$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha' \sharp* P$› ‹$bn\ \alpha \sharp* \alpha'$›
obtain Q'' **where** A: $\alpha \prec Q' = \alpha' \prec Q''$ **and** $PQ' = P \parallel Q''$
by(*metis actionPar2Dest'*)
moreover from ‹$A_Q \sharp* PQ'$› ‹$PQ' = P \parallel Q''$› **have** $A_Q \sharp* Q''$ **by** *simp*
ultimately obtain $p\ \Psi'\ A_Q{}'\ \Psi_Q{}'$ **where** S: *set* $p \subseteq$ *set*$(bn\ \alpha') \times$ *set* $(bn(p \cdot \alpha'))$ **and** *QeqQ'*: $((p \cdot \Psi_Q) \otimes \Psi') \simeq \Psi_Q{}'$
and *distinctPerm* p **and** $(bn(p \cdot \alpha')) \sharp* C'$ **and** *FrQ'*: *extractFrame* $Q'' = \langle A_Q{}', \Psi_Q{}' \rangle$
and $A_Q{}' \sharp* Q''$ **and** $A_Q{}' \sharp* \alpha'\ A_Q{}' \sharp* (p \cdot \alpha')$ **and** $A_Q{}' \sharp* C$

and $(bn(p \cdot \alpha')) \sharp* \alpha'$ **and** $(bn(p \cdot \alpha')) \sharp* Q''$ **and** *distinct* $A_Q{}'$
and $A_Q{}' \sharp* (X \mathbin{@} A_P)$ **and** $A_Q{}' \sharp* (\Psi_P \# Y)$
and $A_Q{}' \sharp* (P \# Z)$ **and** $(bn(p \cdot \alpha')) \sharp* (X \mathbin{@} A_P)$ **and** $(bn(p \cdot \alpha')) \sharp* (\Psi_P \# Y)$
and $(bn(p \cdot \alpha')) \sharp* (P \# Z)$ **using** *cPar2*
by(*rule-tac cPar2*)

then have $A_Q{}' \sharp* P$ **and** $A_Q{}' \sharp* Z$ **and** $A_Q{}' \sharp* A_P$ **and** $A_Q{}' \sharp* X$ **and** $A_Q{}' \sharp* \Psi_P$ **and** $A_Q{}' \sharp* Y$
and $(bn(p \cdot \alpha')) \sharp* A_P$ **and** $(bn(p \cdot \alpha')) \sharp* X$ **and** $(bn(p \cdot \alpha')) \sharp* Y$ **and** $(bn(p \cdot \alpha')) \sharp* Z$ **and** $(bn(p \cdot \alpha')) \sharp* \Psi_P$
and $(bn(p \cdot \alpha')) \sharp* P$
by(*simp del*: *freshChainSimps*)+

from $\langle A_P \sharp* PQ' \rangle$ $\langle PQ' = P \parallel Q'' \rangle$ $\langle A_Q{}' \sharp* A_P \rangle$ *FrQ'* **have** $A_P \sharp* \Psi_Q{}'$
by(*force dest*: *extractFrameFreshChain*)
note *S*
moreover from *QeqQ'* **have** $((p \cdot (\Psi_P \otimes \Psi_Q)) \otimes \Psi') \simeq (p \cdot \Psi_P) \otimes \Psi_Q{}'$
by(*simp add*: *eqvts*) (*metis Composition Associativity AssertionStatEqTrans AssertionStatEqSym Commutativity*)
with $\langle (bn(p \cdot \alpha')) \sharp* \Psi_P \rangle$ $\langle bn\ \alpha' \sharp* \Psi_P \rangle$ *S* **have** $((p \cdot (\Psi_P \otimes \Psi_Q)) \otimes \Psi') \simeq \Psi_P \otimes \Psi_Q{}'$
by *simp*
moreover from $\langle PQ' = P \parallel Q'' \rangle$ $\langle A_Q{}' \sharp* A_P \rangle$ $\langle A_Q{}' \sharp* \Psi_P \rangle$ $\langle A_P \sharp* \Psi_Q{}' \rangle$ $\langle A_P \sharp* PQ' \rangle$ *FrQ' FrP* **have** *extractFrame* $PQ' = \langle (A_P @ A_Q{}'), \Psi_P \otimes \Psi_Q{}' \rangle$
by *simp*
moreover note $\langle distinctPerm\ p \rangle$ $\langle (bn(p \cdot \alpha')) \sharp* C' \rangle$
moreover from $\langle A_Q{}' \sharp* Q'' \rangle$ $\langle A_Q{}' \sharp* P \rangle$ $\langle PQ' = P \parallel Q'' \rangle$ **have** $A_Q{}' \sharp* PQ'$ **by** *simp*
moreover note $\langle A_Q \sharp* PQ' \rangle$ $\langle A_Q{}' \sharp* \alpha' \rangle$ $\langle A_Q \sharp* \alpha' \rangle$ $\langle A_Q{}' \sharp* C \rangle$ $\langle A_Q \sharp* C \rangle$ $\langle (bn(p \cdot \alpha')) \sharp* \alpha' \rangle$
moreover from $\langle bn\ \alpha' \sharp* Q \rangle$ **have** $(bn(p \cdot \alpha')) \sharp* (p \cdot Q)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *bnEqvt*[*symmetric*])
with $\langle bn\ \alpha \sharp* P \rangle$ $\langle (bn(p \cdot \alpha')) \sharp* P \rangle$ *S* **have** $(bn(p \cdot \alpha')) \sharp* P$ **by** *simp*
with $\langle (bn(p \cdot \alpha')) \sharp* Q'' \rangle$ $\langle PQ' = P \parallel Q'' \rangle$ **have** $(bn(p \cdot \alpha')) \sharp* PQ'$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* \alpha' \rangle$ $\langle A_P \sharp* \alpha' \rangle$ **have** $(A_P @ A_Q{}') \sharp* \alpha'$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* C \rangle$ $\langle A_P \sharp* C \rangle$ **have** $(A_P @ A_Q{}') \sharp* C$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* X \rangle$ $\langle A_P \sharp* X \rangle$ **have** $(A_P @ A_Q{}') \sharp* X$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* Y \rangle$ $\langle A_P \sharp* Y \rangle$ **have** $(A_P @ A_Q{}') \sharp* Y$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* Z \rangle$ $\langle A_P \sharp* Z \rangle$ **have** $(A_P @ A_Q{}') \sharp* Z$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* PQ' \rangle$ $\langle A_P \sharp* PQ' \rangle$ **have** $(A_P @ A_Q{}') \sharp* PQ'$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* \alpha' \rangle$ $\langle A_P \sharp* \alpha' \rangle$ **have** $(A_P @ A_Q{}') \sharp* \alpha'$ **by** *simp*
moreover from $\langle A_P \sharp* \alpha' \rangle$ **have** $(p \cdot A_P) \sharp* (p \cdot \alpha')$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *bnEqvt*[*symmetric*])
with *S* $\langle A_P \sharp* \alpha' \rangle$ $\langle bn(p \cdot \alpha') \sharp* A_P \rangle$ **have** $A_P \sharp* (p \cdot \alpha')$
by *simp*
with $\langle A_Q{}' \sharp* (p \cdot \alpha') \rangle$ $\langle A_P \sharp* \alpha' \rangle$ $\langle bn(p \cdot \alpha') \sharp* A_P \rangle$ *S* **have** $(A_P @ A_Q{}') \sharp* (p \cdot \alpha')$

by *simp*
moreover note ‹$(bn(p \cdot \alpha')) \sharp* X$› ‹$(bn(p \cdot \alpha')) \sharp* Y$› ‹$(bn(p \cdot \alpha')) \sharp* Z$›
moreover from ‹$A_Q' \sharp* A_P$› ‹*distinct* A_P› ‹*distinct* A_Q'› **have** *distinct*$(A_P @ A_Q')$ **by** *auto*
ultimately show *?case* **using** *cPar2*
by *metis*
next
case *cComm1*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cComm2*
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cOpen* Ψ P M *xvec1* *xvec2* N P' x A_P Ψ_P C C' α P'' X Y Z)
from ‹$M(\!|\nu*(xvec1 @ x \# xvec2)|\!)\langle N\rangle \prec P' = \alpha \prec P''$› ‹$x \sharp xvec1$› ‹$x \sharp xvec2$› ‹$x \sharp \alpha$› ‹$x \sharp P''$› ‹*distinct*$(bn\ \alpha)$› ‹$A_P \sharp* \alpha$› ‹$x \sharp \alpha$›
obtain *yvec1* y *yvec2* N' **where** *yvecEq*: $bn\ \alpha = yvec1 @ y \# yvec2$ **and** $P'eqP''$: $(\!|\nu*(xvec1 @ xvec2)|\!) N \prec' P' = (\!|\nu*(yvec1 @ yvec2)|\!)([(x, y)] \cdot N') \prec' ([(x, y)] \cdot P')$ **and** $A_P \sharp* N'$ **and** *Subj*: *subject* $\alpha = Some\ M$ **and** $x \sharp N'$ **and** αeq: $\alpha = M(\!|\nu*(yvec1 @ y \# yvec2)|\!)\langle N'\rangle$
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
apply(*rule boundOutputOpenDest*)
by *assumption auto*
note ‹$A_P \sharp* P$› ‹$A_P \sharp* M$›
moreover from *Subj* *yvecEq* ‹$bn\ \alpha \sharp* subject\ \alpha$› **have** *yvec1* $\sharp*$ M *yvec2* $\sharp*$ M **by** *simp+*
moreover from *yvecEq* ‹$A_P \sharp* \alpha$› **have** $A_P \sharp* (yvec1 @ yvec2)$ **by** *simp*
moreover note ‹$A_P \sharp* C$›
moreover from *yvecEq* ‹$bn\ \alpha \sharp* (\!|\nu x|\!) P$› ‹$x \sharp \alpha$› **have** $(yvec1 @ yvec2) \sharp* P$ **by** *simp*
moreover from *yvecEq* ‹$bn\ \alpha \sharp* C'$› ‹$bn\ \alpha \sharp* X$› ‹$bn\ \alpha \sharp* Y$› ‹$bn\ \alpha \sharp* Z$› ‹*distinct*$(bn\ \alpha)$› ‹$x \sharp \alpha$›
have $(yvec1 @ yvec2) \sharp* C'$ **and** $(yvec1 @ yvec2) \sharp* (x \# y \# X)$ **and** $(yvec1 @ yvec2) \sharp* Y$ **and** $(yvec1 @ yvec2) \sharp* Z$
by *simp+*
moreover from ‹$A_P \sharp* X$› ‹$x \sharp A_P$› ‹$A_P \sharp* \alpha$› *yvecEq* **have** $A_P \sharp* (x \# y \# X)$ **by** *simp*
moreover note ‹$A_P \sharp* Y$› ‹$A_P \sharp* Z$›
moreover from ‹$A_P \sharp* N'$› ‹$A_P \sharp* P''$› ‹$x \sharp A_P$› ‹$A_P \sharp* \alpha$› *yvecEq* **have** $A_P \sharp* ([(x, y)] \cdot N')$ **and** $A_P \sharp* ([(x, y)] \cdot P')$
by *simp+*
moreover from *yvecEq* ‹*distinct*$(bn\ \alpha)$› **have** *distinct*$(yvec1 @ yvec2)$ **by** *simp*
moreover from $P'eqP''$ **have** $M(\!|\nu*(xvec1 @ xvec2)|\!)\langle N\rangle \prec P' = M(\!|\nu*(yvec1 @ yvec2)|\!)\langle([(x, y)] \cdot N')\rangle \prec ([(x, y)] \cdot P')$
by(*simp add*: *residualInject*)
ultimately obtain p Ψ' A_P' Ψ_P' **where** S: *set* $p \subseteq$ *set* $(yvec1 @ yvec2) \times$ *set* $(p \cdot (yvec1 @ yvec2))$ **and** *PeqP'*: $((p \cdot \Psi_P) \otimes \Psi') \simeq \Psi_P'$
and *distinctPerm* p **and** $(p \cdot (yvec1 @ yvec2)) \sharp*$

C' **and** *FrP'*: *extractFrame*($[(x,\ y)] \cdot P''$) = $\langle A_P{}',\ \Psi_P{}' \rangle$
and $A_P{}' \sharp* ([(x,\ y)] \cdot P'')$ **and** $A_P{}' \sharp* ([(x,\ y)] \cdot N')$ **and** $A_P{}' \sharp* C$ **and** $(p \cdot (yvec1@yvec2)) \sharp* ([(x,\ y)] \cdot N')$ **and** $A_P{}' \sharp* M$ **and** $(p \cdot (yvec1@yvec2)) \sharp* (yvec1@yvec2)$ **and** $(p \cdot (yvec1@yvec2)) \sharp* M$ **and** *distinct* $A_P{}'$
and $(p \cdot (yvec1@yvec2)) \sharp* ([(x,\ y)] \cdot P'')$ **and** $(yvec1@yvec2) \sharp* A_P{}'$ **and** $(p \cdot (yvec1@yvec2)) \sharp* A_P{}'$
and $A_P{}' \sharp* (x\#y\#X)$ **and** $A_P{}' \sharp* Y$ **and** $A_P{}' \sharp* Z$ **and** $(p \cdot (yvec1@yvec2)) \sharp* (x\#y\#X)$
and $(p \cdot (yvec1@yvec2)) \sharp* Y$ **and** $(p \cdot (yvec1@yvec2)) \sharp* Z$ **using** ‹$A_P \sharp* C'$›
by(*rule-tac cOpen*(*4*)[**where** *bd*=$x\#y\#X$]) (*assumption* | *simp-all*)+

from ‹$A_P{}' \sharp* (x\#y\#X)$› **have** $x \sharp A_P{}'$ **and** $y \sharp A_P{}'$ **and** $A_P{}' \sharp* X$ **by** *simp*+
from ‹$(p \cdot (yvec1@yvec2)) \sharp* (x\#y\#X)$› **have** $x \sharp (p \cdot (yvec1@yvec2))$ **and** $y \sharp (p \cdot (yvec1@yvec2))$ **and** $(p \cdot (yvec1@yvec2)) \sharp* X$ **by** *simp*+

from ‹$x \sharp \alpha$› *yvecEq* **have** $x \sharp yvec1$ **and** $x \neq y$ **and** $x \sharp yvec2$ **by** *simp*+
from ‹*distinct*(*bn* α)› *yvecEq* **have** $yvec1 \sharp* yvec2$ **and** $y \sharp yvec1$ **and** $y \sharp yvec2$ **by** *simp*+
from ‹*bn* $\alpha \sharp* C'$› *yvecEq* **have** $yvec1 \sharp* C'$ **and** $y \sharp C'$ **and** $yvec2 \sharp* C'$ **by** *simp*+

from *S* ‹$x \sharp \alpha$› ‹$x \sharp p \cdot (yvec1@yvec2)$› *yvecEq* **have** $x \sharp p$ **by**(*rule-tac freshAlphaSwap*) (*assumption* | *simp*)+
from *S* ‹*distinct*(*bn* α)› ‹$y \sharp p \cdot (yvec1@yvec2)$› *yvecEq* **have** $y \sharp p$ **by**(*rule-tac freshAlphaSwap*) (*assumption* | *simp*)+

from *yvecEq S* ‹$x \sharp yvec1$› ‹$x \sharp yvec2$› ‹$y \sharp yvec1$› ‹$y \sharp yvec2$› ‹$x \sharp p \cdot (yvec1@yvec2)$› ‹$y \sharp p \cdot (yvec1@yvec2)$›
have *set* $((y,\ x)\#p) \subseteq set(bn\ \alpha) \times set(bn(((y,\ x)\#p) \cdot \alpha))$
apply(*simp add*: *bnEqvt*[*symmetric*])
by(*auto simp add*: *eqvts calc-atm*)

moreover from *PeqP'* **have** $([(y,\ x)] \cdot ((p \cdot \Psi_P) \otimes \Psi')) \simeq [(y,\ x)] \cdot \Psi_P{}'$
by(*simp add*: *AssertionStatEqClosed*)
hence $(((y,\ x)\#p) \cdot \Psi_P) \otimes ([(y,\ x)] \cdot \Psi') \simeq ([(y,\ x)] \cdot \Psi_P{}')$
by(*simp add*: *eqvts*)
moreover from ‹*distinctPerm p*› *S* ‹$x \neq y$› ‹$x \sharp p$› ‹$y \sharp p$› **have** *distinctPerm*$((y,\ x)\#p)$
by *simp*
moreover from *FrP'* **have** $([(x,\ y)] \cdot (extractFrame([(x,\ y)] \cdot P''))) = ([(x,\ y)] \cdot \langle A_P{}',\ \Psi_P{}' \rangle)$
by *simp*
with ‹$x \sharp A_P{}'$› ‹$y \sharp A_P{}'$› **have** *extractFrame* $P'' = \langle A_P{}',\ ([(y,\ x)] \cdot \Psi_P{}') \rangle$
by(*simp add*: *eqvts name-swap*)
moreover from ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp N'$› ‹$(p \cdot (yvec1@yvec2)) \sharp* ([(x,\ y)] \cdot N')$› **have** $([(y,\ x)] \cdot p \cdot (yvec1@yvec2)) \sharp* ([(y,\ x)] \cdot [(x,\ y)] \cdot N')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

hence $(((y,\ x)\#p) \cdot (yvec1@yvec2)) \sharp* N'$ **by**(*simp add*: *name-swap*)
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle x \sharp C\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp N'\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* N'$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *eqvts perm-compose calc-atm freshChainSimps*)
moreover from $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp N'\rangle$ $\langle (p \cdot (yvec1@yvec2)) \sharp* ([(x,\ y)] \cdot P'')\rangle$ **have** $([(y,\ x)] \cdot p \cdot (yvec1@yvec2)) \sharp* ([(y,\ x)] \cdot [(x,\ y)] \cdot P'')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
hence $(((y,\ x)\#p) \cdot (yvec1@yvec2)) \sharp* P''$ **by**(*simp add*: *name-swap*)
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp P''\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* P''$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)
moreover from $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp A_P'\rangle$ $\langle (p \cdot (yvec1@yvec2)) \sharp* A_P'\rangle$ **have** $(p \cdot (yvec1@x\#yvec2)) \sharp* A_P'$
by(*simp add*: *eqvts freshChainSimps*)
hence $([(y,\ x)] \cdot p \cdot (yvec1@x\#yvec2)) \sharp* ([(y,\ x)] \cdot A_P')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle x \sharp A_P'\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle y \sharp A_P'\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* A_P'$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)
moreover from $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp C'\rangle$ $\langle (p \cdot (yvec1@yvec2)) \sharp* C'\rangle$ **have** $(p \cdot (yvec1@x\#yvec2)) \sharp* C'$
by(*simp add*: *eqvts freshChainSimps*)
hence $([(y,\ x)] \cdot p \cdot (yvec1@x\#yvec2)) \sharp* ([(y,\ x)] \cdot C')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle x \sharp C'\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle y \sharp C'\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* C'$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)
moreover from $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp X\rangle$ $\langle (p \cdot (yvec1@yvec2)) \sharp* X\rangle$ **have** $(p \cdot (yvec1@x\#yvec2)) \sharp* X$
by(*simp add*: *eqvts freshChainSimps*)
hence $([(y,\ x)] \cdot p \cdot (yvec1@x\#yvec2)) \sharp* ([(y,\ x)] \cdot X)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle x \sharp X\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle bn\ \alpha \sharp* X\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* X$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)
moreover from $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle x \sharp Y\rangle$ $\langle (p \cdot (yvec1@yvec2)) \sharp* Y\rangle$ **have** $(p \cdot (yvec1@x\#yvec2)) \sharp* Y$
by(*simp add*: *eqvts freshChainSimps*)
hence $([(y,\ x)] \cdot p \cdot (yvec1@x\#yvec2)) \sharp* ([(y,\ x)] \cdot Y)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp yvec1\rangle$ $\langle x \sharp yvec2\rangle$ $\langle x \neq y\rangle$ $\langle x \sharp Y\rangle$ $\langle y \sharp yvec1\rangle$ $\langle y \sharp yvec2\rangle$ $\langle x \sharp p\rangle$ $\langle y \sharp p\rangle$ $\langle bn\ \alpha \sharp* Y\rangle$ *yvecEq*
have $bn(((y,\ x)\#p) \cdot \alpha) \sharp* Y$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)

moreover from ‹$x \sharp p$› ‹$y \sharp p$› ‹$x \sharp Z$› ‹$(p \cdot (yvec1@yvec2)) \sharp* Z$› **have** ($p \cdot (yvec1@x\#yvec2)) \sharp* Z$
by(*simp add*: *eqvts freshChainSimps*)
hence $([(y, x)] \cdot p \cdot (yvec1@x\#yvec2)) \sharp* ([(y, x)] \cdot Z)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp yvec1$› ‹$x \sharp yvec2$› ‹$x \neq y$› ‹$x \sharp Z$› ‹$y \sharp yvec1$› ‹$y \sharp yvec2$› ‹$x \sharp p$› ‹$y \sharp p$› ‹$bn\ \alpha \sharp* Z$› *yvecEq*
have $bn(((y, x)\#p) \cdot \alpha) \sharp* Z$ **by**(*simp add*: *bnEqvt*[*symmetric*]) (*simp add*: *perm-compose calc-atm eqvts freshChainSimps*)
moreover from ‹$(yvec1@yvec2) \sharp* A_P{}'$› ‹$y \sharp A_P{}'$› *yvecEq* **have** $bn\ \alpha \sharp* A_P{}'$
by *simp*
moreover from ‹$A_P{}' \sharp* ([(x, y)] \cdot N')$› **have** $([(x, y)] \cdot A_P{}') \sharp* ([(x, y)] \cdot [(x, y)] \cdot N')$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp A_P{}'$› ‹$y \sharp A_P{}'$› **have** $A_P{}' \sharp* N'$ **by** *simp*
with ‹$A_P{}' \sharp* M$› ‹$(yvec1@yvec2) \sharp* A_P{}'$› ‹$y \sharp A_P{}'$› αeq **have** $A_P{}' \sharp* \alpha$ **by** *simp*
moreover hence $(((y, x)\#p) \cdot A_P{}') \sharp* (((y, x)\#p) \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp A_P{}'$› ‹$y \sharp A_P{}'$› *S* ‹$(yvec1@yvec2) \sharp* A_P{}'$› ‹$(p \cdot (yvec1@yvec2)) \sharp* A_P{}'$›
have $A_P{}' \sharp* (((y, x)\#p) \cdot \alpha)$ **by**(*simp add*: *eqvts*)
moreover from ‹$A_P{}' \sharp* ([(x, y)] \cdot P'')$› **have** $([(x, y)] \cdot A_P{}') \sharp* ([(x, y)] \cdot [(x, y)] \cdot P'')$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp A_P{}'$› ‹$y \sharp A_P{}'$› **have** $A_P{}' \sharp* P''$ **by** *simp*
moreover from *yvecEq* αeq ‹$(p \cdot (yvec1@yvec2)) \sharp* (yvec1@yvec2)$› ‹$y \sharp p$› ‹$x \sharp \alpha$› *S* ‹$(p \cdot (yvec1@yvec2)) \sharp* M$›‹$(p \cdot (yvec1@yvec2)) \sharp* ([(x, y)] \cdot N')$› ‹$y \sharp yvec1$›‹$y \sharp yvec2$› ‹$x \sharp p$›
have $bn(((y, x)\#p) \cdot \alpha) \sharp* \alpha$
apply(*simp add*: *eqvts del*: *set-append*)
apply(*intro conjI*)
apply(*simp add*: *perm-compose eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose eqvts del*: *set-append*)
apply(*simp add*: *perm-compose eqvts swapStarFresh del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *swapStarFresh calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*subst pt-fresh-star-bij*[*symmetric*, *OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=[(*x*, *y*)]])
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)
apply(*subst pt-fresh-star-bij*[*symmetric*, *OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=[(*x*, *y*)]])
by(*simp add*: *perm-compose freshChainSimps*(*6*) *calc-atm eqvts del*: *set-append*)

moreover note ‹$A_P{'} \sharp* C$› ‹$A_P{'} \sharp* X$› ‹$A_P{'} \sharp* Y$› ‹$A_P{'} \sharp* Z$› ‹*distinct* $A_P{'}$›

ultimately show *?case*
by(*rule-tac cOpen*)
next
case(*cScope* Ψ P α P' x A_P Ψ_P C C' α' P'' X Y Z)
from ‹$\alpha \prec (\!|\nu x|\!) P' = \alpha' \prec P''$› ‹$x \sharp \alpha$› ‹$x \sharp \alpha'$›
obtain P''' **where** $\alpha \prec P' = \alpha' \prec P'''$ **and** $P'' = (\!|\nu x|\!) P'''$
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
by(*metis boundOutputScopeDest*)
then obtain p Ψ' $A_P{'}$ $\Psi_P{'}$ **where** S: *set* $p \subseteq$ *set*(*bn* α') $\times$ *set*(*bn*($p \cdot \alpha'$)) **and** *PeqP'*: $((p \cdot \Psi_P) \otimes \Psi') \simeq \Psi_P{'}$
and *distinctPerm* p **and** $bn(p \cdot \alpha') \sharp* C'$ **and** *FrP'*: *extractFrame* $P''' = \langle A_P{'}, \Psi_P{'} \rangle$
and $A_P{'} \sharp* P'''$ **and** $A_P{'} \sharp* \alpha'$ **and** $A_P{'} \sharp* (p \cdot \alpha')$ **and** $A_P{'} \sharp* C$ **and** *distinct* $A_P{'}$
and $bn(p \cdot \alpha') \sharp* P'''$ **and** $A_P{'} \sharp* (x\#X)$ **and** $A_P{'} \sharp* Y$ **and** $bn(p \cdot \alpha') \sharp* \alpha'$
and $A_P{'} \sharp* Z$ **and** $bn(p \cdot \alpha') \sharp* (x\#X)$ **and** $bn(p \cdot \alpha') \sharp* Y$
and $bn(p \cdot \alpha') \sharp* Z$ **using** *cScope*
by(*rule-tac cScope*) (*assumption* | *simp*)+
from ‹$A_P{'} \sharp* (x\#X)$› **have** $x \sharp A_P{'}$ **and** $A_P{'} \sharp* X$ **by** *simp*+
from ‹$bn(p \cdot \alpha') \sharp* (x\#X)$› **have** $x \sharp bn(p \cdot \alpha')$ **and** $bn(p \cdot \alpha') \sharp* X$ **by** *simp*+

note *S PeqP'* ‹*distinctPerm* p› ‹$bn(p \cdot \alpha') \sharp* C'$›
moreover from *FrP'* ‹$P'' = (\!|\nu x|\!) P'''$› **have** *extractFrame* $P'' = \langle (x\#A_P{'}), \Psi_P{'} \rangle$ **by** *simp*
moreover from ‹$A_P{'} \sharp* P'''$› ‹$P'' = (\!|\nu x|\!) P'''$› ‹$x \sharp A_P{'}$› **have** $(x\#A_P{'}) \sharp* P''$ **by**(*simp add*: *abs-fresh*)
moreover from ‹$A_P{'} \sharp* \alpha'$› ‹$A_P{'} \sharp* C$› ‹$x \sharp \alpha'$› ‹$x \sharp C$› **have** $(x\#A_P{'}) \sharp* \alpha'$ **and** $(x\#A_P{'}) \sharp* C$ **by** *simp*+
moreover note ‹$bn(p \cdot \alpha') \sharp* \alpha'$›
moreover from ‹$bn(p \cdot \alpha') \sharp* P'''$› ‹$P'' = (\!|\nu x|\!) P'''$› ‹$x \sharp bn(p \cdot \alpha')$› **have** $bn(p \cdot \alpha') \sharp* P''$ **by** *simp*
moreover from ‹$A_P{'} \sharp* \alpha'$› ‹$x \sharp \alpha'$› **have** $(x\#A_P{'}) \sharp* \alpha'$ **by** *simp*
moreover from ‹$A_P{'} \sharp* (p \cdot \alpha')$› ‹$x \sharp \alpha'$› S ‹$x \sharp bn(p \cdot \alpha')$› **have** $(x\#A_P{'}) \sharp* (p \cdot \alpha')$
by(*simp add*: *subjectEqvt*[*symmetric*] *bnEqvt*[*symmetric*] *objectEqvt*[*symmetric*] *freshChainSimps*)
moreover from ‹$A_P{'} \sharp* X$› ‹$x \sharp X$› **have** $(x\#A_P{'}) \sharp* X$ **by** *simp*+
moreover from ‹$A_P{'} \sharp* Y$› ‹$x \sharp Y$› **have** $(x\#A_P{'}) \sharp* Y$ **by** *simp*+
moreover from ‹$A_P{'} \sharp* Z$› ‹$x \sharp Z$› **have** $(x\#A_P{'}) \sharp* Z$ **by** *simp*+
moreover note ‹$bn(p \cdot \alpha') \sharp* X$› ‹$bn(p \cdot \alpha') \sharp* Y$› ‹$bn(p \cdot \alpha') \sharp* Z$›
moreover from ‹*distinct* $A_P{'}$› ‹$x \sharp A_P{'}$› **have** *distinct*$(x\#A_P{'})$ **by** *simp*
ultimately show *?case* **by**(*rule-tac cScope*)
next

case(*cBang* Ψ P A_P Ψ_P C C' α P' X Y Z)
then obtain p Ψ' A_P' Ψ_P' **where** S: *set* $p \subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$))
and *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$
and *PeqP'*: $(p \cdot (\Psi_P \otimes \mathbf{1})) \otimes \Psi' \simeq \Psi_P'$
and $A_P' \sharp* C$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* \alpha$ **and** $A_P' \sharp* (p \cdot \alpha)$
and $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$ **and** *distinct* A_P'
and *distinctPerm* p **and** (*bn*($p \cdot \alpha$)) $\sharp* \alpha$ **and** (*bn*($p \cdot \alpha$)) $\sharp* P'$
and (*bn*($p \cdot \alpha$)) $\sharp* C'$ **and** (*bn*($p \cdot \alpha$)) $\sharp* X$ **and** (*bn*($p \cdot \alpha$)) $\sharp* Y$ **and** (*bn*($p \cdot \alpha$)) $\sharp* Z$
by(*rule-tac cBang*) (*assumption* | *simp* (*no-asm-use*))+
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $(p \cdot \Psi_P) \simeq (p \cdot \mathbf{1})$
by(*simp add*: *AssertionStatEqClosed*)
hence $(p \cdot \Psi_P) \simeq \mathbf{1}$ **by**(*simp add*: *permBottom*)
with *PeqP'* **have** $(\mathbf{1} \otimes \Psi') \simeq \Psi_P'$
by(*simp add*: *eqvts permBottom*) (*metis Identity AssertionStatEqTrans composition' Commutativity Associativity AssertionStatEqSym*)
ultimately show *?case* **using** *cBang*
apply(*rule-tac cBang*(*18*)) **by**(*assumption* | *simp*)+
qed

with *A* **have** *?thesis* **by** *blast*
}
moreover have *bn* α $\sharp*$ ([]::*name list*) **and** *bn* α $\sharp*$ ([]::'*b list*) **and** *bn* α $\sharp*$ ([]::('*a*, '*b*, '*c*) *psi list*)
and A_P $\sharp*$ ([]::*name list*) **and** A_P $\sharp*$ ([]::'*b list*) **and** A_P $\sharp*$ ([]::('*a*, '*b*, '*c*) *psi list*)
by *simp*+
ultimately show *?thesis* **by** *blast*
qed

lemma *expandTauFrame*:
fixes Ψ :: '*b*
and P :: ('*a*, '*b*,'*c*) *psi*
and P' :: ('*a*, '*b*, '*c*) *psi*
and A_P :: *name list*
and Ψ_P :: '*b*
and C :: '*d*::*fs-name*

assumes $\Psi \rhd P \longmapsto \tau \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and $A_P \sharp* P$
and $A_P \sharp* C$

obtains Ψ' A_P' Ψ_P' **where** *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** $A_P' \sharp* C$ **and** $A_P' \sharp* P'$ **and** *distinct* A_P'

proof −
assume A: $\bigwedge A_P'\ \Psi_P'\ \Psi'$.
⟦*extractFrame* $P' = \langle A_P', \Psi_P'\rangle$; $\Psi_P \otimes \Psi' \simeq \Psi_P'$; $A_P' \sharp* C$; $A_P' \sharp* P'$; *distinct* A_P'⟧
$\Longrightarrow$ *thesis*

from $\langle \Psi \rhd P \longmapsto \tau \prec P' \rangle$ $\langle A_P \sharp* P \rangle$ **have** $A_P \sharp* P'$ **by**(*rule tauFreshChainDerivative*)

{
fix X :: *name list*
and Y :: *'b list*
and Z :: (*'a*, *'b*, *'c*) *psi list*

assume $A_P \sharp* X$
and $A_P \sharp* Y$
and $A_P \sharp* Z$

with *assms* $\langle A_P \sharp* P' \rangle$ **obtain** $\Psi'\ A_P'\ \Psi_P'$ **where** *extractFrame* $P' = \langle A_P', \Psi_P'\rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** $A_P' \sharp* C$
and $A_P' \sharp* P'$ **and** $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$ **and** *distinct* A_P'
proof(*nominal-induct avoiding*: *C X Y Z arbitrary*: *thesis rule*: *tauFrameInduct*)
case(*cAlpha* Ψ P P' A_P Ψ_P p C X Y Z)
then obtain $\Psi'\ A_P'\ \Psi_P'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P'\rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** *distinct* A_P'
and $A_P' \sharp* C$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* Z$
by *metis*

have *S*: *set* $p \subseteq$ *set* $A_P \times$ *set*$(p \cdot A_P)$ **by** *fact*

from *FrP'* **have** $(p \cdot$ *extractFrame* $P') = p \cdot \langle A_P', \Psi_P'\rangle$ **by** *simp*
with $\langle A_P \sharp* P' \rangle$ $\langle (p \cdot A_P) \sharp* P' \rangle$ *S* **have** *extractFrame* $P' = \langle (p \cdot A_P'), (p \cdot \Psi_P')\rangle$ **by**(*simp add*: *eqvts*)
moreover from $\langle \Psi_P \otimes \Psi' \simeq \Psi_P' \rangle$ **have** $(p \cdot (\Psi_P \otimes \Psi')) \simeq (p \cdot \Psi_P')$ **by**(*rule AssertionStatEqClosed*)
hence $(p \cdot \Psi_P) \otimes (p \cdot \Psi') \simeq (p \cdot \Psi_P')$ **by**(*simp add*: *eqvts*)
moreover from $\langle A_P' \sharp* C \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot C)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* C \rangle$ $\langle (p \cdot A_P) \sharp* C \rangle$ *S* **have** $(p \cdot A_P') \sharp* C$ **by** *simp*
moreover from $\langle A_P' \sharp* P' \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot P')$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* P' \rangle$ $\langle (p \cdot A_P) \sharp* P' \rangle$ *S* **have** $(p \cdot A_P') \sharp* P'$ **by** *simp*
moreover from $\langle A_P' \sharp* X \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot X)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* X \rangle$ $\langle (p \cdot A_P) \sharp* X \rangle$ *S* **have** $(p \cdot A_P') \sharp* X$ **by** *simp*
moreover from $\langle A_P' \sharp* Y \rangle$ **have** $(p \cdot A_P') \sharp* (p \cdot Y)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

with ‹A_P ♯* Y› ‹$(p \cdot A_P)$ ♯* Y› S **have** $(p \cdot A_P')$ ♯* Y **by** *simp*
moreover from ‹A_P' ♯* Z› **have** $(p \cdot A_P')$ ♯* $(p \cdot Z)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_P ♯* Z› ‹$(p \cdot A_P)$ ♯* Z› S **have** $(p \cdot A_P')$ ♯* Z **by** *simp*
moreover from ‹*distinct* A_P'› **have** *distinct*$(p \cdot A_P')$ **by** *simp*
ultimately show *?case* **by**(*rule cAlpha*)
next
case(*cCase* Ψ P P' φ *Cs* A_P Ψ_P C B Y Z *thesis*)
then obtain Ψ' A_P' Ψ_P' **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$
and $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** *distinct* A_P'
and A_P' ♯* C **and** A_P' ♯* P'
and A_P' ♯* B **and** A_P' ♯* Y **and** A_P' ♯* Z
by(*rule-tac cCase*) (*assumption* | *simp* (*no-asm-use*))+
with ‹$\Psi_P \simeq \mathbf{1}$› ‹$\Psi_P \otimes \Psi' \simeq \Psi_P'$› **have** $\mathbf{1} \otimes \Psi' \simeq \Psi_P'$
by(*metis Identity AssertionStatEqTrans composition' Commutativity Associativity AssertionStatEqSym*)
thus *?case* **using** *FrP'* ‹A_P' ♯* P'› ‹A_P' ♯* C› ‹A_P' ♯* B› ‹A_P' ♯* Y› ‹A_P' ♯* Z› ‹*distinct* A_P'› **using** *cCase*
by *force*
next
case(*cPar1* Ψ Ψ_Q P P' A_Q Q A_P Ψ_P C X Y Z)
moreover from ‹A_P ♯* X› ‹A_P ♯* A_Q› ‹A_P ♯* Y› ‹A_P ♯* Ψ_Q› ‹A_P ♯* Q› ‹A_P ♯* Z›
have A_P ♯* $(X@A_Q)$ **and** A_P ♯* $(\Psi_Q \# Y)$ **and** A_P ♯* $(Q \# Z)$
by *simp+*
ultimately obtain Ψ' A_P' Ψ_P' **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** *distinct* A_P'
and $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** A_P ♯* P **and** A_P ♯* C **and** A_P' ♯* C **and** A_P' ♯* P'
and A_P' ♯* $(X@A_Q)$ **and** A_P' ♯* $(\Psi_Q \# Y)$ **and** A_P' ♯* $(Q \# Z)$
by *metis*

hence A_P' ♯* X **and** A_P' ♯* A_Q **and** A_P' ♯* Y **and** A_P' ♯* Ψ_Q **and** A_P' ♯* Q **and** A_P' ♯* Z
by *simp+*

from ‹A_P' ♯* A_Q› ‹A_Q ♯* P'› *FrP'* **have** A_Q ♯* Ψ_P' **by**(*force dest*: *extractFrameFreshChain*)
with ‹A_P' ♯* Ψ_Q› ‹A_P' ♯* A_Q› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› *FrP'*
have *extractFrame*$(P' \parallel Q) = \langle (A_P'@A_Q), \Psi_P' \otimes \Psi_Q \rangle$ **by** *simp*

moreover from ‹$\Psi_P \otimes \Psi' \simeq \Psi_P'$›**have** $(\Psi_P \otimes \Psi_Q) \otimes \Psi' \simeq \Psi_P' \otimes \Psi_Q$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)

moreover from ‹A_P' ♯* C› ‹A_Q ♯* C› **have** $(A_P'@A_Q)$ ♯* C **by** *simp*
moreover from ‹A_P' ♯* P'› ‹A_Q ♯* P'› ‹A_P' ♯* Q› ‹A_Q ♯* Q› **have** $(A_P'@A_Q)$ ♯* $(P' \parallel Q)$ **by** *simp*

moreover from ‹$A_P{}' \sharp* X$› ‹$A_Q \sharp* X$› **have** $(A_P{}'@A_Q) \sharp* X$ **by** *simp*
moreover from ‹$A_P{}' \sharp* Y$› ‹$A_Q \sharp* Y$› **have** $(A_P{}'@A_Q) \sharp* Y$ **by** *simp*
moreover from ‹$A_P{}' \sharp* Z$› ‹$A_Q \sharp* Z$› **have** $(A_P{}'@A_Q) \sharp* Z$ **by** *simp*
moreover from ‹$A_P{}' \sharp* A_Q$› ‹*distinct* $A_P{}'$› ‹*distinct* A_Q› **have** *distinct*$(A_P{}'@A_Q)$ **by** *simp*
ultimately show *?case* **by**(*rule cPar1*)
next
case(*cPar2* Ψ Ψ_P Q Q' A_P P A_Q Ψ_Q C X Y Z)
moreover from ‹$A_Q \sharp* X$› ‹$A_P \sharp* A_Q$› ‹$A_Q \sharp* Y$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Z$›
have $A_Q \sharp* (X@A_P)$ **and** $A_Q \sharp* (\Psi_P \# Y)$ **and** $A_Q \sharp* (P \# Z)$
by(*simp add*: *freshChainSimps*)+
ultimately obtain Ψ' $A_Q{}'$ $\Psi_Q{}'$ **where** *FrQ'*: *extractFrame* $Q' = \langle A_Q{}', \Psi_Q{}' \rangle$ **and** *distinct* $A_Q{}'$
and $\Psi_Q \otimes \Psi' \simeq \Psi_Q{}'$**and** $A_Q{}' \sharp* C$ **and** $A_Q{}' \sharp* Q'$
and $A_Q{}' \sharp* (X@A_P)$ **and** $A_Q{}' \sharp* (\Psi_P \# Y)$ **and** $A_Q{}' \sharp* (P \# Z)$
by *metis*

hence $A_Q{}' \sharp* X$ **and** $A_Q{}' \sharp* A_P$ **and** $A_Q{}' \sharp* Y$ **and** $A_Q{}' \sharp* \Psi_P$ **and** $A_Q{}' \sharp* P$ **and** $A_Q{}' \sharp* Z$
by *simp*+

from ‹$A_Q{}' \sharp* A_P$› ‹$A_P \sharp* Q'$› *FrQ'* **have** $A_P \sharp* \Psi_Q{}'$ **by**(*force dest*: *extractFrameFreshChain*)
with ‹$A_Q{}' \sharp* \Psi_P$› ‹$A_Q{}' \sharp* A_P$› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› *FrQ'*
have *extractFrame*$(P \parallel Q') = \langle (A_P@A_Q{}'), \Psi_P \otimes \Psi_Q{}' \rangle$ **by** *simp*

moreover from ‹$\Psi_Q \otimes \Psi' \simeq \Psi_Q{}'$›**have** $(\Psi_P \otimes \Psi_Q) \otimes \Psi' \simeq \Psi_P \otimes \Psi_Q{}'$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
moreover from ‹$A_P \sharp* C$› ‹$A_Q{}' \sharp* C$› **have** $(A_P@A_Q{}') \sharp* C$ **by** *simp*
moreover from ‹$A_P \sharp* P$› ‹$A_Q{}' \sharp* P$› ‹$A_P \sharp* Q'$› ‹$A_Q{}' \sharp* Q'$› **have** $(A_P@A_Q{}') \sharp* (P \parallel Q')$ **by** *simp*
moreover from ‹$A_P \sharp* X$› ‹$A_Q{}' \sharp* X$› **have** $(A_P@A_Q{}') \sharp* X$ **by** *simp*
moreover from ‹$A_P \sharp* Y$› ‹$A_Q{}' \sharp* Y$› **have** $(A_P@A_Q{}') \sharp* Y$ **by** *simp*
moreover from ‹$A_P \sharp* Z$› ‹$A_Q{}' \sharp* Z$› **have** $(A_P@A_Q{}') \sharp* Z$ **by** *simp*
moreover from ‹$A_Q{}' \sharp* A_P$› ‹*distinct* A_P› ‹*distinct* $A_Q{}'$› **have** *distinct*$(A_P@A_Q{}')$ **by** *simp*
ultimately show *?case* **by**(*rule cPar2*)
next
case(*cComm1* Ψ Ψ_Q P M N P' A_P Ψ_P Q K *xvec* Q' A_Q C X Y Z)
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M\langle\!|N|\!\rangle \prec P'$ **and** *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto K\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec Q'$ **by** *fact*+
from *PTrans* ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹*distinct* A_P› ‹$A_P \sharp* P$› ‹$A_P \sharp* N$› ‹$A_P \sharp* M$›
‹$A_P \sharp* Q'$› ‹$A_P \sharp* C$› ‹$A_P \sharp* X$› ‹$A_P \sharp* Y$› ‹$A_P \sharp* Z$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* xvec$›
obtain Ψ' $A_P{}'$ $\Psi_P{}'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P{}', \Psi_P{}' \rangle$ **and** *PeqP'*:

$\Psi_P \otimes \Psi' \simeq \Psi_P{}'$

and $A_P{}' \sharp* Q'$ **and** $A_P{}' \sharp* C$ **and** $A_P{}' \sharp* X$ **and** $A_P{}' \sharp* Y$ **and** *distinct* $A_P{}'$

and $A_P{}' \sharp* Z$ **and** $A_P{}' \sharp* A_Q$ **and** $A_P{}' \sharp* xvec$ **and** $A_P{}' \sharp* P'$

by(*rule-tac* $C=(Q', C, X, Y, Z, A_Q, xvec)$ **and** $C'=(Q', C, X, Y, Z, A_Q, xvec)$ **in** *expandNonTauFrame*) *auto*

moreover from *QTrans* **have** *distinct xvec* **by**(*auto dest*: *boundOutputDistinct*)

from *QTrans* ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹*distinct* A_Q› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* xvec$› ‹$A_Q \sharp* K$› ‹$xvec \sharp* K$› ‹*distinct xvec*›

‹$A_P{}' \sharp* A_Q$› ‹$A_P{}' \sharp* xvec$› ‹$A_Q \sharp* Q$› ‹$xvec \sharp* Q$› ‹$A_Q \sharp* \Psi_P$› ‹$xvec \sharp* \Psi_P$› ‹$A_Q \sharp* N$›

‹$A_Q \sharp* C$› ‹$A_Q \sharp* X$› ‹$A_Q \sharp* Y$› ‹$A_Q \sharp* Z$› ‹$xvec \sharp* P$› ‹$xvec \sharp* C$› ‹$xvec \sharp* X$› ‹$xvec \sharp* Y$› ‹$xvec \sharp* Z$›

obtain $p\ \Psi''\ A_Q{}'\ \Psi_Q{}'$ **where** S: *set* $p \subseteq$ *set* $xvec \times$ *set*$(p \cdot xvec)$ **and** $QeqQ'$: $(p \cdot \Psi_Q) \otimes \Psi'' \simeq \Psi_Q{}'$ **and** FrQ': *extractFrame* $Q' = \langle A_Q', \Psi_Q'\rangle$

and $A_Q{}' \sharp* A_P{}'$ **and** $A_Q{}' \sharp* C$ **and** $A_Q{}' \sharp* X$ **and** $A_Q{}' \sharp* Y$ **and** $A_Q{}' \sharp* Z$ **and** $A_Q{}' \sharp* P$ **and** $A_Q{}' \sharp* N$ **and** *distinct* $A_Q{}'$

and $(p \cdot xvec) \sharp* A_P{}'$ **and** $(p \cdot xvec) \sharp* C$ **and** $(p \cdot xvec) \sharp* X$ **and** $(p \cdot xvec) \sharp* Y$ **and** $(p \cdot xvec) \sharp* P$

and $(p \cdot xvec) \sharp* Z$ **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* \Psi_P$ **and** $(p \cdot xvec) \sharp* A_Q'$**and** $(p \cdot xvec) \sharp* Q'$

and *distinctPerm* p **and** $A_Q{}' \sharp* xvec$ **and** $A_Q{}' \sharp* Q'$

by(*rule-tac* $C=(P, C, X, Y, Z, A_P', \Psi_P)$ **and** $C'=(P, C, X, Y, Z, A_P', \Psi_P)$ **in** *expandNonTauFrame*) (*assumption* | *simp*)+

from *PTrans* ‹$A_Q{}' \sharp* P$› ‹$A_Q{}' \sharp* N$› ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* N$›

have $A_Q{}' \sharp* P'$ **and** $(p \cdot xvec) \sharp* P'$ **by**(*force dest*: *inputFreshChainDerivative*)+

with FrP' ‹$A_Q{}' \sharp* A_P{}'$› ‹$(p \cdot xvec) \sharp* A_P{}'$› **have** $A_Q{}' \sharp* \Psi_P{}'$ **and** $(p \cdot xvec) \sharp* \Psi_P{}'$ **by**(*force dest*: *extractFrameFreshChain*)+

from FrQ' ‹$A_Q{}' \sharp* A_P{}'$› ‹$A_P{}' \sharp* Q'$› ‹$(p \cdot xvec) \sharp* A_Q{}'$› ‹$(p \cdot xvec) \sharp* Q'$› **have** $A_P{}' \sharp* \Psi_Q{}'$ **and** $(p \cdot xvec) \sharp* \Psi_Q{}'$

by(*force dest*: *extractFrameFreshChain*)+

have $extractFrame((\!|\nu*xvec|\!)(P' \parallel Q')) = \langle((p \cdot xvec)@A_P{}'@A_Q{}'), (p \cdot \Psi_P{}') \otimes (p \cdot \Psi_Q{}')\rangle$

proof −

from FrP' FrQ' ‹$A_P{}' \sharp* \Psi_Q{}'$› ‹$A_Q{}' \sharp* A_P{}'$› ‹$A_Q{}' \sharp* \Psi_P{}'$› **have** $extractFrame(P' \parallel Q') = \langle(A_P{}'@A_Q{}'), \Psi_P{}' \otimes \Psi_Q{}'\rangle$

by *simp*

hence $extractFrame((\!|\nu*xvec|\!)(P' \parallel Q')) = \langle(xvec@A_P{}'@A_Q{}'), \Psi_P{}' \otimes \Psi_Q{}'\rangle$

by(*induct xvec*) *auto*

moreover from ‹$(p \cdot xvec) \sharp* \Psi_P{}'$› ‹$(p \cdot xvec) \sharp* \Psi_Q{}'$› S

have $(\!|\nu*xvec|\!)((\!|\nu*(A_P{}'@A_Q{}')|\!)(FAssert\ (\Psi_P{}' \otimes \Psi_Q{}'))) = (\!|\nu*(p \cdot xvec)|\!)(p \cdot (\!|\nu*(A_P{}'@A_Q{}')|\!)(FAssert(\Psi_P{}' \otimes \Psi_Q{}')))$

by(*rule-tac frameChainAlpha*) (*auto simp add*: *fresh-star-def frameResChainFresh*)

hence $(\!|\nu{*}xvec|\!)((\!|\nu{*}(A_P{'}@A_Q{'})|\!)(FAssert\ (\Psi_P{'} \otimes \Psi_Q{'}))) = (\!|\nu{*}(p \cdot xvec)|\!)((\!|\nu{*}(A_P{'}@A_Q{'})|\!)(FAssert((p \cdot \Psi_P{'}) \otimes (p \cdot \Psi_Q{'}))))$
using $\langle A_P{'} \sharp{*}\ xvec\rangle\ \langle (p \cdot xvec) \sharp{*}\ A_P{'}\rangle\ \langle A_Q{'} \sharp{*}\ xvec\rangle\ \langle (p \cdot xvec) \sharp{*}\ A_Q{'}\rangle\ S$
by(*fastforce simp add*: *eqvts*)
ultimately show *?thesis*
by(*simp add*: *frameChainAppend*)
qed

moreover have $(\Psi_P \otimes \Psi_Q) \otimes ((p \cdot \Psi{'}) \otimes (p \cdot \Psi{''})) \simeq (p \cdot \Psi_P{'}) \otimes (p \cdot \Psi_Q{'})$
proof −
have $(\Psi_P \otimes (p \cdot \Psi_Q)) \otimes (\Psi{'} \otimes \Psi{''}) \simeq (\Psi_P \otimes \Psi{'}) \otimes ((p \cdot \Psi_Q) \otimes \Psi{''})$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans*)
moreover from *PeqP' QeqQ'* **have** $(\Psi_P \otimes \Psi{'}) \otimes ((p \cdot \Psi_Q) \otimes \Psi{''}) \simeq \Psi_P{'} \otimes \Psi_Q{'}$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately have $(\Psi_P \otimes (p \cdot \Psi_Q)) \otimes (\Psi{'} \otimes \Psi{''}) \simeq \Psi_P{'} \otimes \Psi_Q{'}$
by(*metis AssertionStatEqTrans*)
hence $(p \cdot ((\Psi_P \otimes (p \cdot \Psi_Q)) \otimes (\Psi{'} \otimes \Psi{''}))) \simeq (p \cdot (\Psi_P{'} \otimes \Psi_Q{'}))$
by(*rule AssertionStatEqClosed*)
with $\langle xvec \sharp{*}\ \Psi_P\rangle\ \langle (p \cdot xvec) \sharp{*}\ \Psi_P\rangle\ S\ \langle distinctPerm\ p\rangle$ **show** *?thesis*
by(*simp add*: *eqvts*)
qed

moreover from $\langle (p \cdot xvec) \sharp{*}\ C\rangle\ \langle A_P{'} \sharp{*}\ C\rangle\ \langle A_Q{'} \sharp{*}\ C\rangle$ **have** $((p \cdot xvec)@A_P{'}@A_Q{'}) \sharp{*}\ C$ **by** *simp*
moreover from $\langle (p \cdot xvec) \sharp{*}\ X\rangle\ \langle A_P{'} \sharp{*}\ X\rangle\ \langle A_Q{'} \sharp{*}\ X\rangle$ **have** $((p \cdot xvec)@A_P{'}@A_Q{'}) \sharp{*}\ X$ **by** *simp*
moreover from $\langle (p \cdot xvec) \sharp{*}\ Y\rangle\ \langle A_P{'} \sharp{*}\ Y\rangle\ \langle A_Q{'} \sharp{*}\ Y\rangle$ **have** $((p \cdot xvec)@A_P{'}@A_Q{'}) \sharp{*}\ Y$ **by** *simp*
moreover from $\langle (p \cdot xvec) \sharp{*}\ Z\rangle\ \langle A_P{'} \sharp{*}\ Z\rangle\ \langle A_Q{'} \sharp{*}\ Z\rangle$ **have** $((p \cdot xvec)@A_P{'}@A_Q{'}) \sharp{*}\ Z$ **by** *simp*
moreover from $\langle (p \cdot xvec) \sharp{*}\ P{'}\rangle\ \langle (p \cdot xvec) \sharp{*}\ Q{'}\rangle\ \langle A_P{'} \sharp{*}\ P{'}\rangle\ \langle A_P{'} \sharp{*}\ Q{'}\rangle\ \langle A_Q{'} \sharp{*}\ P{'}\rangle\ \langle A_Q{'} \sharp{*}\ Q{'}\rangle$
have $((p \cdot xvec)@A_P{'}@A_Q{'}) \sharp{*}\ ((\!|\nu{*}xvec|\!)(P' \parallel Q'))$ **by**(*auto simp add*: *resChainFresh fresh-star-def*)
moreover from $\langle (p \cdot xvec) \sharp{*}\ A_P{'}\rangle\ \langle (p \cdot xvec) \sharp{*}\ A_Q{'}\rangle\ \langle A_Q{'} \sharp{*}\ A_P{'}\rangle\ \langle distinct\ xvec\rangle\ \langle distinct\ A_P{'}\rangle\ \langle distinct\ A_Q{'}\rangle$
have $distinct((p \cdot xvec)@A_P{'}@A_Q{'})$
by *auto* (*simp add*: *name-list-supp fresh-star-def fresh-def*)+

ultimately show *?case* **using** *cComm1*
by *metis*
next
case(*cComm2* $\Psi\ \Psi_Q\ P\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ Q\ K\ Q'\ A_Q\ C\ X\ Y\ Z$)
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$ **and** *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$ **by** *fact*+
from *PTrans* **have** *distinct xvec* **by**(*auto dest*: *boundOutputDistinct*)
from *PTrans* $\langle extractFrame\ P = \langle A_P, \Psi_P\rangle\rangle\ \langle distinct\ A_P\rangle\ \langle A_P \sharp{*}\ P\rangle\ \langle A_P \sharp{*}\ Q\rangle\ \langle xvec \sharp{*}\ Q\rangle\ \langle distinct\ xvec\rangle\ \langle xvec \sharp{*}\ M\rangle$

$\langle A_P \sharp* C\rangle$ $\langle A_P \sharp* X\rangle$ $\langle A_P \sharp* Y\rangle$ $\langle A_P \sharp* Z\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* xvec\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* N\rangle$
$\langle xvec \sharp* P\rangle$ $\langle xvec \sharp* C\rangle$ $\langle xvec \sharp* X\rangle$ $\langle xvec \sharp* Y\rangle$ $\langle xvec \sharp* Z\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle xvec \sharp* \Psi_Q\rangle$
obtain p Ψ' A_P' Ψ_P' **where** S: *set* $p \subseteq$ *set* $xvec \times$ *set*$(p \cdot xvec)$
and *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P'\rangle$ **and** *PeqP'*: $(p \cdot \Psi_P) \otimes \Psi' \simeq \Psi_P'$ **and** *distinct* A_P'
and $A_P' \sharp* C$ **and** $A_P' \sharp* X$ **and** $A_P' \sharp* Y$ **and** $A_P' \sharp* N$ **and** $A_P' \sharp* Q$ **and** $(p \cdot xvec) \sharp* Q$
and $A_P' \sharp* Z$ **and** $A_P' \sharp* A_Q$ **and** $A_P' \sharp* xvec$ **and** $A_P' \sharp* P'$ **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* \Psi_Q$
and $(p \cdot xvec) \sharp* A_P'$ **and** $(p \cdot xvec) \sharp* C$ **and** $(p \cdot xvec) \sharp* X$ **and** $(p \cdot xvec) \sharp* A_Q$
and $(p \cdot xvec) \sharp* Y$ **and** $(p \cdot xvec) \sharp* Z$ **and** $(p \cdot xvec) \sharp* P'$ **and** *distinctPerm p*
by(*rule-tac* C=$(C, X, Y, Z, A_Q, Q, \Psi_Q)$ **and** C'=$(C, X, Y, Z, A_Q, Q, \Psi_Q)$ **in** *expandNonTauFrame*) (*assumption* | *simp*)+

from *QTrans* $\langle$*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle\rangle$ $\langle$*distinct* $A_Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle (p \cdot xvec) \sharp* A_Q\rangle$
$\langle A_P' \sharp* A_Q\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* C\rangle$ $\langle A_Q \sharp* X\rangle$ $\langle A_Q \sharp* Y\rangle$ $\langle A_Q \sharp* Z\rangle$ $\langle A_Q \sharp* N\rangle$ $\langle A_Q \sharp* K\rangle$
obtain Ψ'' A_Q' Ψ_Q' **where** *QeqQ'*: $\Psi_Q \otimes \Psi'' \simeq \Psi_Q'$ **and** *FrQ'*: *extractFrame* $Q' = \langle A_Q', \Psi_Q'\rangle$ **and** *distinct* A_Q'
and $A_Q' \sharp* xvec$ **and** $A_Q' \sharp* Q'$ **and** $A_Q' \sharp* xvec$ **and** $A_Q' \sharp* P'$ **and** $A_Q' \sharp* (p \cdot xvec)$
and $A_Q' \sharp* A_P'$ **and** $A_Q' \sharp* C$ **and** $A_Q' \sharp* X$ **and** $A_Q' \sharp* Y$ **and** $A_Q' \sharp* Z$ **and** $A_Q' \sharp* P$
by(*rule-tac* C=$(P, C, P', X, Y, Z, A_P', xvec, (p \cdot xvec), \Psi_P)$ **and** C'=$(P, C, P', X, Y, Z, A_P', xvec, (p \cdot xvec), \Psi_P)$ **in** *expandNonTauFrame*) (*assumption* | *simp*)+

from *QTrans* $\langle A_P' \sharp* Q\rangle$ $\langle A_P' \sharp* N\rangle$ $\langle (p \cdot xvec) \sharp* Q\rangle$ $\langle (p \cdot xvec) \sharp* N\rangle$
have $A_P' \sharp* Q'$ **and** $(p \cdot xvec) \sharp* Q'$ **by**(*force dest*: *inputFreshChainDerivative*)+
with *FrQ'* $\langle A_Q' \sharp* A_P'\rangle$ $\langle A_Q' \sharp* (p \cdot xvec)\rangle$ **have** $A_P' \sharp* \Psi_Q'$ **and** $(p \cdot xvec) \sharp* \Psi_Q'$ **by**(*force dest*: *extractFrameFreshChain*)+
from *FrP'* $\langle A_Q' \sharp* A_P'\rangle$ $\langle A_Q' \sharp* P'\rangle$ $\langle (p \cdot xvec) \sharp* A_P'\rangle$ $\langle (p \cdot xvec) \sharp* P'\rangle$ **have** $A_Q' \sharp* \Psi_P'$ **and** $(p \cdot xvec) \sharp* \Psi_P'$
by(*force dest*: *extractFrameFreshChain*)+

have *extractFrame*$(\!|\nu*xvec|\!)(P' \parallel Q') = \langle((p \cdot xvec)@A_P'@A_Q'), (p \cdot \Psi_P') \otimes (p \cdot \Psi_Q')\rangle$
proof −
from *FrP' FrQ'* $\langle A_P' \sharp* \Psi_Q'\rangle$ $\langle A_Q' \sharp* A_P'\rangle$ $\langle A_Q' \sharp* \Psi_P'\rangle$ **have** *extractFrame*$(P' \parallel Q') = \langle(A_P'@A_Q'), \Psi_P' \otimes \Psi_Q'\rangle$
by *simp*
hence *extractFrame*$(\!|\nu*xvec|\!)(P' \parallel Q')) = \langle(xvec@A_P'@A_Q'), \Psi_P' \otimes \Psi_Q'\rangle$
by(*induct xvec*) *auto*

moreover from ‹$(p \cdot xvec) \sharp* \Psi_P{}'$› ‹$(p \cdot xvec) \sharp* \Psi_Q{}'$› S
have $(\!|\nu{*}xvec|\!)((\!|\nu{*}(A_P{}'@A_Q{}')|\!)(FAssert\ (\Psi_P{}' \otimes \Psi_Q{}'))) = (\!|\nu{*}(p \cdot xvec)|\!)(p \cdot (\!|\nu{*}(A_P{}'@A_Q{}')|\!)(FAssert(\Psi_P{}' \otimes \Psi_Q{}')))$
by(*rule-tac frameChainAlpha*) (*auto simp add*: *fresh-star-def frameResChainFresh*)
hence $(\!|\nu{*}xvec|\!)((\!|\nu{*}(A_P{}'@A_Q{}')|\!)(FAssert\ (\Psi_P{}' \otimes \Psi_Q{}'))) = (\!|\nu{*}(p \cdot xvec)|\!)((\!|\nu{*}(A_P{}'@A_Q{}')|\!)(FAssert((p \cdot \Psi_P{}') \otimes (p \cdot \Psi_Q{}'))))$
using ‹$A_P{}' \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_P{}'$› ‹$A_Q{}' \sharp* xvec$› ‹$A_Q{}' \sharp* (p \cdot xvec)$› S
by(*fastforce simp add*: *eqvts*)
ultimately show *?thesis*
by(*simp add*: *frameChainAppend*)
qed

moreover have $(\Psi_P \otimes \Psi_Q) \otimes ((p \cdot \Psi') \otimes (p \cdot \Psi'')) \simeq (p \cdot \Psi_P{}') \otimes (p \cdot \Psi_Q{}')$
proof −
have $((p \cdot \Psi_P) \otimes \Psi_Q) \otimes (\Psi' \otimes \Psi'') \simeq ((p \cdot \Psi_P) \otimes \Psi') \otimes (\Psi_Q \otimes \Psi'')$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans*)
moreover from *PeqP' QeqQ'* **have** $((p \cdot \Psi_P) \otimes \Psi') \otimes (\Psi_Q \otimes \Psi'') \simeq \Psi_P{}' \otimes \Psi_Q{}'$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately have $((p \cdot \Psi_P) \otimes \Psi_Q) \otimes (\Psi' \otimes \Psi'') \simeq \Psi_P{}' \otimes \Psi_Q{}'$
by(*metis AssertionStatEqTrans*)
hence $(p \cdot ((p \cdot \Psi_P) \otimes \Psi_Q) \otimes (\Psi' \otimes \Psi'')) \simeq (p \cdot (\Psi_P{}' \otimes \Psi_Q{}'))$
by(*rule AssertionStatEqClosed*)
with ‹*xvec* $\sharp* \Psi_Q$› ‹$(p \cdot xvec) \sharp* \Psi_Q$› S ‹*distinctPerm p*› **show** *?thesis*
by(*simp add*: *eqvts*)
qed

moreover from ‹$(p \cdot xvec) \sharp* C$› ‹$A_P{}' \sharp* C$› ‹$A_Q{}' \sharp* C$› **have** $((p \cdot xvec)@A_P{}'@A_Q{}') \sharp* C$ **by** *simp*
moreover from ‹$(p \cdot xvec) \sharp* X$› ‹$A_P{}' \sharp* X$› ‹$A_Q{}' \sharp* X$› **have** $((p \cdot xvec)@A_P{}'@A_Q{}') \sharp* X$ **by** *simp*
moreover from ‹$(p \cdot xvec) \sharp* Y$› ‹$A_P{}' \sharp* Y$› ‹$A_Q{}' \sharp* Y$› **have** $((p \cdot xvec)@A_P{}'@A_Q{}') \sharp* Y$ **by** *simp*
moreover from ‹$(p \cdot xvec) \sharp* Z$› ‹$A_P{}' \sharp* Z$› ‹$A_Q{}' \sharp* Z$› **have** $((p \cdot xvec)@A_P{}'@A_Q{}') \sharp* Z$ **by** *simp*
moreover from ‹$(p \cdot xvec) \sharp* P'$› ‹$(p \cdot xvec) \sharp* Q'$› ‹$A_P{}' \sharp* P'$› ‹$A_P{}' \sharp* Q'$› ‹$A_Q{}' \sharp* P'$› ‹$A_Q{}' \sharp* Q'$›
have $((p \cdot xvec)@A_P{}'@A_Q{}') \sharp* ((\!|\nu{*}xvec|\!)(P' \parallel Q'))$ **by**(*auto simp add*: *resChainFresh fresh-star-def*)
moreover from ‹$(p \cdot xvec) \sharp* A_P{}'$› ‹ $A_Q{}' \sharp* (p \cdot xvec)$› ‹$A_Q{}' \sharp* A_P{}'$› ‹*distinct xvec*› ‹*distinct* $A_P{}'$› ‹*distinct* $A_Q{}'$›
have *distinct*$((p \cdot xvec)@A_P{}'@A_Q{}')$
by *auto* (*simp add*: *name-list-supp fresh-star-def fresh-def*)+

ultimately show *?case* **using** *cComm2* **by** *metis*
next
case(*cScope* Ψ P P' x A_P Ψ_P C X Y Z)
then obtain Ψ' $A_P{}'$ $\Psi_P{}'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P{}', \Psi_P{}' \rangle$ **and**

distinct $A_P{}'$
and $\Psi_P \otimes \Psi' \simeq \Psi_P{}'$ **and** $A_P{}' \sharp* C$ **and** $A_P{}' \sharp* P'$
and $A_P{}' \sharp* (x\#X)$ **and** $A_P{}' \sharp* Y$ **and** $A_P{}' \sharp* Z$
by(*rule-tac cScope*(*4*)[**where** *ba*=$x\#X$]) *auto*
from ‹$A_P{}' \sharp* (x\#X)$› **have** $x \sharp A_P{}'$ **and** $A_P{}' \sharp* X$ **by** *simp*+
moreover from *FrP'* **have** *extractFrame*($(\!|\nu x|\!)P'$) = $\langle (x\#A_P{}'), \Psi_P{}' \rangle$ **by** *simp*
moreover note ‹$\Psi_P \otimes \Psi' \simeq \Psi_P{}'$›
moreover from ‹$x \sharp C$› ‹$A_P{}' \sharp* C$› **have** $(x\#A_P{}') \sharp* C$ **by** *simp*
moreover from ‹$A_P{}' \sharp* P'$› **have** $(x\#A_P{}') \sharp* ((\!|\nu x|\!)P')$ **by**(*simp add*: *abs-fresh fresh-star-def*)
moreover from ‹$x \sharp X$› ‹$A_P{}' \sharp* X$› **have** $(x\#A_P{}') \sharp* X$ **by** *simp*
moreover from ‹$x \sharp Y$› ‹$A_P{}' \sharp* Y$› **have** $(x\#A_P{}') \sharp* Y$ **by** *simp*
moreover from ‹$x \sharp Z$› ‹$A_P{}' \sharp* Z$› **have** $(x\#A_P{}') \sharp* Z$ **by** *simp*
moreover from ‹$x \sharp A_P{}'$› ‹*distinct* $A_P{}'$› **have** *distinct*($x\#A_P{}'$) **by** *simp*
ultimately show *?case* **by**(*rule-tac cScope*)
next
case(*cBang* Ψ P P' A_P Ψ_P C B Y Z)
then obtain Ψ' $A_P{}'$ $\Psi_P{}'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P{}', \Psi_P{}' \rangle$
and $(\Psi_P \otimes \mathbf{1}) \otimes \Psi' \simeq \Psi_P{}'$
and $A_P{}' \sharp* C$ **and** $A_P{}' \sharp* P'$ **and** *distinct* $A_P{}'$
and $A_P{}' \sharp* B$ **and** $A_P{}' \sharp* Y$ **and** $A_P{}' \sharp* Z$
by(*rule-tac cBang*) (*assumption* | *simp*)+
with ‹$\Psi_P \simeq \mathbf{1}$› ‹$(\Psi_P \otimes \mathbf{1}) \otimes \Psi' \simeq \Psi_P{}'$› **have** $\mathbf{1} \otimes \Psi' \simeq \Psi_P{}'$
by(*metis Identity AssertionStatEqTrans composition' Commutativity Associativity AssertionStatEqSym*)
thus *?case* **using** *FrP'* ‹$A_P{}' \sharp* P'$› ‹$A_P{}' \sharp* C$› ‹$A_P{}' \sharp* B$› ‹$A_P{}' \sharp* Y$› ‹$A_P{}' \sharp* Z$› ‹*distinct* $A_P{}'$›
by(*rule-tac cBang*)
qed
with *A* **have** *?thesis* **by** *simp*
}
moreover have $A_P \sharp* ([]::name\ list)$ **and** $A_P \sharp* ([]::'b\ list)$ **and** $A_P \sharp* ([]::('a, 'b, 'c)\ psi\ list)$ **by** *simp*+
ultimately show *?thesis* **by** *blast*
qed

lemma *expandFrame*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and C :: $'d$::*fs-name*
and C' :: $'e$::*fs-name*

assumes *Trans*: $\Psi \rhd P \longmapsto \alpha \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P

and $bn\ \alpha \sharp* subject\ \alpha$
and $distinct(bn\ \alpha)$
and $A_P \sharp* \alpha$
and $A_P \sharp* P$
and $A_P \sharp* C$
and $A_P \sharp* C'$
and $bn\ \alpha \sharp* P$
and $bn\ \alpha \sharp* C'$

obtains $p\ \Psi'\ {A_P}'\ {\Psi_P}'$ **where** $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $(p \cdot \Psi_P) \otimes \Psi' \simeq {\Psi_P}'$ **and** $distinctPerm\ p$ **and**
$extractFrame\ P' = \langle {A_P}', {\Psi_P}' \rangle$ **and** ${A_P}' \sharp* P'$ **and** ${A_P}' \sharp* \alpha$ **and** ${A_P}' \sharp* (p \cdot \alpha)$ **and**
${A_P}' \sharp* C$ **and** $(bn(p \cdot \alpha)) \sharp* C'$ **and** $(bn(p \cdot \alpha)) \sharp* \alpha$ **and** $(bn(p \cdot \alpha)) \sharp* P'$ **and** $distinct\ {A_P}'$
using *assms*
apply(*cases* α=τ)
by(*auto intro*: *expandTauFrame*[**where** C=C] *expandNonTauFrame*[**where** C=C **and** C'=C'])

abbreviation
$frameImpJudge\ (- \hookrightarrow_F - [80, 80]\ 80)$
where $F \hookrightarrow_F G \equiv FrameStatImp\ F\ G$

lemma *FrameStatEqImpCompose*:
fixes $F :: 'b\ frame$
and $G :: 'b\ frame$
and $H :: 'b\ frame$
and $I :: 'b\ frame$

assumes $F \simeq_F G$
and $G \hookrightarrow_F H$
and $H \simeq_F I$

shows $F \hookrightarrow_F I$
using *assms*
by(*auto simp add*: *FrameStatEq-def*) (*blast intro*: *FrameStatImpTrans*)

lemma *transferNonTauFrame*:
fixes $\Psi_F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ action$
and $P' :: ('a, 'b, 'c)\ psi$
and $A_F :: name\ list$
and $A_G :: name\ list$
and $\Psi_G :: 'b$

assumes $\Psi_F \rhd P \longmapsto \alpha \prec P'$
and $extractFrame\ P = \langle A_P, \Psi_P \rangle$

and $distinct\ A_P$
and $distinct(bn\ \alpha)$
and $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
and $A_F \sharp* P$
and $A_G \sharp* P$
and $A_F \sharp* subject\ \alpha$
and $A_G \sharp* subject\ \alpha$
and $A_P \sharp* A_F$
and $A_P \sharp* A_G$
and $A_P \sharp* \Psi_F$
and $A_P \sharp* \Psi_G$
and $\alpha \neq \tau$

shows $\Psi_G \rhd P \longmapsto \alpha \prec P'$
using *assms*
proof(*nominal-induct* $\Psi_F\ P\ Rs==\alpha \prec P'\ A_P\ \Psi_P$ *avoiding*: $\alpha\ P'\ \Psi_G\ A_F\ A_G$ *rule*: *semanticsFrameInduct*)
case(*cAlpha* $\Psi_F\ P\ A_P\ \Psi_P\ p\ \alpha\ P'\ \Psi_G\ A_F\ A_G$)
from ‹$\langle A_F, \Psi_F \otimes (p \cdot \Psi_P)\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes (p \cdot \Psi_P)\rangle$›
have $(p \cdot (\langle A_F, \Psi_F \otimes (p \cdot \Psi_P)\rangle)) \hookrightarrow_F (p \cdot (\langle A_G, \Psi_G \otimes (p \cdot \Psi_P)\rangle))$
by(*rule FrameStatImpClosed*)
with ‹$A_P \sharp* A_F$› ‹$(p \cdot A_P) \sharp* A_F$› ‹$A_P \sharp* \Psi_F$› ‹$(p \cdot A_P) \sharp* \Psi_F$› ‹$A_P \sharp* A_G$› ‹$(p \cdot A_P) \sharp* A_G$› ‹$A_P \sharp* \Psi_G$› ‹$(p \cdot A_P) \sharp* \Psi_G$›
‹*distinctPerm p*› ‹*set p* $\subseteq$ *set* $A_P \times$ *set* $(p \cdot A_P)$› **have** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*simp add*: *eqvts*)
with *cAlpha* **show** *?case* **by** *force*
next
case(*cInput* $\Psi_F\ M\ K\ xvec\ N\ Tvec\ P\ \alpha\ P'\ \Psi_G\ A_F\ A_G$)
from *cInput* **have** $A_F \sharp* K$ **and** $A_G \sharp* K$ **by**(*auto simp add*: *residualInject*)

from ‹$A_F \sharp* (M\langle\!|\lambda* xvec\ N|\!\rangle.P)$› ‹$A_G \sharp* (M\langle\!|\lambda* xvec\ N|\!\rangle.P)$› **have** $A_F \sharp* M$ **and** $A_G \sharp* M$ **by** *simp+*
from ‹$\Psi_F \vdash M \leftrightarrow K$›
have $\Psi_F \otimes \mathbf{1} \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
with ‹$A_F \sharp* M$› ‹$A_F \sharp* K$›
have $(\langle A_F, \Psi_F \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow K$
by(*force intro*: *frameImpI*)
with ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$›
have $(\langle A_G, \Psi_G \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow K$
by(*simp add*: *FrameStatEq-def FrameStatImp-def*)
with ‹$A_G \sharp* M$› ‹$A_G \sharp* K$›
have $\Psi_G \otimes \mathbf{1} \vdash M \leftrightarrow K$ **by**(*force dest*: *frameImpE*)
hence $\Psi_G \vdash M \leftrightarrow K$ **by**(*blast intro*: *statEqEnt Identity*)
thus *?case* **using** ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec* = *length Tvec*›
using *cInput Input*
by(*force simp add*: *residualInject*)
next

case(*cOutput* Ψ_F M K N P α P' Ψ_G A_F A_G)
from *cOutput* **have** $A_F \sharp* K$ **and** $A_G \sharp* K$ **by**(*auto simp add*: *residualInject*)

from ‹$A_F \sharp* (M\langle N\rangle.P)$› ‹$A_G \sharp* (M\langle N\rangle.P)$› **have** $A_F \sharp* M$ **and** $A_G \sharp* M$ **by** *simp+*
from ‹$\Psi_F \vdash M \leftrightarrow K$›
have $\Psi_F \otimes \mathbf{1} \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
with ‹$A_F \sharp* M$› ‹$A_F \sharp* K$›
have $(\langle A_F, \Psi_F \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow K$
by(*force intro*: *frameImpI*)
with ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$›
have $(\langle A_G, \Psi_G \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow K$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_G \sharp* M$› ‹$A_G \sharp* K$›
have $\Psi_G \otimes \mathbf{1} \vdash M \leftrightarrow K$ **by**(*force dest*: *frameImpE*)
hence $\Psi_G \vdash M \leftrightarrow K$ **by**(*blast intro*: *statEqEnt Identity*)
thus *?case* **using** *cOutput Output* **by**(*force simp add*: *residualInject*)
next
case(*cCase* Ψ_F P φ *Cs* A_P Ψ_P α P' Ψ_G A_F A_G)
from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans*)
moreover note ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_G, \Psi_G \otimes \mathbf{1}\rangle \simeq_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*rule FrameStatEqImpCompose*)
with *cCase* **have** $\Psi_G \rhd P \longmapsto \alpha \prec P'$ **by**(*fastforce dest*: *memFreshChain*)
moreover note ‹(φ, P) *mem Cs*›
moreover from ‹$A_F \sharp* (Cases\ Cs)$›‹$A_G \sharp* (Cases\ Cs)$› ‹(φ, P) *mem Cs*› **have** $A_F \sharp* \varphi$ **and** $A_G \sharp* \varphi$
by(*auto dest*: *memFreshChain*)
from ‹$\Psi_F \vdash \varphi$› **have** $\Psi_F \otimes \mathbf{1} \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
with ‹$A_F \sharp* \varphi$› **have** $(\langle A_F, \Psi_F \otimes \mathbf{1}\rangle) \vdash_F \varphi$ **by**(*force intro*: *frameImpI*)
with ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$› **have** $(\langle A_G, \Psi_G \otimes \mathbf{1}\rangle) \vdash_F \varphi$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_G \sharp* \varphi$› **have** $\Psi_G \otimes \mathbf{1} \vdash \varphi$ **by**(*force dest*: *frameImpE*)
hence $\Psi_G \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity*)
ultimately show *?case* **using** ‹*guarded P*› *cCase Case* **by**(*force intro*: *residualInject*)
next
case(*cPar1* Ψ_F Ψ_Q P α P' A_Q Q A_P Ψ_P α' PQ' Ψ_G A_F A_G)
from ‹$A_F \sharp* (P \parallel Q)$› ‹$A_G \sharp* (P \parallel Q)$› **have** $A_F \sharp* P$ **and** $A_G \sharp* P$ **and** $A_F \sharp* Q$ **and** $A_G \sharp* Q$
by *simp+*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$

by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
moreover note $\langle\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle\rangle$
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
ultimately have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*rule FrameStatEqImpCompose*)
moreover from $\langle A_F \sharp* Q\rangle \langle A_G \sharp* Q\rangle$ *FrQ* $\langle A_Q \sharp* A_F\rangle \langle A_Q \sharp* A_G\rangle$ **have** $A_F \sharp* \Psi_Q$ **and** $A_G \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+
moreover note $\langle A_F \sharp* P\rangle \langle A_G \sharp* P\rangle \langle A_F \sharp* subject\ \alpha'\rangle \langle A_G \sharp* subject\ \alpha'\rangle \langle A_P \sharp* A_F\rangle \langle A_P \sharp* A_G\rangle \langle A_P \sharp* \Psi_F\rangle \langle A_P \sharp* \Psi_Q\rangle \langle A_P \sharp* A_G\rangle \langle A_P \sharp* \Psi_G\rangle$
moreover from $\langle \alpha \prec P' \parallel Q = \alpha' \prec PQ'\rangle \langle bn\ \alpha \sharp* \alpha'\rangle$
obtain $p\ P''$ **where** $\alpha \prec P' = \alpha' \prec P''$ **and** $set\ p \subseteq set(bn\ \alpha') \times set(bn\ \alpha)$ **and** $PQ' = P'' \parallel (p \cdot Q)$
apply(*drule-tac sym*)
by(*rule-tac actionPar1Dest*) (*assumption* | *simp* | *blast dest*: *sym*)+
ultimately have $\Psi_G \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$ **using** $\langle \alpha' \neq \tau\rangle$ **by**(*force intro*: *cPar1*)
thus *?case* **using** *FrQ* $\langle (bn\ \alpha) \sharp* Q\rangle \langle A_Q \sharp* \Psi_G\rangle \langle A_Q \sharp* P\rangle \langle A_Q \sharp* \alpha\rangle$ **using** *cPar1*
by(*metis Par1*)
next
case(*cPar2* $\Psi_F\ \Psi_P\ Q\ \alpha\ Q'\ A_P\ P\ A_Q\ \Psi_Q\ \alpha'\ PQ'\ \Psi_G\ A_F\ A_G$)
from $\langle A_F \sharp* (P \parallel Q)\rangle \langle A_G \sharp* (P \parallel Q)\rangle$ **have** $A_F \sharp* P$ **and** $A_G \sharp* P$ **and** $A_F \sharp* Q$ **and** $A_G \sharp* Q$
by *simp*+
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*
have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$
by(*metis Associativity frameResChainPres frameNilStatEq*)
moreover note $\langle\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle\rangle$
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*metis Associativity AssertionStatEqSym frameResChainPres frameNilStatEq*)
ultimately have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*rule FrameStatEqImpCompose*)
moreover from $\langle A_F \sharp* P\rangle \langle A_G \sharp* P\rangle$ *FrP* $\langle A_P \sharp* A_F\rangle \langle A_P \sharp* A_G\rangle$ **have** $A_F \sharp* \Psi_P$ **and** $A_G \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)+
moreover note $\langle A_F \sharp* Q\rangle \langle A_G \sharp* Q\rangle \langle A_F \sharp* subject\ \alpha'\rangle \langle A_G \sharp* subject\ \alpha'\rangle \langle A_Q \sharp* A_F\rangle \langle A_Q \sharp* A_G\rangle \langle A_Q \sharp* \Psi_F\rangle \langle A_Q \sharp* \Psi_P\rangle \langle A_Q \sharp* A_G\rangle \langle A_Q \sharp* \Psi_G\rangle$
moreover from $\langle \alpha \prec P \parallel Q' = \alpha' \prec PQ'\rangle \langle bn\ \alpha \sharp* \alpha'\rangle$
obtain $p\ Q''$ **where** $\alpha \prec Q' = \alpha' \prec Q''$ **and** $set\ p \subseteq set(bn\ \alpha') \times set(bn\ \alpha)$ **and** $PQ' = (p \cdot P) \parallel Q''$
apply(*drule-tac sym*)
by(*rule-tac actionPar2Dest*) (*assumption* | *simp* | *blast dest*: *sym*)+
ultimately have $\Psi_G \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$ **using** $\langle \alpha' \neq \tau\rangle$ **by**(*force intro*: *cPar2*)
thus *?case* **using** *FrP* $\langle (bn\ \alpha) \sharp* P\rangle \langle A_P \sharp* \Psi_G\rangle \langle A_P \sharp* Q\rangle \langle A_P \sharp* \alpha\rangle$ **using**

cPar2
by(*metis Par2*)
next
case *cComm1*
thus *?case* **by**(*simp add*: *residualInject*)
next
case *cComm2*
thus *?case* **by**(*simp add*: *residualInject*)
next
case(*cOpen* Ψ_F P M *xvec* *yvec* N P' x A_P Ψ_P α P'' Ψ_G A_F A_G)
from ‹$M(\!|\nu*(xvec\ @\ x\ \#\ yvec)|\!)\langle N\rangle \prec P' = \alpha \prec P''$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$x \sharp \alpha$› ‹$x \sharp P''$› ‹*distinct*($bn\ \alpha$)›
obtain *xvec'* x' *yvec'* N' **where** $M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P' = M(\!|\nu*(xvec'@yvec')|\!)\langle([(x, x')] \cdot N')\rangle \prec ([(x, x')] \cdot P'')$
and $\alpha = M(\!|\nu*(xvec'@x'\#yvec')|\!)\langle N'\rangle$
apply(*cases rule*: *actionCases*[**where** α=α])
apply(*auto simp add*: *residualInject*)
apply(*rule boundOutputOpenDest*) **by** *assumption auto*

then have $\Psi_G \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$ **using** *cOpen*
by(*rule-tac cOpen*) (*assumption* | *simp*)+
with ‹$x \in supp\ N$› ‹$x \sharp \Psi_G$› ‹$x \sharp M$› ‹$x \sharp xvec$› ‹$x \sharp yvec$›
have $\Psi_G \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
by(*rule-tac Open*)
thus *?case* **using** ‹$\alpha = M(\!|\nu*(xvec'@x'\#yvec')|\!)\langle N'\rangle$› ‹$M(\!|\nu*(xvec\ @\ x\ \#\ yvec)|\!)\langle N\rangle \prec P' = \alpha \prec P''$›
by *simp*
next
case(*cScope* Ψ_F P α P' x A_P Ψ_P α' xP Ψ_G A_F A_G)
from ‹$\alpha \prec (\!|\nu x|\!)P' = \alpha' \prec xP$› ‹$x \sharp \alpha$› ‹$x \sharp \alpha'$› **obtain** P'' **where** $xP = (\!|\nu x|\!)P''$ **and** $\alpha \prec P' = \alpha' \prec P''$
by(*drule-tac sym*) (*force intro*: *actionScopeDest*)
then have $\Psi_G \rhd P \longmapsto \alpha \prec P'$ **using** *cScope* **by** *auto*
with ‹$x \sharp \Psi_G$› ‹$x \sharp \alpha'$› ‹$\alpha \prec P' = \alpha' \prec P''$› ‹$xP = (\!|\nu x|\!)P''$› **show** *?case*
by(*metis Scope*)
next
case(*cBang* Ψ_F P A_P Ψ_P α P' Ψ_G A_F A_G)
from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_F, \Psi_F \otimes \Psi_P \otimes \mathbf{1}\rangle \simeq_F \langle A_F, \Psi_F \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans*)
moreover note ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_G, \Psi_G \otimes \mathbf{1}\rangle \simeq_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $\langle A_F, \Psi_F \otimes \Psi_P \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \mathbf{1}\rangle$
by(*rule FrameStatEqImpCompose*)
with *cBang* **have** $\Psi_G \rhd P \parallel !P \longmapsto \alpha \prec P'$ **by** *force*
thus *?case* **using** ‹*guarded P*› **using** *cBang* **by**(*metis Bang*)
qed

lemma *transferTauFrame*:
fixes Ψ_F :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*
and A_F :: *name list*
and A_G :: *name list*
and Ψ_G :: *'b*

assumes $\Psi_F \rhd P \longmapsto \tau \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P \rangle$
and *distinct* A_P
and $\langle A_F, \Psi_F \otimes \Psi_P \rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \rangle$
and $A_F \sharp* P$
and $A_G \sharp* P$
and $A_P \sharp* A_F$
and $A_P \sharp* A_G$
and $A_P \sharp* \Psi_F$
and $A_P \sharp* \Psi_G$

shows $\Psi_G \rhd P \longmapsto \tau \prec P'$
using *assms*
proof(*nominal-induct avoiding*: Ψ_G A_F A_G *rule*: *tauFrameInduct*)
case(*cAlpha* Ψ_F P P' A_P Ψ_P p Ψ_G A_F A_G)
from ‹$\langle A_F, \Psi_F \otimes (p \cdot \Psi_P)\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes (p \cdot \Psi_P)\rangle$›
have $(p \cdot (\langle A_F, \Psi_F \otimes (p \cdot \Psi_P)\rangle)) \hookrightarrow_F (p \cdot (\langle A_G, \Psi_G \otimes (p \cdot \Psi_P)\rangle))$
by(*rule FrameStatImpClosed*)
with ‹$A_P \sharp* A_F$› ‹$(p \cdot A_P) \sharp* A_F$› ‹$A_P \sharp* \Psi_F$› ‹$(p \cdot A_P) \sharp* \Psi_F$› ‹$A_P \sharp* A_G$› ‹$(p \cdot A_P) \sharp* A_G$› ‹$A_P \sharp* \Psi_G$› ‹$(p \cdot A_P) \sharp* \Psi_G$›
‹*distinctPerm p*› ‹*set p* ⊆ *set* $A_P \times$ *set* $(p \cdot A_P)$› **have** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*simp add*: *eqvts*)
with *cAlpha* **show** *?case* **by** *blast*
next
case(*cCase* Ψ_F P P' φ *Cs* A_P Ψ_P Ψ_G A_F A_G)
from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_F, \Psi_F \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans*)
moreover note ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_G, \Psi_G \otimes \mathbf{1}\rangle \simeq_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
by(*rule FrameStatEqImpCompose*)
with *cCase* **have** $\Psi_G \rhd P \longmapsto \tau \prec P'$ **by**(*fastforce dest*: *memFreshChain*)
moreover note ‹(φ, P) *mem Cs*›
moreover from ‹$A_F \sharp* (Cases\ Cs)$›‹$A_G \sharp* (Cases\ Cs)$› ‹(φ, P) *mem Cs*› **have** $A_F \sharp* \varphi$ **and** $A_G \sharp* \varphi$
by(*auto dest*: *memFreshChain*)
from ‹$\Psi_F \vdash \varphi$› **have** $\Psi_F \otimes \mathbf{1} \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)

with ‹A_F ♯* φ› **have** ($\langle A_F, \Psi_F \otimes \mathbf{1}\rangle$) $\vdash_F \varphi$ **by**(*force intro*: *frameImpI*)
with ‹$\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle$› **have** ($\langle A_G, \Psi_G \otimes \mathbf{1}\rangle$) $\vdash_F \varphi$
by(*simp add*: *FrameStatImp-def*)
with ‹A_G ♯* φ› **have** $\Psi_G \otimes \mathbf{1} \vdash \varphi$ **by**(*force dest*: *frameImpE*)
hence $\Psi_G \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity*)
ultimately show *?case* **using** ‹*guarded P*› **by**(*rule Case*)
next
case(*cPar1* Ψ_F Ψ_Q P P' A_Q Q A_P Ψ_P Ψ_G A_F A_G)
from ‹A_F ♯* ($P \parallel Q$)› ‹A_G ♯* ($P \parallel Q$)› **have** A_F ♯* P **and** A_G ♯* P **and** A_F ♯* Q **and** A_G ♯* Q
by *simp+*
have *IH*: $\bigwedge \Psi\ A_F\ A_G.\ [\![\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi \otimes \Psi_P\rangle;\ A_F$ ♯* $P;\ A_G$ ♯* $P;$
A_P ♯* $A_F;\ A_P$ ♯* $A_G;\ A_P$ ♯* $(\Psi_F \otimes \Psi_Q);\ A_P$ ♯* $\Psi]\!] \Longrightarrow \Psi \rhd P \longmapsto \tau \prec P'$
by *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
moreover note ‹$\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle$›
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
ultimately have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*rule FrameStatEqImpCompose*)
moreover from ‹A_F ♯* Q› ‹A_G ♯* Q› *FrQ* ‹A_Q ♯* A_F› ‹A_Q ♯* A_G› **have** A_F ♯* Ψ_Q **and** A_G ♯* Ψ_Q
by(*force dest*: *extractFrameFreshChain*)+
moreover note ‹A_F ♯* P› ‹A_G ♯* P› ‹A_P ♯* A_F› ‹A_P ♯* A_G› ‹A_P ♯* Ψ_F› ‹A_P ♯* Ψ_Q› ‹A_P ♯* A_G› ‹A_P ♯* Ψ_G›
ultimately have $\Psi_G \otimes \Psi_Q \rhd P \longmapsto \tau \prec P'$ **by**(*rule-tac IH*) (*assumption* | *auto*)+
thus *?case* **using** *FrQ* ‹A_Q ♯* Ψ_G› ‹A_Q ♯* P›
by(*rule-tac Par1*) *auto*
next
case(*cPar2* Ψ_F Ψ_P Q Q' A_P P A_Q Ψ_Q Ψ_G A_F A_G)
from ‹A_F ♯* ($P \parallel Q$)› ‹A_G ♯* ($P \parallel Q$)› **have** A_F ♯* P **and** A_G ♯* P **and** A_F ♯* Q **and** A_G ♯* Q
by *simp+*
have *IH*: $\bigwedge \Psi\ A_F\ A_G.\ [\![\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi \otimes \Psi_Q\rangle;\ A_F$ ♯* $Q;\ A_G$ ♯* $Q;$
A_Q ♯* $A_F;\ A_Q$ ♯* $A_G;\ A_Q$ ♯* $(\Psi_F \otimes \Psi_P);\ A_Q$ ♯* $\Psi]\!] \Longrightarrow \Psi \rhd Q \longmapsto \tau \prec Q'$
by *fact*
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*
have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$
by(*metis Associativity frameResChainPres frameNilStatEq*)
moreover note ‹$\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle$›

moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*metis Associativity AssertionStatEqSym frameResChainPres frameNilStatEq*)
ultimately have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*rule FrameStatEqImpCompose*)
moreover from $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ *FrP* $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ **have** $A_F \sharp* \Psi_P$ **and** $A_G \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)+
moreover note $\langle A_F \sharp* Q\rangle$ $\langle A_G \sharp* Q\rangle$ $\langle A_Q \sharp* A_F\rangle$ $\langle A_Q \sharp* A_G\rangle$ $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* A_G\rangle$ $\langle A_Q \sharp* \Psi_G\rangle$
ultimately have $\Psi_G \otimes \Psi_P \rhd Q \longmapsto\tau \prec Q'$ **by**(*rule-tac IH*) (*assumption* | *auto*)+
thus *?case* **using** *FrP* $\langle A_P \sharp* \Psi_G\rangle$ $\langle A_P \sharp* Q\rangle$
by(*rule-tac Par2*) *auto*
next
case(*cComm1* Ψ_F Ψ_Q P M N P' A_P Ψ_P Q K *xvec* Q' A_Q Ψ_G A_F A_G)
have *FimpG*: $\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle$ **by** *fact*
from $\langle A_F \sharp* (P \parallel Q)\rangle$ $\langle A_G \sharp* (P \parallel Q)\rangle$ **have** $A_F \sharp* P$ **and** $A_G \sharp* P$ **and** $A_F \sharp* Q$ **and** $A_G \sharp* Q$
by *simp*+
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ *FrP* $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ **have** $A_F \sharp* \Psi_P$ **and** $A_G \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle A_F \sharp* Q\rangle$ $\langle A_G \sharp* Q\rangle$ *FrQ* $\langle A_Q \sharp* A_F\rangle$ $\langle A_Q \sharp* A_G\rangle$ **have** $A_F \sharp* \Psi_Q$ **and** $A_G \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle \Psi_F \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$
obtain K' **where** *KeqK'*: $(\Psi_F \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow K'$ **and** $A_P \sharp* K'$ **and** $A_F \sharp* K'$ **and** $A_G \sharp* K'$
using $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* A_F\rangle$ $\langle A_Q \sharp* A_G\rangle$ $\langle A_F \sharp* Q\rangle$ $\langle A_G \sharp* Q\rangle$ $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle$*xvec* $\sharp* K\rangle$ $\langle$*distinct xvec*$\rangle$
by(*rule-tac* B=A_P@A_F@A_G **in** *obtainPrefix*) *force*+
have $\Psi_G \otimes \Psi_Q \rhd P \longmapsto K'(\!|N|\!) \prec P'$
proof −
from *KeqK'* **have** $\Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash K \leftrightarrow K'$ **by**(*rule statEqEnt*[*OF Associativity*])
with $\langle \Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K\rangle$ **have** $\Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K'$
by(*rule chanEqTrans*)
hence $(\Psi_F \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$
by(*metis statEqEnt AssertionStatEqSym Associativity AssertionStatEqTrans compositionSym Commutativity*)
with $\langle \Psi_F \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$
have $\Psi_F \otimes \Psi_Q \rhd P \longmapsto K'(\!|N|\!) \prec P'$ **using** $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K'\rangle$
by(*force intro*: *inputRenameSubject*)
moreover have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
proof −
have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$

by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
ultimately show *?thesis* **using** *FimpG*
by(*rule-tac FrameStatEqImpCompose*)
qed
ultimately show *?thesis* **using** $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ $\langle A_F \sharp* K'\rangle$ $\langle A_G \sharp* K'\rangle$ $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_G\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$
by(*auto intro*: *transferNonTauFrame*)
qed

moreover from *FrP* $\langle\Psi_F \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'\rangle$ $\langle$*distinct* $A_P\rangle$
obtain M' **where** *MeqM'*: $(\Psi_F \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $A_Q \sharp* M'$ **and** $A_F \sharp* M'$ **and** $A_G \sharp* M'$
using $\langle A_Q \sharp* P\rangle$ $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$
by(*rule-tac* B=A_Q@A_F@A_G **in** *obtainPrefix*) *force*+

have $\Psi_G \otimes \Psi_P \rhd Q \longmapsto M'(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$
proof −
from *MeqM'* **have** $\Psi_F \otimes (\Psi_Q \otimes \Psi_P) \vdash M \leftrightarrow M'$ **by**(*rule statEqEnt*[*OF Associativity*])
with $\langle\Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K\rangle$ **have** $\Psi_F \otimes (\Psi_Q \otimes \Psi_P) \vdash K \leftrightarrow M'$
by(*blast intro*: *chanEqTrans chanEqSym compositionSym Commutativity statEqEnt*)
hence $(\Psi_F \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*blast intro*: *statEqEnt AssertionStatEqSym Associativity AssertionStatEqTrans compositionSym Commutativity*)
with $\langle\Psi_F \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$
have $\Psi_F \otimes \Psi_P \rhd Q \longmapsto M'(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$ **using** $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle A_Q \sharp* M'\rangle$
by(*force intro*: *outputRenameSubject*)
moreover have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
proof −
have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$
by(*metis Associativity frameResChainPres frameNilStatEq*)
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*metis Associativity AssertionStatEqSym frameResChainPres frameNilStatEq*)
ultimately show *?thesis* **using** *FimpG*
by(*rule-tac FrameStatEqImpCompose*)
qed

ultimately show *?thesis* **using** $\langle A_F \sharp* Q\rangle$ $\langle A_G \sharp* Q\rangle$ $\langle A_F \sharp* M'\rangle$ $\langle A_G \sharp* M'\rangle$ $\langle A_Q \sharp* A_F\rangle$ $\langle A_Q \sharp* A_G\rangle$ $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_G\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$ $\langle$*distinct xvec*$\rangle$

by(*auto intro*: *transferNonTauFrame*)
qed

moreover have $\Psi_G \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$
proof −
from *MeqM'* **have** $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash M' \leftrightarrow M$
by(*blast intro*: *chanEqSym Associativity statEqEnt Commutativity compositionSym*)
moreover from *KeqK'* **have** $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow K'$
by(*blast intro*: *chanEqSym Associativity statEqEnt Commutativity compositionSym*)
ultimately have $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$ **using** $\langle \Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K \rangle$
by(*blast intro*: *chanEqSym chanEqTrans*)
thus *?thesis* **using** $\langle A_F \sharp* M' \rangle$ $\langle A_F \sharp* K' \rangle$ $\langle A_G \sharp* M' \rangle$ $\langle A_G \sharp* K' \rangle$ *FimpG*
apply(*auto simp add*: *FrameStatImp-def*)
apply(*erule-tac x=SChanEq' K' M'* **in** *allE*)
by(*force intro*: *frameImpI dest*: *frameImpE*)
qed

ultimately show *?case* **using** $\langle A_P \sharp* \Psi_G \rangle$ $\langle A_P \sharp* P \rangle$ $\langle A_P \sharp* Q \rangle$ $\langle A_P \sharp* A_Q \rangle$ $\langle A_P \sharp* K' \rangle$ $\langle A_Q \sharp* \Psi_G \rangle$ $\langle A_Q \sharp* P \rangle$ $\langle A_Q \sharp* Q \rangle$ $\langle A_Q \sharp* M' \rangle$ $\langle xvec \sharp* P \rangle$ *FrP FrQ*
by(*rule-tac Comm1*) (*assumption* | *simp*)+
next
case(*cComm2* Ψ_F Ψ_Q P M *xvec* N P' A_P Ψ_P Q K Q' A_Q Ψ_G A_F A_G)
have *FimpG*: $\langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q \rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q \rangle$ **by** *fact*
from $\langle A_F \sharp* (P \parallel Q) \rangle$ $\langle A_G \sharp* (P \parallel Q) \rangle$ **have** $A_F \sharp* P$ **and** $A_G \sharp* P$ **and** $A_F \sharp* Q$ **and** $A_G \sharp* Q$
by *simp*+
have *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **by** *fact*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*
from $\langle A_F \sharp* P \rangle$ $\langle A_G \sharp* P \rangle$ *FrP* $\langle A_P \sharp* A_F \rangle$ $\langle A_P \sharp* A_G \rangle$ **have** $A_F \sharp* \Psi_P$ **and** $A_G \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle A_F \sharp* Q \rangle$ $\langle A_G \sharp* Q \rangle$ *FrQ* $\langle A_Q \sharp* A_F \rangle$ $\langle A_Q \sharp* A_G \rangle$ **have** $A_F \sharp* \Psi_Q$ **and** $A_G \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle \Psi_F \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q' \rangle$ *FrQ* $\langle distinct\ A_Q \rangle$
obtain K' **where** *KeqK'*: $(\Psi_F \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow K'$ **and** $A_P \sharp* K'$ **and** $A_F \sharp* K'$ **and** $A_G \sharp* K'$
using $\langle A_P \sharp* Q \rangle$ $\langle A_Q \sharp* A_F \rangle$ $\langle A_Q \sharp* A_G \rangle$ $\langle A_F \sharp* Q \rangle$ $\langle A_G \sharp* Q \rangle$ $\langle A_Q \sharp* \Psi_F \rangle$ $\langle A_Q \sharp* \Psi_P \rangle$ $\langle A_P \sharp* A_Q \rangle$ $\langle A_Q \sharp* Q \rangle$ $\langle A_Q \sharp* K \rangle$
by(*rule-tac* $B=A_P@A_F@A_G$ **in** *obtainPrefix*) *force*+
have $\Psi_G \otimes \Psi_Q \rhd P \longmapsto K'(\!|\nu*xvec|\!)\langle N \rangle \prec P'$
proof −
from *KeqK'* **have** $\Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash K \leftrightarrow K'$ **by**(*rule statEqEnt*[*OF Associativity*])
with $\langle \Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K \rangle$ **have** $\Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K'$
by(*rule chanEqTrans*)

hence $(\Psi_F \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$
by(*metis statEqEnt AssertionStatEqSym Associativity AssertionStatEqTrans compositionSym Commutativity*)
with $\langle \Psi_F \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$
have $\Psi_F \otimes \Psi_Q \rhd P \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **using** $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K'\rangle$
by(*force intro*: *outputRenameSubject*)
moreover have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
proof −
have $\langle A_F, (\Psi_F \otimes \Psi_Q) \otimes \Psi_P\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_Q) \otimes \Psi_P\rangle$
by(*metis Associativity Composition AssertionStatEqSym AssertionStatEqTrans Commutativity frameResChainPres frameNilStatEq*)
ultimately show *?thesis* **using** *FimpG*
by(*rule-tac FrameStatEqImpCompose*)
qed
ultimately show *?thesis* **using** $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ $\langle A_F \sharp* K'\rangle$
$\langle A_G \sharp* K'\rangle$ $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_G\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ *FrP* $\langle$*distinct* $A_P\rangle$
$\langle$*distinct xvec*$\rangle$
by(*auto intro*: *transferNonTauFrame*)
qed

moreover from *FrP* $\langle \Psi_F \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'\rangle$ $\langle$*distinct* $A_P\rangle$
obtain M' **where** *MeqM'*: $(\Psi_F \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $A_Q \sharp* M'$ **and** $A_F \sharp* M'$ **and** $A_G \sharp* M'$
using $\langle A_Q \sharp* P\rangle$ $\langle A_P \sharp* A_F\rangle$ $\langle A_P \sharp* A_G\rangle$ $\langle A_F \sharp* P\rangle$ $\langle A_G \sharp* P\rangle$ $\langle A_P \sharp* \Psi_F\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle xvec \sharp* M\rangle$ $\langle$*distinct xvec*$\rangle$
by(*rule-tac* B=A_Q@A_F@A_G **in** *obtainPrefix*) *force*+

have $\Psi_G \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q'$
proof −
from *MeqM'* **have** $\Psi_F \otimes (\Psi_Q \otimes \Psi_P) \vdash M \leftrightarrow M'$ **by**(*rule statEqEnt*[*OF Associativity*])
with $\langle \Psi_F \otimes (\Psi_P \otimes \Psi_Q) \vdash M \leftrightarrow K\rangle$ **have** $\Psi_F \otimes (\Psi_Q \otimes \Psi_P) \vdash K \leftrightarrow M'$
by(*blast intro*: *chanEqTrans chanEqSym compositionSym Commutativity statEqEnt*)
hence $(\Psi_F \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*blast intro*: *statEqEnt AssertionStatEqSym Associativity AssertionStatEqTrans compositionSym Commutativity*)
with $\langle \Psi_F \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'\rangle$ *FrQ* $\langle$*distinct* $A_Q\rangle$
have $\Psi_F \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q'$ **using** $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* K\rangle$ $\langle A_Q \sharp* M'\rangle$
by(*force intro*: *inputRenameSubject*)
moreover have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
proof −
have $\langle A_F, (\Psi_F \otimes \Psi_P) \otimes \Psi_Q\rangle \simeq_F \langle A_F, \Psi_F \otimes \Psi_P \otimes \Psi_Q\rangle$

by(*metis Associativity frameResChainPres frameNilStatEq*)
moreover have $\langle A_G, \Psi_G \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle A_G, (\Psi_G \otimes \Psi_P) \otimes \Psi_Q\rangle$
by(*metis Associativity AssertionStatEqSym frameResChainPres frameNilStatEq*)
ultimately show *?thesis* **using** *FimpG*
by(*rule-tac FrameStatEqImpCompose*)
qed

ultimately show *?thesis* **using** $\langle A_F \sharp* Q\rangle$ $\langle A_G \sharp* Q\rangle$ $\langle A_F \sharp* M'\rangle$ $\langle A_G \sharp* M'\rangle$ $\langle A_Q \sharp* A_F\rangle$ $\langle A_Q \sharp* A_G\rangle$ $\langle A_Q \sharp* \Psi_F\rangle$ $\langle A_Q \sharp* \Psi_G\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$ *FrQ* ⟨*distinct* A_Q⟩ ⟨*distinct xvec*⟩
by(*auto intro*: *transferNonTauFrame*)
qed

moreover have $\Psi_G \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$
proof −
from *MeqM'* **have** $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash M' \leftrightarrow M$
by(*blast intro*: *chanEqSym Associativity statEqEnt Commutativity compositionSym*)
moreover from *KeqK'* **have** $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow K'$
by(*blast intro*: *chanEqSym Associativity statEqEnt Commutativity compositionSym*)
ultimately have $\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash K' \leftrightarrow M'$ **using** $\langle\Psi_F \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K\rangle$
by(*blast intro*: *chanEqSym chanEqTrans*)
thus *?thesis* **using** $\langle A_F \sharp* M'\rangle$ $\langle A_F \sharp* K'\rangle$ $\langle A_G \sharp* M'\rangle$ $\langle A_G \sharp* K'\rangle$ *FimpG*
apply(*auto simp add*: *FrameStatImp-def*)
apply(*erule-tac x=SChanEq' K' M'* **in** *allE*)
by(*force intro*: *frameImpI dest*: *frameImpE*)
qed

ultimately show *?case* **using** $\langle A_P \sharp* \Psi_G\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* K'\rangle$ $\langle A_Q \sharp* \Psi_G\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_Q \sharp* M'\rangle$ $\langle xvec \sharp* Q\rangle$ *FrP FrQ*
by(*rule-tac Comm2*) (*assumption* | *simp*)+
next
case(*cScope* Ψ_F P P' x A_P Ψ_P Ψ_G A_F A_G)
then have $\Psi_G \rhd P \longmapsto\tau \prec P'$ **by** *auto*
with $\langle x \sharp \Psi_G\rangle$ **show** *?case*
by(*rule-tac Scope*) *auto*
next
case(*cBang* Ψ_F P P' A_P Ψ_P Ψ_G A_F A_G)
from $\langle\Psi_P \simeq \mathbf{1}\rangle$ **have** $\langle A_F, \Psi_F \otimes \Psi_P \otimes \mathbf{1}\rangle \simeq_F \langle A_F, \Psi_F \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans*)
moreover note $\langle\langle A_F, \Psi_F \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \mathbf{1}\rangle\rangle$
moreover from $\langle\Psi_P \simeq \mathbf{1}\rangle$ **have** $\langle A_G, \Psi_G \otimes \mathbf{1}\rangle \simeq_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym Identity AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $\langle A_F, \Psi_F \otimes \Psi_P \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P \otimes \mathbf{1}\rangle$
by(*rule FrameStatEqImpCompose*)

with *cBang* **have** $\Psi_G \rhd P \parallel !P \longmapsto\tau \prec P'$ **by** *force*
thus *?case* **using** ‹*guarded P*› **by**(*rule Bang*)
qed

lemma *transferFrame*:
fixes Ψ_F :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and A_F :: *name list*
and A_G :: *name list*
and Ψ_G :: *'b*

assumes $\Psi_F \rhd P \longmapsto\alpha \prec P'$
and *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *distinct* A_P
and $\langle A_F, \Psi_F \otimes \Psi_P\rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P\rangle$
and $A_F \sharp* P$
and $A_G \sharp* P$
and $A_F \sharp* \mathit{subject}\ \alpha$
and $A_G \sharp* \mathit{subject}\ \alpha$
and $A_P \sharp* A_F$
and $A_P \sharp* A_G$
and $A_P \sharp* \Psi_F$
and $A_P \sharp* \Psi_G$

shows $\Psi_G \rhd P \longmapsto\alpha \prec P'$
using *assms*
proof −
from ‹$\Psi_F \rhd P \longmapsto\alpha \prec P'$› **have** *distinct*(*bn* α) **by**(*auto dest*: *boundOutputDistinct*)
thus *?thesis* **using** *assms*
by(*cases* $\alpha = \tau$) (*auto intro*: *transferNonTauFrame transferTauFrame*)
qed

lemma *parCasesInputFrame*[*consumes 7*, *case-names cPar1 cPar2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and T :: (*'a*, *'b*, *'c*) *psi*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|N|\!) \prec T$
and *extractFrame*$(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$
and *distinct* A_{PQ}
and $A_{PQ} \sharp* \Psi$

and $A_{PQ} \sharp* P$
and $A_{PQ} \sharp* Q$
and $A_{PQ} \sharp* M$
and *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P@A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'; \mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle; \mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle;$
$\mathit{distinct}\ A_P; \mathit{distinct}\ A_Q; A_P \sharp* \Psi; A_P \sharp* P; A_P \sharp* Q; A_P \sharp* M;\ A_Q \sharp* \Psi; A_Q \sharp* P; A_Q \sharp* Q; A_Q \sharp* M;$
$A_P \sharp* \Psi_Q; A_Q \sharp* \Psi_P; A_P \sharp* A_Q; A_{PQ} = A_P@A_Q; \Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow \mathit{Prop}\ (P \parallel Q')$
shows *Prop T*
using *Trans*
proof(*induct rule*: *parInputCases*[*of* - - - - - - (A_{PQ}, Ψ_{PQ})])
case(*cPar1 P' A_Q Ψ_Q*)
from ‹$A_Q \sharp* (A_{PQ}, \Psi_{PQ})$› **have** $A_Q \sharp* A_{PQ}$ **and** $A_Q \sharp* \Psi_{PQ}$ **by** *simp+*
obtain $A_P\ \Psi_P$ **where** *FrP*: $\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle$ **and** $\mathit{distinct}\ A_P$
$A_P \sharp* (P, Q, \Psi, M, A_Q, A_{PQ}, \Psi_Q)$
by(*rule freshFrame*)
hence $A_P \sharp* P$ **and** $A_P \sharp* Q$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* M$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* A_{PQ}$ **and** $A_P \sharp* \Psi_Q$
by *simp+*

have *FrQ*: $\mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*

from ‹$A_Q \sharp* P$› ‹$A_P \sharp* A_Q$› *FrP* **have** $A_Q \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹$\mathit{extractFrame}(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› *FrP FrQ* ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have $\langle (A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$ **by** *simp*
moreover from ‹$\mathit{distinct}\ A_P$› ‹$\mathit{distinct}\ A_Q$› ‹$A_P \sharp* A_Q$› **have** $\mathit{distinct}(A_P@A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain *p* **where** *S*: $\mathit{set}\ p \subseteq \mathit{set}(A_P@A_Q) \times \mathit{set}((p \cdot A_P)@(p \cdot A_Q))$ **and** $\mathit{distinctPerm}\ p$
and *Ψeq*: $\Psi_{PQ} = (p \cdot \Psi_P) \otimes (p \cdot \Psi_Q)$ **and** *Aeq*: $A_{PQ} = (p \cdot A_P)@(p \cdot A_Q)$
using ‹$A_P \sharp* A_{PQ}$› ‹$A_Q \sharp* A_{PQ}$› ‹$\mathit{distinct}\ A_{PQ}$›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$› *S* ‹$A_{PQ} \sharp* P$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_{PQ} \sharp* M$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* M$› *Aeq*
have $(p \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto M(\!|N|\!) \prec P'$
by(*rule-tac inputPermFrame*) *auto*
with *S* ‹$A_{PQ} \sharp* \Psi$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› *Aeq* **have** $\Psi \otimes (p \cdot \Psi_Q) \rhd P \longmapsto M(\!|N|\!)$

$\prec P'$
by(*simp add*: *eqvts*)
moreover from *FrP* **have** $(p \cdot \mathit{extractFrame}\ P) = p \cdot \langle A_P, \Psi_P\rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* P$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› *Aeq* **have** $\mathit{extractFrame}\ P = \langle(p \cdot A_P), p \cdot \Psi_P\rangle$
by(*simp add*: *eqvts*)
moreover from *FrQ* **have** $(p \cdot \mathit{extractFrame}\ Q) = p \cdot \langle A_Q, \Psi_Q\rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* Q$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› *Aeq* **have** $\mathit{extractFrame}\ Q = \langle(p \cdot A_Q), p \cdot \Psi_Q\rangle$
by(*simp add*: *eqvts*)
moreover from ‹*distinct* A_P› ‹*distinct* A_Q› **have** $\mathit{distinct}(p \cdot A_P)$ **and** $\mathit{distinct}(p \cdot A_Q)$
by *simp*+
moreover from ‹$A_P \sharp* A_Q$› **have** $(p \cdot A_P) \sharp* (p \cdot A_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_P \sharp* \Psi_Q$› **have** $(p \cdot A_P) \sharp* (p \cdot \Psi_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_Q \sharp* \Psi_P$› **have** $(p \cdot A_Q) \sharp* (p \cdot \Psi_P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **using** ‹$A_{PQ} \sharp* \Psi$› ‹$A_{PQ} \sharp* P$› ‹$A_{PQ} \sharp* Q$› ‹$A_{PQ} \sharp* M$› *Aeq* Ψ*eq*
by(*rule-tac rPar1*) (*assumption* | *simp*)+
next
case(*cPar2* Q' A_P Ψ_P)
from ‹$A_P \sharp* (A_{PQ}, \Psi_{PQ})$› **have** $A_P \sharp* A_{PQ}$ **and** $A_P \sharp* \Psi_{PQ}$ **by** *simp*+
obtain A_Q Ψ_Q **where** *FrQ*: $\mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle$ **and** *distinct* A_Q
$A_Q \sharp* (P, Q, \Psi, M, A_P, A_{PQ}, \Psi_P)$
by(*rule freshFrame*)
hence $A_Q \sharp* P$ **and** $A_Q \sharp* Q$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* M$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* A_{PQ}$ **and** $A_Q \sharp* \Psi_P$
by *simp*+

have *FrP*: $\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle$ **by** *fact*

from ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› *FrQ* **have** $A_P \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)

from ‹$\mathit{extractFrame}(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› *FrP FrQ* ‹$A_Q \sharp* A_P$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have $\langle(A_P @ A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$ **by** *simp*
moreover from ‹*distinct* A_P› ‹*distinct* A_Q› ‹$A_Q \sharp* A_P$› **have** $\mathit{distinct}(A_P @ A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain p **where** S: $\mathit{set}\ p \subseteq \mathit{set}(A_P @ A_Q) \times \mathit{set}((p \cdot A_P) @ (p \cdot A_Q))$ **and** *distinctPerm* p
and Ψ*eq*: $\Psi_{PQ} = (p \cdot \Psi_P) \otimes (p \cdot \Psi_Q)$ **and** *Aeq*: $A_{PQ} = (p \cdot A_P) @ (p \cdot A_Q)$
using ‹$A_P \sharp* A_{PQ}$› ‹$A_Q \sharp* A_{PQ}$› ‹*distinct* A_{PQ}›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'$› S ‹$A_{PQ} \sharp* Q$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_{PQ} \sharp* M$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* M$› Aeq
have $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto M(\!|N|\!) \prec Q'$
by(*rule-tac inputPermFrame*) *auto*
with S ‹$A_{PQ} \sharp* \Psi$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› Aeq **have** $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto M(\!|N|\!) \prec Q'$
by(*simp add*: *eqvts*)
moreover from *FrP* **have** $(p \cdot extractFrame\ P) = p \cdot \langle A_P, \Psi_P \rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* P$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› Aeq **have** $extractFrame\ P = \langle (p \cdot A_P), p \cdot \Psi_P \rangle$
by(*simp add*: *eqvts*)
moreover from *FrQ* **have** $(p \cdot extractFrame\ Q) = p \cdot \langle A_Q, \Psi_Q \rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* Q$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› Aeq **have** $extractFrame\ Q = \langle (p \cdot A_Q), p \cdot \Psi_Q \rangle$
by(*simp add*: *eqvts*)
moreover from ‹*distinct* A_P› ‹*distinct* A_Q› **have** $distinct(p \cdot A_P)$ **and** $distinct(p \cdot A_Q)$
by *simp+*
moreover from ‹$A_Q \sharp* A_P$› **have** $(p \cdot A_P) \sharp* (p \cdot A_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_P \sharp* \Psi_Q$› **have** $(p \cdot A_P) \sharp* (p \cdot \Psi_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_Q \sharp* \Psi_P$› **have** $(p \cdot A_Q) \sharp* (p \cdot \Psi_P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **using** ‹$A_{PQ} \sharp* \Psi$› ‹$A_{PQ} \sharp* P$› ‹$A_{PQ} \sharp* Q$› ‹$A_{PQ} \sharp* M$› Aeq Ψeq
by(*rule-tac rPar2*) (*assumption* | *simp*)+
qed

lemma *parCasesOutputFrame*[*consumes 11*, *case-names cPar1 cPar2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and T :: (*'a*, *'b*, *'c*) *psi*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec T$
and $xvec \sharp* \Psi$
and $xvec \sharp* P$
and $xvec \sharp* Q$
and $xvec \sharp* M$
and $extractFrame(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ} \rangle$
and $distinct\ A_{PQ}$
and $A_{PQ} \sharp* \Psi$
and $A_{PQ} \sharp* P$
and $A_{PQ} \sharp* Q$

and $A_{PQ} \sharp* M$
and *rPar1*: $\bigwedge P'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_P; *distinct* A_Q; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* M$; $A_Q \sharp* \Psi$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* M$; $A_P \sharp* \Psi_Q$; $A_Q \sharp* \Psi_P$; $A_P \sharp* A_Q$; $A_{PQ} = A_P@A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$ *Prop* $(P' \parallel Q)$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec Q';$ *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_P; *distinct* A_Q; $A_P \sharp* \Psi$; $A_P \sharp* P$; $A_P \sharp* Q$; $A_P \sharp* M$; $A_Q \sharp* \Psi$; $A_Q \sharp* P$; $A_Q \sharp* Q$; $A_Q \sharp* M$; $A_P \sharp* \Psi_Q$; $A_Q \sharp* \Psi_P$; $A_P \sharp* A_Q$; $A_{PQ} = A_P@A_Q$; $\Psi_{PQ} = \Psi_P \otimes \Psi_Q]\!] \Longrightarrow$ *Prop* $(P \parallel Q')$
shows *Prop T*
using *Trans* ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ P› ‹*xvec* $\sharp*$ Q› ‹*xvec* $\sharp*$ M›
proof(*induct rule*: *parOutputCases*[*of* - - - - - - - (A_{PQ}, Ψ_{PQ})])
case(*cPar1* P' A_Q Ψ_Q)
from ‹$A_Q \sharp* (A_{PQ}, \Psi_{PQ})$› **have** $A_Q \sharp* A_{PQ}$ **and** $A_Q \sharp* \Psi_{PQ}$ **by** *simp*+
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** *distinct* A_P $A_P \sharp* (P, Q, \Psi, M, A_Q, A_{PQ}, \Psi_Q)$
by(*rule freshFrame*)
hence $A_P \sharp* P$ **and** $A_P \sharp* Q$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* M$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* A_{PQ}$ **and** $A_P \sharp* \Psi_Q$
by *simp*+

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*

from ‹$A_Q \sharp* P$› ‹$A_P \sharp* A_Q$› *FrP* **have** $A_Q \sharp* \Psi_P$
by(*force dest*: *extractFrameFreshChain*)

from ‹*extractFrame*$(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› *FrP FrQ* ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have $\langle (A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$ **by** *simp*
moreover from ‹*distinct* A_P› ‹*distinct* A_Q› ‹$A_P \sharp* A_Q$› **have** *distinct*$(A_P@A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain p **where** S: *set* $p \subseteq$ *set*$(A_P@A_Q) \times$ *set*$((p \cdot A_P)@(p \cdot A_Q))$ **and** *distinctPerm* p **and** Ψeq: $\Psi_{PQ} = (p \cdot \Psi_P) \otimes (p \cdot \Psi_Q)$ **and** *Aeq*: $A_{PQ} = (p \cdot A_P)@(p \cdot A_Q)$
using ‹$A_P \sharp* A_{PQ}$› ‹$A_Q \sharp* A_{PQ}$› ‹*distinct* A_{PQ}›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'$› S ‹$A_{PQ} \sharp* P$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_{PQ} \sharp* M$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* M$› *Aeq*
have $(p \cdot (\Psi \otimes \Psi_Q)) \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'$
by(*rule-tac outputPermFrame*) (*assumption* | *simp*)+

with S ‹$A_{PQ} \sharp* \Psi$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› *Aeq* **have** $\Psi \otimes (p \cdot \Psi_Q) \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'$

by(*simp add*: *eqvts*)
moreover from *FrP* **have** $(p \cdot \mathit{extractFrame}\ P) = p \cdot \langle A_P, \Psi_P\rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* P$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› *Aeq* **have** $\mathit{extractFrame}\ P = \langle(p \cdot A_P), p \cdot \Psi_P\rangle$
by(*simp add*: *eqvts*)
moreover from *FrQ* **have** $(p \cdot \mathit{extractFrame}\ Q) = p \cdot \langle A_Q, \Psi_Q\rangle$ **by** *simp*
with S ‹$A_{PQ} \sharp* Q$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› *Aeq* **have** $\mathit{extractFrame}\ Q = \langle(p \cdot A_Q), p \cdot \Psi_Q\rangle$
by(*simp add*: *eqvts*)
moreover from ‹$\mathit{distinct}\ A_P$› ‹$\mathit{distinct}\ A_Q$› **have** $\mathit{distinct}(p \cdot A_P)$ **and** $\mathit{distinct}(p \cdot A_Q)$
by *simp*+
moreover from ‹$A_P \sharp* A_Q$› **have** $(p \cdot A_P) \sharp* (p \cdot A_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_P \sharp* \Psi_Q$› **have** $(p \cdot A_P) \sharp* (p \cdot \Psi_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹$A_Q \sharp* \Psi_P$› **have** $(p \cdot A_Q) \sharp* (p \cdot \Psi_P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **using** ‹$A_{PQ} \sharp* \Psi$› ‹$A_{PQ} \sharp* P$› ‹$A_{PQ} \sharp* Q$› ‹$A_{PQ} \sharp* M$› *Aeq* Ψ*eq*
by(*rule-tac rPar1*) (*assumption* | *simp*)+
next
case(*cPar2* Q' A_P Ψ_P)
from ‹$A_P \sharp* (A_{PQ}, \Psi_{PQ})$› **have** $A_P \sharp* A_{PQ}$ **and** $A_P \sharp* \Psi_{PQ}$ **by** *simp*+
obtain A_Q Ψ_Q **where** *FrQ*: $\mathit{extractFrame}\ Q = \langle A_Q, \Psi_Q\rangle$ **and** $\mathit{distinct}\ A_Q$
$A_Q \sharp* (P, Q, \Psi, M, A_P, A_{PQ}, \Psi_P)$
by(*rule freshFrame*)
hence $A_Q \sharp* P$ **and** $A_Q \sharp* Q$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* M$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* A_{PQ}$ **and** $A_Q \sharp* \Psi_P$
by *simp*+

have *FrP*: $\mathit{extractFrame}\ P = \langle A_P, \Psi_P\rangle$ **by** *fact*

from ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› *FrQ* **have** $A_P \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)

from ‹$\mathit{extractFrame}(P \parallel Q) = \langle A_{PQ}, \Psi_{PQ}\rangle$› *FrP FrQ* ‹$A_Q \sharp* A_P$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have $\langle(A_P @ A_Q), \Psi_P \otimes \Psi_Q\rangle = \langle A_{PQ}, \Psi_{PQ}\rangle$ **by** *simp*
moreover from ‹$\mathit{distinct}\ A_P$› ‹$\mathit{distinct}\ A_Q$› ‹$A_Q \sharp* A_P$› **have** $\mathit{distinct}(A_P @ A_Q)$
by(*auto simp add*: *fresh-star-def fresh-def name-list-supp*)
ultimately obtain p **where** S: $\mathit{set}\ p \subseteq \mathit{set}(A_P @ A_Q) \times \mathit{set}((p \cdot A_P) @ (p \cdot A_Q))$ **and** *distinctPerm* p
and Ψ*eq*: $\Psi_{PQ} = (p \cdot \Psi_P) \otimes (p \cdot \Psi_Q)$ **and** *Aeq*: $A_{PQ} = (p \cdot A_P) @ (p \cdot A_Q)$
using ‹$A_P \sharp* A_{PQ}$› ‹$A_Q \sharp* A_{PQ}$› ‹$\mathit{distinct}\ A_{PQ}$›
by(*rule-tac frameChainEq'*) (*assumption* | *simp add*: *eqvts*)+

from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu * \mathit{xvec}|\!)\langle N\rangle \prec Q'$› S ‹$A_{PQ} \sharp* Q$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp*$

$Q\rangle$ ‹A_{PQ} ♯* $M\rangle$ ‹A_P ♯* $M\rangle$ ‹A_Q ♯* $M\rangle$ Aeq
have $(p \cdot (\Psi \otimes \Psi_P)) \rhd Q \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$
by(*rule-tac outputPermFrame*) (*assumption* | *simp*)+
with S ‹A_{PQ} ♯* $\Psi\rangle$ ‹A_P ♯* $\Psi\rangle$ ‹A_Q ♯* $\Psi\rangle$ Aeq **have** $\Psi \otimes (p \cdot \Psi_P) \rhd Q \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$
by(*simp add*: *eqvts*)
moreover from FrP **have** $(p \cdot extractFrame\ P) = p \cdot \langle A_P, \Psi_P\rangle$ **by** *simp*
with S ‹A_{PQ} ♯* $P\rangle$ ‹A_P ♯* $P\rangle$ ‹A_Q ♯* $P\rangle$ Aeq **have** $extractFrame\ P = \langle(p \cdot A_P), p \cdot \Psi_P\rangle$
by(*simp add*: *eqvts*)
moreover from FrQ **have** $(p \cdot extractFrame\ Q) = p \cdot \langle A_Q, \Psi_Q\rangle$ **by** *simp*
with S ‹A_{PQ} ♯* $Q\rangle$ ‹A_P ♯* $Q\rangle$ ‹A_Q ♯* $Q\rangle$ Aeq **have** $extractFrame\ Q = \langle(p \cdot A_Q), p \cdot \Psi_Q\rangle$
by(*simp add*: *eqvts*)
moreover from ‹*distinct* $A_P\rangle$ ‹*distinct* $A_Q\rangle$ **have** $distinct(p \cdot A_P)$ **and** $distinct(p \cdot A_Q)$
by *simp*+
moreover from ‹A_Q ♯* $A_P\rangle$ **have** $(p \cdot A_P)$ ♯* $(p \cdot A_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹A_P ♯* $\Psi_Q\rangle$ **have** $(p \cdot A_P)$ ♯* $(p \cdot \Psi_Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹A_Q ♯* $\Psi_P\rangle$ **have** $(p \cdot A_Q)$ ♯* $(p \cdot \Psi_P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **using** ‹A_{PQ} ♯* $\Psi\rangle$ ‹A_{PQ} ♯* $P\rangle$ ‹A_{PQ} ♯* $Q\rangle$ ‹A_{PQ} ♯* $M\rangle$ Aeq Ψeq
by(*rule-tac rPar2*) (*assumption* | *simp*)+
qed

inductive *bangPred* :: ('*a*, '*b*, '*c*) *psi* ⇒ ('*a*, '*b*, '*c*) *psi* ⇒ *bool*
where
aux1: *bangPred P* (!*P*)
| *aux2*: *bangPred P* (*P* ‖ !*P*)

lemma *bangInduct*[*consumes 1*, *case-names cPar1 cPar2 cComm1 cComm2 cBang*]:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Rs* :: ('*a*, '*b*, '*c*) *residual*
and *Prop* :: '*d*::*fs-name* ⇒ '*b* ⇒ ('*a*, '*b*, '*c*) *psi* ⇒ ('*a*, '*b*, '*c*) *residual* ⇒ *bool*
and *C* :: '*d*

assumes Ψ ▷ !*P* ⟼ *Rs*
and *rPar1*: ⋀α *P'* *C*. ⟦Ψ ▷ *P* ⟼α ≺ *P'*; *bn* α ♯* Ψ; *bn* α ♯* *P*; *bn* α ♯* *subject* α; *bn* α ♯* *C*; *distinct*(*bn* α)⟧ ⟹ *Prop C* Ψ (*P* ‖ !*P*) (α ≺ (*P'* ‖ !*P*))
and *rPar2*: ⋀α *P'* *C*. ⟦Ψ ▷ !*P* ⟼α ≺ *P'*; *bn* α ♯* Ψ; *bn* α ♯* *P*; *bn* α ♯* *subject* α; *bn* α ♯* *C*; *distinct*(*bn* α);
⋀*C*. *Prop C* Ψ (!*P*) (α ≺ *P'*)⟧ ⟹ *Prop C* Ψ (*P* ‖ !*P*) (α ≺ (*P* ‖ *P'*))
and *rComm1*: ⋀*M N P' K xvec P'' C*. ⟦Ψ ▷ *P* ⟼*M*(⦇*N*⦈) ≺ *P'*; Ψ ▷ !*P* ⟼*K*(⦇ν*xvec⦈⟨*N*⟩ ≺ *P''*; ⋀*C*. *Prop C* Ψ (!*P*) (*K*(⦇ν*xvec⦈⟨*N*⟩ ≺ *P''*); Ψ ⊢ *M* ↔

K;
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* C$; $distinct\ xvec]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel P''))$
and $rComm2$: $\bigwedge M\ xvec\ N\ P'\ K\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'; \Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''; \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ (K(\!|N|\!) \prec P''); \Psi \vdash M \leftrightarrow K;$
$xvec \sharp* \Psi$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* C$; $distinct\ xvec]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (\tau \prec (\!|\nu * xvec|\!)(P' \parallel P''))$
and $rBang$: $\bigwedge Rs\ C.\ [\![\Psi \rhd P \parallel !P \longmapsto Rs; \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ Rs; guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ Rs$
shows $Prop\ C\ \Psi\ (!P)\ Rs$
proof −
from ‹$\Psi \rhd !P \longmapsto Rs$› **have** $guarded\ P$
by(*nominal-induct* Ψ $P==!P$ *Rs rule*: *semantics.strong-induct*) (*auto simp add*: *psi.inject*)
{
fix Q :: ('a, 'b, 'c) *psi*
and Ψ' :: 'b

assume $\Psi' \rhd Q \longmapsto Rs$
and $guarded\ Q$
and $bangPred\ P\ Q$
and $\Psi \simeq \Psi'$

hence $Prop\ C\ \Psi\ Q\ Rs$ **using** $rPar1\ rPar1\ rPar2\ rPar2\ rComm1\ rComm2\ rBang$
proof(*nominal-induct avoiding*: $\Psi\ C$ *rule*: *semantics.strong-induct*)
case(*cInput* $\Psi'\ M\ K\ xvec\ N\ Tvec\ Q\ \Psi\ C$)
thus *?case* **by** − (*ind-cases bangPred* P ($M(\!|\lambda * xvec\ N|\!).Q$))
next
case(*Output* $\Psi'\ M\ K\ N\ Q\ \Psi\ C$)
thus *?case* **by** − (*ind-cases bangPred* P ($M\langle N\rangle.Q$))
next
case(*Case* $\Psi'\ Q\ Rs\ \varphi\ Cs\ \Psi\ C$)
thus *?case* **by** − (*ind-cases bangPred* P (*Cases* Cs))
next
case(*cPar1* $\Psi'\ \Psi_R\ Q\ \alpha\ P'\ R\ A_R\ \Psi\ C$)
have $rPar1$: $\bigwedge \alpha\ P'\ C.\ [\![\Psi \rhd P \longmapsto \alpha \prec P'; bn\ \alpha \sharp* \Psi; bn\ \alpha \sharp* P; bn\ \alpha \sharp* subject\ \alpha; bn\ \alpha \sharp* C; distinct(bn\ \alpha)]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (\alpha \prec (P' \parallel !P))$
by *fact*
from ‹$bangPred\ P\ (Q \parallel R)$› **have** $Q = P$ **and** $R = !P$
by − (*ind-cases bangPred* P ($Q \parallel R$), *auto simp add*: *psi.inject*)+
from ‹$R = !P$› ‹$extractFrame\ R = \langle A_R, \Psi_R\rangle$› **have** $A_R = []$ **and** $\Psi_R = \mathbf{1}$ **by** *auto*
from ‹$\Psi' \otimes \Psi_R \rhd Q \longmapsto \alpha \prec P'$› ‹$Q = P$› ‹$\Psi \simeq \Psi'$› ‹$\Psi_R = \mathbf{1}$› **have** $\Psi \rhd P \longmapsto \alpha \prec P'$
by(*metis statEqTransition Identity AssertionStatEqSym*)
hence $Prop\ C\ \Psi\ (P \parallel !P)\ (\alpha \prec (P' \parallel !P))$ **using** ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$Q = P$› ‹$distinct(bn\ \alpha)$›
by(*rule-tac rPar1*) *auto*

with ‹$R = !P$› ‹$Q = P$› **show** *?case* **by** *simp*
next
case(*cPar2* Ψ' Ψ_P R α P' Q A_P Ψ C)
have *rPar2*: $\bigwedge\alpha\ P'\ C.\ [\![\Psi \rhd !P \longmapsto\alpha \prec P'$; *bn* $\alpha \sharp* \Psi$; *bn* $\alpha \sharp* P$; *bn* $\alpha \sharp*$ *subject* α; *bn* $\alpha \sharp* C$; *distinct*(*bn* α)$;$
$\bigwedge C.$ *Prop* C Ψ $(!P)$ $(\alpha \prec P')]\!] \Longrightarrow$ *Prop* C Ψ $(P \parallel !P)$ $(\alpha \prec (P \parallel P'))$
by *fact*
from ‹*bangPred* P $(Q \parallel R)$› **have** $Q = P$ **and** $R = !P$
by $-$ (*ind-cases bangPred* P $(Q \parallel R)$, *auto simp add*: *psi.inject*)+
from ‹$Q = P$› ‹*extractFrame* $Q = \langle A_P, \Psi_P\rangle$› ‹*guarded* P› **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::$*name set*$)$
by(*blast dest*: *guardedStatEq*)+
from ‹$\Psi' \otimes \Psi_P \rhd R \longmapsto\alpha \prec P'$› ‹$R = !P$› ‹$\Psi \simeq \Psi'$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \rhd !P \longmapsto\alpha \prec P'$
by(*metis statEqTransition Identity Composition Commutativity AssertionStatEqSym*)
moreover
{
fix C
have *bangPred* P $(!P)$ **by**(*rule aux1*)
moreover from ‹$\Psi \simeq \Psi'$› ‹$\Psi_P \simeq \mathbf{1}$›
have $\Psi \simeq \Psi' \otimes \Psi_P$ **by**(*metis Composition Identity Commutativity AssertionStatEqSym AssertionStatEqTrans*)
ultimately have $\bigwedge C.$ *Prop* C Ψ $(!P)$ $(\alpha \prec P')$ **using** *cPar2* ‹$R = !P$› ‹*guarded* P› **by** *simp*
}
ultimately have *Prop* C Ψ $(P \parallel !P)$ $(\alpha \prec (P \parallel P'))$ **using** ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha \sharp* C$› ‹$Q = P$› ‹*distinct*(*bn* α)›
by(*rule-tac rPar2*) *auto*
with ‹$R = !P$› ‹$Q = P$› **show** *?case* **by** *simp*
next
case(*cComm1* Ψ' Ψ_R Q M N P' A_P Ψ_P R K *xvec* P'' A_R Ψ C)
have *rComm1*: $\bigwedge M\ N\ P'\ K$ *xvec* $P''\ C.\ [\![\Psi \rhd P \longmapsto M\langle\!(N)\!\rangle \prec P'$; $\Psi \rhd !P \longmapsto K\langle\!(\nu* xvec)\!\rangle\langle N\rangle \prec P''$; $\bigwedge C.$ *Prop* C Ψ $(!P)$ $(K\langle\!(\nu* xvec)\!\rangle\langle N\rangle \prec P'')$; $\Psi \vdash M \leftrightarrow K$;
xvec $\sharp* \Psi$; *xvec* $\sharp* P$; *xvec* $\sharp* M$; *xvec* $\sharp* K$; *xvec* $\sharp* C$; *distinct xvec*$]\!] \Longrightarrow$ *Prop* C Ψ $(P \parallel !P)$ $(\tau \prec \langle\!(\nu* xvec)\!\rangle(P' \parallel P''))$
by *fact*
from ‹*bangPred* P $(Q \parallel R)$› **have** $Q = P$ **and** $R = !P$
by $-$ (*ind-cases bangPred* P $(Q \parallel R)$, *auto simp add*: *psi.inject*)+
from ‹$R = !P$› ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› **have** $A_R = []$ **and** $\Psi_R = \mathbf{1}$ **by** *auto*
from ‹$\Psi' \otimes \Psi_R \rhd Q \longmapsto M\langle\!(N)\!\rangle \prec P'$› ‹$Q = P$› ‹$\Psi \simeq \Psi'$› ‹$\Psi_R = \mathbf{1}$› **have** $\Psi \rhd P \longmapsto M\langle\!(N)\!\rangle \prec P'$
by(*metis statEqTransition Identity AssertionStatEqSym*)
moreover from ‹$Q = P$› ‹*extractFrame* $Q = \langle A_P, \Psi_P\rangle$› ‹*guarded* P› **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::$*name set*$)$
by(*blast dest*: *guardedStatEq*)+
moreover from ‹$\Psi' \otimes \Psi_P \rhd R \longmapsto K\langle\!(\nu* xvec)\!\rangle\langle N\rangle \prec P''$› ‹$R = !P$› ‹$\Psi_P \simeq$

1› ‹$\Psi \simeq \Psi'$› **have** $\Psi \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P''$
by(*metis statEqTransition Identity Composition Commutativity AssertionStatEqSym*)
moreover
{
fix *C*
have *bangPred P* (!*P*) **by**(*rule aux1*)
moreover from ‹$\Psi \simeq \Psi'$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \simeq \Psi' \otimes \Psi_P$ **by**(*metis Composition Identity Commutativity AssertionStatEqSym AssertionStatEqTrans*)
ultimately have $\bigwedge C.$ *Prop C* Ψ (!*P*) ($K(\!|\nu * xvec|\!)\langle N\rangle \prec P''$) **using** *cComm1* ‹$R = !P$› ‹*guarded P*› **by** *simp*
}
moreover from ‹$\Psi' \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› ‹$\Psi \simeq \Psi'$› ‹$\Psi_R = \mathbf{1}$› **have** $\Psi \vdash M \leftrightarrow K$
by(*metis statEqEnt Identity Composition Commutativity AssertionStatEqSym*)
ultimately have *Prop C* Ψ ($P \parallel !P$) ($\tau \prec (\!|\nu * xvec|\!)(P' \parallel P'')$) **using** ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *Q*› ‹*xvec* $\sharp*$ *M*› ‹*xvec* $\sharp*$ *K*› ‹*xvec* $\sharp*$ *C*› ‹$Q = P$› ‹*distinct xvec*›
by(*rule-tac rComm1*) (*assumption* | *auto*)+
with ‹$R = !P$› ‹$Q = P$› **show** *?case* **by** *simp*
next
case(*cComm2* Ψ' Ψ_R *Q M xvec N P'* A_P Ψ_P *R K P''* A_R Ψ *C*)
have *rComm2*: $\bigwedge M$ *xvec N P' K P'' C*. $[\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'; \Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''; \bigwedge C.$ *Prop C* Ψ (!*P*) ($K(\!|N|\!) \prec P''$); $\Psi \vdash M \leftrightarrow K$; *xvec* $\sharp*$ Ψ; *xvec* $\sharp*$ *P*; *xvec* $\sharp*$ *M*; *xvec* $\sharp*$ *K*; *xvec* $\sharp*$ *C*; *distinct xvec*$]\!] \Longrightarrow$ *Prop C* Ψ ($P \parallel !P$) ($\tau \prec (\!|\nu * xvec|\!)(P' \parallel P'')$)
by *fact*
from ‹*bangPred P* ($Q \parallel R$)› **have** $Q = P$ **and** $R = !P$
by − (*ind-cases bangPred P* ($Q \parallel R$), *auto simp add*: *psi.inject*)+
from ‹$R = !P$› ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› **have** $A_R = []$ **and** $\Psi_R = \mathbf{1}$ **by** *auto*
from ‹$\Psi' \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$› ‹$Q = P$› ‹$\Psi \simeq \Psi'$› ‹$\Psi_R = \mathbf{1}$› **have** $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*metis statEqTransition Identity AssertionStatEqSym*)
moreover from ‹$Q = P$› ‹*extractFrame* $Q = \langle A_P, \Psi_P\rangle$› ‹*guarded P*› **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::name\ set)$
by(*blast dest*: *guardedStatEq*)+
moreover from ‹$\Psi' \otimes \Psi_P \rhd R \longmapsto K(\!|N|\!) \prec P''$› ‹$R = !P$› ‹$\Psi_P \simeq \mathbf{1}$› ‹$\Psi \simeq \Psi'$› **have** $\Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''$
by(*metis statEqTransition Identity Composition Commutativity AssertionStatEqSym*)
moreover
{
fix *C*
have *bangPred P* (!*P*) **by**(*rule aux1*)
moreover from ‹$\Psi \simeq \Psi'$› ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \simeq \Psi' \otimes \Psi_P$ **by**(*metis Composition Identity Commutativity AssertionStatEqSym AssertionStatEqTrans*)
ultimately have $\bigwedge C.$ *Prop C* Ψ (!*P*) ($K(\!|N|\!) \prec P''$) **using** *cComm2* ‹$R = !P$› ‹*guarded P*› **by** *simp*

```
    }
    moreover from ‹Ψ′ ⊗ Ψ_P ⊗ Ψ_R ⊢ M ↔ K› ‹Ψ_P ≃ 1› ‹Ψ ≃ Ψ′› ‹Ψ_R = 1›
have Ψ ⊢ M ↔ K
      by(metis statEqEnt Identity Composition Commutativity AssertionStatEqSym)
    ultimately have Prop C Ψ (P ∥ !P) (τ ≺ (⦇νxvec⦈)(P′ ∥ P′′)) using ‹xvec ♯* Ψ› ‹xvec ♯* Q› ‹xvec ♯* M› ‹xvec ♯* K› ‹xvec ♯* C› ‹Q = P› ‹distinct xvec›
      by(rule-tac rComm2) (assumption | auto)+
    with ‹R = !P› ‹Q = P› show ?case by simp
  next
    case(cOpen Ψ Q M xvec yvec N P′ x C)
    thus ?case by − (ind-cases bangPred P (⦇νx⦈Q))
  next
    case(cScope Ψ Q α P′ x C)
    thus ?case by − (ind-cases bangPred P (⦇νx⦈Q))
  next
    case(Bang Ψ′ Q Rs Ψ C)
     have rBang: ⋀Rs C. ⟦Ψ ▷ P ∥ !P ⟼ Rs; ⋀C. Prop C Ψ (P ∥ !P) Rs; guarded P⟧ ⟹ Prop C Ψ (!P) Rs
      by fact
    from ‹bangPred P (!Q)› have P = Q
      by − (ind-cases bangPred P (!Q), auto simp add: psi.inject)
     with ‹Ψ′ ▷ Q ∥ !Q ⟼ Rs› ‹Ψ ≃ Ψ′› have Ψ ▷ P ∥ !P ⟼ Rs by(metis statEqTransition AssertionStatEqSym)
    moreover
    {
      fix C
      have bangPred P (P ∥ !P) by(rule aux2)
      with Bang ‹P = Q› have ⋀C. Prop C Ψ (P ∥ !P) Rs by simp
    }
    moreover from ‹guarded Q› ‹P = Q› have guarded P by simp
    ultimately have Prop C Ψ (!P) Rs by(rule rBang)
    with ‹P = Q› show ?case by simp
  qed
 }
 with ‹guarded P› ‹Ψ ▷ !P ⟼ Rs›
 show ?thesis by(force intro: aux1)
qed

lemma bangInputInduct[consumes 1, case-names cPar1 cPar2 cBang]:
 fixes Ψ  :: 'b
 and   P  :: ('a, 'b, 'c) psi
 and   M  :: 'a
 and   N  :: 'a
 and   P′ :: ('a, 'b, 'c) psi
 and   Prop :: 'b ⇒ ('a, 'b, 'c) psi ⇒ 'a ⇒ 'a ⇒ ('a, 'b, 'c) psi ⇒ bool

 assumes Ψ ▷ !P ⟼M(⦇N⦈) ≺ P′
 and     rPar1: ⋀P′. Ψ ▷ P ⟼M(⦇N⦈) ≺ P′ ⟹ Prop Ψ (P ∥ !P) M N (P′ ∥
```

$!P$)
  and *rPar2*: $\bigwedge P'.\ [\![\Psi \rhd\ !P \longmapsto M(\!|N|\!) \prec P';\ Prop\ \Psi\ (!P)\ M\ N\ P']\!] \Longrightarrow Prop\ \Psi\ (P \parallel !P)\ M\ N\ (P \parallel P')$
  and *rBang*: $\bigwedge P'.\ [\![\Psi \rhd\ P \parallel !P \longmapsto M(\!|N|\!) \prec P';\ Prop\ \Psi\ (P \parallel !P)\ M\ N\ P';\ guarded\ P]\!] \Longrightarrow Prop\ \Psi\ (!P)\ M\ N\ P'$
  shows $Prop\ \Psi\ (!P)\ M\ N\ P'$
using $\langle \Psi \rhd\ !P \longmapsto M(\!|N|\!) \prec P' \rangle$
by(*nominal-induct* $\Psi\ P\ Rs{==}M(\!|N|\!) \prec P'$ *arbitrary*: P' *rule*: *bangInduct*)
  (*auto simp add*: *residualInject intro*: *rPar1 rPar2 rBang*)

lemma *bangOutputInduct*[*consumes 1*, *case-names cPar1 cPar2 cBang*]:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)\ psi$
  and M :: $'a$
  and *xvec* :: *name list*
  and N :: $'a$
  and P' :: $('a, 'b, 'c)\ psi$
  and *Prop* :: $'d$::*fs-name* $\Rightarrow$ $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow 'a \Rightarrow ('a, 'b, 'c)\ boundOutput \Rightarrow bool$
  and C :: $'d$

  assumes $\Psi \rhd\ !P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
  and *rPar1*: $\bigwedge xvec\ N\ P'\ C.\ [\![\Psi \rhd\ P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ xvec \sharp{*} \Psi;\ xvec \sharp{*} P;\ xvec \sharp{*} M;\ xvec \sharp{*} C;\ distinct\ xvec]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ M\ (\!|\nu{*}xvec|\!)N \prec' (P' \parallel !P))$
  and *rPar2*: $\bigwedge xvec\ N\ P'\ C.\ [\![\Psi \rhd\ !P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ M\ (\!|\nu{*}xvec|\!)N \prec' P');\ xvec \sharp{*} \Psi;\ xvec \sharp{*} P;\ xvec \sharp{*} M;\ xvec \sharp{*} C;\ distinct\ xvec]\!] \Longrightarrow$
    $Prop\ C\ \Psi\ (P \parallel !P)\ M\ (\!|\nu{*}xvec|\!)N \prec' (P \parallel P'))$
  and *rBang*: $\bigwedge B\ C.\ [\![\Psi \rhd\ P \parallel !P \longmapsto (ROut\ M\ B);\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel !P)\ M\ B;\ guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ M\ B$

  shows $Prop\ C\ \Psi\ (!P)\ M\ (\!|\nu{*}xvec|\!)N \prec' P'$)
using $\langle \Psi \rhd\ !P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P' \rangle$
apply(*auto simp add*: *residualInject*)
by(*nominal-induct* $\Psi\ P\ Rs{==}ROut\ M\ ((\!|\nu{*}xvec|\!)N \prec' P')$ *avoiding*: C *arbitrary*: *xvec* $N\ P'$ *rule*: *bangInduct*)
  (*force simp add*: *residualInject intro*: *rPar1 rPar2 rBang*)+

lemma *bangTauInduct*[*consumes 1*, *case-names cPar1 cPar2 cComm1 cComm2 cBang*]:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)\ psi$
  and P' :: $('a, 'b, 'c)\ psi$
  and *Prop* :: $'d$::*fs-name* $\Rightarrow$ $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow bool$
  and C :: $'d$

  assumes $\Psi \rhd\ !P \longmapsto \tau \prec P'$
  and *rPar1*: $\bigwedge P'\ C.\ \Psi \rhd\ P \longmapsto \tau \prec P' \Longrightarrow Prop\ C\ \Psi\ (P \parallel !P)\ (P' \parallel !P)$

and *rPar2*: $\bigwedge P'\ C.\ [\![\Psi \rhd\ !P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ P']\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel\ !P)\ (P \parallel P')$
and *rComm1*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \Psi \rhd\ !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'';\ \Psi \vdash M \leftrightarrow K;$
$xvec \sharp * \Psi;\ xvec \sharp * P;\ xvec \sharp * M;\ xvec \sharp * K;\ xvec \sharp * C]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel\ !P)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
and *rComm2*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P';\ \Psi \rhd\ !P \longmapsto K(\!|N|\!) \prec P'';\ \Psi \vdash M \leftrightarrow K;$
$xvec \sharp * \Psi;\ xvec \sharp * P;\ xvec \sharp * M;\ xvec \sharp * K;\ xvec \sharp * C]\!] \Longrightarrow Prop\ C\ \Psi\ (P \parallel\ !P)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
and *rBang*: $\bigwedge P'\ C.\ [\![\Psi \rhd P \parallel\ !P \longmapsto \tau \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (P \parallel\ !P)\ P';\ guarded\ P]\!] \Longrightarrow Prop\ C\ \Psi\ (!P)\ P'$

shows $Prop\ C\ \Psi\ (!P)\ P'$
using $\langle \Psi \rhd\ !P \longmapsto \tau \prec P' \rangle$
by(*nominal-induct* $\Psi\ P\ Rs == \tau \prec P'$ *avoiding*: *C arbitrary*: P' *rule*: *bangInduct*)
(*auto simp add*: *residualInject intro*: *rPar1 rPar2 rComm1 rComm2 rBang*)

lemma *bangInduct'*[*consumes 2*, *case-names cAlpha cPar1 cPar2 cComm1 cComm2 cBang*]:
fixes Ψ :: *'b*
and P :: *('a, 'b, 'c) psi*
and α :: *'a action*
and P' :: *('a, 'b, 'c) psi*
and *Prop* :: *'d::fs-name* ⇒ *'b* ⇒ *('a, 'b, 'c) psi* ⇒ *'a action* ⇒ *('a, 'b, 'c) psi* ⇒ *bool*
and C :: *'d::fs-name*

assumes $\Psi \rhd\ !P \longmapsto \alpha \prec P'$
and $bn\ \alpha \sharp * subject\ \alpha$
and *rAlpha*: $\bigwedge \alpha\ P'\ p\ C.\ [\![bn\ \alpha \sharp * \Psi;\ bn\ \alpha \sharp * P;\ bn\ \alpha \sharp * subject\ \alpha;\ bn\ \alpha \sharp * C;$
$set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha));\ distinctPerm\ p;$
$bn(p \cdot \alpha) \sharp * \alpha;\ bn(p \cdot \alpha) \sharp * P';\ Prop\ C\ \Psi\ (P \parallel\ !P)\ \alpha\ P']\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel\ !P)\ (p \cdot \alpha)\ (p \cdot P')$
and *rPar1*: $\bigwedge \alpha\ P'\ C.$
$[\![\Psi \rhd P \longmapsto \alpha \prec P';\ bn\ \alpha \sharp * \Psi;\ bn\ \alpha \sharp * P;\ bn\ \alpha \sharp * subject\ \alpha;\ bn\ \alpha \sharp * C;\ distinct(bn\ \alpha)]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel\ !P)\ \alpha\ (P' \parallel\ !P)$
and *rPar2*: $\bigwedge \alpha\ P'\ C.$
$[\![\Psi \rhd\ !P \longmapsto \alpha \prec P';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ \alpha\ P';$
$bn\ \alpha \sharp * \Psi;\ bn\ \alpha \sharp * P;\ bn\ \alpha \sharp * subject\ \alpha;\ bn\ \alpha \sharp * C;\ distinct(bn\ \alpha)]\!] \Longrightarrow$
$Prop\ C\ \Psi\ (P \parallel\ !P)\ \alpha\ (P \parallel P')$
and *rComm1*: $\bigwedge M\ N\ P'\ K\ xvec\ P''\ C.\ [\![\Psi \rhd P \longmapsto M(\!|N|\!) \prec P';\ \Psi \rhd\ !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'';\ \bigwedge C.\ Prop\ C\ \Psi\ (!P)\ (K(\!|\nu * xvec|\!)\langle N\rangle)\ P'';\ \Psi \vdash M \leftrightarrow K;$
$xvec \sharp * \Psi;\ xvec \sharp * P;\ xvec \sharp * M;\ xvec \sharp * K;$

xvec $\sharp*$ *C*; *distinct xvec*$\rrbracket \Longrightarrow$ *Prop C* Ψ (*P* $\parallel$!*P*) (τ) ($(\!|\nu{*}xvec|\!)$(*P'* $\parallel$ *P''*))
and *rComm2*: $\bigwedge$*M xvec N P' K P'' C*. $\llbracket\Psi \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$; $\Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''$; $\bigwedge C$. *Prop C* Ψ (!*P*) (*K*$(\!|N|\!)$) *P''*; $\Psi \vdash M \leftrightarrow K$; *xvec* $\sharp*$ Ψ; *xvec* $\sharp*$ *P*; *xvec* $\sharp*$ *M*; *xvec* $\sharp*$ *K*; *xvec* $\sharp*$ *C*; *distinct xvec*$\rrbracket \Longrightarrow$ *Prop C* Ψ (*P* $\parallel$!*P*) (τ) ($(\!|\nu{*}xvec|\!)$(*P'* $\parallel$ *P''*))
and *rBang*: $\bigwedge\alpha$ *P' C*. $\llbracket\Psi \rhd P \parallel !P \longmapsto\alpha \prec P'$; *guarded P*; $\bigwedge C$. *Prop C* Ψ (*P* $\parallel$!*P*) α *P'*; *guarded P*; *distinct*(*bn* α)$\rrbracket \Longrightarrow$ *Prop C* Ψ (!*P*) α *P'*
shows *Prop C* Ψ (!*P*) α *P'*
proof −
from ‹$\Psi \rhd !P \longmapsto\alpha \prec P'$› **have** *distinct*(*bn* α) **by**(*rule boundOutputDistinct*)
with ‹$\Psi \rhd !P \longmapsto\alpha \prec P'$› ‹*bn* α $\sharp*$ *subject* α› **show** *?thesis*
proof(*nominal-induct* Ψ *P Rs*==$\alpha \prec P'$ *avoiding*: *C* α *P'* *rule*: *bangInduct*)
case(*cPar1* α *P' C* α' *P''*)
note ‹$\alpha \prec (P' \parallel !P) = \alpha' \prec P''$›
moreover from ‹*bn* α $\sharp*$ α'› **have** *bn* α $\sharp*$ *bn* α' **by** *simp*
moreover note ‹*distinct*(*bn* α)› ‹*distinct*(*bn* α')›
moreover from ‹*bn* α $\sharp*$ *subject* α› **have** *bn* α $\sharp*$ ($\alpha \prec P' \parallel !P$) **by** *simp*
moreover from ‹*bn* α' $\sharp*$ *subject* α'› **have** *bn* α' $\sharp*$ ($\alpha' \prec P''$) **by** *simp*
ultimately obtain *p* **where** *S*: *set p* $\subseteq$ *set*(*bn* α) $\times$ *set*(*bn*(*p* $\cdot$ α)) **and** *distinctPerm p* **and** $\alpha' = p \cdot \alpha$ **and** *P'eq*: *P''* = *p* $\cdot$ (*P'* $\parallel$!*P*) **and** *bn*(*p* $\cdot$ α) $\sharp*$ α **and** *bn*(*p* $\cdot$ α) $\sharp*$ (*P'* $\parallel$!*P*)
by(*rule-tac residualEq*)

from ‹$\Psi \rhd P \longmapsto\alpha \prec P'$› ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ *P*› ‹*bn* α $\sharp*$ *subject* α› ‹*bn* α $\sharp*$ *C*› ‹*distinct*(*bn* α)›
have *Prop C* Ψ (*P* $\parallel$!*P*) α (*P'* $\parallel$!*P*)
by(*rule rPar1*)
with ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ *P*› ‹*bn* α $\sharp*$ *subject* α› ‹*bn* α $\sharp*$ *C*› *S* ‹*distinctPerm p*› ‹*bn* α $\sharp*$ α'› ‹*bn* α $\sharp*$ *P''*› ‹$\alpha' = (p \cdot \alpha)$› *P'eq* ‹*bn*(*p* $\cdot$ α) $\sharp*$ α› ‹*bn*(*p* $\cdot$ α) $\sharp*$ (*P'* $\parallel$!*P*)›
have *Prop C* Ψ (*P* $\parallel$!*P*) (*p* $\cdot$ α) (*p* $\cdot$ (*P'* $\parallel$!*P*))
by(*rule-tac rAlpha*)
with *P'eq* ‹$\alpha' = p \cdot \alpha$› ‹*distinctPerm p*› **show** *?case* **by** *simp*
next
case(*cPar2* α *P' C* α' *P''*)
note ‹$\alpha \prec (P \parallel P') = \alpha' \prec P''$›
moreover from ‹*bn* α $\sharp*$ α'› **have** *bn* α $\sharp*$ *bn* α' **by** *simp*
moreover note ‹*distinct*(*bn* α)› ‹*distinct*(*bn* α')›
moreover from ‹*bn* α $\sharp*$ *subject* α› **have** *bn* α $\sharp*$ ($\alpha \prec P \parallel P'$) **by** *simp*
moreover from ‹*bn* α' $\sharp*$ *subject* α'› **have** *bn* α' $\sharp*$ ($\alpha' \prec P''$) **by** *simp*
ultimately obtain *p* **where** *S*: *set p* $\subseteq$ *set*(*bn* α) $\times$ *set*(*bn*(*p* $\cdot$ α)) **and** *distinctPerm p* **and** $\alpha' = p \cdot \alpha$ **and** *P'eq*: *P''* = *p* $\cdot$ (*P* $\parallel$ *P'*) **and** *bn*(*p* $\cdot$ α) $\sharp*$ α **and** *bn*(*p* $\cdot$ α) $\sharp*$ (*P* $\parallel$ *P'*)
by(*rule-tac residualEq*)

note ‹$\Psi \rhd !P \longmapsto \alpha \prec P'$›
moreover from ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$distinct(bn\ \alpha)$› **have** $\bigwedge C.\ Prop\ C\ \Psi\ (!P)\ \alpha\ P'$ **by**(*rule-tac cPar2*) *auto*
moreover note ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$distinct(bn\ \alpha)$›
ultimately have $Prop\ C\ \Psi\ (P \parallel !P)\ \alpha\ (P \parallel P')$
by(*rule rPar2*)
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$bn\ \alpha \sharp* C$› S ‹$distinctPerm\ p$› ‹$bn\ \alpha \sharp* \alpha'$› ‹$bn\ \alpha \sharp* P''$› ‹$\alpha' = (p \cdot \alpha)$› $P'eq$ ‹$bn(p \cdot \alpha) \sharp* \alpha$› ‹$bn(p \cdot \alpha) \sharp* (P \parallel P')$›
have $Prop\ C\ \Psi\ (P \parallel !P)\ (p \cdot \alpha)\ (p \cdot (P \parallel P'))$
by(*rule-tac rAlpha*)
with $P'eq$ ‹$\alpha' = p \cdot \alpha$› **show** *?case* **by** *simp*
next
case($cComm1\ M\ N\ P'\ K\ xvec\ P''\ C\ \alpha\ P'''$)
hence $Prop\ C\ \Psi\ (P \parallel !P)\ (\tau)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
by(*rule-tac rComm1*) (*assumption* | *simp*)+
thus *?case* **using** ‹$\tau \prec (\!|\nu * xvec|\!)(P' \parallel P'') = \alpha \prec P'''$›
by(*simp add*: *residualInject*)
next
case($cComm2\ M\ xvec\ N\ P'\ K\ P''\ C\ \alpha\ P'''$)
hence $Prop\ C\ \Psi\ (P \parallel !P)\ (\tau)\ ((\!|\nu * xvec|\!)(P' \parallel P''))$
by(*rule-tac rComm2*) (*assumption* | *simp*)+
thus *?case* **using** ‹$\tau \prec (\!|\nu * xvec|\!)(P' \parallel P'') = \alpha \prec P'''$›
by(*simp add*: *residualInject*)
next
case($cBang\ C\ \alpha\ P'$)
thus *?case* **by**(*auto intro*: *rBang*)
qed
qed

lemma *comm1Aux*:
fixes Ψ :: $'b$
and Ψ_Q :: $'b$
and R :: $('a, 'b, 'c)\ psi$
and K :: $'a$
and $xvec$ :: *name list*
and N :: $'a$
and R' :: $('a, 'b, 'c)\ psi$
and A_R :: *name list*
and Ψ_R :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and M :: $'a$
and L :: $'a$
and P' :: $('a, 'b, 'c)\ psi$
and A_P :: *name list*
and Ψ_P :: $'b$
and A_Q :: *name list*

assumes *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec R'$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
and *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|L|\!) \prec P'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$
and *PeqQ*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *distinct* A_P
and *distinct* A_R
and $A_R \sharp* A_P$
and $A_R \sharp* A_Q$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
and $A_R \sharp* R$
and $A_R \sharp* K$
and $A_P \sharp* \Psi$
and $A_P \sharp* R$
and $A_P \sharp* P$
and $A_P \sharp* M$
and $A_Q \sharp* R$
and $A_Q \sharp* M$

obtains K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
proof −
from $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$ $\langle A_R \sharp* A_P\rangle$ $\langle A_R \sharp* A_Q\rangle$ *FrP FrQ* **have** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+
assume *Assumptions*: $\bigwedge K'.\ [\![\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec R';\ \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K';\ A_R \sharp* K']\!] \Longrightarrow$ *thesis*
{
fix Ψ' ::*'b*
and *zvec* ::*name list*

assume *A*: $\Psi \otimes \Psi_Q \simeq \Psi'$
assume $\Psi' \rhd R \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec R'$
hence $\Psi' \rhd R \longmapsto ROut\ K\ ((\!|\nu * xvec|\!)N \prec' R')$ **by**(*simp add*: *residualInject*)
moreover note *FrR* ‹*distinct* A_R› *PTrans*
moreover from $\langle \Psi' \rhd R \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec R'\rangle$ **have** *distinct xvec* **by**(*auto dest*: *boundOutputDistinct*)
moreover assume $\Psi' \otimes \Psi_R \vdash M \leftrightarrow K$ **and** $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
and $A_R \sharp* zvec$ **and** $A_P \sharp* zvec$ **and** $zvec \sharp* R$ **and** $zvec \sharp* P$
and $A_R \sharp* \Psi'$
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu * xvec|\!)N \prec' R') \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge zvec \sharp* K' \wedge A_R \sharp* K'$
using *FrP* $\langle A_P \sharp* R\rangle$ $\langle A_Q \sharp* R\rangle$ $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ ‹*distinct* A_P› $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* R\rangle$

$\langle A_P \sharp* M\rangle$ $\langle A_Q \sharp* M\rangle$ $\langle A_R \sharp* K\rangle$ $\langle A_R \sharp* A_P\rangle$ $\langle A_R \sharp* A_Q\rangle$ $\langle A_R \sharp* \Psi_P\rangle$
proof(*nominal-induct* Ψ' R K $B{=}{=}(\!|\nu{*}xvec|\!)N \prec' R'$ A_R Ψ_R *avoiding*: Ψ P A_P Ψ_P A_Q *zvec xvec* N R' *arbitrary*: M *rule*: *outputFrameInduct*)
case(*cAlpha* Ψ' R K A_R Ψ_R p Ψ P A_P Ψ_P A_Q *zvec xvec* N R' M)
have S: *set* $p \subseteq$ *set* $A_R \times$ *set* $(p \cdot A_R)$ **by** *fact*
from $\langle \Psi' \otimes (p \cdot \Psi_R) \vdash M \leftrightarrow K\rangle$ **have** $(p \cdot (\Psi' \otimes (p \cdot \Psi_R))) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$
by(*rule chanEqClosed*)
with $\langle A_R \sharp* \Psi'\rangle$ $\langle (p \cdot A_R) \sharp* \Psi'\rangle$ $\langle A_R \sharp* K\rangle$ $\langle (p \cdot A_R) \sharp* K\rangle$ S $\langle$*distinctPerm* $p\rangle$
have $\Psi' \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K$ **by**(*simp add*: *eqvts*)
moreover from $\langle \Psi \otimes (p \cdot \Psi_R) \rhd P \longmapsto M(\!|L|\!) \prec P'\rangle$ S $\langle A_R \sharp* P\rangle$ $\langle (p \cdot A_R) \sharp* P\rangle$ **have** $(p \cdot (\Psi \otimes (p \cdot \Psi_R))) \rhd P \longmapsto (p \cdot M)(\!|L|\!) \prec P'$
by(*rule-tac inputPermFrameSubject*) *auto*
with $\langle A_R \sharp* \Psi\rangle$ $\langle (p \cdot A_R) \sharp* \Psi\rangle$ S $\langle$*distinctPerm* $p\rangle$ **have** $\Psi \otimes \Psi_R \rhd P \longmapsto (p \cdot M)(\!|L|\!) \prec P'$
by(*simp add*: *eqvts*)
moreover from $\langle A_P \sharp* M\rangle$ **have** $(p \cdot A_P) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_R \sharp* A_P\rangle$ $\langle (p \cdot A_R) \sharp* A_P\rangle$ S **have** $A_P \sharp* (p \cdot M)$ **by** *simp*
moreover from $\langle A_Q \sharp* M\rangle$ **have** $(p \cdot A_Q) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_R \sharp* A_Q\rangle$ $\langle (p \cdot A_R) \sharp* A_Q\rangle$ S **have** $A_Q \sharp* (p \cdot M)$ **by** *simp*
moreover from $\langle \langle A_Q, \Psi' \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle\rangle$
have $(p \cdot \langle A_Q, \Psi' \otimes (p \cdot \Psi_R)\rangle) \hookrightarrow_F (p \cdot \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle)$
by(*rule FrameStatImpClosed*)
with $\langle A_R \sharp* A_P\rangle$ $\langle (p \cdot A_R) \sharp* A_P\rangle$ $\langle A_R \sharp* \Psi'\rangle$ $\langle (p \cdot A_R) \sharp* \Psi'\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle (p \cdot A_R) \sharp* \Psi_P\rangle$ $\langle A_R \sharp* A_Q\rangle$
$\langle (p \cdot A_R) \sharp* A_Q\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle (p \cdot A_R) \sharp* \Psi\rangle$ S $\langle$*distinctPerm* $p\rangle$
have $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$ **by**(*simp add*: *eqvts*)
ultimately obtain K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu{*}xvec|\!)N \prec' R')$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K'$ **and** *zvec* $\sharp*$ K' **and** $A_R \sharp* K'$
using *cAlpha*
by *metis*
from $\langle \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu{*}xvec|\!)N \prec' R')\rangle$ S $\langle A_R \sharp* R\rangle$ $\langle (p \cdot A_R) \sharp* R\rangle$
have $(p \cdot (\Psi \otimes \Psi_P)) \rhd R \longmapsto (ROut\ (p \cdot K')\ ((\!|\nu{*}xvec|\!)N \prec' R'))$ **using** *outputPermFrameSubject*
by(*auto simp add*: *residualInject*)
with S $\langle A_R \sharp* \Psi\rangle$ $\langle (p \cdot A_R) \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle (p \cdot A_R) \sharp* \Psi_P\rangle$ **have** $\Psi \otimes \Psi_P \rhd R \longmapsto (ROut\ (p \cdot K')\ ((\!|\nu{*}xvec|\!)N \prec' R'))$
by(*simp add*: *eqvts*)
moreover from $\langle \Psi \otimes \Psi_P \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K'\rangle$ **have** $(p \cdot (\Psi \otimes \Psi_P \otimes \Psi_R)) \vdash (p \cdot p \cdot M) \leftrightarrow (p \cdot K')$
by(*rule chanEqClosed*)
with S $\langle A_R \sharp* \Psi\rangle$ $\langle (p \cdot A_R) \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle (p \cdot A_R) \sharp* \Psi_P\rangle$ $\langle$*distinctPerm* $p\rangle$ **have** $\Psi \otimes \Psi_P \otimes (p \cdot \Psi_R) \vdash M \leftrightarrow (p \cdot K')$
by(*simp add*: *eqvts*)
moreover from $\langle$*zvec* $\sharp*$ $K'\rangle$ **have** $(p \cdot zvec) \sharp* (p \cdot K')$

by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* zvec$› ‹$(p \cdot A_R) \sharp* zvec$› S **have** $zvec \sharp* (p \cdot K')$ **by** *simp*
moreover from ‹$A_R \sharp* K'$› **have** $(p \cdot A_R) \sharp* (p \cdot K')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **by** *blast*
next
case(*cOutput* Ψ' M' K N R Ψ P A_P Ψ_P A_Q *zvec xvec* N' R' M)
from ‹$A_P \sharp* (M'\langle N\rangle.R)$› ‹$A_Q \sharp* (M'\langle N\rangle.R)$› ‹$zvec \sharp* (M'\langle N\rangle.R)$›
have $A_P \sharp* M'$ **and** $A_Q \sharp* M'$ **and** $zvec \sharp* M'$ **by** *simp+*

from ‹$\Psi' \vdash M' \leftrightarrow K$› **have** $\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow K$ **by**(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
hence $\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow M'$ **by**(*blast intro*: *chanEqSym chanEqTrans*)
with ‹$A_Q \sharp* M'$› **have** $(\langle A_Q, \Psi' \otimes \mathbf{1}\rangle) \vdash_F M' \leftrightarrow M'$ **by**(*force intro*: *frameImpI*)

with ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$› **have** $(\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F M' \leftrightarrow M'$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_P \sharp* M'$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash M' \leftrightarrow M'$ **by**(*force dest*: *frameImpE*)
hence $\Psi \otimes \Psi_P \vdash M' \leftrightarrow M'$ **by**(*blast intro*: *statEqEnt Identity*) **hence** $\Psi \otimes \Psi_P \rhd M'\langle N\rangle.R \longmapsto M'\langle N\rangle \prec R$
by(*rule Output*)

moreover from ‹$\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow K$› ‹$\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow K$›
have $\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'$ **by**(*metis chanEqSym chanEqTrans*)
with ‹$A_Q \sharp* M$› ‹$A_Q \sharp* M'$›
have $(\langle A_Q, \Psi' \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow M'$
by(*force intro*: *frameImpI*)
with ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$›
have $(\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow M'$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_P \sharp* M$› ‹$A_P \sharp* M'$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*force dest*: *frameImpE*)
hence $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity*)
ultimately show *?case* **using** *cOutput* **by**(*auto simp add*: *residualInject*)
next
case(*cCase* Ψ' R M' φ *Cs* A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec xvec* N R' M)
from ‹*guarded R*› ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› **have** $\Psi_R \simeq \mathbf{1}$
by(*metis guardedStatEq*)
with ‹$\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'$› **have** $\Psi' \otimes \Psi_R \vdash M \leftrightarrow M'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition*)
moreover have $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
proof −
from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_Q, \ \Psi' \otimes \Psi_R\rangle \simeq_F \langle A_Q, \Psi' \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
moreover note ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$›

moreover from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$

by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)

ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)

qed

moreover from ‹$\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|L|\!) \prec P'$› ‹$\Psi_R \simeq \mathbf{1}$›

have $\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|L|\!) \prec P'$ **by**(*metis statEqTransition Identity Commutativity AssertionStatEqSym Composition*)

moreover from ‹*zvec* ♯* (*Cases Cs*)› ‹A_P ♯* (*Cases Cs*)› ‹A_Q ♯* (*Cases Cs*)› ‹(φ, R) *mem Cs*›

have A_P ♯* R **and** A_Q ♯* R **and** *zvec* ♯* R **and** A_P ♯* φ **and** A_Q ♯* φ

by(*auto dest*: *memFreshChain*)

ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ (\!|\nu{*}xvec|\!)N \prec' R') \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge$ *zvec* ♯* $K' \wedge A_R$ ♯* K' **using** *cCase*

by(*rule-tac cCase*) (*assumption* | *simp*)+

then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ (\!|\nu{*}xvec|\!)N \prec' R'$)

and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** *zvec* ♯* K' **and** A_R ♯* K'

by *metis*

note *RTrans* ‹(φ, R) *mem Cs*›

moreover from ‹$\Psi' \vdash \varphi$› **have** $\Psi' \otimes \mathbf{1} \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)

with ‹A_Q ♯* φ› **have** $(\langle A_Q, \Psi' \otimes \mathbf{1}\rangle) \vdash_F \varphi$ **by**(*force intro*: *frameImpI*)

with ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$› **have** $(\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F \varphi$

by(*simp add*: *FrameStatImp-def*)

with ‹A_P ♯* φ› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash \varphi$ **by**(*force dest*: *frameImpE*)

hence $\Psi \otimes \Psi_P \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity*)

ultimately have $\Psi \otimes \Psi_P \rhd Cases\ Cs \longmapsto ROut\ K'\ (\!|\nu{*}xvec|\!)N \prec' R'$) **using** ‹*guarded R*› **by**(*rule Case*)

moreover from *MeqK'* ‹$\Psi_R \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K'$

by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition AssertionStatEqTrans*)

ultimately show *?case* **using** ‹*zvec* ♯* K'›

by *fastforce*

next

case(*cPar1* Ψ' Ψ_{R2} R_1 M' *xvec* N' R_1' A_{R2} R_2 A_{R1} Ψ_{R1} Ψ P A_P Ψ_P A_Q *zvec yvec* N R' M)

have *FrR2*: *extractFrame* $R_2 = \langle A_{R2}, \Psi_{R2}\rangle$ **by** *fact*

from ‹$\Psi' \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow M'$› **have** $(\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1} \vdash M \leftrightarrow M'$

by(*metis statEqEnt Associativity Composition Commutativity*)

moreover have $\langle A_Q, (\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1}\rangle \hookrightarrow_F \langle A_P, ((\Psi \otimes \Psi_{R2}) \otimes \Psi_P) \otimes \Psi_{R1}\rangle$

proof −

have $\langle A_Q, (\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1}\rangle \simeq_F \langle A_Q, \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$

by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)

moreover note $\langle\langle A_Q,\ \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P,\ (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle\rangle$

moreover have $\langle A_P,\ (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \simeq_F \langle A_P,\ ((\Psi \otimes \Psi_{R2}) \otimes \Psi_P) \otimes \Psi_{R1}\rangle$

by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)

ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)

qed

moreover from $\langle\Psi \otimes \Psi_{R1} \otimes \Psi_{R2} \rhd P \longmapsto M(\!|L|\!) \prec P'\rangle$ **have** $(\Psi \otimes \Psi_{R2}) \otimes \Psi_{R1} \rhd P \longmapsto M(\!|L|\!) \prec P'$

by(*metis statEqTransition Associativity Composition Commutativity*)

moreover from $\langle A_{R1} \sharp\ast A_P\rangle\ \langle A_{R2} \sharp\ast A_P\rangle\ \langle A_P \sharp\ast (R_1 \parallel R_2)\rangle\ \langle extractFrame\ R_1 = \langle A_{R1},\ \Psi_{R1}\rangle\rangle$ *FrR2* **have** $A_P \sharp\ast \Psi_{R1}$ **and** $A_P \sharp\ast \Psi_{R2}$

by(*force dest*: *extractFrameFreshChain*)+

moreover from $\langle(\!|\nu\ast xvec|\!)N' \prec' (R_1' \parallel R_2) = (\!|\nu\ast yvec|\!)N \prec' R'\rangle\ \langle xvec \sharp\ast yvec\rangle$

obtain $p\ T$ **where** $(\!|\nu\ast xvec|\!)N' \prec' R_1' = (\!|\nu\ast yvec|\!)N \prec' T$ **and** $R' = T \parallel (p \cdot R_2)$ **and** *set* $p \subseteq$ *set yvec* $\times$ *set xvec*

apply(*drule-tac sym*)

by(*rule-tac boundOutputPar1Dest'*) (*assumption* | *simp* | *blast dest*: *sym*)+

ultimately have $\exists K'.\ (\Psi \otimes \Psi_{R2}) \otimes \Psi_P \rhd R_1 \longmapsto ROut\ K'\ ((\!|\nu\ast yvec|\!)N \prec' T) \wedge (\Psi \otimes \Psi_{R2}) \otimes \Psi_P \otimes \Psi_{R1} \vdash M \leftrightarrow K' \wedge (A_{R2}@zvec) \sharp\ast K' \wedge A_{R1} \sharp\ast K'$ **using** *cPar1*

apply(*rule-tac cPar1*(*6*)) **by**(*assumption* | *simp* | *fastforce*)+

then obtain K' **where** *RTrans*: $(\Psi \otimes \Psi_{R2}) \otimes \Psi_P \rhd R_1 \longmapsto K'(\!|\nu\ast xvec|\!)\langle N'\rangle \prec R_1'$

and *MeqK'*: $(\Psi \otimes \Psi_{R2}) \otimes \Psi_P \otimes \Psi_{R1} \vdash M \leftrightarrow K'$ **and** $A_{R2} \sharp\ast K'$ **and** $A_{R1} \sharp\ast K'$ **and** $zvec \sharp\ast K'$

using $\langle(\!|\nu\ast xvec|\!)N' \prec' R_1' = (\!|\nu\ast yvec|\!)N \prec' T\rangle$ **by**(*auto simp add*: *residualInject*)

from *RTrans* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_{R2} \rhd R_1 \longmapsto K'(\!|\nu\ast xvec|\!)\langle N'\rangle \prec R_1'$

by(*metis statEqTransition Associativity Composition Commutativity*)

hence $\Psi \otimes \Psi_P \rhd (R_1 \parallel R_2) \longmapsto K'(\!|\nu\ast xvec|\!)\langle N'\rangle \prec (R_1' \parallel R_2)$ **using** *FrR2* $\langle xvec \sharp\ast R_2\rangle\ \langle A_{R2} \sharp\ast \Psi\rangle\ \langle A_{R2} \sharp\ast \Psi_P\rangle\ \langle A_{R2} \sharp\ast K'\rangle\ \langle A_{R2} \sharp\ast R_1\rangle\ \langle A_{R2} \sharp\ast xvec\rangle\ \langle A_{R2} \sharp\ast N'\rangle$

by(*force intro*: *Par1*)

moreover from *MeqK'* **have** $\Psi \otimes \Psi_P \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow K'$

by(*metis statEqEnt Associativity Composition Commutativity*)

ultimately show *?case* **using** $\langle zvec \sharp\ast K'\rangle\ \langle A_{R1} \sharp\ast K'\rangle\ \langle A_{R2} \sharp\ast K'\rangle\ \langle(\!|\nu\ast xvec|\!)N' \prec' (R_1' \parallel R_2) = (\!|\nu\ast yvec|\!)N \prec' R'\rangle$

by(*auto simp add*: *residualInject*)

next

case(*cPar2* $\Psi'\ \Psi_{R1}\ R_2\ M'\ xvec\ N'\ R_2'\ A_{R1}\ R_1\ A_{R2}\ \Psi_{R2}\ \Psi\ P\ A_P\ \Psi_P\ A_Q\ zvec\ yvec\ N\ R'\ M$)

have *FrR1*: *extractFrame* $R_1 = \langle A_{R1},\ \Psi_{R1}\rangle$ **by** *fact*

from $\langle\Psi' \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow M'\rangle$ **have** $(\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2} \vdash M \leftrightarrow M'$

by(*metis statEqEnt Associativity Composition Commutativity*)

moreover have $\langle A_Q, (\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P, ((\Psi \otimes \Psi_{R1}) \otimes \Psi_P) \otimes \Psi_{R2}\rangle$

proof −

have $\langle A_Q, (\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2}\rangle \simeq_F \langle A_Q, \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$

by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)

moreover note $\langle\langle A_Q, \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle\rangle$

moreover have $\langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \simeq_F \langle A_P, ((\Psi \otimes \Psi_{R1}) \otimes \Psi_P) \otimes \Psi_{R2}\rangle$

by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)

ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)

qed

moreover from $\langle\Psi \otimes \Psi_{R1} \otimes \Psi_{R2} \rhd P \longmapsto M(\!|L|\!) \prec P'\rangle$ **have** $(\Psi \otimes \Psi_{R1}) \otimes \Psi_{R2} \rhd P \longmapsto M(\!|L|\!) \prec P'$

by(*metis statEqTransition Associativity Composition Commutativity*)

moreover from $\langle A_{R1} \sharp* A_P\rangle$ $\langle A_{R2} \sharp* A_P\rangle$ $\langle A_P \sharp* (R_1 \parallel R_2)\rangle$ *FrR1* $\langle$*extractFrame* $R_2 = \langle A_{R2}, \Psi_{R2}\rangle\rangle$ **have** $A_P \sharp* \Psi_{R1}$ **and** $A_P \sharp* \Psi_{R2}$

by(*force dest*: *extractFrameFreshChain*)+

moreover from $\langle(\!|\nu*xvec|\!)N' \prec' (R_1 \parallel R_2') = (\!|\nu*yvec|\!)N \prec' R'\rangle$ $\langle$*xvec* $\sharp*$ *yvec*$\rangle$

obtain p T **where** $(\!|\nu*xvec|\!)N' \prec' R_2' = (\!|\nu*yvec|\!)N \prec' T$ **and** $R' = (p \cdot R_1) \parallel T$ **and** *set* $p \subseteq$ *set yvec* $\times$ *set xvec*

apply(*drule-tac sym*)

by(*rule-tac boundOutputPar2Dest'*) (*assumption* | *simp* | *blast dest*: *sym*)+

ultimately have $\exists K'.\ (\Psi \otimes \Psi_{R1}) \otimes \Psi_P \rhd R_2 \longmapsto ROut\ K'\ ((\!|\nu*yvec|\!)N \prec' T) \wedge (\Psi \otimes \Psi_{R1}) \otimes \Psi_P \otimes \Psi_{R2} \vdash M \leftrightarrow K' \wedge (A_{R1}@zvec) \sharp* K' \wedge A_{R2} \sharp* K'$ **using** *cPar2*

by(*rule-tac cPar2*(*6*)) (*assumption* | *simp* | *fastforce*)+

then obtain K' **where** *RTrans*: $(\Psi \otimes \Psi_{R1}) \otimes \Psi_P \rhd R_2 \longmapsto K'(\!|\nu*xvec|\!)\langle N'\rangle \prec R_2'$

and *MeqK'*: $(\Psi \otimes \Psi_{R1}) \otimes \Psi_P \otimes \Psi_{R2} \vdash M \leftrightarrow K'$ **and** $A_{R1} \sharp* K'$ **and** *zvec* $\sharp*$ K' **and** $A_{R2} \sharp* K'$

using $\langle(\!|\nu*xvec|\!)N' \prec' R_2' = (\!|\nu*yvec|\!)N \prec' T\rangle$ **by**(*auto simp add*: *residualInject*)

from *RTrans* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_{R1} \rhd R_2 \longmapsto K'(\!|\nu*xvec|\!)\langle N'\rangle \prec R_2'$

by(*metis statEqTransition Associativity Composition Commutativity*)

hence $\Psi \otimes \Psi_P \rhd (R_1 \parallel R_2) \longmapsto K'(\!|\nu*xvec|\!)\langle N'\rangle \prec (R_1 \parallel R_2')$ **using** *FrR1* $\langle$*xvec* $\sharp*$ $R_1\rangle$ $\langle A_{R1} \sharp* \Psi\rangle$ $\langle A_{R1} \sharp* \Psi_P\rangle$ $\langle A_{R1} \sharp* K'\rangle\langle A_{R1} \sharp* xvec\rangle$ $\langle A_{R1} \sharp* N'\rangle$ $\langle A_{R1} \sharp* R_2\rangle$

by(*force intro*: *Par2*)

moreover from *MeqK'* **have** $\Psi \otimes \Psi_P \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow K'$

by(*metis statEqEnt Associativity Composition Commutativity*)

ultimately show *?case* **using** $\langle zvec \sharp* K'\rangle$ $\langle A_{R1} \sharp* K'\rangle$ $\langle A_{R2} \sharp* K'\rangle$ $\langle(\!|\nu*xvec|\!)N' \prec' (R_1 \parallel R_2') = (\!|\nu*yvec|\!)N \prec' R'\rangle$

by(*auto simp add*: *residualInject*)

next

case(*cOpen* Ψ' R M' *xvec yvec* N' R' x A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec zvec2* N R'' M)

from ‹$(\!|\nu*(xvec\ @\ x\ \#\ yvec)|\!)N' \prec' R' = (\!|\nu*zvec2|\!)N \prec' R''$› ‹$x \sharp xvec$› ‹$x \sharp yvec$› ‹$x \sharp zvec2$› ‹$x \sharp R''$› ‹$x \sharp N$› ‹*distinct zvec2*›

obtain *xvec' x' yvec'* **where** *A*: $(\!|\nu*(xvec@yvec)|\!)N' \prec' R' = (\!|\nu*(xvec'@yvec')|\!)([(x, x')] \cdot N) \prec' ([(x, x')] \cdot R'')$

and *B*: *zvec2* = $(xvec'@x'\#yvec')$

by(*rule-tac boundOutputOpenDest*) *auto*

then have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu*(xvec'@yvec')|\!)([(x, x')] \cdot N) \prec' ([(x, x')] \cdot R'')) \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge (zvec) \sharp* K' \wedge A_R \sharp* K'$ **using** *cOpen*

by(*rule-tac cOpen*(*4*)) (*assumption* | *simp*)+

then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|\nu*(xvec@yvec)|\!)\langle N'\rangle \prec R'$

and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$

using *A* **by**(*auto simp add*: *residualInject*)

from ‹$A_R \sharp* A_P$› ‹$A_P \sharp* ((\!|\nu x|\!)R)$› ‹$x \sharp A_P$› ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› **have** $A_P \sharp* \Psi_R$

by(*force dest*: *extractFrameFreshChain*)+

from ‹$\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|L|\!) \prec P'$› ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$› ‹*distinct* A_P› ‹$zvec \sharp* P$› ‹$A_P \sharp* \Psi_R$› ‹$x \sharp A_P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* P$› ‹$A_P \sharp* zvec$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* zvec$› ‹$A_R \sharp* P$› ‹$A_R \sharp* A_P$› ‹$x \sharp A_P$› ‹$x \sharp P$›

obtain K'' **where** *MeqK''*: $(\Psi \otimes \Psi_R) \otimes \Psi_P \vdash M \leftrightarrow K''$ **and** $A_R \sharp* K''$ **and** $zvec \sharp* K''$ **and** $x \sharp K''$

by(*rule-tac B*=$(x\#A_R@zvec)$ **in** *obtainPrefix*) (*assumption* | *simp* | *force*)+

from *MeqK'' MeqK'* **have** *KeqK''*: $(\Psi \otimes \Psi_P) \otimes \Psi_R \vdash K' \leftrightarrow K''$

by(*metis statEqEnt Associativity Composition Commutativity chanEqSym chanEqTrans*)

with *RTrans* ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› ‹*distinct* A_R› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_P$› ‹$A_R \sharp* K'$› ‹$A_R \sharp* K''$› ‹$A_R \sharp* R$›

have $\Psi \otimes \Psi_P \rhd R \longmapsto K''(\!|\nu*(xvec@yvec)|\!)\langle N'\rangle \prec R'$

by(*rule-tac outputRenameSubject*) (*assumption* | *force*)+

hence $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)R \longmapsto K''(\!|\nu*(xvec@x\#yvec)|\!)\langle N'\rangle \prec R'$

using ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› ‹$x \sharp K''$› ‹$x \sharp xvec$› ‹ $x \sharp yvec$› ‹$x \in supp\ N'$› ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* \Psi_P$› ‹$xvec \sharp* R$› ‹$x \sharp K''$›

by(*rule-tac Open*) (*assumption* | *force*)+

moreover from *MeqK''* **have** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K''$

by(*metis statEqEnt Associativity Composition Commutativity*)

ultimately show *?case* **using** ‹$zvec \sharp* K''$› ‹$x \sharp K''$› ‹$A_R \sharp* K''$› *B* ‹$(\!|\nu*(xvec\ @\ x\ \#\ yvec)|\!)N' \prec' R' = (\!|\nu*zvec2|\!)N \prec' R''$›

by(*auto simp add*: *residualInject*)

next

case(*cScope* Ψ' R M' *xvec* N' R' x A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec yvec* N R'' M)

from ‹$(\!|\nu*xvec|\!)N' \prec' (\!|\nu x|\!)R' = (\!|\nu*yvec|\!)N \prec' R''$› ‹$x \sharp xvec$› ‹$x \sharp yvec$›

obtain R''' **where** $R'' = (\!|\nu x|\!)R'''$ **and** $(\!|\nu{*}xvec|\!)N' \prec' R' = (\!|\nu{*}yvec|\!)N \prec' R'''$
apply(*drule-tac sym*)
by(*rule boundOutputScopeDest*) (*assumption* | *auto*)+
then have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu{*}yvec|\!)N \prec' R''') \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge zvec \sharp* K' \wedge A_R \sharp* K'$ **using** *cScope*
by(*rule-tac cScope*(*4*)) (*assumption* | *simp*)+
then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|\nu{*}xvec|\!)\langle N'\rangle \prec R'$
and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$
using $\langle (\!|\nu{*}xvec|\!)N' \prec' R' = (\!|\nu{*}yvec|\!)N \prec' R'''\rangle$
by(*auto simp add*: *residualInject*)
from $\langle A_R \sharp* A_P\rangle$ $\langle A_P \sharp* ((\!|\nu x|\!)R)\rangle$ $\langle x \sharp A_P\rangle$ $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ **have** $A_P \sharp* \Psi_R$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle \Psi \otimes \Psi_R \rhd P \longmapsto M(\!|L|\!) \prec P'\rangle$ $\langle extractFrame\ P = \langle A_P, \Psi_P\rangle\rangle$ $\langle distinct\ A_P\rangle$ $\langle x \sharp P\rangle$ $\langle zvec \sharp* P\rangle$ $\langle A_P \sharp* \Psi_R\rangle$ $\langle x \sharp A_P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* zvec\rangle$ $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* zvec\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* A_P\rangle$
obtain K'' **where** *MeqK''*: $(\Psi \otimes \Psi_R) \otimes \Psi_P \vdash M \leftrightarrow K''$ **and** $x \sharp K''$ **and** $A_R \sharp* K''$ **and** $zvec \sharp* K''$
by(*rule-tac B*=$(x\#A_R@zvec)$ **in** *obtainPrefix*) (*assumption* | *force*)+

from *MeqK''* *MeqK'* **have** *KeqK''*: $(\Psi \otimes \Psi_P) \otimes \Psi_R \vdash K' \leftrightarrow K''$
by(*metis statEqEnt Associativity Composition Commutativity chanEqSym chanEqTrans*)
with *RTrans* $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ $\langle distinct\ A_R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle A_R \sharp* K'\rangle$ $\langle A_R \sharp* K''\rangle$ $\langle A_R \sharp* R\rangle$
have $\Psi \otimes \Psi_P \rhd R \longmapsto K''(\!|\nu{*}xvec|\!)\langle N'\rangle \prec R'$
by(*rule-tac outputRenameSubject*) (*assumption* | *force*)+
hence $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)R \longmapsto K''(\!|\nu{*}xvec|\!)\langle N'\rangle \prec (\!|\nu x|\!)R'$ **using** $\langle x \sharp \Psi\rangle$ $\langle x \sharp \Psi_P\rangle$ $\langle x \sharp K''\rangle$ $\langle x \sharp xvec\rangle$ $\langle x \sharp N'\rangle$ $\langle xvec \sharp* \Psi\rangle$ $\langle xvec \sharp* \Psi_P\rangle$ $\langle xvec \sharp* R\rangle$ $\langle x \sharp K''\rangle$
by(*rule-tac Scope*) (*assumption* | *force*)+
moreover from *MeqK''* **have** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K''$
by(*metis statEqEnt Associativity Composition Commutativity*)
ultimately show *?case* **using** $\langle zvec \sharp* K''\rangle$ $\langle x \sharp K''\rangle$ $\langle A_R \sharp* K''\rangle$ $\langle (\!|\nu{*}xvec|\!)N' \prec' (\!|\nu x|\!)R' = (\!|\nu{*}yvec|\!)N \prec' R''\rangle$
by(*auto simp add*: *residualInject*)
next
case(*cBang* Ψ' R M' A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec xvec N R' M*)
from $\langle guarded\ R\rangle$ $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ **have** $\Psi_R \simeq \mathbf{1}$
by(*metis guardedStatEq*)
with $\langle \Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'\rangle$ **have** $\Psi' \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition*)
moreover have $\langle A_Q, \Psi' \otimes \Psi_R \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \otimes \mathbf{1}\rangle$
proof −
from $\langle \Psi_R \simeq \mathbf{1}\rangle$ **have** $\langle A_Q,\ \Psi' \otimes \Psi_R \otimes \mathbf{1}\rangle \simeq_F \langle A_Q, \Psi' \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)

moreover note ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
moreover from ‹$\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|L|\!) \prec P'$› ‹$\Psi_R \simeq \mathbf{1}$›
have $\Psi \otimes \Psi_R \otimes \mathbf{1} \rhd P \longmapsto M(\!|L|\!) \prec P'$ **by**(*metis statEqTransition Identity Commutativity AssertionStatEqSym Composition*)
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \parallel !R \longmapsto ROut\ K'\ ((\!|\nu * xvec|\!)N \prec' R') \wedge \Psi \otimes \Psi_P \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K' \wedge zvec \sharp* K' \wedge A_R \sharp* K'$ **using** *cBang*
by(*rule-tac cBang*(*5*)) (*assumption* |*simp*)+
then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \parallel !R \longmapsto ROut\ K'\ ((\!|\nu * xvec|\!)N \prec' R')$
and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$
by *metis*
from *RTrans* ‹*guarded R*› **have** $\Psi \otimes \Psi_P \rhd !R \longmapsto ROut\ K'\ ((\!|\nu * xvec|\!)N \prec' R')$ **by**(*rule Bang*)
moreover from *MeqK'* ‹$\Psi_R \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition AssertionStatEqTrans*)
ultimately show *?case* **using** ‹$zvec \sharp* K'$›
by *force*
qed
}
note *Goal* = *this*
have $\Psi \otimes \Psi_Q \simeq \Psi \otimes \Psi_Q$ **by** *simp*
moreover note *RTrans*
moreover from *MeqK* **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_R \vdash M \leftrightarrow K$
by(*metis statEqEnt Associativity Commutativity*)
moreover note *PeqQ* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_Q$›
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto ROut\ K'\ ((\!|\nu * xvec|\!)N \prec' R') \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge ([]::name\ list) \sharp* K' \wedge A_R \sharp* K'$
by(*rule-tac Goal*) (*assumption* | *force simp add*: *residualInject*)+
with *Assumptions* **show** *?thesis*
by(*force simp add*: *residualInject*)
qed

lemma *comm2Aux*:
fixes Ψ :: *'b*
and Ψ_Q :: *'b*
and R :: (*'a*, *'b*, *'c*) *psi*
and K :: *'a*
and N :: *'a*
and R' :: (*'a*, *'b*, *'c*) *psi*
and A_R :: *name list*
and Ψ_R :: *'b*

and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and A_Q :: *name list*

assumes *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
and *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$
and *QimpP*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *distinct* A_P
and *distinct* A_R
and $A_R \sharp* A_P$
and $A_R \sharp* A_Q$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
and $A_R \sharp* R$
and $A_R \sharp* K$
and $A_P \sharp* \Psi$
and $A_P \sharp* R$
and $A_P \sharp* P$
and $A_P \sharp* M$
and $A_Q \sharp* R$
and $A_Q \sharp* M$
and $A_R \sharp* xvec$
and $xvec \sharp* M$

obtains K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
proof −
from ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* A_P$› ‹$A_R \sharp* A_Q$› *FrP FrQ* **have** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+
assume *Assumptions*: $\bigwedge K'.\ [\![\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R';\ \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K';\ A_R \sharp* K']\!] \Longrightarrow thesis$
{
fix Ψ'::$'b$
fix *zvec*::*name list*
assume $A_R \sharp* \Psi'$
assume $A_R \sharp* zvec$
assume $A_P \sharp* zvec$
assume $zvec \sharp* R$
assume $zvec \sharp* P$

assume A: $\Psi \otimes \Psi_Q \simeq \Psi'$
with *RTrans* **have** $\Psi' \rhd R \longmapsto K\langle\!|N|\!\rangle \prec R'$
by(*rule statEqTransition*)
moreover note *FrR* ‹*distinct* A_R›
moreover from ‹$\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_Q) \otimes \Psi_R \vdash M \leftrightarrow K$
by(*blast intro*: *statEqEnt Associativity AssertionStatEqSym*)
with A **have** $\Psi' \otimes \Psi_R \vdash M \leftrightarrow K$ **by**(*rule statEqEnt*[*OF Composition*])
moreover have $\langle A_Q, \Psi' \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle$ **using** A
by(*blast dest*: *frameIntComposition FrameStatEqTrans FrameStatEqSym*)
with *QimpP* **have** $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
by(*force intro*: *FrameStatEqImpCompose*)
moreover from *PTrans* **have** *distinct xvec* **by**(*auto dest*: *boundOutputDistinct*)
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto K'\langle\!|N|\!\rangle \prec R' \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge$ *zvec* $\sharp*$ $K' \wedge A_R \sharp* K'$
using *PTrans FrP* ‹$A_R \sharp* K$› ‹$A_R \sharp* \Psi'$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* R$› ‹$A_R \sharp* \Psi_P$› ‹$A_P \sharp* R$› ‹$A_Q \sharp* R$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp*$ *zvec*› ‹$A_Q \sharp* M$› ‹$A_R \sharp*$ *zvec*› ‹*zvec* $\sharp* R$› ‹*zvec* $\sharp* P$› ‹*distinct* A_P› ‹$A_R \sharp* A_P$› ‹$A_R \sharp* A_Q$› ‹$A_R \sharp*$ *xvec*› ‹*xvec* $\sharp* M$›
proof(*nominal-induct avoiding*: Ψ P A_P Ψ_P A_Q *zvec xvec arbitrary*: M *rule*: *inputFrameInduct*)
case(*cAlpha* Ψ' R K N R' A_R Ψ_R p Ψ P A_P Ψ_P A_Q *zvec xvec* M)
have S: *set* $p \subseteq$ *set* $A_R \times$ *set* $(p \cdot A_R)$ **by** *fact*
from ‹$\Psi' \otimes (p \cdot \Psi_R) \vdash M \leftrightarrow K$› **have** $(p \cdot (\Psi' \otimes (p \cdot \Psi_R))) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$
by(*rule chanEqClosed*)
with ‹$A_R \sharp* \Psi'$› ‹$(p \cdot A_R) \sharp* \Psi'$› ‹$A_R \sharp* K$› ‹$(p \cdot A_R) \sharp* K$› S ‹*distinctPerm* p›
have $\Psi' \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K$ **by**(*simp add*: *eqvts*)
moreover from ‹$\Psi \otimes (p \cdot \Psi_R) \rhd P \longmapsto M\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$› S ‹$A_R \sharp* P$› ‹$(p \cdot A_R) \sharp* P$› ‹$A_R \sharp*$ *xvec*› ‹$(p \cdot A_R) \sharp*$ *xvec*› ‹*xvec* $\sharp* M$›
have $(p \cdot (\Psi \otimes (p \cdot \Psi_R))) \rhd P \longmapsto (p \cdot M)\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$
using *outputPermFrameSubject* **by**(*auto simp add*: *residualInject*)
with ‹$A_R \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* \Psi$› S ‹*distinctPerm* p› **have** $\Psi \otimes \Psi_R \rhd P \longmapsto (p \cdot M)\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$
by(*simp add*: *eqvts*)
moreover from ‹$A_P \sharp* M$› **have** $(p \cdot A_P) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* A_P$› ‹$(p \cdot A_R) \sharp* A_P$› S **have** $A_P \sharp* (p \cdot M)$ **by** *simp*
moreover from ‹$A_Q \sharp* M$› **have** $(p \cdot A_Q) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* A_Q$› ‹$(p \cdot A_R) \sharp* A_Q$› S **have** $A_Q \sharp* (p \cdot M)$ **by** *simp*

moreover from ‹$\langle A_Q, \Psi' \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle$›
have $(p \cdot \langle A_Q, \Psi' \otimes (p \cdot \Psi_R)\rangle) \hookrightarrow_F (p \cdot \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle)$
by(*rule FrameStatImpClosed*)
with ‹$A_R \sharp* A_P$› ‹$(p \cdot A_R) \sharp* A_P$› ‹$A_R \sharp* \Psi'$› ‹$(p \cdot A_R) \sharp* \Psi'$› ‹$A_R \sharp* \Psi_P$› ‹$(p \cdot A_R) \sharp* \Psi_P$› ‹$A_R \sharp* A_Q$›

$\langle(p \cdot A_R) \sharp* A_Q\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle(p \cdot A_R) \sharp* \Psi\rangle$ S $\langle distinctPerm\ p\rangle$
have $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$ **by**(*simp add*: *eqvts*)
moreover from $\langle xvec \sharp* M\rangle$ **have** $(p \cdot xvec) \sharp* (p \cdot M)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with S $\langle A_R \sharp* xvec\rangle$ $\langle(p \cdot A_R) \sharp* xvec\rangle$ **have** $xvec \sharp* (p \cdot M)$ **by** *simp*
ultimately obtain K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'\langle\!(N)\!\rangle \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$
using *cAlpha*
by(*fastforce simp del*: *freshChainSimps*)
from $\langle\Psi \otimes \Psi_P \rhd R \longmapsto K'\langle\!(N)\!\rangle \prec R'\rangle$ S $\langle A_R \sharp* R\rangle$ $\langle(p \cdot A_R) \sharp* R\rangle$ **have** $(p \cdot (\Psi \otimes \Psi_P)) \rhd R \longmapsto (p \cdot K')\langle\!(N)\!\rangle \prec R'$
by(*rule-tac inputPermFrameSubject*) *auto*
with S $\langle A_R \sharp* \Psi\rangle$ $\langle(p \cdot A_R) \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle(p \cdot A_R) \sharp* \Psi_P\rangle$ **have** $\Psi \otimes \Psi_P \rhd R \longmapsto (p \cdot K')\langle\!(N)\!\rangle \prec R'$
by(*simp add*: *eqvts*)
moreover from $\langle\Psi \otimes \Psi_P \otimes \Psi_R \vdash (p \cdot M) \leftrightarrow K'\rangle$ **have** $(p \cdot (\Psi \otimes \Psi_P \otimes \Psi_R)) \vdash (p \cdot p \cdot M) \leftrightarrow (p \cdot K')$
by(*rule chanEqClosed*)
with S $\langle A_R \sharp* \Psi\rangle$ $\langle(p \cdot A_R) \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle(p \cdot A_R) \sharp* \Psi_P\rangle$ $\langle distinctPerm\ p\rangle$ **have** $\Psi \otimes \Psi_P \otimes (p \cdot \Psi_R) \vdash M \leftrightarrow (p \cdot K')$
by(*simp add*: *eqvts*)
moreover from $\langle zvec \sharp* K'\rangle$ **have** $(p \cdot zvec) \sharp* (p \cdot K')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_R \sharp* zvec\rangle$ $\langle(p \cdot A_R) \sharp* zvec\rangle$ S **have** $zvec \sharp* (p \cdot K')$ **by** *simp*
moreover from $\langle A_R \sharp* K'\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot K')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case* **by** *blast*
next
case(*cInput* Ψ' M' K $xvec$ N $Tvec$ R Ψ P A_P Ψ_P A_Q $zvec$ $yvec$ M)
from $\langle A_P \sharp* (M'\langle\!(\lambda* xvec\ N)\!\rangle.R)\rangle$ $\langle A_Q \sharp* (M'\langle\!(\lambda* xvec\ N)\!\rangle.R)\rangle$ $\langle zvec \sharp* (M'\langle\!(\lambda* xvec\ N)\!\rangle.R)\rangle$
have $A_P \sharp* M'$ **and** $A_Q \sharp* M'$ **and** $zvec \sharp* M'$ **by** *simp+*

from $\langle\Psi' \vdash M' \leftrightarrow K\rangle$
have $\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow K$
by(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
hence $\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow M'$
by(*blast intro*: *chanEqSym chanEqTrans*)
with $\langle A_Q \sharp* M'\rangle$
have $(\langle A_Q, \Psi' \otimes \mathbf{1}\rangle) \vdash_F M' \leftrightarrow M'$
by(*force intro*: *frameImpI*)

with $\langle\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle\rangle$
have $(\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F M' \leftrightarrow M'$
by(*simp add*: *FrameStatImp-def*)
with $\langle A_P \sharp* M'\rangle$ **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash M' \leftrightarrow M'$
by(*force dest*: *frameImpE*)
hence $\Psi \otimes \Psi_P \vdash M' \leftrightarrow M'$ **by**(*blast intro*: *statEqEnt Identity*)
hence $\Psi \otimes \Psi_P \rhd M'\langle\!(\lambda* xvec\ N)\!\rangle.R \longmapsto M'\langle\!(N[xvec::=Tvec])\!\rangle \prec R[xvec::=Tvec]$

using ‹*distinct xvec*› ‹*set xvec* ⊆ *supp N*› ‹*length xvec* = *length Tvec*›
by(*rule Input*)

moreover from ‹$\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow K$› ‹$\Psi' \otimes \mathbf{1} \vdash M' \leftrightarrow K$›
have $\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'$ **by**(*metis chanEqSym chanEqTrans*)
with ‹$A_Q \sharp* M$› ‹$A_Q \sharp* M'$›
have $(\langle A_Q, \Psi' \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow M'$
by(*force intro*: *frameImpI*)
with ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$ ›
have $(\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F M \leftrightarrow M'$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_P \sharp* M$› ‹$A_P \sharp* M'$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*force dest*: *frameImpE*)
hence $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity*)
ultimately show *?case* **using** ‹*zvec* $\sharp* M'$›
by *force*
next
case(*cCase* Ψ' *R* M' *N* R' φ *Cs* A_R Ψ_R Ψ *P* A_P Ψ_P A_Q *zvec xvec M*)
from ‹*guarded R*› ‹*extractFrame R* = $\langle A_R, \Psi_R\rangle$› **have** $\Psi_R \simeq \mathbf{1}$
by(*metis guardedStatEq*)
with ‹$\Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'$› **have** $\Psi' \otimes \Psi_R \vdash M \leftrightarrow M'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition*)
moreover have $\langle A_Q, \Psi' \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
proof −
from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_Q, \Psi' \otimes \Psi_R\rangle \simeq_F \langle A_Q, \Psi' \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
moreover note ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
moreover from ‹$\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$› ‹$\Psi_R \simeq \mathbf{1}$›
have $\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by**(*metis statEqTransition Identity Commutativity AssertionStatEqSym Composition*)
moreover from ‹*zvec* $\sharp*$ (*Cases Cs*)› ‹$A_P \sharp*$ (*Cases Cs*)› ‹$A_Q \sharp*$ (*Cases Cs*)› ‹(φ, *R*) *mem Cs*›
have $A_P \sharp* R$ **and** $A_Q \sharp* R$ **and** *zvec* $\sharp* R$ **and** $A_P \sharp* \varphi$ **and** $A_Q \sharp* \varphi$
by(*auto dest*: *memFreshChain*)
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R' \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge$ *zvec* $\sharp* K' \wedge A_R \sharp* K'$ **using** *cCase*
by(*rule-tac cCase*) (*assumption* |*simp*)+
then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R'$
and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** *zvec* $\sharp* K'$ **and** $A_R \sharp* K'$

by *metis*
note *RTrans* ‹$(\varphi,\ R)$ *mem Cs*›
moreover from ‹$\Psi' \vdash \varphi$› **have** $\Psi' \otimes \mathbf{1} \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity AssertionStatEqSym*)
with ‹$A_Q \sharp^* \varphi$› **have** $(\langle A_Q,\ \Psi' \otimes \mathbf{1}\rangle) \vdash_F \varphi$ **by**(*force intro*: *frameImpI*)
with ‹$\langle A_Q,\ \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P,\ (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$› **have** $(\langle A_P,\ (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle) \vdash_F \varphi$
by(*simp add*: *FrameStatImp-def*)
with ‹$A_P \sharp^* \varphi$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \vdash \varphi$ **by**(*force dest*: *frameImpE*)
hence $\Psi \otimes \Psi_P \vdash \varphi$ **by**(*blast intro*: *statEqEnt Identity*)
ultimately have $\Psi \otimes \Psi_P \rhd \mathit{Cases}\ \mathit{Cs} \longmapsto K'\langle\!|N|\!\rangle \prec R'$ **using** ‹*guarded R*› **by**(*rule Case*)
moreover from *MeqK'* ‹$\Psi_R \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition AssertionStatEqTrans*)
ultimately show *?case* **using** ‹*zvec* $\sharp^*$ K'›
by *force*
next
case(*cPar1* $\Psi'\ \Psi_{R2}\ R_1\ M'\ N\ {R_1}'\ A_{R2}\ R_2\ A_{R1}\ \Psi_{R1}\ \Psi\ P\ A_P\ \Psi_P\ A_Q$ *zvec xvec M*)
have *FrR2*: *extractFrame* $R_2 = \langle A_{R2},\ \Psi_{R2}\rangle$ **by** *fact*
from ‹$\Psi' \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow M'$› **have** $(\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1} \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity Composition Commutativity*)
moreover have $\langle A_Q,\ (\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1}\rangle \hookrightarrow_F \langle A_P,\ ((\Psi \otimes \Psi_{R2}) \otimes \Psi_P) \otimes \Psi_{R1}\rangle$
proof −
have $\langle A_Q,\ (\Psi' \otimes \Psi_{R2}) \otimes \Psi_{R1}\rangle \simeq_F \langle A_Q,\ \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
moreover note ‹$\langle A_Q,\ \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P,\ (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$›
moreover have $\langle A_P,\ (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \simeq_F \langle A_P,\ ((\Psi \otimes \Psi_{R2}) \otimes \Psi_P) \otimes \Psi_{R1}\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
moreover from ‹$\Psi \otimes \Psi_{R1} \otimes \Psi_{R2} \rhd P \longmapsto M\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec P'$› **have** $(\Psi \otimes \Psi_{R2}) \otimes \Psi_{R1} \rhd P \longmapsto M\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec P'$
by(*metis statEqTransition Associativity Composition Commutativity*)
moreover from ‹$A_{R1} \sharp^* A_P$› ‹$A_{R2} \sharp^* A_P$› ‹$A_P \sharp^* (R_1 \parallel R_2)$› ‹*extractFrame* $R_1 = \langle A_{R1},\ \Psi_{R1}\rangle$› *FrR2* **have** $A_P \sharp^* \Psi_{R1}$ **and** $A_P \sharp^* \Psi_{R2}$
by(*force dest*: *extractFrameFreshChain*)+
moreover note ‹*distinct xvec*›

ultimately have $\exists K'.\ (\Psi \otimes \Psi_{R2}) \otimes \Psi_P \rhd R_1 \longmapsto K'\langle\!|N|\!\rangle \prec {R_1}' \wedge (\Psi \otimes \Psi_{R2}) \otimes \Psi_P \otimes \Psi_{R1} \vdash M \leftrightarrow K' \wedge (A_{R2}@zvec) \sharp^* K' \wedge A_{R1} \sharp^* K'$ **using** *cPar1*
by(*rule-tac cPar1*(*6*)[**where** *bf*=*xvec*]) (*assumption* | *simp* | *fastforce*)+
then obtain K' **where** *RTrans*: $(\Psi \otimes \Psi_{R2}) \otimes \Psi_P \rhd R_1 \longmapsto K'\langle\!|N|\!\rangle \prec {R_1}'$

and *MeqK′*: $(\Psi \otimes \Psi_{R2}) \otimes \Psi_P \otimes \Psi_{R1} \vdash M \leftrightarrow K'$ **and** $A_{R2} \sharp* K'$ **and** *zvec* $\sharp* K'$ **and** $A_{R1} \sharp* K'$
by *force*

from *RTrans* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_{R2} \rhd R_1 \longmapsto K'\langle\!|N|\!\rangle \prec R_1'$
by(*metis statEqTransition Associativity Composition Commutativity*)
hence $\Psi \otimes \Psi_P \rhd (R_1 \parallel R_2) \longmapsto K'\langle\!|N|\!\rangle \prec (R_1' \parallel R_2)$ **using** *FrR2* ‹$A_{R2} \sharp* \Psi$› ‹$A_{R2} \sharp* \Psi_P$› ‹$A_{R2} \sharp* K'$› ‹$A_{R2} \sharp* R_1$› ‹$A_{R2} \sharp* N$›
by(*force intro*: *Par1*)
moreover from *MeqK′* **have** $\Psi \otimes \Psi_P \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow K'$
by(*metis statEqEnt Associativity Composition Commutativity*)
ultimately show *?case* **using** ‹*zvec* $\sharp* K'$› ‹$A_{R1} \sharp* K'$› ‹$A_{R2} \sharp* K'$›
by *force*
next
case(*cPar2* Ψ' Ψ_{R1} R_2 M' N R_2' A_{R1} R_1 A_{R2} Ψ_{R2} Ψ P A_P Ψ_P A_Q *zvec xvec M*)
have *FrR1*: *extractFrame* $R_1 = \langle A_{R1}, \Psi_{R1}\rangle$ **by** *fact*
from ‹$\Psi' \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow M'$› **have** $(\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2} \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity Composition Commutativity*)
moreover have $\langle A_Q, (\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P, ((\Psi \otimes \Psi_{R1}) \otimes \Psi_P) \otimes \Psi_{R2}\rangle$
proof −
have $\langle A_Q, (\Psi' \otimes \Psi_{R1}) \otimes \Psi_{R2}\rangle \simeq_F \langle A_Q, \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
moreover note ‹$\langle A_Q, \Psi' \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle$›
moreover have $\langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_{R1} \otimes \Psi_{R2}\rangle \simeq_F \langle A_P, ((\Psi \otimes \Psi_{R1}) \otimes \Psi_P) \otimes \Psi_{R2}\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
moreover from ‹$\Psi \otimes \Psi_{R1} \otimes \Psi_{R2} \rhd P \longmapsto M\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$› **have** $(\Psi \otimes \Psi_{R1}) \otimes \Psi_{R2} \rhd P \longmapsto M\langle\!|\nu*xvec|\!\rangle\langle N\rangle \prec P'$
by(*metis statEqTransition Associativity Composition Commutativity*)
moreover from ‹$A_{R1} \sharp* A_P$› ‹$A_{R2} \sharp* A_P$› ‹$A_P \sharp* (R_1 \parallel R_2)$› *FrR1* ‹*extractFrame* $R_2 = \langle A_{R2}, \Psi_{R2}\rangle$› **have** $A_P \sharp* \Psi_{R1}$ **and** $A_P \sharp* \Psi_{R2}$
by(*force dest*: *extractFrameFreshChain*)+
ultimately have $\exists K'.\ (\Psi \otimes \Psi_{R1}) \otimes \Psi_P \rhd R_2 \longmapsto K'\langle\!|N|\!\rangle \prec R_2' \wedge (\Psi \otimes \Psi_{R1}) \otimes \Psi_P \otimes \Psi_{R2} \vdash M \leftrightarrow K' \wedge (A_{R1}@zvec) \sharp* K' \wedge A_{R2} \sharp* K'$ **using** ‹*distinct xvec*› *cPar2*
by(*rule-tac cPar2*(6)) (*assumption* | *simp* | *fastforce*)+
then obtain K' **where** *RTrans*: $(\Psi \otimes \Psi_{R1}) \otimes \Psi_P \rhd R_2 \longmapsto K'\langle\!|N|\!\rangle \prec R_2'$
and *MeqK′*: $(\Psi \otimes \Psi_{R1}) \otimes \Psi_P \otimes \Psi_{R2} \vdash M \leftrightarrow K'$ **and** $A_{R1} \sharp* K'$ **and** *zvec* $\sharp* K'$ **and** $A_{R2} \sharp* K'$
by *force*

from *RTrans* **have** $(\Psi \otimes \Psi_P) \otimes \Psi_{R1} \rhd R_2 \longmapsto K'\langle\!|N|\!\rangle \prec R_2'$

by(*metis statEqTransition Associativity Composition Commutativity*)
hence $\Psi \otimes \Psi_P \rhd (R_1 \parallel R_2) \longmapsto K'(\!|N|\!) \prec (R_1 \parallel R_2')$ **using** *FrR1* $\langle A_{R1} \sharp* \Psi\rangle$ $\langle A_{R1} \sharp* \Psi_P\rangle$ $\langle A_{R1} \sharp* K'\rangle$ $\langle A_{R1} \sharp* R_2\rangle$ $\langle A_{R1} \sharp* N\rangle$
by(*force intro*: *Par2*)
moreover from *MeqK'* **have** $\Psi \otimes \Psi_P \otimes \Psi_{R1} \otimes \Psi_{R2} \vdash M \leftrightarrow K'$
by(*metis statEqEnt Associativity Composition Commutativity*)
ultimately show *?case* **using** $\langle zvec \sharp* K'\rangle$ $\langle A_{R1} \sharp* K'\rangle$ $\langle A_{R2} \sharp* K'\rangle$
by *force*
next
case(*cScope* Ψ' R M' N R' x A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec xvec* M)
then have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R' \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge zvec \sharp* K' \wedge A_R \sharp* K'$
by(*rule-tac cScope*(*4*)) (*assumption* | *simp del*: *freshChainSimps*)+
then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R'$
and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$
by *metis*
from $\langle A_R \sharp* A_P\rangle$ $\langle A_P \sharp* ((\!|\nu x|\!)R)\rangle$ $\langle x \sharp A_P\rangle$ $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ **have** $A_P \sharp* \Psi_R$
by(*force dest*: *extractFrameFreshChain*)+
from $\langle \Psi \otimes \Psi_R \rhd P \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec P'\rangle$ $\langle extractFrame\ P = \langle A_P, \Psi_P\rangle\rangle$ $\langle distinct\ A_P\rangle$ $\langle x \sharp P\rangle$ $\langle zvec \sharp* P\rangle$ $\langle A_P \sharp* \Psi_R\rangle$ $\langle x \sharp A_P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* zvec\rangle$ $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* zvec\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* A_P\rangle$ $\langle xvec \sharp* M\rangle$ $\langle distinct\ xvec\rangle$
obtain K'' **where** *MeqK''*: $(\Psi \otimes \Psi_R) \otimes \Psi_P \vdash M \leftrightarrow K''$ **and** $x \sharp K''$ **and** $A_R \sharp* K''$ **and** $zvec \sharp* K''$
by(*rule-tac* B=($x\#A_R@zvec$) **in** *obtainPrefix*) (*assumption* | *simp* | *force* | *metis freshChainSym*)+

from *MeqK''* *MeqK'* **have** *KeqK''*: $(\Psi \otimes \Psi_P) \otimes \Psi_R \vdash K' \leftrightarrow K''$
by(*metis statEqEnt Associativity Composition Commutativity chanEqSym chanEqTrans*)
with *RTrans* $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ $\langle distinct\ A_R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle A_R \sharp* K'\rangle$ $\langle A_R \sharp* K''\rangle$ $\langle A_R \sharp* R\rangle$
have $\Psi \otimes \Psi_P \rhd R \longmapsto K''(\!|N|\!) \prec R'$
by(*rule-tac inputRenameSubject*) (*assumption* | *force*)+
hence $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)R \longmapsto K''(\!|N|\!) \prec (\!|\nu x|\!)R'$ **using** $\langle x \sharp \Psi\rangle$ $\langle x \sharp \Psi_P\rangle$ $\langle x \sharp K''\rangle$ $\langle x \sharp N\rangle$
by(*rule-tac Scope*) (*assumption* | *force*)+
moreover from *MeqK''* **have** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K''$
by(*metis statEqEnt Associativity Composition Commutativity*)
ultimately show *?case* **using** $\langle zvec \sharp* K''\rangle$ $\langle x \sharp K''\rangle$ $\langle A_R \sharp* K''\rangle$
by *force*
next
case(*cBang* Ψ' R M' N R' A_R Ψ_R Ψ P A_P Ψ_P A_Q *zvec xvec* M)
from $\langle guarded\ R\rangle$ $\langle extractFrame\ R = \langle A_R, \Psi_R\rangle\rangle$ **have** $\Psi_R \simeq \mathbf{1}$
by(*metis guardedStatEq*)
with $\langle \Psi' \otimes \mathbf{1} \vdash M \leftrightarrow M'\rangle$ **have** $\Psi' \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow M'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composi-*

tion)
moreover have $\langle A_Q, \Psi' \otimes \Psi_R \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \otimes \mathbf{1}\rangle$
proof −
from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_Q, \Psi' \otimes \Psi_R \otimes \mathbf{1}\rangle \simeq_F \langle A_Q, \Psi' \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
moreover note ‹$\langle A_Q, \Psi' \otimes \mathbf{1}\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle$›
moreover from ‹$\Psi_R \simeq \mathbf{1}$› **have** $\langle A_P, (\Psi \otimes \Psi_P) \otimes \mathbf{1}\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R \otimes \mathbf{1}\rangle$
by(*metis Identity Commutativity AssertionStatEqSym Composition frameResChainPres frameNilStatEq AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
moreover from ‹$\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$› ‹$\Psi_R \simeq \mathbf{1}$›
have $\Psi \otimes \Psi_R \otimes \mathbf{1} \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by**(*metis statEqTransition Identity Commutativity AssertionStatEqSym Composition*)
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \parallel\ !R \longmapsto K'(\!|N|\!) \prec R' \wedge \Psi \otimes \Psi_P \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K' \wedge zvec \sharp* K' \wedge A_R \sharp* K'$ **using** *cBang*
by(*rule-tac cBang*(*5*)) (*assumption* | *simp del*: *freshChainSimps*)+
then obtain K' **where** *RTrans*: $\Psi \otimes \Psi_P \rhd R \parallel\ !R \longmapsto K'(\!|N|\!) \prec R'$
and *MeqK'*: $\Psi \otimes \Psi_P \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K'$ **and** $zvec \sharp* K'$ **and** $A_R \sharp* K'$
by *metis*
from *RTrans* ‹*guarded R*› **have** $\Psi \otimes \Psi_P \rhd !R \longmapsto K'(\!|N|\!) \prec R'$ **by**(*rule Bang*)
moreover from *MeqK'* ‹$\Psi_R \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K'$
by(*metis Identity Commutativity statEqEnt AssertionStatEqSym Composition AssertionStatEqTrans*)
ultimately show *?case* **using** ‹$zvec \sharp* K'$›
by *force*
qed
}
note *Goal* = *this*
have $\Psi \otimes \Psi_Q \simeq \Psi \otimes \Psi_Q$ **by**(*simp add*: *AssertionStatEqRefl*)
moreover from ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_Q$› **have** $A_R \sharp* (\Psi \otimes \Psi_Q)$ **by** *force*
ultimately have $\exists K'.\ \Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R' \wedge \Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K' \wedge ([]::name\ list) \sharp* K' \wedge A_R \sharp* K'$
by(*rule-tac Goal*) (*assumption* | *force*)+
with *Assumptions* **show** *?thesis*
by *blast*
qed

end

end

theory *Simulation*
imports *Semantics*
begin

context *env* **begin**

definition
simulation :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow$
$('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set \Rightarrow$
$('a, 'b, 'c)\ psi \Rightarrow bool$ (- ▷ - ⇝[-] - [80, 80, 80, 80] 80)
where
$\Psi \rhd P \rightsquigarrow[Rel]\ Q \equiv \forall \alpha\ Q'.\ \Psi \rhd Q \longmapsto \alpha \prec Q' \longrightarrow bn\ \alpha \sharp* \Psi \longrightarrow bn\ \alpha \sharp* P \longrightarrow (\exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel)$

abbreviation
simulationNilJudge (- ⇝[-] - [80, 80, 80] 80) **where** $P \rightsquigarrow[Rel]\ Q \equiv SBottom' \rhd P \rightsquigarrow[Rel]\ Q$

lemma *simI*[*consumes 1*, *case-names cSim*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)\ psi$
and C :: $'d$::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and *Sim*: $\bigwedge \alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q'; bn\ \alpha \sharp* P; bn\ \alpha \sharp* Q; bn\ \alpha \sharp* \Psi; distinct(bn\ \alpha);$
$bn\ \alpha \sharp* (subject\ \alpha); bn\ \alpha \sharp* C]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
proof(*auto simp add*: *simulation-def*)
fix $\alpha\ Q'$
assume $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **and** $bn\ \alpha \sharp* \Psi$ **and** $bn\ \alpha \sharp* P$
thus $\exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel$
proof(*nominal-induct* α *rule*: *action.strong-induct*)
case(*In M N*)
thus *?case* **by**(*rule-tac Sim*) *auto*
next
case(*Out M xvec N*)
moreover {
fix *M xvec N Q'*
assume (*xvec*::*name list*) $\sharp*$ Ψ **and** *xvec* $\sharp*$ *P*
obtain *p* **where** *xvecFreshPsi*: $((p::name\ prm) \cdot (xvec::name\ list)) \sharp* \Psi$
and *xvecFreshM*: $(p \cdot xvec) \sharp* (M::'a)$
and *xvecFreshN*: $(p \cdot xvec) \sharp* (N::'a)$
and *xvecFreshP*: $(p \cdot xvec) \sharp* P$
and *xvecFreshQ*: $(p \cdot xvec) \sharp* Q$
and *xvecFreshQ'*: $(p \cdot xvec) \sharp* (Q'::('a, 'b, 'c)\ psi)$
and *xvecFreshC*: $(p \cdot xvec) \sharp* C$
and *xvecFreshxvec*: $(p \cdot xvec) \sharp* xvec$
and *S*: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$

```
            and dpr: distinctPerm p
    by(rule-tac xvec=xvec and c=(Ψ, M, Q, N, P, Q', xvec, C) in name-list-avoiding)
       (auto simp add: eqvts fresh-star-prod)

    from ‹(p • xvec) ♯* M› ‹distinctPerm p› have xvec ♯* (p • M)
      by(subst pt-fresh-star-bij[OF pt-name-inst, OF at-name-inst, where pi=p,
symmetric]) simp

    assume Trans: Ψ ▷ Q ⟼M⦇ν*xvec⦈⟨N⟩ ≺ Q'
    with xvecFreshN xvecFreshQ' S
    have Ψ ▷ Q ⟼M⦇ν*(p • xvec)⦈⟨(p • N)⟩ ≺ (p • Q')
      by(simp add: boundOutputChainAlpha'' residualInject)
    moreover hence distinct(p • xvec)  by(auto dest: boundOutputDistinct)

    moreover note xvecFreshPsi xvecFreshP xvecFreshQ xvecFreshM xvecFreshC
    ultimately obtain P' where PTrans: Ψ ▷ P ⟼M⦇ν*(p • xvec)⦈⟨(p • N)⟩
≺ P'
                          and P'RelQ': (Ψ, P', p • Q') ∈ Rel
      by(drule-tac Sim) auto
    hence (p • Ψ) ▷ (p • P) ⟼(p • (M⦇ν*(p • xvec)⦈⟨(p • N)⟩ ≺ P'))
      by(rule-tac semantics.eqvt)
    with ‹xvec ♯* Ψ› xvecFreshPsi ‹xvec ♯* P› xvecFreshP S dpr
    have Ψ ▷ P ⟼(p • M)⦇ν*xvec⦈⟨N⟩ ≺ (p • P')
      by(simp add: eqvts name-set-fresh-fresh)
    with ‹xvec ♯* Ψ› xvecFreshPsi ‹xvec ♯* P› xvecFreshP S ‹xvec ♯* (p • M)›
    have Ψ ▷ P ⟼(p • p • M)⦇ν*xvec⦈⟨N⟩ ≺ (p • P')
     by(rule-tac outputPermSubject)
       (simp add: fresh-star-def | assumption)+

    with dpr have Ψ ▷ P ⟼M⦇ν*xvec⦈⟨N⟩ ≺ (p • P')
      by simp

    moreover from P'RelQ' Eqvt have (p • Ψ, p • P', p • p • Q') ∈ Rel
      apply(simp add: eqvt-def eqvts)
      apply(erule-tac x=(Ψ, P', p • Q') in ballE)
      apply(erule-tac x=p in allE)
      by(auto simp add: eqvts)


    with ‹xvec ♯* Ψ› xvecFreshPsi S dpr have (Ψ, p • P', Q') ∈ Rel by simp
    ultimately have ∃ P'. Ψ ▷ P ⟼ M⦇ν*xvec⦈⟨N⟩ ≺ P' ∧ (Ψ, P', Q') ∈ Rel
      by blast
   }
   ultimately show ?case by force
  next
   case Tau
   thus ?case by(rule-tac Sim) auto
  qed
qed
```

lemma *simI2*[*case-names cSim*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and C :: *'d*::*fs-name*

assumes *Sim*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* \Psi;\ distinct(bn\ \alpha)]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
using *assms*
by(*auto simp add*: *simulation-def dest*: *boundOutputDistinct*)

lemma *simIChainFresh*[*consumes 4*, *case-names cSim*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and *xvec* :: *name list*
and C :: *'d*::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and *xvec* $\sharp*$ Ψ
and *xvec* $\sharp*$ P
and *xvec* $\sharp*$ Q
and *Sim*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* subject\ \alpha;\ distinct(bn\ \alpha);\ bn\ \alpha \sharp* C;\ xvec \sharp* \alpha;\ xvec \sharp* Q]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
using ‹*eqvt Rel*›
proof(*induct rule*: *simI*[**where** *C*=(*xvec*, *C*)])
case(*cSim* α Q')
from ‹*bn* α $\sharp*$ (*xvec*, *C*)› **have** *bn* α $\sharp*$ *xvec* **and** *bn* α $\sharp*$ *C* **by**(*simp add*: *freshChainSym*)+
obtain *p*::*name prm* **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$
and $(p \cdot xvec) \sharp* \alpha$ **and** *S*: *set p* ⊆ *set xvec* × *set*(*p* · *xvec*)
and *distinctPerm p*
by(*rule-tac c*=(Ψ, P, Q, α) **and** *xvec*=*xvec* **in** *name-list-avoiding*) *auto*
show *?case*
proof(*cases rule*: *actionCases*[**where** α=α])
case(*cInput M N*)
from ‹$\Psi \rhd Q \longmapsto\alpha \prec Q'$› ‹$\alpha$=*M*⟮*N*⟯› **have** $(p \cdot \Psi) \rhd (p \cdot Q) \longmapsto (p \cdot (M⟮N⟯ \prec Q'))$
by(*fastforce intro*: *semantics.eqvt*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec) \sharp* \Psi$› ‹*xvec* $\sharp*$ Q› ‹$(p \cdot xvec) \sharp* Q$› *S*

have *QTrans*: $\Psi \rhd Q \longmapsto (p \cdot M)(\!|(p \cdot N)|\!) \prec (p \cdot Q')$
by(*simp add*: *eqvts*)
moreover from ‹$(p \cdot xvec) \sharp* \alpha$› **have** $(p \cdot (p \cdot xvec)) \sharp* (p \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› ‹α=$M(\!|N|\!)$› **have** *xvec* $\sharp*$ $(p \cdot M)$ **and** *xvec* $\sharp*$ $(p \cdot N)$ **by** *simp+*
moreover with *QTrans* ‹*xvec* $\sharp*$ *Q*› **have** *xvec* $\sharp*$ $(p \cdot Q')$ **by**(*rule-tac input-FreshChainDerivative*)
ultimately have $\exists P'.\ \Psi \rhd P \longmapsto (p \cdot M)(\!|(p \cdot N)|\!) \prec P' \wedge (\Psi, P', (p \cdot Q')) \in Rel$
by(*rule-tac Sim*) (*assumption* | *simp*)+
then obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto (p \cdot M)(\!|(p \cdot N)|\!) \prec P'$ **and** *P'RelQ'*: $(\Psi, P', (p \cdot Q')) \in Rel$
by *blast*
from *PTrans* **have** $(p \cdot \Psi) \rhd (p \cdot P) \longmapsto (p \cdot ((p \cdot M)(\!|(p \cdot N)|\!) \prec P'))$
by(*rule semantics.eqvt*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec) \sharp* \Psi$› ‹*xvec* $\sharp*$ *P*› ‹$(p \cdot xvec) \sharp* P$› *S* ‹*distinctPerm p*›
have $\Psi \rhd P \longmapsto M(\!|N|\!) \prec (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from *P'RelQ'* ‹*eqvt Rel*› **have** $(p \cdot \Psi, p \cdot P', p \cdot p \cdot Q') \in Rel$
by(*auto simp add*: *eqvt-def*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec) \sharp* \Psi$› *S* ‹*distinctPerm p*›
have $(\Psi, p \cdot P', Q') \in Rel$ **by** *simp*
ultimately show *?thesis* **using** ‹α=$M(\!|N|\!)$› **by** *blast*
next
case(*cOutput M yvec N*)
from ‹*distinct*(*bn* α)› ‹*bn* α $\sharp*$ *subject* α› ‹α=$M(\!|\nu*yvec|\!)\langle N\rangle$› **have** *distinct yvec* **and** *yvec* $\sharp*$ *M* **by** *simp+*
from ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹$\alpha$=$M(\!|\nu*yvec|\!)\langle N\rangle$› **have** $(p \cdot \Psi) \rhd (p \cdot Q) \longmapsto (p \cdot (M(\!|\nu*yvec|\!)\langle N\rangle \prec Q'))$
by(*fastforce intro*: *semantics.eqvt*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec) \sharp* \Psi$› ‹*xvec* $\sharp*$ *Q*› ‹$(p \cdot xvec) \sharp* Q$› *S*
have *QTrans*: $\Psi \rhd Q \longmapsto (p \cdot M)(\!|\nu*(p \cdot yvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot Q')$
by(*simp add*: *eqvts*)
with *S* ‹*bn* α $\sharp*$ *xvec*› ‹$(p \cdot xvec) \sharp* \alpha$› ‹$\alpha$=$M(\!|\nu*yvec|\!)\langle N\rangle$› **have** $\Psi \rhd Q \longmapsto (p \cdot M)(\!|\nu*yvec|\!)\langle(p \cdot N)\rangle \prec (p \cdot Q')$
by *simp*
moreover from ‹$(p \cdot xvec) \sharp* \alpha$› **have** $(p \cdot (p \cdot xvec)) \sharp* (p \cdot \alpha)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› ‹α=$M(\!|\nu*yvec|\!)\langle N\rangle$› **have** *xvec* $\sharp*$ $(p \cdot M)$ **and** *xvec* $\sharp*$ $(p \cdot N)$ **and** *xvec* $\sharp*$ $(p \cdot yvec)$ **by** *simp+*
moreover with *QTrans* ‹*xvec* $\sharp*$ *Q*› ‹*distinct yvec*› ‹*yvec* $\sharp*$ *M*› **have** *xvec* $\sharp*$ $(p \cdot Q')$
by(*drule-tac outputFreshChainDerivative*(*2*)) (*auto simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹*yvec* $\sharp*$ *M*› *S* ‹*bn* α $\sharp*$ *xvec*› ‹$(p \cdot xvec) \sharp* \alpha$› ‹$\alpha$=$M(\!|\nu*yvec|\!)\langle N\rangle$› ‹*distinctPerm p*›
have *yvec* $\sharp*$ $(p \cdot M)$ **by**(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, **where** *pi*=*p*]) *simp*

ultimately have $\exists P'.\ \Psi \rhd P \longmapsto (p \cdot M)\langle\!|\nu{*}yvec|\!\rangle\langle(p \cdot N)\rangle \prec P' \wedge (\Psi, P', (p \cdot Q')) \in Rel$
using ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$›‹$bn\ \alpha \sharp* xvec$› ‹$bn\ \alpha \sharp* C$› ‹$yvec \sharp* M$› ‹$\alpha{=}M\langle\!|\nu{*}yvec|\!\rangle\langle N\rangle$› ‹$distinct\ yvec$›
by(*rule-tac Sim*) *auto*
then obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto (p \cdot M)\langle\!|\nu{*}yvec|\!\rangle\langle(p \cdot N)\rangle \prec P'$ **and** *P'RelQ'*: $(\Psi, P', (p \cdot Q')) \in Rel$
by *blast*
from *PTrans* **have** $(p \cdot \Psi) \rhd (p \cdot P) \longmapsto (p \cdot ((p \cdot M)\langle\!|\nu{*}yvec|\!\rangle\langle(p \cdot N)\rangle \prec P'))$
by(*rule semantics.eqvt*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$xvec \sharp* P$› ‹$(p \cdot xvec) \sharp* P$› *S* ‹$distinctPerm\ p$› ‹$bn\ \alpha \sharp* xvec$› ‹$(p \cdot xvec) \sharp* \alpha$› ‹$\alpha{=}M\langle\!|\nu{*}yvec|\!\rangle\langle N\rangle$›
have $\Psi \rhd P \longmapsto M\langle\!|\nu{*}yvec|\!\rangle\langle N\rangle \prec (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from *P'RelQ'* ‹*eqvt Rel*› **have** $(p \cdot \Psi, p \cdot P', p \cdot p \cdot Q') \in Rel$
by(*auto simp add*: *eqvt-def*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› *S* ‹*distinctPerm p*›
have $(\Psi, p \cdot P', Q') \in Rel$ **by** *simp*
ultimately show *?thesis* **using** ‹$\alpha{=}M\langle\!|\nu{*}yvec|\!\rangle\langle N\rangle$› **by** *blast*
next
case *cTau*
from ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹$\alpha = \tau$› ‹$xvec \sharp* Q$› **have** $xvec \sharp* Q'$
by(*blast dest*: *tauFreshChainDerivative*)
with ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹$\alpha = \tau$›
show *?thesis* **by**(*drule-tac Sim*) *auto*
qed
qed

lemma *simIFresh*[*consumes 4*, *case-names cSim*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and x :: *name*
and C :: *'d*::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and $x \sharp \Psi$
and $x \sharp P$
and $x \sharp Q$
and $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ bn\ \alpha \sharp* \Psi;$
$bn\ \alpha \sharp* subject\ \alpha;\ distinct(bn\ \alpha);\ bn\ \alpha \sharp* C;\ x \sharp \alpha;\ x \sharp Q']\!] \Longrightarrow$
$\exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
using *assms*
by(*rule-tac xvec*=[x] **and** $C{=}C$ **in** *simIChainFresh*) *auto*

lemma *simE*:

fixes F :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd P \rightsquigarrow[Rel]\ Q$

shows $\bigwedge \alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \longmapsto \alpha \prec P' \wedge (\Psi, P', Q') \in Rel$
using *assms*
by(*auto simp add*: *simulation-def*)

lemma *simClosedAux*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and p :: *name prm*

assumes *EqvtRel*: *eqvt Rel*
and *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$

shows $(p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
using *EqvtRel*
proof(*induct rule*: *simI*[*of* - - - - (Ψ, P, p)])
case(*cSim* $\alpha\ Q'$)
from ‹$p \cdot \Psi \rhd p \cdot Q \longmapsto \alpha \prec Q'$›
have $(rev\ p \cdot p \cdot \Psi) \rhd (rev\ p \cdot p \cdot Q) \longmapsto (rev\ p \cdot (\alpha \prec Q'))$
by(*blast dest*: *semantics.eqvt*)
hence $\Psi \rhd Q \longmapsto (rev\ p \cdot \alpha) \prec (rev\ p \cdot Q')$
by(*simp add*: *eqvts*)
moreover with ‹$bn\ \alpha \sharp* (\Psi, P, p)$› **have** $bn\ \alpha \sharp* \Psi$ **and** $bn\ \alpha \sharp* P$ **and** $bn\ \alpha \sharp* p$ **by** *simp*+
ultimately obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto (rev\ p \cdot \alpha) \prec P'$
and *P'RelQ'*: $(\Psi, P', rev\ p \cdot Q') \in Rel$
using *PSimQ*
by(*force dest*: *simE freshChainPermSimp simp add*: *eqvts*)
from *PTrans* **have** $(p \cdot \Psi) \rhd (p \cdot P) \longmapsto (p \cdot ((rev\ p \cdot \alpha) \prec P'))$
by(*rule semantics.eqvt*)
with ‹$bn\ \alpha \sharp* p$› **have** $(p \cdot \Psi) \rhd (p \cdot P) \longmapsto \alpha \prec (p \cdot P')$
by(*simp add*: *eqvts freshChainPermSimp*)
moreover from *P'RelQ' EqvtRel* **have** $(p \cdot (\Psi, P', rev\ p \cdot Q')) \in Rel$
by(*simp only*: *eqvt-def*)
hence $(p \cdot \Psi, p \cdot P', Q') \in Rel$ **by** *simp*
ultimately show *?case* **by** *blast*
qed

lemma *simClosed*:
fixes Ψ :: $'b$

and $P :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $p :: name\ prm$

assumes *EqvtRel*: $eqvt\ Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ Q \Longrightarrow (p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
and $P \rightsquigarrow[Rel]\ Q \Longrightarrow (p \cdot P) \rightsquigarrow[Rel]\ (p \cdot Q)$
using *EqvtRel*
by(*force dest*: *simClosedAux simp add*: *permBottom*)+

lemma *reflexive*:
fixes $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

assumes $\{(\Psi, P, P) \mid \Psi\ P.\ True\} \subseteq Rel$

shows $\Psi \rhd P \rightsquigarrow[Rel]\ P$
using *assms*
by(*auto simp add*: *simulation-def*)

lemma *transitive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $Rel' :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $R :: ('a, 'b, 'c)\ psi$
and $Rel'' :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
and *QSimR*: $\Psi \rhd Q \rightsquigarrow[Rel']\ R$
and *Eqvt*: $eqvt\ Rel''$
and *Set*: $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in Rel \wedge (\Psi, Q, R) \in Rel'\} \subseteq Rel''$

shows $\Psi \rhd P \rightsquigarrow[Rel'']\ R$
using ‹*eqvt Rel''*›
proof(*induct rule*: *simI*[**where** *C*=*Q*])
case(*cSim* $\alpha\ R'$)
from *QSimR* ‹$\Psi \rhd R \longmapsto \alpha \prec R'$› ‹$(bn\ \alpha) \sharp* \Psi$› ‹$(bn\ \alpha) \sharp* Q$›
obtain Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **and** $Q'Rel'R'$: $(\Psi, Q', R') \in Rel'$
by(*blast dest*: *simE*)
from *PSimQ QTrans* ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$›
obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *simE*)
with *PTrans* $Q'Rel'R'$ $P'RelQ'$ *Set*

show *?case* **by** *blast*
qed

lemma *statEqSim*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and Ψ' :: $'b$

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
and *eqvt Rel'*
and $\Psi \simeq \Psi'$
and *C1*: $\bigwedge \Psi'' R S \Psi'''. [\![(\Psi'', R, S) \in Rel; \Psi'' \simeq \Psi''']\!] \Longrightarrow (\Psi''', R, S) \in Rel'$

shows $\Psi' \rhd P \rightsquigarrow[Rel']\ Q$
using ‹*eqvt Rel'*›
proof(*induct rule*: *simI*[*of* - - - - Ψ])
case(*cSim* α Q')
from ‹$\Psi' \rhd Q \longmapsto \alpha \prec Q'$› ‹$\Psi \simeq \Psi'$›
have $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **by**(*metis statEqTransition AssertionStatEqSym*)
with *PSimQ* ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$›
obtain P' **where** $\Psi \rhd P \longmapsto \alpha \prec P'$ **and** $(\Psi, P', Q') \in Rel$
by(*blast dest*: *simE*)

from ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹$\Psi \simeq \Psi'$› **have** $\Psi' \rhd P \longmapsto \alpha \prec P'$
by(*rule statEqTransition*)
moreover from ‹$(\Psi, P', Q') \in Rel$› ‹$\Psi \simeq \Psi'$› **have** $(\Psi', P', Q') \in Rel'$
by(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *monotonic*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and A :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and B :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*

assumes $\Psi \rhd P \rightsquigarrow[A]\ Q$
and $A \subseteq B$

shows $\Psi \rhd P \rightsquigarrow[B]\ Q$
using *assms*
by(*simp* (*no-asm*) *add*: *simulation-def*) (*auto dest*: *simE*)

end

end

theory *Tau-Chain*
  imports *Semantics*
begin

context *env* **begin**

abbreviation *tauChain* :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ▷ - $\Longrightarrow\hat{}_\tau$ - [*80*, *80*, *80*] *80*)
where Ψ ▷ *P* $\Longrightarrow\hat{}_\tau$ *P'* ≡ (*P*, *P'*) ∈ {(*P*, *P'*). Ψ ▷ *P* ⟼τ ≺ *P'*}⁀*

abbreviation *tauStepChain* :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ▷ - $\Longrightarrow_\tau$ - [*80*, *80*, *80*] *80*)
where Ψ ▷ *P* $\Longrightarrow_\tau$ *P'* ≡ (*P*, *P'*) ∈ {(*P*, *P'*). Ψ ▷ *P* ⟼τ ≺ *P'*}⁀+

abbreviation *tauContextChain* :: (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- $\Longrightarrow\hat{}_\tau$ - [*80*, *80*] *80*)
where *P* $\Longrightarrow\hat{}_\tau$ *P'* ≡ **1** ▷ *P* $\Longrightarrow\hat{}_\tau$ *P'*
abbreviation *tauContextStepChain* :: (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- $\Longrightarrow_\tau$ - [*80*, *80*] *80*)
where *P* $\Longrightarrow_\tau$ *P'* ≡ **1** ▷ *P* $\Longrightarrow_\tau$ *P'*

lemmas *tauChainInduct*[*consumes 1*, *case-names TauBase TauStep*] = *rtrancl.induct*[*of* - - {(*P*, *P'*). Ψ ▷ *P* ⟼τ ≺ *P'*}, *simplified*] **for** Ψ
lemmas *tauStepChainInduct*[*consumes 1*, *case-names TauBase TauStep*] = *trancl.induct*[*of* - - {(*P*, *P'*). Ψ ▷ *P* ⟼τ ≺ *P'*}, *simplified*] **for** Ψ

lemma *tauActTauStepChain*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *P'* :: (*'a*, *'b*, *'c*) *psi*

  assumes Ψ ▷ *P* ⟼τ ≺ *P'*

  shows Ψ ▷ *P* $\Longrightarrow_\tau$ *P'*
using *assms* **by** *auto*

lemma *tauActTauChain*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *P'* :: (*'a*, *'b*, *'c*) *psi*

  assumes Ψ ▷ *P* ⟼τ ≺ *P'*

  shows Ψ ▷ *P* $\Longrightarrow\hat{}_\tau$ *P'*
using *assms* **by**(*auto simp add*: *rtrancl-eq-or-trancl*)

lemma *tauStepChainEqvt*[*eqvt*]:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*
  and P' :: (*'a*, *'b*, *'c*) *psi*
  and p :: *name prm*

  assumes $\Psi \rhd P \Longrightarrow_\tau P'$

  shows $(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow_\tau (p \cdot P')$
using *assms*
proof(*induct rule*: *tauStepChainInduct*)
  case(*TauBase P P'*)
  hence $\Psi \rhd P \longmapsto\tau \prec P'$ **by** *simp*
  thus *?case* **by**(*force dest*: *semantics.eqvt simp add*: *eqvts*)
next
  case(*TauStep P P' P''*)
  hence $\Psi \rhd P' \longmapsto\tau \prec P''$ **by** *simp*
  hence $(p \cdot \Psi) \rhd (p \cdot P') \longmapsto\tau \prec (p \cdot P'')$ **by**(*force dest*: *semantics.eqvt simp add*: *eqvts*)
  with ‹$(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow_\tau (p \cdot P')$› **show** *?case*
    by(*subst trancl.trancl-into-trancl*) *auto*
qed

lemma *tauChainEqvt*[*eqvt*]:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*
  and P' :: (*'a*, *'b*, *'c*) *psi*
  and p :: *name prm*

  assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$

  shows $(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow\hat{}_\tau (p \cdot P')$
using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl eqvts*)

lemma *tauStepChainEqvt'*[*eqvt*]:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*
  and P' :: (*'a*, *'b*, *'c*) *psi*
  and p :: *name prm*

  shows $(p \cdot (\Psi \rhd P \Longrightarrow_\tau P')) = (p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow_\tau (p \cdot P')$
apply(*auto simp add*: *eqvts perm-set-def pt-bij*[*OF pt-name-inst*, *OF at-name-inst*])
by(*drule-tac p*=*rev p* **in** *tauStepChainEqvt*) *auto*

lemma *tauChainEqvt'*[*eqvt*]:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*
  and P' :: (*'a*, *'b*, *'c*) *psi*

and $p :: name\ prm$

shows $(p \cdot (\Psi \rhd P \Longrightarrow\hat{}_\tau\ P')) = (p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow\hat{}_\tau\ (p \cdot P')$
apply(*auto simp add*: *eqvts perm-set-def pt-bij*[*OF pt-name-inst*, *OF at-name-inst*] *rtrancl-eq-or-trancl*)
by(*drule-tac p*=*rev p* **in** *tauStepChainEqvt*) *auto*

lemma *tauStepChainFresh*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$
and $x :: name$

assumes $\Psi \rhd P \Longrightarrow_\tau P'$
and $x \sharp P$

shows $x \sharp P'$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto dest*: *tauFreshDerivative*)

lemma *tauChainFresh*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$
and $x :: name$

assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$
and $x \sharp P$

shows $x \sharp P'$
using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl intro*: *tauStepChainFresh*)

lemma *tauStepChainFreshChain*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$
and $xvec :: name\ list$

assumes $\Psi \rhd P \Longrightarrow_\tau P'$
and $xvec \sharp* P$

shows $xvec \sharp* P'$
using *assms*
by(*induct xvec*) (*auto intro*: *tauStepChainFresh*)

lemma *tauChainFreshChain*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

and P' :: $('a, 'b, 'c)$ *psi*
and *xvec* :: *name list*

assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$
and *xvec* $\sharp*$ P

shows *xvec* $\sharp*$ P'
using *assms*
by(*induct xvec*) (*auto intro*: *tauChainFresh*)

lemma *tauStepChainCase*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*
and φ :: $'c$
and *Cs* :: $('c \times ('a, 'b, 'c)$ *psi*$)$ *list*

assumes $\Psi \rhd P \Longrightarrow_\tau P'$
and (φ, P) *mem Cs*
and $\Psi \vdash \varphi$
and *guarded P*

shows $\Psi \rhd$ (*Cases Cs*) $\Longrightarrow_\tau P'$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto intro*: *Case trancl-into-trancl*)

lemma *tauStepChainResPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*
and x :: *name*

assumes $\Psi \rhd P \Longrightarrow_\tau P'$
and $x \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!) P \Longrightarrow_\tau (\!|\nu x|\!) P'$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto dest*: *Scope trancl-into-trancl*)

lemma *tauChainResPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*
and x :: *name*

assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$
and $x \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!) P \Longrightarrow\hat{}_\tau (\!|\nu x|\!) P'$

using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl intro*: *tauStepChainResPres*)

lemma *tauStepChainResChainPres*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and P' :: $('a, 'b, 'c)$ *psi*
  and *xvec* :: *name list*

  assumes $\Psi \rhd P \Longrightarrow_\tau P'$
  and *xvec* $\sharp*$ Ψ

  shows $\Psi \rhd (\!|\nu*xvec|\!)P \Longrightarrow_\tau (\!|\nu*xvec|\!)P'$
using *assms*
by(*induct xvec*) (*auto intro*: *tauStepChainResPres*)

lemma *tauChainResChainPres*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and P' :: $('a, 'b, 'c)$ *psi*
  and *xvec* :: *name list*

  assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$
  and *xvec* $\sharp*$ Ψ

  shows $\Psi \rhd (\!|\nu*xvec|\!)P \Longrightarrow\hat{}_\tau (\!|\nu*xvec|\!)P'$
using *assms*
by(*induct xvec*) (*auto intro*: *tauChainResPres*)

lemma *tauStepChainPar1*:
  fixes Ψ :: $'b$
  and Ψ_Q :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and P' :: $('a, 'b, 'c)$ *psi*
  and Q :: $('a, 'b, 'c)$ *psi*
  and A_Q :: *name list*

  assumes $\Psi \otimes \Psi_Q \rhd P \Longrightarrow_\tau P'$
  and *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
  and $A_Q \sharp* \Psi$
  and $A_Q \sharp* P$

  shows $\Psi \rhd P \parallel Q \Longrightarrow_\tau P' \parallel Q$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto dest*: *Par1 tauStepChainFreshChain trancl-into-trancl*)

lemma *tauChainPar1*:
  fixes Ψ :: $'b$
  and Ψ_Q :: $'b$

and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $A_Q :: name\ list$

assumes $\Psi \otimes \Psi_Q \rhd P \Longrightarrow\hat{}_\tau\ P'$
and $extractFrame\ Q = \langle A_Q, \Psi_Q \rangle$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$

shows $\Psi \rhd P \parallel Q \Longrightarrow\hat{}_\tau\ P' \parallel Q$
using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl intro*: *tauStepChainPar1*)

lemma *tauStepChainPar2*:
fixes $\Psi :: 'b$
and $\Psi_P :: 'b$
and $Q :: ('a, 'b, 'c)\ psi$
and $Q' :: ('a, 'b, 'c)\ psi$
and $P :: ('a, 'b, 'c)\ psi$
and $A_P :: name\ list$

assumes $\Psi \otimes \Psi_P \rhd Q \Longrightarrow_\tau\ Q'$
and $extractFrame\ P = \langle A_P, \Psi_P \rangle$
and $A_P \sharp* \Psi$
and $A_P \sharp* Q$

shows $\Psi \rhd P \parallel Q \Longrightarrow_\tau\ P \parallel Q'$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto dest*: *Par2 trancl-into-trancl tauStepChain-FreshChain*)

lemma *tauChainPar2*:
fixes $\Psi :: 'b$
and $\Psi_P :: 'b$
and $Q :: ('a, 'b, 'c)\ psi$
and $Q' :: ('a, 'b, 'c)\ psi$
and $P :: ('a, 'b, 'c)\ psi$
and $A_P :: name\ list$

assumes $\Psi \otimes \Psi_P \rhd Q \Longrightarrow\hat{}_\tau\ Q'$
and $extractFrame\ P = \langle A_P, \Psi_P \rangle$
and $A_P \sharp* \Psi$
and $A_P \sharp* Q$

shows $\Psi \rhd P \parallel Q \Longrightarrow\hat{}_\tau\ P \parallel Q'$
using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl intro*: *tauStepChainPar2*)

lemma *tauStepChainBang*:
fixes Ψ :: *'b*
and P :: *('a, 'b, 'c) psi*
and P' :: *('a, 'b, 'c) psi*

assumes $\Psi \rhd P \parallel !P \Longrightarrow_\tau P'$
and *guarded P*

shows $\Psi \rhd !P \Longrightarrow_\tau P'$
using *assms*
by(*induct x1==P* $\parallel$ *!P P' rule*: *trancl.induct*) (*auto intro*: *Bang dest*: *Bang trancl-into-trancl*)

lemma *tauStepChainStatEq*:
fixes Ψ :: *'b*
and P :: *('a, 'b, 'c) psi*
and P' :: *('a, 'b, 'c) psi*
and Ψ' :: *'b*

assumes $\Psi \rhd P \Longrightarrow_\tau P'$
and $\Psi \simeq \Psi'$

shows $\Psi' \rhd P \Longrightarrow_\tau P'$
using *assms*
by(*induct rule*: *trancl.induct*) (*auto dest*: *statEqTransition trancl-into-trancl*)

lemma *tauChainStatEq*:
fixes Ψ :: *'b*
and P :: *('a, 'b, 'c) psi*
and P' :: *('a, 'b, 'c) psi*
and Ψ' :: *'b*

assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$
and $\Psi \simeq \Psi'$

shows $\Psi' \rhd P \Longrightarrow\hat{}_\tau P'$
using *assms*
by(*auto simp add*: *rtrancl-eq-or-trancl intro*: *tauStepChainStatEq*)

definition *weakTransition* :: *'b* $\Rightarrow$ *('a, 'b, 'c) psi* $\Rightarrow$ *('a, 'b, 'c) psi* $\Rightarrow$ *'a action* $\Rightarrow$ *('a, 'b, 'c) psi* $\Rightarrow$ *bool* (- : - $\rhd$ - $\Longrightarrow$- $\prec$ - [*80*, *80*, *80*, *80*, *80*] *80*)
where
$\Psi : Q \rhd P \Longrightarrow\alpha \prec P' \equiv \exists P''.\ \Psi \rhd P \Longrightarrow\hat{}_\tau P'' \wedge$ (*insertAssertion* (*extractFrame* Q) Ψ) $\hookrightarrow_F$ (*insertAssertion* (*extractFrame* P'') Ψ) $\wedge$
$\Psi \rhd P'' \longmapsto\alpha \prec P'$

lemma *weakTransitionI*:
fixes Ψ :: *'b*
and Q :: *('a, 'b, 'c) psi*
and P :: *('a, 'b, 'c) psi*

and P'' :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$
and *insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') Ψ
and $\Psi \rhd P'' \longmapsto \alpha \prec P'$

shows $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
using *assms*
by(*auto simp add*: *weakTransition-def*)

lemma *weakTransitionE*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*

assumes $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$

obtains P'' **where** $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$ **and** *insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') Ψ
and $\Psi \rhd P'' \longmapsto \alpha \prec P'$
using *assms*
by(*auto simp add*: *weakTransition-def*)

lemma *weakTransitionClosed*[*eqvt*]:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*
and p :: *name prm*

assumes $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$

shows $(p \cdot \Psi) : (p \cdot Q) \rhd (p \cdot P) \Longrightarrow (p \cdot \alpha) \prec (p \cdot P')$
proof −
from *assms* **obtain** P'' **where** $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$ **and** *insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') Ψ
and $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)

from ‹$\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$› **have** $(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow\hat{}_{\tau} (p \cdot P'')$
by(*rule tauChainEqvt*)
moreover from ‹*insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') Ψ›

have $(p \cdot (insertAssertion\ (extractFrame\ Q)\ \Psi)) \hookrightarrow_F (p \cdot (insertAssertion\ (extractFrame\ P'')\ \Psi))$
by(*rule FrameStatImpClosed*)
hence $insertAssertion\ (extractFrame(p \cdot Q))\ (p \cdot \Psi) \hookrightarrow_F insertAssertion\ (extractFrame(p \cdot P''))\ (p \cdot \Psi)$ **by**(*simp add*: *eqvts*)
moreover from $\langle \Psi \rhd P'' \longmapsto \alpha \prec P' \rangle$ **have** $(p \cdot \Psi) \rhd (p \cdot P'') \longmapsto (p \cdot (\alpha \prec P'))$
by(*rule semantics.eqvt*)
hence $(p \cdot \Psi) \rhd (p \cdot P'') \longmapsto (p \cdot \alpha) \prec (p \cdot P')$ **by**(*simp add*: *eqvts*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakOutputAlpha*:
fixes $\Psi :: 'b$
and $Q :: ('a, 'b, 'c)\ psi$
and $P :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $xvec :: name\ list$
and $N :: 'a$
and $P' :: ('a, 'b, 'c)\ psi$
and $p :: name\ prm$
and $yvec :: name\ list$

assumes $PTrans$: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * (p \cdot xvec)|\!)\langle (p \cdot N)\rangle \prec P'$
and S: $set\ p \subseteq set\ xvec \times set(p \cdot xvec)$
and $distinctPerm\ p$
and $xvec \sharp* P$
and $xvec \sharp* (p \cdot xvec)$
and $(p \cdot xvec) \sharp* M$
and $distinct\ xvec$

shows $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec (p \cdot P')$
proof −
from $PTrans$ **obtain** P'' **where** $PChain$: $\Psi \rhd P \Longrightarrow\hat{}_\tau P''$ **and** $QeqP''$: $insertAssertion\ (extractFrame\ Q)\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame\ P'')\ \Psi$
and $P''Trans$: $\Psi \rhd P'' \longmapsto M(\!|\nu * (p \cdot xvec)|\!)\langle (p \cdot N)\rangle \prec P'$
by(*rule weakTransitionE*)

note $PChain\ QeqP''$
moreover from $PChain\ \langle xvec \sharp* P\rangle$ **have** $xvec \sharp* P''$ **by**(*rule tauChainFreshChain*)
with $P''Trans\ \langle xvec \sharp* (p \cdot xvec)\rangle\ \langle distinct\ xvec\rangle\ \langle (p \cdot xvec) \sharp* M\rangle$ **have** $xvec \sharp* (p \cdot N)$ **and** $xvec \sharp* P'$
by(*force intro*: *outputFreshChainDerivative*)+
hence $(p \cdot xvec) \sharp* (p \cdot p \cdot N)$ **and** $(p \cdot xvec) \sharp* (p \cdot P')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])+
with $\langle distinctPerm\ p\rangle$ **have** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* (p \cdot P')$ **by** *simp*+
with $P''Trans\ S\ \langle distinctPerm\ p\rangle$ **have** $\Psi \rhd P'' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec (p \cdot P')$
apply(*simp add*: *residualInject*)

by(*subst boundOutputChainAlpha*) *auto*

ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakFreshDerivative*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and x :: *name*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
and $x \sharp P$
and $x \sharp \alpha$
and $bn\ \alpha \sharp* subject\ \alpha$
and $distinct(bn\ \alpha)$

shows $x \sharp P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_{\tau}\ P''$ **and** P''*Trans*: $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$x \sharp P$› **have** $x \sharp P''$ **by**(*rule tauChainFresh*)
with P''*Trans* **show** $x \sharp P'$ **using** ‹$x \sharp \alpha$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$distinct(bn\ \alpha)$›
by(*force intro*: *freeFreshDerivative*)
qed

lemma *weakFreshChainDerivative*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and *yvec* :: *name list*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
and *yvec* $\sharp* P$
and *yvec* $\sharp* \alpha$
and $bn\ \alpha \sharp* subject\ \alpha$
and $distinct(bn\ \alpha)$

shows *yvec* $\sharp* P'$
using *assms*
by(*induct yvec*) (*auto intro*: *weakFreshDerivative*)

lemma *weakInputFreshDerivative*:

fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and x :: *name*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|N|\!) \prec P'$
and $x \sharp P$
and $x \sharp N$

shows $x \sharp P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$ **and** P''*Trans*: $\Psi \rhd P'' \longmapsto M(\!|N|\!) \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$x \sharp P$› **have** $x \sharp P''$ **by**(*rule tauChainFresh*)
with P''*Trans* **show** $x \sharp P'$ **using** ‹$x \sharp N$›
by(*force intro*: *inputFreshDerivative*)
qed

lemma *weakInputFreshChainDerivative*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and *xvec* :: *name list*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|N|\!) \prec P'$
and *xvec* $\sharp*$ P
and *xvec* $\sharp*$ N

shows *xvec* $\sharp*$ P'
using *assms*
by(*induct xvec*) (*auto intro*: *weakInputFreshDerivative*)

lemma *weakOutputFreshDerivative*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and x :: *name*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and $x \sharp P$
and $x \sharp xvec$
and $xvec \sharp* M$
and *distinct xvec*

shows $x \sharp N$
and $x \sharp P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$ **and** P''*Trans*: $\Psi \rhd P'' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$x \sharp P$› **have** $x \sharp P''$ **by**(*rule tauChainFresh*)
with P''*Trans* **show** $x \sharp N$ **and** $x \sharp P'$ **using** ‹$x \sharp xvec$› ‹$xvec \sharp* M$› ‹*distinct xvec*›
by(*force intro*: *outputFreshDerivative*)+
qed

lemma *weakOutputFreshChainDerivative*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*
and *yvec* :: *name list*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and $yvec \sharp* P$
and $xvec \sharp* yvec$
and $xvec \sharp* M$
and *distinct xvec*

shows $yvec \sharp* N$
and $yvec \sharp* P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_{\tau} P''$ **and** P''*Trans*: $\Psi \rhd P'' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$yvec \sharp* P$› **have** $yvec \sharp* P''$ **by**(*rule tauChainFreshChain*)
with P''*Trans* **show** $yvec \sharp* N$ **and** $yvec \sharp* P'$ **using** ‹$xvec \sharp* yvec$› ‹$xvec \sharp* M$› ‹*distinct xvec*›
by(*force intro*: *outputFreshChainDerivative*)+
qed

lemma *weakOutputPermSubject*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and P' :: (*'a*, *'b*, *'c*) *psi*
and p :: *name prm*
and *yvec* :: *name list*
and *zvec* :: *name list*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and S: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
and *yvec* $\sharp*$ Ψ
and *zvec* $\sharp*$ Ψ
and *yvec* $\sharp*$ P
and *zvec* $\sharp*$ P

shows $\Psi : Q \rhd P \Longrightarrow (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P''$ **and** *QeqP''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) Ψ
and *P''Trans*: $\Psi \rhd P'' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹*yvec* $\sharp*$ *P*› ‹*zvec* $\sharp*$ *P*› **have** *yvec* $\sharp*$ P'' **and** *zvec* $\sharp*$ P''
by(*force intro*: *tauChainFreshChain*)+

note *PChain QeqP''*
moreover from *P''Trans S* ‹*yvec* $\sharp*$ Ψ› ‹*zvec* $\sharp*$ Ψ› ‹*yvec* $\sharp*$ *P''*› ‹*zvec* $\sharp*$ *P''*›
have $\Psi \rhd P'' \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule-tac outputPermSubject*) (*assumption* | *auto*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakInputPermSubject*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and N :: *'a*
and P' :: (*'a*, *'b*, *'c*) *psi*
and p :: *name prm*
and *yvec* :: *name list*
and *zvec* :: *name list*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|N|\!) \prec P'$
and S: *set p* $\subseteq$ *set yvec* $\times$ *set zvec*
and *yvec* $\sharp*$ Ψ

and $zvec \sharp* \Psi$
and $yvec \sharp* P$
and $zvec \sharp* P$

shows $\Psi : Q \rhd P \Longrightarrow (p \cdot M)(\!|N|\!) \prec P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P''$ **and** *QeqP''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) Ψ
and *P''Trans*: $\Psi \rhd P'' \longmapsto M(\!|N|\!) \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹*yvec* $\sharp*$ *P*› ‹*zvec* $\sharp*$ *P*› **have** *yvec* $\sharp*$ P'' **and** *zvec* $\sharp*$ P''
by(*force intro*: *tauChainFreshChain*)+

note *PChain QeqP''*
moreover from *P''Trans S* ‹*yvec* $\sharp*$ Ψ› ‹*zvec* $\sharp*$ Ψ› ‹*yvec* $\sharp*$ *P''*› ‹*zvec* $\sharp*$ *P''*›
have $\Psi \rhd P'' \longmapsto (p \cdot M)(\!|N|\!) \prec P'$
by(*rule-tac inputPermSubject*) *auto*
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakInput*:
fixes Ψ :: *'b*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *M* :: *'a*
and *K* :: *'a*
and *xvec* :: *name list*
and *N* :: *'a*
and *Tvec* :: *'a list*
and *P* :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \vdash M \leftrightarrow K$
and *distinct xvec*
and *set xvec* ⊆ *supp N*
and *length xvec* = *length Tvec*
and *QeqΨ*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F \langle \varepsilon, \Psi \otimes \mathbf{1} \rangle$

shows $\Psi : Q \rhd M(\!|\lambda* xvec\ N|\!).P \Longrightarrow K(\!|(N[xvec ::= Tvec])|\!) \prec P[xvec ::= Tvec]$
proof −
have $\Psi \rhd M(\!|\lambda* xvec\ N|\!).P \Longrightarrow\hat{}_\tau M(\!|\lambda* xvec\ N|\!).P$ **by** *simp*
moreover from *QeqΨ* **have** *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M(\!|\lambda* xvec\ N|\!).P$)) Ψ
by *auto*
moreover from *assms* **have** $\Psi \rhd M(\!|\lambda* xvec\ N|\!).P \longmapsto K(\!|(N[xvec ::= Tvec])|\!) \prec P[xvec ::= Tvec]$
by(*rule-tac Input*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakOutput*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and K :: $'a$
and N :: $'a$
and P :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \vdash M \leftrightarrow K$
and $Qeq\Psi$: *insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F \langle\varepsilon, \Psi \otimes \mathbf{1}\rangle$

shows $\Psi : Q \rhd M\langle N\rangle.P \Longrightarrow K\langle N\rangle \prec P$
proof −
have $\Psi \rhd M\langle N\rangle.P \Longrightarrow\hat{}_\tau M\langle N\rangle.P$ **by** *simp*
moreover from $Qeq\Psi$ **have** *insertAssertion* (*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M\langle N\rangle.P$)) Ψ
by *auto*
moreover have *insertAssertion* (*extractFrame*($M\langle N\rangle.P$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M\langle N\rangle.P$)) Ψ **by** *simp*
moreover from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \rhd M\langle N\rangle.P \longmapsto K\langle N\rangle \prec P$
by(*rule Output*)
ultimately show *?thesis* **by**(*rule-tac weakTransitionI*) *auto*
qed

lemma *insertGuardedAssertion*:
fixes P :: $('a, 'b, 'c)$ *psi*

assumes *guarded* P

shows *insertAssertion*(*extractFrame* P) $\Psi \simeq_F \langle\varepsilon, \Psi \otimes \mathbf{1}\rangle$
proof −
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **by**(*rule freshFrame*)
from ‹*guarded* P› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **and** *supp* $\Psi_P = (\{\}::name\ set)$
by(*blast dest*: *guardedStatEq*)+

from *FrP* ‹$A_P \sharp* \Psi$› ‹$\Psi_P \simeq \mathbf{1}$› **have** *insertAssertion*(*extractFrame* P) $\Psi \simeq_F \langle A_P, \Psi \otimes \mathbf{1}\rangle$
by *simp* (*metis frameIntCompositionSym*)
moreover from ‹$A_P \sharp* \Psi$› **have** $\langle A_P, \Psi \otimes \mathbf{1}\rangle \simeq_F \langle\varepsilon, \Psi \otimes \mathbf{1}\rangle$
by(*rule-tac frameResFreshChain*) *auto*
ultimately show *?thesis* **by**(*rule FrameStatEqTrans*)
qed

lemma *weakCase*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*

and P' :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow\alpha \prec P'$
and (φ, P) *mem CsP*
and $\Psi \vdash \varphi$
and *guarded P*
and *RImpQ*: *insertAssertion* (*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q*) Ψ
and *ImpR*: *insertAssertion* (*extractFrame R*) $\Psi \hookrightarrow_F \langle\varepsilon, \Psi\rangle$

shows $\Psi : R \rhd$ *Cases CsP* $\Longrightarrow\alpha \prec P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P''$
and *QeqP''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') Ψ
and *P''Trans*: $\Psi \rhd P'' \longmapsto\alpha \prec P'$
by(*rule weakTransitionE*)
show *?thesis*
proof(*case-tac* $P = P''$)
assume $P = P''$
have $\Psi \rhd$ *Cases CsP* $\Longrightarrow\hat{}_\tau$ *Cases CsP* **by** *simp*
moreover from *ImpR AssertionStatEq-def* **have** *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame*(*Cases CsP*)) Ψ
by(*rule-tac FrameStatImpTrans*) (*auto intro*: *Identity*)+

moreover from *P''Trans* ‹(φ, P) *mem CsP*› ‹$\Psi \vdash \varphi$› ‹*guarded P*› ‹$P = P''$›
have $\Psi \rhd$ *Cases CsP* $\longmapsto\alpha \prec P'$
by(*blast intro*: *Case*)
ultimately show *?thesis*
by(*rule weakTransitionI*)
next
assume $P \neq P''$
with *PChain* **have** $\Psi \rhd P \Longrightarrow_\tau P''$ **by**(*simp add*: *rtrancl-eq-or-trancl*)
hence $\Psi \rhd$ *Cases CsP* $\Longrightarrow_\tau P''$ **using** ‹(φ, P) *mem CsP*› ‹$\Psi \vdash \varphi$› ‹*guarded P*›
by(*rule tauStepChainCase*)
hence $\Psi \rhd$ *Cases CsP* $\Longrightarrow\hat{}_\tau P''$ **by** *simp*
moreover from *RImpQ QeqP''* **have** *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* P'') Ψ
by(*rule FrameStatImpTrans*)
ultimately show *?thesis* **using** *P''Trans* **by**(*rule weakTransitionI*)
qed
qed

lemma *weakOpen*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*

and M :: $'a$
and $xvec$ :: $name\ list$
and $yvec$ :: $name\ list$
and N :: $'a$
and P' :: $('a, 'b, 'c)\ psi$

assumes $PTrans$: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$
and $x \in supp\ N$
and $x \sharp \Psi$
and $x \sharp M$
and $x \sharp xvec$
and $x \sharp yvec$

shows $\Psi : (\!|\nu x|\!)Q \rhd (\!|\nu x|\!)P \Longrightarrow M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
proof −
from $PTrans$ **obtain** P'' **where** $PChain$: $\Psi \rhd P \Longrightarrow^{\hat{}}_{\tau} P''$ **and** $QeqP''$: $insertAssertion\ (extractFrame\ Q)\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame\ P'')\ \Psi$
and $P''Trans$: $\Psi \rhd P'' \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle N\rangle \prec P'$
by(*rule weakTransitionE*)

from $PChain\ \langle x \sharp \Psi\rangle$ **have** $\Psi \rhd (\!|\nu x|\!)P \Longrightarrow^{\hat{}}_{\tau} (\!|\nu x|\!)P''$ **by**(*rule tauChainResPres*)
moreover from $QeqP''\ \langle x \sharp \Psi\rangle$ **have** $insertAssertion\ (extractFrame((\!|\nu x|\!)Q))\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame((\!|\nu x|\!)P''))\ \Psi$ **by**(*force intro*: *frameImpResPres*)
moreover from $P''Trans\ \langle x \in supp\ N\rangle\ \langle x \sharp \Psi\rangle\ \langle x \sharp M\rangle\ \langle x \sharp xvec\rangle\ \langle x \sharp yvec\rangle$ **have** $\Psi \rhd (\!|\nu x|\!)P'' \longmapsto M(\!|\nu*(xvec@x\#yvec)|\!)\langle N\rangle \prec P'$
by(*rule Open*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakScope*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)\ psi$
and P :: $('a, 'b, 'c)\ psi$
and α :: $'a\ action$
and P' :: $('a, 'b, 'c)\ psi$

assumes $PTrans$: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
and $x \sharp \Psi$
and $x \sharp \alpha$

shows $\Psi : (\!|\nu x|\!)Q \rhd (\!|\nu x|\!)P \Longrightarrow \alpha \prec (\!|\nu x|\!)P'$
proof −
from $PTrans$ **obtain** P'' **where** $PChain$: $\Psi \rhd P \Longrightarrow^{\hat{}}_{\tau} P''$ **and** $QeqP''$: $insertAssertion\ (extractFrame\ Q)\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame\ P'')\ \Psi$
and $P''Trans$: $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)

from $PChain\ \langle x \sharp \Psi\rangle$ **have** $\Psi \rhd (\!|\nu x|\!)P \Longrightarrow^{\hat{}}_{\tau} (\!|\nu x|\!)P''$ **by**(*rule tauChainResPres*)
moreover from $QeqP''\ \langle x \sharp \Psi\rangle$ **have** $insertAssertion\ (extractFrame((\!|\nu x|\!)Q))\ \Psi$

$\hookrightarrow_F$ *insertAssertion* (*extractFrame*($(\!|\nu x|\!)P''$)) Ψ **by**(*force intro*: *frameImpResPres*)
moreover from *P''Trans* ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› **have** $\Psi \rhd (\!|\nu x|\!)P'' \longmapsto \alpha \prec (\!|\nu x|\!)P'$
by(*rule Scope*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakPar1*:
fixes Ψ :: *'b*
and R :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and A_Q :: *name list*
and Ψ_Q :: *'b*

assumes *PTrans*: $\Psi \otimes \Psi_Q : R \rhd P \Longrightarrow \alpha \prec P'$
and *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$
and *bn* $\alpha \sharp* Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* \alpha$
and $A_Q \sharp* R$

shows $\Psi : R \parallel Q \rhd P \parallel Q \Longrightarrow \alpha \prec P' \parallel Q$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_Q \rhd P \Longrightarrow\hat{}_\tau P''$
and *ReqP''*: *insertAssertion* (*extractFrame* R) ($\Psi \otimes \Psi_Q$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') ($\Psi \otimes \Psi_Q$)
and *P''Trans*: $\Psi \otimes \Psi_Q \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$A_Q \sharp* P$› **have** $A_Q \sharp* P''$ **by**(*rule tauChainFreshChain*)
from *PChain FrQ* ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› **have** $\Psi \rhd P \parallel Q \Longrightarrow\hat{}_\tau P'' \parallel Q$ **by**(*rule tauChainPar1*)
moreover have *insertAssertion* (*extractFrame*($R \parallel Q$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($P'' \parallel Q$)) Ψ
proof −
obtain A_R Ψ_R **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **and** $A_R \sharp* A_Q$ **and** $A_R \sharp* \Psi_Q$ **and** $A_R \sharp* \Psi$
by(*rule-tac* C=(A_Q, Ψ_Q, Ψ) **in** *freshFrame*) *auto*
obtain A_P'' Ψ_P'' **where** *FrP''*: *extractFrame* $P'' = \langle A_P'', \Psi_P'' \rangle$ **and** $A_P'' \sharp* A_Q$ **and** $A_P'' \sharp* \Psi_Q$ **and** $A_P'' \sharp* \Psi$
by(*rule-tac* C=(A_Q, Ψ_Q, Ψ) **in** *freshFrame*) *auto*

from *FrR FrP''* ‹$A_Q \sharp* R$› ‹$A_Q \sharp* P''$› ‹$A_R \sharp* A_Q$› ‹$A_P'' \sharp* A_Q$› **have** $A_Q \sharp* \Psi_R$ **and** $A_Q \sharp* \Psi_P''$
by(*force dest*: *extractFrameFreshChain*)+
have $\langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q \rangle \simeq_F \langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle$

by(*metis frameNilStatEq frameResChainPres Associativity Commutativity Composition AssertionStatEqTrans*)
moreover from *ReqP''* *FrR* *FrP''* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_Q$› ‹$A_P'' \sharp* \Psi$› ‹$A_P'' \sharp* \Psi_Q$›
have $\langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P'', (\Psi \otimes \Psi_Q) \otimes \Psi_P''\rangle$ **using** *freshCompChain* **by** *auto*
moreover have $\langle A_P'', (\Psi \otimes \Psi_Q) \otimes \Psi_P''\rangle \simeq_F \langle A_P'', \Psi \otimes \Psi_P'' \otimes \Psi_Q\rangle$
by(*metis frameNilStatEq frameResChainPres Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately have $\langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P'', \Psi \otimes \Psi_P'' \otimes \Psi_Q\rangle$
by(*force dest*: *FrameStatImpTrans simp add*: *FrameStatEq-def*)

hence $\langle (A_R @ A_Q), \Psi \otimes \Psi_R \otimes \Psi_Q\rangle \hookrightarrow_F \langle (A_P'' @ A_Q), \Psi \otimes \Psi_P'' \otimes \Psi_Q\rangle$
apply(*simp add*: *frameChainAppend*)
apply(*drule-tac xvec*=A_Q **in** *frameImpResChainPres*)
by(*metis frameImpChainComm FrameStatImpTrans*)
with *FrR* *FrQ* *FrP''* ‹$A_R \sharp* A_Q$› ‹$A_R \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_R$› ‹$A_P'' \sharp* A_Q$› ‹$A_P'' \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P''$› ‹$A_R \sharp* \Psi$› ‹$A_P'' \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› *ReqP''*
show *?thesis* **by** *simp*
qed
moreover from *P''Trans* *FrQ* ‹*bn* $\alpha \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P''$› ‹$A_Q \sharp* \alpha$› **have** $\Psi \rhd P'' \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$
by(*rule Par1*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakPar2*:
fixes Ψ :: *'b*
and R :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and Q' :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and A_P :: *name list*
and Ψ_P :: *'b*

assumes *QTrans*: $\Psi \otimes \Psi_P : R \rhd Q \Longrightarrow \alpha \prec Q'$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *bn* $\alpha \sharp* P$
and $A_P \sharp* \Psi$
and $A_P \sharp* Q$
and $A_P \sharp* \alpha$
and $A_P \sharp* R$

shows $\Psi : P \parallel R \rhd P \parallel Q \Longrightarrow \alpha \prec P \parallel Q'$
proof −
from *QTrans* **obtain** Q'' **where** *QChain*: $\Psi \otimes \Psi_P \rhd Q \Longrightarrow\hat{}_\tau Q''$

and *ReqQ''*: *insertAssertion* (*extractFrame R*) ($\Psi \otimes \Psi_P$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame Q''*) ($\Psi \otimes \Psi_P$)
and *Q''Trans*: $\Psi \otimes \Psi_P \rhd Q'' \longmapsto \alpha \prec Q'$
by(*rule weakTransitionE*)

from *QChain* ‹$A_P \sharp* Q$› **have** $A_P \sharp* Q''$ **by**(*rule tauChainFreshChain*)

from *QChain FrP* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› **have** $\Psi \rhd P \parallel Q \Longrightarrow\hat{}_\tau P \parallel Q''$ **by**(*rule tauChainPar2*)
moreover have *insertAssertion* (*extractFrame*($P \parallel R$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($P \parallel Q''$)) Ψ
proof −
obtain A_R Ψ_R **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* A_P$ **and** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* \Psi$
by(*rule-tac* C=(A_P, Ψ_P, Ψ) **in** *freshFrame*) *auto*
obtain A_Q'' Ψ_Q'' **where** *FrQ''*: *extractFrame* $Q'' = \langle A_Q'', \Psi_Q''\rangle$ **and** $A_Q'' \sharp* A_P$ **and** $A_Q'' \sharp* \Psi_P$ **and** $A_Q'' \sharp* \Psi$
by(*rule-tac* C=(A_P, Ψ_P, Ψ) **in** *freshFrame*) *auto*

from *FrR FrQ''* ‹$A_P \sharp* R$› ‹$A_P \sharp* Q''$› ‹$A_R \sharp* A_P$› ‹$A_Q'' \sharp* A_P$› **have** $A_P \sharp* \Psi_R$ **and** $A_P \sharp* \Psi_Q''$
by(*force dest*: *extractFrameFreshChain*)+
have $\langle A_R, \Psi \otimes \Psi_P \otimes \Psi_R\rangle \simeq_F \langle A_R, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
by(*metis frameNilStatEq frameResChainPres Associativity Commutativity Composition AssertionStatEqTrans*)

moreover from *ReqQ'' FrR FrQ''* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_P$› ‹$A_Q'' \sharp* \Psi$› ‹$A_Q'' \sharp* \Psi_P$›
have $\langle A_R, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_Q'', (\Psi \otimes \Psi_P) \otimes \Psi_Q''\rangle$ **using** *freshCompChain* **by** *simp*
moreover have $\langle A_Q'', (\Psi \otimes \Psi_P) \otimes \Psi_Q''\rangle \simeq_F \langle A_Q'', \Psi \otimes \Psi_P \otimes \Psi_Q''\rangle$
by(*metis frameNilStatEq frameResChainPres Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately have $\langle A_R, \Psi \otimes \Psi_P \otimes \Psi_R\rangle \hookrightarrow_F \langle A_Q'', \Psi \otimes \Psi_P \otimes \Psi_Q''\rangle$
by(*force dest*: *FrameStatImpTrans simp add*: *FrameStatEq-def*)
hence $\langle (A_P@A_R), \Psi \otimes \Psi_P \otimes \Psi_R\rangle \hookrightarrow_F \langle (A_P@A_Q''), \Psi \otimes \Psi_P \otimes \Psi_Q''\rangle$
apply(*simp add*: *frameChainAppend*)
apply(*drule-tac xvec*=A_P **in** *frameImpResChainPres*)
by(*metis frameImpChainComm FrameStatImpTrans*)
with *FrR FrP FrQ''* ‹$A_R \sharp* A_P$› ‹$A_R \sharp* \Psi_P$› ‹$A_P \sharp* \Psi_R$› ‹$A_Q'' \sharp* A_P$› ‹$A_Q'' \sharp* \Psi_P$› ‹$A_P \sharp* \Psi_Q''$› ‹$A_R \sharp* \Psi$› ‹$A_Q'' \sharp* \Psi$› ‹$A_P \sharp* \Psi$› *ReqQ''*
show *?thesis* **by** *simp*
qed
moreover from *Q''Trans FrP* ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q''$› ‹$A_P \sharp* \alpha$› **have** $\Psi \rhd P \parallel Q'' \longmapsto \alpha \prec (P \parallel Q')$
by(*rule-tac Par2*) *auto*
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *weakComm1*:
fixes Ψ :: *'b*
and R :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and A_Q :: *name list*
and Ψ_Q :: *'b*

assumes *PTrans*: $\Psi \otimes \Psi_Q : R \rhd P \Longrightarrow M(\!|N|\!) \prec P'$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
and *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$
and *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *MeqK*: $\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
and $A_R \sharp* R$
and $A_R \sharp* M$
and $A_R \sharp* A_Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* R$
and $A_Q \sharp* K$
and $xvec \sharp* P$

shows $\Psi \rhd P \parallel Q \Longrightarrow_\tau (\!|\nu * xvec|\!)(P' \parallel Q')$
proof −
from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* R$› ‹$A_Q \sharp* K$› ‹$A_R \sharp* A_Q$›
obtain A_Q' **where** *FrQ'*: *extractFrame* $Q = \langle A_Q', \Psi_Q\rangle$ **and** *distinct* A_Q' **and** $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* P$
and $A_Q' \sharp* Q$ **and** $A_Q' \sharp* R$ **and** $A_Q' \sharp* K$ **and** $A_R \sharp* A_Q'$
by(*rule-tac* C=(Ψ, P, Q, R, K, A_R) **in** *distinctFrame*) *auto*

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_Q \rhd P \Longrightarrow\hat{}_\tau P''$
and *RimpP''*: *insertAssertion* (*extractFrame* R) ($\Psi \otimes \Psi_Q$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') ($\Psi \otimes \Psi_Q$)
and P''*Trans*: $\Psi \otimes \Psi_Q \rhd P'' \longmapsto M(\!|N|\!) \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$A_Q' \sharp* P$› **have** $A_Q' \sharp* P''$ **by**(*rule tauChainFreshChain*)
obtain A_P'' Ψ_P'' **where** *FrP''*: *extractFrame* $P'' = \langle A_P'', \Psi_P''\rangle$ **and** $A_P'' \sharp*$ (Ψ, A_Q', Ψ_Q, A_R, Ψ_R, M, N, K, R, Q, P'', $xvec$) **and** *distinct* A_P''
by(*rule freshFrame*)
hence $A_P'' \sharp* \Psi$ **and** $A_P'' \sharp* A_Q'$ **and** $A_P'' \sharp* \Psi_Q$ **and** $A_P'' \sharp* M$ **and** $A_P'' \sharp* R$ **and** $A_P'' \sharp* Q$

and $A_P'' \sharp* N$ **and** $A_P'' \sharp* K$ **and** $A_P'' \sharp* A_R$ **and** $A_P'' \sharp* P''$ **and** $A_P'' \sharp* xvec$ **and** $A_P'' \sharp* \Psi_R$
by *simp+*
from *FrR* $\langle A_R \sharp* A_Q' \rangle$ $\langle A_Q' \sharp* R \rangle$ **have** $A_Q' \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from *FrQ'* $\langle A_R \sharp* A_Q' \rangle$ $\langle A_R \sharp* Q \rangle$ **have** $A_R \sharp* \Psi_Q$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from *PChain* $\langle xvec \sharp* P \rangle$ **have** $xvec \sharp* P''$ **by**(*force intro*: *tauChainFreshChain*)+

have $\langle A_R, (\Psi \otimes \Psi_R) \otimes \Psi_Q \rangle \simeq_F \langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover with *RimpP'' FrP'' FrR* $\langle A_P'' \sharp* \Psi \rangle$ $\langle A_R \sharp* \Psi \rangle$ $\langle A_P'' \sharp* \Psi_Q \rangle$ $\langle A_R \sharp* \Psi_Q \rangle$
have $\langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle \hookrightarrow_F \langle A_P'', (\Psi \otimes \Psi_Q) \otimes \Psi_P'' \rangle$ **using** *freshCompChain*
by(*simp add*: *freshChainSimps*)
moreover have $\langle A_P'', (\Psi \otimes \Psi_Q) \otimes \Psi_P'' \rangle \simeq_F \langle A_P'', (\Psi \otimes \Psi_P'') \otimes \Psi_Q \rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately have *RImpP''*: $\langle A_R, (\Psi \otimes \Psi_R) \otimes \Psi_Q \rangle \hookrightarrow_F \langle A_P'', (\Psi \otimes \Psi_P'') \otimes \Psi_Q \rangle$
by(*rule FrameStatEqImpCompose*)

from *PChain FrQ'* $\langle A_Q' \sharp* \Psi \rangle$ $\langle A_Q' \sharp* P \rangle$ **have** $\Psi \rhd P \parallel Q \Longrightarrow\!\!\hat{}_\tau\ P'' \parallel Q$ **by**(*rule tauChainPar1*)
moreover from *QTrans FrR P''Trans MeqK RImpP'' FrP'' FrQ'* $\langle$*distinct* $A_P''\rangle$ $\langle$*distinct* $A_Q'\rangle$ $\langle A_P'' \sharp* A_Q' \rangle$ $\langle A_R \sharp* A_Q' \rangle$
$\langle A_Q' \sharp* \Psi \rangle$ $\langle A_Q' \sharp* P'' \rangle$ $\langle A_Q' \sharp* Q \rangle$ $\langle A_Q' \sharp* R \rangle$ $\langle A_Q' \sharp* K \rangle$ $\langle A_P'' \sharp* \Psi \rangle$ $\langle A_P'' \sharp* R \rangle$ $\langle A_P'' \sharp* Q \rangle$
$\langle A_P'' \sharp* P'' \rangle$ $\langle A_P'' \sharp* M \rangle$ $\langle A_Q \sharp* R \rangle$ $\langle A_R \sharp* Q \rangle$ $\langle A_R \sharp* M \rangle$
obtain K' **where** $\Psi \otimes \Psi_P'' \rhd Q \longmapsto K' \langle\!(\nu * xvec)\!\rangle \langle N \rangle \prec Q'$ **and** $\Psi \otimes \Psi_P'' \otimes \Psi_Q \vdash M \leftrightarrow K'$ **and** $A_Q' \sharp* K'$
by(*rule-tac comm1Aux*) (*assumption* | *simp*)+
with *P''Trans FrP''* **have** $\Psi \rhd P'' \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$ **using** *FrQ'* $\langle A_Q' \sharp* \Psi \rangle$ $\langle A_Q' \sharp* P'' \rangle$ $\langle A_Q' \sharp* Q \rangle$
$\langle xvec \sharp* P'' \rangle$ $\langle A_P'' \sharp* \Psi \rangle$ $\langle A_P'' \sharp* P'' \rangle$ $\langle A_P'' \sharp* Q \rangle$ $\langle A_P'' \sharp* M \rangle$ $\langle A_P'' \sharp* A_Q' \rangle$
by(*rule-tac Comm1*)
ultimately show *?thesis*
by(*drule-tac tauActTauStepChain*) *auto*
qed

lemma *weakComm2*:
fixes Ψ :: *'b*
and R :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

and A_Q :: *name list*
and Ψ_Q :: *'b*

assumes *PTrans*: $\Psi \otimes \Psi_Q : R \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$
and *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
and *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$
and *MeqK*: $\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
and $A_R \sharp* R$
and $A_R \sharp* M$
and $A_R \sharp* A_Q$
and $A_Q \sharp* \Psi$
and $A_Q \sharp* P$
and $A_Q \sharp* Q$
and $A_Q \sharp* R$
and $A_Q \sharp* K$
and *xvec* $\sharp* Q$
and *xvec* $\sharp* M$
and *xvec* $\sharp* A_Q$
and *xvec* $\sharp* A_R$

shows $\Psi \rhd P \parallel Q \Longrightarrow_\tau (\nu * xvec)(P' \parallel Q')$
proof −
from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* R$› ‹$A_Q \sharp* K$› ‹$A_R \sharp* A_Q$› ‹*xvec* $\sharp* A_Q$›
obtain A_Q' **where** *FrQ'*: *extractFrame* $Q = \langle A_Q', \Psi_Q\rangle$ **and** *distinct* A_Q' **and** $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* P$
and $A_Q' \sharp* Q$ **and** $A_Q' \sharp* R$ **and** $A_Q' \sharp* K$ **and** $A_R \sharp* A_Q'$ **and** $A_Q' \sharp* xvec$
by(*rule-tac C*=(Ψ, *P*, *Q*, *R*, *K*, A_R, *xvec*) **in** *distinctFrame*) *auto*

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_Q \rhd P \Longrightarrow\hat{}_\tau P''$
and *RimpP''*: *insertAssertion* (*extractFrame R*) ($\Psi \otimes \Psi_Q$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') ($\Psi \otimes \Psi_Q$)
and P''*Trans*: $\Psi \otimes \Psi_Q \rhd P'' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$A_Q' \sharp* P$› **have** $A_Q' \sharp* P''$ **by**(*rule tauChainFreshChain*)
obtain A_P'' Ψ_P'' **where** *FrP''*: *extractFrame* $P'' = \langle A_P'', \Psi_P''\rangle$ **and** $A_P'' \sharp*$ (Ψ, A_Q', Ψ_Q, A_R, Ψ_R, *M*, *N*, *K*, *R*, *Q*, P'', *xvec*) **and** *distinct* A_P''
by(*rule freshFrame*)
hence $A_P'' \sharp* \Psi$ **and** $A_P'' \sharp* A_Q'$ **and** $A_P'' \sharp* \Psi_Q$ **and** $A_P'' \sharp* M$ **and** $A_P'' \sharp* R$ **and** $A_P'' \sharp* Q$
and $A_P'' \sharp* N$ **and** $A_P'' \sharp* K$ **and** $A_P'' \sharp* A_R$ **and** $A_P'' \sharp* P''$ **and** $A_P'' \sharp* xvec$ **and** $A_P'' \sharp* \Psi_R$
by *simp*+

from *FrR* ‹$A_R \sharp* A_Q{}'$› ‹$A_Q{}' \sharp* R$› **have** $A_Q{}' \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from *FrQ'* ‹$A_R \sharp* A_Q{}'$› ‹$A_R \sharp* Q$› **have** $A_R \sharp* \Psi_Q$ **by**(*drule-tac extractFrameFreshChain*) *auto*

have $\langle A_R, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \simeq_F \langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover with *RimpP'' FrP'' FrR* ‹$A_P{}'' \sharp* \Psi$› ‹$A_R \sharp* \Psi$› ‹$A_P{}'' \sharp* \Psi_Q$› ‹$A_R \sharp* \Psi_Q$›
have $\langle A_R, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_Q) \otimes \Psi_P{}''\rangle$ **using** *freshCompChain*
by(*simp add*: *freshChainSimps*)
moreover have $\langle A_P{}'', (\Psi \otimes \Psi_Q) \otimes \Psi_P{}''\rangle \simeq_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'') \otimes \Psi_Q\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately have *RImpP''*: $\langle A_R, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'') \otimes \Psi_Q\rangle$
by(*rule FrameStatEqImpCompose*)

from *PChain FrQ'* ‹$A_Q{}' \sharp* \Psi$› ‹$A_Q{}' \sharp* P$› **have** $\Psi \rhd P \parallel Q \Longrightarrow\hat{}_\tau\ P'' \parallel Q$ **by**(*rule tauChainPar1*)
moreover from *QTrans FrR P''Trans MeqK RImpP'' FrP'' FrQ'* ‹*distinct* $A_P{}''$› ‹*distinct* $A_Q{}'$› ‹$A_P{}'' \sharp* A_Q{}'$› ‹$A_R \sharp* A_Q{}'$›
‹$A_Q{}' \sharp* \Psi$› ‹$A_Q{}' \sharp* P''$› ‹$A_Q{}' \sharp* Q$› ‹$A_Q{}' \sharp* R$› ‹$A_Q{}' \sharp* K$› ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* Q$›
‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* M$› ‹$A_Q \sharp* R$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* M$› ‹*xvec* $\sharp*$ A_R› ‹*xvec* $\sharp*$ M› ‹$A_Q{}' \sharp* xvec$›
obtain K' **where** $\Psi \otimes \Psi_P{}'' \rhd Q \longmapsto K'\langle\!|N|\!\rangle \prec Q'$ **and** $\Psi \otimes \Psi_P{}'' \otimes \Psi_Q \vdash M \leftrightarrow K'$ **and** $A_Q{}' \sharp* K'$
by(*rule-tac comm2Aux*) (*assumption* | *simp*)+
with *P''Trans FrP''* **have** $\Psi \rhd P'' \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$ **using** *FrQ'* ‹$A_Q{}' \sharp* \Psi$› ‹$A_Q{}' \sharp* P''$› ‹$A_Q{}' \sharp* Q$›
‹*xvec* $\sharp*$ Q› ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* Q$› ‹$A_P{}'' \sharp* M$› ‹$A_P{}'' \sharp* A_Q{}'$›
by(*rule-tac Comm2*)
ultimately show *?thesis*
by(*drule-tac tauActTauStepChain*) *auto*
qed

lemma *frameImpIntComposition*:
fixes Ψ :: *'b*
and Ψ' :: *'b*
and A_F :: *name list*
and Ψ_F :: *'b*

assumes $\Psi \simeq \Psi'$

shows $\langle A_F, \Psi \otimes \Psi_F\rangle \hookrightarrow_F \langle A_F, \Psi' \otimes \Psi_F\rangle$
proof −

from *assms* **have** $\langle A_F,\ \Psi \otimes \Psi_F\rangle \simeq_F \langle A_F,\ \Psi' \otimes \Psi_F\rangle$ **by**(*rule frameIntComposition*)
thus *?thesis* **by**(*simp add*: *FrameStatEq-def*)
qed

lemma *insertAssertionStatImp*:
fixes F :: *'b frame*
and Ψ :: *'b*
and G :: *'b frame*
and Ψ' :: *'b*

assumes *FeqG*: *insertAssertion* $F\ \Psi \hookrightarrow_F$ *insertAssertion* $G\ \Psi$
and $\Psi \simeq \Psi'$

shows *insertAssertion* $F\ \Psi' \hookrightarrow_F$ *insertAssertion* $G\ \Psi'$
proof −
obtain $A_F\ \Psi_F$ **where** *FrF*: $F = \langle A_F,\ \Psi_F\rangle$ **and** $A_F \sharp* \Psi$ **and** $A_F \sharp* \Psi'$
by(*rule-tac* $C=(\Psi,\ \Psi')$ **in** *freshFrame*) *auto*
obtain $A_G\ \Psi_G$ **where** *FrG*: $G = \langle A_G,\ \Psi_G\rangle$ **and** $A_G \sharp* \Psi$ **and** $A_G \sharp* \Psi'$
by(*rule-tac* $C=(\Psi,\ \Psi')$ **in** *freshFrame*) *auto*

from $\langle \Psi \simeq \Psi'\rangle$ **have** $\langle A_F,\ \Psi' \otimes \Psi_F\rangle \simeq_F \langle A_F,\ \Psi \otimes \Psi_F\rangle$ **by** (*metis frameIntComposition FrameStatEqSym*)
moreover from $\langle \Psi \simeq \Psi'\rangle$ **have** $\langle A_G,\ \Psi \otimes \Psi_G\rangle \simeq_F \langle A_G,\ \Psi' \otimes \Psi_G\rangle$ **by**(*rule frameIntComposition*)
ultimately have $\langle A_F,\ \Psi' \otimes \Psi_F\rangle \hookrightarrow_F \langle A_G,\ \Psi' \otimes \Psi_G\rangle$ **using** *FeqG FrF FrG* $\langle A_F \sharp* \Psi\rangle$ $\langle A_G \sharp* \Psi\rangle$ $\langle \Psi \simeq \Psi'\rangle$
by(*force simp add*: *FrameStatEq-def dest*: *FrameStatImpTrans*)
with *FrF FrG* $\langle A_F \sharp* \Psi'\rangle$ $\langle A_G \sharp* \Psi'\rangle$ **show** *?thesis* **by** *simp*
qed

lemma *insertAssertionStatEq*:
fixes F :: *'b frame*
and Ψ :: *'b*
and G :: *'b frame*
and Ψ' :: *'b*

assumes *FeqG*: *insertAssertion* $F\ \Psi \simeq_F$ *insertAssertion* $G\ \Psi$
and $\Psi \simeq \Psi'$

shows *insertAssertion* $F\ \Psi' \simeq_F$ *insertAssertion* $G\ \Psi'$
proof −
obtain $A_F\ \Psi_F$ **where** *FrF*: $F = \langle A_F,\ \Psi_F\rangle$ **and** $A_F \sharp* \Psi$ **and** $A_F \sharp* \Psi'$
by(*rule-tac* $C=(\Psi,\ \Psi')$ **in** *freshFrame*) *auto*
obtain $A_G\ \Psi_G$ **where** *FrG*: $G = \langle A_G,\ \Psi_G\rangle$ **and** $A_G \sharp* \Psi$ **and** $A_G \sharp* \Psi'$
by(*rule-tac* $C=(\Psi,\ \Psi')$ **in** *freshFrame*) *auto*

from *FeqG FrF FrG* $\langle A_F \sharp* \Psi\rangle$ $\langle A_G \sharp* \Psi\rangle$ $\langle \Psi \simeq \Psi'\rangle$
have $\langle A_F,\ \Psi' \otimes \Psi_F\rangle \simeq_F \langle A_G,\ \Psi' \otimes \Psi_G\rangle$

by *simp* (*metis frameIntComposition FrameStatEqTrans FrameStatEqSym*)
with *FrF FrG* ‹$A_F \sharp* \Psi'$› ‹$A_G \sharp* \Psi'$› **show** *?thesis* **by** *simp*
qed

lemma *weakTransitionStatEq*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*
and Ψ' :: *'b*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
and $\Psi \simeq \Psi'$

shows $\Psi' : Q \rhd P \Longrightarrow \alpha \prec P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P''$
and *QeqP''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) Ψ
and *P''Trans*: $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)

from *PChain* ‹$\Psi \simeq \Psi'$› **have** $\Psi' \rhd P \Longrightarrow\hat{}_\tau P''$ **by**(*rule tauChainStatEq*)
moreover from *QeqP''* ‹$\Psi \simeq \Psi'$› **have** *insertAssertion* (*extractFrame Q*) Ψ' $\hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) Ψ'
by(*rule insertAssertionStatImp*)
moreover from *P''Trans* ‹$\Psi \simeq \Psi'$› **have** $\Psi' \rhd P'' \longmapsto \alpha \prec P'$
by(*rule statEqTransition*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed

lemma *transitionWeakTransition*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \longmapsto \alpha \prec P'$
and *insertAssertion*(*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame P*) Ψ

shows $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
using *assms*
by(*fastforce intro*: *weakTransitionI*)

lemma *weakPar1Guarded*:
fixes Ψ :: *'b*

and R :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes *PTrans*: $\Psi : R \rhd P \Longrightarrow \alpha \prec P'$
and $bn\ \alpha \sharp* Q$
and *guarded* Q

shows $\Psi : (R \parallel Q) \rhd P \parallel Q \Longrightarrow \alpha \prec P' \parallel Q$
proof −
obtain $A_Q\ \Psi_Q$ **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* P$ **and** $A_Q \sharp* \alpha$ **and** $A_Q \sharp* R$
by(*rule-tac C*=(Ψ, P, α, R) **in** *freshFrame*) *auto*
from ‹*guarded* Q› *FrQ* **have** $\Psi_Q \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
with *PTrans* **have** $\Psi \otimes \Psi_Q : R \rhd P \Longrightarrow \alpha \prec P'$ **by**(*metis weakTransitionStatEq Identity AssertionStatEqSym compositionSym*)
thus *?thesis* **using** *FrQ* ‹$bn\ \alpha \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* \alpha$› ‹$A_Q \sharp* R$›
by(*rule weakPar1*)
qed

lemma *weakBang*:
fixes Ψ :: $'b$
and R :: $('a, 'b, 'c)$ *psi*
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes *PTrans*: $\Psi : R \rhd P \parallel !P \Longrightarrow \alpha \prec P'$
and *guarded* P

shows $\Psi : R \rhd !P \Longrightarrow \alpha \prec P'$
proof −
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \parallel !P \Longrightarrow\hat{}_\tau\ P''$
and *RImpP''*: *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* P'') Ψ
and P''*Trans*: $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*rule weakTransitionE*)
moreover obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **and** $A_P \sharp* \Psi$ **by**(*rule freshFrame*)
moreover from ‹*guarded* P› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
ultimately show *?thesis*
proof(*auto simp add*: *rtrancl-eq-or-trancl*)
have $\Psi \rhd !P \Longrightarrow\hat{}_\tau\ !P$ **by** *simp*
moreover assume *RimpP*: *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F \langle A_P, \Psi \otimes \Psi_P \otimes \mathbf{1} \rangle$

have *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame*(!P)) Ψ
proof −
from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle A_P, \Psi \otimes \Psi_P \otimes \mathbf{1}\rangle \simeq_F \langle A_P, \Psi \otimes \mathbf{1}\rangle$
by(*metis frameIntCompositionSym frameIntAssociativity frameIntCommutativity frameIntIdentity FrameStatEqTrans FrameStatEqSym*)
moreover from ‹$A_P \sharp* \Psi$› **have** $\langle A_P, \Psi \otimes \mathbf{1}\rangle \simeq_F \langle \varepsilon, \Psi \otimes \mathbf{1}\rangle$
by(*force intro*: *frameResFreshChain*)
ultimately show *?thesis* **using** *RimpP* **by**(*auto simp add*: *FrameStatEq-def dest*: *FrameStatImpTrans*)
qed
moreover assume $\Psi \rhd P \parallel !P \longmapsto\alpha \prec P'$
hence $\Psi \rhd !P \longmapsto\alpha \prec P'$ **using** ‹*guarded P*› **by**(*rule Bang*)
ultimately show *?thesis* **by**(*rule weakTransitionI*)
next
fix P'''
assume $\Psi \rhd P \parallel !P \Longrightarrow_\tau P''$
hence $\Psi \rhd !P \Longrightarrow_\tau P''$ **using** ‹*guarded P*› **by**(*rule tauStepChainBang*)
hence $\Psi \rhd !P \Longrightarrow\hat{}_\tau P''$ **by** *simp*
moreover assume *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame P''*) Ψ
and $\Psi \rhd P'' \longmapsto\alpha \prec P'$
ultimately show *?thesis* **by**(*rule weakTransitionI*)
qed
qed

lemma *weakTransitionFrameImp*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and *xvec* :: *name list*
and N :: *'a*
and P' :: (*'a*, *'b*, *'c*) *psi*
and R :: (*'a*, *'b*, *'c*) *psi*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow\alpha \prec P'$
and *insertAssertion*(*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame Q*) Ψ

shows $\Psi : R \rhd P \Longrightarrow\alpha \prec P'$
using *assms*
by(*auto simp add*: *weakTransition-def intro*: *FrameStatImpTrans*)

lemma *guardedFrameStatEq*:
fixes P :: (*'a*, *'b*, *'c*) *psi*

assumes *guarded P*

shows *extractFrame P* $\simeq_F$ $\langle \varepsilon, \mathbf{1} \rangle$
proof −
obtain A_P Ψ_P **where** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P \rangle$ **by**(*rule freshFrame*)
from ‹*guarded P*› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
hence $\langle A_P, \Psi_P \rangle \simeq_F \langle A_P, \mathbf{1} \rangle$ **by**(*rule-tac frameResChainPres*) *auto*
moreover have $\langle A_P, \mathbf{1} \rangle \simeq_F \langle \varepsilon, \mathbf{1} \rangle$ **by**(*rule-tac frameResFreshChain*) *auto*
ultimately show *?thesis* **using** *FrP* **by**(*force intro*: *FrameStatEqTrans*)
qed

lemma *weakGuardedTransition*:
fixes Ψ :: *'b*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and *P'* :: (*'a*, *'b*, *'c*) *psi*
and *R* :: (*'a*, *'b*, *'c*) *psi*

assumes *PTrans*: Ψ : *Q* $\rhd$ *P* $\Longrightarrow\alpha \prec$ *P'*
and *guarded Q*

shows Ψ : $\mathbf{0}$ $\rhd$ *P* $\Longrightarrow\alpha \prec$ *P'*
proof −
obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame Q* = $\langle A_Q, \Psi_Q \rangle$ **and** $A_Q \sharp* \Psi$ **by**(*rule freshFrame*)
moreover from ‹*guarded Q*› *FrQ* **have** $\Psi_Q \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
hence $\langle A_Q, \Psi \otimes \Psi_Q \rangle \simeq_F \langle A_Q, \Psi \otimes \mathbf{1} \rangle$ **by**(*metis frameIntCompositionSym*)
moreover from ‹$A_Q \sharp* \Psi$› **have** $\langle A_Q, \Psi \otimes \mathbf{1} \rangle \simeq_F \langle \varepsilon, \Psi \otimes \mathbf{1} \rangle$ **by**(*rule-tac frameResFreshChain*) *auto*
ultimately have *insertAssertion*(*extractFrame Q*) $\Psi \simeq_F$ *insertAssertion* (*extractFrame* (**0**)) Ψ
using *FrQ* ‹$A_Q \sharp* \Psi$› **by** *simp* (*blast intro*: *FrameStatEqTrans*)
with *PTrans* **show** *?thesis* **by**(*rule-tac weakTransitionFrameImp*) (*auto simp add*: *FrameStatEq-def*)
qed

lemma *expandTauChainFrame*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *P'* :: (*'a*, *'b*, *'c*) *psi*
and A_P :: *name list*
and Ψ_P :: *'b*
and *C* :: *'d*::*fs-name*

assumes *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau$ *P'*
and *FrP*: *extractFrame P* = $\langle A_P, \Psi_P \rangle$
and *distinct* A_P
and $A_P \sharp* P$
and $A_P \sharp* C$

obtains $\Psi'\ A_P'\ \Psi_P'$ **where** *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$ **and** $A_P' \sharp* P'$ **and** $A_P' \sharp* C$ **and** *distinct* A_P'
using *PChain FrP* ‹$A_P \sharp* P$›
proof(*induct arbitrary*: *thesis rule*: *tauChainInduct*)
case(*TauBase P*)
have $\Psi_P \otimes SBottom' \simeq \Psi_P$ **by**(*rule Identity*)
with ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› **show** *?case* **using** ‹$A_P \sharp* P$› ‹$A_P \sharp* C$› ‹*distinct* A_P› **by**(*rule TauBase*)
next
case(*TauStep P P' P''*)
from ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹$A_P \sharp* P$›
obtain $\Psi'\ A_P'\ \Psi_P'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$
and $A_P' \sharp* P'$ **and** $A_P' \sharp* C$ **and** *distinct* A_P'
by(*rule-tac TauStep*)
from ‹$\Psi \rhd P' \longmapsto \tau \prec P''$› *FrP'* ‹*distinct* A_P'› ‹$A_P' \sharp* P'$› ‹$A_P' \sharp* C$›
obtain $\Psi''\ A_P''\ \Psi_P''$ **where** *FrP''*: *extractFrame* $P'' = \langle A_P'', \Psi_P'' \rangle$ **and** $\Psi_P' \otimes \Psi'' \simeq \Psi_P''$
and $A_P'' \sharp* P''$ **and** $A_P'' \sharp* C$ **and** *distinct* A_P''
by(*rule expandTauFrame*)
from ‹$\Psi_P \otimes \Psi' \simeq \Psi_P'$› **have** $(\Psi_P \otimes \Psi') \otimes \Psi'' \simeq \Psi_P' \otimes \Psi''$ **by**(*rule Composition*)
with ‹$\Psi_P' \otimes \Psi'' \simeq \Psi_P''$› **have** $\Psi_P \otimes \Psi' \otimes \Psi'' \simeq \Psi_P''$
by(*metis AssertionStatEqTrans Associativity Commutativity*)
with *FrP''* **show** *?case* **using** ‹$A_P'' \sharp* P''$› ‹$A_P'' \sharp* C$› ‹*distinct* A_P''›
by(*rule TauStep*)
qed

lemma *frameIntImpComposition*:
fixes Ψ :: *'b*
and Ψ' :: *'b*
and A_F :: *name list*
and Ψ_F :: *'b*

assumes $\Psi \simeq \Psi'$

shows $\langle A_F, \Psi \otimes \Psi_F \rangle \hookrightarrow_F \langle A_F, \Psi' \otimes \Psi_F \rangle$
using *assms frameIntComposition*
by(*simp add*: *FrameStatEq-def*)

lemma *tauChainInduct2*[*consumes 1*, *case-names TauBase TauStep*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes *PChain*: $\Psi \rhd P \Longrightarrow^{\hat{}}_\tau P'$
and *cBase*: $\bigwedge P.\ Prop\ P\ P$
and *cStep*: $\bigwedge P\ P'\ P''.\ [\![\Psi \rhd P' \longmapsto \tau \prec P''; \Psi \rhd P \Longrightarrow^{\hat{}}_\tau P'; Prop\ P\ P']\!] \Longrightarrow Prop\ P\ P''$

shows *Prop P P′*
using *assms*
by(*rule tauChainInduct*)

lemma *tauStepChainInduct2*[*consumes 1*, *case-names TauBase TauStep*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*

assumes *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$
and *cBase*: $\bigwedge P\ P'.\ \Psi \rhd P \longmapsto \tau \prec P' \Longrightarrow$ *Prop P P′*
and *cStep*: $\bigwedge P\ P'\ P''.\ [\![\Psi \rhd P' \longmapsto \tau \prec P''; \Psi \rhd P \Longrightarrow_\tau P'; \mathit{Prop}\ P\ P']\!] \Longrightarrow$ *Prop P P″*

shows *Prop P P′*
using *assms*
by(*rule tauStepChainInduct*)

lemma *weakTransferTauChainFrame*:
fixes Ψ_F :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*
and A_P :: *name list*
and Ψ_P :: $'b$
and A_F :: *name list*
and A_G :: *name list*
and Ψ_G :: $'b$

assumes *PChain*: $\Psi_F \rhd P \Longrightarrow\hat{}_\tau P'$
and *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$
and *distinct* A_P
and *FeqG*: $\bigwedge\Psi.$ *insertAssertion* $(\langle A_F, \Psi_F \otimes \Psi_P\rangle)\ \Psi \hookrightarrow_F$ *insertAssertion* $(\langle A_G, \Psi_G \otimes \Psi_P\rangle)\ \Psi$
and $A_F \sharp* \Psi_G$
and $A_G \sharp* \Psi$
and $A_G \sharp* \Psi_F$
and $A_F \sharp* A_G$
and $A_F \sharp* P$
and $A_G \sharp* P$
and $A_P \sharp* A_F$
and $A_P \sharp* A_G$
and $A_P \sharp* \Psi_F$
and $A_P \sharp* \Psi_G$
and $A_P \sharp* P$

shows $\Psi_G \rhd P \Longrightarrow\hat{}_\tau P'$
using *PChain FrP* ‹$A_F \sharp* P$› ‹$A_G \sharp* P$› ‹$A_P \sharp* P$›
proof(*induct rule*: *tauChainInduct2*)

case *TauBase*
thus *?case* **by** *simp*
next
case(*TauStep P P′ P″*)
have *FrP*: *extractFrame P* = ⟨A_P, Ψ_P⟩ **by** *fact*
then have *PChain*: $\Psi_G \rhd P \Longrightarrow\hat{}_\tau\ P'$ **using** ‹A_F ♯* *P*› ‹A_G ♯* *P*› ‹A_P ♯* *P*› **by**(*rule TauStep*)
then obtain A_P' Ψ_P' Ψ' **where** *FrP′*: *extractFrame P′* = ⟨A_P', Ψ_P'⟩ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P'$
and A_P' ♯* A_F **and** A_P' ♯* A_G **and** A_P' ♯* Ψ_F **and** A_P' ♯* Ψ_G
and *distinct* A_P'

using *FrP* ‹*distinct* A_P› ‹A_P ♯* *P*› ‹A_P ♯* A_F› ‹A_P ♯* A_G› ‹A_P ♯* Ψ_F› ‹A_P ♯* Ψ_G›
by(*rule-tac C*=(A_F, A_G, Ψ_F, Ψ_G) **in** *expandTauChainFrame*) *auto*

from *PChain* ‹A_F ♯* *P*› ‹A_G ♯* *P*› **have** A_F ♯* *P′* **and** A_G ♯* *P′* **by**(*blast dest*: *tauChainFreshChain*)+

with ‹A_F ♯* *P*› ‹A_G ♯* *P*› ‹A_P ♯* A_F› ‹A_P ♯* A_G›‹A_P' ♯* A_F› ‹A_P' ♯* A_G› *FrP FrP′*
have A_F ♯* Ψ_P **and** A_G ♯* Ψ_P **and** A_F ♯* Ψ_P' **and** A_G ♯* Ψ_P'
by(*auto dest*: *extractFrameFreshChain*)

from *FeqG* **have** *FeqG*: *insertAssertion* (⟨A_F, $\Psi_F \otimes \Psi_P$⟩) $\Psi' \hookrightarrow_F$ *insertAssertion* (⟨A_G, $\Psi_G \otimes \Psi_P$⟩) Ψ'
by *blast*
obtain *p*::*name prm* **where** (*p* ∙ A_F) ♯* Ψ_F **and** (*p* ∙ A_F) ♯* Ψ_P **and** (*p* ∙ A_F) ♯* Ψ_P' **and** (*p* ∙ A_F) ♯* Ψ'
and *Sp*: (*set p*) ⊆ *set* A_F × *set*(*p* ∙ A_F) **and** *distinctPerm p*
by(*rule-tac xvec*=A_F **and** *c*=(Ψ_F, Ψ_P, Ψ', Ψ_P') **in** *name-list-avoiding*) *auto*
obtain *q*::*name prm* **where** (*q* ∙ A_G) ♯* Ψ_G **and** (*q* ∙ A_G) ♯* Ψ_P **and** (*q* ∙ A_G) ♯* Ψ_P' **and** (*q* ∙ A_G) ♯* Ψ'
and *Sq*: (*set q*) ⊆ *set* A_G × *set*(*q* ∙ A_G) **and** *distinctPerm q*
by(*rule-tac xvec*=A_G **and** *c*=(Ψ_G, Ψ_P, Ψ', Ψ_P') **in** *name-list-avoiding*) *auto*
from ‹$\Psi_P \otimes \Psi' \simeq \Psi_P'$› **have** ⟨(*p* ∙ A_F), ((*p* ∙ Ψ_F) ⊗ Ψ_P')⟩ ≃$_F$ ⟨(*p* ∙ A_F), (*p* ∙ Ψ_F) ⊗ (Ψ_P ⊗ Ψ')⟩
by(*rule frameIntCompositionSym*[*OF AssertionStatEqSym*])
hence ⟨(*p* ∙ A_F), (*p* ∙ Ψ_F) ⊗ Ψ_P'⟩ ≃$_F$ ⟨(*p* ∙ A_F), Ψ' ⊗ ((*p* ∙ Ψ_F) ⊗ Ψ_P)⟩
by(*metis frameIntAssociativity FrameStatEqTrans frameIntCommutativity FrameStatEqSym*)
moreover from *FeqG* ‹A_F ♯* Ψ_P› ‹(*p* ∙ A_F) ♯* Ψ_P› ‹(*p* ∙ A_F) ♯* Ψ_F› ‹(*p* ∙ A_F) ♯* Ψ'› *Sp*
have ⟨(*p* ∙ A_F), Ψ' ⊗ ((*p* ∙ Ψ_F) ⊗ Ψ_P)⟩ ↪$_F$ *insertAssertion* (⟨A_G, $\Psi_G \otimes \Psi_P$⟩) Ψ'
apply(*erule-tac rev-mp*) **by**(*subst frameChainAlpha*) (*auto simp add*: *eqvts*)
hence ⟨(*p* ∙ A_F), Ψ' ⊗ ((*p* ∙ Ψ_F) ⊗ Ψ_P)⟩ ↪$_F$ (⟨(*q* ∙ A_G), Ψ' ⊗ (*q* ∙ Ψ_G) ⊗ Ψ_P⟩)

using ‹$A_G \sharp* \Psi_P$› ‹$(q \cdot A_G) \sharp* \Psi_P$› ‹$(q \cdot A_G) \sharp* \Psi_G$› ‹$(q \cdot A_G) \sharp* \Psi'$› *Sq*
apply(*erule-tac rev-mp*) **by**(*subst frameChainAlpha*) (*auto simp add*: *eqvts*)
moreover have $\langle (q \cdot A_G), \Psi' \otimes ((q \cdot \Psi_G) \otimes \Psi_P) \rangle \simeq_F \langle (q \cdot A_G), (q \cdot \Psi_G) \otimes (\Psi_P \otimes \Psi') \rangle$
by(*metis frameIntAssociativity FrameStatEqTrans frameIntCommutativity FrameStatEqSym*)
hence $\langle (q \cdot A_G), \Psi' \otimes ((q \cdot \Psi_G) \otimes \Psi_P) \rangle \simeq_F \langle (q \cdot A_G), (q \cdot \Psi_G) \otimes \Psi_P' \rangle$ **using** ‹$\Psi_P \otimes \Psi' \simeq \Psi_P'$›
by(*blast intro*: *FrameStatEqTrans frameIntCompositionSym*)
ultimately have $\langle (p \cdot A_F), (p \cdot \Psi_F) \otimes \Psi_P' \rangle \hookrightarrow_F \langle (q \cdot A_G), (q \cdot \Psi_G) \otimes \Psi_P' \rangle$
by(*rule FrameStatEqImpCompose*)
with ‹$A_F \sharp* \Psi_P'$› ‹$(p \cdot A_F) \sharp* \Psi_P'$› ‹$(p \cdot A_F) \sharp* \Psi_F$› *Sp* **have** $\langle A_F, \Psi_F \otimes \Psi_P' \rangle \hookrightarrow_F \langle (q \cdot A_G), (q \cdot \Psi_G) \otimes \Psi_P' \rangle$
by(*subst frameChainAlpha*) (*auto simp add*: *eqvts*)
with ‹$A_G \sharp* \Psi_P'$› ‹$(q \cdot A_G) \sharp* \Psi_P'$› ‹$(q \cdot A_G) \sharp* \Psi_G$› *Sq* **have** $\langle A_F, \Psi_F \otimes \Psi_P' \rangle \hookrightarrow_F \langle A_G, \Psi_G \otimes \Psi_P' \rangle$
by(*subst frameChainAlpha*) (*auto simp add*: *eqvts*)

with ‹$\Psi_F \rhd P' \longmapsto \tau \prec P''$› *FrP'* ‹*distinct* A_P'›
‹$A_F \sharp* P'$› ‹$A_G \sharp* P'$› ‹$A_F \sharp* \Psi_G$› ‹$A_G \sharp* \Psi_F$› ‹$A_P' \sharp* A_F$› ‹$A_P' \sharp* A_G$› ‹$A_P' \sharp* \Psi_F$› ‹$A_P' \sharp* \Psi_G$›
have $\Psi_G \rhd P' \longmapsto \tau \prec P''$ **by**(*rule-tac transferTauFrame*)
with *PChain* **show** *?case* **by**(*simp add*: *r-into-rtrancl rtrancl-into-rtrancl*)
qed

coinductive *quiet* :: ('*a*, '*b*, '*c*) *psi* ⇒ *bool*
where $[\![\forall \Psi.\ (insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F \langle \varepsilon, \Psi \rangle \wedge (\forall Rs.\ \Psi \rhd P \longmapsto Rs \longrightarrow (\exists P'.\ Rs = \tau \prec P' \wedge quiet\ P')))]\!] \Longrightarrow quiet\ P$

lemma *quietFrame*:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*

assumes *quiet P*

shows *insertAssertion* (*extractFrame P*) $\Psi \simeq_F \langle \varepsilon, \Psi \rangle$
using *assms*
by(*erule-tac quiet.cases*) *force*

lemma *quietTransition*:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Rs* :: ('*a*, '*b*, '*c*) *residual*

assumes *quiet P*
and $\Psi \rhd P \longmapsto Rs$

obtains *P'* **where** $Rs = \tau \prec P'$ **and** *quiet P'*

using *assms*
by(*erule-tac quiet.cases*) *force*

lemma *quietEqvt*:
  fixes P :: ($'a$, $'b$, $'c$) *psi*
  and p :: *name prm*

  assumes *quiet* P

  shows *quiet*($p \cdot P$)
proof −
  let *?X* = $\lambda P.\ \exists\, p$::*name prm*. *quiet*($p \cdot P$)
  from *assms* **have** *?X* ($p \cdot P$) **by**(*rule-tac x*=*rev p* **in** *exI*) *auto*
  thus *?thesis*
    apply *coinduct*
    apply(*clarify*)
    apply(*rule-tac x*=*x* **in** *exI*)
    apply *auto*
    apply(*drule-tac* Ψ=$p \cdot \Psi$ **in** *quietFrame*)
    apply(*drule-tac p*=*rev p* **in** *FrameStatEqClosed*)
    apply(*simp add*: *eqvts*)
    apply(*drule-tac pi*=*p* **in** *semantics.eqvt*)
    apply(*erule-tac quietTransition*)
    apply *assumption*
    apply(*rule-tac x*=*rev* ($p \cdot P'$) **in** *exI*)
    apply *auto*
    apply(*drule-tac pi*=*rev p* **in** *pt-bij3*)
    apply(*simp add*: *eqvts*)
    apply(*rule-tac x*=*p* **in** *exI*)
    by *simp*
qed

lemma *quietOutput*:
  fixes Ψ :: $'b$
  and P :: ($'a$, $'b$, $'c$) *psi*
  and M :: $'a$
  and *xvec* :: *name list*
  and N :: $'a$
  and P' :: ($'a$, $'b$, $'c$) *psi*

  assumes $\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
  and *quiet* P

  shows *False*
using *assms*
apply(*erule-tac quiet.cases*)
by(*force simp add*: *residualInject*)

lemma *quietInput*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and M :: $'a$
  and N :: $'a$
  and P' :: $('a, 'b, 'c)$ *psi*

  assumes $\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$
  and *quiet P*

  shows *False*
using *assms*
by(*erule-tac quiet.cases*) (*force simp add*: *residualInject*)

lemma *quietTau*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and P' :: $('a, 'b, 'c)$ *psi*

  assumes $\Psi \rhd P \longmapsto \tau \prec P'$
  and *insertAssertion* (*extractFrame P*) $\Psi \simeq_F \langle \varepsilon, \Psi \rangle$
  and *quiet P*

  shows *quiet* P'
using *assms*
by(*erule-tac quiet.cases*) (*force simp add*: *residualInject*)

lemma *tauChainCases*[*consumes 1*, *case-names TauBase TauStep*]:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and P' :: $('a, 'b, 'c)$ *psi*

  assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$
  and $P = P' \Longrightarrow Prop$
  and $\Psi \rhd P \Longrightarrow_\tau P' \Longrightarrow Prop$

  shows *Prop*
using *assms*
by(*blast elim*: *rtranclE dest*: *rtrancl-into-trancl1*)

end

end

theory *Weak-Simulation*
  imports *Simulation Tau-Chain*
begin

context *env* **begin**

definition
weakSimulation :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
(*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set* ⇒
(*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ▷ - ⇝<-> - [*80*, *80*, *80*, *80*] *80*)
where
Ψ ▷ *P* ⇝<*Rel*> *Q* ≡ (∀ Ψ′ α *Q*′. Ψ ▷ *Q* ⟼α ≺ *Q*′ ⟶ *bn* α ♯* Ψ ⟶ *bn* α ♯* *P* ⟶ α ≠ τ ⟶
(∃ *P*′′. Ψ : *Q* ▷ *P* ⟹α ≺ *P*′′ ∧ (∃ *P*′. Ψ ⊗ Ψ′ ▷ *P*′′ ⟹^τ *P*′ ∧ (Ψ ⊗ Ψ′, *P*′, *Q*′) ∈ *Rel*))) ∧
(∀ *Q*′. Ψ ▷ *Q* ⟼τ ≺ *Q*′ ⟶ (∃ *P*′. Ψ ▷ *P* ⟹^τ *P*′ ∧ (Ψ, *P*′, *Q*′) ∈ *Rel*))

abbreviation
weakSimulationNilJudge (- ⇝<-> - [*80*, *80*, *80*] *80*) **where** *P* ⇝<*Rel*> *Q* ≡ *SBottom*′ ▷ *P* ⇝<*Rel*> *Q*

lemma *weakSimI*[*consumes 1*, *case-names cAct cTau*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *C* :: *'d*::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and *rAct*: ⋀Ψ′ α *Q*′. ⟦Ψ ▷ *Q* ⟼α ≺ *Q*′; *bn* α ♯* Ψ; *bn* α ♯* *P*; *bn* α ♯* *Q*; *bn* α ♯* *subject* α; *bn* α ♯* *C*; *distinct*(*bn* α); α ≠ τ⟧ ⟹
∃ *P*′′. Ψ : *Q* ▷ *P* ⟹α ≺ *P*′′ ∧ (∃ *P*′. Ψ ⊗ Ψ′ ▷ *P*′′ ⟹^τ *P*′ ∧ (Ψ ⊗ Ψ′, *P*′, *Q*′) ∈ *Rel*)
and *rTau*: ⋀*Q*′. Ψ ▷ *Q* ⟼τ ≺ *Q*′ ⟹ ∃ *P*′. Ψ ▷ *P* ⟹^τ *P*′ ∧ (Ψ, *P*′, *Q*′) ∈ *Rel*

shows Ψ ▷ *P* ⇝<*Rel*> *Q*
proof(*auto simp add*: *weakSimulation-def*)
fix Ψ′ α *Q*′
assume Ψ ▷ *Q* ⟼α ≺ *Q*′ **and** *bn* α ♯* Ψ **and** *bn* α ♯* *P* **and** α ≠ τ
thus ∃ *P*′′. Ψ : *Q* ▷ *P* ⟹α ≺ *P*′′ ∧ (∃ *P*′. Ψ ⊗ Ψ′ ▷ *P*′′ ⟹^τ *P*′ ∧ (Ψ ⊗ Ψ′, *P*′, *Q*′) ∈ *Rel*)
proof(*nominal-induct* α *rule*: *action.strong-induct*)
case(*In M N*)
thus *?case* **by**(*rule-tac rAct*) *auto*
next
case(*Out M xvec N*)
from ‹*bn*(*M*⟪ν∗*xvec*⟫⟨*N*⟩) ♯* Ψ› ‹*bn*(*M*⟪ν∗*xvec*⟫⟨*N*⟩) ♯* *P*› **have** *xvec* ♯* Ψ **and** *xvec* ♯* *P* **by** *simp*+
from ‹Ψ ▷ *Q* ⟼*M*⟪ν∗*xvec*⟫⟨*N*⟩ ≺ *Q*′› **have** *distinct xvec* **by**(*force dest*: *boundOutputDistinct*)
obtain *p* **where** (*p* ∙ *xvec*) ♯* Ψ **and** (*p* ∙ *xvec*) ♯* *M* **and** (*p* ∙ *xvec*) ♯* *xvec*

and$(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* Q'$ **and** $(p \cdot xvec) \sharp* \Psi'$
and $(p \cdot xvec) \sharp* C$ **and** $(p \cdot xvec) \sharp* xvec$ **and** $(p \cdot xvec) \sharp* N$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm p*
by(*rule-tac xvec=xvec* **and** $c=(\Psi, M, Q, N, P, Q', xvec, C, \Psi')$ **in** *name-list-avoiding*)
(*auto simp add*: *eqvts fresh-star-prod*)
from ‹*distinct xvec*› **have** *distinct*$(p \cdot xvec)$ **by** *simp*
from $\langle \Psi \rhd Q \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec Q'\rangle$ $\langle (p \cdot xvec) \sharp* N\rangle$ $\langle (p \cdot xvec) \sharp* Q'\rangle$ S
have $\Psi \rhd Q \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle (p \cdot N)\rangle \prec (p \cdot Q')$ **by**(*simp add*: *boundOutputChainAlpha''* *residualInject*)

then obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu*(p \cdot xvec)|\!)\langle (p \cdot N)\rangle \prec P''$
and $P''Chain$: $\Psi \otimes (p \cdot \Psi') \rhd P'' \Longrightarrow\hat{}_\tau P'$
and $P'RelQ'$: $(\Psi \otimes (p \cdot \Psi'), P', p \cdot Q') \in Rel$
using $\langle (p \cdot xvec) \sharp* \Psi\rangle$ $\langle (p \cdot xvec) \sharp* P\rangle$ $\langle (p \cdot xvec) \sharp* Q\rangle$ $\langle (p \cdot xvec) \sharp* M\rangle$ $\langle (p \cdot xvec) \sharp* C\rangle$ ‹*distinct*$(p \cdot xvec)$›
by(*drule-tac* $\Psi'=p \cdot \Psi'$ **in** *rAct*) *auto*

from *PTrans* S ‹*distinctPerm p*› ‹*xvec* $\sharp* P$› ‹$(p \cdot xvec) \sharp* xvec$› ‹$(p \cdot xvec) \sharp* M$› ‹*distinct xvec*›
have $\Psi : Q \rhd P \Longrightarrow M(\!|\nu* xvec|\!)\langle N\rangle \prec (p \cdot P'')$
by(*rule-tac weakOutputAlpha*) *auto*
moreover from $P''Chain$ **have** $(p \cdot (\Psi \otimes (p \cdot \Psi'))) \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau (p \cdot P')$ **by**(*rule eqvts*)
with ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi'$› S ‹*distinctPerm p*›
have $\Psi \otimes \Psi' \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from $P'RelQ'$ *Eqvt* S ‹*distinctPerm p*› **have** $(p \cdot (\Psi \otimes (p \cdot \Psi')), p \cdot P', p \cdot p \cdot Q') \in Rel$
apply(*simp add*: *eqvt-def eqvts*)
apply(*erule-tac* $x=(\Psi \otimes (p \cdot \Psi'), P', p \cdot Q')$ **in** *ballE*)
apply(*erule-tac* $x=p$ **in** *allE*)
by(*auto simp add*: *eqvts*)
with ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S ‹*distinctPerm p*›
have $(\Psi \otimes \Psi', p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
case *Tau*
thus *?case* **by** *simp*
qed
next
fix Q'
assume $\Psi \rhd Q \longmapsto \tau \prec Q'$
thus $\exists P'.\ \Psi \rhd P \Longrightarrow\hat{}_\tau P' \wedge (\Psi, P', Q') \in Rel$
by(*rule rTau*)
qed

lemma *weakSimI2*[*case-names cAct cTau*]:
fixes Ψ :: *'b*

and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and C :: $'d$::*fs-name*

assumes *rOutput*: $\bigwedge \Psi' \alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ \alpha \neq \tau]\!] \Longrightarrow$
$\exists P''.\ \Psi : Q \rhd P \Longrightarrow \alpha \prec P'' \wedge (\exists P'.\ \Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_{\tau}\ P' \wedge (\Psi \otimes \Psi', P', Q') \in Rel)$
and *rTau*: $\bigwedge Q'.\ \Psi \rhd Q \longmapsto \tau \prec Q' \Longrightarrow \exists P'.\ \Psi \rhd P \Longrightarrow\hat{}_{\tau}\ P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel> Q$
using *assms* **by**(*simp add*: *weakSimulation-def*)

lemma *weakSimIChainFresh*[*consumes 4*, *case-names cOutput cInput*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and *yvec* :: *name list*
and C :: $'d$::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and *yvec* $\sharp*$ Ψ
and *yvec* $\sharp*$ P
and *yvec* $\sharp*$ Q
and *rAct*: $\bigwedge \Psi' \alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ \alpha \neq \tau;$
$bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C;\ yvec \sharp* \alpha;\ yvec \sharp* Q';\ yvec \sharp* \Psi]\!] \Longrightarrow$
$\exists P''.\ \Psi : Q \rhd P \Longrightarrow \alpha \prec P'' \wedge (\exists P'.\ \Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_{\tau}\ P' \wedge (\Psi \otimes \Psi', P', Q') \in Rel)$
and *rTau*: $\bigwedge Q'.\ [\![\Psi \rhd Q \longmapsto \tau \prec Q';\ yvec \sharp* Q']\!] \Longrightarrow \exists P'.\ \Psi \rhd P \Longrightarrow\hat{}_{\tau}\ P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel> Q$
using ‹*eqvt Rel*›
proof(*induct rule*: *weakSimI*[*of* - - - - (*yvec*, *C*)])
case(*cAct* $\Psi'\ \alpha\ Q'$)
obtain *p*::*name prm* **where** $(p \cdot yvec) \sharp* \Psi$ **and** $(p \cdot yvec) \sharp* P$ **and** $(p \cdot yvec) \sharp* Q$
and $(p \cdot yvec) \sharp* \alpha$ **and** $(p \cdot yvec) \sharp* \Psi'$
and $(p \cdot yvec) \sharp* Q'$ **and** *S*: $(set\ p) \subseteq set\ yvec \times set(p \cdot yvec)$
and *distinctPerm p*
by(*rule-tac c*=$(\Psi, P, Q, \alpha, Q', \Psi')$ **and** *xvec*=*yvec* **in** *name-list-avoiding*) *auto*
from ‹$bn\ \alpha \sharp* (yvec, C)$› **have** $bn\ \alpha \sharp* yvec$ **and** $bn\ \alpha \sharp* C$ **by**(*simp add*: *freshChainSym*)+
show *?case*

proof(*cases rule*: *actionCases*[**where** $\alpha = \alpha$])
case(*cInput M N*)
from ‹$(p \cdot yvec) \sharp* \alpha$› ‹$\alpha = M(\!|N|\!)$› **have** $(p \cdot yvec) \sharp* M$ **and** $(p \cdot yvec) \sharp* N$ **by** *simp+*
from ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹$\alpha = M(\!|N|\!)$› ‹$yvec \sharp* \Psi$› ‹$(p \cdot yvec) \sharp* \Psi$› ‹$yvec \sharp* Q$› ‹$(p \cdot yvec) \sharp* Q$› *S*
have $\Psi \rhd Q \longmapsto (p \cdot M)(\!|(p \cdot N)|\!) \prec (p \cdot Q')$
by(*drule-tac pi=p* **in** *semantics.eqvt*) (*simp add*: *eqvts*)
moreover from ‹$(p \cdot yvec) \sharp* M$› **have** $(p \cdot (p \cdot yvec)) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $yvec \sharp* (p \cdot M)$ **by** *simp*
moreover from ‹$(p \cdot yvec) \sharp* N$› **have** $(p \cdot p \cdot yvec) \sharp* (p \cdot N)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $yvec \sharp* (p \cdot N)$ **by** *simp*
moreover from ‹$(p \cdot yvec) \sharp* Q'$› **have** $(p \cdot p \cdot yvec) \sharp* (p \cdot Q')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $yvec \sharp* (p \cdot Q')$ **by** *simp*
moreover from ‹$(p \cdot yvec) \sharp* \Psi'$› **have** $(p \cdot p \cdot yvec) \sharp* (p \cdot \Psi')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $yvec \sharp* (p \cdot \Psi')$ **by** *simp*
ultimately obtain P' P'' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow (p \cdot M)(\!|(p \cdot N)|\!) \prec P''$ **and** *P''Chain*: $\Psi \otimes (p \cdot \Psi') \rhd P'' \Longrightarrow\hat{}_\tau P'$
and *P'RelQ'*: $(\Psi \otimes (p \cdot \Psi'), P', (p \cdot Q')) \in Rel$
by(*auto dest*: *rAct*)
from *PTrans* **have** $(p \cdot \Psi) : (p \cdot Q) \rhd (p \cdot P) \Longrightarrow p \cdot ((p \cdot M)(\!|(p \cdot N)|\!)) \prec (p \cdot P'')$
by(*rule weakTransitionClosed*)
with *S* ‹$yvec \sharp* \Psi$› ‹$(p \cdot yvec) \sharp* \Psi$› ‹$yvec \sharp* Q$› ‹$(p \cdot yvec) \sharp* Q$› ‹$yvec \sharp* P$› ‹$(p \cdot yvec) \sharp* P$› ‹*distinctPerm p*›
have $\Psi : Q \rhd P \Longrightarrow M(\!|N|\!) \prec (p \cdot P'')$ **by**(*simp add*: *eqvts*)
moreover from *P''Chain* **have** $(p \cdot (\Psi \otimes (p \cdot \Psi'))) \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau (p \cdot P')$ **by**(*rule eqvts*)
with ‹$yvec \sharp* \Psi$› ‹$(p \cdot yvec) \sharp* \Psi$› *S* ‹*distinctPerm p*›
have $\Psi \otimes \Psi' \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from *P'RelQ' Eqvt* **have** $(p \cdot (\Psi \otimes (p \cdot \Psi')), p \cdot P', p \cdot p \cdot Q') \in Rel$
apply(*simp add*: *eqvt-def eqvts*)
apply(*erule-tac* $x=(\Psi \otimes (p \cdot \Psi'), P', p \cdot Q')$ **in** *ballE*)
apply(*erule-tac* $x=p$ **in** *allE*)
by(*auto simp add*: *eqvts*)
with ‹$yvec \sharp* \Psi$› ‹$(p \cdot yvec) \sharp* \Psi$› *S* ‹*distinctPerm p*› **have** $(\Psi \otimes \Psi', p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
ultimately show *?thesis* **using** ‹$\alpha = M(\!|N|\!)$› **by** *blast*
next
case(*cOutput M xvec N*)
from ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp* Q$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$\alpha = M(\!|\nu * xvec|\!)\langle N\rangle$› ‹$bn\ \alpha \sharp* yvec$›
‹$(p \cdot yvec) \sharp* \alpha$› ‹$bn\ \alpha \sharp* C$› ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$distinct(bn\ \alpha)$›
have $xvec \sharp* \Psi$ **and** $xvec \sharp* P$ **and** $xvec \sharp* Q$ **and** $xvec \sharp* M$ **and** $yvec \sharp* xvec$

and $(p \cdot yvec) \sharp* M$ **and** $(p \cdot yvec) \sharp* xvec$ **and** $xvec \sharp* C$ **and** $xvec \sharp* M$ **and** $(p \cdot yvec) \sharp* N$
and *distinct xvec* **by** *simp+*
from $\langle \Psi \rhd Q \longmapsto \alpha \prec Q' \rangle$ $\langle yvec \sharp* \Psi \rangle$ $\langle (p \cdot yvec) \sharp* \Psi \rangle$ $\langle yvec \sharp* Q \rangle$ $\langle (p \cdot yvec) \sharp* Q \rangle$ S $\langle \alpha = M(\!|\nu * xvec|\!)\langle N \rangle \rangle$
have $\Psi \rhd Q \longmapsto (p \cdot M)(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$
by(*rule-tac outputPermSubject*) (*assumption* | *simp add*: *fresh-star-def*)+
moreover note $\langle xvec \sharp* \Psi \rangle$ $\langle xvec \sharp* P \rangle$
moreover note $\langle xvec \sharp* Q \rangle$
moreover from $\langle xvec \sharp* M \rangle$ **have** $(p \cdot xvec) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle yvec \sharp* xvec \rangle$ $\langle (p \cdot yvec) \sharp* xvec \rangle$ S **have** $xvec \sharp* (p \cdot M)$
by *simp*
moreover note $\langle xvec \sharp* C \rangle$
moreover from $\langle (p \cdot yvec) \sharp* M \rangle$ **have** $(p \cdot (p \cdot yvec)) \sharp* (p \cdot M)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle distinctPerm\ p \rangle$ **have** $yvec \sharp* (p \cdot M)$ **by** *simp*
moreover note $\langle yvec \sharp* xvec \rangle$
moreover from $\langle \Psi \rhd Q \longmapsto \alpha \prec Q' \rangle$ $\langle yvec \sharp* Q \rangle$ $\langle yvec \sharp* xvec \rangle$ $\langle \alpha = M(\!|\nu * xvec|\!)\langle N \rangle \rangle$ $\langle xvec \sharp* M \rangle$ $\langle distinct\ xvec \rangle$
have $yvec \sharp* N$ **and** $yvec \sharp* Q'$ **by**(*force dest*: *outputFreshChainDerivative*)+
moreover from $\langle (p \cdot yvec) \sharp* \Psi' \rangle$ **have** $(p \cdot p \cdot yvec) \sharp* (p \cdot \Psi')$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle distinctPerm\ p \rangle$ **have** $yvec \sharp* (p \cdot \Psi')$ **by** *simp*
ultimately obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow (p \cdot M)(\!|\nu * xvec|\!)\langle N \rangle \prec P''$
and *PChain*: $\Psi \otimes (p \cdot \Psi') \rhd P'' \Longrightarrow\!\!\hat{}_{\tau}\ P'$
and $P'RelQ'$: $(\Psi \otimes (p \cdot \Psi'), P', Q') \in Rel$
by(*drule-tac rAct*) *auto*
from *PTrans* **have** $(p \cdot \Psi) : (p \cdot Q) \rhd (p \cdot P) \Longrightarrow (p \cdot ((p \cdot M)(\!|\nu * xvec|\!)\langle N \rangle)) \prec (p \cdot P'')$
by(*rule eqvts*)
with S $\langle yvec \sharp* \Psi \rangle$ $\langle (p \cdot yvec) \sharp* \Psi \rangle$ $\langle yvec \sharp* P \rangle$ $\langle (p \cdot yvec) \sharp* P \rangle$ $\langle yvec \sharp* Q \rangle$ $\langle (p \cdot yvec) \sharp* Q \rangle$ $\langle yvec \sharp* xvec \rangle$ $\langle (p \cdot yvec) \sharp* xvec \rangle$
$\langle yvec \sharp* N \rangle$ $\langle (p \cdot yvec) \sharp* N \rangle$ $\langle distinctPerm\ p \rangle$ **have** $\Psi : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N \rangle \prec (p \cdot P'')$
by(*simp add*: *eqvts*)
moreover from *PChain* **have** $(p \cdot (\Psi \otimes (p \cdot \Psi'))) \rhd (p \cdot P'') \Longrightarrow\!\!\hat{}_{\tau}\ (p \cdot P')$ **by**(*rule eqvts*)
with S $\langle yvec \sharp* \Psi \rangle$ $\langle (p \cdot yvec) \sharp* \Psi \rangle$ $\langle distinctPerm\ p \rangle$ **have** $\Psi \otimes \Psi' \rhd (p \cdot P'') \Longrightarrow\!\!\hat{}_{\tau}\ (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from $P'RelQ'$ $\langle eqvt\ Rel \rangle$ **have** $p \cdot (\Psi \otimes (p \cdot \Psi'), P', Q') \in Rel$
by(*simp add*: *eqvt-def*) *auto*
with $\langle yvec \sharp* \Psi \rangle$ $\langle (p \cdot yvec) \sharp* \Psi \rangle$ $\langle yvec \sharp* Q' \rangle$ $\langle (p \cdot yvec) \sharp* Q' \rangle$ S $\langle distinctPerm\ p \rangle$
have $(\Psi \otimes \Psi', p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
ultimately show *?thesis* **using** $\langle \alpha = M(\!|\nu * xvec|\!)\langle N \rangle \rangle$ **by** *blast*
next
case *cTau*

from ‹$\alpha = \tau$› ‹$\alpha \neq \tau$› **have** *False* **by** *simp*
thus *?thesis* **by** *simp*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$› ‹*yvec* $\sharp*$ *Q*› **have** *yvec* $\sharp*$ *Q'*
by(*force dest*: *tauFreshChainDerivative*)
with ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$› **show** *?case*
by(*rule rTau*)
qed

lemma *weakSimIFresh*[*consumes 4*, *case-names cAct cTau*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *x* :: *name*
and *C* :: *'d*::*fs-name*

assumes *Eqvt*: *eqvt Rel*
and $x \sharp \Psi$
and $x \sharp P$
and $x \sharp Q$
and $\bigwedge \alpha\ Q'\ \Psi'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q;\ \alpha \neq \tau;$
$bn\ \alpha \sharp* subject\ \alpha;\ bn\ \alpha \sharp* C;\ x \sharp \alpha;\ x \sharp Q';\ x \sharp \Psi]\!] \Longrightarrow$
$\exists P''.\ \Psi : Q \rhd P \Longrightarrow \alpha \prec P'' \wedge (\exists P'.\ \Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$
$\wedge\ (\Psi \otimes \Psi',\ P',\ Q') \in Rel)$
and $\bigwedge Q'.\ [\![\Psi \rhd Q \longmapsto \tau \prec Q';\ x \sharp Q]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \Longrightarrow\hat{}_\tau\ P' \wedge (\Psi,\ P',$
$Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel> Q$
using *assms*
by(*rule-tac yvec*=[*x*] **and** *C*=*C* **in** *weakSimIChainFresh*) *auto*

lemma *weakSimE*:
fixes *F* :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \rightsquigarrow <Rel> Q$

shows $\bigwedge \Psi'\ \alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto \alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P;\ \alpha \neq \tau]\!] \Longrightarrow$
$\exists P''.\ \Psi : Q \rhd P \Longrightarrow \alpha \prec P'' \wedge (\exists P'.\ \Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau$
$P' \wedge (\Psi \otimes \Psi',\ P',\ Q') \in Rel)$
and $\bigwedge Q'.\ \Psi \rhd Q \longmapsto \tau \prec Q' \Longrightarrow \exists P'.\ \Psi \rhd P \Longrightarrow\hat{}_\tau\ P' \wedge (\Psi,\ P',\ Q') \in Rel$
using *assms*
by(*auto simp add*: *weakSimulation-def*)

lemma *weakSimClosedAux*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and p :: *name prm*

assumes *EqvtRel*: *eqvt Rel*
and *PSimQ*: $\Psi \rhd P \rightsquigarrow <Rel> Q$

shows $(p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow <Rel> (p \cdot Q)$
using *EqvtRel*
proof(*induct rule*: *weakSimI*[*of* - - - - (Ψ, P, p)])
case(*cAct* Ψ' α Q')
from ‹$p \cdot \Psi \rhd p \cdot Q \longmapsto \alpha \prec Q'$›
have $(rev\ p \cdot p \cdot \Psi) \rhd (rev\ p \cdot p \cdot Q) \longmapsto (rev\ p \cdot (\alpha \prec Q'))$
by(*blast dest*: *semantics.eqvt*)
hence $\Psi \rhd Q \longmapsto (rev\ p \cdot \alpha) \prec (rev\ p \cdot Q')$
by(*simp add*: *eqvts*)
moreover with ‹*bn* α $\sharp*$ (Ψ, P, p)› **have** *bn* α $\sharp*$ Ψ **and** *bn* α $\sharp*$ P **and** *bn* α $\sharp*$ p **by** *simp+*
moreover from ‹$\alpha \neq \tau$› **have** $(rev\ p \cdot \alpha) \neq \tau$
by(*cases rule*: *actionCases*[**where** α=α]) *auto*
ultimately obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow (rev\ p \cdot \alpha) \prec P''$
and *P''Chain*: $\Psi \otimes (rev\ p \cdot \Psi') \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: $(\Psi \otimes (rev\ p \cdot \Psi'), P', rev\ p \cdot Q') \in Rel$
using ‹$\alpha \neq \tau$› *PSimQ*
by(*drule-tac* Ψ'=*rev p* · Ψ' **in** *weakSimE*(*1*)) (*auto simp add*: *freshChainPerm-Simp bnEqvt*[*symmetric*])

from *PTrans* **have** $(p \cdot \Psi) : (p \cdot Q) \rhd (p \cdot P) \Longrightarrow (p \cdot (rev\ p \cdot \alpha)) \prec (p \cdot P'')$
by(*rule eqvts*)
hence $(p \cdot \Psi) : (p \cdot Q) \rhd (p \cdot P) \Longrightarrow \alpha \prec (p \cdot P'')$ **by**(*simp add*: *eqvts freshChain-PermSimp*)
moreover from *P''Chain* **have** $(p \cdot (\Psi \otimes (rev\ p \cdot \Psi'))) \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau\ (p \cdot P')$ **by**(*rule eqvts*)
hence $(p \cdot \Psi) \otimes \Psi' \rhd (p \cdot P'') \Longrightarrow\hat{}_\tau\ (p \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from *P'RelQ' EqvtRel* **have** $(p \cdot ((\Psi \otimes (rev\ p \cdot \Psi')), P', rev\ p \cdot Q')) \in Rel$
by(*simp only*: *eqvt-def*)
hence $((p \cdot \Psi) \otimes \Psi', p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
case(*cTau* Q')
from ‹$p \cdot \Psi \rhd p \cdot Q \longmapsto \tau \prec Q'$›
have $(rev\ p \cdot p \cdot \Psi) \rhd (rev\ p \cdot p \cdot Q) \longmapsto (rev\ p \cdot (\tau \prec Q'))$
by(*blast dest*: *semantics.eqvt*)
hence $\Psi \rhd Q \longmapsto \tau \prec (rev\ p \cdot Q')$ **by**(*simp add*: *eqvts*)
with *PSimQ* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: (Ψ, P',

rev $p \cdot Q') \in Rel$
  by(*blast dest*: *weakSimE*)
 from *PChain* **have** $(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow^{\hat{}}_{\tau} (p \cdot P')$ **by**(*rule tauChainEqvt*)
 moreover from *P'RelQ' EqvtRel* **have** $(p \cdot (\Psi, P', rev\ p \cdot Q')) \in Rel$
  by(*simp only*: *eqvt-def*)
 hence $(p \cdot \Psi, p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
 ultimately show *?case*
  by *blast*
qed

lemma *weakSimClosed*:
 fixes Ψ :: *'b*
 and P :: *('a, 'b, 'c) psi*
 and *Rel* :: *('b* $\times$ *('a, 'b, 'c) psi* $\times$ *('a, 'b, 'c) psi) set*
 and Q :: *('a, 'b, 'c) psi*
 and p :: *name prm*

 assumes *EqvtRel*: *eqvt Rel*

 shows $\Psi \rhd P \rightsquigarrow$*<Rel>* $Q \Longrightarrow (p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow$*<Rel>* $(p \cdot Q)$
 and $P \rightsquigarrow$*<Rel>* $Q \Longrightarrow (p \cdot P) \rightsquigarrow$*<Rel>* $(p \cdot Q)$
using *EqvtRel*
by(*force dest*: *weakSimClosedAux simp add*: *permBottom*)+

lemma *weakSimReflexive*:
 fixes *Rel* :: *('b* $\times$ *('a, 'b, 'c) psi* $\times$ *('a, 'b, 'c) psi) set*
 and Ψ :: *'b*
 and P :: *('a, 'b, 'c) psi*

 assumes $\{(\Psi, P, P) \mid \Psi\ P.\ True\} \subseteq Rel$

 shows $\Psi \rhd P \rightsquigarrow$*<Rel>* P
using *assms*
by(*auto simp add*: *weakSimulation-def weakTransition-def dest*: *rtrancl-into-rtrancl*)
force+

lemma *weakSimTauChain*:
 fixes Ψ :: *'b*
 and Q :: *('a, 'b, 'c) psi*
 and Q' :: *('a, 'b, 'c) psi*
 and P :: *('a, 'b, 'c) psi*
 and *Rel* :: *('b* $\times$ *('a, 'b, 'c) psi* $\times$ *('a, 'b, 'c) psi) set*

 assumes $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$
 and $(\Psi, P, Q) \in Rel$
 and *Sim*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow$*<Rel>* S

 obtains P' **where** $\Psi \rhd P \Longrightarrow^{\hat{}}_{\tau} P'$ **and** $(\Psi, P', Q') \in Rel$
proof −

assume A: $\bigwedge P'.\ [\![\Psi \rhd P \Longrightarrow\hat{}_\tau\ P';\ (\Psi,\ P',\ Q') \in Rel]\!] \Longrightarrow thesis$
from $\langle \Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q' \rangle$ $\langle (\Psi,\ P,\ Q) \in Rel \rangle$ A **show** *?thesis*
proof(*induct arbitrary*: P *thesis rule*: *tauChainInduct*)
case(*TauBase* Q P)
moreover have $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P$ **by** *simp*
ultimately show *?case* **by** *blast*
next
case(*TauStep* Q Q' Q'' P)
from $\langle (\Psi,\ P,\ Q) \in Rel \rangle$ **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** $(\Psi,\ P',\ Q') \in Rel$
by(*rule TauStep*)
from $\langle (\Psi,\ P',\ Q') \in Rel \rangle$ **have** $\Psi \rhd P' \rightsquigarrow <Rel>\ Q'$ **by**(*rule Sim*)
then obtain P'' **where** *P'Chain*: $\Psi \rhd P' \Longrightarrow\hat{}_\tau\ P''$ **and** $(\Psi,\ P'',\ Q'') \in Rel$
using $\langle \Psi \rhd Q' \longmapsto \tau \prec Q'' \rangle$ **by**(*blast dest*: *weakSimE*)
from *PChain P'Chain* **have** $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''$ **by** *simp*
thus *?case* **using** $\langle (\Psi,\ P'',\ Q'') \in Rel \rangle$ **by**(*rule TauStep*)
qed
qed

lemma *weakSimE2*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Rel :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes *PRelQ*: $(\Psi,\ P,\ Q) \in Rel$
and *Sim*: $\bigwedge \Psi'\ R\ S.\ (\Psi',\ R,\ S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow <Rel>\ S$
and *QTrans*: $\Psi : R \rhd Q \Longrightarrow \alpha \prec Q'$
and $bn\ \alpha \sharp* \Psi$
and $bn\ \alpha \sharp* P$
and $\alpha \neq \tau$

obtains P'' P' **where** $\Psi : R \rhd P \Longrightarrow \alpha \prec P''$ **and** $\Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **and** $(\Psi \otimes \Psi',\ P',\ Q') \in Rel$
proof −
assume A: $\bigwedge P''\ P'.\ [\![\Psi : R \rhd P \Longrightarrow \alpha \prec P'';\ \Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P';\ (\Psi \otimes \Psi',\ P',\ Q') \in Rel]\!] \Longrightarrow thesis$
from *QTrans* **obtain** Q''
where *QChain*: $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q''$
and *ReqQ''*: *insertAssertion* (*extractFrame* R) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'') Ψ
and *Q''Trans*: $\Psi \rhd Q'' \longmapsto \alpha \prec Q'$
by(*rule weakTransitionE*)

from *QChain PRelQ Sim*
obtain P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''$ **and** *P''RelQ''*: $(\Psi,\ P'',\ Q'') \in Rel$ **by**(*rule weakSimTauChain*)

from *PChain* $\langle bn\ \alpha \sharp* P \rangle$ **have** $bn\ \alpha \sharp* P''$ **by**(*rule tauChainFreshChain*)

from $P''RelQ''$ **have** $\Psi \rhd P'' \rightsquigarrow <Rel> Q''$ **by**(*rule Sim*)
with $Q''Trans$ ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P''$› ‹$\alpha \neq \tau$›
obtain $P'''\ P'$ **where** $P''Trans$: $\Psi : Q'' \rhd P'' \Longrightarrow \alpha \prec P'''$ **and** $P'''Chain$: $\Psi \otimes \Psi' \rhd P''' \Longrightarrow\hat{}_\tau\ P'$
and $P'RelQ'$: $(\Psi \otimes \Psi', P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from $P''Trans$ **obtain** P'''' **where** $P''Chain$: $\Psi \rhd P'' \Longrightarrow\hat{}_\tau\ P''''$
and $Q''eqP''''$: $insertAssertion\ (extractFrame\ Q'')\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame\ P'''')\ \Psi$
and $P''''Trans$: $\Psi \rhd P'''' \longmapsto \alpha \prec P'''$
by(*rule weakTransitionE*)
from $PChain\ P''Chain$ **have** $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''''$ **by** *simp*
moreover from $ReqQ''\ Q''eqP''''$ **have** $insertAssertion\ (extractFrame\ R)\ \Psi \hookrightarrow_F insertAssertion\ (extractFrame\ P'''')\ \Psi$
by(*rule FrameStatImpTrans*)
ultimately have $\Psi : R \rhd P \Longrightarrow \alpha \prec P'''$ **using** $P''''Trans$ **by**(*rule weakTransitionI*)
with $P'''Chain\ P'RelQ'\ A$ **show** *?thesis* **by** *blast*
qed

lemma *weakSimTransitive*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)\ psi$
and Rel' :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and T :: $('a, 'b, 'c)\ psi$
and Rel'' :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $PRelQ$: $(\Psi, P, Q) \in Rel$
and $QSimR$: $\Psi \rhd Q \rightsquigarrow <Rel'> R$
and $Eqvt$: $eqvt\ Rel''$
and Set: $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in Rel \wedge (\Psi, Q, R) \in Rel'\} \subseteq Rel''$
and Sim: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow <Rel> S$

shows $\Psi \rhd P \rightsquigarrow <Rel''> R$
using ‹$eqvt\ Rel''$›
proof(*induct rule*: *weakSimI*[*of* - - - - *Q*])
case($cAct\ \Psi'\ \alpha\ R'$)
from $QSimR$ ‹$\Psi \rhd R \longmapsto \alpha \prec R'$› ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* Q$› ‹$\alpha \neq \tau$›
obtain $Q''\ Q'$ **where** $QTrans$: $\Psi : R \rhd Q \Longrightarrow \alpha \prec Q''$ **and** $Q''Chain$: $\Psi \otimes \Psi' \rhd Q'' \Longrightarrow\hat{}_\tau\ Q'$
and $Q'RelR'$: $(\Psi \otimes \Psi', Q', R') \in Rel'$
by(*blast dest*: *weakSimE*)
with *PRelQ Sim QTrans* ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$\alpha \neq \tau$›
obtain $P'''\ P''$ **where** $PTrans$: $\Psi : R \rhd P \Longrightarrow \alpha \prec P'''$
and $P'''Chain$: $\Psi \otimes \Psi' \rhd P''' \Longrightarrow\hat{}_\tau\ P''$ **and** $P''RelQ''$: $(\Psi \otimes \Psi',$

P'', Q'') $\in$ *Rel*
by(*drule-tac weakSimE2*) *auto*

note *PTrans*
moreover from *Q''Chain P''RelQ'' Sim* **obtain** P' **where** *P''Chain*: $\Psi \otimes \Psi'$ $\rhd$ $P'' \Longrightarrow^{\hat{}}_{\tau} P'$ **and** *P'RelQ'*: ($\Psi \otimes \Psi'$, P', Q') $\in$ *Rel*
by(*rule weakSimTauChain*)
from *P'''Chain P''Chain* **have** $\Psi \otimes \Psi' \rhd P''' \Longrightarrow^{\hat{}}_{\tau} P'$ **by** *simp*
moreover from *P'RelQ' Q'RelR' Set* **have** ($\Psi \otimes \Psi'$, P', R') $\in$ *Rel''* **by** *blast*
ultimately show *?case* **by** *blast*
next
case(*cTau R'*)
from *QSimR* ‹$\Psi \rhd R \longmapsto \tau \prec R'$› **obtain** Q' **where** *QChain*: $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$ **and** *Q'RelR'*: (Ψ, Q', R') $\in$ *Rel'*
by(*blast dest*: *weakSimE*)
from *QChain PRelQ Sim* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow^{\hat{}}_{\tau} P'$ **and** *P'RelQ'*: (Ψ, P', Q') $\in$ *Rel*
by(*rule weakSimTauChain*)
note *PChain*
moreover from *P'RelQ' Q'RelR' Set* **have** (Ψ, P', R') $\in$ *Rel''* **by** *blast*
ultimately show *?case* **by** *blast*
qed

lemma *weakSimStatEq*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and Ψ' :: *'b*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *eqvt Rel'*
and $\Psi \simeq \Psi'$
and *C1*: $\bigwedge \Psi' R S \Psi''. [\![(\Psi', R, S) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', R, S) \in Rel'$

shows $\Psi' \rhd P \rightsquigarrow <Rel'> Q$
using ‹*eqvt Rel'*›
proof(*induct rule*: *weakSimI*[*of* - - - - Ψ])
case(*cAct* Ψ'' α Q')
from ‹$\Psi' \rhd Q \longmapsto \alpha \prec Q'$› ‹$\Psi \simeq \Psi'$›
have $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **by**(*metis statEqTransition AssertionStatEqSym*)
with *PSimQ* ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P› ‹$\alpha \neq \tau$›
obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$ **and** *P''Chain*: $\Psi \otimes \Psi''$ $\rhd$ $P'' \Longrightarrow^{\hat{}}_{\tau} P'$
and *P'RelQ'*: ($\Psi \otimes \Psi''$, P', Q') $\in$ *Rel*
by(*blast dest*: *weakSimE*)

from *PTrans* ‹$\Psi \simeq \Psi'$› **have** $\Psi' : Q \rhd P \Longrightarrow \alpha \prec P''$ **by**(*rule weakTransitionStatEq*)

moreover from $P''Chain$ $\langle\Psi \simeq \Psi'\rangle$ **have** $\Psi' \otimes \Psi'' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **by**(*metis tauChainStatEq Composition*)
moreover from $P'RelQ'$ $\langle\Psi \simeq \Psi'\rangle$ **have** $(\Psi' \otimes \Psi'', P', Q') \in Rel'$
by(*metis C1 Composition*)
ultimately show *?case*
by *blast*
next
case(*cTau* Q')
from $\langle\Psi' \rhd Q \longmapsto\tau \prec Q'\rangle$ $\langle\Psi \simeq \Psi'\rangle$
have $\Psi \rhd Q \longmapsto\tau \prec Q'$ **by**(*metis statEqTransition AssertionStatEqSym*)
with *PSimQ* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from *PChain* $\langle\Psi \simeq \Psi'\rangle$ **have** $\Psi' \rhd P \Longrightarrow\hat{}_\tau\ P'$ **by**(*rule tauChainStatEq*)
moreover from $\langle(\Psi, P', Q') \in Rel\rangle$ $\langle\Psi \simeq \Psi'\rangle$ **have** $(\Psi', P', Q') \in Rel'$
by(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *weakSimMonotonic*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $A :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $B :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $\Psi \rhd P \rightsquigarrow <A> Q$
and $A \subseteq B$

shows $\Psi \rhd P \rightsquigarrow <B> Q$
using *assms*
by(*simp* (*no-asm*) *add*: *weakSimulation-def*) (*blast dest*: *weakSimE*)+

lemma *strongSimWeakSim*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PRelQ*: $(\Psi, P, Q) \in Rel$
and *StatImp*: $\bigwedge\Psi'\ R\ S.\ (\Psi', R, S) \in Rel \Longrightarrow insertAssertion(extractFrame\ S)\ \Psi' \hookrightarrow_F insertAssertion(extractFrame\ R)\ \Psi'$
and *Sim*: $\bigwedge\Psi'\ R\ S.\ (\Psi', R, S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow[Rel]\ S$
and *Ext*: $\bigwedge\Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in Rel \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel> Q$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ Q'$)

from *PRelQ* **have** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$ **by**(*rule Sim*)
with $\langle \Psi \rhd Q \longmapsto \alpha \prec Q' \rangle$ $\langle bn\ \alpha \sharp* \Psi \rangle$ $\langle bn\ \alpha \sharp* P \rangle$
obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *simE*)

from *PRelQ* **have** *insertAssertion*(*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame P*) Ψ **by**(*rule StatImp*)
with *PTrans* **have** $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$ **by**(*rule transitionWeakTransition*)
moreover from $P'RelQ'$ **have** $\forall \Psi'.\ \exists P''.\ \Psi \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ P'' \wedge (\Psi \otimes \Psi', P'', Q') \in Rel$
by(*force intro*: *Ext*)
ultimately show *?case* **by** *blast*
next
case(*cTau Q'*)
from *PRelQ* **have** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$ **by**(*rule Sim*)
with $\langle \Psi \rhd Q \longmapsto \tau \prec Q' \rangle$ **obtain** P' **where** *PTrans*: $\Psi \rhd P \longmapsto \tau \prec P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*force dest*: *simE*)
with *PTrans* **have** $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **by** *auto*
thus *?case* **using** $P'RelQ'$ **by** *blast*
qed

lemma *strongAppend*:
fixes Ψ :: *'b*
and *P* :: *('a, 'b, 'c) psi*
and *Q* :: *('a, 'b, 'c) psi*
and *R* :: *('a, 'b, 'c) psi*
and *Rel* :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*
and *Rel'* :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*
and *Rel''* :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow<Rel> Q$
and *QSimR*: $\Psi \rhd Q \rightsquigarrow[Rel']\ R$
and *Eqvt''*: *eqvt Rel''*
and *RimpQ*: *insertAssertion* (*extractFrame R*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q*) Ψ
and *Set*: $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in Rel \wedge (\Psi, Q, R) \in Rel'\} \subseteq Rel''$
and *C1*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in Rel' \Longrightarrow (\Psi \otimes \Psi', P, Q) \in Rel'$

shows $\Psi \rhd P \rightsquigarrow<Rel''> R$
proof −
from *Eqvt''* **show** *?thesis*
proof(*induct rule*: *weakSimI*[*of* - - - - *Q*])
case(*cAct* $\Psi'\ \alpha\ R'$)
from $\langle \Psi \rhd Q \rightsquigarrow[Rel'] R \rangle$ $\langle \Psi \rhd R \longmapsto \alpha \prec R' \rangle$ $\langle bn\ \alpha \sharp* \Psi \rangle$ $\langle bn\ \alpha \sharp* Q \rangle$
obtain Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **and** $(\Psi, Q', R') \in Rel'$
by(*blast dest*: *simE*)

from ‹$(\Psi, Q', R') \in Rel'$› **have** $Q'RelR'$: $(\Psi \otimes \Psi', Q', R') \in Rel'$ **by**(*rule C1*)

from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› $QTrans$ ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$\alpha \neq \tau$›
obtain $P''\ P'$ **where** $PTrans$: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$ **and** $P''Chain$: $\Psi \otimes \Psi' \rhd P'' \hat{\Longrightarrow}_\tau P'$
and $P'RelQ'$: $(\Psi \otimes \Psi', P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from $PTrans$ $RimpQ$ **have** $\Psi : R \rhd P \Longrightarrow \alpha \prec P''$ **by**(*rule weakTransitionFrameImp*)
moreover note $P''Chain$
moreover from $P'RelQ'$ $Q'RelR'$ Set **have** $(\Psi \otimes \Psi', P', R') \in Rel''$ **by** *blast*
ultimately show *?case* **by** *blast*
next
case(*cTau* R')
from ‹$\Psi \rhd Q \rightsquigarrow [Rel']\ R$› ‹$\Psi \rhd R \longmapsto \tau \prec R'$›
obtain Q' **where** $QTrans$: $\Psi \rhd Q \longmapsto \tau \prec Q'$ **and** $Q'RelR'$: $(\Psi, Q', R') \in Rel'$
by(*force dest*: *simE*)

from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› $QTrans$
obtain P' **where** $PTrans$: $\Psi \rhd P \hat{\Longrightarrow}_\tau P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

note $PTrans$
moreover from $P'RelQ'$ $Q'RelR'$ Set **have** $(\Psi, P', R') \in Rel''$ **by** *blast*
ultimately show *?case* **by** *blast*
qed
qed

lemma *quietSim*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

assumes *quiet P*
and *eqvt Rel*
and *cQuiet*: $\bigwedge P.$ *quiet P* $\Longrightarrow (\Psi, \mathbf{0}, P) \in Rel$

shows $\Psi \rhd \mathbf{0} \rightsquigarrow <Rel> P$
using ‹*eqvt Rel*›
proof(*induct rule*: *weakSimI*[*of* - - - - ()])
case(*cAct* $\Psi'\ \alpha\ P'$)
from ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹$\alpha \neq \tau$› **have** *False* **using** ‹*quiet P*›
by(*cases rule*: *actionCases*[**where** α=α]) (*auto intro*: *quietOutput quietInput*)
thus *?case* **by** *simp*
next
case(*cTau* P')
from ‹$\Psi \rhd P \longmapsto \tau \prec P'$› ‹*quiet P*› **have** *quiet* P'
by(*erule-tac quiet.cases*) (*force simp add*: *residualInject*)
have $\Psi \rhd P \hat{\Longrightarrow}_\tau P$ **by** *simp*

moreover from ‹*quiet P'*› **have** $(\Psi, \mathbf{0}, P') \in Rel$ **by**(*rule cQuiet*)
ultimately show *?case* **by** *blast*
qed

end

end

theory *Weak-Stat-Imp*
imports *Tau-Chain*
begin

context *env* **begin**

definition
weakStatImp :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow$
$('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set \Rightarrow$
$('a, 'b, 'c)\ psi \Rightarrow bool$ (- ▷ - $\lessapprox$<-> - [80, 80, 80, 80] 80)
where $\Psi \rhd P \lessapprox <Rel> Q \equiv \forall \Psi'.\ \exists Q'\ Q''.\ \Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q' \wedge$ *insertAssertion*(*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame Q'*) $\Psi \wedge \Psi \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q'' \wedge (\Psi \otimes \Psi', P, Q'') \in Rel$

lemma *weakStatImpMonotonic*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $A :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $B :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $\Psi \rhd P \lessapprox <A> Q$
and $A \subseteq B$

shows $\Psi \rhd P \lessapprox <B> Q$
using *assms*
by(*auto simp add*: *weakStatImp-def*) *blast*

lemma *weakStatImpI*[*case-names cStatImp*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi' :: 'b$

assumes $\bigwedge \Psi'.\ \exists Q'\ Q''.\ \Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q' \wedge$ *insertAssertion*(*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame Q'*) $\Psi \wedge \Psi \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q'' \wedge (\Psi \otimes \Psi', P, Q'') \in Rel$

shows $\Psi \rhd P \lessapprox <Rel> Q$

using *assms*
by(*auto simp add*: *weakStatImp-def*)

lemma *weakStatImpE*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
  and *Q* :: (*'a*, *'b*, *'c*) *psi*
  and Ψ' :: *'b*

  assumes $\Psi \rhd P \lessapprox <Rel> Q$

  obtains Q' Q'' **where** $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$ **and** *insertAssertion*(*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame Q'*) Ψ **and** $\Psi \otimes \Psi' \rhd Q' \Longrightarrow^{\hat{}}_{\tau} Q''$ **and** ($\Psi \otimes \Psi'$, *P*, *Q''*) ∈ *Rel*

using *assms*
by(*auto simp add*: *weakStatImp-def*) *blast*

lemma *weakStatImpClosed*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
  and *Q* :: (*'a*, *'b*, *'c*) *psi*
  and *p* :: *name prm*

  assumes *EqvtRel*: *eqvt Rel*
  and *PStatImpQ*: $\Psi \rhd P \lessapprox <Rel> Q$

  shows $(p \cdot \Psi) \rhd (p \cdot P) \lessapprox <Rel> (p \cdot Q)$
proof(*induct rule*: *weakStatImpI*)
  case(*cStatImp* Ψ')
  from *PStatImpQ* **obtain** Q' Q'' **where** *QChain*: $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$
                and *PimpQ'*: *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q'*) Ψ
                and *Q'Chain*: $\Psi \otimes (rev(p::name\ prm) \cdot \Psi') \rhd Q' \Longrightarrow^{\hat{}}_{\tau} Q''$ **and** ($\Psi \otimes (rev\ p \cdot \Psi')$, *P*, *Q''*) ∈ *Rel*
    by(*rule weakStatImpE*)
  from *QChain* **have** $(p \cdot \Psi) \rhd (p \cdot Q) \Longrightarrow^{\hat{}}_{\tau} (p \cdot Q')$ **by**(*rule tauChainEqvt*)
  moreover from *PimpQ'* **have** *insertAssertion* (*extractFrame* $(p \cdot P)$) $(p \cdot \Psi) \hookrightarrow_F$ *insertAssertion* (*extractFrame*$(p \cdot Q')$) $(p \cdot \Psi)$
    by(*drule-tac p*=*p* **in** *FrameStatImpClosed*) (*simp add*: *eqvts*)
  moreover from *Q'Chain* **have** $(p \cdot \Psi) \otimes \Psi' \rhd (p \cdot Q') \Longrightarrow^{\hat{}}_{\tau} (p \cdot Q'')$ **by**(*drule-tac p*=*p* **in** *tauChainEqvt*) (*simp add*: *eqvts*)
  moreover from ‹($\Psi \otimes (rev\ p \cdot \Psi')$, *P*, *Q''*) ∈ *Rel*› *EqvtRel* **have** (($p \cdot \Psi$) ⊗ Ψ', ($p \cdot P$), ($p \cdot Q''$)) ∈ *Rel*
    by(*drule-tac p*=*p* **in** *eqvtI*) (*auto simp add*: *eqvts*)
  ultimately show *?case*
    by *blast*

qed

lemma *weakStatImpReflexive*:
  fixes *Rel* :: ('*b* × ('*a*, '*b*, '*c*) *psi* × ('*a*, '*b*, '*c*) *psi*) *set*
  and Ψ :: '*b*
  and *P* :: ('*a*, '*b*, '*c*) *psi*

  assumes {(Ψ, *P*, *P*) | Ψ *P*. *True*} ⊆ *Rel*

  shows Ψ ▷ *P* ≲≈<*Rel*> *P*
using *assms*
by(*auto simp add*: *weakStatImp-def weakTransition-def dest*: *rtrancl-into-rtrancl*) *force*+

lemma *weakStatImpTransitive*:
  fixes Ψ :: '*b*
  and *P* :: ('*a*, '*b*, '*c*) *psi*
  and *Rel* :: ('*b* × ('*a*, '*b*, '*c*) *psi* × ('*a*, '*b*, '*c*) *psi*) *set*
  and *Q* :: ('*a*, '*b*, '*c*) *psi*
  and *Rel'* :: ('*b* × ('*a*, '*b*, '*c*) *psi* × ('*a*, '*b*, '*c*) *psi*) *set*
  and *R* :: ('*a*, '*b*, '*c*) *psi*
  and *Rel''* :: ('*b* × ('*a*, '*b*, '*c*) *psi* × ('*a*, '*b*, '*c*) *psi*) *set*

  assumes *PStatImpQ*: Ψ ▷ *P* ≲≈<*Rel*> *Q*
  and *QRelR*: (Ψ, *Q*, *R*) ∈ *Rel'*
  and *Set*: {(Ψ', *S*, *U*) | Ψ' *S U*. ∃ *T*. (Ψ', *S*, *T*) ∈ *Rel* ∧ (Ψ', *T*, *U*) ∈ *Rel'*} ⊆ *Rel''*
  and *C1*: ⋀Ψ' *S T*. (Ψ', *S*, *T*) ∈ *Rel'* ⟹ Ψ' ▷ *S* ≲≈<*Rel'*> *T*
  and *C2*: ⋀Ψ' *S T S'*. ⟦(Ψ', *S*, *T*) ∈ *Rel'*; Ψ' ▷ *S* ⟹^ τ *S'*⟧ ⟹ ∃ *T'*. Ψ' ▷ *T* ⟹^ τ *T'* ∧ (Ψ', *S'*, *T'*) ∈ *Rel'*

  shows Ψ ▷ *P* ≲≈<*Rel''*> *R*
proof(*induct rule*: *weakStatImpI*)
  case(*cStatImp* Ψ')
  from ‹Ψ ▷ *P* ≲≈<*Rel*> *Q*› **obtain** *Q'* *Q''* **where** *QChain*: Ψ ▷ *Q* ⟹^ τ *Q'*
                                and *PimpQ'*: *insertAssertion* (*extractFrame P*) Ψ ↪_F *insertAssertion* (*extractFrame Q'*) Ψ
                                and *Q'Chain*: Ψ ⊗ Ψ' ▷ *Q'* ⟹^ τ *Q''* **and** (Ψ ⊗ Ψ', *P*, *Q''*) ∈ *Rel*
    by(*rule weakStatImpE*)
  from *QChain* ‹(Ψ, *Q*, *R*) ∈ *Rel'*› **obtain** *R'* **where** *RChain*: Ψ ▷ *R* ⟹^ τ *R'* **and** (Ψ, *Q'*, *R'*) ∈ *Rel'*
    by(*metis C2*)
  from ‹(Ψ, *Q'*, *R'*) ∈ *Rel'*› **obtain** *R''* *R'''* **where** *R'Chain*: Ψ ▷ *R'* ⟹^ τ *R''*
                                and *Q'impR''*: *insertAssertion* (*extractFrame Q'*) Ψ ↪_F *insertAssertion* (*extractFrame R''*) Ψ
                                and *R''Chain*: Ψ ⊗ Ψ' ▷ *R''* ⟹^ τ *R'''* **and** (Ψ ⊗ Ψ', *Q'*, *R'''*) ∈ *Rel'*
    by(*blast dest*: *C1 weakStatImpE*)

from *RChain R'Chain* **have** $\Psi \rhd R \Longrightarrow\hat{}_\tau\ R''$ **by** *auto*
moreover from *PimpQ' Q'impR''* **have** *insertAssertion* (*extractFrame P*) Ψ $\hookrightarrow_F$ *insertAssertion* (*extractFrame R''*) Ψ
by(*rule FrameStatImpTrans*)
moreover from *Q'Chain* $\langle(\Psi \otimes \Psi', Q', R''') \in Rel'\rangle$ **obtain** *R''''* **where** *R'''Chain*: $\Psi \otimes \Psi' \rhd R''' \Longrightarrow\hat{}_\tau\ R''''$ **and** $(\Psi \otimes \Psi', Q'', R'''') \in Rel'$
by(*metis C2*)
from *R''Chain R'''Chain* **have** $\Psi \otimes \Psi' \rhd R'' \Longrightarrow\hat{}_\tau\ R''''$ **by** *auto*
moreover from $\langle(\Psi \otimes \Psi', P, Q'') \in Rel\rangle$ $\langle(\Psi \otimes \Psi', Q'', R'''') \in Rel'\rangle$ *Set* **have** $(\Psi \otimes \Psi', P, R'''') \in Rel''$ **by** *blast*
ultimately show *?case*
by *blast*
qed

lemma *weakStatImpStatEq*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and Ψ' :: *'b*

assumes *PSimQ*: $\Psi \rhd P \lessapprox <Rel> Q$
and $\Psi \simeq \Psi'$
and *C1*: $\bigwedge \Psi' R S \Psi''. [\![(\Psi', R, S) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', R, S) \in Rel'$

shows $\Psi' \rhd P \lessapprox <Rel'> Q$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ'')
from $\langle\Psi \rhd P \lessapprox <Rel> Q\rangle$ **obtain** *Q' Q''* **where** *QChain*: $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q'$
and *PimpQ*: *insertAssertion* (*extractFrame P*) Ψ $\hookrightarrow_F$ *insertAssertion* (*extractFrame Q'*) Ψ
and *Q'Chain*: $\Psi \otimes \Psi'' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''$ **and** $(\Psi \otimes \Psi'', P, Q'') \in Rel$
by(*rule weakStatImpE*)
from *QChain* $\langle\Psi \simeq \Psi'\rangle$ **have** $\Psi' \rhd Q \Longrightarrow\hat{}_\tau\ Q'$ **by**(*rule tauChainStatEq*)
moreover from *PimpQ* $\langle\Psi \simeq \Psi'\rangle$ **have** *insertAssertion* (*extractFrame P*) Ψ' $\hookrightarrow_F$ *insertAssertion* (*extractFrame Q'*) Ψ'
by(*rule insertAssertionStatImp*)
moreover from *Q'Chain* $\langle\Psi \simeq \Psi'\rangle$ **have** $\Psi' \otimes \Psi'' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''$ **by**(*metis tauChainStatEq Composition*)
moreover from $\langle(\Psi \otimes \Psi'', P, Q'') \in Rel\rangle$ $\langle\Psi \simeq \Psi'\rangle$ **have** $(\Psi' \otimes \Psi'', P, Q'') \in Rel'$ **by**(*blast intro*: *Composition C1*)
ultimately show *?case*
by *blast*
qed

lemma *statImpWeakStatImp*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*

and Q :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*

assumes *PImpQ*: *insertAssertion*(*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q) Ψ
and *C1*: $\bigwedge \Psi'.\ (\Psi \otimes \Psi', P, Q) \in Rel$

shows $\Psi \rhd P \lessapprox <Rel> Q$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd Q \Longrightarrow\!\!\hat{}_\tau\ Q$ **by** *simp*
moreover note *PImpQ*
moreover have $\Psi \otimes \Psi' \rhd Q \Longrightarrow\!\!\hat{}_\tau\ Q$ **by** *simp*
moreover have $(\Psi \otimes \Psi', P, Q) \in Rel$ **by**(*rule C1*)
ultimately show *?case*
by *blast*
qed

end

end

theory *Bisimulation*
imports *Simulation*
begin

context *env* **begin**

lemma *monoCoinduct*: $\bigwedge x\ y\ xa\ xb\ xc\ P\ Q\ \Psi.$
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \rightsquigarrow[\{(xc, xb, xa).\ x\ xc\ xb\ xa\}]\ P) \longrightarrow$
$(\Psi \rhd Q \rightsquigarrow[\{(xb, xa, xc).\ y\ xb\ xa\ xc\}]\ P)$
apply *auto*
apply(*rule monotonic*)
by(*auto dest*: *le-funE*)

coinductive-set *bisim* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
where
step: $[\![(insertAssertion\ (extractFrame\ P))\ \Psi \simeq_F (insertAssertion\ (extractFrame\ Q)\ \Psi);$
$\Psi \rhd P \rightsquigarrow[bisim]\ Q;$
$\forall \Psi'.\ (\Psi \otimes \Psi',\ P, Q) \in bisim; (\Psi, Q, P) \in bisim]\!] \Longrightarrow (\Psi, P, Q) \in bisim$
monos *monoCoinduct*

abbreviation
bisimJudge $(- \rhd - \sim - [70, 70, 70]\ 65)$ **where** $\Psi \rhd P \sim Q \equiv (\Psi, P, Q) \in bisim$
abbreviation
bisimNilJudge $(- \sim - [70, 70]\ 65)$ **where** $P \sim Q \equiv SBottom' \rhd P \sim Q$

lemma *bisimCoinductAux*[*consumes 1*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi \wedge (\Psi \rhd P \rightsquigarrow[(X \cup bisim)]\ Q) \wedge (\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X \vee (\Psi \otimes \Psi', P, Q) \in bisim) \wedge ((\Psi, Q, P) \in X \vee (\Psi, Q, P) \in bisim)$

shows $(\Psi, P, Q) \in bisim$
proof −
have $X \cup bisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in bisim\}$ **by** *auto*
with *assms* **show** *?thesis*
by *coinduct simp*
qed

lemma *bisimCoinduct*[*consumes 1*, *case-names cStatEq cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow insertAssertion\ (extractFrame\ R)\ \Psi' \simeq_F insertAssertion\ (extractFrame\ S)\ \Psi'$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow[(X \cup bisim)]\ S$
and $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee (\Psi' \otimes \Psi'', R, S) \in bisim$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee (\Psi', S, R) \in bisim$

shows $(\Psi, P, Q) \in bisim$
proof −
have $X \cup bisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in bisim\}$ **by** *auto*
with *assms* **show** *?thesis*
by *coinduct simp*
qed

lemma *bisimWeakCoinductAux*[*consumes 1*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$

and $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \mathit{insertAssertion}\ (\mathit{extractFrame}\ P)\ \Psi \simeq_F \mathit{insertAssertion}\ (\mathit{extractFrame}\ Q)\ \Psi \wedge$
$\Psi \rhd P \rightsquigarrow [X]\ Q \wedge$
$(\forall \Psi'.\ (\Psi \otimes \Psi',\ P,\ Q) \in X) \wedge (\Psi,\ Q,\ P) \in X$

shows $(\Psi,\ P,\ Q) \in \mathit{bisim}$
using *assms*
by(*coinduct rule*: *bisimCoinductAux*) (*blast intro*: *monotonic*)

lemma *bisimWeakCoinduct*[*consumes 1*, *case-names cStatEq cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $Q :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $X :: ('b \times ('a,\ 'b,\ 'c)\ \mathit{psi} \times ('a,\ 'b,\ 'c)\ \mathit{psi})\ \mathit{set}$

assumes $(\Psi,\ P,\ Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \mathit{insertAssertion}\ (\mathit{extractFrame}\ P)\ \Psi \simeq_F \mathit{insertAssertion}\ (\mathit{extractFrame}\ Q)\ \Psi$
and $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow [X]\ Q$
and $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in X$

shows $(\Psi,\ P,\ Q) \in \mathit{bisim}$
proof −
have $X \cup \mathit{bisim} = \{(\Psi,\ P,\ Q).\ (\Psi,\ P,\ Q) \in X \vee (\Psi,\ P,\ Q) \in \mathit{bisim}\}$ **by** *auto*
with *assms* **show** *?thesis*
by(*coinduct rule*: *bisimCoinduct*) (*blast intro*: *monotonic*)+
qed

lemma *bisimE*:
fixes $P :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $Q :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $\Psi :: 'b$
and $\Psi' :: 'b$

assumes $(\Psi,\ P,\ Q) \in \mathit{bisim}$

shows $\mathit{insertAssertion}\ (\mathit{extractFrame}\ P)\ \Psi \simeq_F \mathit{insertAssertion}\ (\mathit{extractFrame}\ Q)\ \Psi$
and $\Psi \rhd P \rightsquigarrow [\mathit{bisim}]\ Q$
and $(\Psi \otimes \Psi',\ P,\ Q) \in \mathit{bisim}$
and $(\Psi,\ Q,\ P) \in \mathit{bisim}$
using *assms*
by(*auto simp add*: *intro*: *bisim.cases*)

lemma *bisimI*:
fixes $P :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $Q :: ('a,\ 'b,\ 'c)\ \mathit{psi}$
and $\Psi :: 'b$

assumes *insertAssertion* (*extractFrame P*) Ψ ≃$_F$ *insertAssertion* (*extractFrame Q*) Ψ
and Ψ ▷ *P* ⇝[*bisim*] *Q*
and ∀Ψ′. (Ψ ⊗ Ψ′, *P*, *Q*) ∈ *bisim*
and (Ψ, *Q*, *P*) ∈ *bisim*

shows (Ψ, *P*, *Q*) ∈ *bisim*
using *assms*
by(*auto intro*: *bisim.step*)

lemma *bisimReflexive*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*

shows Ψ ▷ *P* ∼ *P*
proof −
let *?X* = {(Ψ, *P*, *P*) | Ψ *P*. *True*}
have (Ψ, *P*, *P*) ∈ *?X* **by** *simp*
thus *?thesis*
by(*coinduct rule*: *bisimWeakCoinduct*, *auto intro*: *reflexive*)
qed

lemma *bisimClosed*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *p* :: *name prm*

assumes *PBisimQ*: Ψ ▷ *P* ∼ *Q*

shows (*p* ∙ Ψ) ▷ (*p* ∙ *P*) ∼ (*p* ∙ *Q*)
proof −
let *?X* = {(*p* ∙ Ψ, *p* ∙ *P*, *p* ∙ *Q*) | (*p*::*name prm*) Ψ *P Q*. Ψ ▷ *P* ∼ *Q*}
from *PBisimQ* **have** (*p* ∙ Ψ, *p* ∙ *P*, *p* ∙ *Q*) ∈ *?X* **by** *blast*
thus *?thesis*
proof(*coinduct rule*: *bisimWeakCoinduct*)
case(*cStatEq* Ψ *P Q*)
have ⋀Ψ *P Q* (*p*::*name prm*). *insertAssertion* (*extractFrame P*) Ψ ≃$_F$ *insertAssertion* (*extractFrame Q*) Ψ ⟹
insertAssertion (*extractFrame*(*p* ∙ *P*)) (*p* ∙ Ψ) ≃$_F$ *insertAssertion* (*extractFrame*(*p* ∙ *Q*)) (*p* ∙ Ψ)
by(*drule-tac p* = *p* **in** *FrameStatEqClosed*) (*simp add*: *eqvts*)

with ‹(Ψ, *P*, *Q*) ∈ *?X*› **show** *?case* **by**(*blast dest*: *bisimE*)
next
case(*cSim* Ψ *P Q*)
{

```
    fix p :: name prm
    fix Ψ P Q
    have eqvt ?X
      apply(auto simp add: eqvt-def)
      apply(rule-tac x=pa@p in exI)
      by(auto simp add: pt2[OF pt-name-inst])
    moreover assume Ψ ▷ P ⇝[bisim] Q
    hence Ψ ▷ P ⇝[?X] Q
      apply(rule-tac A=bisim in monotonic, auto)
      by(rule-tac x=[]::name prm in exI) auto
    ultimately have ((p::name prm) · Ψ) ▷ (p · P) ⇝[?X] (p · Q)
      by(rule-tac simClosed)
  }
  with ‹(Ψ, P, Q) ∈ ?X› show ?case
    by(blast dest: bisimE)
next
  case(cExt Ψ P Q Ψ')
  {
    fix p :: name prm
    fix Ψ P Q Ψ'
    assume ∀Ψ'. (Ψ ⊗ Ψ', P, Q) ∈ bisim
    hence ((p · Ψ) ⊗ Ψ', p · P, p · Q) ∈ ?X
      apply(auto, rule-tac x=p in exI)
      apply(rule-tac x=Ψ ⊗ (rev p · Ψ') in exI)
      by(auto simp add: eqvts)
  }
  with ‹(Ψ, P, Q) ∈ ?X› show ?case
    by(blast dest: bisimE)
next
  case(cSym Ψ P Q)
  thus ?case
    by(blast dest: bisimE)
qed
qed

lemma bisimEqvt[simp]:
  shows eqvt bisim
by(auto simp add: eqvt-def bisimClosed)

lemma statEqBisim:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   Q  :: ('a, 'b, 'c) psi
  and   Ψ' :: 'b

  assumes Ψ ▷ P ∼ Q
  and     Ψ ≃ Ψ'

  shows Ψ' ▷ P ∼ Q
```

proof −
let *?X* = $\{(\Psi', P, Q) \mid \Psi\ P\ Q\ \Psi'.\ \Psi \rhd P \sim Q \wedge \Psi \simeq \Psi'\}$
from ‹$\Psi \rhd P \sim Q$› ‹$\Psi \simeq \Psi'$› **have** $(\Psi', P, Q) \in$ *?X* **by** *auto*
thus *?thesis*
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* $\Psi'\ P\ Q$)
from ‹$(\Psi', P, Q) \in$ *?X*› **obtain** Ψ **where** $\Psi \rhd P \sim Q$ **and** $\Psi \simeq \Psi'$
by *auto*
from ‹$\Psi \rhd P \sim Q$› **have** *PeqQ*: *insertAssertion* (*extractFrame P*) $\Psi \simeq_F$ *insertAssertion* (*extractFrame Q*) Ψ
by(*rule bisimE*)

obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* \Psi'$
by(*rule-tac C*=(Ψ, Ψ') **in** *freshFrame*) *auto*
obtain $A_Q\ \Psi_Q$ **where** *FrQ*: *extractFrame Q* = $\langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* \Psi'$
by(*rule-tac C*=(Ψ, Ψ') **in** *freshFrame*) *auto*

from *PeqQ FrP FrQ* ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$\Psi \simeq \Psi'$›
have $\langle A_P, \Psi' \otimes \Psi_P\rangle \simeq_F \langle A_Q, \Psi' \otimes \Psi_Q\rangle$
by *simp* (*metis frameIntComposition FrameStatEqTrans FrameStatEqSym*)
with *FrP FrQ* ‹$A_P \sharp* \Psi'$› ‹$A_Q \sharp* \Psi'$› **show** *?case* **by** *simp*
next
case(*cSim* $\Psi'\ P\ Q$)
from ‹$(\Psi', P, Q) \in$ *?X*› **obtain** Ψ **where** $\Psi \rhd P \sim Q$ **and** $\Psi \simeq \Psi'$
by *auto*
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \rhd P \rightsquigarrow[bisim]\ Q$ **by**(*blast dest*: *bisimE*)
moreover have *eqvt ?X*
by(*auto simp add*: *eqvt-def*) (*metis bisimClosed AssertionStatEqClosed*)
hence *eqvt*(*?X* ∪ *bisim*) **by** *auto*
moreover note ‹$\Psi \simeq \Psi'$›
moreover have $\bigwedge\Psi\ P\ Q\ \Psi'.\ [\![\Psi \rhd P \sim Q; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in$ *?X* ∪ *bisim*
by *auto*
ultimately show *?case*
by(*rule statEqSim*)
next
case(*cExt* $\Psi'\ P\ Q\ \Psi''$)
from ‹$(\Psi', P, Q) \in$ *?X*› **obtain** Ψ **where** $\Psi \rhd P \sim Q$ **and** $\Psi \simeq \Psi'$
by *auto*
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \otimes \Psi'' \rhd P \sim Q$ **by**(*rule bisimE*)
moreover from ‹$\Psi \simeq \Psi'$› **have** $\Psi \otimes \Psi'' \simeq \Psi' \otimes \Psi''$ **by**(*rule Composition*)
ultimately show *?case* **by** *blast*
next
case(*cSym* $\Psi'\ P\ Q$)
from ‹$(\Psi', P, Q) \in$ *?X*› **obtain** Ψ **where** $\Psi \rhd P \sim Q$ **and** $\Psi \simeq \Psi'$
by *auto*
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \rhd Q \sim P$ **by**(*rule bisimE*)

```
    thus ?case using ‹Ψ ≃ Ψ'› by auto
  qed
qed

lemma bisimTransitive:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   R :: ('a, 'b, 'c) psi

  assumes PQ: Ψ ▷ P ∼ Q
  and     QR: Ψ ▷ Q ∼ R

  shows Ψ ▷ P ∼ R
proof −
  let ?X = {(Ψ, P, R) | Ψ P Q R. Ψ ▷ P ∼ Q ∧ Ψ ▷ Q ∼ R}
  from PQ QR have (Ψ, P, R) ∈ ?X by auto
  thus ?thesis
  proof(coinduct rule: bisimCoinduct)
    case(cStatEq Ψ P R)
    thus ?case by(blast dest: bisimE FrameStatEqTrans)
  next
    case(cSim Ψ P R)
    {
      fix Ψ P Q R
      assume Ψ ▷ P ⇝[bisim] Q and Ψ ▷ Q ⇝[bisim] R
      moreover have eqvt ?X
        by(force simp add: eqvt-def dest: bisimClosed)
      with bisimEqvt have eqvt (?X ∪ bisim) by blast
      moreover have ?X ⊆ ?X ∪ bisim by auto
      ultimately have Ψ ▷ P ⇝[(?X ∪ bisim)] R
        by(force intro: transitive)
    }
    with ‹(Ψ, P, R) ∈ ?X› show ?case
      by(blast dest: bisimE)
  next
    case(cExt Ψ P R Ψ')
    thus ?case by(blast dest: bisimE)
  next
    case(cSym Ψ P R)
    thus ?case by(blast dest: bisimE)
  qed
qed

lemma weakTransitiveCoinduct[case-names cStatEq cSim cExt cSym, case-conclusion
bisim step, consumes 2]:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
```

and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes p: $(\Psi, P, Q) \in X$
and *Eqvt*: $eqvt\ X$
and *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})]\ Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

shows $\Psi \rhd P \sim Q$
proof −
let *?X* = $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
from p **have** $(\Psi, P, Q) \in ?X$
by(*blast intro*: *bisimReflexive*)
thus *?thesis*
proof(*coinduct rule*: *bisimWeakCoinduct*)
case(*cStatEq* $\Psi\ P\ Q$)
thus *?case*
by(*blast dest*: *rStatEq bisimE FrameStatEqTrans*)
next
case(*cSim* $\Psi\ P\ Q$)
{
fix $\Psi\ P\ P'\ Q'\ Q$
assume $\Psi \rhd P \rightsquigarrow[bisim]\ P'$
moreover assume *P'RelQ'*: $(\Psi, P', Q') \in X$
hence $\Psi \rhd P' \rightsquigarrow[?X]\ Q'$ **by**(*rule rSim*)
moreover from ‹*eqvt X*› *P'RelQ'* **have** *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*drule-tac p*=*p* **in** *bisimClosed*)
apply(*drule-tac p*=*p* **in** *bisimClosed*)
apply(*rule-tac x*=*p* · *P'a* **in** *exI*, *simp*)
by(*rule-tac x*=*p* · *Q'a* **in** *exI*, *auto*)
ultimately have $\Psi \rhd P \rightsquigarrow[?X]\ Q'$
by(*force intro*: *transitive dest*: *bisimTransitive*)
moreover assume $\Psi \rhd Q' \rightsquigarrow[bisim]\ Q$
ultimately have $\Psi \rhd P \rightsquigarrow[?X]\ Q$ **using** ‹*eqvt ?X*›
by(*force intro*: *transitive dest*: *bisimTransitive*)
}
with ‹$(\Psi, P, Q) \in ?X$› **show** *?case*
by(*blast dest*: *bisimE*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*blast dest*: *bisimE intro*: *rExt*)
next

```
    case(cSym Ψ P Q)
    thus ?case by(blast dest: bisimE intro: rSym)
  qed
qed

lemma weakTransitiveCoinduct'[case-names cStatEq cSim cExt cSym, case-conclusion
bisim step, consumes 2]:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   X :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  assumes p: (Ψ, P, Q) ∈ X
  and Eqvt: eqvt X
  and rStatEq: ⋀Ψ P Q. (Ψ, P, Q) ∈ X ⟹ insertAssertion (extractFrame P) Ψ
≃_F insertAssertion (extractFrame Q) Ψ
  and rSim: ⋀Ψ P Q. (Ψ, P, Q) ∈ X ⟹ Ψ ▷ P ⇝[({(Ψ, P, Q) | Ψ P P' Q' Q.
Ψ ▷ P ∼ P' ∧
                                                    (Ψ, P', Q') ∈ X ∧
                                                    Ψ ▷ Q' ∼ Q})] Q
  and rExt: ⋀Ψ P Q Ψ'. (Ψ, P, Q) ∈ X ⟹ (Ψ ⊗ Ψ', P, Q) ∈ X
  and rSym: ⋀Ψ P Q. (Ψ, P, Q) ∈ X ⟹
                    (Ψ, Q, P) ∈ {(Ψ, P, Q) | Ψ P P' Q' Q. Ψ ▷ P ∼ P' ∧ (Ψ, P',
Q') ∈ X ∧ Ψ ▷ Q' ∼ Q}

  shows Ψ ▷ P ∼ Q
proof −
  let ?X = {(Ψ, P, Q) | Ψ P P' Q' Q. Ψ ▷ P ∼ P' ∧ (Ψ, P', Q') ∈ X ∧ Ψ ▷ Q'
∼ Q}
  from p have (Ψ, P, Q) ∈ ?X
    by(blast intro: bisimReflexive)
  thus ?thesis
  proof(coinduct rule: bisimWeakCoinduct)
    case(cStatEq Ψ P Q)
    thus ?case
      by(blast dest: rStatEq bisimE FrameStatEqTrans)
  next
    case(cSim Ψ P Q)
    {
      fix Ψ P P' Q' Q
      assume Ψ ▷ P ⇝[bisim] P'
      moreover assume P'RelQ': (Ψ, P', Q') ∈ X
      hence Ψ ▷ P' ⇝[?X] Q' by(rule rSim)
      moreover from ‹eqvt X› P'RelQ' have eqvt ?X
        apply(auto simp add: eqvt-def)
        apply(drule-tac p=p in bisimClosed)
        apply(drule-tac p=p in bisimClosed)
        apply(rule-tac x=p • P'a in exI, simp)
        by(rule-tac x=p • Q'a in exI, auto)
```

ultimately have $\Psi \rhd P \rightsquigarrow[?X]\ Q'$
by(*force intro*: *transitive dest*: *bisimTransitive*)
moreover assume $\Psi \rhd Q' \rightsquigarrow[bisim]\ Q$
ultimately have $\Psi \rhd P \rightsquigarrow[?X]\ Q$ **using** ‹*eqvt ?X*›
by(*force intro*: *transitive dest*: *bisimTransitive*)
}
with ‹$(\Psi, P, Q) \in ?X$› **show** *?case*
by(*blast dest*: *bisimE*)
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*blast dest*: *bisimE intro*: *rExt*)
next
case(*cSym* Ψ P Q)
thus *?case*
apply *auto*
apply(*drule rSym*)
apply *auto*
by(*metis bisimTransitive bisimE*(*4*))
qed
qed

lemma *weakTransitiveCoinduct''*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *p*: $(\Psi, P, Q) \in X$
and *Eqvt*: *eqvt X*
and *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$
and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})]\ Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\} \Longrightarrow (\Psi \otimes \Psi', P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\} \Longrightarrow (\Psi, Q, P) \in \{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$

shows $\Psi \rhd P \sim Q$
proof −
let *?X* = $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$

```
  from p have (Ψ, P, Q) ∈ ?X
    by(blast intro: bisimReflexive)
  thus ?thesis
  proof(coinduct rule: bisimWeakCoinduct)
    case(cStatEq Ψ P Q)
    thus ?case
      by(blast dest: rStatEq bisimE FrameStatEqTrans)
  next
    case(cSim Ψ P Q)
    {
      fix Ψ P P' Q' Q
      assume Ψ ▷ P ⇝[bisim] P'
      moreover assume P'RelQ': (Ψ, P', Q') ∈ X
      hence Ψ ▷ P' ⇝[?X] Q' by(rule rSim)
      moreover from ‹eqvt X› P'RelQ' have eqvt ?X
        apply(auto simp add: eqvt-def)
        apply(drule-tac p=p in bisimClosed)
        apply(drule-tac p=p in bisimClosed)
        apply(rule-tac x=p • P'a in exI, simp)
        by(rule-tac x=p • Q'a in exI, auto)
      ultimately have Ψ ▷ P ⇝[?X] Q'
        by(force intro: transitive dest: bisimTransitive)
      moreover assume Ψ ▷ Q' ⇝[bisim] Q
      ultimately have Ψ ▷ P ⇝[?X] Q using ‹eqvt ?X›
        by(force intro: transitive dest: bisimTransitive)
    }
    with ‹(Ψ, P, Q) ∈ ?X› show ?case
      by(blast dest: bisimE)
  next
    case(cExt Ψ P Q Ψ')
    thus ?case by(rule-tac rExt)
  next
    case(cSym Ψ P Q)
    thus ?case by(rule-tac rSym)
  qed
qed

lemma transitiveCoinduct[case-names cStatEq cSim cExt cSym, case-conclusion
bisim step, consumes 2]:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   X :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  assumes p: (Ψ, P, Q) ∈ X
  and Eqvt: eqvt X
  and rStatEq: ⋀Ψ' R S. (Ψ', R, S) ∈ X ⟹ insertAssertion (extractFrame R) Ψ'
≃F insertAssertion (extractFrame S) Ψ'
  and rSim: ⋀Ψ' R S. (Ψ', R, S) ∈ X ⟹ Ψ' ▷ R ⇝[({(Ψ', R, S) | Ψ' R R' S'
```

$S.\ \Psi' \rhd R \sim R' \wedge$
$((\Psi', R', S') \in X \vee \Psi' \rhd R' \sim S') \wedge$
$\Psi' \rhd S' \sim S\})]\ S$
and *rExt*: $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee \Psi' \otimes \Psi'' \rhd R \sim S$
and *rSym*: $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee \Psi' \rhd S \sim R$

shows $\Psi \rhd P \sim Q$
proof −
from *p* **have** $(\Psi, P, Q) \in (X \cup bisim)$
by *blast*
moreover from ‹*eqvt X*› *bisimEqvt* **have** *eqvt* $(X \cup bisim)$
by *auto*
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct'*)
case(*cStatEq* $\Psi\ P\ Q$)
thus *?case*
by(*blast intro*: *rStatEq dest*: *bisimE*)
next
case(*cSim* $\Psi\ P\ Q$)
thus *?case*
apply *auto*
apply(*blast intro*: *rSim*)
apply(*drule bisimE*(*2*))
apply(*rule-tac A*=*bisim* **in** *monotonic*, *simp*)
by(*force intro*: *bisimReflexive*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case*
by(*blast dest*: *bisimE rExt*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by**(*blast dest*: *bisimE rSym intro*: *bisimReflexive*)
qed
qed

lemma *transitiveCoinduct'*[*case-names cStatEq cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *p*: $(\Psi, P, Q) \in X$
and *Eqvt*: *eqvt X*
and *rStatEq*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow insertAssertion\ (extractFrame\ P)\ \Psi \simeq_F insertAssertion\ (extractFrame\ Q)\ \Psi$

and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow[(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in (X \cup bisim) \wedge \Psi \rhd Q' \sim Q\})]\ Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X \vee \Psi \otimes \Psi' \rhd P \sim Q$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in \{(\Psi,\ P,\ Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge ((\Psi,\ P',\ Q') \in (X \cup bisim)) \wedge \Psi \rhd Q' \sim Q\}$

shows $\Psi \rhd P \sim Q$
proof −
from *p* **have** $(\Psi,\ P,\ Q) \in (X \cup bisim)$
by *blast*
moreover from ‹*eqvt X*› *bisimEqvt* **have** *eqvt* $(X \cup bisim)$
by *auto*
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct'*)
case(*cStatEq* $\Psi\ P\ Q$)
thus *?case*
by(*blast intro*: *rStatEq dest*: *bisimE*)
next
case(*cSim* $\Psi\ P\ Q$)
thus *?case*
apply −
apply(*case-tac* $(\Psi,\ P,\ Q) \in X$ **for** X)
apply(*rule-tac rSim*)
apply *simp*
apply(*clarify*)
apply(*drule bisimE*(*2*))
apply(*rule-tac A*=*bisim* **in** *monotonic*, *simp*)
by(*force intro*: *bisimReflexive*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case*
by(*blast dest*: *bisimE rExt*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case*
apply *auto*
apply(*drule rSym*)
apply *auto*
apply(*rule-tac x*=*Q* **in** *exI*)
apply(*auto intro*: *bisimReflexive*)
apply(*rule-tac x*=*P* **in** *exI*)
by(*auto intro*: *bisimReflexive dest*: *bisimE*(*4*))
qed
qed

lemma *bisimSymmetric*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$

  assumes $\Psi \rhd P \sim Q$

  shows $\Psi \rhd Q \sim P$
using *assms*
by(*rule bisimE*)

lemma *eqvtTrans*[*intro*]:
  assumes *eqvt X*

  shows *eqvt* $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge ((\Psi, P', Q') \in X \vee \Psi \rhd P' \sim Q') \wedge \Psi \rhd Q' \sim Q\}$
using *assms*
apply(*auto simp add*: *eqvt-def eqvts*)
apply(*erule-tac x*=(*a*, *P'*, *Q'*) **in** *ballE*, *auto*)
by(*blast dest*: *bisimClosed*)+

lemma *eqvtWeakTrans*[*intro*]:
  assumes *eqvt X*

  shows *eqvt* $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
using *assms*
apply(*auto simp add*: *eqvt-def eqvts*)
apply(*erule-tac x*=(*a*, *P'*, *Q'*) **in** *ballE*, *auto*)
by(*blast dest*: *bisimClosed*)+

end

end

theory *Sim-Pres*
  imports *Simulation*
begin

context *env* **begin**

lemma *inputPres*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)\ psi$
  and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
  and Q :: $('a, 'b, 'c)\ psi$
  and M :: $'a$
  and $xvec$ :: *name list*
  and N :: $'a$

assumes *PRelQ*: $\bigwedge Tvec.\ length\ xvec = length\ Tvec \Longrightarrow (\Psi,\ P[xvec::=Tvec],\ Q[xvec::=Tvec]) \in Rel$

shows $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \rightsquigarrow[Rel]\ M(\!|\lambda * xvec\ N|\!).Q$
proof(*auto simp add*: *simulation-def residual.inject psi.inject*)
fix $\alpha\ Q'$
assume $\Psi \rhd M(\!|\lambda * xvec\ N|\!).Q \longmapsto \alpha \prec Q'$
thus $\exists P'.\ \Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P' \wedge (\Psi,\ P',\ Q') \in Rel$
by(*induct rule*: *inputCases*) (*auto intro*: *Input PRelQ*)
qed

lemma *outputPres*:
fixes Ψ :: $'b$
and P :: $('a,\ 'b,\ 'c)\ psi$
and Rel :: $('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$
and Q :: $('a,\ 'b,\ 'c)\ psi$
and M :: $'a$
and N :: $'a$

assumes *PRelQ*: $(\Psi,\ P,\ Q) \in Rel$

shows $\Psi \rhd M\langle N\rangle.P \rightsquigarrow[Rel]\ M\langle N\rangle.Q$
proof(*auto simp add*: *simulation-def residual.inject psi.inject*)
fix $\alpha\ Q'$
assume $\Psi \rhd M\langle N\rangle.Q \longmapsto \alpha \prec Q'$
thus $\exists P'.\ \Psi \rhd M\langle N\rangle.P \longmapsto \alpha \prec P' \wedge (\Psi,\ P',\ Q') \in Rel$
by(*induct rule*: *outputCases*) (*auto intro*: *Output PRelQ*)
qed

lemma *casePres*:
fixes Ψ :: $'b$
and CsP :: $('c \times ('a,\ 'b,\ 'c)\ psi)\ list$
and Rel :: $('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$
and CsQ :: $('c \times ('a,\ 'b,\ 'c)\ psi)\ list$
and M :: $'a$
and N :: $'a$

assumes *PRelQ*: $\bigwedge \varphi\ Q.\ (\varphi,\ Q)\ mem\ CsQ \Longrightarrow \exists P.\ (\varphi,\ P)\ mem\ CsP \wedge guarded\ P \wedge (\Psi,\ P,\ Q) \in Rel$
and *Sim*: $\bigwedge \Psi'\ R\ S.\ (\Psi',\ R,\ S) \in Rel \Longrightarrow \Psi' \rhd R \rightsquigarrow[Rel]\ S$
and $Rel \subseteq Rel'$

shows $\Psi \rhd Cases\ CsP \rightsquigarrow[Rel']\ Cases\ CsQ$
proof(*auto simp add*: *simulation-def residual.inject psi.inject*)
fix $\alpha\ Q'$
assume $\Psi \rhd Cases\ CsQ \longmapsto \alpha \prec Q'$ **and** $bn\ \alpha \sharp * CsP$ **and** $bn\ \alpha \sharp * \Psi$
thus $\exists P'.\ \Psi \rhd Cases\ CsP \longmapsto \alpha \prec P' \wedge (\Psi,\ P',\ Q') \in Rel'$
proof(*induct rule*: *caseCases*)

case(*cCase* φ Q)
from ‹(φ, Q) *mem CsQ*› **obtain** P **where** (φ, P) *mem CsP* **and** *guarded P* **and** (Ψ, P, Q) $\in$ *Rel*
by(*metis PRelQ*)
from ‹(Ψ, P, Q) $\in$ *Rel*› **have** $\Psi \rhd P \rightsquigarrow[Rel]\ Q$ **by**(*rule Sim*)
moreover from ‹*bn* α $\sharp*$ *CsP*› ‹(φ, P) *mem CsP*› **have** *bn* α $\sharp*$ P **by**(*auto dest*: *memFreshChain*)
moreover note ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* α $\sharp*$ Ψ›
ultimately obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec P'$ **and** $P'RelQ'$: (Ψ, P', Q') $\in$ *Rel*
by(*blast dest*: *simE*)
from *PTrans* ‹(φ, P) *mem CsP*› ‹$\Psi \vdash \varphi$› ‹*guarded P*› **have** $\Psi \rhd$ *Cases CsP* $\longmapsto \alpha \prec P'$
by(*rule Case*)
moreover from $P'RelQ'$ ‹*Rel* $\subseteq$ *Rel'*› **have** (Ψ, P', Q') $\in$ *Rel'* **by** *blast*
ultimately show *?case* **by** *blast*
qed
qed

lemma *resPres*:
fixes Ψ :: *'b*
and P :: ('a, 'b, 'c) *psi*
and *Rel* :: ('b × ('a, 'b, 'c) *psi* × ('a, 'b, 'c) *psi*) *set*
and Q :: ('a, 'b, 'c) *psi*
and x :: *name*
and *Rel'* :: ('b × ('a, 'b, 'c) *psi* × ('a, 'b, 'c) *psi*) *set*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
and *eqvt Rel'*
and $x \sharp \Psi$
and *Rel* $\subseteq$ *Rel'*
and *C1*: $\bigwedge \Psi'\ R\ S\ y.\ [\![(\Psi', R, S) \in Rel; y \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)R, (\!|\nu y|\!)S) \in Rel'$

shows $\Psi \rhd (\!|\nu x|\!)P \rightsquigarrow[Rel'] (\!|\nu x|\!)Q$
proof −
note ‹*eqvt Rel'*› ‹$x \sharp \Psi$›
moreover have $x \sharp (\!|\nu x|\!)P$ **and** $x \sharp (\!|\nu x|\!)Q$ **by**(*simp add*: *abs-fresh*)+
ultimately show *?thesis*
proof(*induct rule*: *simIFresh*[**where** C=()])
case(*cSim* α Q')
from ‹*bn* α $\sharp*$ $(\!|\nu x|\!)P$› ‹*bn* α $\sharp*$ $(\!|\nu x|\!)Q$› ‹$x \sharp \alpha$› **have** *bn* α $\sharp*$ P **and** *bn* α $\sharp*$ Q **by** *simp*+
from ‹$\Psi \rhd (\!|\nu x|\!)Q \longmapsto \alpha \prec Q'$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp Q'$› ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ Q› ‹*bn* α $\sharp*$ *subject* α›
‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P› ‹$x \sharp \alpha$›
show *?case*
proof(*induct rule*: *resCases*)
case(*cOpen M xvec1 xvec2 y N Q'*)

from ‹*bn* $(M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle)$ $\sharp*$ Ψ› **have** *xvec1* $\sharp*$ Ψ **and** $y \sharp \Psi$ **and** *xvec2* $\sharp*$ Ψ **by** *simp+*

from ‹*bn* $(M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle)$ $\sharp*$ P› **have** *xvec1* $\sharp*$ P **and** $y \sharp P$ **and** *xvec2* $\sharp*$ P **by** *simp+*

from ‹$x \sharp (M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle)$› **have** $x \sharp$ *xvec1* **and** $x \neq y$ **and** $x \sharp$ *xvec2* **and** $x \sharp M$ **by** *simp+*

from *PSimQ* ‹$\Psi \rhd Q \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec ([(x,\ y)] \cdot Q')$›
‹*xvec1* $\sharp*$ Ψ› ‹*xvec2* $\sharp*$ Ψ› ‹*xvec1* $\sharp*$ P› ‹*xvec2* $\sharp*$ P›

obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec P'$ **and** *P'RelQ'*: $(\Psi,\ P',\ ([(x,\ y)] \cdot Q')) \in Rel$
by(*force dest*: *simE*)

from ‹$y \in supp\ N$› ‹$x \neq y$› **have** $x \in supp([(x,\ y)] \cdot N)$
by(*drule-tac pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=[(*x*, *y*)]]) (*simp add*: *eqvts calc-atm*)

with *PTrans* ‹$x \sharp \Psi$› ‹$x \sharp M$› ‹$x \sharp$ *xvec1*› ‹$x \sharp$ *xvec2*›
have $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec1@x\#xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec P'$
by(*rule-tac Open*)

hence $([(x,\ y)] \cdot \Psi) \rhd ([(x,\ y)] \cdot (\!|\nu x|\!)P) \longmapsto ([(x,\ y)] \cdot (M(\!|\nu*(xvec1@x\#xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec P'))$
by(*rule eqvts*)

with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› ‹$y \sharp P$› ‹$x \sharp M$› ‹$y \sharp M$› ‹$x \sharp$ *xvec1*› ‹$y \sharp$ *xvec1*› ‹$x \sharp$ *xvec2*› ‹$y \sharp$ *xvec2*› ‹$x \neq y$›
have $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec ([(x,\ y)] \cdot P')$ **by**(*simp add*: *eqvts calc-atm alphaRes*)

moreover from *P'RelQ'* ‹*Rel* $\subseteq$ *Rel'*› ‹*eqvt Rel'*› **have** $([(y,\ x)] \cdot \Psi,\ [(y,\ x)] \cdot P',\ [(y,\ x)] \cdot [(x,\ y)] \cdot Q') \in Rel'$
by(*force simp add*: *eqvt-def*)

with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $(\Psi,\ [(x,\ y)] \cdot P',\ Q') \in Rel'$ **by**(*simp add*: *name-swap*)

ultimately show *?case* **by** *blast*

next
case(*cRes Q'*)
from *PSimQ* ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P›
obtain P' **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec P'$ **and** *P'RelQ'*: $(\Psi,\ P',\ Q') \in Rel$
by(*blast dest*: *simE*)

from *PTrans* ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› **have** $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec (\!|\nu x|\!)P'$
by(*rule Scope*)

moreover from *P'RelQ'* ‹$x \sharp \Psi$› **have** $(\Psi,\ (\!|\nu x|\!)P',\ (\!|\nu x|\!)Q') \in Rel'$ **by**(*rule C1*)

ultimately show *?case* **by** *blast*

qed

qed

qed

lemma *resChainPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

and *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and *Q* :: $('a, 'b, 'c)\ psi$
and *xvec* :: *name list*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
and *eqvt Rel*
and $xvec \sharp* \Psi$
and *C1*: $\bigwedge \Psi'\ R\ S\ y.\ [\![(\Psi', R, S) \in Rel; y \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)R, (\!|\nu y|\!)S) \in Rel$

shows $\Psi \rhd (\!|\nu* xvec|\!)P \rightsquigarrow[Rel]\ (\!|\nu* xvec|\!)Q$
using ‹*xvec* $\sharp*$ Ψ›
proof(*induct xvec*)
case *Nil*
from *PSimQ* **show** *?case* **by** *simp*
next
case(*Cons x xvec*)
from ‹$(x\#xvec) \sharp* \Psi$› **have** $x \sharp \Psi$ **and** $xvec \sharp* \Psi$ **by** *simp+*
from ‹$xvec \sharp* \Psi$› **have** $\Psi \rhd (\!|\nu* xvec|\!)P \rightsquigarrow[Rel]\ (\!|\nu* xvec|\!)Q$ **by**(*rule Cons*)
moreover note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover have $Rel \subseteq Rel$ **by** *simp*
ultimately have $\Psi \rhd (\!|\nu x|\!)((\!|\nu* xvec|\!)P) \rightsquigarrow[Rel]\ (\!|\nu x|\!)((\!|\nu* xvec|\!)Q)$ **using** *C1*
by(*rule resPres*)
thus *?case* **by** *simp*
qed

lemma *parPres*:
fixes Ψ :: *'b*
and *P* :: $('a, 'b, 'c)\ psi$
and *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and *Q* :: $('a, 'b, 'c)\ psi$
and *R* :: $('a, 'b, 'c)\ psi$
and *Rel'* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PRelQ*: $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R\rangle; A_R \sharp* \Psi; A_R \sharp* P; A_R \sharp* Q]\!] \Longrightarrow (\Psi \otimes \Psi_R, P, Q) \in Rel$
and *Eqvt*: *eqvt Rel*
and *Eqvt'*: *eqvt Rel'*

and *StatImp*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow insertAssertion\ (extractFrame\ T)\ \Psi' \hookrightarrow_F insertAssertion\ (extractFrame\ S)\ \Psi'$
and *Sim*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow[Rel]\ T$
and *Ext*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel]\!] \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$

and *C1*: $\bigwedge \Psi'\ S\ T\ A_U\ \Psi_U\ U.\ [\![(\Psi' \otimes \Psi_U, S, T) \in Rel; extractFrame\ U = \langle A_U, \Psi_U\rangle; A_U \sharp* \Psi'; A_U \sharp* S; A_U \sharp* T]\!] \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel'$
and *C2*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel'; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu* xvec|\!)S, (\!|\nu* xvec|\!)T) \in Rel'$

and *C3*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'', S, T) \in Rel$

shows $\Psi \rhd P \parallel R \rightsquigarrow [Rel']\ Q \parallel R$
using *Eqvt'*
proof(*induct rule*: *simI*[*of* - - - - ()])
case(*cSim* α *QR*)
from ‹*bn* $\alpha \sharp* (P \parallel R)$› ‹*bn* $\alpha \sharp* (Q \parallel R)$›
have *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* Q$ **and** *bn* $\alpha \sharp* R$
by *simp+*
from ‹$\Psi \rhd Q \parallel R \longmapsto \alpha \prec QR$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp* R$› ‹*bn* $\alpha \sharp*$ *subject* α›
show *?case*
proof(*induct rule*: *parCases*[**where** $C = (P, R)$])
case(*cPar1* $Q'\ A_R\ \Psi_R$)
from ‹$A_R \sharp* (P, R)$› **have** $A_R \sharp* P$ **by** *simp*
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact*
from ‹$A_R \sharp* \alpha$› ‹*bn* $\alpha \sharp* R$› *FrR*
have *bn* $\alpha \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow [Rel]\ Q$
by(*blast intro*: *Sim PRelQ*)
moreover have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto \alpha \prec Q'$ **by** *fact*
ultimately obtain P' **where** *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto \alpha \prec P'$
and $P'RelQ'$: $(\Psi \otimes \Psi_R, P', Q') \in Rel$
using ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* \Psi_R$› ‹*bn* $\alpha \sharp* P$›
by(*force dest*: *simE*)
from *PTrans QTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* \alpha$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinct*(*bn* α)› **have** $A_R \sharp* P'$ **and** $A_R \sharp* Q'$
by(*blast dest*: *freeFreshChainDerivative*)+
from *PTrans* ‹*bn* $\alpha \sharp* R$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* \alpha$› **have** $\Psi \rhd P \parallel R \longmapsto \alpha \prec (P' \parallel R)$
by(*rule-tac Par1*)
moreover from $P'RelQ'$ *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$› **have** $(\Psi, P' \parallel R, Q' \parallel R) \in Rel'$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2* $R'\ A_Q\ \Psi_Q$)
from ‹$A_Q \sharp* (P, R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp+*
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **and** $A_P \sharp* (\Psi, A_Q, \Psi_Q, \alpha, R)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* \alpha$ **and** $A_P \sharp* R$
by *simp+*

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*
from ‹$A_Q \sharp* P$› *FrP* ‹$A_P \sharp* A_Q$› **have** $A_Q \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*

from *FrP FrQ* ‹*bn* $\alpha \sharp* P$› ‹*bn* $\alpha \sharp* Q$› ‹$A_P \sharp* \alpha$› ‹$A_Q \sharp* \alpha$›
have *bn* $\alpha \sharp* \Psi_P$ **and** *bn* $\alpha \sharp* \Psi_Q$

by(*force dest*: *extractFrameFreshChain*)+

obtain A_R Ψ_R **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp^* (\Psi, P, Q, A_Q, A_P, \Psi_Q, \Psi_P, \alpha, R)$ **and** *distinct* A_R
by(*rule freshFrame*)
then have $A_R \sharp^* \Psi$ **and** $A_R \sharp^* P$ **and** $A_R \sharp^* Q$ **and** $A_R \sharp^* A_Q$ **and** $A_R \sharp^* A_P$ **and** $A_R \sharp^* \Psi_Q$ **and** $A_R \sharp^* \Psi_P$ **and** $A_R \sharp^* \alpha$ **and** $A_R \sharp^* R$
by *simp*+

from $\langle A_Q \sharp^* R\rangle$ *FrR* $\langle A_R \sharp^* A_Q\rangle$ **have** $A_Q \sharp^* \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*
from $\langle A_P \sharp^* R\rangle$ $\langle A_R \sharp^* A_P\rangle$ *FrR* **have** $A_P \sharp^* \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*

have *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$ **by** *fact*
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
proof −
have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym FrameStatEqSym*)
moreover from *FrR* $\langle A_R \sharp^* \Psi\rangle$ $\langle A_R \sharp^* P\rangle$ $\langle A_R \sharp^* Q\rangle$
have (*insertAssertion* (*extractFrame Q*) $(\Psi \otimes \Psi_R)$) $\hookrightarrow_F$ (*insertAssertion* (*extractFrame P*) $(\Psi \otimes \Psi_R)$)
by(*blast intro*: *PRelQ StatImp*)
with *FrP FrQ* $\langle A_P \sharp^* \Psi\rangle$ $\langle A_Q \sharp^* \Psi\rangle$ $\langle A_P \sharp^* \Psi_R\rangle$ $\langle A_Q \sharp^* \Psi_R\rangle$
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle$ **using** *freshCompChain* **by** *auto*
moreover have $\langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym frameIntAssociativity*[*THEN FrameStatEqSym*])
ultimately show *?thesis*
by(*rule FrameStatEqImpCompose*)
qed

ultimately have $\Psi \otimes \Psi_P \rhd R \longmapsto \alpha \prec R'$
using $\langle A_P \sharp^* \Psi\rangle$ $\langle A_P \sharp^* \Psi_Q\rangle$ $\langle A_Q \sharp^* \Psi\rangle$ $\langle A_Q \sharp^* \Psi_P\rangle$ $\langle A_P \sharp^* R\rangle$ $\langle A_Q \sharp^* R\rangle$ $\langle A_P \sharp^* \alpha\rangle$ $\langle A_Q \sharp^* \alpha\rangle$
$\langle A_R \sharp^* A_P\rangle$ $\langle A_R \sharp^* A_Q\rangle$ $\langle A_R \sharp^* \Psi_P\rangle$ $\langle A_R \sharp^* \Psi_Q\rangle$ $\langle A_R \sharp^* \Psi\rangle$ *FrR* ⟨*distinct* A_R⟩
by(*force intro*: *transferFrame*)
with ⟨*bn* $\alpha \sharp^* P\rangle$ $\langle A_P \sharp^* \Psi\rangle$ $\langle A_P \sharp^* R\rangle$ $\langle A_P \sharp^* \alpha\rangle$ *FrP* **have** $\Psi \rhd P \parallel R \longmapsto \alpha \prec (P \parallel R')$
by(*force intro*: *Par2*)
moreover obtain A_R' Ψ_R' **where** *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$ **and** $A_R' \sharp^* \Psi$ **and** $A_R' \sharp^* P$ **and** $A_R' \sharp^* Q$
by(*rule-tac freshFrame*[**where** $C=(\Psi, P, Q)$]) *auto*

moreover from *RTrans FrR* ⟨*distinct* $A_R\rangle$ $\langle A_R \sharp^* \Psi\rangle$ $\langle A_R \sharp^* P\rangle$ $\langle A_R \sharp^* Q\rangle$

$\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* \alpha\rangle$ $\langle bn\ \alpha \sharp* \Psi\rangle$ $\langle bn\ \alpha \sharp* P\rangle$ $\langle bn\ \alpha \sharp* Q\rangle$ $\langle bn\ \alpha \sharp* R\rangle$ $\langle bn\ \alpha \sharp* subject\ \alpha\rangle$ $\langle distinct(bn\ \alpha)\rangle$
obtain $p\ \Psi'\ A_R'\ \Psi_R'$ **where** S: $set\ p \subseteq set(bn\ \alpha) \times set(bn(p \cdot \alpha))$ **and** $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** FrR': $extractFrame\ R' = \langle A_R', \Psi_R'\rangle$
and $bn(p \cdot \alpha) \sharp* R$ **and** $bn(p \cdot \alpha) \sharp* \Psi$ **and** $bn(p \cdot \alpha) \sharp* P$ **and** $bn(p \cdot \alpha) \sharp* Q$ **and** $bn(p \cdot \alpha) \sharp* R$
and $A_R' \sharp* \Psi$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q$
by(*rule-tac* C=(Ψ, P, Q, R) **and** C'=(Ψ, P, Q, R) **in** *expandFrame*) (*assumption* | *simp*)+

from $\langle A_R \sharp* \Psi\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle bn\ \alpha \sharp* \Psi\rangle$ $\langle bn(p \cdot \alpha) \sharp* \Psi\rangle$ S **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
from $\langle A_R \sharp* P\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle bn\ \alpha \sharp* P\rangle$ $\langle bn(p \cdot \alpha) \sharp* P\rangle$ S **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
from $\langle A_R \sharp* Q\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle bn\ \alpha \sharp* Q\rangle$ $\langle bn(p \cdot \alpha) \sharp* Q\rangle$ S **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*

from *FrR* **have** $(p \cdot extractFrame\ R) = p \cdot \langle A_R, \Psi_R\rangle$ **by** *simp*
with $\langle bn\ \alpha \sharp* R\rangle$ $\langle bn(p \cdot \alpha) \sharp* R\rangle$ S **have** $extractFrame\ R = \langle (p \cdot A_R), (p \cdot \Psi_R)\rangle$
by(*simp add*: *eqvts*)

with $\langle (p \cdot A_R) \sharp* \Psi\rangle$ $\langle (p \cdot A_R) \sharp* P\rangle$ $\langle (p \cdot A_R) \sharp* Q\rangle$ **have** $(\Psi \otimes (p \cdot \Psi_R), P, Q) \in Rel$ **by**(*rule-tac PRelQ*)

hence $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', P, Q) \in Rel$ **by**(*rule Ext*)
with $\langle (p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'\rangle$ **have** $(\Psi \otimes \Psi_R', P, Q) \in Rel$ **by**(*blast intro*: *C3 Associativity compositionSym*)
with *FrR'* $\langle A_R' \sharp* \Psi\rangle$ $\langle A_R' \sharp* P\rangle$ $\langle A_R' \sharp* Q\rangle$ **have** $(\Psi, P \parallel R', Q \parallel R') \in Rel'$ **by**(*rule-tac C1*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* $\Psi_R\ M\ N\ Q'\ A_Q\ \Psi_Q\ K\ xvec\ R'\ A_R$)
have *FrQ*: $extractFrame\ Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from $\langle A_Q \sharp* (P, R)\rangle$ **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp*+

have *FrR*: $extractFrame\ R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from $\langle A_R \sharp* (P, R)\rangle$ **have** $A_R \sharp* P$ **and** $A_R \sharp* R$ **by** *simp*+

from $\langle xvec \sharp* (P, R)\rangle$ **have** $xvec \sharp* P$ **and** $xvec \sharp* R$ **by** *simp*+

obtain $A_P\ \Psi_P$ **where** *FrP*: $extractFrame\ P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* (\Psi, A_Q, \Psi_Q, A_R, M, N, K, R, P, xvec)$ **and** $distinct\ A_P$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* M$ **and** $A_P \sharp* R$ **and** $A_P \sharp* N$ **and** $A_P \sharp* K$ **and** $A_P \sharp* A_R$ **and** $A_P \sharp* P$ **and** $A_P \sharp* xvec$

by *simp*+

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|N|\!) \prec Q'$ **and** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec R'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$ **by** *fact*+

from *FrP FrR* ‹$A_Q \sharp* P$› ‹$A_P \sharp* R$› ‹$A_R \sharp* P$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* A_R$› ‹$A_P \sharp* xvec$› ‹$xvec \sharp* P$›
have $A_P \sharp* \Psi_R$ **and** $A_Q \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_P$ **and** $xvec \sharp* \Psi_P$
by(*fastforce dest*!: *extractFrameFreshChain*)+

from *RTrans FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* xvec$› ‹$xvec \sharp* R$› ‹$xvec \sharp* Q$› ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* \Psi_Q$› ‹$A_R \sharp* Q$›
‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_Q$› ‹$xvec \sharp* K$› ‹$A_R \sharp* K$› ‹$A_R \sharp* N$› ‹$A_R \sharp* R$› ‹$xvec \sharp* R$› ‹$A_R \sharp* P$› ‹$xvec \sharp* P$› ‹$A_P \sharp* A_R$› ‹$A_P \sharp* xvec$›
‹$A_Q \sharp* A_R$› ‹$A_Q \sharp* xvec$› ‹$A_R \sharp* \Psi_P$› ‹$xvec \sharp* \Psi_P$› ‹*distinct xvec*› ‹$xvec \sharp* M$›
obtain p Ψ' A_R' Ψ_R' **where** S: *set* $p \subseteq$ *set* $xvec \times$ *set*$(p \cdot xvec)$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$
and $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* Q$ **and** $A_R' \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi_Q$ **and** $(p \cdot xvec) \sharp* K$ **and** $(p \cdot xvec) \sharp* R$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* A_P$ **and** $(p \cdot xvec) \sharp* A_Q$ **and** $(p \cdot xvec) \sharp* \Psi_P$
and $A_R' \sharp* P$ **and** $A_R' \sharp* N$
by(*rule-tac* C=(Ψ, Q, Ψ_Q, K, R, P, A_P, A_Q, Ψ_P) **and** C'=(Ψ, Q, Ψ_Q, K, R, P, A_P, A_Q, Ψ_P) **in** *expandFrame*)
(*assumption* | *simp*)+

from ‹$A_R \sharp* \Psi$› **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
from ‹$A_R \sharp* P$› **have** $(p \cdot A_R) \sharp* (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* P$› ‹$(p \cdot xvec) \sharp* P$› S **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
from ‹$A_R \sharp* Q$› **have** $(p \cdot A_R) \sharp* (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* Q$› ‹$(p \cdot xvec) \sharp* Q$› S **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
from ‹$A_R \sharp* R$› **have** $(p \cdot A_R) \sharp* (p \cdot R)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* R$› ‹$(p \cdot xvec) \sharp* R$› S **have** $(p \cdot A_R) \sharp* R$ **by** *simp*
from ‹$A_R \sharp* K$› **have** $(p \cdot A_R) \sharp* (p \cdot K)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* K$› ‹$(p \cdot xvec) \sharp* K$› S **have** $(p \cdot A_R) \sharp* K$ **by** *simp*

from ‹$A_P \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_P$› ‹$A_P \sharp* M$› S **have** $A_P \sharp* (p \cdot M)$ **by**(*simp add*: *freshChainSimps*)
from ‹$A_Q \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_Q$› ‹$A_Q \sharp* M$› S **have** $A_Q \sharp* (p \cdot M)$ **by**(*simp*

add: *freshChainSimps*)
from ‹$A_P \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_P$› ‹$A_P \sharp* A_R$› S **have** $(p \cdot A_R) \sharp* A_P$ **by**(*simp add*: *freshChainSimps*)
from ‹$A_Q \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_Q$› ‹$A_Q \sharp* A_R$› S **have** $(p \cdot A_R) \sharp* A_Q$ **by**(*simp add*: *freshChainSimps*)

from *QTrans* S ‹$xvec \sharp* Q$› ‹$(p \cdot xvec) \sharp* Q$› **have** $(p \cdot (\Psi \otimes \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*rule-tac inputPermFrameSubject*) (*assumption* | *auto simp add*: *fresh-star-def*)+
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S **have** *QTrans*: $(\Psi \otimes (p \cdot \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*simp add*: *eqvts*)

from *FrR* **have** $(p \cdot extractFrame\ R) = p \cdot \langle A_R, \Psi_R\rangle$ **by** *simp*
with ‹$xvec \sharp* R$› ‹$(p \cdot xvec) \sharp* R$› S **have** *FrR*: $extractFrame\ R = \langle(p \cdot A_R), (p \cdot \Psi_R)\rangle$
by(*simp add*: *eqvts*)

note *RTrans FrR*
moreover from *FrR* ‹$(p \cdot A_R) \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* P$› ‹$(p \cdot A_R) \sharp* Q$› **have** $\Psi \otimes (p \cdot \Psi_R) \rhd P \rightsquigarrow[Rel]\ Q$
by(*metis Sim PRelQ*)
with *QTrans* **obtain** P' **where** *PTrans*: $\Psi \otimes (p \cdot \Psi_R) \rhd P \longmapsto (p \cdot M)(\!|N|\!) \prec P'$ **and** $P' RelQ'$: $(\Psi \otimes (p \cdot \Psi_R), P', Q') \in Rel$
by(*force dest*: *simE*)
from *PTrans QTrans* ‹$A_R' \sharp* P$› ‹$A_R' \sharp* Q$› ‹$A_R' \sharp* N$› **have** $A_R' \sharp* P'$ **and** $A_R' \sharp* Q'$
by(*blast dest*: *inputFreshChainDerivative*)+

note *PTrans*
moreover from *MeqK* **have** $(p \cdot (\Psi \otimes \Psi_Q \otimes \Psi_R)) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$ **by**(*rule chanEqClosed*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$xvec \sharp* \Psi_Q$› ‹$(p \cdot xvec) \sharp* \Psi_Q$› ‹$xvec \sharp* K$› ‹$(p \cdot xvec) \sharp* K$› S
have *MeqK*: $\Psi \otimes \Psi_Q \otimes (p \cdot \Psi_R) \vdash (p \cdot M) \leftrightarrow K$ **by**(*simp add*: *eqvts*)

moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle$
proof −
have $\langle A_P, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes (p \cdot \Psi_R)\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover from *FrR* ‹$(p \cdot A_R) \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* P$› ‹$(p \cdot A_R) \sharp* Q$›
have $(insertAssertion\ (extractFrame\ Q)\ (\Psi \otimes (p \cdot \Psi_R))) \hookrightarrow_F (insertAssertion\ (extractFrame\ P)\ (\Psi \otimes (p \cdot \Psi_R)))$
by(*metis PRelQ StatImp*)
with *FrP FrQ* ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P \sharp* \Psi_R$› ‹$A_Q \sharp* \Psi_R$› ‹$A_P \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_P$› ‹$A_Q \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_Q$› S
have $\langle A_Q, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P\rangle$ **using** *freshCompChain*

by(*simp add*: *freshChainSimps*)
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes (p \cdot \Psi_R)\rangle \simeq_F \langle A_Q, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately show *?thesis* **by**(*rule-tac FrameStatEqImpCompose*)
qed
moreover note *FrP FrQ* ‹*distinct* A_P›
moreover from ‹*distinct* A_R› **have** *distinct*$(p \cdot A_R)$ **by** *simp*
moreover note ‹$(p \cdot A_R) \sharp* A_P$› ‹$(p \cdot A_R) \sharp* A_Q$› ‹$(p \cdot A_R) \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* P$› ‹$(p \cdot A_R) \sharp* Q$› ‹$(p \cdot A_R) \sharp* R$› ‹$(p \cdot A_R) \sharp* K$›
‹$A_P \sharp* \Psi$› ‹$A_P \sharp* R$› ‹$A_P \sharp* P$› ‹$A_P \sharp* (p \cdot M)$› ‹$A_Q \sharp* R$› ‹$A_Q \sharp* (p \cdot M)$› ‹$A_P \sharp* xvec$› ‹$xvec \sharp* P$› ‹$A_P \sharp* R$›
ultimately obtain K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec R'$ **and** $\Psi \otimes \Psi_P \otimes (p \cdot \Psi_R) \vdash (p \cdot M) \leftrightarrow K'$ **and** $(p \cdot A_R) \sharp* K'$
by(*rule-tac comm1Aux*)

with *PTrans FrP* **have** $\Psi \rhd P \parallel R \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel R')$ **using** *FrR* ‹$(p \cdot A_R) \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* P$› ‹$(p \cdot A_R) \sharp* R$›
‹$xvec \sharp* P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* R$› ‹$A_P \sharp* (p \cdot M)$› ‹$(p \cdot A_R) \sharp* K'$› ‹$(p \cdot A_R) \sharp* A_P$›
by(*rule-tac Comm1*) (*assumption* | *simp*)+

moreover from *P'RelQ'* **have** $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', P', Q') \in Rel$ **by**(*rule Ext*)
with ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R{}'$› **have** $(\Psi \otimes \Psi_R{}', P', Q') \in Rel$ **by**(*metis C3 Associativity compositionSym*)
with *FrR'* ‹$A_R{}' \sharp* P'$› ‹$A_R{}' \sharp* Q'$› ‹$A_R{}' \sharp* \Psi$› **have** $(\Psi, P' \parallel R', Q' \parallel R') \in Rel'$ **by**(*rule-tac C1*)
with ‹$xvec \sharp* \Psi$› **have** $(\Psi, (\!|\nu * xvec|\!)(P' \parallel R'), (\!|\nu * xvec|\!)(Q' \parallel R')) \in Rel'$ **by**(*rule-tac C2*)
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_R M *xvec* N Q' A_Q Ψ_Q K R' A_R)
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from ‹$A_Q \sharp* (P, R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp*+

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$A_R \sharp* (P, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* R$ **by** *simp*+

from ‹$xvec \sharp* (P, R)$› **have** $xvec \sharp* P$ **and** $xvec \sharp* R$ **by** *simp*+

obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* (\Psi, A_Q, \Psi_Q, A_R, M, N, K, R, P, xvec)$ **and** *distinct* A_P
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* M$ **and** $A_P \sharp* R$ **and** $A_P \sharp* N$ $A_P \sharp* K$ **and** $A_P \sharp* A_R$ **and** $A_P \sharp* P$ **and** $A_P \sharp* xvec$
by *simp*+

from *FrP FrR* ‹$A_Q \sharp* P$› ‹$A_P \sharp* R$› ‹$A_R \sharp* P$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* A_R$› ‹A_P

$\sharp*$ *xvec*› ‹*xvec* $\sharp*$ *P*›
have $A_P \sharp* \Psi_R$ **and** $A_Q \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_P$ **and** *xvec* $\sharp* \Psi_P$
by(*fastforce dest*!: *extractFrameFreshChain*)+

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$ **by** *fact*

note ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$› *FrR* ‹$\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$›
moreover from *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow[Rel]\ Q$ **by**(*metis PRelQ Sim*)
with *QTrans* **obtain** P' **where** *PTrans*: $\Psi \otimes \Psi_R \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$ **and** $P'RelQ'$: $(\Psi \otimes \Psi_R, P', Q') \in Rel$
using ‹*xvec* $\sharp* \Psi$› ‹*xvec* $\sharp* \Psi_R$› ‹*xvec* $\sharp* P$›
by(*force dest*: *simE*)
from *PTrans QTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp*$ *xvec*› ‹*xvec* $\sharp* M$› ‹*distinct xvec*› **have** $A_R \sharp* P'$ **and** $A_R \sharp* Q'$
by(*blast dest*: *outputFreshChainDerivative*)+
note *PTrans* ‹$\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$›
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
proof −
have $\langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_P) \otimes \Psi_R\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover from *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$›
have (*insertAssertion* (*extractFrame Q*) ($\Psi \otimes \Psi_R$)) $\hookrightarrow_F$ (*insertAssertion* (*extractFrame P*) ($\Psi \otimes \Psi_R$))
by(*metis PRelQ StatImp*)
with *FrP FrQ* ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P \sharp* \Psi_R$› ‹$A_Q \sharp* \Psi_R$›
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle$ **using** *freshCompChain* **by** *simp*
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately show *?thesis* **by**(*rule-tac FrameStatEqImpCompose*)
qed
moreover note *FrP FrQ* ‹*distinct* A_P› ‹*distinct* A_R›
moreover from ‹$A_P \sharp* A_R$› ‹$A_Q \sharp* A_R$› **have** $A_R \sharp* A_P$ **and** $A_R \sharp* A_Q$ **by** *simp*+
moreover note ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* K$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$›
‹$A_P \sharp* R$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* R$› ‹$A_Q \sharp* M$› ‹$A_R \sharp*$ *xvec*› ‹*xvec* $\sharp*$ *M*›
ultimately obtain K' **where** $\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R'$ **and** $\Psi \otimes \Psi_P \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
by(*rule-tac comm2Aux*) *assumption*+

with *PTrans FrP* **have** $\Psi \rhd P \parallel R \longmapsto \tau \prec (\!|\nu*xvec|\!)(P' \parallel R')$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* R$›
‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* R$› **and** ‹*xvec* $\sharp* R$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* R$› ‹$A_P \sharp* A_R$› ‹$A_P \sharp* M$› ‹$A_R \sharp* K'$›

by(*force intro*: *Comm2*)

moreover from ‹$\Psi \otimes \Psi_P \rhd R \longmapsto K'(\!|N|\!) \prec R'$› *FrR* ‹*distinct* A_R› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$› ‹$A_R \sharp* N$› ‹$A_R \sharp* K'$›
obtain Ψ' A_R' Ψ_R' **where** *ReqR'*: $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R' \rangle$
and $A_R' \sharp* \Psi$ **and** $A_R' \sharp* P'$ **and** $A_R' \sharp* Q'$
by(*rule-tac* C=(Ψ, P', Q') **and** C'=Ψ **in** *expandFrame*) *auto*

from *P'RelQ'* **have** $((\Psi \otimes \Psi_R) \otimes \Psi', P', Q') \in Rel$ **by**(*rule Ext*)
with *ReqR'* **have** $(\Psi \otimes \Psi_R', P', Q') \in Rel$ **by**(*metis C3 Associativity compositionSym*)
with *FrR'* ‹$A_R' \sharp* P'$› ‹$A_R' \sharp* Q'$› ‹$A_R' \sharp* \Psi$› **have** $(\Psi, P' \parallel R', Q' \parallel R') \in Rel'$
by(*rule-tac C1*)
with ‹*xvec* $\sharp* \Psi$› **have** $(\Psi, (\!|\nu*xvec|\!)(P' \parallel R'), (\!|\nu*xvec|\!)(Q' \parallel R')) \in Rel'$ **by**(*rule-tac C2*)
ultimately show *?case* **by** *blast*
qed
qed
no-notation *relcomp* (**infixr** *O* 75)
lemma *bangPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and R :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Rel'* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

assumes $(\Psi, P, Q) \in Rel$
and *eqvt Rel'*
and *guarded P*
and *guarded Q*
and *cSim*: $\bigwedge \Psi' S T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow [Rel]\ T$
and *cExt*: $\bigwedge \Psi' S T \Psi''.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$
and *cSym*: $\bigwedge \Psi' S T.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', T, S) \in Rel$
and *StatEq*: $\bigwedge \Psi' S T \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'', S, T) \in Rel$
and *Closed*: $\bigwedge \Psi' S T p.\ (\Psi', S, T) \in Rel \Longrightarrow ((p::name\ prm) \cdot \Psi', p \cdot S, p \cdot T) \in Rel$
and *Assoc*: $\bigwedge \Psi' S T U.\ (\Psi', S \parallel (T \parallel U), (S \parallel T) \parallel U) \in Rel$
and *ParPres*: $\bigwedge \Psi' S T U.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel$
and *FrameParPres*: $\bigwedge \Psi' \Psi_U S T U A_U.\ [\![(\Psi' \otimes \Psi_U, S, T) \in Rel; extractFrame\ U = \langle A_U, \Psi_U \rangle; A_U \sharp* \Psi'; A_U \sharp* S; A_U \sharp* T]\!] \Longrightarrow (\Psi', U \parallel S, U \parallel T) \in Rel$
and *ResPres*: $\bigwedge \Psi' S T xvec.\ [\![(\Psi', S, T) \in Rel; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu*xvec|\!)S, (\!|\nu*xvec|\!)T) \in Rel$
and *ScopeExt*: $\bigwedge xvec\ \Psi' S T.\ [\![xvec \sharp* \Psi'; xvec \sharp* T]\!] \Longrightarrow (\Psi', (\!|\nu*xvec|\!)(S \parallel T), ((\!|\nu*xvec|\!)S) \parallel T) \in Rel$

and *Trans*: $\bigwedge \Psi'\ S\ T\ U.\ [\![(\Psi',\ S,\ T) \in Rel;\ (\Psi',\ T,\ U) \in Rel]\!] \Longrightarrow (\Psi',\ S,\ U) \in Rel$
and *Compose*: $\bigwedge \Psi'\ S\ T\ U\ O.\ [\![(\Psi',\ S,\ T) \in Rel;\ (\Psi',\ T,\ U) \in Rel';\ (\Psi',\ U,\ O) \in Rel]\!] \Longrightarrow (\Psi',\ S,\ O) \in Rel'$
and *C1*: $\bigwedge \Psi\ S\ T\ U.\ [\![(\Psi,\ S,\ T) \in Rel;\ guarded\ S;\ guarded\ T]\!] \Longrightarrow (\Psi,\ U \parallel\ !S,\ U \parallel\ !T) \in Rel'$
and *Der*: $\bigwedge \Psi'\ S\ \alpha\ S'\ T.\ [\![\Psi' \rhd\ !S \longmapsto \alpha \prec S';\ (\Psi',\ S,\ T) \in Rel;\ bn\ \alpha \sharp* \Psi';\ bn\ \alpha \sharp* S;\ bn\ \alpha \sharp* T;\ guarded\ T;\ bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow$
$\exists\, T'\ U\ O.\ \ \Psi' \rhd\ !T \longmapsto \alpha \prec T' \wedge (\Psi',\ S',\ U \parallel\ !S) \in Rel \wedge (\Psi',\ T',\ O \parallel\ !T) \in Rel \wedge$
$(\Psi',\ U,\ O) \in Rel \wedge ((supp\ U)::name\ set) \subseteq supp\ S' \wedge$
$((supp\ O)::name\ set) \subseteq supp\ T'$

shows $\Psi \rhd R \parallel\ !P \rightsquigarrow [Rel']\ R \parallel\ !Q$
using ‹*eqvt Rel'*›
proof(*induct rule*: *simI*[*of* - - - - ()])
case(*cSim* α *RQ'*)
from ‹*bn* $\alpha \sharp*$ $(R \parallel\ !P)$› ‹*bn* $\alpha \sharp*$ $(R \parallel\ !Q)$› **have** *bn* $\alpha \sharp*$ P **and** *bn* $\alpha \sharp*$ $(!Q)$ **and** *bn* $\alpha \sharp*$ Q **and** *bn* $\alpha \sharp*$ R **by** *simp+*
from ‹$\Psi \rhd R \parallel\ !Q \longmapsto \alpha \prec RQ'$› ‹*bn* $\alpha \sharp*$ Ψ› ‹*bn* $\alpha \sharp*$ R› ‹*bn* $\alpha \sharp*$ $!Q$› ‹*bn* $\alpha \sharp*$ *subject* α› **show** *?case*
proof(*induct rule*: *parCases*[**where** *C*=*P*])
case(*cPar1 R' A*$_Q$ Ψ_Q)
from ‹*extractFrame* $(!Q) = \langle A_Q,\ \Psi_Q\rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp+*
with ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$› ‹*bn* $\alpha \sharp*$ P› **have** $\Psi \rhd R \parallel\ !P \longmapsto \alpha \prec (R' \parallel\ !P)$
by(*rule-tac Par1*) (*assumption* | *simp*)+
moreover from ‹$(\Psi,\ P,\ Q) \in Rel$› ‹*guarded P*› ‹*guarded Q*› **have** $(\Psi,\ R' \parallel\ !P,\ R' \parallel\ !Q) \in Rel'$
by(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 Q' A*$_R$ Ψ_R)
have *QTrans*: $\Psi \otimes \Psi_R \rhd\ !Q \longmapsto \alpha \prec Q'$ **and** *FrR*: *extractFrame R* $= \langle A_R,\ \Psi_R\rangle$ **by** *fact+*
with ‹*bn* $\alpha \sharp*$ R› ‹$A_R \sharp*$ α› **have** *bn* $\alpha \sharp*$ Ψ_R **by**(*force dest*: *extractFrame-FreshChain*)
with *QTrans* ‹$(\Psi,\ P,\ Q) \in Rel$› ‹*bn* $\alpha \sharp*$ Ψ›‹*bn* $\alpha \sharp*$ P› ‹*bn* $\alpha \sharp*$ Q› ‹*guarded P*› ‹*bn* $\alpha \sharp*$ *subject* α›
obtain $P'\ S\ T$ **where** *PTrans*: $\Psi \otimes \Psi_R \rhd\ !P \longmapsto \alpha \prec P'$ **and** $(\Psi \otimes \Psi_R,\ P',\ T \parallel\ !P) \in Rel$
and $(\Psi \otimes \Psi_R,\ Q',\ S \parallel\ !Q) \in Rel$ **and** $(\Psi \otimes \Psi_R,\ S,\ T) \in Rel$
and *suppT*: $((supp\ T)::name\ set) \subseteq supp\ P'$ **and** *suppS*: $((supp\ S)::name\ set) \subseteq supp\ Q'$
by(*drule-tac cSym*) (*auto dest*: *Der cExt*)
from *PTrans FrR* ‹$A_R \sharp*$ Ψ› ‹$A_R \sharp*$ P› ‹$A_R \sharp*$ α› ‹*bn* $\alpha \sharp*$ R› **have** $\Psi \rhd R \parallel\ !P \longmapsto \alpha \prec (R \parallel P')$

by(*rule-tac Par2*) *auto*
moreover
{
from ‹$A_R \sharp* P$› ‹$A_R \sharp* (!Q)$› ‹$A_R \sharp* \alpha$› *PTrans QTrans* ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinct*(*bn* α)› **have** $A_R \sharp* P'$ **and** $A_R \sharp* Q'$
by(*force dest*: *freeFreshChainDerivative*)+
from ‹$(\Psi \otimes \Psi_R, P', T \parallel !P) \in Rel$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* P$› *suppT* **have** $(\Psi, R \parallel P', R \parallel (T \parallel !P)) \in Rel$
by(*rule-tac FrameParPres*) (*auto simp add*: *fresh-star-def fresh-def psi.supp*)
hence $(\Psi, R \parallel P', (R \parallel T) \parallel !P) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹*guarded* P› ‹*guarded* Q› **have** $(\Psi, (R \parallel T) \parallel !P, (R \parallel T) \parallel !Q) \in Rel'$
by(*rule C1*)
moreover from ‹$(\Psi \otimes \Psi_R, Q', S \parallel !Q) \in Rel$› ‹$(\Psi \otimes \Psi_R, S, T) \in Rel$› **have** $(\Psi \otimes \Psi_R, Q', T \parallel !Q) \in Rel$
by(*blast intro*: *ParPres Trans*)
with *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$› ‹$A_R \sharp* (!Q)$› *suppT suppS* **have** $(\Psi, R \parallel Q', R \parallel (T \parallel !Q)) \in Rel$
by(*rule-tac FrameParPres*) (*auto simp add*: *fresh-star-def fresh-def psi.supp*)
hence $(\Psi, R \parallel Q', (R \parallel T) \parallel !Q) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
ultimately have $(\Psi, R \parallel P', R \parallel Q') \in Rel'$ **by**(*blast intro*: *cSym Compose*)
}
ultimately show *?case* **by** *blast*
next
case(*cComm1* Ψ_Q M N R' A_R Ψ_R K *xvec* Q' A_Q)
from ‹*extractFrame* $(!Q) = \langle A_Q, \Psi_Q \rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp*+
have *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$ **and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact*+
moreover have *QTrans*: $\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$ **by** *fact*
from *FrR* ‹*xvec* $\sharp* R$› ‹$A_R \sharp* xvec$› **have** *xvec* $\sharp* \Psi_R$ **by**(*force dest*: *extractFrameFreshChain*)
with *QTrans* ‹$(\Psi, P, Q) \in Rel$› ‹*xvec* $\sharp* \Psi$›‹*xvec* $\sharp* P$› ‹*xvec* $\sharp* (!Q)$› ‹*guarded* P› ‹*xvec* $\sharp* K$›
obtain P' S T **where** *PTrans*: $\Psi \otimes \Psi_R \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec P'$ **and** $(\Psi \otimes \Psi_R, P', T \parallel !P) \in Rel$
and $(\Psi \otimes \Psi_R, Q', S \parallel !Q) \in Rel$ **and** $(\Psi \otimes \Psi_R, S, T) \in Rel$
and *suppT*: $((supp\ T)::name\ set) \subseteq supp\ P'$ **and** *suppS*: $((supp\ S)::name\ set) \subseteq supp\ Q'$
by(*drule-tac cSym*) (*fastforce dest*: *Der intro*: *cExt*)
note ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$›
ultimately have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (\!|\nu * xvec|\!)(R' \parallel P')$
using *PTrans* ‹$\Psi_Q = SBottom'$› ‹*xvec* $\sharp* R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_R \sharp* P$›
by(*rule-tac Comm1*) (*assumption* | *simp*)+

moreover from ‹$A_R \sharp* P$› ‹$A_R \sharp* (!Q)$› ‹$A_R \sharp* xvec$› *PTrans QTrans* ‹*xvec* $\sharp* K$› ‹*distinct xvec*›
have $A_R \sharp* P'$ **and** $A_R \sharp* Q'$ **by**(*force dest*: *outputFreshChainDerivative*)+

moreover with *RTrans FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* N$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* (!Q)$› ‹$A_R \sharp* M$›
obtain $\Psi'\ A_R'\ \Psi_R'$ **where** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R' \rangle$ **and** $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* \Psi$
and $A_R' \sharp* P'$ **and** $A_R' \sharp* Q'$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q$
by(*rule-tac* C=(Ψ, P, P', Q, Q') **and** C'=Ψ **in** *expandFrame*) *auto*

moreover
{
from ‹$(\Psi \otimes \Psi_R, P', T \parallel !P) \in Rel$›
have $((\Psi \otimes \Psi_R) \otimes \Psi', P', T \parallel !P) \in Rel$ **by**(*rule cExt*)
with ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$›
have $(\Psi \otimes \Psi_R', P', T \parallel !P) \in Rel$
by(*metis Associativity StatEq compositionSym*)
with *FrR'* ‹$A_R' \sharp* \Psi$› ‹$A_R' \sharp* P'$› ‹$A_R' \sharp* P$› *suppT*
have $(\Psi, R' \parallel P', R' \parallel (T \parallel !P)) \in Rel$
by(*rule-tac FrameParPres*) (*auto simp add*: *fresh-star-def fresh-def psi.supp*)
hence $(\Psi, R' \parallel P', (R' \parallel T) \parallel !P) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
with ‹*xvec* $\sharp* \Psi$› ‹*xvec* $\sharp* P$›
have $(\Psi, (\!|\nu* xvec|\!)(R' \parallel P'), ((\!|\nu* xvec|\!)(R' \parallel T)) \parallel !P) \in Rel$
by(*metis ResPres psiFreshVec ScopeExt Trans*)
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹*guarded* P› ‹*guarded* Q›
have $(\Psi, ((\!|\nu* xvec|\!)(R' \parallel T)) \parallel !P, ((\!|\nu* xvec|\!)(R' \parallel T)) \parallel !Q) \in Rel'$
by(*rule C1*)
moreover from ‹$(\Psi \otimes \Psi_R, Q', S \parallel !Q) \in Rel$› ‹$(\Psi \otimes \Psi_R, S, T) \in Rel$›
have $(\Psi \otimes \Psi_R, Q', T \parallel !Q) \in Rel$
by(*blast intro*: *ParPres Trans*)
hence $((\Psi \otimes \Psi_R) \otimes \Psi', Q', T \parallel !Q) \in Rel$ **by**(*rule cExt*)
with ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$›
have $(\Psi \otimes \Psi_R', Q', T \parallel !Q) \in Rel$
by(*metis Associativity StatEq compositionSym*)
with *FrR'* ‹$A_R' \sharp* \Psi$› ‹$A_R' \sharp* P'$› ‹$A_R' \sharp* Q'$› ‹$A_R' \sharp* Q$› *suppT suppS*
have $(\Psi, R' \parallel Q', R' \parallel (T \parallel !Q)) \in Rel$
by(*rule-tac FrameParPres*) (*auto simp add*: *fresh-star-def fresh-def psi.supp*)
hence $(\Psi, R' \parallel Q', (R' \parallel T) \parallel !Q) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
with ‹*xvec* $\sharp* \Psi$› ‹*xvec* $\sharp* (!Q)$›
have $(\Psi, (\!|\nu* xvec|\!)(R' \parallel Q'), ((\!|\nu* xvec|\!)(R' \parallel T)) \parallel !Q) \in Rel$
by(*metis ResPres psiFreshVec ScopeExt Trans*)
ultimately have $(\Psi, (\!|\nu* xvec|\!)(R' \parallel P'), (\!|\nu* xvec|\!)(R' \parallel Q')) \in Rel'$ **by**(*blast intro*: *cSym Compose*)
}
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_Q M *xvec* N R' A_R Ψ_R K Q' A_Q)
from ‹*extractFrame* $(!Q) = \langle A_Q, \Psi_Q \rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp+*
have *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu* xvec|\!)\langle N \rangle \prec R'$ **and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact+*
then obtain $p\ \Psi'\ A_R'\ \Psi_R'$ **where** S: *set* $p \subseteq$ *set xvec* $\times$ *set*$(p \cdot xvec)$
and *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R' \rangle$ **and** $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* \Psi$

and $A_R{'} \sharp* N$ **and** $A_R{'} \sharp* R'$ **and** $A_R{'} \sharp* P$ **and** $A_R{'} \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $xvec \sharp* A_R{'}$ **and** $(p \cdot xvec) \sharp* A_R{'}$
and *distinctPerm p* **and** $(p \cdot xvec) \sharp* R'$ **and** $(p \cdot xvec) \sharp* N$
using ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_R \sharp* xvec$› ‹$A_R \sharp* N$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* (!Q)$›
‹$xvec \sharp* \Psi$› ‹$xvec \sharp* P$› ‹$xvec \sharp* (!Q)$› ‹$xvec \sharp* R$› ‹$xvec \sharp* M$› ‹*distinct xvec*›
by(*rule-tac* C=(Ψ, P, Q) **and** C'=(Ψ, P, Q) **in** *expandFrame*) (*assumption* | *simp*)+

from *RTrans S* ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* R'$› **have** $\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$
apply(*simp add*: *residualInject*)
by(*subst boundOutputChainAlpha''*[*symmetric*]) *auto*

moreover have *QTrans*: $\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|N|\!) \prec Q'$ **by** *fact*
with *QTrans S* ‹$(p \cdot xvec) \sharp* N$› **have** $\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|(p \cdot N)|\!) \prec (p \cdot Q')$ **using** ‹*distinctPerm p*› ‹*xvec* $\sharp*$ $(!Q)$› ‹$(p \cdot xvec) \sharp* Q$›
by(*rule-tac inputAlpha*) *auto*
with ‹$(\Psi, P, Q) \in Rel$› ‹*guarded P*›
obtain P' S T **where** *PTrans*: $\Psi \otimes \Psi_R \rhd !P \longmapsto K(\!|(p \cdot N)|\!) \prec P'$ **and** $(\Psi \otimes \Psi_R, P', T \parallel !P) \in Rel$
and $(\Psi \otimes \Psi_R, (p \cdot Q'), S \parallel !Q) \in Rel$ **and** $(\Psi \otimes \Psi_R, S, T) \in Rel$
and *suppT*: $((supp\ T)::name\ set) \subseteq supp\ P'$ **and** *suppS*: $((supp\ S)::name\ set) \subseteq supp(p \cdot Q')$
by(*drule-tac cSym*) (*auto dest*: *Der cExt*)
note ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$›
ultimately have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel P')$
using *PTrans FrR* ‹$\Psi_Q = SBottom'$› ‹$(p \cdot xvec) \sharp* P$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_R \sharp* P$›
by(*rule-tac Comm2*) (*assumption* | *simp*)+

moreover from ‹$A_R{'} \sharp* P$› ‹$A_R{'} \sharp* Q$› ‹$A_R{'} \sharp* N$› *S* ‹*xvec* $\sharp*$ $A_R{'}$› ‹$(p \cdot xvec) \sharp* A_R{'}$› *PTrans QTrans* ‹*distinctPerm p*› **have** $A_R{'} \sharp* P'$ **and** $A_R{'} \sharp* Q'$
apply −
apply(*drule-tac inputFreshChainDerivative*, *auto*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, *of - - p*], *simp*)
by(*force dest*: *inputFreshChainDerivative*)+
from ‹*xvec* $\sharp*$ P› ‹$(p \cdot xvec) \sharp* N$› *PTrans* ‹*distinctPerm p*› **have** $(p \cdot xvec) \sharp* (p \cdot P')$
apply(*drule-tac inputFreshChainDerivative*, *simp*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, *of - - p*], *simp*)
by(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, *of - - p*], *simp*)

{
from ‹$(\Psi \otimes \Psi_R,\ P',\ T \parallel !P) \in Rel$› **have** $(p \cdot (\Psi \otimes \Psi_R),\ (p \cdot P'),\ p \cdot (T \parallel !P)) \in Rel$
by(*rule Closed*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$xvec \sharp* P$› ‹$(p \cdot xvec) \sharp* P$› S **have** $(\Psi \otimes (p \cdot \Psi_R),\ p \cdot P',\ (p \cdot T) \parallel !P) \in Rel$
by(*simp add*: *eqvts*)
hence $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi',\ p \cdot P',\ (p \cdot T) \parallel !P) \in Rel$ **by**(*rule cExt*)
with ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq {\Psi_R}'$› **have** $(\Psi \otimes {\Psi_R}',\ (p \cdot P'),\ (p \cdot T) \parallel !P) \in Rel$
by(*metis Associativity StatEq compositionSym*)
with *FrR'* ‹${A_R}' \sharp* \Psi$› ‹${A_R}' \sharp* P'$› ‹${A_R}' \sharp* P$› ‹$xvec \sharp* {A_R}'$› ‹$(p \cdot xvec) \sharp* {A_R}'$› S ‹*distinctPerm p*› *suppT*
have $(\Psi,\ R' \parallel (p \cdot P'),\ R' \parallel ((p \cdot T) \parallel !P)) \in Rel$
apply(*rule-tac FrameParPres*)
apply(*assumption* | *simp add*: *freshChainSimps*)+
by(*auto simp add*: *fresh-star-def fresh-def*)
hence $(\Psi,\ R' \parallel (p \cdot P'),\ (R' \parallel (p \cdot T)) \parallel !P) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
with ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* P$› **have** $(\Psi,\ (\!|\nu* xvec|\!)(R' \parallel (p \cdot P')),\ ((\!|\nu* xvec|\!)(R' \parallel (p \cdot T))) \parallel !P) \in Rel$
by(*metis ResPres psiFreshVec ScopeExt Trans*)
hence $(\Psi,\ (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel P'),\ ((\!|\nu* xvec|\!)(R' \parallel (p \cdot T))) \parallel !P) \in Rel$
using ‹$(p \cdot xvec) \sharp* R'$› ‹$(p \cdot xvec) \sharp* (p \cdot P')$› S ‹*distinctPerm p*›
apply(*erule-tac rev-mp*) **by**(*subst resChainAlpha*[*of p*]) *auto*
moreover from ‹$(\Psi,\ P,\ Q) \in Rel$› ‹*guarded P*› ‹*guarded Q*› **have** $(\Psi,\ ((\!|\nu* xvec|\!)(R' \parallel (p \cdot T))) \parallel !P,\ ((\!|\nu* xvec|\!)(R' \parallel (p \cdot T))) \parallel !Q) \in Rel'$
by(*rule C1*)
moreover from ‹$(\Psi \otimes \Psi_R,\ (p \cdot Q'),\ S \parallel !Q) \in Rel$› ‹$(\Psi \otimes \Psi_R,\ S,\ T) \in Rel$› **have** $(\Psi \otimes \Psi_R,\ (p \cdot Q'),\ T \parallel !Q) \in Rel$
by(*blast intro*: *ParPres Trans*)
hence $(p \cdot (\Psi \otimes \Psi_R),\ p \cdot p \cdot Q',\ p \cdot (T \parallel !Q)) \in Rel$ **by**(*rule Closed*)
with S ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹$xvec \sharp* (!Q)$› ‹$(p \cdot xvec) \sharp* Q$› ‹*distinctPerm p*›
have $(\Psi \otimes (p \cdot \Psi_R),\ Q',\ (p \cdot T) \parallel !Q) \in Rel$ **by**(*simp add*: *eqvts*)
hence $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi',\ Q',\ (p \cdot T) \parallel !Q) \in Rel$ **by**(*rule cExt*)
with ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq {\Psi_R}'$› **have** $(\Psi \otimes {\Psi_R}',\ Q',\ (p \cdot T) \parallel !Q) \in Rel$
by(*metis Associativity StatEq compositionSym*)
with *FrR'* ‹${A_R}' \sharp* \Psi$› ‹${A_R}' \sharp* P'$› ‹${A_R}' \sharp* Q'$› ‹${A_R}' \sharp* Q$› *suppT suppS* ‹$xvec \sharp* {A_R}'$› ‹$(p \cdot xvec) \sharp* {A_R}'$› S ‹*distinctPerm p*›
have $(\Psi,\ R' \parallel Q',\ R' \parallel ((p \cdot T) \parallel !Q)) \in Rel$
apply(*rule-tac FrameParPres*)
apply(*assumption* | *simp*)+
apply(*simp add*: *freshChainSimps*)
by(*auto simp add*: *fresh-star-def fresh-def*)
hence $(\Psi,\ R' \parallel Q',\ (R' \parallel (p \cdot T)) \parallel !Q) \in Rel$ **by**(*blast intro*: *Assoc Trans*)
with ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* (!Q)$› **have** $(\Psi,\ (\!|\nu* xvec|\!)(R' \parallel Q'),\ ((\!|\nu* xvec|\!)(R' \parallel (p \cdot T))) \parallel !Q) \in Rel$
by(*metis ResPres psiFreshVec ScopeExt Trans*)

ultimately have $(\Psi, (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel P'), (\!|\nu*xvec|\!)(R' \parallel Q')) \in Rel'$
by(*blast intro*: *cSym Compose*)
}
ultimately show *?case* **by** *blast*
qed
qed
notation *relcomp* (**infixr** *O* 75)
end

end

theory *Bisim-Pres*
imports *Bisimulation Sim-Pres*
begin

context *env* **begin**

lemma *bisimInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $\bigwedge Tvec.\ length\ xvec = length\ Tvec \Longrightarrow \Psi \rhd P[xvec::=Tvec] \sim Q[xvec::=Tvec]$

shows $\Psi \rhd M(\!|\lambda*xvec\ N|\!).P \sim M(\!|\lambda*xvec\ N|\!).Q$
proof −
let *?X* = $\{(\Psi, M(\!|\lambda*xvec\ N|\!).P, M(\!|\lambda*xvec\ N|\!).Q) \mid \Psi\ M\ xvec\ N\ P\ Q.\ \forall\ Tvec.$
$length\ xvec = length\ Tvec \longrightarrow \Psi \rhd P[xvec::=Tvec] \sim Q[xvec::=Tvec]\}$

from *assms* **have** $(\Psi, M(\!|\lambda*xvec\ N|\!).P, M(\!|\lambda*xvec\ N|\!).Q) \in ?X$ **by** *blast*
thus *?thesis*
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* Ψ P Q)
thus *?case* **by** *auto*
next
case(*cSim* Ψ P Q)
thus *?case* **by**(*blast intro*: *inputPres*)
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*blast dest*: *bisimE*)
next
case(*cSym* Ψ P Q)
thus *?case* **by**(*blast dest*: *bisimE*)
qed
qed

lemma *bisimOutputPres*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$
  and $M :: 'a$
  and $N :: 'a$

  assumes $\Psi \rhd P \sim Q$

  shows $\Psi \rhd M\langle N\rangle.P \sim M\langle N\rangle.Q$
proof −
  let *?X* = $\{(\Psi, M\langle N\rangle.P, M\langle N\rangle.Q) \mid \Psi\ M\ N\ P\ Q.\ \Psi \rhd P \sim Q\}$
  from ‹$\Psi \rhd P \sim Q$› **have** $(\Psi, M\langle N\rangle.P, M\langle N\rangle.Q) \in$ *?X* **by** *auto*
  thus *?thesis*
    by(*coinduct rule*: *bisimCoinduct*, *auto*) (*blast intro*: *outputPres dest*: *bisimE*)+
qed

lemma *bisimCasePres*:
  fixes Ψ :: $'b$
  and *CsP* :: $('c \times ('a, 'b, 'c)\ psi)\ list$
  and *CsQ* :: $('c \times ('a, 'b, 'c)\ psi)\ list$

  assumes $\bigwedge\varphi\ P.\ (\varphi, P)\ mem\ CsP \Longrightarrow \exists\, Q.\ (\varphi, Q)\ mem\ CsQ \wedge guarded\ Q \wedge \Psi \rhd P \sim Q$
  and $\bigwedge\varphi\ Q.\ (\varphi, Q)\ mem\ CsQ \Longrightarrow \exists\, P.\ (\varphi, P)\ mem\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim Q$

  shows $\Psi \rhd Cases\ CsP \sim Cases\ CsQ$
proof −
  let *?X* = $\{(\Psi, Cases\ CsP, Cases\ CsQ) \mid \Psi\ CsP\ CsQ.\ (\forall\varphi\ P.\ (\varphi, P)\ mem\ CsP \longrightarrow (\exists\, Q.\ (\varphi, Q)\ mem\ CsQ \wedge guarded\ Q \wedge \Psi \rhd P \sim Q)) \wedge (\forall\varphi\ Q.\ (\varphi, Q)\ mem\ CsQ \longrightarrow (\exists\, P.\ (\varphi, P)\ mem\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim Q))\}$
  from *assms* **have** $(\Psi, Cases\ CsP, Cases\ CsQ) \in$ *?X* **by** *auto*
  thus *?thesis*
  proof(*coinduct rule*: *bisimCoinduct*)
    case(*cStatEq* Ψ *P Q*)
    thus *?case* **by** *auto*
  next
    case(*cSim* Ψ *CasesP CasesQ*)
    then obtain *CsP CsQ* **where** *C1*: $\bigwedge\varphi\ Q.\ (\varphi, Q)\ mem\ CsQ \Longrightarrow \exists\, P.\ (\varphi, P)\ mem\ CsP \wedge guarded\ P \wedge \Psi \rhd P \sim Q$
      and *A*: *CasesP* = *Cases CsP* **and** *B*: *CasesQ* = *Cases CsQ*
      by *auto*
    note *C1*
    moreover have $\bigwedge\Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow \Psi \rhd P \rightsquigarrow[bisim]\ Q$ **by**(*rule bisimE*)
    moreover have *bisim* $\subseteq$ *?X* $\cup$ *bisim* **by** *blast*
    ultimately have $\Psi \rhd Cases\ CsP \rightsquigarrow[(?X \cup bisim)]\ Cases\ CsQ$
      by(*rule casePres*)

```
    thus ?case using A B by blast
  next
    case(cExt Ψ P Q)
    thus ?case by(blast dest: bisimE)
  next
    case(cSym Ψ P Q)
    thus ?case by(blast dest: bisimE)
  qed
qed

lemma bisimResPres:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   x :: name

  assumes Ψ ▷ P ∼ Q
  and     x ♯ Ψ

  shows Ψ ▷ (|νx|)P ∼ (|νx|)Q
proof −
  let ?X = {(Ψ, (|νx|)P, (|νx|)Q) | Ψ x P Q. Ψ ▷ P ∼ Q ∧ x ♯ Ψ}

  from assms have (Ψ, (|νx|)P, (|νx|)Q) ∈ ?X by auto
  thus ?thesis
  proof(coinduct rule: bisimCoinduct)
    case(cStatEq Ψ xP xQ)
    from ‹(Ψ, xP, xQ) ∈ ?X› obtain x P Q where Ψ ▷ P ∼ Q and x ♯ Ψ and
xP = (|νx|)P and xQ = (|νx|)Q
      by auto
    moreover from ‹Ψ ▷ P ∼ Q› have PeqQ: insertAssertion(extractFrame P)
Ψ ≃_F insertAssertion(extractFrame Q) Ψ
      by(rule bisimE)
    ultimately show ?case by(auto intro: frameResPres)
  next
    case(cSim Ψ xP xQ)
    from ‹(Ψ, xP, xQ) ∈ ?X› obtain x P Q where Ψ ▷ P ∼ Q and x ♯ Ψ and
xP = (|νx|)P and xQ = (|νx|)Q
      by auto
    from ‹Ψ ▷ P ∼ Q› have Ψ ▷ P ⇝[bisim] Q by(rule bisimE)
    moreover have eqvt ?X
      by(force simp add: eqvt-def bisimClosed pt-fresh-bij[OF pt-name-inst, OF
at-name-inst])
    hence eqvt(?X ∪ bisim) by auto
    moreover note ‹x ♯ Ψ›
    moreover have bisim ⊆ ?X ∪ bisim by auto
    moreover have ⋀Ψ P Q x. ⟦(Ψ, P, Q) ∈ bisim; x ♯ Ψ⟧ ⟹ (Ψ, (|νx|)P, (|νx|)Q)
∈ ?X ∪ bisim
      by auto
```

ultimately have $\Psi \rhd (\!|\nu x|\!) P \rightsquigarrow [(?X \cup bisim)]\ (\!|\nu x|\!) Q$
by(*rule resPres*)
with ‹$xP = (\!|\nu x|\!) P$› ‹$xQ = (\!|\nu x|\!) Q$› **show** *?case*
by *simp*
next
case(*cExt* Ψ xP xQ Ψ')
from ‹$(\Psi, xP, xQ) \in ?X$› **obtain** x P Q **where** $\Psi \rhd P \sim Q$ **and** $x \sharp \Psi$ **and** $xP = (\!|\nu x|\!) P$ **and** $xQ = (\!|\nu x|\!) Q$
by *auto*
obtain y::*name* **where** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp \Psi$ **and** $y \sharp \Psi'$
by(*generate-fresh name*, *auto simp add*: *fresh-prod*)
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \otimes ([(x, y)] \cdot \Psi') \rhd P \sim Q$
by(*rule bisimE*)
hence $([(x, y)] \cdot (\Psi \otimes ([(x, y)] \cdot \Psi'))) \rhd ([(x, y)] \cdot P) \sim ([(x, y)] \cdot Q)$
by(*rule bisimClosed*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \otimes \Psi' \rhd ([(x, y)] \cdot P) \sim ([(x, y)] \cdot Q)$
by(*simp add*: *eqvts*)
with ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu y|\!)([(x, y)] \cdot P), (\!|\nu y|\!)([(x, y)] \cdot Q)) \in ?X$
by *auto*
moreover from ‹$y \sharp P$› ‹$y \sharp Q$› **have** $(\!|\nu x|\!) P = (\!|\nu y|\!)([(x, y)] \cdot P)$ **and** $(\!|\nu x|\!) Q = (\!|\nu y|\!)([(x, y)] \cdot Q)$
by(*simp add*: *alphaRes*)+
ultimately show *?case* **using** ‹$xP = (\!|\nu x|\!) P$› ‹$xQ = (\!|\nu x|\!) Q$› **by** *simp*
next
case(*cSym* Ψ P Q)
thus *?case*
by(*blast dest*: *bisimE*)
qed
qed

lemma *bisimResChainPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and *xvec* :: *name list*

assumes $\Psi \rhd P \sim Q$
and *xvec* $\sharp*$ Ψ

shows $\Psi \rhd (\!|\nu * xvec|\!) P \sim (\!|\nu * xvec|\!) Q$
using *assms*
by(*induct xvec*) (*auto intro*: *bisimResPres*)

lemma *bisimParPresAux*:
fixes Ψ :: $'b$
and Ψ_R :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

and $R :: ('a, 'b, 'c)\ psi$
and $A_R :: name\ list$

assumes $\Psi \otimes \Psi_R \rhd P \sim Q$
and *FrR*: $extractFrame\ R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$

shows $\Psi \rhd P \parallel R \sim Q \parallel R$
proof −
let $?X = \{(\Psi, (\!|\nu * xvec|\!)(P \parallel R), (\!|\nu * xvec|\!)(Q \parallel R)) \mid xvec\ \Psi\ P\ Q\ R.\ xvec \sharp* \Psi \wedge (\forall A_R\ \Psi_R.\ (extractFrame\ R = \langle A_R, \Psi_R \rangle \wedge A_R \sharp* \Psi \wedge A_R \sharp* P \wedge A_R \sharp* Q) \longrightarrow \Psi \otimes \Psi_R \rhd P \sim Q)\}$
{
fix $xvec :: name\ list$
and $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assume $xvec \sharp* \Psi$
and $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R \rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q]\!] \Longrightarrow \Psi \otimes \Psi_R \rhd P \sim Q$

hence $(\Psi, (\!|\nu * xvec|\!)(P \parallel R), (\!|\nu * xvec|\!)(Q \parallel R)) \in ?X$
apply *auto*
by *blast*
}

note $XI = this$
{
fix $xvec :: name\ list$
and $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$
and $C :: 'd::fs\text{-}name$

assume $xvec \sharp* \Psi$
and *A*: $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R \rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q;\ A_R \sharp* C]\!] \Longrightarrow \Psi \otimes \Psi_R \rhd P \sim Q$

from ‹$xvec \sharp* \Psi$› **have** $(\Psi, (\!|\nu * xvec|\!)(P \parallel R), (\!|\nu * xvec|\!)(Q \parallel R)) \in ?X$
proof(*rule XI*)
fix $A_R\ \Psi_R$
assume *FrR*: $extractFrame\ R = \langle A_R, \Psi_R \rangle$
obtain $p::name\ prm$ **where** $(p \cdot A_R) \sharp* \Psi$ **and** $(p \cdot A_R) \sharp* P$ **and** $(p \cdot A_R)$

$\sharp*\ Q$ **and** $(p \cdot A_R) \sharp*\ R$ **and** $(p \cdot A_R) \sharp*\ C$
and $(p \cdot A_R) \sharp*\ \Psi_R$ **and** S: $(set\ p) \subseteq (set\ A_R) \times (set(p \cdot A_R))$ **and** *distinctPerm p*
by(*rule-tac c*=$(\Psi, P, Q, R, \Psi_R, C)$ **in** *name-list-avoiding*) *auto*
from *FrR* ‹$(p \cdot A_R) \sharp*\ \Psi_R$› S **have** *extractFrame* $R = \langle (p \cdot A_R), p \cdot \Psi_R \rangle$ **by**(*simp add*: *frameChainAlpha'*)

moreover assume $A_R \sharp*\ \Psi$
hence $(p \cdot A_R) \sharp*\ (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp*\ \Psi$› ‹$(p \cdot A_R) \sharp*\ \Psi$› S **have** $(p \cdot A_R) \sharp*\ \Psi$ **by** *simp*
moreover assume $A_R \sharp*\ P$
hence $(p \cdot A_R) \sharp*\ (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp*\ P$› ‹$(p \cdot A_R) \sharp*\ P$› S **have** $(p \cdot A_R) \sharp*\ P$ **by** *simp*
moreover assume $A_R \sharp*\ Q$
hence $(p \cdot A_R) \sharp*\ (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp*\ Q$› ‹$(p \cdot A_R) \sharp*\ Q$› S **have** $(p \cdot A_R) \sharp*\ Q$ **by** *simp*
ultimately have $\Psi \otimes (p \cdot \Psi_R) \rhd P \sim Q$ **using** ‹$(p \cdot A_R) \sharp*\ C$› A **by** *blast*
hence $(p \cdot (\Psi \otimes (p \cdot \Psi_R))) \rhd (p \cdot P) \sim (p \cdot Q)$ **by**(*rule bisimClosed*)
with ‹$A_R \sharp*\ \Psi$› ‹$(p \cdot A_R) \sharp*\ \Psi$› ‹$A_R \sharp*\ P$› ‹$(p \cdot A_R) \sharp*\ P$› ‹$A_R \sharp*\ Q$› ‹$(p \cdot A_R) \sharp*\ Q$› S ‹*distinctPerm p*›
show $\Psi \otimes \Psi_R \rhd P \sim Q$ **by**(*simp add*: *eqvts*)
qed
}
note $XI' = this$

have *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*rule-tac x*=$p \cdot xvec$ **in** *exI*)
apply(*rule-tac x*=$p \cdot P$ **in** *exI*)
apply(*rule-tac x*=$p \cdot Q$ **in** *exI*)
apply(*rule-tac x*=$p \cdot R$ **in** *exI*)
apply(*simp add*: *eqvts*)
apply(*simp add*: *fresh-star-bij*)
apply(*clarify*)
apply(*erule-tac x*=$(rev\ p) \cdot A_R$ **in** *allE*)
apply(*erule-tac x*=$(rev\ p) \cdot \Psi_R$ **in** *allE*)
apply(*drule mp*)
apply(*rule conjI*)
apply(*rule-tac pi*=p **in** *pt-bij4*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *eqvts*)
defer
apply(*drule-tac p*=p **in** *bisimClosed*)
apply(*simp add*: *eqvts*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
apply *simp*

apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
apply *simp*
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
by *simp*

moreover have *Res*: $\bigwedge \Psi\ P\ Q\ x.\ [\![(\Psi, P, Q) \in ?X \cup bisim;\ x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X \cup bisim$
proof −
fix $\Psi\ P\ Q\ x$
assume $(\Psi, P, Q) \in ?X \cup bisim$ **and** $(x::name) \sharp \Psi$
show $(\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X \cup bisim$
proof(*case-tac* $(\Psi, P, Q) \in ?X$)
assume $(\Psi, P, Q) \in ?X$
with ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X$
apply *auto*
by(*rule-tac x*=*x*#*xvec* **in** *exI*) *auto*
thus *?thesis* **by** *simp*
next
assume $\neg(\Psi, P, Q) \in ?X$
with ‹$(\Psi, P, Q) \in ?X \cup bisim$› **have** $\Psi \rhd P \sim Q$
by *blast*
hence $\Psi \rhd (\!|\nu x|\!)P \sim (\!|\nu x|\!)Q$ **using** ‹$x \sharp \Psi$›
by(*rule bisimResPres*)
thus *?thesis*
by *simp*
qed
qed

have $(\Psi, P \parallel R, Q \parallel R) \in ?X$
proof −
{
fix A_R' :: *name list*
and Ψ_R' :: *'b*

assume *FrR'*: *extractFrame* $R = \langle A_R', \Psi_R' \rangle$
and $A_R' \sharp* \Psi$
and $A_R' \sharp* P$
and $A_R' \sharp* Q$

obtain p **where** $(p \cdot A_R') \sharp* A_R$ **and** $(p \cdot A_R') \sharp* \Psi_R'$ **and** $(p \cdot A_R') \sharp* \Psi$ **and** $(p \cdot A_R') \sharp* P$ **and** $(p \cdot A_R') \sharp* Q$
and *Sp*: $(set\ p) \subseteq (set\ A_R') \times (set(p \cdot A_R'))$ **and** *distinctPerm p*
by(*rule-tac c*=$(A_R, \Psi, \Psi_R', P, Q)$ **in** *name-list-avoiding*) *auto*

from *FrR'* ‹$(p \cdot A_R') \sharp* \Psi_R'$› *Sp* **have** *extractFrame* $R = \langle (p \cdot A_R'), p \cdot \Psi_R' \rangle$
by(*simp add*: *frameChainAlpha eqvts*)

with *FrR* ‹$(p \cdot A_R') \sharp* A_R$› **obtain** *q*::*name prm*
where *Sq*: *set q* ⊆ *set*$(p \cdot A_R') \times$ *set* A_R **and** *distinctPerm q* **and** $\Psi_R = q \cdot p \cdot \Psi_R'$
by(*force elim*: *frameChainEq*)

from ‹$\Psi \otimes \Psi_R \rhd P \sim Q$› ‹$\Psi_R = q \cdot p \cdot \Psi_R'$› **have** $\Psi \otimes (q \cdot p \cdot \Psi_R') \rhd P \sim Q$ **by** *simp*
hence $(q \cdot (\Psi \otimes (q \cdot p \cdot \Psi_R'))) \rhd (q \cdot P) \sim (q \cdot Q)$ **by**(*rule bisimClosed*)
with *Sq* ‹$A_R \sharp* \Psi$› ‹$(p \cdot A_R') \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$(p \cdot A_R') \sharp* P$› ‹$A_R \sharp* Q$› ‹$(p \cdot A_R') \sharp* Q$› ‹*distinctPerm q*›
have $\Psi \otimes (p \cdot \Psi_R') \rhd P \sim Q$ **by**(*simp add*: *eqvts*)
hence $(p \cdot (\Psi \otimes (p \cdot \Psi_R'))) \rhd (p \cdot P) \sim (p \cdot Q)$ **by**(*rule bisimClosed*)
with *Sp* ‹$A_R' \sharp* \Psi$› ‹$(p \cdot A_R') \sharp* \Psi$› ‹$A_R' \sharp* P$› ‹$(p \cdot A_R') \sharp* P$› ‹$A_R' \sharp* Q$› ‹$(p \cdot A_R') \sharp* Q$› ‹*distinctPerm p*›
have $\Psi \otimes \Psi_R' \rhd P \sim Q$ **by**(*simp add*: *eqvts*)
}
thus *?thesis*
apply *auto*
apply(*rule-tac x*=[] **in** *exI*)
by *auto blast*
qed
thus *?thesis*
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* Ψ *PR QR*)
from ‹$(\Psi, PR, QR) \in ?X$›
obtain *xvec P Q R* A_R Ψ_R **where** *PFrR*: $PR = (\!|\nu * xvec|\!)(P \parallel R)$ **and** *QFrR*: $QR = (\!|\nu * xvec|\!)(Q \parallel R)$
and *xvec* $\sharp*$ Ψ **and** *FrR*: *extractFrame R* = $\langle A_R, \Psi_R\rangle$ **and** *PSimQ*: $\Psi \otimes \Psi_R \rhd P \sim Q$
and $A_R \sharp* xvec$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$
apply *auto*
apply(*subgoal-tac* $\exists A_R\ \Psi_R.$ *extractFrame R* = $\langle A_R, \Psi_R\rangle \wedge A_R \sharp* (xvec, \Psi, P, Q, R)$)
apply *auto*
apply(*rule-tac F*=*extractFrame R* **and** *C*=(*xvec*, Ψ, *P*, *Q*, *R*) **in** *freshFrame*)
by *auto*

obtain A_P Ψ_P **where** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* A_R$ **and** $A_P \sharp* \Psi_R$
by(*rule-tac C*=(Ψ, A_R, Ψ_R) **in** *freshFrame*) *auto*
obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame Q* = $\langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* A_R$ **and** $A_Q \sharp* \Psi_R$
by(*rule-tac C*=(Ψ, A_R, Ψ_R) **in** *freshFrame*) *auto*
from ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_P \sharp* A_R$› ‹$A_Q \sharp* A_R$› *FrP FrQ* **have** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_Q$
by(*force dest*: *extractFrameFreshChain*)+

have $\langle (A_P @ A_R), \Psi \otimes \Psi_P \otimes \Psi_R\rangle \simeq_F \langle (A_Q @ A_R), \Psi \otimes \Psi_Q \otimes \Psi_R\rangle$

proof −
have $\langle A_P, \Psi \otimes \Psi_P \otimes \Psi_R\rangle \simeq_F \langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle$
by(*metis frameResChainPres frameNilStatEq Associativity Commutativity AssertionStatEqTrans Composition*)
moreover from *PSimQ* **have** *insertAssertion*(*extractFrame P*) $(\Psi \otimes \Psi_R) \simeq_F$ *insertAssertion*(*extractFrame Q*) $(\Psi \otimes \Psi_R)$
by(*rule bisimE*)
with *FrP FrQ freshCompChain* $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* \Psi_R\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$ **have** $\langle A_P, (\Psi \otimes \Psi_R) \otimes \Psi_P\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by *auto*
moreover have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \simeq_F \langle A_Q, \Psi \otimes \Psi_Q \otimes \Psi_R\rangle$
by(*metis frameResChainPres frameNilStatEq Associativity Commutativity AssertionStatEqTrans Composition*)
ultimately have $\langle A_P, \Psi \otimes \Psi_P \otimes \Psi_R\rangle \simeq_F \langle A_Q, \Psi \otimes \Psi_Q \otimes \Psi_R\rangle$
by(*blast intro*: *FrameStatEqTrans*)
hence $\langle (A_R@A_P), \Psi \otimes \Psi_P \otimes \Psi_R\rangle \simeq_F \langle (A_R@A_Q), \Psi \otimes \Psi_Q \otimes \Psi_R\rangle$
by(*drule-tac frameResChainPres*) (*simp add*: *frameChainAppend*)
thus *?thesis*
apply(*simp add*: *frameChainAppend*)
by(*metis frameResChainComm FrameStatEqTrans*)
qed
moreover from $\langle A_P \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi\rangle$ **have** $(A_P@A_R) \sharp* \Psi$ **by** *simp*
moreover from $\langle A_Q \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi\rangle$ **have** $(A_Q@A_R) \sharp* \Psi$ **by** *simp*
ultimately have *PFrRQR*: *insertAssertion*(*extractFrame*($P \parallel R$)) $\Psi \simeq_F$ *insertAssertion*(*extractFrame*($Q \parallel R$)) Ψ
using *FrP FrQ FrR* $\langle A_P \sharp* A_R\rangle$ $\langle A_P \sharp* \Psi_R\rangle$ $\langle A_Q \sharp* A_R\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$
by *simp*

from $\langle xvec \sharp* \Psi\rangle$ **have** *insertAssertion* (*extractFrame*($(\!|\nu*xvec|\!)P \parallel R$)) $\Psi \simeq_F (\!|\nu*xvec|\!)$(*insertAssertion* (*extractFrame*($P \parallel R$)) Ψ)
by(*rule insertAssertionExtractFrameFresh*)
moreover from *PFrRQR* **have** $(\!|\nu*xvec|\!)$(*insertAssertion* (*extractFrame*($P \parallel R$)) Ψ) $\simeq_F (\!|\nu*xvec|\!)$(*insertAssertion* (*extractFrame*($Q \parallel R$)) Ψ)
by(*induct xvec*) (*auto intro*: *frameResPres*)
moreover from $\langle xvec \sharp* \Psi\rangle$ **have** $(\!|\nu*xvec|\!)$(*insertAssertion* (*extractFrame*($Q \parallel R$)) Ψ) $\simeq_F$ *insertAssertion* (*extractFrame*($(\!|\nu*xvec|\!)Q \parallel R$)) Ψ
by(*rule FrameStatEqSym*[*OF insertAssertionExtractFrameFresh*])
ultimately show *?case* **using** *PFrR QFrR*
by(*blast intro*: *FrameStatEqTrans*)
next
case(*cSim* Ψ *PR QR*)
{
fix Ψ *P Q R xvec*
assume $\bigwedge A_R\ \Psi_R.$ ⟦*extractFrame R* = $\langle A_R, \Psi_R\rangle$; $A_R \sharp* \Psi$; $A_R \sharp* P$; $A_R \sharp* Q$⟧ $\Longrightarrow \Psi \otimes \Psi_R \rhd P \sim Q$
moreover have *eqvt bisim* **by** *simp*
moreover from $\langle eqvt\ ?X\rangle$ **have** *eqvt*(*?X* ∪ *bisim*) **by** *auto*
moreover from *bisimE*(*1*) **have** $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow$ *insertAsser-*

tion (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P*) Ψ **by**(*simp add*: *FrameStatEq-def*)
moreover note *bisimE*(*2*) *bisimE*(*3*)
moreover
{
fix Ψ *P Q* A_R Ψ_R *R*
assume *PSimQ*: $\Psi \otimes \Psi_R \rhd P \sim Q$
and *FrR*: *extractFrame R* = $\langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
hence $(\Psi, P \parallel R, Q \parallel R) \in ?X$
proof −
have $P \parallel R = (\!|\nu*[]|\!)(P \parallel R)$ **by** *simp*
moreover have $Q \parallel R = (\!|\nu*[]|\!)(Q \parallel R)$ **by** *simp*
moreover have ([]::*name list*) $\sharp* \Psi$ **by** *simp*
moreover
{
fix $A_R{'}$ $\Psi_R{'}$

assume *FrR'*: *extractFrame R* = $\langle A_R{'}, \Psi_R{'} \rangle$
and $A_R{'} \sharp* \Psi$
and $A_R{'} \sharp* P$
and $A_R{'} \sharp* Q$
obtain *p* **where** $(p \cdot A_R{'}) \sharp* A_R$
and $(p \cdot A_R{'}) \sharp* \Psi_R{'}$
and $(p \cdot A_R{'}) \sharp* \Psi$
and $(p \cdot A_R{'}) \sharp* P$
and $(p \cdot A_R{'}) \sharp* Q$
and *S*: (*set p*) ⊆ (*set* $A_R{'}$) × (*set*($p \cdot A_R{'}$)) **and** *distinctPerm p*
by(*rule-tac c*=(A_R, Ψ, $\Psi_R{'}$, *P*, *Q*) **in** *name-list-avoiding*) *auto*

from ‹($p \cdot A_R{'}$) $\sharp* \Psi_R{'}$› *S* **have** $\langle A_R{'}, \Psi_R{'} \rangle = \langle p \cdot A_R{'}, p \cdot \Psi_R{'} \rangle$
by(*simp add*: *frameChainAlpha*)

with *FrR'* **have** *FrR''*: *extractFrame R* = $\langle p \cdot A_R{'}, p \cdot \Psi_R{'} \rangle$ **by** *simp*
with *FrR* ‹($p \cdot A_R{'}$) $\sharp* A_R$›
obtain *q* **where** $p \cdot \Psi_R{'} = (q::name\ prm) \cdot \Psi_R$ **and** *S'*: *set q* ⊆ (*set* A_R) × *set*($p \cdot A_R{'}$) **and** *distinctPerm q*
apply *auto*
apply(*drule-tac sym*) **apply** *simp*
by(*drule-tac frameChainEq*) *auto*
from *PSimQ* **have** $(q \cdot (\Psi \otimes \Psi_R)) \rhd (q \cdot P) \sim (q \cdot Q)$
by(*rule bisimClosed*)
with ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹($p \cdot A_R{'}$) $\sharp* \Psi$› ‹($p \cdot A_R{'}$) $\sharp* P$› ‹($p \cdot A_R{'}$) $\sharp* Q$› *S'*
have $\Psi \otimes (q \cdot \Psi_R) \rhd P \sim Q$ **by**(*simp add*: *eqvts*)
hence $(p \cdot (\Psi \otimes (q \cdot \Psi_R))) \rhd (p \cdot P) \sim (p \cdot Q)$ **by**(*rule bisimClosed*)

with ‹$A_R{}'$ $\sharp*$ Ψ› ‹$A_R{}'$ $\sharp*$ P› ‹$A_R{}'$ $\sharp*$ Q› ‹$(p \cdot A_R{}')$ $\sharp*$ Ψ› ‹$(p \cdot A_R{}')$ $\sharp*$ P› ‹$(p \cdot A_R{}')$ $\sharp*$ Q› S ‹*distinctPerm* p› ‹$(p \cdot \Psi_R{}') = q \cdot \Psi_R$›
have $\Psi \otimes \Psi_R{}' \rhd P \sim Q$
by(*drule-tac sym*) (*simp add*: *eqvts*)
}
ultimately show *?thesis*
by *blast*
qed
hence $(\Psi, P \parallel R, Q \parallel R) \in ?X \cup bisim$
by *simp*
}
moreover have $\bigwedge \Psi\ P\ Q\ xvec.\ [\![(\Psi, P, Q) \in ?X \cup bisim; (xvec::name\ list) \sharp* \Psi]\!] \Longrightarrow (\Psi, (\!|\nu*xvec|\!)P, (\!|\nu*xvec|\!)Q) \in ?X \cup bisim$
proof −
fix $\Psi\ P\ Q\ xvec$
assume $(\Psi, P, Q) \in ?X \cup bisim$
assume $(xvec::name\ list) \sharp* \Psi$
thus $(\Psi, (\!|\nu*xvec|\!)P, (\!|\nu*xvec|\!)Q) \in ?X \cup bisim$
proof(*induct xvec*)
case *Nil*
thus *?case* **using** ‹$(\Psi, P, Q) \in ?X \cup bisim$› **by** *simp*
next
case(*Cons x xvec*)
thus *?case* **by**(*simp only*: *resChain.simps*) (*rule-tac Res*, *auto*)
qed
qed
ultimately have $\Psi \rhd P \parallel R \rightsquigarrow[(?X \cup bisim)]\ Q \parallel R$ **using** *statEqBisim*
by(*rule parPres*)
moreover assume $(xvec::name\ list) \sharp* \Psi$
moreover from ‹*eqvt ?X*› **have** *eqvt*$(?X \cup bisim)$ **by** *auto*
ultimately have $\Psi \rhd (\!|\nu*xvec|\!)(P \parallel R) \rightsquigarrow[(?X \cup bisim)]\ (\!|\nu*xvec|\!)(Q \parallel R)$ **using** *Res*
by(*rule-tac resChainPres*)
}
with ‹$(\Psi, PR, QR) \in ?X$› **show** *?case* **by** *blast*
next
case(*cExt* $\Psi\ PR\ QR\ \Psi'$)

from ‹$(\Psi, PR, QR) \in ?X$›
obtain $xvec\ P\ Q\ R\ A_R\ \Psi_R$ **where** *PFrR*: $PR = (\!|\nu*xvec|\!)(P \parallel R)$ **and** *QFrR*: $QR = (\!|\nu*xvec|\!)(Q \parallel R)$
and $xvec \sharp* \Psi$ **and** *A*: $\forall A_R\ \Psi_R.\ (extractFrame\ R = \langle A_R, \Psi_R\rangle \wedge A_R \sharp* \Psi \wedge A_R \sharp* P \wedge A_R \sharp* Q) \longrightarrow \Psi \otimes \Psi_R \rhd P \sim Q$
by *auto*

obtain p **where** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* Q$
and $(p \cdot xvec) \sharp* R$

and $(p \cdot xvec) \sharp* \Psi'$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm p*
by(*rule-tac c*=(Ψ, P, Q, R, Ψ') **in** *name-list-avoiding*) *auto*

from $\langle(p \cdot xvec) \sharp* P\rangle$ $\langle(p \cdot xvec) \sharp* R\rangle$ S **have** $(\!|\nu*xvec|\!)(P \parallel R) = (\!|\nu*(p \cdot xvec)|\!)(p \cdot (P \parallel R))$
by(*subst resChainAlpha*) *auto*
hence *PRAlpha*: $(\!|\nu*xvec|\!)(P \parallel R) = (\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel (p \cdot R))$
by(*simp add*: *eqvts*)

from $\langle(p \cdot xvec) \sharp* Q\rangle$ $\langle(p \cdot xvec) \sharp* R\rangle$ S **have** $(\!|\nu*xvec|\!)(Q \parallel R) = (\!|\nu*(p \cdot xvec)|\!)(p \cdot (Q \parallel R))$
by(*subst resChainAlpha*) *auto*
hence *QRAlpha*: $(\!|\nu*xvec|\!)(Q \parallel R) = (\!|\nu*(p \cdot xvec)|\!)((p \cdot Q) \parallel (p \cdot R))$
by(*simp add*: *eqvts*)

from $\langle(p \cdot xvec) \sharp* \Psi\rangle$ $\langle(p \cdot xvec) \sharp* \Psi'\rangle$ **have** $(\Psi \otimes \Psi', (\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel (p \cdot R)), (\!|\nu*(p \cdot xvec)|\!)((p \cdot Q) \parallel (p \cdot R))) \in ?X$
proof(*rule-tac C2*=$(\Psi, (p \cdot P), (p \cdot Q), R, \Psi', xvec, p \cdot xvec)$ **in** *XI'*, *auto*)
fix A_R Ψ_R
assume *FrR*: *extractFrame* $(p \cdot R) = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* \Psi'$ **and** $A_R \sharp* (p \cdot P)$ **and** $A_R \sharp* (p \cdot Q)$
from *FrR* **have** $(p \cdot (extractFrame\ (p \cdot R))) = (p \cdot \langle A_R, \Psi_R\rangle)$ **by** *simp*
with $\langle distinctPerm\ p\rangle$ **have** $extractFrame\ R = \langle p \cdot A_R, p \cdot \Psi_R\rangle$ **by**(*simp add*: *eqvts*)
moreover from $\langle A_R \sharp* \Psi\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* \Psi\rangle$ $\langle(p \cdot xvec) \sharp* \Psi\rangle$ S **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
moreover from $\langle A_R \sharp* (p \cdot P)\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle distinctPerm\ p\rangle$ **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
moreover from $\langle A_R \sharp* (p \cdot Q)\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle distinctPerm\ p\rangle$ **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
ultimately have $\Psi \otimes (p \cdot \Psi_R) \rhd P \sim Q$ **using** *A* **by** *blast*
hence $(\Psi \otimes (p \cdot \Psi_R)) \otimes (p \cdot \Psi') \rhd P \sim Q$ **by**(*rule bisimE*)
moreover have $(\Psi \otimes (p \cdot \Psi_R)) \otimes (p \cdot \Psi') \simeq (\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R)$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $(\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R) \rhd P \sim Q$
by(*rule statEqBisim*)
hence $(p \cdot ((\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R))) \rhd (p \cdot P) \sim (p \cdot Q)$
by(*rule bisimClosed*)
with $\langle distinctPerm\ p\rangle$ $\langle xvec \sharp* \Psi\rangle$ $\langle(p \cdot xvec) \sharp* \Psi\rangle$ S **show** $(\Psi \otimes \Psi') \otimes \Psi_R \rhd (p \cdot P) \sim (p \cdot Q)$
by(*simp add*: *eqvts*)
qed
with *PFrR QFrR PRAlpha QRAlpha* **show** *?case* **by** *simp*
next

```
    case(cSym Ψ PR QR)
    thus ?case by(blast dest: bisimE)
  qed
qed

lemma bisimParPres:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   R :: ('a, 'b, 'c) psi

  assumes Ψ ▷ P ∼ Q

  shows Ψ ▷ P ∥ R ∼ Q ∥ R
proof −
  obtain A_R Ψ_R where extractFrame R = ⟨A_R, Ψ_R⟩ and A_R ♯* Ψ and A_R ♯* P
and A_R ♯* Q
    by(rule-tac C=(Ψ, P, Q) in freshFrame) auto
  moreover from ‹Ψ ▷ P ∼ Q› have Ψ ⊗ Ψ_R ▷ P ∼ Q by(rule bisimE)
  ultimately show ?thesis by(rule-tac bisimParPresAux)
qed

end

end

theory Sim-Struct-Cong
  imports Simulation
begin

lemma partitionListLeft:
  assumes xs@ys=xs'@y#ys'
  and     y mem xs
  and     distinct(xs@ys)

  obtains zs where xs = xs'@y#zs and ys'=zs@ys
using assms
by(auto simp add: append-eq-append-conv2 append-eq-Cons-conv)

lemma partitionListRight:
  assumes xs@ys=xs'@y#ys'
  and     y mem ys
  and     distinct(xs@ys)

  obtains zs where xs' = xs@zs and ys=zs@y#ys'
using assms
by(force simp add: append-eq-append-conv2 append-eq-Cons-conv)

context env begin
```

lemma *resComm*:
  fixes Ψ :: *'b*
  and *x* :: *name*
  and *y* :: *name*
  and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
  and *P* :: (*'a*, *'b*, *'c*) *psi*

  assumes $x \sharp \Psi$
  and $y \sharp \Psi$
  and *eqvt Rel*
  and *R1*: $\bigwedge \Psi' Q.\ (\Psi', Q, Q) \in Rel$
  and *R2*: $\bigwedge \Psi' a b Q.\ [\![a \sharp \Psi'; b \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu a|\!)((\!|\nu b|\!) Q), (\!|\nu b|\!)((\!|\nu a|\!) Q)) \in Rel$

  shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!) P) \leadsto [Rel]\ (\!|\nu y|\!)((\!|\nu x|\!) P)$
proof(*case-tac* $x{=}y$)
  assume $x = y$
  thus *?thesis* **using** *R1*
    by(*force intro*: *reflexive*)
next
  assume $x \neq y$
  note ‹*eqvt Rel*›
  moreover from ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $[x, y] \sharp* \Psi$ **by**(*simp add*: *fresh-star-def*)
  moreover have $[x, y] \sharp* (\!|\nu x|\!)((\!|\nu y|\!) P)$ **by**(*simp add*: *abs-fresh*)
  moreover have $[x, y] \sharp* (\!|\nu y|\!)((\!|\nu x|\!) P)$ **by**(*simp add*: *abs-fresh*)
  ultimately show *?thesis*
  proof(*induct rule*: *simIChainFresh*[**where** $C{=}(x, y)$])
    case(*cSim* α P')
    from ‹$bn\ \alpha \sharp* (x, y)$› ‹$bn\ \alpha \sharp* ((\!|\nu x|\!)((\!|\nu y|\!) P))$› **have** $x \sharp bn\ \alpha$ **and** $y \sharp bn\ \alpha$ **and** $bn\ \alpha \sharp* P$ **by** *simp+*
    from ‹$[x, y] \sharp* \alpha$› **have** $x \sharp \alpha$ **and** $y \sharp \alpha$ **by** *simp+*
    from ‹$[x, y] \sharp* P'$› **have** $x \sharp P'$ **and** $y \sharp P'$ **by** *simp+*
    from ‹$bn\ \alpha \sharp* P$› ‹$x \sharp \alpha$› **have** $bn\ \alpha \sharp* (\!|\nu x|\!) P$ **by**(*simp add*: *abs-fresh*)
    with ‹$\Psi \rhd (\!|\nu y|\!)((\!|\nu x|\!) P) \longmapsto \alpha \prec P'$› ‹$y \sharp \Psi$› ‹$y \sharp \alpha$› ‹$y \sharp P'$› ‹$bn\ \alpha \sharp* \Psi$›
    show *?case* **using** ‹$bn\ \alpha \sharp*$ *subject* α› ‹$x \sharp \alpha$› ‹$x \sharp P'$› ‹$bn\ \alpha \sharp* \Psi$› ‹$bn\ \alpha \sharp* P$› ‹$bn\ \alpha \sharp*$ *subject* α› ‹$y \sharp \alpha$›
    proof(*induct rule*: *resCases'*)
      case(*cOpen M yvec1 yvec2 y' N P'*)
    from ‹*yvec1* $\sharp*$ *yvec2*› ‹*distinct yvec1*› ‹*distinct yvec2*› **have** *distinct*(*yvec1*@*yvec2*) **by** *auto*
      from ‹$x \sharp M(\!|\nu*(yvec1\ @\ y'\ \#\ yvec2)|\!)\langle N\rangle$› **have** $x \sharp M$ **and** $x \sharp yvec1$ **and** $x \neq y'$ **and** $x \sharp yvec2$ **and** $x \sharp N$
        by *simp+*
      from ‹$y \sharp M(\!|\nu*(yvec1\ @\ y'\ \#\ yvec2)|\!)\langle N\rangle$› **have** $y \sharp M$ **and** $y \sharp yvec1$ **and** $y \sharp yvec2$
        by *simp+*
      from ‹$\Psi \rhd ([(y, y')] \cdot (\!|\nu x|\!) P) \longmapsto M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle \prec P'$› ‹$x \neq y$› ‹$x \neq y'$›

have $\Psi \rhd (\!|\nu x|\!)([(y,\ y')] \cdot P) \longmapsto M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle \prec P'$ **by**(*simp add*: *eqvts*)
moreover note ‹$x \sharp \Psi$›
moreover from ‹$x \sharp N$› ‹$x \sharp yvec1$› ‹$x \sharp yvec2$› ‹$x \sharp M$› **have** $x \sharp M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle$ **by** *simp*
moreover note ‹$x \sharp P'$›
moreover from ‹$yvec1 \sharp* \Psi$› ‹$yvec2 \sharp* \Psi$› **have** $bn(M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle) \sharp* \Psi$ **by** *simp*
moreover from ‹$yvec1 \sharp* (\!|\nu x|\!)P$› ‹$yvec2 \sharp* (\!|\nu x|\!)P$› ‹$y \sharp yvec1$› ‹$y' \sharp yvec1$› ‹$y \sharp yvec2$› ‹$y' \sharp yvec2$› ‹$x \sharp yvec1$› ‹$x \sharp yvec2$›
have $bn(M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle) \sharp* ([(y,\ y')] \cdot P)$ **by** *simp*
moreover from ‹$yvec1 \sharp* M$› ‹$yvec2 \sharp* M$› **have** $bn(M(\!|\nu*(yvec1\ @\ yvec2)|\!)\langle N\rangle) \sharp* subject(M(\!|\nu*(yvec1\ @\ yvec2)|\!)\langle N\rangle)$
by *simp*
moreover have $bn(M(\!|\nu*(yvec1\ @\ yvec2)|\!)\langle N\rangle) = yvec1@yvec2$ **by** *simp*
moreover have $subject(M(\!|\nu*(yvec1\ @\ yvec2)|\!)\langle N\rangle) = Some\ M$ **by** *simp*
moreover have $object(M(\!|\nu*(yvec1\ @\ yvec2)|\!)\langle N\rangle) = Some\ N$ **by** *simp*
ultimately show *?case*
proof(*induct rule*: *resCases'*)
case(*cOpen M' xvec1 xvec2 x' N' P'*)
from ‹$bn(M'(\!|\nu*(xvec1\ @\ x'\ \#\ xvec2)|\!)\langle N'\rangle) = yvec1\ @\ yvec2$› **have** $yvec1@yvec2 = xvec1@x'\#xvec2$ **by** *simp*
from ‹$subject(M'(\!|\nu*(xvec1\ @\ x'\ \#\ xvec2)|\!)\langle N'\rangle) = Some\ M$› **have** $M = M'$ **by** *simp*
from ‹$object(M'(\!|\nu*(xvec1\ @\ x'\ \#\ xvec2)|\!)\langle N'\rangle) = Some\ N$› **have** $N = N'$ **by** *simp*
from ‹$x \sharp yvec1$› ‹$x \sharp yvec2$› ‹$y' \sharp yvec1$› ‹$y' \sharp yvec2$› ‹$y \sharp yvec1$› ‹$y \sharp yvec2$›
have $x \sharp (yvec1@yvec2)$ **and** $y \sharp (yvec1@yvec2)$ **and** $y' \sharp (yvec1@yvec2)$ **by** *simp+*
with ‹$yvec1@yvec2 = xvec1@x'\#xvec2$›
have $x \sharp xvec1$ **and** $x \neq x'$ **and** $x \sharp xvec2$ **and** $y \sharp xvec1$ **and** $y \neq x'$ **and** $y \sharp xvec2$
and $y' \sharp xvec1$ **and** $x' \neq y'$ **and** $y' \sharp xvec2$
by *auto*

show *?case*
proof(*case-tac x' mem yvec1*)
assume x' *mem yvec1*

with ‹$yvec1@yvec2 = xvec1@x'\#xvec2$› ‹*distinct* $(yvec1@yvec2)$›
obtain *xvec2'* **where** *Eq1*: $yvec1=xvec1@x'\#xvec2'$
and *Eq*: $xvec2=xvec2'@yvec2$
by(*rule-tac partitionListLeft*)
from ‹$\Psi \rhd ([(x,\ x')] \cdot [(y,\ y')] \cdot P) \longmapsto M'(\!|\nu*(xvec1@xvec2)|\!)\langle N'\rangle \prec P'$› ‹$y' \in supp\ N$› ‹$y' \sharp \Psi$› ‹$y' \sharp M$› ‹$y' \sharp xvec1$› ‹$y' \sharp xvec2$› *Eq* ‹$M=M'$› ‹$N = N'$›
have $\Psi \rhd (\!|\nu y'|\!)([(x,\ x')] \cdot [(y,\ y')] \cdot P) \longmapsto M'(\!|\nu*((xvec1@xvec2')@y'\#yvec2)|\!)\langle N'\rangle \prec P'$
by(*rule-tac Open*) *auto*
then have $\Psi \rhd (\!|\nu x'|\!)((\!|\nu y'|\!)([(x,\ x')] \cdot [(y,\ y')] \cdot P)) \longmapsto M(\!|\nu*(xvec1@x'\#xvec2'@y'\#yvec2)|\!)\langle N\rangle$

$\prec P'$
using ‹$x' \in$ *supp* N'› ‹$x' \sharp \Psi$› ‹$x' \sharp M'$› ‹$x' \sharp$ *xvec1*› ‹$x' \sharp$ *xvec2*› ‹$x' \neq y'$› *Eq* ‹$M{=}M'$› ‹$N{=}N'$›
by(*rule-tac Open*) *auto*
with ‹$x' \neq y'$› ‹$x \neq y'$› ‹$x' \sharp [(y, y')] \cdot P$›
have $\Psi \rhd (\!|\nu x|\!)((\!|\nu y'|\!)([(y, y')] \cdot P)) \longmapsto M(\!|\nu*(xvec1@x'\#xvec2'@y'\#yvec2)|\!)\langle N\rangle \prec P'$
by(*subst alphaRes*[**where** $y{=}x'$]) (*simp add*: *calc-atm eqvts abs-fresh*)+
with *Eq1* ‹$y' \sharp (\!|\nu x|\!)P$› ‹$x \neq y'$› *R1* **show** *?case*
by(*fastforce simp add*: *alphaRes abs-fresh*)
next
assume $\neg x'$ *mem yvec1*
hence $x' \sharp$ *yvec1* **by**(*simp add*: *fresh-def*)
from ‹$\neg x'$ *mem yvec1*› ‹*yvec1*@*yvec2* = *xvec1*@x'\#*xvec2*›
have x' *mem yvec2*
by(*fastforce simp add*: *append-eq-append-conv2 append-eq-Cons-conv fresh-list-append fresh-list-cons*)
with ‹*yvec1*@*yvec2* = *xvec1*@x'\#*xvec2*› ‹*distinct* (*yvec1*@*yvec2*)›
obtain *xvec2'* **where** *Eq*: *xvec1*=*yvec1*@*xvec2'* **and** *Eq1*: *yvec2*=*xvec2'*@x'\#*xvec2*
by(*rule-tac partitionListRight*)
from ‹$\Psi \rhd ([(x, x')] \cdot [(y, y')] \cdot P) \longmapsto M'(\!|\nu*(xvec1@xvec2)|\!)\langle N'\rangle \prec P'$› ‹$y' \in$ *supp* N› ‹$y' \sharp \Psi$› ‹$y' \sharp M$› ‹$y' \sharp$ *xvec1*› ‹$y' \sharp$ *xvec2*› *Eq* ‹$M{=}M'$› ‹$N = N'$›
have $\Psi \rhd (\!|\nu y'|\!)([(x, x')] \cdot [(y, y')] \cdot P) \longmapsto M'(\!|\nu*(yvec1@y'\#xvec2'@xvec2)|\!)\langle N'\rangle \prec P'$
by(*rule-tac Open*) (*assumption* | *simp*)+
then have $\Psi \rhd (\!|\nu x'|\!)((\!|\nu y'|\!)([(x, x')] \cdot [(y, y')] \cdot P)) \longmapsto M(\!|\nu*((yvec1@y'\#xvec2')@x'\#xvec2)|\!)\langle N\rangle \prec P'$
using ‹$x' \in$ *supp* N'› ‹$x' \sharp \Psi$› ‹$x' \sharp M'$› ‹$x' \sharp$ *xvec1*› ‹$x' \sharp$ *xvec2*› ‹$x' \neq y'$› *Eq* ‹$M{=}M'$› ‹$N{=}N'$›
by(*rule-tac Open*) *auto*
with ‹$x' \neq y'$› ‹$x \neq y'$› ‹$x' \sharp [(y, y')] \cdot P$›
have $\Psi \rhd (\!|\nu x|\!)((\!|\nu y'|\!)([(y, y')] \cdot P)) \longmapsto M(\!|\nu*((yvec1@y'\#xvec2')@x'\#xvec2)|\!)\langle N\rangle \prec P'$
by(*subst alphaRes*[**where** $y{=}x'$]) (*simp add*: *calc-atm eqvts abs-fresh*)+
with *Eq1* ‹$y' \sharp (\!|\nu x|\!)P$› ‹$x \neq y'$› *R1* **show** *?case*
by(*fastforce simp add*: *alphaRes abs-fresh*)
qed
next
case(*cRes P'*)
from ‹$\Psi \rhd ([(y, y')] \cdot P) \longmapsto M(\!|\nu*(yvec1@yvec2)|\!)\langle N\rangle \prec P'$› ‹$y' \in$ *supp* N› ‹$y' \sharp \Psi$› ‹$y' \sharp M$› ‹$y' \sharp$ *yvec1*› ‹$y' \sharp$ *yvec2*›
have $\Psi \rhd (\!|\nu y'|\!)([(y, y')] \cdot P) \longmapsto M(\!|\nu*(yvec1@y'\#yvec2)|\!)\langle N\rangle \prec P'$ **by**(*rule Open*)
with ‹$y' \sharp (\!|\nu x|\!)P$› ‹$x \neq y'$›**have** $\Psi \rhd (\!|\nu y|\!)P \longmapsto M(\!|\nu*(yvec1@y'\#yvec2)|\!)\langle N\rangle \prec P'$ **by**(*simp add*: *alphaRes abs-fresh*)
hence $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \longmapsto M(\!|\nu*(yvec1@y'\#yvec2)|\!)\langle N\rangle \prec (\!|\nu x|\!)P'$ **using** ‹$x \sharp \Psi$› ‹$x \sharp M$› ‹$x \sharp$ *yvec1*› ‹$x \neq y'$› ‹$x \sharp$ *yvec2*› ‹$x \sharp N$›
by(*rule-tac Scope*) *auto*

```
      moreover have (Ψ, (|νx|)P', (|νx|)P') ∈ Rel by(rule R1)
      ultimately show ?case by blast
    qed
  next
    case(cRes P')
    from ‹x ♯ (|νy|)P'› ‹x ≠ y› have x ♯ P' by(simp add: abs-fresh)
    with ‹Ψ ▷ (|νx|)P ⟼α ≺ P'› ‹x ♯ Ψ› ‹x ♯ α›
    show ?case using ‹bn α ♯* Ψ› ‹bn α ♯* P› ‹bn α ♯* subject α› ‹y ♯ α›
    proof(induct rule: resCases')
      case(cOpen M xvec1 xvec2 x' N P')
    from ‹y ♯ M(|ν*(xvec1@x'#xvec2)|)⟨N⟩› have y ≠ x' and y ♯ M(|ν*(xvec1@xvec2)|)⟨N⟩
by simp+
      from ‹Ψ ▷ ([(x, x')] • P) ⟼M(|ν*(xvec1@xvec2)|)⟨N⟩ ≺ P'› ‹y ♯ Ψ› ‹y ♯
M(|ν*(xvec1@xvec2)|)⟨N⟩›
      have Ψ ▷ (|νy|)([(x, x')] • P) ⟼M(|ν*(xvec1@xvec2)|)⟨N⟩ ≺ (|νy|)P'
        by(rule Scope)
       hence Ψ ▷ (|νx'|)((|νy|)([(x, x')] • P)) ⟼M(|ν*(xvec1@x'#xvec2)|)⟨N⟩ ≺
(|νy|)P'
        using ‹x' ∈ supp N› ‹x' ♯ Ψ› ‹x' ♯ M› ‹x' ♯ xvec1› ‹x' ♯ xvec2›
        by(rule Open)
    with ‹y ≠ x'› ‹x ≠ y› ‹x' ♯ P› have Ψ ▷ (|νx|)((|νy|)P) ⟼M(|ν*(xvec1@x'#xvec2)|)⟨N⟩
≺ (|νy|)P'
      by(subst alphaRes[where y=x']) (simp add: abs-fresh eqvts calc-atm)+
      moreover have (Ψ, (|νy|)P', (|νy|)P') ∈ Rel by(rule R1)
      ultimately show ?case by blast
    next
      case(cRes P')
      from ‹Ψ ▷ P ⟼α ≺ P'› ‹y ♯ Ψ› ‹y ♯ α›
      have Ψ ▷ (|νy|)P ⟼α ≺ (|νy|)P' by(rule Scope)
      hence Ψ ▷ (|νx|)((|νy|)P) ⟼α ≺ (|νx|)((|νy|)P') using ‹x ♯ Ψ› ‹x ♯ α›
        by(rule Scope)
      moreover from ‹x ♯ Ψ› ‹y ♯ Ψ› have (Ψ, (|νx|)((|νy|)P'), (|νy|)((|νx|)P')) ∈
Rel
        by(rule R2)
      ultimately show ?case by blast
    qed
  qed
qed

lemma parAssocLeft:
  fixes Ψ  :: 'b
  and   P  :: ('a, 'b, 'c) psi
  and   Q  :: ('a, 'b, 'c) psi
  and   R  :: ('a, 'b, 'c) psi
  and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  assumes eqvt Rel
  and     C1: ⋀Ψ' S T U. (Ψ, (S ∥ T) ∥ U, S ∥ (T ∥ U)) ∈ Rel
```

and *C2*: $\bigwedge xvec\ \Psi'\ S\ T\ U.\ [\![xvec\ \sharp*\ \Psi';\ xvec\ \sharp*\ S]\!] \Longrightarrow (\Psi',\ (\!|\nu{*}xvec|\!)((S \parallel T) \parallel U),\ S \parallel ((\!|\nu{*}xvec|\!)(T \parallel U))) \in Rel$
and *C3*: $\bigwedge xvec\ \Psi'\ S\ T\ U.\ [\![xvec\ \sharp*\ \Psi';\ xvec\ \sharp*\ U]\!] \Longrightarrow (\Psi',\ ((\!|\nu{*}xvec|\!)(S \parallel T)) \parallel U,\ (\!|\nu{*}xvec|\!)(S \parallel (T \parallel U))) \in Rel$
and *C4*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi',\ S,\ T) \in Rel;\ xvec\ \sharp*\ \Psi']\!] \Longrightarrow (\Psi',\ (\!|\nu{*}xvec|\!)S,\ (\!|\nu{*}xvec|\!)\,T) \in Rel$

shows $\Psi \rhd (P \parallel Q) \parallel R \rightsquigarrow[Rel]\ P \parallel (Q \parallel R)$
using ‹*eqvt Rel*›
proof(*induct rule*: *simI*[*of* - - - - ()])
case(*cSim* α *PQR*)
from ‹$bn\ \alpha\ \sharp*\ (P \parallel Q \parallel R)$› **have** $bn\ \alpha\ \sharp*\ P$ **and** $bn\ \alpha\ \sharp*\ Q$ **and** $bn\ \alpha\ \sharp*\ R$ **by** *simp+*
hence $bn\ \alpha\ \sharp*\ (Q \parallel R)$ **by** *simp*
with ‹$\Psi \rhd P \parallel (Q \parallel R) \longmapsto \alpha \prec PQR$› ‹$bn\ \alpha\ \sharp*\ \Psi$› ‹$bn\ \alpha\ \sharp*\ P$›
show *?case* **using** ‹$bn\ \alpha\ \sharp*\ subject\ \alpha$›
proof(*induct rule*: *parCases*[**where** $C = (\Psi,\ P,\ Q,\ R,\ \alpha)$])
case(*cPar1* $P'\ A_{QR}\ \Psi_{QR}$)
from ‹$A_{QR}\ \sharp*\ (\Psi,\ P,\ Q,\ R,\ \alpha)$› **have** $A_{QR}\ \sharp*\ Q$ **and** $A_{QR}\ \sharp*\ R$
by *simp+*
with ‹*extractFrame*$(Q \parallel R) = \langle A_{QR},\ \Psi_{QR}\rangle$› ‹*distinct* A_{QR}›
obtain $A_Q\ \Psi_Q\ A_R\ \Psi_R$ **where** $A_{QR} = A_Q@A_R$ **and** $\Psi_{QR} = \Psi_Q \otimes \Psi_R$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q,\ \Psi_Q\rangle$ **and** *FrR*: *extractFrame* $R = \langle A_R,\ \Psi_R\rangle$
and $A_Q\ \sharp*\ \Psi_R$ **and** $A_R\ \sharp*\ \Psi_Q$
by(*rule-tac mergeFrameE*) (*auto dest*: *extractFrameFreshChain*)

from ‹$A_{QR} = A_Q@A_R$› ‹$A_{QR}\ \sharp*\ \Psi$› ‹$A_{QR}\ \sharp*\ P$› ‹$A_{QR}\ \sharp*\ Q$› ‹$A_{QR}\ \sharp*\ \alpha$›
have $A_Q\ \sharp*\ \Psi$ **and** $A_R\ \sharp*\ \Psi$ **and** $A_Q\ \sharp*\ P$ **and** $A_R\ \sharp*\ P$ **and** $A_Q\ \sharp*\ Q$ **and** $A_R\ \sharp*\ Q$ **and** $A_Q\ \sharp*\ \alpha$ **and** $A_R\ \sharp*\ \alpha$
by *simp+*

from ‹$\Psi \otimes \Psi_{QR} \rhd P \longmapsto \alpha \prec P'$› ‹$\Psi_{QR} = \Psi_Q \otimes \Psi_R$› **have** $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$
by(*metis statEqTransition Associativity Commutativity Composition*)
hence $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$ **using** *FrQ* ‹$bn\ \alpha\ \sharp*\ Q$› ‹$A_Q\ \sharp*\ \Psi$› ‹$A_Q\ \sharp*\ \Psi_R$› ‹$A_Q\ \sharp*\ P$› ‹$A_Q\ \sharp*\ \alpha$›
by(*rule-tac Par1*) *auto*
hence $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \alpha \prec ((P' \parallel Q) \parallel R)$ **using** *FrR* ‹$bn\ \alpha\ \sharp*\ R$› ‹$A_R\ \sharp*\ \Psi$› ‹$A_R\ \sharp*\ P$› ‹$A_R\ \sharp*\ Q$› ‹$A_R\ \sharp*\ \alpha$›
by(*rule-tac Par1*) *auto*
moreover have $(\Psi,\ (P' \parallel Q) \parallel R,\ P' \parallel (Q \parallel R)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 QR* $A_P\ \Psi_P$)
from ‹$A_P\ \sharp*\ (\Psi,\ P,\ Q,\ R,\ \alpha)$› **have** $A_P\ \sharp*\ Q$ **and** $A_P\ \sharp*\ R$ **and** $A_P\ \sharp*\ \alpha$
by *simp+*
have *FrP*: *extractFrame* $P = \langle A_P,\ \Psi_P\rangle$ **by** *fact*
with ‹$bn\ \alpha\ \sharp*\ P$› ‹$A_P\ \sharp*\ \alpha$› **have** $bn\ \alpha\ \sharp*\ \Psi_P$ **by**(*auto dest*: *extractFrameFreshChain*)

with ‹$bn\ \alpha\ \sharp*\ \Psi$› **have** $bn\ \alpha\ \sharp*\ (\Psi \otimes \Psi_P)$ **by** *force*
with ‹$\Psi \otimes \Psi_P \rhd Q \parallel R \longmapsto \alpha \prec QR$›
show *?case* **using** ‹$bn\ \alpha\ \sharp*\ Q$› ‹$bn\ \alpha\ \sharp*\ R$› ‹$bn\ \alpha\ \sharp*$ *subject* α› ‹$A_P\ \sharp*\ Q$› ‹$A_P\ \sharp*\ R$›
proof(*induct rule*: *parCasesSubject*[**where** $C = (A_P, \Psi_P, P, Q, R, \Psi)$])
case(*cPar1* $Q'\ A_R\ \Psi_R$)
from ‹$A_R\ \sharp*\ (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $A_R\ \sharp*\ A_P$ **and** $A_R\ \sharp*\ P$ **and** $A_R\ \sharp*\ Q$ **and** $A_R\ \sharp*\ \Psi_P$ **and** $A_R\ \sharp*\ \Psi$
by *simp+*
from ‹$A_P\ \sharp*\ R$› ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› ‹$A_R\ \sharp*\ A_P$› **have** $A_P\ \sharp*\ \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*
from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_R \rhd Q \longmapsto \alpha \prec Q'$› **have** $(\Psi \otimes \Psi_R) \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$
by(*metis statEqTransition Associativity Commutativity Composition*)
hence $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$
using *FrP* ‹$bn\ \alpha\ \sharp*\ P$› ‹$A_P\ \sharp*\ \Psi$› ‹$A_P\ \sharp*\ \Psi_R$› ‹$A_P\ \sharp*\ Q$› ‹$A_P\ \sharp*\ \alpha$›
by(*rule-tac Par2*) (*assumption* | *force*)+
hence $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \alpha \prec ((P \parallel Q') \parallel R)$
using ‹*extractFrame* $R = \langle A_R, \Psi_R\rangle$› ‹$bn\ \alpha\ \sharp*\ R$› ‹$A_R\ \sharp*\ \Psi$› ‹$A_R\ \sharp*\ P$› ‹$A_R\ \sharp*\ Q$› ‹$A_R\ \sharp*\ \alpha$›
by(*rule-tac Par1*) (*assumption* | *simp*)+
moreover have $(\Psi, (P \parallel Q') \parallel R, P \parallel (Q' \parallel R)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2* $R'\ A_Q\ \Psi_Q$)
from ‹$A_Q\ \sharp*\ (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $A_Q\ \sharp*\ A_P$ **and** $A_Q\ \sharp*\ R$ **and** $A_Q\ \sharp*\ \Psi_P$ **and** $A_Q\ \sharp*\ \Psi$
by *simp+*
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from ‹$A_P\ \sharp*\ Q$› *FrQ* ‹$A_Q\ \sharp*\ A_P$› **have** $A_P\ \sharp*\ \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$›
have $\Psi \otimes (\Psi_P \otimes \Psi_Q) \rhd R \longmapsto \alpha \prec R'$
by(*blast intro*: *statEqTransition Associativity*)
moreover from *FrP FrQ* ‹$A_Q\ \sharp*\ A_P$› ‹$A_P\ \sharp*\ \Psi_Q$› ‹$A_Q\ \sharp*\ \Psi_P$›
have *extractFrame*$(P \parallel Q) = \langle (A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle$ **by** *simp*
moreover from ‹$bn\ \alpha\ \sharp*\ P$› ‹$bn\ \alpha\ \sharp*\ Q$› **have** $bn\ \alpha\ \sharp*\ (P \parallel Q)$ **by** *simp*
moreover from ‹$A_P\ \sharp*\ \Psi$› ‹$A_Q\ \sharp*\ \Psi$› **have** $(A_P@A_Q)\ \sharp*\ \Psi$ **by** *simp*
moreover from ‹$A_P\ \sharp*\ R$› ‹$A_Q\ \sharp*\ R$› **have** $(A_P@A_Q)\ \sharp*\ R$ **by** *simp*
moreover from ‹$A_P\ \sharp*\ \alpha$› ‹$A_Q\ \sharp*\ \alpha$› **have** $(A_P@A_Q)\ \sharp*\ \alpha$ **by** *simp*
ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \alpha \prec ((P \parallel Q) \parallel R')$
by(*rule Par2*)
moreover have $(\Psi, (P \parallel Q) \parallel R', P \parallel (Q \parallel R')) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* $\Psi_R\ M\ N\ Q'\ A_Q\ \Psi_Q\ K\ xvec\ R'\ A_R$)
from ‹$A_Q\ \sharp*\ (A_P, \Psi_P, P, Q, R, \Psi)$›
have $A_Q\ \sharp*\ P$ **and** $A_Q\ \sharp*\ Q$ **and** $A_Q\ \sharp*\ R$ **and** $A_Q\ \sharp*\ A_P$ **and** $A_Q\ \sharp*\ \Psi_P$ **and** $A_Q\ \sharp*\ \Psi$ **by** *simp+*

from ‹$A_R \sharp* (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **and** $A_R \sharp* A_P$ **and** $A_R \sharp* \Psi$ **by** *simp+*

from ‹$xvec \sharp* (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $xvec \sharp* A_P$ **and** $xvec \sharp* P$ **and** $xvec \sharp* Q$ **and** $xvec \sharp* \Psi$ **by** *simp+*

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*

with ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› **have** $A_P \sharp* \Psi_Q$

by(*drule-tac extractFrameFreshChain*) *auto*

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact*

with ‹$A_P \sharp* R$› ‹$A_R \sharp* A_P$› **have** $A_P \sharp* \Psi_R$

by(*drule-tac extractFrameFreshChain*) *auto*

from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec R'$› ‹$A_P \sharp* R$› ‹$xvec \sharp* A_P$› ‹$xvec \sharp* K$› ‹*distinct xvec*› **have** $A_P \sharp* N$

by(*rule-tac outputFreshChainDerivative*) *auto*

from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_R \rhd Q \longmapsto M(\!|N|\!) \prec Q'$› **have** $(\Psi \otimes \Psi_R) \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'$

by(*metis statEqTransition Associativity Commutativity Composition*)

hence $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto M(\!|N|\!) \prec (P \parallel Q')$ **using** *FrP* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_R$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* N$›

by(*rule-tac Par2*) *auto*

moreover from *FrP FrQ* ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* A_P$› ‹$A_Q \sharp* \Psi_P$› **have** *extractFrame*$(P \parallel Q) = \langle (A_P @ A_Q), \Psi_P \otimes \Psi_Q \rangle$

by *simp*

moreover from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec R'$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec R'$

by(*metis statEqTransition Associativity*)

moreover note ‹*extractFrame* $R = \langle A_R, \Psi_R \rangle$›

moreover from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$› **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \otimes \Psi_R \vdash M \leftrightarrow K$

by(*metis statEqEnt Associativity Commutativity Composition*)

ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \tau \prec (\!|\nu*xvec|\!)((P \parallel Q') \parallel R')$

using ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_R \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_R \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_R \sharp* Q$› ‹$A_P \sharp* R$› ‹$A_Q \sharp* R$› ‹$A_R \sharp* R$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* M$› ‹$A_R \sharp* K$› ‹$A_R \sharp* A_P$› ‹$A_Q \sharp* A_R$› ‹$xvec \sharp* P$› ‹$xvec \sharp* Q$›

by(*rule-tac Comm1*) (*assumption* | *simp*)+

moreover from ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* P$› **have** $(\Psi, (\!|\nu*xvec|\!)((P \parallel Q') \parallel R'), P \parallel ((\!|\nu*xvec|\!)(Q' \parallel R'))) \in Rel$

by(*rule C2*)

ultimately show *?case* **by** *blast*

next

case(*cComm2* $\Psi_R\ M\ xvec\ N\ Q'\ A_Q\ \Psi_Q\ K\ R'\ A_R$)

from ‹$A_Q \sharp* (A_P, \Psi_P, P, Q, R, \Psi)$›

have $A_Q \sharp* P$ **and** $A_Q \sharp* Q$ **and** $A_Q \sharp* R$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* \Psi_P$ **by** *simp+*

from ‹$A_R \sharp* (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **and** $A_R \sharp* A_P$**and** $A_R \sharp* \Psi$ **by** *simp+*

from ‹$xvec \sharp* (A_P, \Psi_P, P, Q, R, \Psi)$› **have** $xvec \sharp* A_P$ **and** $xvec \sharp* P$ **and**

xvec $\sharp*$ *Q* **and** *xvec* $\sharp*$ Ψ **by** *simp+*

have *FrQ*: *extractFrame Q* = $\langle A_Q, \Psi_Q\rangle$ **by** *fact*
with ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› **have** $A_P \sharp* \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
have *FrR*: *extractFrame R* = $\langle A_R, \Psi_R\rangle$ **by** *fact*
with ‹$A_P \sharp* R$› ‹$A_R \sharp* A_P$› **have** $A_P \sharp* \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*

from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$› ‹$A_P \sharp* Q$› ‹*xvec* $\sharp*$ A_P› ‹*xvec* $\sharp*$ *M*› ‹*distinct xvec*› **have** $A_P \sharp* N$
by(*rule-tac outputFreshChainDerivative*) *auto*

from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$› **have** $(\Psi \otimes \Psi_R) \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$
by(*metis statEqTransition Associativity Commutativity Composition*)
hence $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec (P \parallel Q')$ **using** *FrP* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_R$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* N$› ‹*xvec* $\sharp*$ *P*› ‹*xvec* $\sharp*$ A_P›
by(*rule-tac Par2*) *auto*
moreover from *FrP FrQ* ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* A_P$› ‹$A_Q \sharp* \Psi_P$› **have** *extractFrame*$(P \parallel Q) = \langle (A_P @ A_Q), \Psi_P \otimes \Psi_Q\rangle$
by *simp+*
moreover from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$
by(*metis statEqTransition Associativity*)
moreover note ‹*extractFrame R* = $\langle A_R, \Psi_R\rangle$›
moreover from ‹$(\Psi \otimes \Psi_P) \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$› **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \otimes \Psi_R \vdash M \leftrightarrow K$
by(*metis statEqEnt Associativity Commutativity Composition*)
ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \tau \prec (\!|\nu*xvec|\!)((P \parallel Q') \parallel R')$
using ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_R \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_R \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_R \sharp* Q$› ‹$A_P \sharp* R$› ‹$A_Q \sharp* R$› ‹$A_R \sharp* R$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* M$› ‹$A_R \sharp* K$› ‹$A_R \sharp* A_P$› ‹$A_Q \sharp* A_R$› ‹*xvec* $\sharp*$ *R*›
by(*rule-tac Comm2*) (*assumption* | *simp*)+
moreover from ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *P*› **have** $(\Psi, (\!|\nu*xvec|\!)((P \parallel Q') \parallel R'), P \parallel ((\!|\nu*xvec|\!)(Q' \parallel R'))) \in Rel$
by(*rule C2*)
ultimately show *?case* **by** *blast*
qed
next
case(*cComm1* Ψ_{QR} *M N P'* A_P Ψ_P *K xvec QR'* A_{QR})
from ‹*xvec* $\sharp*$ (Ψ, P, Q, R, α)› **have** *xvec* $\sharp*$ *Q* **and** *xvec* $\sharp*$ *R* **by** *simp+*
from ‹$A_{QR} \sharp* (\Psi, P, Q, R, \alpha)$› **have** $A_{QR} \sharp* Q$ **and** $A_{QR} \sharp* R$ **and** $A_{QR} \sharp* \Psi$ **by** *simp+*
from ‹$A_P \sharp* (Q \parallel R)$› **have** $A_P \sharp* Q$ **and** $A_P \sharp* R$ **by** *simp+*
have *PTrans*: $\Psi \otimes \Psi_{QR} \rhd P \longmapsto M(\!|N|\!) \prec P'$ **and** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P\rangle$ **and** *MeqK*: $\Psi \otimes \Psi_P \otimes \Psi_{QR} \vdash M \leftrightarrow K$ **by** *fact+*
note ‹$\Psi \otimes \Psi_P \rhd Q \parallel R \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec QR'$›
moreover from ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ Ψ_P› **have** *xvec* $\sharp*$ $(\Psi \otimes \Psi_P)$ **by** *force*

moreover note ‹*xvec* ♯* *Q*›‹*xvec* ♯* *R*› ‹*xvec* ♯* *K*›
‹*extractFrame*(*Q* ∥ *R*) = ⟨*A*$_{QR}$, Ψ$_{QR}$⟩› ‹*distinct A*$_{QR}$›
moreover from ‹*A*$_{QR}$ ♯* Ψ› ‹*A*$_{QR}$ ♯* Ψ$_P$› **have** *A*$_{QR}$ ♯* (Ψ ⊗ Ψ$_P$) **by** *force*
ultimately show *?case* **using** ‹*A*$_{QR}$ ♯* *Q*› ‹*A*$_{QR}$ ♯* *R*› ‹*A*$_{QR}$ ♯* *K*›
proof(*induct rule*: *parCasesOutputFrame*)
case(*cPar1 Q*′ *A*$_Q$ Ψ$_Q$ *A*$_R$ Ψ$_R$)
have *Aeq*: *A*$_{QR}$ = *A*$_Q$@*A*$_R$ **and** Ψ*eq*: Ψ$_{QR}$ = Ψ$_Q$ ⊗ Ψ$_R$ **by** *fact*+
from *PTrans* Ψ*eq* **have** (Ψ ⊗ Ψ$_R$) ⊗ Ψ$_Q$ ▷ *P* ⟼*M*(⦇*N*⦈) ≺ *P*′
by(*metis statEqTransition Associativity Commutativity Composition*)
moreover note *FrP*
moreover from ‹(Ψ ⊗ Ψ$_P$) ⊗ Ψ$_R$ ▷ *Q* ⟼*K*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *Q*′› **have** (Ψ ⊗ Ψ$_R$) ⊗ Ψ$_P$ ▷ *Q* ⟼*K*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *Q*′
by(*metis statEqTransition Associativity Commutativity Composition*)
moreover note ‹*extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩›
moreover from *MeqK* Ψ*eq* **have** (Ψ ⊗ Ψ$_R$) ⊗ Ψ$_P$ ⊗ Ψ$_Q$ ⊢ *M* ↔ *K*
by(*metis statEqEnt Associativity Commutativity Composition*)
moreover from ‹*A*$_P$ ♯* *R*› ‹*A*$_P$ ♯* *A*$_{QR}$› *Aeq* ‹*extractFrame R* = ⟨*A*$_R$, Ψ$_R$⟩› **have** *A*$_P$ ♯* *A*$_Q$ **and** *A*$_P$ ♯* Ψ$_R$
by(*auto dest*: *extractFrameFreshChain*)
moreover from ‹*A*$_{QR}$ ♯* *P*› ‹*A*$_{QR}$ ♯* Ψ› *Aeq* **have** *A*$_Q$ ♯* *P* **and** *A*$_Q$ ♯* Ψ **by** *simp*+
ultimately have Ψ ⊗ Ψ$_R$ ▷ *P* ∥ *Q* ⟼τ ≺ (⦇ν*xvec⦈)(*P*′ ∥ *Q*′)
using ‹*A*$_P$ ♯* Ψ› ‹*A*$_P$ ♯* *P*› ‹*A*$_P$ ♯* *Q*› ‹*A*$_P$ ♯* *M*› ‹*A*$_P$ ♯* *A*$_Q$› ‹*A*$_Q$ ♯* Ψ› ‹*A*$_Q$ ♯* Ψ$_R$› ‹*A*$_Q$ ♯* *P*› ‹*A*$_Q$ ♯* *Q*› ‹*A*$_Q$ ♯* *K*› ‹*xvec* ♯* *P*›
by(*rule-tac Comm1*) (*assumption* | *force*)+
moreover from ‹*A*$_{QR}$ ♯* Ψ› *Aeq* **have** *A*$_R$ ♯* Ψ **by** *simp*
moreover from ‹*A*$_{QR}$ ♯* *P*› *Aeq* **have** *A*$_R$ ♯* *P* **by** *simp*
ultimately have Ψ ▷ (*P* ∥ *Q*) ∥ *R* ⟼τ ≺ ((⦇ν*xvec⦈)(*P*′ ∥ *Q*′)) ∥ *R* **using** ‹*extractFrame R* = ⟨*A*$_R$, Ψ$_R$⟩› ‹*A*$_R$ ♯* *Q*›
by(*rule-tac Par1*) (*assumption* | *simp*)+
moreover from ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *R*› **have** (Ψ, ((⦇ν*xvec⦈)(*P*′ ∥ *Q*′)) ∥ *R*, (⦇ν*xvec⦈)(*P*′ ∥ (*Q*′ ∥ *R*))) ∈ *Rel*
by(*rule C3*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 R*′ *A*$_Q$ Ψ$_Q$ *A*$_R$ Ψ$_R$)
have *Aeq*: *A*$_{QR}$ = *A*$_Q$@*A*$_R$ **and** Ψ*eq*: Ψ$_{QR}$ = Ψ$_Q$ ⊗ Ψ$_R$ **by** *fact*+
from ‹*A*$_{QR}$ ♯* Ψ› ‹*A*$_{QR}$ ♯* *P*› ‹*A*$_{QR}$ ♯* Ψ$_P$› ‹*A*$_P$ ♯* *A*$_{QR}$› *Aeq* **have** *A*$_R$ ♯* Ψ **and** *A*$_R$ ♯* Ψ$_P$ **and** *A*$_P$ ♯* *A*$_R$ **and** *A*$_P$ ♯* *A*$_Q$ **and** *A*$_R$ ♯* *P* **by** *simp*+
from ‹*A*$_{QR}$ ♯* Ψ› *Aeq* **have** *A*$_Q$ ♯* Ψ **by** *simp*
from ‹*A*$_{QR}$ ♯* *P*› ‹*A*$_P$ ♯* *A*$_{QR}$› *Aeq FrP* **have** *A*$_Q$ ♯* Ψ$_P$ **by**(*auto dest*: *extractFrameFreshChain*)
from ‹*A*$_P$ ♯* *A*$_{QR}$› ‹*extractFrame R* = ⟨*A*$_R$, Ψ$_R$⟩› ‹*extractFrame Q* = ⟨*A*$_Q$, Ψ$_Q$⟩› *Aeq* ‹*A*$_P$ ♯* *Q*› ‹*A*$_P$ ♯* *R*› **have** *A*$_P$ ♯* Ψ$_Q$ **and** *A*$_P$ ♯* Ψ$_R$ **by**(*auto dest*: *extractFrameFreshChain*)
have *RTrans*: (Ψ ⊗ Ψ$_P$) ⊗ Ψ$_Q$ ▷ *R* ⟼*K*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *R*′ **and** *FrR*: *extractFrame R* = ⟨*A*$_R$, Ψ$_R$⟩ **by** *fact*+
then obtain *K*′ **where** *KeqK*′: ((Ψ ⊗ Ψ$_P$) ⊗ Ψ$_Q$) ⊗ Ψ$_R$ ⊢ *K* ↔ *K*′ **and** *A*$_P$ ♯* *K*′ **and** *A*$_Q$ ♯* *K*′

using $\langle A_P \sharp* R\rangle$ $\langle A_Q \sharp* R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* A_R\rangle$ $\langle A_P \sharp* A_R\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* K\rangle$ $\langle distinct\ A_R\rangle$ $\langle xvec \sharp* K\rangle$ $\langle distinct\ xvec\rangle$

by(*rule-tac* $B{=}A_P@A_Q$ **in** *obtainPrefix*) (*assumption* | *force*)+

from *PTrans* Ψeq **have** $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$

by(*metis statEqTransition Associativity Commutativity Composition*)

moreover from *MeqK KeqK'* Ψeq **have** *MeqK'*: $((\Psi \otimes \Psi_R) \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$

by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans*)

ultimately have $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto K'(\!|N|\!) \prec P'$ **using** *FrP* $\langle distinct\ A_P\rangle$ $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* K'\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_P \sharp* \Psi_R\rangle$

by(*rule-tac inputRenameSubject*) *auto*

moreover from $\langle A_{QR} \sharp* P\rangle$ $\langle A_{QR} \sharp* N\rangle$ *Aeq* **have** $A_Q \sharp* P$ **and** $A_Q \sharp* N$ **by** *simp*+

ultimately have $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto K'(\!|N|\!) \prec P' \parallel Q$ **using** $\langle extractFrame\ Q = \langle A_Q, \Psi_Q\rangle\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$ $\langle A_Q \sharp* K'\rangle$ $\langle A_Q \sharp* \Psi\rangle$

by(*rule-tac Par1*) (*assumption* | *force*)+

moreover from *FrP* $\langle extractFrame\ Q = \langle A_Q, \Psi_Q\rangle\rangle$ $\langle A_P \sharp* A_Q\rangle$ $\langle A_P \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* \Psi_P\rangle$

have $extractFrame(P \parallel Q) = \langle (A_P@A_Q), \Psi_P \otimes \Psi_Q\rangle$ **by** *simp*+

moreover from *RTrans* **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \rhd R \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec R'$ **by**(*metis Associativity statEqTransition*)

moreover note *FrR*

moreover from *MeqK' KeqK'* **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \otimes \Psi_R \vdash K' \leftrightarrow K$

by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)

ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \tau \prec (\!|\nu{*}xvec|\!)((P' \parallel Q) \parallel R')$

using $\langle A_P \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* Q\rangle$ $\langle A_P \sharp* R\rangle$ $\langle A_Q \sharp* R\rangle$ $\langle A_P \sharp* K'\rangle$ $\langle A_Q \sharp* K'\rangle$ $\langle A_P \sharp* A_R\rangle$ $\langle A_Q \sharp* A_R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* K\rangle$ $\langle xvec \sharp* P\rangle$ $\langle xvec \sharp* Q\rangle$

by(*rule-tac Comm1*) (*assumption* | *simp*)+

moreover from $\langle xvec \sharp* \Psi\rangle$ **have** $(\Psi, (\!|\nu{*}xvec|\!)((P' \parallel Q) \parallel R'), (\!|\nu{*}xvec|\!)(P' \parallel (Q \parallel R'))) \in Rel$

by(*metis C1 C4*)

ultimately show *?case* **by** *blast*

qed

next

case(*cComm2* Ψ_{QR} *M xvec N P'* A_P Ψ_P *K QR'* A_{QR})

from $\langle A_{QR} \sharp* (\Psi, P, Q, R, \alpha)\rangle$ **have** $A_{QR} \sharp* Q$ **and** $A_{QR} \sharp* R$ **and** $A_{QR} \sharp* \Psi$ **by** *simp*+

from $\langle A_P \sharp* (Q \parallel R)\rangle$ $\langle xvec \sharp* (Q \parallel R)\rangle$ **have** $A_P \sharp* Q$ **and** $A_P \sharp* R$ **and** $xvec \sharp* Q$ **and** $xvec \sharp* R$ **by** *simp*+

have *PTrans*: $\Psi \otimes \Psi_{QR} \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$ **and** *FrP*: $extractFrame\ P = \langle A_P, \Psi_P\rangle$ **and** *MeqK*: $\Psi \otimes \Psi_P \otimes \Psi_{QR} \vdash M \leftrightarrow K$ **by** *fact*+

note $\langle \Psi \otimes \Psi_P \rhd Q \parallel R \longmapsto K(\!|N|\!) \prec QR'\rangle$ $\langle extractFrame(Q \parallel R) = \langle A_{QR}, \Psi_{QR}\rangle\rangle$ $\langle distinct\ A_{QR}\rangle$

moreover from $\langle A_{QR} \sharp* \Psi\rangle$ $\langle A_{QR} \sharp* \Psi_P\rangle$ **have** $A_{QR} \sharp* (\Psi \otimes \Psi_P)$ **by** *force*

ultimately show *?case* **using** $\langle A_{QR} \sharp* Q\rangle$ $\langle A_{QR} \sharp* R\rangle$ $\langle A_{QR} \sharp* K\rangle$

proof(*induct rule*: *parCasesInputFrame*)
case(*cPar1* Q' A_Q Ψ_Q A_R Ψ_R)
have *Aeq*: $A_{QR} = A_Q @ A_R$ **and** Ψeq: $\Psi_{QR} = \Psi_Q \otimes \Psi_R$ **by** *fact+*
from *PTrans* Ψeq **have** $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*metis statEqTransition Associativity Commutativity Composition*)
moreover note *FrP*
moreover from $\langle(\Psi \otimes \Psi_P) \otimes \Psi_R \rhd Q \longmapsto K(\!|N|\!) \prec Q'\rangle$ **have** $(\Psi \otimes \Psi_R) \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
by(*metis statEqTransition Associativity Commutativity Composition*)
moreover note $\langle$*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle\rangle$
moreover from *MeqK* Ψeq **have** $(\Psi \otimes \Psi_R) \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$
by(*metis statEqEnt Associativity Commutativity Composition*)
moreover from $\langle A_P \sharp * Q\rangle$ $\langle A_P \sharp * R\rangle$ $\langle A_P \sharp * A_{QR}\rangle$ *Aeq* $\langle$*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle\rangle$ $\langle$*extractFrame* $R = \langle A_R, \Psi_R\rangle\rangle$
have $A_P \sharp * A_Q$ **and** $A_P \sharp * \Psi_R$ **by**(*auto dest*: *extractFrameFreshChain*)
moreover from $\langle A_{QR} \sharp * \Psi\rangle$ $\langle A_{QR} \sharp * P\rangle$ *Aeq* **have** $A_Q \sharp * \Psi$ **and** $A_Q \sharp * P$ **by** *simp+*
ultimately have $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q')$
using $\langle A_P \sharp * \Psi\rangle$ $\langle A_P \sharp * P\rangle$ $\langle A_P \sharp * Q\rangle$ $\langle A_P \sharp * M\rangle$ $\langle A_P \sharp * A_Q\rangle$ $\langle A_Q \sharp * \Psi\rangle$ $\langle A_Q \sharp * \Psi_R\rangle$ $\langle A_Q \sharp * P\rangle$ $\langle A_Q \sharp * Q\rangle$ $\langle A_Q \sharp * K\rangle$ $\langle xvec \sharp * Q\rangle$
by(*rule-tac Comm2*) (*assumption* | *force*)*+*
moreover from $\langle A_{QR} \sharp * \Psi\rangle$ $\langle A_{QR} \sharp * P\rangle$ *Aeq* **have** $A_R \sharp * \Psi$ **and** $A_R \sharp * P$ **by** *simp+*
ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \tau \prec ((\!|\nu * xvec|\!)(P' \parallel Q')) \parallel R$ **using** $\langle$*extractFrame* $R = \langle A_R, \Psi_R\rangle\rangle$ $\langle A_R \sharp * Q\rangle$
by(*rule-tac Par1*) (*assumption* | *simp*)*+*
moreover from $\langle xvec \sharp * \Psi\rangle$ $\langle xvec \sharp * R\rangle$ **have** $(\Psi, ((\!|\nu * xvec|\!)(P' \parallel Q')) \parallel R, (\!|\nu * xvec|\!)(P' \parallel (Q' \parallel R))) \in Rel$
by(*rule C3*)
ultimately show *?case* **by** *blast*
next
case(*cPar2* R' A_Q Ψ_Q A_R Ψ_R)
have *Aeq*: $A_{QR} = A_Q @ A_R$ **and** Ψeq: $\Psi_{QR} = \Psi_Q \otimes \Psi_R$ **by** *fact+*
from $\langle A_{QR} \sharp * \Psi\rangle$ $\langle A_{QR} \sharp * P\rangle$ $\langle A_{QR} \sharp * \Psi_P\rangle$ $\langle A_P \sharp * A_{QR}\rangle$ *Aeq*
have $A_R \sharp * \Psi$ **and** $A_R \sharp * \Psi_P$ **and** $A_P \sharp * A_R$ **and** $A_P \sharp * A_Q$ **and** $A_R \sharp * P$ **and** $A_Q \sharp * \Psi$ **and** $A_Q \sharp * \Psi_P$ **by** *simp+*
have *RTrans*: $(\Psi \otimes \Psi_P) \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$ **and** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact+*
then obtain K' **where** *KeqK'*: $((\Psi \otimes \Psi_P) \otimes \Psi_Q) \otimes \Psi_R \vdash K \leftrightarrow K'$ **and** $A_P \sharp * K'$ **and** $A_Q \sharp * K'$
using $\langle A_P \sharp * R\rangle$ $\langle A_Q \sharp * R\rangle$ $\langle A_R \sharp * \Psi\rangle$ $\langle A_R \sharp * \Psi_P\rangle$ $\langle A_R \sharp * \Psi_Q\rangle$ $\langle A_Q \sharp * A_R\rangle$ $\langle A_P \sharp * A_R\rangle$ $\langle A_R \sharp * R\rangle$ $\langle A_R \sharp * R\rangle$ $\langle A_R \sharp * K\rangle$ $\langle$*distinct* $A_R\rangle$
by(*rule-tac* $B{=}A_P @ A_Q$ **in** *obtainPrefix*) (*assumption* | *force*)*+*
from *PTrans* Ψeq **have** $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*metis statEqTransition Associativity Commutativity Composition*)
moreover from *MeqK KeqK'* Ψeq **have** *MeqK'*: $((\Psi \otimes \Psi_R) \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow K'$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans*)
moreover from $\langle A_P \sharp * R\rangle$ $\langle A_P \sharp * Q\rangle$ $\langle A_P \sharp * A_{QR}\rangle$ *FrR* $\langle$*extractFrame* $Q =$

$\langle A_Q, \Psi_Q\rangle\rangle$ *Aeq* **have** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* \Psi_R$
by(*auto dest*: *extractFrameFreshChain*)
ultimately have $(\Psi \otimes \Psi_R) \otimes \Psi_Q \rhd P \longmapsto K'(\!|\nu* xvec|\!)\langle N\rangle \prec P'$ **using** *FrP* ‹*distinct* A_P› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* K'$›
by(*rule-tac outputRenameSubject*) *auto*
moreover from ‹$A_{QR} \sharp* P$› ‹$A_{QR} \sharp* N$› ‹$A_{QR} \sharp* xvec$› *Aeq* **have** $A_Q \sharp* P$ **and** $A_Q \sharp* N$ **and** $A_Q \sharp* xvec$ **by** *simp+*
ultimately have $\Psi \otimes \Psi_R \rhd P \parallel Q \longmapsto K'(\!|\nu* xvec|\!)\langle N\rangle \prec (P' \parallel Q)$ **using** ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_Q \sharp* \Psi_R$› ‹$A_Q \sharp* K'$› ‹*xvec* $\sharp* Q$› ‹$A_Q \sharp* \Psi$›
by(*rule-tac Par1*) (*assumption* | *force*)+
moreover from *FrP* ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$›
have *extractFrame*$(P \parallel Q) = \langle (A_P @ A_Q), \Psi_P \otimes \Psi_Q\rangle$ **by** *simp+*
moreover from *RTrans* **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \rhd R \longmapsto K(\!|N|\!) \prec R'$ **by**(*metis Associativity statEqTransition*)
moreover note *FrR*
moreover from *MeqK' KeqK'* **have** $\Psi \otimes (\Psi_P \otimes \Psi_Q) \otimes \Psi_R \vdash K' \leftrightarrow K$
by(*metis statEqEnt Associativity Commutativity Composition chanEqTrans chanEqSym*)
ultimately have $\Psi \rhd (P \parallel Q) \parallel R \longmapsto \tau \prec (\!|\nu* xvec|\!)((P' \parallel Q) \parallel R')$
using ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* Q$› ‹$A_P \sharp* R$› ‹$A_Q \sharp* R$› ‹$A_P \sharp* K'$› ‹$A_Q \sharp* K'$› ‹$A_P \sharp* A_R$› ‹$A_Q \sharp* A_R$›
‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* K$› ‹*xvec* $\sharp* R$›
by(*rule-tac Comm2*) (*assumption* | *simp*)+
moreover from ‹*xvec* $\sharp* \Psi$› **have** $(\Psi, (\!|\nu* xvec|\!)((P' \parallel Q) \parallel R'), (\!|\nu* xvec|\!)(P' \parallel (Q \parallel R'))) \in Rel$
by(*metis C1 C4*)
ultimately show *?case* **by** *blast*
qed
qed
qed

lemma *parNilLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *eqvt Rel*
and *C1*: $\bigwedge Q.\ (\Psi, Q \parallel \mathbf{0}, Q) \in Rel$

shows $\Psi \rhd (P \parallel \mathbf{0}) \rightsquigarrow[Rel]\ P$
using ‹*eqvt Rel*›
proof(*induct rule*: *simI*[*of* - - - - ()])
case(*cSim* α P')
from ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› **have** $\Psi \otimes SBottom' \rhd P \longmapsto \alpha \prec P'$
by(*metis statEqTransition Identity AssertionStatEqSym*)
hence $\Psi \rhd (P \parallel \mathbf{0}) \longmapsto \alpha \prec (P' \parallel \mathbf{0})$
by(*rule-tac Par1*) *auto*
moreover have $(\Psi, P' \parallel \mathbf{0}, P') \in Rel$ **by**(*rule C1*)

```
  ultimately show ?case by blast
qed

lemma parNilRight:
  fixes Ψ :: 'b
  and   P   :: ('a, 'b, 'c) psi
  and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  assumes eqvt Rel
  and     C1: ⋀Q. (Ψ, Q, (Q ∥ 𝟬)) ∈ Rel

  shows Ψ ▷ P ⇝[Rel] (P ∥ 𝟬)
using ‹eqvt Rel›
proof(induct rule: simI[of - - - - ()])
  case(cSim α P')
  note ‹Ψ ▷ P ∥ 𝟬 ⟼α ≺ P'› ‹bn α ♯* Ψ› ‹bn α ♯* P›
  moreover have bn α ♯* 𝟬 by simp
  ultimately show ?case using ‹bn α ♯* subject α›
  proof(induct rule: parCases[where C=()])
    case(cPar1 P' A_Q Ψ_Q)
    from ‹extractFrame(𝟬) = ⟨A_Q, Ψ_Q⟩› have Ψ_Q = SBottom' by auto
    with ‹Ψ ⊗ Ψ_Q ▷ P ⟼α ≺ P'› have Ψ ▷ P ⟼α ≺ P'
      by(metis statEqTransition Identity)
    moreover have (Ψ, P', P' ∥ 𝟬) ∈ Rel by(rule C1)
    ultimately show ?case by blast
  next
    case(cPar2 Q' A_P Ψ_P)
    from ‹Ψ ⊗ Ψ_P ▷ 𝟬 ⟼α ≺ Q'› have False
      by auto
    thus ?case by simp
  next
    case(cComm1 Ψ_Q M N P' A_P Ψ_P K xvec Q' A_Q)
    from ‹Ψ ⊗ Ψ_P ▷ 𝟬 ⟼K(⦇ν*xvec⦈)⟨N⟩ ≺ Q'› have False by auto
    thus ?case by simp
  next
    case(cComm2 Ψ_Q M xvec N P' A_P Ψ_P K Q' A_Q)
    from ‹Ψ ⊗ Ψ_P ▷ 𝟬 ⟼K(⦇N⦈) ≺ Q'› have False
      by auto
    thus ?case by simp
  qed
qed

lemma resNilLeft:
  fixes x   :: name
  and   Ψ   :: 'b
  and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  shows Ψ ▷ (⦇νx⦈)𝟬 ⇝[Rel] 𝟬
by(auto simp add: simulation-def)
```

lemma *resNilRight*:
fixes x :: *name*
and Ψ :: $'b$
and *Rel* :: ($'b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

shows $\Psi \rhd \mathbf{0} \rightsquigarrow[Rel]\ (\!|\nu x|\!)\mathbf{0}$
apply(*auto simp add*: *simulation-def*)
by(*cases rule*: *semantics.cases*) (*auto simp add*: *psi.inject alpha'*)

lemma *inputPushResLeft*:
fixes Ψ :: $'b$
and x :: *name*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and P :: ($'a, 'b, 'c$) *psi*

assumes *eqvt Rel*
and $x \sharp \Psi$
and $x \sharp M$
and $x \sharp xvec$
and $x \sharp N$
and *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

shows $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \rightsquigarrow[Rel]\ M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
proof −
note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover have $x \sharp (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P)$ **by**(*simp add*: *abs-fresh*)
moreover from ‹$x \sharp M$› ‹$x \sharp N$› **have** $x \sharp M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
by(*auto simp add*: *inputChainFresh abs-fresh*)
ultimately show *?thesis*
proof(*induct rule*: *simIFresh*[*of* - - - - - ()])
case(*cSim* $\alpha\ P'$)
from ‹$\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P \longmapsto \alpha \prec P'$› ‹$x \sharp \alpha$› **show** *?case*
proof(*induct rule*: *inputCases*)
case(*cInput K Tvec*)
from ‹$x \sharp K(\!|N[xvec::=Tvec]|\!)$› **have** $x \sharp K$ **and** $x \sharp N[xvec::=Tvec]$ **by** *simp*+
from ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* ⊆ *supp N*› ‹*length xvec* = *length Tvec*›
have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto K(\!|(N[xvec::=Tvec])|\!) \prec P[xvec::=Tvec]$
by(*rule Input*)
hence $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \longmapsto K(\!|(N[xvec::=Tvec])|\!) \prec (\!|\nu x|\!)(P[xvec::=Tvec])$
using ‹$x \sharp \Psi$› ‹$x \sharp K$› ‹$x \sharp N[xvec::=Tvec]$›
by(*rule-tac Scope*) *auto*
moreover from ‹*length xvec* = *length Tvec*› ‹*distinct xvec*› ‹*set xvec* ⊆ *supp N*› ‹$x \sharp N[xvec::=Tvec]$› **have** $x \sharp Tvec$
by(*rule substTerm.subst3*)
with ‹$x \sharp xvec$› **have** $(\Psi, (\!|\nu x|\!)(P[xvec::=Tvec]), ((\!|\nu x|\!)P)[xvec::=Tvec]) \in Rel$

```
      by(force intro: C1)
    ultimately show ?case by blast
  qed
 qed
qed

lemma inputPushResRight:
 fixes Ψ   :: 'b
 and   x   :: name
 and   M   :: 'a
 and   xvec :: name list
 and   N   :: 'a
 and   P   :: ('a, 'b, 'c) psi

 assumes eqvt Rel
 and     x ♯ Ψ
 and     x ♯ M
 and     x ♯ xvec
 and     x ♯ N
 and     C1: ⋀Q. (Ψ, Q, Q) ∈ Rel

 shows Ψ ▷ M⦇λ*xvec N⦈.⦇νx⦈P ⇝[Rel] ⦇νx⦈(M⦇λ*xvec N⦈.P)
proof −
 note ‹eqvt Rel› ‹x ♯ Ψ›
 moreover from ‹x ♯ M› ‹x ♯ N› have x ♯ M⦇λ*xvec N⦈.⦇νx⦈P
   by(auto simp add: inputChainFresh abs-fresh)
 moreover have x ♯ ⦇νx⦈(M⦇λ*xvec N⦈.P) by(simp add: abs-fresh)
 ultimately show ?thesis
 proof(induct rule: simIFresh[of - - - - - ()])
   case(cSim α P')
   note ‹Ψ ▷ ⦇νx⦈(M⦇λ*xvec N⦈.P) ⟼α ≺ P'› ‹x ♯ Ψ› ‹x ♯ α› ‹x ♯ P'› ‹bn α
♯* Ψ›
    moreover from ‹bn α ♯* (⦇νx⦈(M⦇λ*xvec N⦈.P))› ‹x ♯ α› have  bn α ♯*
(M⦇λ*xvec N⦈.P)
     by simp
   ultimately show ?case using ‹bn α ♯* subject α›
   proof(induct rule: resCases)
     case(cRes P')
     from ‹Ψ ▷ M⦇λ*xvec N⦈.P ⟼α ≺ P'› ‹x ♯ α› show ?case
     proof(induct rule: inputCases)
       case(cInput K Tvec)
         from ‹x ♯ K⦇N[xvec::=Tvec]⦈› have x ♯ K and x ♯ N[xvec::=Tvec] by
simp+
        from ‹Ψ ⊢ M ↔ K› ‹distinct xvec› ‹set xvec ⊆ supp N› ‹length xvec =
length Tvec›
     have Ψ ▷ M⦇λ*xvec N⦈.(⦇νx⦈P) ⟼K⦇(N[xvec::=Tvec])⦈ ≺ (⦇νx⦈P)[xvec::=Tvec]
        by(rule Input)
      moreover from ‹length xvec = length Tvec› ‹distinct xvec› ‹set xvec ⊆ supp
N› ‹x ♯ N[xvec::=Tvec]› have x ♯ Tvec
```

```
        by(rule substTerm.subst3)
       with ‹x ♯ xvec› have (Ψ, ((⦇νx⦈)P)[xvec::=Tvec], (⦇νx⦈)(P[xvec::=Tvec])) ∈
Rel
        by(force intro: C1)
       ultimately show ?case by blast
     qed
   next
     case cOpen
     then have False by auto
     thus ?case by simp
   qed
 qed
qed

lemma outputPushResLeft:
 fixes Ψ   :: 'b
 and   x   :: name
 and   M   :: 'a
 and   N   :: 'a
 and   P   :: ('a, 'b, 'c) psi

 assumes eqvt Rel
 and     x ♯ Ψ
 and     x ♯ M
 and     x ♯ N
 and     C1: ⋀Q. (Ψ, Q, Q) ∈ Rel

 shows Ψ ▷ (⦇νx⦈)(M⟨N⟩.P) ⇝[Rel] M⟨N⟩.(⦇νx⦈)P
proof −
 note ‹eqvt Rel› ‹x ♯ Ψ›
 moreover have x ♯ (⦇νx⦈)(M⟨N⟩.P) by(simp add: abs-fresh)
 moreover from ‹x ♯ M› ‹x ♯ N› have x ♯ M⟨N⟩.(⦇νx⦈)P
   by(auto simp add: abs-fresh)
 ultimately show ?thesis
 proof(induct rule: simIFresh[of - - - - - ()])
   case(cSim α P')
   from ‹Ψ ▷ M⟨N⟩.(⦇νx⦈)P ⟼α ≺ P'› ‹x ♯ α›
   show ?case
   proof(induct rule: outputCases)
     case(cOutput K)
     from ‹Ψ ⊢ M ↔ K› have Ψ ▷ M⟨N⟩.P ⟼K⟨N⟩ ≺ P
       by(rule Output)
     hence Ψ ▷ (⦇νx⦈)(M⟨N⟩.P) ⟼K⟨N⟩ ≺ (⦇νx⦈)P using ‹x ♯ Ψ› ‹x ♯ K⟨N⟩›
       by(rule Scope)
     moreover have (Ψ, (⦇νx⦈)P, (⦇νx⦈)P) ∈ Rel by(rule C1)
     ultimately show ?case by blast
   qed
 qed
qed
```

lemma *outputPushResRight*:
fixes Ψ :: $'b$
and x :: *name*
and M :: $'a$
and N :: $'a$
and P :: $('a, 'b, 'c)$ *psi*

assumes *eqvt Rel*
and $x \sharp \Psi$
and $x \sharp M$
and $x \sharp N$
and *C1*: $\bigwedge Q.\ (\Psi, Q, Q) \in Rel$

shows $\Psi \rhd M\langle N\rangle.(\!|\nu x|\!)P \rightsquigarrow[Rel]\ (\!|\nu x|\!)(M\langle N\rangle.P)$
proof −
note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover from ‹$x \sharp M$› ‹$x \sharp N$› **have** $x \sharp M\langle N\rangle.(\!|\nu x|\!)P$
by(*auto simp add*: *abs-fresh*)
moreover have $x \sharp (\!|\nu x|\!)(M\langle N\rangle.P)$ **by**(*simp add*: *abs-fresh*)
ultimately show *?thesis*
proof(*induct rule*: *simIFresh*[*of* - - - - - (M, N)])
case(*cSim* α P')
note ‹$\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \longmapsto \alpha \prec P'$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp P'$› ‹*bn* $\alpha \sharp* \Psi$›
moreover from ‹*bn* $\alpha \sharp* ((\!|\nu x|\!)(M\langle N\rangle.P))$› ‹$x \sharp \alpha$› **have** *bn* $\alpha \sharp* (M\langle N\rangle.P)$
by *simp*
ultimately show *?case* **using** ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha \sharp* (M, N)$› ‹$x \sharp \alpha$›
proof(*induct rule*: *resCases*)
case(*cOpen K xvec1 xvec2 y N' P'*)
from ‹$bn(K(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N'\rangle) \sharp* (M, N)$› **have** $y \sharp N$ **by** *simp+*
from ‹$\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|\nu*(xvec1@xvec2)|\!)\langle([(x, y)] \cdot N')\rangle \prec ([(x, y)] \cdot P')$›
have $N = ([(x, y)] \cdot N')$
apply −
by(*ind-cases* $\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|\nu*(xvec1@xvec2)|\!)\langle([(x, y)] \cdot N')\rangle \prec ([(x, y)] \cdot P')$)
(*auto simp add*: *residualInject psi.inject*)
with ‹$x \sharp N$› ‹$y \sharp N$› ‹$x \neq y$› **have** $N = N'$
by(*subst pt-bij*[*OF pt-name-inst*, *OF at-name-inst*, *symmetric*, **where** *pi*=$[(x, y)]$])
(*simp add*: *fresh-left calc-atm*)
with ‹$y \in supp\ N'$› ‹$y \sharp N$› **have** *False* **by**(*simp add*: *fresh-def*)
thus *?case* **by** *simp*
next
case(*cRes P'*)
from ‹$\Psi \rhd M\langle N\rangle.P \longmapsto \alpha \prec P'$› **show** *?case*
proof(*induct rule*: *outputCases*)
case(*cOutput K*)
from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \rhd M\langle N\rangle.((\!|\nu x|\!)P) \longmapsto K\langle N\rangle \prec (\!|\nu x|\!)P$

```
        by(rule Output)
       moreover have (Ψ, (|νx|)P, (|νx|)P) ∈ Rel by(rule C1)
       ultimately show ?case by force
      qed
    qed
  qed
qed

lemma casePushResLeft:
  fixes Ψ  :: 'b
  and   x  :: name
  and   Cs :: ('c × ('a, 'b, 'c) psi) list

  assumes eqvt Rel
  and     x ♯ Ψ
  and     x ♯ map fst Cs
  and     C1: ⋀Q. (Ψ, Q, Q) ∈ Rel

  shows Ψ ▷ (|νx|)(Cases Cs) ⇝[Rel] Cases (map (λ(φ, P). (φ, (|νx|)P)) Cs)
proof −
  note ‹eqvt Rel› ‹x ♯ Ψ›
  moreover have x ♯ (|νx|)(Cases Cs) by(simp add: abs-fresh)
  moreover from ‹x ♯ map fst Cs› have x ♯ Cases (map (λ(φ, P). (φ, (|νx|)P))
Cs)
    by(induct Cs) (auto simp add: abs-fresh)
  ultimately show ?thesis
  proof(induct rule: simIFresh[of - - - - - Cs])
    case(cSim α P'')
    from ‹Ψ ▷ Cases (map (λ(φ, P). (φ, (|νx|)P)) Cs) ⟼α ≺ P''›
    show ?case
    proof(induct rule: caseCases)
      case(cCase φ P')
      from ‹(φ, P') mem (map (λ(φ, P). (φ, (|νx|)P)) Cs)›
      obtain P where (φ, P) mem Cs and P' = (|νx|)P
        by(induct Cs) auto
      from ‹guarded P'› ‹P' = (|νx|)P› have guarded P by simp
      from ‹Ψ ▷ P' ⟼α ≺ P''› ‹P' = (|νx|)P› have Ψ ▷ (|νx|)P ⟼α ≺ P''
        by simp
      moreover note ‹x ♯ Ψ› ‹x ♯ α› ‹x ♯ P''› ‹bn α ♯* Ψ›
      moreover from ‹bn α ♯* Cs› ‹(φ, P) mem Cs›
      have bn α ♯* P by(auto dest: memFreshChain)
      ultimately show ?case using ‹bn α ♯* subject α› ‹x ♯ α› ‹bn α ♯* Cs›
      proof(induct rule: resCases)
        case(cOpen M xvec1 xvec2 y N P')
        from ‹x ♯ M(|ν*(xvec1@y#xvec2)|)⟨N⟩› have x ♯ xvec1 and x ♯ xvec2 and
x ♯ M by simp+
        from ‹bn(M(|ν*(xvec1@y#xvec2)|)⟨N⟩) ♯* Cs› have y ♯ Cs by simp
       from ‹Ψ ▷ P ⟼M(|ν*(xvec1@xvec2)|)⟨([(x, y)] • N)⟩ ≺ ([(x, y)] • P')› ‹(φ,
P) mem Cs› ‹Ψ ⊢ φ› ‹guarded P›
```

```
      have Ψ ▷ Cases Cs ⟼M⦇ν*(xvec1@xvec2)⦈⟨([(x, y)] • N)⟩ ≺ ([(x, y)] •
P') by(rule Case)
    hence ([(x, y)] • Ψ) ▷ ([(x, y)] • (Cases Cs)) ⟼([(x, y)] • (M⦇ν*(xvec1@xvec2)⦈⟨([(x,
y)] • N)⟩ ≺ ([(x, y)] • P')))
      by(rule semantics.eqvt)
     with ‹x ♯ Ψ› ‹x ♯ M› ‹y ♯ xvec1› ‹y ♯ xvec2› ‹y ♯ Ψ› ‹y ♯ M› ‹x ♯ xvec1›
‹x ♯ xvec2›
    have Ψ ▷ ([(x, y)] • (Cases Cs)) ⟼M⦇ν*(xvec1@xvec2)⦈⟨N⟩ ≺ P' by(simp
add: eqvts)
     hence Ψ ▷ ⦇νy⦈([(x, y)] • (Cases Cs)) ⟼M⦇ν*(xvec1@y#xvec2)⦈⟨N⟩ ≺
P' using ‹y ∈ supp N› ‹y ♯ Ψ› ‹y ♯ M› ‹y ♯ xvec1› ‹y ♯ xvec2›
      by(rule Open)
     hence Ψ ▷ ⦇νx⦈(Cases Cs) ⟼M⦇ν*(xvec1@y#xvec2)⦈⟨N⟩ ≺ P' using
‹y ♯ Cs›
      by(simp add: alphaRes)
     moreover have (Ψ, P', P') ∈ Rel by(rule C1)
     ultimately show ?case by blast
    next
     case(cRes P')
     from ‹Ψ ▷ P ⟼α ≺ P'› ‹(φ, P) mem Cs› ‹Ψ ⊢ φ› ‹guarded P›
     have Ψ ▷ Cases Cs ⟼α ≺ P' by(rule Case)
     hence Ψ ▷ ⦇νx⦈(Cases Cs) ⟼α ≺ ⦇νx⦈P' using ‹x ♯ Ψ› ‹x ♯ α›
      by(rule Scope)
     moreover have (Ψ, ⦇νx⦈P', ⦇νx⦈P') ∈ Rel by(rule C1)
     ultimately show ?case by blast
    qed
   qed
  qed
qed

lemma casePushResRight:
  fixes Ψ  :: 'b
  and   x  :: name
  and   Cs :: ('c × ('a, 'b, 'c) psi) list

  assumes eqvt Rel
  and     x ♯ Ψ
  and     x ♯ map fst Cs
  and     C1: ⋀Q. (Ψ, Q, Q) ∈ Rel

  shows Ψ ▷ Cases (map (λ(φ, P). (φ, ⦇νx⦈P)) Cs) ⇝[Rel] ⦇νx⦈(Cases Cs)
proof −
  note ‹eqvt Rel› ‹x ♯ Ψ›
  moreover from ‹x ♯ map fst Cs› have x ♯ Cases (map (λ(φ, P). (φ, ⦇νx⦈P))
Cs)
    by(induct Cs) (auto simp add: abs-fresh)
  moreover have x ♯ ⦇νx⦈(Cases Cs) by(simp add: abs-fresh)
  ultimately show ?thesis
  proof(induct rule: simIFresh[of - - - - - Cs])
```

case(*cSim* α P'')
note ‹$\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \longmapsto \alpha \prec P''$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp P''$› ‹$bn\ \alpha \sharp* \Psi$›
moreover from ‹$bn\ \alpha \sharp* (\!|\nu x|\!)(Cases\ Cs)$› ‹$x \sharp \alpha$› **have** $bn\ \alpha \sharp* (Cases\ Cs)$ **by** *simp*
ultimately show *?case* **using** ‹$bn\ \alpha \sharp* subject\ \alpha$› ‹$x \sharp \alpha$› ‹$bn\ \alpha \sharp* Cs$›
proof(*induct rule*: *resCases*)
case(*cOpen M xvec1 xvec2 y N P'*)
from ‹$x \sharp M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle$› **have** $x \sharp xvec1$ **and** $x \sharp xvec2$ **and** $x \sharp M$ **by** *simp+*
from ‹$bn(M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle) \sharp* Cs$› **have** $y \sharp Cs$ **by** *simp*
from ‹$\Psi \rhd Cases\ Cs \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P')$›
show *?case*
proof(*induct rule*: *caseCases*)
case(*cCase* φ P)
from ‹$\Psi \rhd P \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P')$›
have $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \longmapsto([(x, y)] \cdot (M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P')))$
by(*rule semantics.eqvt*)
with ‹$x \sharp \Psi$› ‹$x \sharp M$› ‹$y \sharp xvec1$› ‹$y \sharp xvec2$› ‹$y \sharp \Psi$› ‹$y \sharp M$› ‹$x \sharp xvec1$› ‹$x \sharp xvec2$›
have $\Psi \rhd ([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle N\rangle \prec P'$ **by**(*simp add*: *eqvts*)
hence $\Psi \rhd (\!|\nu y|\!)([(x, y)] \cdot P) \longmapsto M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec P'$ **using** ‹$y \in supp\ N$› ‹$y \sharp \Psi$› ‹$y \sharp M$› ‹$y \sharp xvec1$› ‹$y \sharp xvec2$›
by(*rule Open*)
hence $\Psi \rhd (\!|\nu x|\!)P \longmapsto M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec P'$ **using** ‹$y \sharp Cs$› ‹$(\varphi, P)\ mem\ Cs$›
by(*subst alphaRes*, *auto dest*: *memFresh*)
moreover from ‹$(\varphi, P)\ mem\ Cs$› **have** $(\varphi, (\!|\nu x|\!)P)\ mem\ (map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
by(*induct Cs*) *auto*
moreover note ‹$\Psi \vdash \varphi$›
moreover from ‹$guarded\ P$› **have** $guarded((\!|\nu x|\!)P)$ **by** *simp*
ultimately have $\Psi \rhd (Cases\ (map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)) \longmapsto M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec P'$
by(*rule Case*)
moreover have $(\Psi, P', P') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed
next
case(*cRes P'*)
from ‹$\Psi \rhd Cases\ Cs \longmapsto \alpha \prec P'$›
show *?case*
proof(*induct rule*: *caseCases*)
case(*cCase* φ P)
from ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$›
have $\Psi \rhd (\!|\nu x|\!)P \longmapsto \alpha \prec (\!|\nu x|\!)P'$ **by**(*rule Scope*)
moreover from ‹$(\varphi, P)\ mem\ Cs$› **have** $(\varphi, (\!|\nu x|\!)P)\ mem\ (map\ (\lambda(\varphi, P).$

```
(φ, (|νx|)P)) Cs)
        by(induct Cs) auto
      moreover note ‹Ψ ⊢ φ›
      moreover from ‹guarded P› have guarded((|νx|)P) by simp
       ultimately have Ψ ▷ (Cases (map (λ(φ, P). (φ, (|νx|)P)) Cs)) ⟼α ≺
(|νx|)P'
        by(rule Case)
      moreover have (Ψ, (|νx|)P', (|νx|)P') ∈ Rel by(rule C1)
      ultimately show ?case by blast
    qed
  qed
 qed
qed

lemma resInputCases[consumes 5, case-names cRes]:
 fixes Ψ    :: 'b
 and   x    :: name
 and   P    :: ('a, 'b, 'c) psi
 and   M    :: 'a
 and   N    :: 'a
 and   P'   :: ('a, 'b, 'c) psi
 and   C    :: 'd::fs-name

 assumes Trans: Ψ ▷ (|νx|)P ⟼M(|N|) ≺ P'
 and     x ♯ Ψ
 and     x ♯ M
 and     x ♯ N
 and     x ♯ P'
 and     rScope: ⋀P'. [[Ψ ▷ P ⟼M(|N|) ≺ P']] ⟹ Prop ((|νx|)P')

 shows Prop P'
proof −
 note Trans ‹x ♯ Ψ›
 moreover from ‹x ♯ M› ‹x ♯ N› have x ♯ (M(|N|)) by simp
 moreover note ‹x ♯ P'›
 moreover have bn(M(|N|)) ♯* Ψ and bn(M(|N|)) ♯* P and bn(M(|N|)) ♯* sub-
ject(M(|N|)) and bn(M(|N|)) = [] by simp+
 ultimately show ?thesis
   by(induct rule: resCases) (auto intro: rScope)
qed

lemma scopeExtLeft:
 fixes x   :: name
 and   P   :: ('a, 'b, 'c) psi
 and   Ψ   :: 'b
 and   Q   :: ('a, 'b, 'c) psi
 and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

 assumes x ♯ P
```

and $x \sharp \Psi$
and *eqvt Rel*
and *C1*: $\bigwedge \Psi'\ R.\ (\Psi', R, R) \in Rel$
and *C2*: $\bigwedge y\ \Psi'\ R\ S\ zvec.\ [\![y \sharp \Psi'; y \sharp R; zvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)((\!|\nu* zvec|\!)(R \parallel S)), (\!|\nu* zvec|\!)(R \parallel (\!|\nu y|\!)S)) \in Rel$
and *C3*: $\bigwedge \Psi'\ zvec\ R\ y.\ [\![y \sharp \Psi'; zvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)((\!|\nu* zvec|\!)R), (\!|\nu* zvec|\!)((\!|\nu y|\!)R)) \in Rel$

shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \rightsquigarrow [Rel]\ P \parallel (\!|\nu x|\!)Q$
proof −
note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover have $x \sharp (\!|\nu x|\!)(P \parallel Q)$ **by**(*simp add*: *abs-fresh*)
moreover from ‹$x \sharp P$› **have** $x \sharp P \parallel (\!|\nu x|\!)Q$ **by**(*simp add*: *abs-fresh*)
ultimately show *?thesis*
proof(*induct rule*: *simIFresh*[*of* - - - - - x])
case(*cSim* α PQ)
from ‹$x \sharp \alpha$› ‹*bn* $\alpha \sharp* (P \parallel (\!|\nu x|\!)Q)$› **have** *bn* $\alpha \sharp* Q$ **by** *simp*
note ‹$\Psi \rhd P \parallel (\!|\nu x|\!)Q \longmapsto \alpha \prec PQ$› ‹*bn* $\alpha \sharp* \Psi$›
moreover from ‹*bn* $\alpha \sharp* (P \parallel (\!|\nu x|\!)Q)$› **have** *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* (\!|\nu x|\!)Q$ **by** *simp+*
ultimately show *?case* **using** ‹*bn* $\alpha \sharp*$ *subject* α› ‹$x \sharp PQ$›
proof(*induct rule*: *parCases*[**where** C=x])
case(*cPar1* P' A_Q Ψ_Q)
from ‹$x \sharp P' \parallel (\!|\nu x|\!)Q$› **have** $x \sharp P'$ **by** *simp*
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$ **by** *fact*
from ‹*extractFrame*$((\!|\nu x|\!)Q) = \langle A_Q, \Psi_Q\rangle$› **have** *FrxQ*: $(\!|\nu x|\!)(extractFrame\ Q) = \langle A_Q, \Psi_Q\rangle$ **by** *simp*
then obtain $y\ A_Q{'}$ **where** *A*: $A_Q = y\#A_Q{'}$ **by**(*case-tac* A_Q) *auto*
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* \alpha$›
have $A_Q{'} \sharp* \Psi$ **and** $A_Q{'} \sharp* P$ **and** $A_Q{'} \sharp* \alpha$
and $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp \alpha$
by *simp+*
from *PTrans* ‹$y \sharp P$› ‹$y \sharp \alpha$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinct*(*bn* α)› **have** $y \sharp P'$
by(*auto intro*: *freeFreshDerivative*)
note *PTrans*
moreover from *A* ‹$A_Q \sharp* x$› *FrxQ* **have** *extractFrame*$([(y, x)] \cdot Q) = \langle A_Q{'}, \Psi_Q\rangle$
by(*simp add*: *frame.inject alpha' fresh-list-cons eqvts*)
moreover from ‹*bn* $\alpha \sharp* Q$› **have** $([(y, x)] \cdot (bn\ \alpha)) \sharp* ([(y, x)] \cdot Q)$
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$x \sharp \alpha$› ‹$A_Q \sharp* \alpha$› *A* **have** *bn* $\alpha \sharp* ([(y, x)] \cdot Q)$ **by** *simp*
ultimately have $\Psi \rhd P \parallel ([(y, x)] \cdot Q) \longmapsto \alpha \prec (P' \parallel ([(y, x)] \cdot Q))$
using ‹$A_Q{'} \sharp* \Psi$› ‹$A_Q{'} \sharp* P$› ‹$A_Q{'} \sharp* \alpha$›
by(*rule Par1*)
hence $\Psi \rhd (\!|\nu y|\!)(P \parallel ([(y, x)] \cdot Q)) \longmapsto \alpha \prec (\!|\nu y|\!)(P' \parallel ([(y, x)] \cdot Q))$
using ‹$y \sharp \Psi$› ‹$y \sharp \alpha$›
by(*rule Scope*)
hence $([(y, x)] \cdot \Psi) \rhd ([(y, x)] \cdot ((\!|\nu y|\!)(P \parallel ([(y, x)] \cdot Q)))) \longmapsto ([(y, x)] \cdot (\alpha \prec (\!|\nu y|\!)(P' \parallel ([(y, x)] \cdot Q))))$

by(*rule semantics.eqvt*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› ‹$x \sharp P$› ‹$y \sharp P$› ‹$x \sharp \alpha$› ‹$y \sharp \alpha$› ‹$x \sharp P'$› ‹$y \sharp P'$›
have $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto \alpha \prec (\!|\nu x|\!)(P' \parallel Q)$
by(*simp add*: *eqvts calc-atm*)
moreover from ‹$x \sharp \Psi$› ‹$x \sharp P'$› **have** $(\Psi, (\!|\nu x|\!)((\!|\nu *[]|\!)(P' \parallel Q)), (\!|\nu *[]|\!)(P' \parallel (\!|\nu x|\!)Q)) \in Rel$
by(*rule-tac C2*) *auto*
ultimately show *?case*
by *force*
next
case(*cPar2 xQ' A_P Ψ_P*)
from ‹$A_P \sharp* (\!|\nu x|\!)Q$› ‹$A_P \sharp* x$› **have** $A_P \sharp* Q$ **by** *simp*
note ‹$\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)Q \longmapsto \alpha \prec xQ'$›
moreover have *FrP*: *extractFrame P* = $\langle A_P, \Psi_P\rangle$ **by** *fact*
with ‹$x \sharp P$› ‹$A_P \sharp* x$› **have** $x \sharp \Psi_P$ **and** $x \sharp A_P$
by(*force dest*: *extractFrameFresh*)+
with ‹$x \sharp \Psi$› **have** $x \sharp \Psi \otimes \Psi_P$ **by** *force*
moreover note ‹$x \sharp \alpha$›
moreover from ‹$x \sharp P \parallel xQ'$› **have** $x \sharp xQ'$ **by** *simp*
moreover from *FrP* ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \alpha$› **have** *bn* $\alpha \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*
with ‹*bn* $\alpha \sharp* \Psi$› **have** *bn* $\alpha \sharp* (\Psi \otimes \Psi_P)$ **by** *force*
ultimately show *?case* **using** ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$x \sharp \alpha$› ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \alpha$› ‹*bn* $\alpha \sharp* \Psi$›
proof(*induct rule*: *resCases'*)
case(*cOpen M xvec1 xvec2 y N Q'*)
from ‹$x \sharp M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle$› **have** $x \sharp$ *xvec1* **and** $x \neq y$ **and** $x \sharp$ *xvec2* **by** *simp*+
from ‹*bn*$(M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle) \sharp* \Psi$› **have** $y \sharp \Psi$ **by** *simp*
note ‹$\Psi \otimes \Psi_P \rhd ([(x, y)] \cdot Q) \longmapsto M(\!|\nu *(xvec1 @ xvec2)|\!)\langle N\rangle \prec Q'$› *FrP*
moreover from ‹*bn*$(M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle) \sharp* P$› **have** $(xvec1 @ xvec2) \sharp* P$ **and** $y \sharp P$ **by** *simp*+
moreover from ‹$A_P \sharp* (M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle)$› **have** $A_P \sharp* (M(\!|\nu *(xvec1 @ xvec2)|\!)\langle N\rangle)$ **and** $y \sharp A_P$ **by** *simp*+
moreover from ‹$A_P \sharp* Q$› ‹$x \sharp A_P$› ‹$y \sharp A_P$› **have** $A_P \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
ultimately have $\Psi \rhd P \parallel ([(x, y)] \cdot Q) \longmapsto M(\!|\nu *(xvec1 @ xvec2)|\!)\langle N\rangle \prec (P \parallel Q')$
using ‹$A_P \sharp* \Psi$›
by(*rule-tac Par2*) (*assumption* | *simp*)+
hence $\Psi \rhd (\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) \longmapsto M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle \prec (P \parallel Q')$
using ‹$y \in$ *supp N*› ‹$y \sharp \Psi$› ‹$y \sharp M$› ‹$y \sharp$ *xvec1*› ‹$y \sharp$ *xvec2*›
by(*rule Open*)
with ‹$x \sharp P$› ‹$y \sharp P$› ‹$y \sharp Q$› **have** $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto M(\!|\nu *(xvec1 @ y \# xvec2)|\!)\langle N\rangle \prec (P \parallel Q')$
by(*subst alphaRes*[**where** *y*=*y*]) (*simp add*: *fresh-left calc-atm eqvts*)+
moreover have $(\Psi, P \parallel Q', P \parallel Q') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*

next
case(*cRes* Q')
from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› *FrP* ‹*bn* $\alpha \sharp^* P$›
have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$ **using** ‹$A_P \sharp^* \Psi$› ‹$A_P \sharp^* Q$› ‹$A_P \sharp^* \alpha$›
by(*rule Par2*)
hence $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto \alpha \prec (\!|\nu x|\!)(P \parallel Q')$ **using** ‹$x \sharp \Psi$› ‹$x \sharp \alpha$›
by(*rule Scope*)
moreover from ‹$x \sharp \Psi$› ‹$x \sharp P$› **have** $(\Psi, (\!|\nu x|\!)((\!|\nu * []|\!)(P \parallel Q')), (\!|\nu * []|\!)(P \parallel (\!|\nu x|\!) Q')) \in Rel$
by(*rule-tac C2*) *auto*
ultimately show *?case*
by *force*
qed
next
case(*cComm1* Ψ_Q M N P' A_P Ψ_P K *xvec* xQ' A_Q)
have *QTrans*: $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!) Q \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec xQ'$ **and** *FrQ*: *extractFrame*$((\!|\nu x|\!) Q) = \langle A_Q, \Psi_Q\rangle$ **by** *fact+*
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$ **and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact+*
have $x \sharp (\!|\nu x|\!) Q$ **by**(*simp add*: *abs-fresh*)
with *QTrans* **have** $x \sharp N$ **and** $x \sharp xQ'$ **using** ‹*xvec* $\sharp^* x$› ‹*xvec* $\sharp^* K$› ‹*distinct xvec*›
by(*force intro*: *outputFreshDerivative*)+
from *PTrans* ‹$x \sharp P$› ‹$x \sharp N$› **have** $x \sharp P'$ **by**(*rule inputFreshDerivative*)
from ‹$x \sharp (\!|\nu x|\!) Q$› *FrQ* ‹$A_Q \sharp^* x$› **have** $x \sharp \Psi_Q$
by(*drule-tac extractFrameFresh*) *auto*
from ‹$x \sharp P$› *FrP* ‹$A_P \sharp^* x$› **have** $x \sharp \Psi_P$
by(*drule-tac extractFrameFresh*) *auto*
from ‹$A_P \sharp^* (\!|\nu x|\!) Q$› ‹$A_P \sharp^* x$› **have** $A_P \sharp^* Q$ **by** *simp*
from ‹$A_Q \sharp^* (\!|\nu x|\!) Q$› ‹$A_Q \sharp^* x$› **have** $A_Q \sharp^* Q$ **by** *simp*
from *PTrans FrP* ‹*distinct* A_P› ‹$x \sharp P$› ‹$A_Q \sharp^* P$› ‹*xvec* $\sharp^* P$› ‹$A_P \sharp^* \Psi$› ‹$A_P \sharp^* \Psi_Q$› ‹$A_P \sharp^* x$› ‹$A_P \sharp^* A_Q$› ‹$A_P \sharp^* P$› ‹$A_P \sharp^* M$› ‹$A_P \sharp^*$ *xvec*›
obtain M' **where** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $x \sharp M'$ **and** $A_Q \sharp^* M'$ **and** *xvec* $\sharp^* M'$
by(*rule-tac* B=x#*xvec*@A_Q **in** *obtainPrefix*) (*assumption* | *force simp add*: *fresh-star-list-cons*)+
hence *MeqM'*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow M'$ **by**(*metis statEqEnt Associativity Commutativity Composition*)
with ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*blast intro*: *chanEqTrans chanEqSym*)
hence $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$ **by**(*metis statEqEnt Associativity Commutativity Composition*)
with *QTrans FrQ* ‹*distinct* A_Q› ‹$A_Q \sharp^* \Psi$› ‹$A_Q \sharp^* \Psi_P$› ‹$A_Q \sharp^* ((\!|\nu x|\!) Q)$› ‹$A_Q \sharp^* K$› ‹$A_Q \sharp^* M'$›
have $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!) Q \longmapsto M'(\!|\nu * xvec|\!)\langle N\rangle \prec xQ'$
by(*force intro*: *outputRenameSubject*)
moreover from ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› **have** $x \sharp \Psi \otimes \Psi_P$ **by** *force*
moreover from ‹*xvec* $\sharp^* x$› **have** $x \sharp$ *xvec* **by** *simp*
with ‹$x \sharp M'$› ‹$x \sharp N$› **have** $x \sharp M'(\!|\nu * xvec|\!)\langle N\rangle$ **by** *simp*

moreover note ‹$x \sharp xQ'$›
moreover from ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* \Psi_P$› **have** $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* (\Psi \otimes \Psi_P)$ **by** *force*
moreover from ‹$xvec \sharp* \langle\!|\nu x|\!\rangle Q$› ‹$x \sharp xvec$› **have** $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* Q$ **by** *simp*
moreover from ‹$xvec \sharp* P$› **have** $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* P$ **by** *simp*
from ‹$xvec \sharp* \Psi$› **have** $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* \Psi$ **by** *simp*
from ‹$A_Q \sharp* xvec$› ‹$A_Q \sharp* M'$› ‹$A_Q \sharp* N$› **have** $A_Q \sharp* (M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle)$ **by** *simp*
have $object(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = Some\ N$ **by** *simp*
have $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = xvec$ **by** *simp*
have $subject(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = Some\ M'$ **by** *simp*
from ‹$xvec \sharp* M'$› **have** $bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* subject(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle)$ **by** *simp*
ultimately show *?case*
using ‹$x \sharp M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle$› ‹$bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* P$› ‹$bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) \sharp* \Psi$› ‹$object(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = Some\ N$›
‹$bn(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = xvec$› ‹$subject(M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle) = Some\ M'$› ‹$A_Q \sharp* (M'\langle\!|\nu * xvec|\!\rangle\langle N\rangle)$›
proof(*induct rule*: *resCases*)
case(*cOpen M'' xvec1 xvec2 y N' Q'*)
from ‹$x \sharp M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle$› **have** $x \sharp xvec1$ **and** $x \neq y$ **and** $x \sharp xvec2$ **and** $x \sharp M''$
by *simp+*
from ‹$bn(M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle) \sharp* P$› **have** $(xvec1 @ xvec2) \sharp* P$ **and** $y \sharp P$ **by** *simp+*
from ‹$A_Q \sharp* (M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle)$› **have** $(xvec1 @ xvec2) \sharp* A_Q$ **and** $y \sharp A_Q$ **and** $A_Q \sharp* M''$ **by** *simp+*
from ‹$bn(M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle) \sharp* \Psi$› **have** $(xvec1 @ xvec2) \sharp* \Psi$ **and** $y \sharp \Psi$ **by** *simp+*
from ‹$object(M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle) = Some\ N$› **have** $N = N'$ **by** *simp*
from ‹$bn(M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle) = xvec$› **have** $xvec = xvec1 @ y \# xvec2$ **by** *simp*
from ‹$subject(M''\langle\!|\nu * (xvec1 @ y \# xvec2)|\!\rangle\langle N'\rangle) = Some\ M'$› **have** $M' = M''$ **by** *simp*
from ‹$x \sharp P$› ‹$y \sharp P$› ‹$x \neq y$› ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M\langle\!|N|\!\rangle \prec P'$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M\langle\!|([(y, x)] \cdot N)|\!\rangle \prec ([(y, x)] \cdot P')$
by(*rule-tac xvec*=[*y*] **in** *inputAlpha*) (*auto simp add*: *calc-atm*)
hence *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M\langle\!|([(x, y)] \cdot N)|\!\rangle \prec ([(x, y)] \cdot P')$
by(*simp add*: *name-swap*)
have *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto M''\langle\!|\nu * (xvec1 @ xvec2)|\!\rangle\langle([(x, y)] \cdot N')\rangle \prec ([(x, y)] \cdot Q')$ **by** *fact*
with ‹$A_Q \sharp* x$› ‹$y \sharp A_Q$› ‹*distinct xvec1*› ‹*distinct xvec2*› ‹$xvec1 \sharp* xvec2$› ‹$xvec1 \sharp* M''$› ‹$xvec2 \sharp* M''$›
‹$(xvec1 @ xvec2) \sharp* A_Q$›
have $A_Q \sharp* ([(x, y)] \cdot Q')$ **using** ‹$A_Q \sharp* Q$›
by(*rule-tac outputFreshChainDerivative*(*2*)) (*assumption* | *simp*)+

from ‹*extractFrame*($(\!|\nu x|\!)\,Q$) = $\langle A_Q,\ \Psi_Q\rangle$› **have** *FrxQ*: $(\!|\nu x|\!)$(*extractFrame* Q) = $\langle A_Q,\ \Psi_Q\rangle$ **by** *simp*
then obtain $z\ A_Q{'}$ **where** *A*: $A_Q = z\#A_Q{'}$ **by**(*case-tac* A_Q) *auto*
with ‹A_Q ♯* Ψ› ‹A_Q ♯* P› ‹A_Q ♯* P'› ‹A_Q ♯* Ψ_P› ‹A_Q ♯* Q› ‹(*xvec1*@*xvec2*) ♯* A_Q› ‹A_Q ♯* M''› ‹A_Q ♯* ([(x, y)] · Q')› ‹y ♯ A_Q› ‹A_Q ♯* N›
have $A_Q{'}$ ♯* Ψ **and** $A_Q{'}$ ♯* P **and** $A_Q{'}$ ♯* Ψ_P **and** $A_Q{'}$ ♯* Q
and z ♯ Ψ **and** z ♯ P **and** z ♯ P' **and** z ♯ Ψ_P **and** z ♯ Q **and** z ♯ *xvec1* **and** z ♯ *xvec2*
and z ♯ M'' **and** z ♯ ([(x, y)] · Q') **and** $A_Q{'}$ ♯* M'' **and** $z \neq y$ **and** z ♯ (*xvec1*@*xvec2*)
by *auto*
from A ‹A_P ♯* A_Q› **have** A_P ♯* $A_Q{'}$ **and** z ♯ A_P **by** *simp+*
from A ‹A_Q ♯* x› **have** $x \neq z$ **and** x ♯ $A_Q{'}$ **by** *simp+*

from ‹*distinct* A_Q› A **have** z ♯ $A_Q{'}$
by(*induct* $A_Q{'}$) (*auto simp add*: *fresh-list-nil fresh-list-cons*)
from *PTrans* ‹x ♯ P› ‹z ♯ P› ‹$x \neq z$› **have** $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|([(x,\ z)] \cdot [(x,\ y)] \cdot N)|\!) \prec ([(x,\ z)] \cdot [(x,\ y)] \cdot P')$
by(*rule-tac xvec*=[x] **in** *inputAlpha*) (*auto simp add*: *calc-atm*)
moreover note *FrP*
moreover from *QTrans* **have** ([(x, z)] · ($\Psi \otimes \Psi_P$)) $\rhd$ ([(x, z)] · Q) $\longmapsto$([(x, z)] · ($M''(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N')\rangle \prec ([(x,\ y)] \cdot Q')$))
by(*rule semantics.eqvt*)
with ‹x ♯ Ψ› ‹z ♯ Ψ› ‹x ♯ Ψ_P› ‹z ♯ Ψ_P› ‹x ♯ M''› ‹z ♯ M''› ‹x ♯ *xvec1*› ‹x ♯ *xvec2*› ‹z ♯ *xvec1*› ‹z ♯ *xvec2*›
have $\Psi \otimes \Psi_P \rhd ([(x,\ z)] \cdot Q) \longmapsto M''(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ z)] \cdot [(x,\ y)] \cdot N')\rangle \prec ([(x,\ z)] \cdot [(x,\ y)] \cdot Q')$
by(*simp add*: *eqvts*)
moreover from A ‹A_Q ♯* x› *FrxQ* **have** *extractFrame*([(x, z)] · Q) = $\langle A_Q{'},\ \Psi_Q\rangle$
by(*clarsimp simp add*: *alpha' eqvts frame.inject fresh-list-cons name-swap*)
moreover from ‹A_P ♯* Q› **have** ([(x, z)] · A_P) ♯* ([(x, z)] · Q)
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹A_P ♯* x› ‹z ♯ A_P› **have** A_P ♯* ([(x, z)] · Q) **by** *simp*
moreover from ‹$A_Q{'}$ ♯* Q› **have** ([(x, z)] · $A_Q{'}$) ♯* ([(x, z)] · Q)
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹x ♯ $A_Q{'}$› ‹z ♯ $A_Q{'}$› **have** $A_Q{'}$ ♯* ([(x, z)] · Q) **by** *simp*
ultimately have $\Psi \rhd (P \parallel ([(x,\ z)] \cdot Q)) \longmapsto \tau \prec (\!|\nu*(xvec1@xvec2)|\!)(([(x,\ z)] \cdot [(x,\ y)] \cdot P') \parallel ([(x,\ z)] \cdot [(x,\ y)] \cdot Q'))$
using *MeqM'* ‹M'=M''› ‹N=N'› ‹A_P ♯* Ψ› ‹A_P ♯* P› ‹A_P ♯* $A_Q{'}$› ‹$A_Q{'}$ ♯* Ψ› ‹$A_Q{'}$ ♯* P› ‹(*xvec1*@*xvec2*) ♯* P› ‹A_P ♯* M› ‹$A_Q{'}$ ♯* M''›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
with‹z ♯ Ψ› **have** $\Psi \rhd (\!|\nu z|\!)(P \parallel ([(x,\ z)] \cdot Q)) \longmapsto \tau \prec (\!|\nu z|\!)((\!|\nu*(xvec1@xvec2)|\!)(([(x,\ z)] \cdot [(x,\ y)] \cdot P') \parallel ([(x,\ z)] \cdot [(x,\ y)] \cdot Q')))$
by(*rule-tac Scope*) *auto*
moreover from ‹x ♯ P› ‹z ♯ P› ‹z ♯ Q› **have** $(\!|\nu z|\!)(P \parallel ([(x,\ z)] \cdot Q)) = (\!|\nu x|\!)([(x,\ z)] \cdot (P \parallel ([(x,\ z)] \cdot Q)))$
by(*subst alphaRes*[*of x*]) (*auto simp add*: *calc-atm fresh-left name-swap*)
with ‹x ♯ P› ‹z ♯ P› **have** $(\!|\nu z|\!)(P \parallel ([(x,\ z)] \cdot Q)) = (\!|\nu x|\!)(P \parallel Q)$

by(*simp add*: *eqvts*)
moreover from ‹$z \neq y$› ‹$x \neq z$› ‹$z \sharp P'$› ‹$z \sharp [(x, y)] \cdot Q'$› **have** $(\!|\nu z|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(([(x, z)] \cdot [(x, y)] \cdot P') \parallel ([(x, z)] \cdot [(x, y)] \cdot Q'))) = (\!|\nu x|\!)([(x, z)] \cdot ((\!|\nu{*}(xvec1@xvec2)|\!)(([(x, z)] \cdot [(x, y)] \cdot P') \parallel ([(x, z)] \cdot [(x, y)] \cdot Q'))))$
by(*subst alphaRes*[*of x*]) (*auto simp add*: *resChainFresh fresh-left calc-atm name-swap*)
with ‹$x \sharp xvec1$› ‹$x \sharp xvec2$› ‹$z \sharp xvec1$› ‹$z \sharp xvec2$› **have** $(\!|\nu z|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(([(x, z)] \cdot [(x, y)] \cdot P') \parallel ([(x, z)] \cdot [(x, y)] \cdot Q'))) = (\!|\nu x|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(([(x, y)] \cdot P') \parallel ([(x, y)] \cdot Q')))$
by(*simp add*: *eqvts*)
moreover from ‹$x \sharp P'$› ‹$x \sharp Q'$› ‹$x \sharp xvec1$› ‹$x \sharp xvec2$› ‹$y \sharp xvec1$› ‹$y \sharp xvec2$›
have $(\!|\nu x|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(([(x, y)] \cdot P') \parallel ([(x, y)] \cdot Q'))) = (\!|\nu y|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(P' \parallel Q'))$
by(*subst alphaRes*[*of y*]) (*auto simp add*: *resChainFresh calc-atm eqvts fresh-left name-swap*)
ultimately have $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto \tau \prec (\!|\nu y|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(P' \parallel Q'))$
by *simp*
moreover from ‹$y \sharp \Psi$› ‹$(xvec1@xvec2) \sharp* \Psi$› ‹$xvec{=}xvec1@y\#xvec2$›
have $(\Psi, (\!|\nu y|\!)((\!|\nu{*}(xvec1@xvec2)|\!)(P' \parallel Q')), (\!|\nu{*}xvec|\!)(P' \parallel Q')) \in Rel$
by(*force intro*: *C3 simp add*: *resChainAppend*)
ultimately show *?case* **by** *blast*
next
case(*cRes Q'*)
have *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$ **by** *fact*
with ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* xvec$› ‹$xvec \sharp* M'$› ‹*distinct xvec*› **have** $A_Q \sharp* Q'$
by(*force dest*: *outputFreshChainDerivative*)

with ‹$extractFrame((\!|\nu x|\!)Q) = \langle A_Q, \Psi_Q\rangle$› **have** *FrxQ*: $(\!|\nu x|\!)(extractFrame\ Q) = \langle A_Q, \Psi_Q\rangle$ **by** *simp*
then obtain $y\ A_Q'$ **where** *A*: $A_Q = y\#A_Q'$ **by**(*case-tac* A_Q) *auto*
with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* P'$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* xvec$› ‹$A_Q \sharp* M'$› ‹$A_Q \sharp* Q'$›
have $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* P$ **and** $A_Q' \sharp* \Psi_P$ **and** $A_Q' \sharp* Q$ **and** $A_Q \sharp* xvec$ **and** $A_Q \sharp* Q'$
and $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp P'$ **and** $y \sharp \Psi_P$ **and** $y \sharp Q$ **and** $y \sharp xvec$ **and** $y \sharp M'$ **and** $y \sharp Q'$
and $A_Q' \sharp* M'$
by(*simp*)+
from *A* ‹$A_P \sharp* A_Q$› **have** $A_P \sharp* A_Q'$ **and** $y \sharp A_P$ **by**(*simp add*: *fresh-star-list-cons*)+
from *A* ‹$A_Q \sharp* x$› **have** $x \neq y$ **and** $x \sharp A_Q'$ **by**(*simp add*: *fresh-list-cons*)+

with *A* ‹*distinct* A_Q›
by(*induct* A_Q') (*auto simp add*: *fresh-list-nil fresh-list-cons*)

from ‹$x \sharp P$› ‹$y \sharp P$› ‹$x \neq y$› ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$›
have $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|([(y, x)] \cdot N)|\!) \prec [(y, x)] \cdot P'$

by(*rule-tac xvec*=[*y*] **in** *inputAlpha*) (*auto simp add*: *calc-atm*)
hence $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|([(x, y)] \cdot N)|\!) \prec [(x, y)] \cdot P'$
by(*simp add*: *name-swap*)
moreover note *FrP*
moreover from $\langle \Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|\nu * xvec|\!)\langle N\rangle \prec Q'\rangle$ **have** $([(x, y)] \cdot (\Psi \otimes \Psi_P)) \rhd ([(x, y)] \cdot Q) \longmapsto ([(x, y)] \cdot (M'(\!|\nu * xvec|\!)\langle N\rangle \prec Q'))$
by(*rule semantics.eqvt*)
with $\langle x \sharp \Psi\rangle$ $\langle y \sharp \Psi\rangle$ $\langle x \sharp \Psi_P\rangle$ $\langle y \sharp \Psi_P\rangle$ $\langle x \sharp M'\rangle$ $\langle y \sharp M'\rangle$ $\langle x \sharp xvec\rangle$ $\langle y \sharp xvec\rangle$
have $\Psi \otimes \Psi_P \rhd ([(x, y)] \cdot Q) \longmapsto M'(\!|\nu * xvec|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot Q')$
by(*simp add*: *eqvts*)
moreover from A $\langle A_Q \sharp* x\rangle$ *FrxQ* **have** *FrQ*: *extractFrame*$([(x, y)] \cdot Q) = \langle A_Q{}', \Psi_Q\rangle$
by(*clarsimp simp add*: *alpha' eqvts frame.inject fresh-list-cons name-swap*)
moreover from $\langle A_P \sharp* Q\rangle$ **have** $([(x, y)] \cdot A_P) \sharp* ([(x, y)] \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* x\rangle$ $\langle y \sharp A_P\rangle$ **have** $A_P \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* Q\rangle$ **have** $([(x, y)] \cdot A_Q{}') \sharp* ([(x, y)] \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp A_Q{}'\rangle$ $\langle y \sharp A_Q{}'\rangle$ **have** $A_Q{}' \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
ultimately have $\Psi \rhd (P \parallel ([(x, y)] \cdot Q)) \longmapsto \tau \prec (\!|\nu * xvec|\!)([(x, y)] \cdot P') \parallel ([(x, y)] \cdot Q')$
using *MeqM'* $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* A_Q{}'\rangle$ $\langle A_Q{}' \sharp* \Psi\rangle$ $\langle A_Q{}' \sharp* P\rangle$ $\langle xvec \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_Q{}' \sharp* M'\rangle$
by(*rule-tac Comm1*) (*assumption* | *simp*)+
with$\langle y \sharp \Psi\rangle$ **have** $\Psi \rhd (\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) \longmapsto \tau \prec (\!|\nu y|\!)((\!|\nu * xvec|\!)(([(x, y)] \cdot P') \parallel ([(x, y)] \cdot Q')))$
by(*rule-tac Scope*) *auto*
moreover from $\langle x \sharp P\rangle$ $\langle y \sharp P\rangle$ $\langle y \sharp Q\rangle$ **have** $(\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) = (\!|\nu x|\!)([(x, y)] \cdot (P \parallel ([(x, y)] \cdot Q)))$
by(*subst alphaRes*[*of x*]) (*auto simp add*: *calc-atm fresh-left name-swap*)
with $\langle x \sharp P\rangle$ $\langle y \sharp P\rangle$ **have** $(\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) = (\!|\nu x|\!)(P \parallel Q)$
by(*simp add*: *eqvts*)
moreover from $\langle y \sharp P'\rangle$ $\langle y \sharp Q'\rangle$ $\langle x \sharp xvec\rangle$ $\langle y \sharp xvec\rangle$ **have** $(\!|\nu y|\!)((\!|\nu * xvec|\!)(([(x, y)] \cdot P') \parallel ([(x, y)] \cdot Q'))) = (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))$
by(*subst alphaRes*[*of y*]) (*auto simp add*: *resChainFresh calc-atm eqvts fresh-left name-swap*)
ultimately have $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto \tau \prec (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))$
by *simp*
moreover from $\langle x \sharp \Psi\rangle$ $\langle x \sharp P'\rangle$ $\langle xvec \sharp* \Psi\rangle$ **have** $(\Psi, (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q')), (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!)Q')) \in Rel$
by(*rule C2*)
ultimately show *?case* **by** *blast*
qed
next
case(*cComm2* Ψ_Q M *xvec* N P' A_P Ψ_P K xQ' A_Q)
have *QTrans*: $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)Q \longmapsto K(\!|N|\!) \prec xQ'$ **and** *FrQ*: *extractFrame*$((\!|\nu x|\!)Q) = \langle A_Q, \Psi_Q\rangle$ **by** *fact+*

have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*+

from *PTrans* ‹$x \sharp P$› **have** $x \sharp N$ **and** $x \sharp P'$ **using** ‹*xvec* $\sharp*$ *x*› ‹*xvec* $\sharp*$ *M*› ‹*distinct xvec*›

by(*force intro*: *outputFreshDerivative*)+

have $x \sharp (\!|\nu x|\!) Q$ **by**(*simp add*: *abs-fresh*)

with *FrQ* ‹$A_Q \sharp* x$› **have** $x \sharp \Psi_Q$

by(*drule-tac extractFrameFresh*) *auto*

from ‹$x \sharp P$› *FrP* ‹$A_P \sharp* x$› **have** $x \sharp \Psi_P$

by(*drule-tac extractFrameFresh*) *auto*

from ‹$A_P \sharp* (\!|\nu x|\!) Q$› ‹$A_P \sharp* x$› **have** $A_P \sharp* Q$ **by** *simp*

from ‹$A_Q \sharp* (\!|\nu x|\!) Q$› ‹$A_Q \sharp* x$› **have** $A_Q \sharp* Q$ **by** *simp*

from ‹*xvec* $\sharp*$ *x*› ‹*xvec* $\sharp*$ $(\!|\nu x|\!) Q$› **have** *xvec* $\sharp*$ *Q* **by** *simp*

from *PTrans FrP* ‹*distinct* A_P› ‹$x \sharp P$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_Q$› ‹$A_P \sharp* x$› ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹*xvec* $\sharp*$ *M*› ‹*distinct xvec*›

obtain M' **where** $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $x \sharp M'$ **and** $A_Q \sharp* M'$

by(*rule-tac* $B{=}x\#A_Q$ **in** *obtainPrefix*) (*assumption* | *force simp add*: *fresh-star-list-cons*)+

hence *MeqM'*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow M'$

by(*metis statEqEnt Associativity Commutativity Composition*)

with ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M'$

by(*blast intro*: *chanEqTrans chanEqSym*)

hence $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$

by(*metis statEqEnt Associativity Commutativity Composition*)

with *QTrans FrQ* ‹*distinct* A_Q› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* (\!|\nu x|\!) Q)$› ‹$A_Q \sharp* K$› ‹$A_Q \sharp* M'$›

have $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!) Q \longmapsto M'(\!|N|\!) \prec xQ'$ **by**(*force intro*: *inputRenameSubject*)

moreover from ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› **have** $x \sharp \Psi \otimes \Psi_P$ **by** *force*

moreover note ‹$x \sharp M'$› ‹$x \sharp N$›

moreover from *QTrans* ‹$x \sharp N$› **have** $x \sharp xQ'$ **by**(*force dest*: *inputFreshDerivative simp add*: *abs-fresh*)

ultimately show *?case*

proof(*induct rule*: *resInputCases*)

case(*cRes* Q')

have *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q'$ **by** *fact*

with ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* N$› **have** $A_Q \sharp* Q'$

by(*rule-tac inputFreshChainDerivative*)

with ‹*extractFrame*$(\!|\nu x|\!) Q) = \langle A_Q, \Psi_Q\rangle$› **have** *FrxQ*: $(\!|\nu x|\!)$(*extractFrame* $Q) = \langle A_Q, \Psi_Q\rangle$ **by** *simp*

then obtain *y* $A_Q{'}$ **where** *A*: $A_Q = y\#A_Q{'}$ **by**(*case-tac* A_Q) *auto*

with ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* P'$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp*$ *xvec*› ‹$A_Q \sharp* M'$› ‹$A_Q \sharp* Q'$› ‹$A_Q \sharp* N$›

have $A_Q{'} \sharp* \Psi$ **and** $A_Q{'} \sharp* P$ **and** $A_Q{'} \sharp* \Psi_P$ **and** $A_Q{'} \sharp* Q$ **and** $A_Q \sharp*$ *xvec* **and** $A_Q \sharp* Q'$

and $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp P'$ **and** $y \sharp \Psi_P$ **and** $y \sharp Q$ **and** $y \sharp$ *xvec* **and** $y \sharp M'$ **and** $y \sharp Q'$ **and** $y \sharp N$

and $A_Q{}' \sharp* M'$
by(*simp*)+
from $A \langle A_P \sharp* A_Q \rangle$ **have** $A_P \sharp* A_Q{}'$ **and** $y \sharp A_P$ **by**(*simp add*: *fresh-star-list-cons*)+
from $A \langle A_Q \sharp* x \rangle$ **have** $x \neq y$ **and** $x \sharp A_Q{}'$ **by**(*simp add*: *fresh-list-cons*)+

with $A \langle$*distinct* $A_Q \rangle$ **have** $y \sharp A_Q{}'$
by(*induct* $A_Q{}'$) (*auto simp add*: *fresh-list-nil fresh-list-cons*)

note *PTrans FrP*
moreover from $\langle \Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q' \rangle$ **have** $([(x, y)] \cdot (\Psi \otimes \Psi_P)) \rhd ([(x, y)] \cdot Q) \longmapsto ([(x, y)] \cdot (M'(\!|N|\!) \prec Q'))$
by(*rule semantics.eqvt*)
with $\langle x \sharp \Psi \rangle \langle y \sharp \Psi \rangle \langle x \sharp \Psi_P \rangle \langle y \sharp \Psi_P \rangle \langle x \sharp M' \rangle \langle y \sharp M' \rangle \langle x \sharp N \rangle \langle y \sharp N \rangle$
have $\Psi \otimes \Psi_P \rhd ([(x, y)] \cdot Q) \longmapsto M'(\!|N|\!) \prec ([(x, y)] \cdot Q')$
by(*simp add*: *eqvts*)
moreover from $A \langle A_Q \sharp* x \rangle$ *FrxQ* **have** *FrQ*: $extractFrame([(x, y)] \cdot Q) = \langle A_Q{}', \Psi_Q \rangle$ **and** $y \sharp extractFrame\ Q$
by(*clarsimp simp add*: *alpha' eqvts frame.inject fresh-list-cons name-swap*)+
moreover from $\langle A_P \sharp* Q \rangle$ **have** $([(x, y)] \cdot A_P) \sharp* ([(x, y)] \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle A_P \sharp* x \rangle \langle y \sharp A_P \rangle$ **have** $A_P \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
moreover from $\langle A_Q{}' \sharp* Q \rangle$ **have** $([(x, y)] \cdot A_Q{}') \sharp* ([(x, y)] \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle x \sharp A_Q{}' \rangle \langle y \sharp A_Q{}' \rangle$ **have** $A_Q{}' \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
moreover from $\langle xvec \sharp* Q \rangle$ **have** $([(x, y)] \cdot xvec) \sharp* ([(x, y)] \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* x \rangle \langle y \sharp xvec \rangle$ **have** $xvec \sharp* ([(x, y)] \cdot Q)$ **by** *simp*
ultimately have $\Psi \rhd (P \parallel ([(x, y)] \cdot Q)) \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel ([(x, y)] \cdot Q'))$
using *MeqM'* $\langle A_P \sharp* \Psi \rangle \langle A_P \sharp* P \rangle \langle A_P \sharp* A_Q{}' \rangle \langle A_Q{}' \sharp* \Psi \rangle \langle A_Q{}' \sharp* P \rangle \langle A_P \sharp* M \rangle \langle A_Q{}' \sharp* M' \rangle$
by(*rule-tac Comm2*) (*assumption* | *simp*)+
with$\langle y \sharp \Psi \rangle$ **have** $\Psi \rhd (\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) \longmapsto \tau \prec (\!|\nu y|\!)((\!|\nu * xvec|\!)(P' \parallel ([(x, y)] \cdot Q')))$
by(*rule-tac Scope*) *auto*
moreover from $\langle x \sharp P \rangle \langle y \sharp P \rangle \langle y \sharp Q \rangle$ **have** $(\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) = (\!|\nu x|\!)([(x, y)] \cdot (P \parallel ([(x, y)] \cdot Q)))$
by(*subst alphaRes*[*of x*]) (*auto simp add*: *calc-atm fresh-left name-swap*)
with $\langle x \sharp P \rangle \langle y \sharp P \rangle$ **have** $(\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) = (\!|\nu x|\!)(P \parallel Q)$
by(*simp add*: *eqvts*)
moreover from $\langle x \sharp P' \rangle \langle y \sharp P' \rangle \langle y \sharp Q' \rangle \langle xvec \sharp* x \rangle \langle y \sharp xvec \rangle$ **have** $(\!|\nu y|\!)((\!|\nu * xvec|\!)(P' \parallel ([(x, y)] \cdot Q'))) = (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))$
by(*subst alphaRes*[*of y*]) (*auto simp add*: *resChainFresh calc-atm eqvts fresh-left name-swap*)
ultimately have $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \longmapsto \tau \prec (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))$
by *simp*
moreover from $\langle x \sharp \Psi \rangle \langle x \sharp P' \rangle \langle xvec \sharp* \Psi \rangle$ **have** $(\Psi, (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q')), (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!)Q')) \in Rel$
by(*rule C2*)

```
      ultimately show ?case by blast
    qed
  qed
 qed
qed

lemma scopeExtRight:
 fixes x   :: name
 and   P   :: ('a, 'b, 'c) psi
 and   Ψ   :: 'b
 and   Q   :: ('a, 'b, 'c) psi
 and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

 assumes x ♯ P
 and     x ♯ Ψ
 and     eqvt Rel
 and     C1: ⋀Ψ' R. (Ψ, R, R) ∈ Rel
 and     C2: ⋀y Ψ' R S zvec. ⟦y ♯ Ψ'; y ♯ R; zvec ♯* Ψ⟧ ⟹ (Ψ', (|ν*zvec|)(R ∥
(|νy|)S), (|νy|)((|ν*zvec|)(R ∥ S))) ∈ Rel

 shows Ψ ▷ P ∥ (|νx|)Q ⇝[Rel] (|νx|)(P ∥ Q)
proof −
 note ‹eqvt Rel› ‹x ♯ Ψ›
 moreover from ‹x ♯ P› have x ♯ P ∥ (|νx|)Q by(simp add: abs-fresh)
 moreover from ‹x ♯ P› have x ♯ (|νx|)(P ∥ Q) by(simp add: abs-fresh)
 ultimately show ?thesis
 proof(induct rule: simIFresh[of - - - - - ()])
  case(cSim α xPQ)
   from ‹bn α ♯* (P ∥ (|νx|)Q)› ‹x ♯ α› have bn α ♯* P and bn α ♯* Q by
simp+
  note ‹Ψ ▷ (|νx|)(P ∥ Q) ⟼α ≺ xPQ› ‹x ♯ Ψ› ‹x ♯ α› ‹x ♯ xPQ› ‹bn α ♯* Ψ›
  moreover from ‹bn α ♯* P› ‹bn α ♯* Q› have bn α ♯* (P ∥ Q) by simp
  ultimately show ?case using ‹bn α ♯* subject α› ‹x ♯ α›
  proof(induct rule: resCases)
   case(cOpen M xvec1 xvec2 y N PQ)
   from ‹x ♯ M(|ν*(xvec1@y#xvec2)|)⟨N⟩› have x ♯ xvec1 and x ≠ y and x ♯
xvec2 and x ♯ M by simp+
   from ‹xvec1 ♯* (P ∥ Q)› ‹xvec2 ♯* (P ∥ Q)› ‹y ♯ (P ∥ Q)›
   have (xvec1@xvec2) ♯* P and (xvec1@xvec2) ♯* Q and y ♯ P and y ♯ Q
    by simp+
  from ‹Ψ ▷ P ∥ Q ⟼ M(|ν*(xvec1@xvec2)|)⟨([(x, y)] • N)⟩ ≺ ([(x, y)] • PQ)›
  have ([(x, y)] • Ψ) ▷ ([(x, y)] • (P ∥ Q)) ⟼ ([(x, y)] • (M(|ν*(xvec1@xvec2)|)⟨([(x,
y)] • N)⟩ ≺ ([(x, y)] • PQ)))
    by(rule semantics.eqvt)
  with ‹x ♯ Ψ› ‹y ♯ Ψ› ‹x ♯ P› ‹y ♯ P› ‹x ♯ M› ‹y ♯ M› ‹x ♯ xvec1› ‹x ♯ xvec2›
‹y ♯ xvec1› ‹y ♯ xvec2›
   have Ψ ▷ P ∥ ([(x, y)] • Q) ⟼ M(|ν*(xvec1@xvec2)|)⟨N⟩ ≺ PQ
    by(simp add: eqvts)
   moreover from ‹xvec1 ♯* Ψ› ‹xvec2 ♯* Ψ› have (xvec1@xvec2) ♯* Ψ by
```

simp
moreover note ‹(*xvec1*@*xvec2*) ♯* *P*›
moreover from ‹(*xvec1*@*xvec2*) ♯* *Q*› **have** ([(*x*, *y*)] ∙ (*xvec1*@*xvec2*)) ♯* ([(*x*, *y*)] ∙ *Q*)
by(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*x* ♯ *xvec1*› ‹*x* ♯ *xvec2*› ‹*y* ♯ *xvec1*› ‹*y* ♯ *xvec2*› **have** (*xvec1*@*xvec2*) ♯* ([(*x*, *y*)] ∙ *Q*)
by(*auto simp add*: *eqvts*)
moreover from ‹*xvec1* ♯* *M*› ‹*xvec2* ♯* *M*› **have** (*xvec1*@*xvec2*) ♯* *M* **by** *simp*
ultimately show *?case*
proof(*induct rule*: *parOutputCases*[**where** *C*=*y*])
case(*cPar1 P′ A*$_Q$ Ψ$_Q$)
from ‹*y* ♯ *xvec1*› ‹*y* ♯ *xvec2*› **have** *y* ♯ (*xvec1*@*xvec2*) **by**(*auto simp add*: *fresh-list-append*)
with ‹Ψ ⊗ Ψ$_Q$ ▷ *P* ⟼*M*⦇*ν**(*xvec1*@*xvec2*)⦈⟨*N*⟩ ≺ *P′*› ‹(*xvec1*@*xvec2*) ♯* *M*› ‹*y* ♯ *P*›
‹*distinct xvec1*› ‹*distinct xvec2*› ‹*xvec1* ♯* *xvec2*›
have *y* ♯ *N* **by**(*force intro*: *outputFreshDerivative*)
with ‹*y* ∈ *supp N*› **have** *False* **by**(*simp add*: *fresh-def*)
thus *?case* **by** *simp*
next
case(*cPar2 Q′ A*$_P$ Ψ$_P$)
have *FrP*: *extractFrame P* = ⟨*A*$_P$, Ψ$_P$⟩ **by** *fact*
with ‹*y* ♯ *P*› ‹*A*$_P$ ♯* *y*› **have** *y* ♯ Ψ$_P$
apply(*drule-tac extractFrameFresh*)
by(*simp add*: *frameResChainFresh*) (*simp add*: *fresh-def name-list-supp*)
from ‹Ψ ⊗ Ψ$_P$ ▷ ([(*x*, *y*)] ∙ *Q*) ⟼*M*⦇*ν**(*xvec1*@*xvec2*)⦈⟨*N*⟩ ≺ *Q′*› ‹*y* ∈ *supp N*› ‹*y* ♯ Ψ› ‹*y* ♯ Ψ$_P$› ‹*y* ♯ *M*› ‹*y* ♯ *xvec1*› ‹*y* ♯ *xvec2*›
have Ψ ⊗ Ψ$_P$ ▷ ⦇*νy*⦈(([(*x*, *y*)] ∙ *Q*)) ⟼*M*⦇*ν**(*xvec1*@*y*#*xvec2*)⦈⟨*N*⟩ ≺ *Q′* **by**(*force intro*: *Open*)
with ‹*y* ♯ *Q*› **have** Ψ ⊗ Ψ$_P$ ▷ ⦇*νx*⦈*Q* ⟼*M*⦇*ν**(*xvec1*@*y*#*xvec2*)⦈⟨*N*⟩ ≺ *Q′*
by(*simp add*: *alphaRes*)
moreover from ‹*A*$_P$ ♯* ([(*x*, *y*)] ∙ *Q*)› **have** *A*$_P$ ♯* ⦇*νy*⦈([(*x*, *y*)] ∙ *Q*) **by**(*auto simp add*: *fresh-star-def abs-fresh*)
with ‹*y* ♯ *Q*› **have** *A*$_P$ ♯* ⦇*νx*⦈*Q* **by**(*simp add*: *alphaRes*)
ultimately have Ψ ▷ *P* ∥ (⦇*νx*⦈*Q*) ⟼*M*⦇*ν**(*xvec1*@*y*#*xvec2*)⦈⟨*N*⟩ ≺ (*P* ∥ *Q′*)
using *FrP* ‹(*xvec1*@*xvec2*) ♯* *P*› ‹*A*$_P$ ♯* Ψ› ‹*A*$_P$ ♯* *M*› ‹*y* ♯ *P*› ‹*A*$_P$ ♯* (*xvec1*@*xvec2*)› ‹*A*$_P$ ♯* *y*› ‹*A*$_P$ ♯* *N*›
by(*rule-tac Par2*) *auto*
moreover have (Ψ, *P* ∥ *Q′*, *P* ∥ *Q′*) ∈ *Rel* **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed
next
case(*cRes PQ*)
from ‹Ψ ▷ *P* ∥ *Q* ⟼α ≺ *PQ*› ‹*bn* α ♯* Ψ› ‹*bn* α ♯* *P*› ‹*bn* α ♯* *Q*› ‹*bn* α ♯* *subject* α›

show *?case*
proof(*induct rule*: *parCases*[**where** *C*=*x*])
case(*cPar1 P′ A*$_Q$ Ψ_Q)
note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$›
moreover with ‹$x \sharp P$› ‹$x \sharp \alpha$› ‹*bn* $\alpha \sharp* subject\ \alpha$› ‹*distinct*(*bn* α)›
have $x \sharp P'$ **by**(*force dest*: *freeFreshDerivative*)
with ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› **have** *extractFrame*($(\!|\nu x|\!)\, Q$) = $\langle (x \# A_Q), \Psi_Q \rangle$
by *simp*
moreover from ‹*bn* $\alpha \sharp* Q$› **have** *bn* $\alpha \sharp* (\!|\nu x|\!)\, Q$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$x \sharp \Psi$› ‹$A_Q \sharp* \Psi$› **have** $(x \# A_Q) \sharp* \Psi$ **by** *simp*
moreover from ‹$x \sharp P$› ‹$A_Q \sharp* P$› **have** $(x \# A_Q) \sharp* P$ **by** *simp*
moreover from ‹$x \sharp \alpha$› ‹$A_Q \sharp* \alpha$› **have** $(x \# A_Q) \sharp* \alpha$ **by** *simp*
ultimately have $\Psi \rhd P \parallel (\!|\nu x|\!)\, Q \longmapsto \alpha \prec (P' \parallel (\!|\nu x|\!)\, Q)$
by(*rule Par1*)
moreover from ‹$x \sharp P'$› ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu * []|\!)(P' \parallel ((\!|\nu x|\!)\, Q)), (\!|\nu x|\!)((\!|\nu * []|\!)(P' \parallel Q))) \in Rel$
by(*rule-tac C2*) *auto*
ultimately show *?case*
by *force*
next
case(*cPar2 Q′ A*$_P$ Ψ_P)
have *FrP*: *extractFrame P* = $\langle A_P, \Psi_P \rangle$ **by** *fact*
with ‹$x \sharp P$› ‹$A_P \sharp* x$› **have** $x \sharp \Psi_P$
apply(*drule-tac extractFrameFresh*)
by(*simp add*: *frameResChainFresh*) (*simp add*: *fresh-def name-list-supp*)
from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› ‹$x \sharp \alpha$›
have $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)\, Q \longmapsto \alpha \prec (\!|\nu x|\!)\, Q'$
by(*rule-tac Scope*) *auto*
moreover note *FrP* ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \Psi$›
moreover from ‹$A_P \sharp* Q$› **have** $A_P \sharp* (\!|\nu x|\!)\, Q$ **by**(*simp add*: *fresh-star-def abs-fresh*)
ultimately have $\Psi \rhd P \parallel (\!|\nu x|\!)\, Q \longmapsto \alpha \prec (P \parallel (\!|\nu x|\!)\, Q')$ **using** ‹$A_P \sharp* \alpha$›
by(*rule Par2*)
moreover from ‹$x \sharp P$› ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu * []|\!)(P \parallel ((\!|\nu x|\!)\, Q')), (\!|\nu x|\!)((\!|\nu * []|\!)(P \parallel Q'))) \in Rel$
by(*rule-tac C2*) *auto*
ultimately show *?case*
by *force*
next
case(*cComm1* Ψ_Q *M N P′ A*$_P$ Ψ_P *K xvec Q′ A*$_Q$)
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$ **and** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P \rangle$ **by** *fact*+
have *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$ **and** *FrQ*: *extractFrame Q* = $\langle A_Q, \Psi_Q \rangle$ **by** *fact*+
from *FrP* ‹$x \sharp P$› **have** $x \sharp \langle A_P, \Psi_P \rangle$ **by**(*drule-tac extractFrameFresh*) *simp*
with ‹$A_P \sharp* x$› **have** $x \sharp \Psi_P$ **by**(*simp add*: *frameResChainFresh*) (*simp add*: *fresh-def name-list-supp*)

from *PTrans FrP* ‹*distinct* A_P› ‹$x \sharp P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_Q$› ‹$A_P \sharp* x$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* A_Q$›
obtain M' **where** *MeqM'*: $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $x \sharp M'$ **and** $A_Q \sharp* M'$
by(*rule-tac* $B{=}x\#A_Q$ **in** *obtainPrefix*) (*assumption* | *force simp add*: *fresh-star-list-cons*)+

from *MeqM'* **have** *MeqM'*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity Composition Commutativity*)
with ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*blast intro*: *chanEqTrans chanEqSym*)
hence $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*metis statEqEnt Associativity Composition Commutativity*)
with *QTrans FrQ* ‹*distinct* A_Q› **have** *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto M'\langle\!|\nu{*}xvec|\!\rangle\langle N\rangle \prec Q'$ **using** ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹$A_Q \sharp* M'$›
by(*rule-tac outputRenameSubject*) (*assumption* | *force*)+
show *?case*
proof(*case-tac* $x \in supp\ N$)
note *PTrans FrP*
moreover assume $x \in supp\ N$
hence $\Psi \otimes \Psi_P \rhd \langle\!|\nu x|\!\rangle Q \longmapsto M'\langle\!|\nu{*}([]@(x\#xvec))|\!\rangle\langle N\rangle \prec Q'$ **using** *QTrans* ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› ‹$x \sharp M'$› ‹$xvec \sharp* x$›
by(*rule-tac Open*) (*assumption* | *force simp add*: *fresh-list-nil*)+
hence *QTrans*: $\Psi \otimes \Psi_P \rhd \langle\!|\nu x|\!\rangle Q \longmapsto M'\langle\!|\nu{*}(x\#xvec)|\!\rangle\langle N\rangle \prec Q'$ **by** *simp*
moreover from ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› **have** $extractFrame(\langle\!|\nu x|\!\rangle Q) = \langle(x\#A_Q), \Psi_Q\rangle$
by *simp*
moreover note *MeqM'* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$›
moreover from ‹$A_P \sharp* Q$› **have** $A_P \sharp* (\langle\!|\nu x|\!\rangle Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* x$› **have** $A_P \sharp* (x\#A_Q)$
by(*simp add*: *fresh-star-def fresh-list-cons*) (*auto simp add*: *fresh-def name-list-supp*)
moreover from ‹$x \sharp \Psi$› ‹$A_Q \sharp* \Psi$› **have** $(x\#A_Q) \sharp* \Psi$ **by** *simp*
moreover from ‹$x \sharp P$› ‹$A_Q \sharp* P$› **have** $(x\#A_Q) \sharp* P$ **by** *simp*
moreover from ‹$x \sharp M'$› ‹$A_Q \sharp* M'$› **have** $(x\#A_Q) \sharp* M'$ **by** *simp*
moreover from ‹$A_Q \sharp* Q$› **have** $(x\#A_Q) \sharp* (\langle\!|\nu x|\!\rangle Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$x \sharp P$› ‹$xvec \sharp* P$› **have** $(x\#xvec) \sharp* P$ **by**(*simp*)
ultimately have $\Psi \rhd P \parallel \langle\!|\nu x|\!\rangle Q \longmapsto \tau \prec \langle\!|\nu{*}(x\#xvec)|\!\rangle(P' \parallel Q')$ **using** ‹$A_P \sharp* M$›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
moreover have $(\Psi, \langle\!|\nu x|\!\rangle(\langle\!|\nu{*}xvec|\!\rangle(P' \parallel Q')), \langle\!|\nu x|\!\rangle(\langle\!|\nu{*}xvec|\!\rangle(P' \parallel Q'))) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *force*
next
note *PTrans FrP*
moreover assume $x \notin supp\ N$
hence $x \sharp N$ **by**(*simp add*: *fresh-def*)

with *QTrans* ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› ‹$x \sharp M'$› ‹$xvec \sharp* x$›
have *QTrans*: $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!)\, Q \longmapsto M'(\!|\nu * xvec|\!)\langle N\rangle \prec (\!|\nu x|\!)\, Q'$
by(*rule-tac Scope*) (*assumption* | *force*)+
moreover from *PTrans* ‹$x \sharp P$› ‹$x \sharp N$› **have** $x \sharp P'$ **by**(*rule inputFreshDerivative*)
with ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$› **have** *extractFrame*$((\!|\nu x|\!)\, Q) = \langle (x \# A_Q), \Psi_Q\rangle$
by *simp*
moreover note *MeqM'* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$›
moreover from ‹$A_P \sharp* Q$› **have** $A_P \sharp* ((\!|\nu x|\!)\, Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$A_P \sharp* A_Q$› ‹$A_P \sharp* x$› **have** $A_P \sharp* (x \# A_Q)$
by(*simp add*: *fresh-star-def fresh-list-cons*) (*auto simp add*: *fresh-def name-list-supp*)
moreover from ‹$x \sharp \Psi$› ‹$A_Q \sharp* \Psi$› **have** $(x \# A_Q) \sharp* \Psi$ **by** *simp*
moreover from ‹$x \sharp P$› ‹$A_Q \sharp* P$› **have** $(x \# A_Q) \sharp* P$ **by** *simp*
moreover from ‹$x \sharp M'$› ‹$A_Q \sharp* M'$› **have** $(x \# A_Q) \sharp* M'$ **by** *simp*
moreover from ‹$A_Q \sharp* Q$› **have** $(x \# A_Q) \sharp* ((\!|\nu x|\!)\, Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$x \sharp P$› ‹$xvec \sharp* P$› **have** $(x \# xvec) \sharp* P$ **by**(*simp*)
ultimately have $\Psi \rhd P \parallel (\!|\nu x|\!)\, Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!)\, Q')$ **using** ‹$A_P \sharp* M$›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
moreover from ‹$x \sharp \Psi$› ‹$x \sharp P'$› ‹$xvec \sharp* \Psi$› **have** $(\Psi, (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!)\, Q'), (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))) \in Rel$ **by**(*rule C2*)
ultimately show *?case* **by** *blast*
qed
next
case(*cComm2* Ψ_Q M *xvec* N P' A_P Ψ_P K Q' A_Q)
have *PTrans*: $\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **and** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **by** *fact*+
have *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$ **and** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*+
from *FrP* ‹$x \sharp P$› **have** $x \sharp \langle A_P, \Psi_P\rangle$ **by**(*drule-tac extractFrameFresh*) *simp*
with ‹$A_P \sharp* x$› **have** $x \sharp \Psi_P$ **by**(*simp add*: *frameResChainFresh*) (*simp add*: *fresh-def name-list-supp*)
from *PTrans FrP* ‹*distinct* A_P› ‹$x \sharp P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi_Q$› ‹$A_P \sharp* x$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_Q \sharp* P$› ‹$A_P \sharp* A_Q$› ‹*xvec* $\sharp* M$› ‹*distinct xvec*›
obtain M' **where** *MeqM'*: $(\Psi \otimes \Psi_Q) \otimes \Psi_P \vdash M \leftrightarrow M'$ **and** $x \sharp M'$ **and** $A_Q \sharp* M'$
by(*rule-tac* $B{=}x \# A_Q$ **in** *obtainPrefix*) (*assumption* | *force simp add*: *fresh-star-list-cons*)+

from *MeqM'* **have** *MeqM'*: $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition*)
with ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*blast intro*: *chanEqTrans chanEqSym*)
hence $(\Psi \otimes \Psi_P) \otimes \Psi_Q \vdash K \leftrightarrow M'$
by(*metis statEqEnt Associativity Commutativity Composition*)

with *QTrans FrQ* ‹*distinct* A_Q› **have** *QTrans*: $\Psi \otimes \Psi_P \rhd Q \longmapsto M'(\!|N|\!) \prec Q'$ **using** ‹$A_Q \sharp^* \Psi$› ‹$A_Q \sharp^* \Psi_P$› ‹$A_Q \sharp^* Q$› ‹$A_Q \sharp^* K$› ‹$A_Q \sharp^* M'$›
by(*rule-tac inputRenameSubject*) (*assumption* | *force*)+

from *PTrans* ‹$x \sharp P$› ‹*xvec* $\sharp^*$ x› ‹*distinct xvec*› ‹*xvec* $\sharp^*$ M›
have $x \sharp N$ **and** $x \sharp P'$ **by**(*force intro*: *outputFreshDerivative*)+
from *QTrans* ‹$x \sharp \Psi$› ‹$x \sharp \Psi_P$› ‹$x \sharp M'$› ‹$x \sharp N$› **have** *QTrans*: $\Psi \otimes \Psi_P \rhd (\!|\nu x|\!) Q \longmapsto M'(\!|N|\!) \prec (\!|\nu x|\!) Q'$
by(*rule-tac Scope*) (*assumption* | *force*)+

note *PTrans FrP QTrans*
moreover with ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› **have** *extractFrame*$((\!|\nu x|\!) Q) = \langle (x \# A_Q), \Psi_Q \rangle$
by *simp*
moreover note *MeqM'* ‹$A_P \sharp^* \Psi$› ‹$A_P \sharp^* P$›
moreover from ‹$A_P \sharp^* Q$› **have** $A_P \sharp^* ((\!|\nu x|\!) Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹$A_P \sharp^* A_Q$› ‹$A_P \sharp^* x$› **have** $A_P \sharp^* (x \# A_Q)$
by(*simp add*: *fresh-star-def fresh-list-cons*) (*auto simp add*: *fresh-def name-list-supp*)
moreover from ‹$x \sharp \Psi$› ‹$A_Q \sharp^* \Psi$› **have** $(x \# A_Q) \sharp^* \Psi$ **by** *simp*
moreover from ‹$x \sharp P$› ‹$A_Q \sharp^* P$› **have** $(x \# A_Q) \sharp^* P$ **by** *simp*
moreover from ‹$x \sharp M'$› ‹$A_Q \sharp^* M'$› **have** $(x \# A_Q) \sharp^* M'$ **by** *simp*
moreover from ‹$A_Q \sharp^* Q$› **have** $(x \# A_Q) \sharp^* ((\!|\nu x|\!) Q)$ **by**(*simp add*: *fresh-star-def abs-fresh*)
moreover from ‹*xvec* $\sharp^*$ Q› **have** $(x \# xvec) \sharp^* ((\!|\nu x|\!) Q)$ **by**(*simp add*: *abs-fresh fresh-star-def*)
ultimately have $\Psi \rhd P \parallel (\!|\nu x|\!) Q \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!) Q')$ **using** ‹$A_P \sharp^* M$›
by(*rule-tac Comm2*) (*assumption* | *simp*)+
moreover from ‹$x \sharp \Psi$› ‹$x \sharp P'$› ‹*xvec* $\sharp^*$ Ψ› **have** $(\Psi, (\!|\nu * xvec|\!)(P' \parallel (\!|\nu x|\!) Q'), (\!|\nu x|\!)((\!|\nu * xvec|\!)(P' \parallel Q'))) \in Rel$ **by**(*rule C2*)
ultimately show *?case* **by** *blast*
qed
qed
qed
qed

lemma *simParComm*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

assumes *eqvt Rel*
and *C1*: $\bigwedge \Psi' R S.\ (\Psi', R \parallel S, S \parallel R) \in Rel$
and *C2*: $\bigwedge \Psi' R S\ xvec.\ [\![(\Psi', R, S) \in Rel; xvec \sharp^* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * xvec|\!) R, (\!|\nu * xvec|\!) S) \in Rel$

shows $\Psi \rhd P \parallel Q \rightsquigarrow[Rel]\ Q \parallel P$
using ‹*eqvt Rel*›
proof(*induct rule*: *simI*[*of* - - - - ()])
case(*cSim* α *PQ*)
from ‹*bn* $\alpha \sharp* (P \parallel Q)$› **have** *bn* $\alpha \sharp* Q$ **and** *bn* $\alpha \sharp* P$ **by** *simp*+
with ‹$\Psi \rhd Q \parallel P \longmapsto \alpha \prec PQ$› ‹*bn* $\alpha \sharp* \Psi$› **show** *?case* **using** ‹*bn* $\alpha \sharp*$ *subject* α›
proof(*induct rule*: *parCases*[**where** *C*=()])
case(*cPar1* Q' A_P Ψ_P)
from ‹$\Psi \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹*bn* $\alpha \sharp* P$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* \alpha$›
have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P \parallel Q')$ **by**(*rule Par2*)
moreover have $(\Psi, P \parallel Q', Q' \parallel P) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2* P' A_Q Ψ_Q)
from ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹*bn* $\alpha \sharp* Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* \alpha$›
have $\Psi \rhd P \parallel Q \longmapsto \alpha \prec (P' \parallel Q)$ **by**(*rule Par1*)
moreover have $(\Psi, P' \parallel Q, Q \parallel P') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* Ψ_P *M N* Q' A_Q Ψ_Q *K xvec* P' A_P)
note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto K(\!|\nu* xvec|\!)\langle N \rangle \prec P'$› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$›
‹$\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|N|\!) \prec Q'$› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$›
moreover from ‹$\Psi \otimes \Psi_Q \otimes \Psi_P \vdash M \leftrightarrow K$›
have $\Psi \otimes \Psi_Q \otimes \Psi_P \vdash K \leftrightarrow M$
by(*rule chanEqSym*)
hence $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M$
by(*blast intro*: *statEqEnt compositionSym Commutativity*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu* xvec|\!)(P' \parallel Q')$
using ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹*xvec* $\sharp* Q$› ‹$A_P \sharp* K$› ‹$A_Q \sharp* M$›
by(*rule-tac Comm2*) (*assumption* | *simp*)+
moreover have $(\Psi, P' \parallel Q', Q' \parallel P') \in Rel$ **by**(*rule C1*)
hence $(\Psi, (\!|\nu* xvec|\!)(P' \parallel Q'), (\!|\nu* xvec|\!)(Q' \parallel P')) \in Rel$ **using** ‹*xvec* $\sharp* \Psi$› **by**(*rule C2*)
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_P *M xvec N* Q' A_Q Ψ_Q *K* P' A_P)
note ‹$\Psi \otimes \Psi_Q \rhd P \longmapsto K(\!|N|\!) \prec P'$› ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$›
‹$\Psi \otimes \Psi_P \rhd Q \longmapsto M(\!|\nu* xvec|\!)\langle N \rangle \prec Q'$› ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$›
moreover from ‹$\Psi \otimes \Psi_Q \otimes \Psi_P \vdash M \leftrightarrow K$›
have $\Psi \otimes \Psi_Q \otimes \Psi_P \vdash K \leftrightarrow M$
by(*rule chanEqSym*)
hence $\Psi \otimes \Psi_P \otimes \Psi_Q \vdash K \leftrightarrow M$
by(*blast intro*: *statEqEnt compositionSym Commutativity*)
ultimately have $\Psi \rhd P \parallel Q \longmapsto \tau \prec (\!|\nu* xvec|\!)(P' \parallel Q')$
using ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* P$›

‹A_Q ♯* Q› ‹$xvec$ ♯* P› ‹A_P ♯* K› ‹A_Q ♯* M›
by(*rule-tac Comm1*) (*assumption* | *simp add*: *freshChainSym*)+
moreover have (Ψ, P' ‖ Q', Q' ‖ P') ∈ *Rel* **by**(*rule C1*)
hence (Ψ, ⦇ν*$xvec$⦈(P' ‖ Q'), ⦇ν*$xvec$⦈(Q' ‖ P')) ∈ *Rel* **using** ‹$xvec$ ♯* Ψ›
by(*rule C2*)
ultimately show *?case* **by** *blast*
qed
qed

lemma *bangExtLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *guarded P*
and ⋀Ψ' Q. (Ψ', Q, Q) ∈ *Rel*

shows Ψ ▷ !P ⇝[*Rel*] P ‖ !P
using *assms*
by(*auto simp add*: *simulation-def dest*: *Bang*)

lemma *bangExtRight*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *C1*: ⋀Ψ' Q. (Ψ', Q, Q) ∈ *Rel*

shows Ψ ▷ P ‖ !P ⇝[*Rel*] !P
proof(*auto simp add*: *simulation-def*)
fix α P'
assume Ψ ▷ !P ⟼α ≺ P'
hence Ψ ▷ P ‖ !P ⟼α ≺ P'
apply −
by(*ind-cases* Ψ ▷ !P ⟼α ≺ P') (*auto simp add*: *psi.inject*)
moreover have (Ψ, P', P') ∈ *Rel* **by**(*rule C1*)
ultimately show ∃P''. Ψ ▷ P ‖ !P ⟼α ≺ P'' ∧ (Ψ, P'', P') ∈ *Rel*
by *blast*
qed

end

end

theory *Structural-Congruence*
imports *Agent*
begin

inductive *structCong* :: ((*'a*::*fs-name*), (*'b*::*fs-name*), (*'c*::*fs-name*)) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ≡$_s$ - [*70*, *70*] *70*)

where
  Refl: $P \equiv_s P$
| *Sym*: $P \equiv_s Q \Longrightarrow Q \equiv_s P$
| *Trans*: $[\![P \equiv_s Q;\ Q \equiv_s R]\!] \Longrightarrow P \equiv_s R$

| *ParComm*: $P \parallel Q \equiv_s Q \parallel P$
| *ParAssoc*: $(P \parallel Q) \parallel R \equiv_s P \parallel (Q \parallel R)$
| *ParId*: $P \parallel \mathbf{0} \equiv_s P$

| *ResNil*: $(\!|\nu x|\!)\mathbf{0} \equiv_s \mathbf{0}$
| *ResComm*: $(\!|\nu x|\!)((\!|\nu y|\!)P) \equiv_s (\!|\nu y|\!)((\!|\nu x|\!)P)$
| *ScopeExtPar*: $x \sharp P \Longrightarrow (\!|\nu x|\!)(P \parallel Q) \equiv_s P \parallel (\!|\nu x|\!)Q$
| *InputRes*: $[\![x \sharp M;\ x \sharp xvec;\ x \sharp N]\!] \Longrightarrow (\!|\nu x|\!)(M(\!|\lambda{*}xvec\ N|\!).P) \equiv_s M(\!|\lambda{*}xvec\ N|\!).((\!|\nu x|\!)P)$
| *OutputRes*: $[\![x \sharp M;\ x \sharp N]\!] \Longrightarrow (\!|\nu x|\!)(M\langle N\rangle.P) \equiv_s M\langle N\rangle.((\!|\nu x|\!)P)$
| *CaseRes*: $x \sharp (map\ fst\ Cs) \Longrightarrow (\!|\nu x|\!)(Cases\ Cs) \equiv_s Cases(map\ (\lambda(\varphi,\ P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$

| *BangUnfold*: $guarded\ P \Longrightarrow !P \equiv_s P \parallel !P$

end

theory *Bisim-Struct-Cong*
  imports *Bisim-Pres Sim-Struct-Cong Structural-Congruence*
begin

context *env* **begin**

lemma *bisimParComm*:
  fixes $\Psi :: 'b$
  and $P :: ('a,\ 'b,\ 'c)\ psi$
  and $Q :: ('a,\ 'b,\ 'c)\ psi$

  shows $\Psi \rhd P \parallel Q \sim Q \parallel P$
proof −
  let *?X* = $\{((\Psi::'b), (\!|\nu{*}xvec|\!)((P::('a,\ 'b,\ 'c)\ psi) \parallel Q), (\!|\nu{*}xvec|\!)(Q \parallel P)) \mid xvec\ \Psi\ P\ Q.\ xvec \sharp* \Psi\}$

  have *eqvt ?X*
    by(*force simp add*: *eqvt-def pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *eqvts*)

  have $(\Psi,\ P \parallel Q,\ Q \parallel P) \in ?X$
    apply *auto* **by**(*rule-tac x*=[] **in** *exI*) *auto*
  thus *?thesis*
  proof(*coinduct rule*: *bisimWeakCoinduct*)
    case(*cStatEq* Ψ *PQ QP*)
    from ‹$(\Psi,\ PQ,\ QP) \in ?X$›
     obtain *xvec P Q* **where** *PFrQ*: $PQ = (\!|\nu{*}xvec|\!)(P \parallel Q)$ **and** *QFrP*: $QP =$

$(\!|\nu{*}xvec|\!)(Q \parallel P)$ **and** $xvec \sharp* \Psi$
by *auto*

obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* Q$
by(*rule-tac* C=(Ψ, Q) **in** *freshFrame*) *auto*
obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* \Psi_P$
by(*rule-tac* C=(Ψ, A_P, Ψ_P) **in** *freshFrame*) *auto*
from *FrQ* ‹$A_Q \sharp* A_P$› ‹$A_P \sharp* Q$› **have** $A_P \sharp* \Psi_Q$ **by**(*force dest*: *extractFrameFreshChain*)
have $\langle(xvec@A_P@A_Q), \Psi \otimes \Psi_P \otimes \Psi_Q\rangle \simeq_F \langle(xvec@A_Q@A_P), \Psi \otimes \Psi_Q \otimes \Psi_P\rangle$
by(*simp add*: *frameChainAppend*)
(*metis frameResChainPres frameResChainComm frameNilStatEq compositionSym Associativity Commutativity FrameStatEqTrans*)
with *FrP FrQ PFrQ QFrP* ‹$A_P \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_P$› ‹$A_Q \sharp* A_P$› ‹$xvec \sharp* \Psi$› ‹$A_P \sharp* \Psi$› ‹$A_Q \sharp* \Psi$›
show *?case* **by**(*auto simp add*: *frameChainAppend*)
next
case(*cSim* Ψ PQ QP)
from ‹$(\Psi, PQ, QP) \in ?X$›
obtain *xvec* P Q **where** *PFrQ*: $PQ = (\!|\nu{*}xvec|\!)(P \parallel Q)$ **and** *QFrP*: $QP = (\!|\nu{*}xvec|\!)(Q \parallel P)$
and $xvec \sharp* \Psi$
by *auto*
moreover have $\Psi \rhd (\!|\nu{*}xvec|\!)(P \parallel Q) \rightsquigarrow[?X] (\!|\nu{*}xvec|\!)(Q \parallel P)$
proof −
have $\Psi \rhd P \parallel Q \rightsquigarrow[?X] Q \parallel P$
proof −
note ‹*eqvt ?X*›
moreover have $\bigwedge\Psi\ P\ Q.\ (\Psi, P \parallel Q, Q \parallel P) \in ?X$
apply *auto* **by**(*rule-tac* x=[] **in** *exI*) *auto*
moreover have $\bigwedge\Psi\ P\ Q\ xvec.\ [\![(\Psi, P, Q) \in ?X; xvec \sharp* \Psi]\!] \Longrightarrow (\Psi, (\!|\nu{*}xvec|\!)P, (\!|\nu{*}xvec|\!)Q) \in ?X$
apply(*induct xvec*, *auto*)
by(*rule-tac* x=*xvec@xveca* **in** *exI*) (*auto simp add*: *resChainAppend*)
ultimately show *?thesis* **by**(*rule simParComm*)
qed
moreover note ‹*eqvt ?X*› ‹$xvec \sharp* \Psi$›
moreover have $\bigwedge\Psi\ P\ Q\ x.\ [\![(\Psi, P, Q) \in ?X; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X$
apply *auto*
by(*rule-tac* x=*x#xvec* **in** *exI*) *auto*
ultimately show *?thesis* **by**(*rule resChainPres*)
qed
ultimately show *?case* **by** *simp*
next
case(*cExt* Ψ PQ QP Ψ')
from ‹$(\Psi, PQ, QP) \in ?X$›

obtain *xvec P Q* **where** *PFrQ*: $PQ = (\!|\nu * xvec|\!)(P \parallel Q)$ **and** *QFrP*: $QP = (\!|\nu * xvec|\!)(Q \parallel P)$
and $xvec \sharp* \Psi$
by *auto*

obtain p **where** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* Q$
and $(p \cdot xvec) \sharp* \Psi'$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm p*
by(*rule-tac c*=(Ψ, P, Q, Ψ') **in** *name-list-avoiding*) *auto*

from $\langle(p \cdot xvec) \sharp* P\rangle$ $\langle(p \cdot xvec) \sharp* Q\rangle$ S **have** $(\!|\nu * xvec|\!)(P \parallel Q) = (\!|\nu * (p \cdot xvec)|\!)(p \cdot (P \parallel Q))$
by(*subst resChainAlpha*) *auto*
hence *PQAlpha*: $(\!|\nu * xvec|\!)(P \parallel Q) = (\!|\nu * (p \cdot xvec)|\!)((p \cdot P) \parallel (p \cdot Q))$
by(*simp add*: *eqvts*)

from $\langle(p \cdot xvec) \sharp* P\rangle$ $\langle(p \cdot xvec) \sharp* Q\rangle$ S **have** $(\!|\nu * xvec|\!)(Q \parallel P) = (\!|\nu * (p \cdot xvec)|\!)(p \cdot (Q \parallel P))$
by(*subst resChainAlpha*) *auto*
hence *QPAlpha*: $(\!|\nu * xvec|\!)(Q \parallel P) = (\!|\nu * (p \cdot xvec)|\!)((p \cdot Q) \parallel (p \cdot P))$
by(*simp add*: *eqvts*)

from $\langle(p \cdot xvec) \sharp* \Psi\rangle$ $\langle(p \cdot xvec) \sharp* \Psi'\rangle$ **have** $(\Psi \otimes \Psi', (\!|\nu * (p \cdot xvec)|\!)((p \cdot P) \parallel (p \cdot Q)), (\!|\nu * (p \cdot xvec)|\!)((p \cdot Q) \parallel (p \cdot P))) \in ?X$
by *auto*
with *PFrQ QFrP PQAlpha QPAlpha* **show** *?case* **by** *simp*
next
case(*cSym* Ψ *PR QR*)
thus *?case* **by** *blast*
qed
qed

lemma *bisimResComm*:
fixes $x :: name$
and $\Psi :: 'b$
and $y :: name$
and $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \sim (\!|\nu y|\!)((\!|\nu x|\!)P)$
proof(*cases* x=y)
case *True*
thus *?thesis* **by**(*blast intro*: *bisimReflexive*)
next
case *False*
{
fix x::*name* **and** y::*name* **and** P::$('a, 'b, 'c)$ *psi*
assume $x \sharp \Psi$ **and** $y \sharp \Psi$

let *?X* = {((Ψ::'*b*), (⦇*νx*⦈)(⦇*νy*⦈)(*P*::('*a*, '*b*, '*c*) *psi*)), (⦇*νy*⦈)(⦇*νx*⦈)*P*)) | Ψ *x y P*. *x* ♯ Ψ ∧ *y* ♯ Ψ}
from ‹*x* ♯ Ψ› ‹*y* ♯ Ψ› **have** (Ψ, (⦇*νx*⦈)(⦇*νy*⦈)*P*), (⦇*νy*⦈)(⦇*νx*⦈)*P*)) ∈ *?X* **by** *auto*
hence Ψ ▷ (⦇*νx*⦈)(⦇*νy*⦈)*P*) ∼ (⦇*νy*⦈)(⦇*νx*⦈)*P*
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* Ψ *xyP yxP*)
from ‹(Ψ, *xyP*, *yxP*) ∈ *?X*› **obtain** *x y P* **where** *x* ♯ Ψ **and** *y* ♯ Ψ **and** *xyP* = (⦇*νx*⦈)(⦇*νy*⦈)*P*) **and** *yxP* = (⦇*νy*⦈)(⦇*νx*⦈)*P*) **by** *auto*
moreover obtain A_P Ψ_P **where** *extractFrame P* = ⟨A_P, Ψ_P⟩ **and** A_P ♯* Ψ **and** *x* ♯ A_P **and** *y* ♯ A_P
by(*rule-tac C*=(*x*, *y*, Ψ) **in** *freshFrame*) *auto*
ultimately show *?case* **by**(*force intro*: *frameResComm FrameStatEqTrans*)
next
case(*cSim* Ψ *xyP yxP*)
from ‹(Ψ, *xyP*, *yxP*) ∈ *?X*› **obtain** *x y P* **where** *x* ♯ Ψ **and** *y* ♯ Ψ **and** *xyP* = (⦇*νx*⦈)(⦇*νy*⦈)*P*) **and** *yxP* = (⦇*νy*⦈)(⦇*νx*⦈)*P*) **by** *auto*
note ‹*x* ♯ Ψ› ‹*y* ♯ Ψ›
moreover have *eqvt ?X* **by**(*force simp add*: *eqvt-def pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
hence *eqvt*(*?X* ∪ *bisim*) **by** *auto*
moreover have ⋀Ψ *P*. (Ψ, *P*, *P*) ∈ *?X* ∪ *bisim* **by**(*blast intro*: *bisimReflexive*)
moreover have ⋀Ψ *x y P*. ⟦*x* ♯ Ψ; *y* ♯ Ψ⟧ ⟹ (Ψ, (⦇*νx*⦈)(⦇*νy*⦈)*P*), (⦇*νy*⦈)(⦇*νx*⦈)*P*)) ∈ *?X* ∪ *bisim* **by** *auto*
ultimately have Ψ ▷ (⦇*νx*⦈)(⦇*νy*⦈)*P*) ⇝[(*?X* ∪ *bisim*)] (⦇*νy*⦈)(⦇*νx*⦈)*P*) **by**(*rule resComm*)
with ‹*xyP* = (⦇*νx*⦈)(⦇*νy*⦈)*P*)› ‹*yxP* = (⦇*νy*⦈)(⦇*νx*⦈)*P*)› **show** *?case*
by *simp*
next
case(*cExt* Ψ *xyP yxP* Ψ′)
from ‹(Ψ, *xyP*, *yxP*) ∈ *?X*› **obtain** *x y P* **where** *x* ♯ Ψ **and** *y* ♯ Ψ **and** *xyPeq*: *xyP* = (⦇*νx*⦈)(⦇*νy*⦈)*P*) **and** *yxPeq*: *yxP* = (⦇*νy*⦈)(⦇*νx*⦈)*P*) **by** *auto*
show *?case*
proof(*case-tac x*=*y*)
assume *x* = *y*
with *xyPeq yxPeq* **show** *?case*
by(*blast intro*: *bisimReflexive*)
next
assume *x* ≠ *y*
obtain *x*′ **where** *x*′ ♯ Ψ **and** *x*′ ♯ Ψ′ **and** *x*′ ≠ *x* **and** *x*′ ≠ *y* **and** *x*′ ♯ *P* **by**(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
obtain *y*′ **where** *y*′ ♯ Ψ **and** *y*′ ♯ Ψ′ **and** *y*′ ≠ *x* **and** *x*′ ≠ *y*′ **and** *y*′ ≠ *y* **and** *y*′ ♯ *P* **by**(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
with *xyPeq* ‹*y*′ ♯ *P*› ‹*x*′ ♯ *P*› ‹*x* ≠ *y*› ‹*x*′ ≠ *y*› ‹*y*′ ≠ *x*› **have** (⦇*νx*⦈)(⦇*νy*⦈)*P*) = (⦇*νx*′⦈)(⦇*νy*′⦈)([(*x*, *x*′)] ∙ [(*y*, *y*′)] ∙ *P*))
apply(*subst alphaRes*[*of x*′]) **apply**(*simp add*: *abs-fresh*) **by**(*subst alphaRes*[*of y*′ - *y*]) (*auto simp add*: *eqvts calc-atm*)
moreover with *yxPeq* ‹*y*′ ♯ *P*› ‹*x*′ ♯ *P*› ‹*x* ≠ *y*› ‹*x*′ ≠ *y*› ‹*y*′ ≠ *x*› ‹*x*′ ≠ *y*′› **have** (⦇*νy*⦈)(⦇*νx*⦈)*P*) = (⦇*νy*′⦈)(⦇*νx*′⦈)([(*y*, *y*′)] ∙ [(*x*, *x*′)] ∙ *P*))
apply(*subst alphaRes*[*of y*′]) **apply**(*simp add*: *abs-fresh*) **by**(*subst al-*

phaRes[*of* x' - x]) (*auto simp add*: *eqvts calc-atm*)
with ‹$x \neq y$› ‹$x' \neq y$› ‹$y' \neq y$› ‹$x' \neq x$› ‹$y' \neq x$› ‹$x' \neq y'$› **have** $(\!|\nu y|\!)((\!|\nu x|\!)P) = (\!|\nu y'|\!)((\!|\nu x'|\!)([(x, x')] \cdot [(y, y')] \cdot P))$
by(*subst perm-compose*) (*simp add*: *calc-atm*)
moreover from ‹$x' \sharp \Psi$› ‹$x' \sharp \Psi'$› ‹$y' \sharp \Psi$› ‹$y' \sharp \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu x'|\!)((\!|\nu y'|\!)([(x, x')] \cdot [(y, y')] \cdot P)), (\!|\nu y'|\!)((\!|\nu x'|\!)([(x, x')] \cdot [(y, y')] \cdot P))) \in ?X$
by *auto*
ultimately show *?case* **using** *xyPeq yxPeq* **by** *simp*
qed
next
case(*cSym* Ψ *xyP yxP*)
thus *?case* **by** *auto*
qed
}
moreover obtain x'::*name* **where** $x' \sharp \Psi$ **and** $x' \sharp P$ **and** $x' \neq x$ **and** $x' \neq y$
by(*generate-fresh name*) *auto*
moreover obtain y'::*name* **where** $y' \sharp \Psi$ **and** $y' \sharp P$ **and** $y' \neq x$ **and** $y' \neq y$ **and** $y' \neq x'$
by(*generate-fresh name*) *auto*
ultimately have $\Psi \rhd (\!|\nu x'|\!)((\!|\nu y'|\!)([(y, y'), (x, x')] \cdot P)) \sim (\!|\nu y'|\!)((\!|\nu x'|\!)([(y, y'), (x, x')] \cdot P))$ **by** *auto*
thus *?thesis* **using** ‹$x' \sharp P$› ‹$x' \neq x$› ‹$x' \neq y$› ‹$y' \sharp P$› ‹$y' \neq x$› ‹$y' \neq y$› ‹$y' \neq x'$› ‹$x \neq y$›
apply(*subst alphaRes*[**where** x=x **and** y=x' **and** P=P], *auto*)
apply(*subst alphaRes*[**where** x=y **and** y=y' **and** P=P], *auto*)
apply(*subst alphaRes*[**where** x=x **and** y=x' **and** P=$(\!|\nu y'|\!)([(y, y')] \cdot P)$], *auto simp add*: *abs-fresh fresh-left*)
apply(*subst alphaRes*[**where** x=y **and** y=y' **and** P=$(\!|\nu x'|\!)([(x, x')] \cdot P)$], *auto simp add*: *abs-fresh fresh-left*)
by(*subst perm-compose*) (*simp add*: *eqvts calc-atm*)
qed

lemma *bisimResComm'*:
fixes x :: *name*
and Ψ :: $'b$
and *xvec* :: *name list*
and P :: ($'a$, $'b$, $'c$) *psi*

assumes $x \sharp \Psi$
and *xvec* $\sharp*$ Ψ

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!)P) \sim (\!|\nu * xvec|\!)((\!|\nu x|\!)P)$
using *assms*
by(*induct xvec*) (*auto intro*: *bisimResComm bisimReflexive bisimResPres bisimTransitive*)

lemma *bisimScopeExt*:
fixes x :: *name*
and Ψ :: $'b$

```
  and    P :: ('a, 'b, 'c) psi
  and    Q :: ('a, 'b, 'c) psi

  assumes x ♯ P

  shows Ψ ▷ (|νx|)(P ∥ Q) ∼ P ∥ (|νx|)Q
proof −
  {
    fix x::name and Q :: ('a, 'b, 'c) psi
    assume x ♯ Ψ and x ♯ P
     let ?X1 = {((Ψ::'b), (|ν∗xvec|)((|νx|)((P::('a, 'b, 'c) psi) ∥ Q)), (|ν∗xvec|)(P ∥
(|νx|)Q)) | Ψ xvec x P Q. x ♯ Ψ ∧ x ♯ P ∧ xvec ♯∗ Ψ}
    let ?X2 = {((Ψ::'b), (|ν∗xvec|)((P::('a, 'b, 'c) psi) ∥ (|νx|)Q), (|ν∗xvec|)((|νx|)(P
∥ Q))) | Ψ xvec x P Q. x ♯ Ψ ∧ x ♯ P ∧ xvec ♯∗ Ψ}
    let ?X = ?X1 ∪ ?X2

    from ‹x ♯ Ψ› ‹x ♯ P› have (Ψ, (|νx|)(P ∥ Q), P ∥ (|νx|)Q) ∈ ?X
      by(auto, rule-tac x=[] in exI) (auto simp add: fresh-list-nil)
    moreover have eqvt ?X
      by(rule eqvtUnion)
   (fastforce simp add: eqvt-def eqvts pt-fresh-star-bij[OF pt-name-inst, OF at-name-inst]
pt-fresh-bij[OF pt-name-inst, OF at-name-inst])+
    ultimately have Ψ ▷ (|νx|)(P ∥ Q) ∼ P ∥ (|νx|)Q
    proof(coinduct rule: transitiveCoinduct)
      case(cStatEq Ψ R T)
      show ?case
      proof(case-tac (Ψ, R, T) ∈ ?X1)
        assume (Ψ, R, T) ∈ ?X1
         then obtain xvec x P Q where R = (|ν∗xvec|)((|νx|)(P ∥ Q)) and T =
(|ν∗xvec|)(P ∥ (|νx|)Q) and xvec ♯∗ Ψ and x ♯ P and x ♯ Ψ
          by auto
        moreover obtain A_P Ψ_P where FrP: extractFrame P = ⟨A_P, Ψ_P⟩ and
A_P ♯∗ Ψ and x ♯ A_P and A_P ♯∗ Q
          by(rule-tac C=(Ψ, x, Q) in freshFrame) auto
        moreover obtain A_Q Ψ_Q where FrQ: extractFrame Q = ⟨A_Q, Ψ_Q⟩ and
A_Q ♯∗ Ψ and x ♯ A_Q and A_Q ♯∗ A_P and A_Q ♯∗ Ψ_P
          by(rule-tac C=(Ψ, x, A_P, Ψ_P) in freshFrame) auto
        moreover from FrQ ‹A_P ♯∗ Q› ‹A_Q ♯∗ A_P› have A_P ♯∗ Ψ_Q
          by(drule-tac extractFrameFreshChain) auto
        moreover from ‹x ♯ P› ‹x ♯ A_P› FrP have x ♯ Ψ_P by(drule-tac extract-
FrameFresh) auto
        ultimately show ?case
         by(force simp add: frameChainAppend intro: frameResComm′ FrameStatE-
qTrans frameResChainPres)
      next
        assume (Ψ, R, T) ∉ ?X1
        with ‹(Ψ, R, T) ∈ ?X› have (Ψ, R, T) ∈ ?X2 by blast
         then obtain xvec x P Q where T = (|ν∗xvec|)((|νx|)(P ∥ Q)) and R =
(|ν∗xvec|)(P ∥ (|νx|)Q) and xvec ♯∗ Ψ and x ♯ P and x ♯ Ψ
```

by *auto*
moreover obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $x \sharp A_P$ **and** $A_P \sharp* Q$
by(*rule-tac* $C=(\Psi, x, Q)$ **in** *freshFrame*) *auto*
moreover obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $x \sharp A_Q$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* \Psi_P$
by(*rule-tac* $C=(\Psi, x, A_P, \Psi_P)$ **in** *freshFrame*) *auto*
moreover from *FrQ* $\langle A_P \sharp* Q\rangle$ $\langle A_Q \sharp* A_P\rangle$ **have** $A_P \sharp* \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
moreover from $\langle x \sharp P\rangle$ $\langle x \sharp A_P\rangle$ *FrP* **have** $x \sharp \Psi_P$ **by**(*drule-tac extractFrameFresh*) *auto*
ultimately show *?case*
apply *auto*
by(*force simp add*: *frameChainAppend intro*: *frameResComm*′ *FrameStatEqTrans frameResChainPres FrameStatEqSym*)
qed
next
case(*cSim* Ψ R T)
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge ((\Psi, P', Q') \in ?X \vee \Psi \rhd P' \sim Q') \wedge \Psi \rhd Q' \sim Q\}$
from $\langle$*eqvt ?X*$\rangle$ **have** *eqvt ?Y* **by** *blast*
have *C1*: $\bigwedge \Psi\ R\ T\ y.\ [\![(\Psi, R, T) \in ?Y;\ (y::name) \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu y|\!)R, (\!|\nu y|\!)T) \in ?Y$
proof −
fix Ψ R T y
assume $(\Psi, R, T) \in ?Y$
then obtain R' T' **where** $\Psi \rhd R \sim R'$ **and** $(\Psi, R', T') \in (?X \cup bisim)$ **and** $\Psi \rhd T' \sim T$ **by** *fastforce*
assume $(y::name) \sharp \Psi$
show $(\Psi, (\!|\nu y|\!)R, (\!|\nu y|\!)T) \in ?Y$
proof(*case-tac* $(\Psi, R', T') \in ?X$)
assume $(\Psi, R', T') \in ?X$
show *?thesis*
proof(*case-tac* $(\Psi, R', T') \in ?X1$)
assume $(\Psi, R', T') \in ?X1$
then obtain *xvec* *x* *P* *Q* **where** *R′eq*: $R' = (\!|\nu{*}xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** *T′eq*: $T' = (\!|\nu{*}xvec|\!)(P \parallel (\!|\nu x|\!)Q)$
and *xvec* $\sharp* \Psi$ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
from $\langle \Psi \rhd R \sim R'\rangle$ $\langle y \sharp \Psi\rangle$ **have** $\Psi \rhd (\!|\nu y|\!)R \sim (\!|\nu y|\!)R'$ **by**(*rule bisimResPres*)
moreover from $\langle$*xvec* $\sharp* \Psi\rangle$ $\langle y \sharp \Psi\rangle$ $\langle x \sharp P\rangle$ $\langle x \sharp \Psi\rangle$ **have** $(\Psi, (\!|\nu{*}(y\#xvec)|\!)(\!|\nu x|\!)(P \parallel Q), (\!|\nu{*}(y\#xvec)|\!)(P \parallel (\!|\nu x|\!)Q)) \in ?X1$
by(*force simp del*: *resChain.simps*)
with *R′eq* *T′eq* **have** $(\Psi, (\!|\nu y|\!)R', (\!|\nu y|\!)T') \in ?X \cup bisim$ **by** *simp*
moreover from $\langle \Psi \rhd T' \sim T\rangle$ $\langle y \sharp \Psi\rangle$ **have** $\Psi \rhd (\!|\nu y|\!)T' \sim (\!|\nu y|\!)T$ **by**(*rule bisimResPres*)
ultimately show *?thesis* **by** *blast*
next

assume $(\Psi, R', T') \notin ?X1$
with ‹$(\Psi, R', T') \in ?X$› **have** $(\Psi, R', T') \in ?X2$ **by** *blast*
then obtain *xvec* x P Q **where** $T'eq$: $T' = (\!|\nu * xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** $R'eq$: $R' = (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)$ **and** *xvec* $\sharp*$ Ψ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
from ‹$\Psi \rhd R \sim R'$› ‹$y \sharp \Psi$› **have** $\Psi \rhd (\!|\nu y|\!)R \sim (\!|\nu y|\!)R'$ **by**(*rule bisimResPres*)
moreover from ‹*xvec* $\sharp*$ Ψ› ‹$y \sharp \Psi$› ‹$x \sharp P$› ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu * (y\#xvec)|\!)(P \parallel (\!|\nu x|\!)Q), (\!|\nu * (y\#xvec)|\!)((\!|\nu x|\!)(P \parallel Q))) \in ?X2$
by(*force simp del*: *resChain.simps*)
with $R'eq$ $T'eq$ **have** $(\Psi, (\!|\nu y|\!)R', (\!|\nu y|\!)T') \in ?X \cup bisim$ **by** *simp*
moreover from ‹$\Psi \rhd T' \sim T$› ‹$y \sharp \Psi$› **have** $\Psi \rhd (\!|\nu y|\!)T' \sim (\!|\nu y|\!)T$ **by**(*rule bisimResPres*)
ultimately show *?thesis* **by** *blast*
qed
next
assume $(\Psi, R', T') \notin ?X$
with ‹$(\Psi, R', T') \in ?X \cup bisim$› **have** $\Psi \rhd R' \sim T'$ **by** *blast*
with ‹$\Psi \rhd R \sim R'$› ‹$\Psi \rhd T' \sim T$› ‹$y \sharp \Psi$› **show** *?thesis*
by(*blast dest*: *bisimResPres*)
qed
qed

show *?case*
proof(*case-tac* $(\Psi, R, T) \in ?X1$)
assume $(\Psi, R, T) \in ?X1$
then obtain *xvec* x P Q **where** Req: $R = (\!|\nu * xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** Teq: $T = (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)$ **and** *xvec* $\sharp*$ Ψ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
have $\Psi \rhd (\!|\nu * xvec|\!)((\!|\nu x|\!)(P \parallel Q)) \rightsquigarrow [?Y]\ (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)$
proof −
have $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \rightsquigarrow [?Y]\ P \parallel (\!|\nu x|\!)Q$
proof −
note ‹$x \sharp P$› ‹$x \sharp \Psi$› ‹*eqvt* *?Y*›
moreover have $\bigwedge \Psi\ P.\ (\Psi, P, P) \in ?Y$ **by**(*blast intro*: *bisimReflexive*)
moreover have $\bigwedge x\ \Psi\ P\ Q\ xvec.\ [\![x \sharp \Psi; x \sharp P; xvec \sharp* \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)((\!|\nu * xvec|\!)(P \parallel Q)), (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)) \in ?Y$
proof −
fix $x\ \Psi\ P\ Q$ *xvec*
assume $(x::name) \sharp (\Psi::'b)$ **and** $x \sharp (P::('a, 'b, 'c)\ psi)$ **and** $(xvec::name\ list) \sharp* \Psi$
from ‹$x \sharp \Psi$› ‹*xvec* $\sharp*$ Ψ› **have** $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!)(P \parallel Q)) \sim (\!|\nu * xvec|\!)((\!|\nu x|\!)(P \parallel Q))$
by(*rule bisimResComm'*)
moreover from ‹*xvec* $\sharp*$ Ψ› ‹$x \sharp \Psi$› ‹$x \sharp P$› **have** $(\Psi, (\!|\nu * xvec|\!)((\!|\nu x|\!)(P \parallel Q)), (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)) \in ?X \cup bisim$
by *blast*
ultimately show $(\Psi, (\!|\nu x|\!)((\!|\nu * xvec|\!)(P \parallel Q)), (\!|\nu * xvec|\!)(P \parallel (\!|\nu x|\!)Q)) \in ?Y$

by(*blast intro*: *bisimReflexive*)
qed
moreover have $\bigwedge\Psi\ xvec\ P\ x.\ [\![x \sharp \Psi;\ xvec \sharp* \Psi]\!] \Longrightarrow (\Psi,\ (\!|\nu x|\!)((\!|\nu*xvec|\!)P),\ (\!|\nu*xvec|\!)((\!|\nu x|\!)P)) \in ?Y$
by(*blast intro*: *bisimResComm′ bisimReflexive*)
ultimately show *?thesis* **by**(*rule scopeExtLeft*)
qed
thus *?thesis* **using** ‹*eqvt ?Y*› ‹*xvec* $\sharp*$ Ψ› *C1*
by(*rule resChainPres*)
qed
with *Req Teq* **show** *?case* **by** *simp*
next
assume $(\Psi,\ R,\ T) \notin ?X1$
with ‹$(\Psi,\ R,\ T) \in ?X$› **have** $(\Psi,\ R,\ T) \in ?X2$ **by** *blast*
then obtain *xvec x P Q* **where** *Teq*: $T = (\!|\nu*xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** *Req*: $R = (\!|\nu*xvec|\!)(P \parallel (\!|\nu x|\!)Q)$ **and** *xvec* $\sharp*$ Ψ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
have $\Psi \rhd (\!|\nu*xvec|\!)(P \parallel (\!|\nu x|\!)Q) \rightsquigarrow[?Y]\ (\!|\nu*xvec|\!)((\!|\nu x|\!)(P \parallel Q))$
proof −
have $\Psi \rhd P \parallel (\!|\nu x|\!)Q \rightsquigarrow[?Y]\ (\!|\nu x|\!)(P \parallel Q)$
proof −
note ‹$x \sharp P$› ‹$x \sharp \Psi$› ‹*eqvt ?Y*›
moreover have $\bigwedge\Psi\ P.\ (\Psi,\ P,\ P) \in ?Y$ **by**(*blast intro*: *bisimReflexive*)
moreover have $\bigwedge x\ \Psi\ P\ Q\ xvec.\ [\![x \sharp \Psi;\ x \sharp P;\ xvec \sharp* \Psi]\!] \Longrightarrow (\Psi,\ (\!|\nu*xvec|\!)(P \parallel (\!|\nu x|\!)Q),\ (\!|\nu x|\!)((\!|\nu*xvec|\!)(P \parallel Q))) \in ?Y$
proof −
fix *x* Ψ *P Q xvec*
assume (*x*::*name*) $\sharp$ (Ψ::*′b*) **and** $x \sharp$ (*P*::(*′a*, *′b*, *′c*) *psi*) **and** (*xvec*::*name list*) $\sharp*$ Ψ
from ‹*xvec* $\sharp*$ Ψ› ‹$x \sharp \Psi$› ‹$x \sharp P$› **have** $(\Psi,\ (\!|\nu*xvec|\!)(P \parallel (\!|\nu x|\!)Q),\ (\!|\nu*xvec|\!)((\!|\nu x|\!)(P \parallel Q))) \in ?X \cup bisim$
by *blast*
moreover from ‹$x \sharp \Psi$› ‹*xvec* $\sharp*$ Ψ› **have** $\Psi \rhd (\!|\nu*xvec|\!)((\!|\nu x|\!)(P \parallel Q)) \sim (\!|\nu x|\!)((\!|\nu*xvec|\!)(P \parallel Q))$
by(*blast intro*: *bisimResComm′ bisimE*)
ultimately show $(\Psi,\ (\!|\nu*xvec|\!)(P \parallel (\!|\nu x|\!)Q),\ (\!|\nu x|\!)((\!|\nu*xvec|\!)(P \parallel Q))) \in ?Y$
by(*blast intro*: *bisimReflexive*)
qed
ultimately show *?thesis* **by**(*rule scopeExtRight*)
qed
thus *?thesis* **using** ‹*eqvt ?Y*› ‹*xvec* $\sharp*$ Ψ› *C1*
by(*rule resChainPres*)
qed
with *Req Teq* **show** *?case* **by** *simp*
qed
next
case(*cExt* Ψ *R T* Ψ')
show *?case*

proof(*case-tac* $(\Psi, R, T) \in ?X1$)
assume $(\Psi, R, T) \in ?X1$
then obtain *xvec* x P Q **where** *Req*: $R = (\!|\nu{*}xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** *Teq*: $T = (\!|\nu{*}xvec|\!)(P \parallel (\!|\nu x|\!)Q)$ **and** $xvec \sharp* \Psi$ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
obtain y::*name* **where** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp xvec$ **and** $y \sharp \Psi$ **and** $y \sharp \Psi'$
by(*generate-fresh name*, *auto simp add*: *fresh-prod*)

obtain p **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi'$
and $x \sharp (p \cdot xvec)$ **and** $y \sharp (p \cdot xvec)$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm p*
by(*rule-tac* c=$(\Psi, P, Q, x, y, \Psi')$ **in** *name-list-avoiding*) *auto*

from ‹$y \sharp P$› **have** $(p \cdot y) \sharp (p \cdot P)$ **by**(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with S ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› **have** $y \sharp (p \cdot P)$ **by** *simp*
with ‹$(p \cdot xvec) \sharp* \Psi$› ‹$y \sharp \Psi$› ‹$(p \cdot xvec) \sharp* \Psi'$› ‹$y \sharp \Psi'$›
have $(\Psi \otimes \Psi', (\!|\nu{*}(p \cdot xvec)|\!)((\!|\nu y|\!)((p \cdot P) \parallel (p \cdot [(x, y)] \cdot Q))), (\!|\nu{*}(p \cdot xvec)|\!)((p \cdot P) \parallel ((\!|\nu y|\!)(p \cdot [(x, y)] \cdot Q)))) \in ?X$
by *auto*
moreover from *Req* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› ‹$x \sharp (p \cdot xvec)$› ‹$y \sharp P$› ‹$y \sharp Q$› ‹$x \sharp P$› S
have $R = (\!|\nu{*}(p \cdot xvec)|\!)((\!|\nu y|\!)((p \cdot P) \parallel (p \cdot [(x, y)] \cdot Q)))$
apply(*erule-tac rev-mp*)
apply(*subst alphaRes*[*of y*])
apply(*clarsimp simp add*: *eqvts*)
apply(*subst resChainAlpha*[*of p*])
by(*auto simp add*: *eqvts*)
moreover from *Teq* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› ‹$x \sharp (p \cdot xvec)$› ‹$y \sharp P$› ‹$y \sharp Q$› ‹$x \sharp P$› S
have $T = (\!|\nu{*}(p \cdot xvec)|\!)((p \cdot P) \parallel (\!|\nu y|\!)(p \cdot [(x, y)] \cdot Q))$
apply(*erule-tac rev-mp*)
apply(*subst alphaRes*[*of y*])
apply(*clarsimp simp add*: *eqvts*)
apply(*subst resChainAlpha*[*of p*])
by(*auto simp add*: *eqvts*)
ultimately show *?case*
by *blast*
next
assume $(\Psi, R, T) \notin ?X1$
with ‹$(\Psi, R, T) \in ?X$› **have** $(\Psi, R, T) \in ?X2$ **by** *blast*
then obtain *xvec* x P Q **where** *Teq*: $T = (\!|\nu{*}xvec|\!)((\!|\nu x|\!)(P \parallel Q))$ **and** *Req*: $R = (\!|\nu{*}xvec|\!)(P \parallel (\!|\nu x|\!)Q)$ **and** $xvec \sharp* \Psi$ **and** $x \sharp P$ **and** $x \sharp \Psi$
by *auto*
obtain y::*name* **where** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp xvec$ **and** $y \sharp \Psi$ **and** $y \sharp \Psi'$

by(*generate-fresh name*, *auto simp add*: *fresh-prod*)

obtain p **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi'$
and $x \sharp (p \cdot xvec)$ **and** $y \sharp (p \cdot xvec)$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm* p
by(*rule-tac* c=$(\Psi, P, Q, x, y, \Psi')$ **in** *name-list-avoiding*) *auto*

from ‹$y \sharp P$› **have** $(p \cdot y) \sharp (p \cdot P)$ **by**(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with S ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› **have** $y \sharp (p \cdot P)$ **by** *simp*
with ‹$(p \cdot xvec) \sharp* \Psi$› ‹$y \sharp \Psi$› ‹$(p \cdot xvec) \sharp* \Psi'$› ‹$y \sharp \Psi'$›
have $(\Psi \otimes \Psi', (\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel (\!|\nu y|\!)(p \cdot [(x, y)] \cdot Q)), (\!|\nu*(p \cdot xvec)|\!)((\!|\nu y|\!)((p \cdot P) \parallel (p \cdot [(x, y)] \cdot Q)))) \in$ *?X2*
by *auto*
moreover from *Teq* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› ‹$x \sharp (p \cdot xvec)$› ‹$y \sharp P$› ‹$y \sharp Q$› ‹$x \sharp P$› S
have $T = (\!|\nu*(p \cdot xvec)|\!)((\!|\nu y|\!)((p \cdot P) \parallel (p \cdot [(x, y)] \cdot Q)))$
apply(*erule-tac rev-mp*)
apply(*subst alphaRes*[*of y*])
apply(*clarsimp simp add*: *eqvts*)
apply(*subst resChainAlpha*[*of p*])
by(*auto simp add*: *eqvts*)
moreover from *Req* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› ‹$x \sharp (p \cdot xvec)$› ‹$y \sharp P$› ‹$y \sharp Q$› ‹$x \sharp P$› S
have $R = (\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel (\!|\nu y|\!)(p \cdot [(x, y)] \cdot Q))$
apply(*erule-tac rev-mp*)
apply(*subst alphaRes*[*of y*])
apply(*clarsimp simp add*: *eqvts*)
apply(*subst resChainAlpha*[*of p*])
by(*auto simp add*: *eqvts*)
ultimately show *?case*
by *blast*
qed
next
case(*cSym* Ψ P Q)
thus *?case*
by(*blast dest*: *bisimE*)
qed
}
moreover obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp P$ $y \sharp Q$
by(*generate-fresh name*) *auto*
ultimately have $\Psi \rhd (\!|\nu y|\!)(P \parallel ([(x, y)] \cdot Q)) \sim P \parallel (\!|\nu y|\!)([(x, y)] \cdot Q)$ **by** *auto*
thus *?thesis* **using** *assms* ‹$y \sharp P$› ‹$y \sharp Q$›
apply(*subst alphaRes*[**where** x=x **and** y=y **and** P=Q], *auto*)
by(*subst alphaRes*[**where** x=x **and** y=y **and** P=$P \parallel Q$]) *auto*
qed

lemma *bisimScopeExtChain*:

fixes *xvec* :: *name list*
and Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes *xvec* $\sharp*$ Ψ
and *xvec* $\sharp*$ P

shows $\Psi \rhd (\!|\nu * xvec|\!)(P \parallel Q) \sim P \parallel ((\!|\nu * xvec|\!) Q)$
using *assms*
by(*induct xvec*) (*auto intro*: *bisimScopeExt bisimReflexive bisimTransitive bisimResPres*)

lemma *bisimParAssoc*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*

shows $\Psi \rhd (P \parallel Q) \parallel R \sim P \parallel (Q \parallel R)$
proof −
let *?X* = $\{(\Psi, (\!|\nu * xvec|\!)((P \parallel Q) \parallel R), (\!|\nu * xvec|\!)(P \parallel (Q \parallel R))) \mid \Psi\ xvec\ P\ Q\ R.\ xvec \sharp* \Psi\}$
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in ?X \wedge \Psi \rhd Q' \sim Q\}$

have $(\Psi, (P \parallel Q) \parallel R, P \parallel (Q \parallel R)) \in ?X$
by(*auto*, *rule-tac x*=[] **in** *exI*) *auto*
moreover have *eqvt ?X* **by**(*force simp add*: *eqvt-def simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*] *eqvts*)
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct'*)
case(*cStatEq* Ψ *PQR PQR'*)
from ‹$(\Psi, PQR, PQR') \in ?X$› **obtain** *xvec P Q R* **where** *xvec* $\sharp*$ Ψ **and** $PQR = (\!|\nu * xvec|\!)((P \parallel Q) \parallel R)$ **and** $PQR' = (\!|\nu * xvec|\!)(P \parallel (Q \parallel R))$
by *auto*
moreover obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* Q$ **and** $A_P \sharp* R$
by(*rule-tac C*=(Ψ, *Q*, *R*) **in** *freshFrame*) *auto*
moreover obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* A_P$ **and** $A_Q \sharp* \Psi_P$ **and** $A_Q \sharp* R$
by(*rule-tac C*=(Ψ, A_P, Ψ_P, *R*) **in** *freshFrame*) *auto*
moreover obtain A_R Ψ_R **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* A_P$ **and** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* A_Q$ **and** $A_R \sharp* \Psi_Q$
by(*rule-tac C*=(Ψ, A_P, Ψ_P, A_Q, Ψ_Q) **in** *freshFrame*) *auto*
moreover from *FrQ* ‹$A_P \sharp* Q$› ‹$A_Q \sharp* A_P$› **have** $A_P \sharp* \Psi_Q$
by(*drule-tac extractFrameFreshChain*) *auto*
moreover from *FrR* ‹$A_P \sharp* R$› ‹$A_R \sharp* A_P$› **have** $A_P \sharp* \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*

moreover from *FrR* ‹$A_Q \sharp* R$› ‹$A_R \sharp* A_Q$› **have** $A_Q \sharp* \Psi_R$
by(*drule-tac extractFrameFreshChain*) *auto*
ultimately show *?case* **using** *freshCompChain*
by *auto* (*metis frameChainAppend compositionSym Associativity frameNilStatEq frameResChainPres*)
next
case(*cSim* Ψ T S)
from ‹$(\Psi, T, S) \in ?X$› **obtain** *xvec* P Q R **where** *xvec* $\sharp* \Psi$ **and** *TEq*: $T = (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$
and *SEq*: $S = (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))$
by *auto*
from ‹*eqvt ?X*›**have** *eqvt ?Y* **by** *blast*
have *C1*: $\bigwedge \Psi\ T\ S\ yvec.\ [\![(\Psi, T, S) \in ?Y;\ yvec \sharp* \Psi]\!] \Longrightarrow (\Psi, (\!|\nu*yvec|\!)T, (\!|\nu*yvec|\!)S) \in ?Y$
proof −
fix Ψ T S *yvec*
assume $(\Psi, T, S) \in ?Y$
then obtain T' S' **where** $\Psi \rhd T \sim T'$ **and** $(\Psi, T', S') \in ?X$ **and** $\Psi \rhd S' \sim S$ **by** *fastforce*
assume (*yvec*::*name list*) $\sharp* \Psi$
from ‹$(\Psi, T', S') \in ?X$› **obtain** *xvec* P Q R **where** $T'eq$: $T' = (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$ **and** $S'eq$: $S' = (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))$
and *xvec* $\sharp* \Psi$
by *auto*
from ‹$\Psi \rhd T \sim T'$› ‹*yvec* $\sharp* \Psi$› **have** $\Psi \rhd (\!|\nu*yvec|\!)T \sim (\!|\nu*yvec|\!)T'$ **by**(*rule bisimResChainPres*)
moreover from ‹*xvec* $\sharp* \Psi$› ‹*yvec* $\sharp* \Psi$› **have** $(\Psi, (\!|\nu*(yvec@xvec)|\!)((P \parallel Q) \parallel R), (\!|\nu*(yvec@xvec)|\!)(P \parallel (Q \parallel R))) \in ?X$
by *force*
with $T'eq$ $S'eq$ **have** $(\Psi, (\!|\nu*yvec|\!)T', (\!|\nu*yvec|\!)S') \in ?X$ **by**(*simp add*: *resChainAppend*)
moreover from ‹$\Psi \rhd S' \sim S$› ‹*yvec* $\sharp* \Psi$› **have** $\Psi \rhd (\!|\nu*yvec|\!)S' \sim (\!|\nu*yvec|\!)S$ **by**(*rule bisimResChainPres*)
ultimately show $(\Psi, (\!|\nu*yvec|\!)T, (\!|\nu*yvec|\!)S) \in ?Y$ **by** *blast*
qed
have *C2*: $\bigwedge \Psi\ T\ S\ y.\ [\![(\Psi, T, S) \in ?Y;\ y \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu y|\!)T, (\!|\nu y|\!)S) \in ?Y$
by(*drule-tac yvec2*=[*y*] **in** *C1*) *auto*

have $\Psi \rhd (\!|\nu*xvec|\!)((P \parallel Q) \parallel R) \rightsquigarrow[?Y] (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))$
proof −
have $\Psi \rhd (P \parallel Q) \parallel R \rightsquigarrow[?Y] P \parallel (Q \parallel R)$
proof −
note ‹*eqvt ?Y*›
moreover have $\bigwedge \Psi\ P\ Q\ R.\ (\Psi, (P \parallel Q) \parallel R, P \parallel (Q \parallel R)) \in ?Y$
proof −
fix Ψ P Q R
have $(\Psi::'b, ((P::('a, 'b, 'c)\ psi) \parallel Q) \parallel R, P \parallel (Q \parallel R)) \in ?X$
by(*auto*, *rule-tac x*=[] **in** *exI*) *auto*
thus $(\Psi, (P \parallel Q) \parallel R, P \parallel (Q \parallel R)) \in ?Y$

by(*blast intro*: *bisimReflexive*)
qed
moreover have $\bigwedge xvec\ \Psi\ P\ Q\ R.\ [\![xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ P]\!] \Longrightarrow (\Psi, (\!|\nu*xvec|\!)((P \parallel Q) \parallel R), P \parallel ((\!|\nu*xvec|\!)(Q \parallel R))) \in ?Y$
proof −
fix *xvec* Ψ *P Q R*
assume (*xvec*::*name list*) $\sharp*$ (Ψ::*'b*) **and** *xvec* $\sharp*$ (*P*::(*'a*, *'b*, *'c*) *psi*)
from ‹*xvec* $\sharp*$ Ψ› **have** $(\Psi, (\!|\nu*xvec|\!)((P \parallel Q) \parallel R), (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))) \in ?X$ **by** *blast*
moreover from ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *P*› **have** $\Psi \rhd (\!|\nu*xvec|\!)(P \parallel (Q \parallel R)) \sim P \parallel ((\!|\nu*xvec|\!)(Q \parallel R))$
by(*rule bisimScopeExtChain*)
ultimately show $(\Psi, (\!|\nu*xvec|\!)((P \parallel Q) \parallel R), P \parallel ((\!|\nu*xvec|\!)(Q \parallel R))) \in ?Y$
by(*blast intro*: *bisimReflexive*)
qed
moreover have $\bigwedge xvec\ \Psi\ P\ Q\ R.\ [\![xvec\ \sharp*\ \Psi;\ xvec\ \sharp*\ R]\!] \Longrightarrow (\Psi, ((\!|\nu*xvec|\!)(P \parallel Q)) \parallel R, (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))) \in ?Y$
proof −
fix *xvec* Ψ *P Q R*
assume (*xvec*::*name list*) $\sharp*$ (Ψ::*'b*) **and** *xvec* $\sharp*$ (*R*::(*'a*, *'b*, *'c*) *psi*)
have $\Psi \rhd ((\!|\nu*xvec|\!)(P \parallel Q)) \parallel R \sim R \parallel ((\!|\nu*xvec|\!)(P \parallel Q))$ **by**(*rule bisimParComm*)
moreover from ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *R*› **have** $\Psi \rhd (\!|\nu*xvec|\!)(R \parallel (P \parallel Q)) \sim R \parallel ((\!|\nu*xvec|\!)(P \parallel Q))$ **by**(*rule bisimScopeExtChain*)
hence $\Psi \rhd R \parallel ((\!|\nu*xvec|\!)(P \parallel Q)) \sim (\!|\nu*xvec|\!)(R \parallel (P \parallel Q))$ **by**(*rule bisimE*)
moreover from ‹*xvec* $\sharp*$ Ψ› **have** $\Psi \rhd (\!|\nu*xvec|\!)(R \parallel (P \parallel Q)) \sim (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$
by(*metis bisimResChainPres bisimParComm*)
moreover from ‹*xvec* $\sharp*$ Ψ› **have** $(\Psi, (\!|\nu*xvec|\!)((P \parallel Q) \parallel R), (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))) \in ?X$ **by** *blast*
ultimately show $(\Psi, ((\!|\nu*xvec|\!)(P \parallel Q)) \parallel R, (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))) \in ?Y$ **by**(*blast dest*: *bisimTransitive intro*: *bisimReflexive*)
qed
ultimately show *?thesis* **using** *C1*
by(*rule parAssocLeft*)
qed
thus *?thesis* **using** ‹*eqvt ?Y*› ‹*xvec* $\sharp*$ Ψ› *C2*
by(*rule resChainPres*)
qed
with *TEq SEq* **show** *?case* **by** *simp*
next
case(*cExt* Ψ *T S* Ψ')
from ‹$(\Psi, T, S) \in ?X$› **obtain** *xvec P Q R* **where** *xvec* $\sharp*$ Ψ **and** *TEq*: $T = (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$
and *SEq*: $S = (\!|\nu*xvec|\!)(P \parallel (Q \parallel R))$
by *auto*
obtain *p* **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and**

$(p \cdot xvec) \sharp* R$ **and** $(p \cdot xvec) \sharp* \Psi'$
and S: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm* p
by(*rule-tac* c=(Ψ, P, Q, R, Ψ') **in** *name-list-avoiding*) *auto*

from ‹$(p \cdot xvec) \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi'$› **have** ($\Psi \otimes \Psi'$, $(\!|\nu*(p \cdot xvec)|\!)(((p \cdot P) \parallel (p \cdot Q)) \parallel (p \cdot R))$, $(\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel ((p \cdot Q) \parallel (p \cdot R)))) \in ?X$
by *auto*
moreover from *TEq* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$(p \cdot xvec) \sharp* R$› S **have** $T = (\!|\nu*(p \cdot xvec)|\!)(((p \cdot P) \parallel (p \cdot Q)) \parallel (p \cdot R))$
apply *auto* **by**(*subst resChainAlpha*[*of p*]) *auto*
moreover from *SEq* ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* Q$› ‹$(p \cdot xvec) \sharp* R$› S **have** $S = (\!|\nu*(p \cdot xvec)|\!)((p \cdot P) \parallel ((p \cdot Q) \parallel (p \cdot R)))$
apply *auto* **by**(*subst resChainAlpha*[*of p*]) *auto*
ultimately show *?case* **by** *simp*
next
case(*cSym* Ψ T S)
from ‹$(\Psi, T, S) \in ?X$› **obtain** *xvec* P Q R **where** $xvec \sharp* \Psi$ **and** *TEq*: $T = (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$
and *SEq*: $(\!|\nu*xvec|\!)(P \parallel (Q \parallel R)) = S$
by *auto*

from ‹$xvec \sharp* \Psi$› **have** $\Psi \rhd (\!|\nu*xvec|\!)(P \parallel (Q \parallel R)) \sim (\!|\nu*xvec|\!)((R \parallel Q) \parallel P)$
by(*metis bisimParComm bisimParPres bisimTransitive bisimResChainPres*)
moreover from ‹$xvec \sharp* \Psi$› **have** (Ψ, $(\!|\nu*xvec|\!)((R \parallel Q) \parallel P)$, $(\!|\nu*xvec|\!)(R \parallel (Q \parallel P))) \in ?X$ **by** *blast*
moreover from ‹$xvec \sharp* \Psi$› **have** $\Psi \rhd (\!|\nu*xvec|\!)(R \parallel (Q \parallel P)) \sim (\!|\nu*xvec|\!)((P \parallel Q) \parallel R)$
by(*metis bisimParComm bisimParPres bisimTransitive bisimResChainPres*)
ultimately show *?case* **using** *TEq SEq* **by**(*blast dest*: *bisimTransitive*)
qed
qed

lemma *bisimParNil*:
fixes $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \parallel \mathbf{0} \sim P$
proof −
let *?X1* = $\{(\Psi, P \parallel \mathbf{0}, P) \mid \Psi\ P.\ True\}$
let *?X2* = $\{(\Psi, P, P \parallel \mathbf{0}) \mid \Psi\ P.\ True\}$
let *?X* = *?X1* ∪ *?X2*
have *eqvt ?X* **by**(*auto simp add*: *eqvt-def*)
have $(\Psi, P \parallel \mathbf{0}, P) \in ?X$ **by** *simp*
thus *?thesis*
proof(*coinduct rule*: *bisimWeakCoinduct*)
case(*cStatEq* Ψ Q R)
show *?case*
proof(*case-tac* $(\Psi, Q, R) \in ?X1$)
assume $(\Psi, Q, R) \in ?X1$
then obtain P **where** $Q = P \parallel \mathbf{0}$ **and** $R = P$ **by** *auto*

```
    moreover obtain A_P Ψ_P where extractFrame P = ⟨A_P, Ψ_P⟩ and A_P ♯* Ψ
      by(rule freshFrame)
    ultimately show ?case
      apply auto by(metis frameResChainPres frameNilStatEq Identity Associativity AssertionStatEqTrans Commutativity)
  next
    assume (Ψ, Q, R) ∉ ?X1
    with ‹(Ψ, Q, R) ∈ ?X› have (Ψ, Q, R) ∈ ?X2 by blast
    then obtain P where Q = P and R = P ∥ 0 by auto
    moreover obtain A_P Ψ_P where extractFrame P = ⟨A_P, Ψ_P⟩ and A_P ♯* Ψ
      by(rule freshFrame)
    ultimately show ?case
      apply auto by(metis frameResChainPres frameNilStatEq Identity Associativity AssertionStatEqTrans AssertionStatEqSym Commutativity)
  qed
 next
  case(cSim Ψ Q R)
  thus ?case using ‹eqvt ?X›
    by(auto intro: parNilLeft parNilRight)
 next
  case(cExt Ψ Q R Ψ')
  thus ?case by auto
 next
  case(cSym Ψ Q R)
  thus ?case by auto
 qed
qed

lemma bisimResNil:
 fixes x :: name
 and   Ψ :: 'b

 shows Ψ ▷ (|νx|)0 ∼ 0
proof −
 {
  fix x::name
  assume x ♯ Ψ
  have Ψ ▷ (|νx|)0 ∼ 0
  proof −
   let ?X1 = {(Ψ, (|νx|)0, 0) | Ψ x. x ♯ Ψ}
   let ?X2 = {(Ψ, 0, (|νx|)0) | Ψ x. x ♯ Ψ}
   let ?X = ?X1 ∪ ?X2

   from ‹x ♯ Ψ› have (Ψ, (|νx|)0, 0) ∈ ?X by auto
   thus ?thesis
   proof(coinduct rule: bisimWeakCoinduct)
    case(cStatEq Ψ P Q)
   thus ?case using freshComp by(force intro: frameResFresh FrameStatEqSym)
   next
```

```
      case(cSim Ψ P Q)
      thus ?case
        by(force intro: resNilLeft resNilRight)
    next
      case(cExt Ψ P Q Ψ')
      obtain y where y ♯ Ψ and y ♯ Ψ' and y ≠ x
        by(generate-fresh name) (auto simp add: fresh-prod)
      show ?case
      proof(case-tac (Ψ, P, Q) ∈ ?X1)
        assume (Ψ, P, Q) ∈ ?X1
        then obtain x where P = (|νx|)0 and Q = 0 by auto
        moreover have (|νx|)0 = (|νy|) 0 by(subst alphaRes) auto
        ultimately show ?case using ‹y ♯ Ψ› ‹y ♯ Ψ'› by auto
      next
        assume (Ψ, P, Q) ∉ ?X1
        with ‹(Ψ, P, Q) ∈ ?X› have (Ψ, P, Q) ∈ ?X2 by auto
        then obtain x where Q = (|νx|)0 and P = 0 by auto
        moreover have (|νx|)0 = (|νy|) 0 by(subst alphaRes) auto
        ultimately show ?case using ‹y ♯ Ψ› ‹y ♯ Ψ'› by auto
      qed
    next
      case(cSym Ψ P Q)
      thus ?case by auto
    qed
  qed
  }
  moreover obtain y::name where y ♯ Ψ by(generate-fresh name) auto
  ultimately have Ψ ▷ (|νy|)0 ∼ 0 by auto
  thus ?thesis by(subst alphaRes[where x=x and y=y]) auto
qed

lemma bisimOutputPushRes:
  fixes x :: name
  and   Ψ :: 'b
  and   M :: 'a
  and   N :: 'a
  and   P :: ('a, 'b, 'c) psi

  assumes x ♯ M
  and     x ♯ N

  shows Ψ ▷ (|νx|)(M⟨N⟩.P) ∼ M⟨N⟩.(|νx|)P
proof -
  {
    fix x::name and P::('a, 'b, 'c) psi
    assume x ♯ Ψ and x ♯ M and x ♯ N
    let ?X1 = {(Ψ, (|νx|)(M⟨N⟩.P), M⟨N⟩.(|νx|)P) | Ψ x M N P. x ♯ Ψ ∧ x ♯ M ∧
x ♯ N}
    let ?X2 = {(Ψ, M⟨N⟩.(|νx|)P, (|νx|)(M⟨N⟩.P)) | Ψ x M N P. x ♯ Ψ ∧ x ♯ M ∧
```

```
x ♯ N}
  let ?X = ?X1 ∪ ?X2

  have eqvt ?X by(rule-tac eqvtUnion) (force simp add: eqvt-def pt-fresh-bij[OF
pt-name-inst, OF at-name-inst] eqvts)+
  from ‹x ♯ Ψ› ‹x ♯ M› ‹x ♯ N› have (Ψ, (|νx|)(M⟨N⟩.P), M⟨N⟩.(|νx|)P) ∈ ?X
by auto
  hence Ψ ▷ (|νx|)(M⟨N⟩.P) ∼ M⟨N⟩.(|νx|)P
  proof(coinduct rule: bisimCoinduct)
    case(cStatEq Ψ Q R)
   thus ?case using freshComp by(force intro: frameResFresh FrameStatEqSym)
  next
    case(cSim Ψ Q R)
    thus ?case using ‹eqvt ?X›
      by(fastforce intro: outputPushResLeft outputPushResRight bisimReflexive)
  next
    case(cExt Ψ Q R Ψ')
    show ?case
    proof(case-tac (Ψ, Q, R) ∈ ?X1)
      assume (Ψ, Q, R) ∈ ?X1
       then obtain x M N P where Qeq: Q = (|νx|)(M⟨N⟩.P) and Req: R =
M⟨N⟩.(|νx|)P and x ♯ Ψ and x ♯ M and x ♯ N by auto
      obtain y::name where y ♯ Ψ and y ♯ Ψ' and y ♯ M and y ♯ N and y ♯ P
        by(generate-fresh name) (auto simp add: fresh-prod)

      moreover hence (Ψ ⊗ Ψ', (|νy|)(M⟨N⟩.([(x, y)] • P)), M⟨N⟩.(|νy|)([(x, y)]
• P)) ∈ ?X by auto
      moreover from Qeq ‹x ♯ M› ‹y ♯ M› ‹x ♯ N› ‹y ♯ N› ‹y ♯ P› have Q =
(|νy|)(M⟨N⟩.([(x, y)] • P))
        apply auto by(subst alphaRes[of y]) (auto simp add: eqvts)
      moreover from Req ‹y ♯ P› have R = M⟨N⟩.(|νy|)([(x, y)] • P)
        apply auto by(subst alphaRes[of y]) (auto simp add: eqvts)
      ultimately show ?case by blast
    next
      assume (Ψ, Q, R) ∉ ?X1
      with ‹(Ψ, Q, R) ∈ ?X› have (Ψ, Q, R) ∈ ?X2 by blast
       then obtain x M N P where Req: R = (|νx|)(M⟨N⟩.P) and Qeq: Q =
M⟨N⟩.(|νx|)P and x ♯ Ψ and x ♯ M and x ♯ N by auto
      obtain y::name where y ♯ Ψ and y ♯ Ψ' and y ♯ M and y ♯ N and y ♯ P
        by(generate-fresh name) (auto simp add: fresh-prod)

      moreover hence (Ψ ⊗ Ψ', (|νy|)(M⟨N⟩.([(x, y)] • P)), M⟨N⟩.(|νy|)([(x, y)]
• P)) ∈ ?X by auto
      moreover from Req ‹x ♯ M› ‹y ♯ M› ‹x ♯ N› ‹y ♯ N› ‹y ♯ P› have R =
(|νy|)(M⟨N⟩.([(x, y)] • P))
        apply auto by(subst alphaRes[of y]) (auto simp add: eqvts)
      moreover from Qeq ‹y ♯ P› have Q = M⟨N⟩.(|νy|)([(x, y)] • P)
        apply auto by(subst alphaRes[of y]) (auto simp add: eqvts)
      ultimately show ?case by blast
```

```
    qed
  next
    case(cSym Ψ R Q)
    thus ?case by blast
  qed
 }
 moreover obtain y::name where y ♯ Ψ and y ♯ M and y ♯ N y ♯ P
  by(generate-fresh name) auto
 ultimately have Ψ ▷ (|νy|)(M⟨N⟩.([(x, y)] · P)) ∼ M⟨N⟩.(|νy|)([(x, y)] · P) by
auto
 thus ?thesis using assms ‹y ♯ P› ‹y ♯ M› ‹y ♯ N›
  apply(subst alphaRes[where x=x and y=y and P=P], auto)
  by(subst alphaRes[where x=x and y=y and P=M⟨N⟩.P]) auto
qed

lemma bisimInputPushRes:
 fixes x    :: name
 and   Ψ    :: 'b
 and   M    :: 'a
 and   xvec :: name list
 and   N    :: 'a
 and   P    :: ('a, 'b, 'c) psi

 assumes x ♯ M
 and     x ♯ xvec
 and     x ♯ N

 shows Ψ ▷ (|νx|)(M(|λ∗xvec N|).P) ∼ M(|λ∗xvec N|).(|νx|)P
proof −
 {
  fix x::name and P::('a, 'b, 'c) psi
  assume x ♯ Ψ and x ♯ M and x ♯ N and x ♯ xvec
  let ?X1 = {(Ψ, (|νx|)(M(|λ∗xvec N|).P), M(|λ∗xvec N|).(|νx|)P) | Ψ x M xvec N
P. x ♯ Ψ ∧ x ♯ M ∧ x ♯ xvec ∧ x ♯ N}
  let ?X2 = {(Ψ, M(|λ∗xvec N|).(|νx|)P, (|νx|)(M(|λ∗xvec N|).P)) | Ψ x M xvec N
P. x ♯ Ψ ∧ x ♯ M ∧ x ♯ xvec ∧ x ♯ N}
  let ?X = ?X1 ∪ ?X2

  have eqvt ?X by(rule-tac eqvtUnion) (force simp add: eqvt-def pt-fresh-bij[OF
pt-name-inst, OF at-name-inst] eqvts)+

  from ‹x ♯ Ψ› ‹x ♯ M› ‹x ♯ xvec› ‹x ♯ N› have (Ψ, (|νx|)(M(|λ∗xvec N|).P),
M(|λ∗xvec N|).(|νx|)P) ∈ ?X by blast
  hence Ψ ▷ (|νx|)(M(|λ∗xvec N|).P) ∼ M(|λ∗xvec N|).(|νx|)P
  proof(coinduct rule: bisimCoinduct)
   case(cStatEq Ψ Q R)
  thus ?case using freshComp by(force intro: frameResFresh FrameStatEqSym)
  next
   case(cSim Ψ Q R)
```

thus *?case* **using** ‹*eqvt ?X*›
by(*fastforce intro*: *inputPushResLeft inputPushResRight bisimReflexive*)
next
case(*cExt* Ψ *Q R* Ψ′)
show *?case*
proof(*case-tac* (Ψ, *Q*, *R*) ∈ *?X1*)
assume (Ψ, *Q*, *R*) ∈ *?X1*
then obtain *x M xvec N P* **where** *Qeq*: $Q = (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P)$ **and** *Req*: $R = M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$ **and** $x \sharp \Psi$
and $x \sharp M$ **and** $x \sharp xvec$ **and** $x \sharp N$ **by** *auto*
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp \Psi'$ **and** $y \sharp M$ **and** $y \sharp N$ **and** $y \sharp P$ **and** $y \sharp xvec$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)

moreover hence $(\Psi \otimes \Psi', (\!|\nu y|\!)(M(\!|\lambda * xvec\ N|\!).([(x,\ y)] \cdot P)), M(\!|\lambda * xvec\ N|\!).(\!|\nu y|\!)([(x,\ y)] \cdot P)) \in ?X$ **by** *fastforce*
moreover from *Qeq* ‹$x \sharp M$› ‹$y \sharp M$› ‹$x \sharp xvec$› ‹$y \sharp xvec$› ‹$x \sharp N$› ‹$y \sharp N$› ‹$y \sharp P$› **have** $Q = (\!|\nu y|\!)(M(\!|\lambda * xvec\ N|\!).([(x,\ y)] \cdot P))$
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts inputChainFresh*)
moreover from *Req* ‹$y \sharp P$› **have** $R = M(\!|\lambda * xvec\ N\ |\!).(\!|\nu y|\!)([(x,\ y)] \cdot P)$
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
assume (Ψ, *Q*, *R*) ∉ *?X1*
with ‹(Ψ, *Q*, *R*) ∈ *?X*› **have** (Ψ, *Q*, *R*) ∈ *?X2* **by** *blast*
then obtain *x M xvec N P* **where** *Req*: $R = (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P)$ **and** *Qeq*: $Q = M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$ **and** $x \sharp \Psi$
and $x \sharp M$ **and** $x \sharp xvec$ **and** $x \sharp N$ **by** *auto*
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp \Psi'$ **and** $y \sharp M$ **and** $y \sharp N$ **and** $y \sharp P$ **and** $y \sharp xvec$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)

moreover hence $(\Psi \otimes \Psi', (\!|\nu y|\!)(M(\!|\lambda * xvec\ N|\!).([(x,\ y)] \cdot P)), M(\!|\lambda * xvec\ N|\!).(\!|\nu y|\!)([(x,\ y)] \cdot P)) \in ?X$ **by** *fastforce*
moreover from *Req* ‹$x \sharp M$› ‹$y \sharp M$› ‹$x \sharp xvec$› ‹$y \sharp xvec$› ‹$x \sharp N$› ‹$y \sharp N$› ‹$y \sharp P$› **have** $R = (\!|\nu y|\!)(M(\!|\lambda * xvec\ N|\!).([(x,\ y)] \cdot P))$
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts inputChainFresh*)
moreover from *Qeq* ‹$y \sharp P$› **have** $Q = M(\!|\lambda * xvec\ N\ |\!).(\!|\nu y|\!)([(x,\ y)] \cdot P)$
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
qed
next
case(*cSym* Ψ *R Q*)
thus *?case* **by** *blast*
qed
}
moreover obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp M$ **and** $y \sharp N$ **and** $y \sharp P$ **and** $y \sharp xvec$
by(*generate-fresh name*) *auto*

ultimately have $\Psi \rhd (\!|\nu y|\!)(M(\!|\lambda * xvec\ N|\!).([(x,\ y)] \cdot P)) \sim M(\!|\lambda * xvec\ N|\!).(\!|\nu y|\!)([(x,\ y)] \cdot P)$ **by** *auto*
thus *?thesis* **using** *assms* ‹$y \sharp P$› ‹$y \sharp M$› ‹$y \sharp N$› ‹$y \sharp xvec$›
apply(*subst alphaRes*[**where** $x{=}x$ **and** $y{=}y$ **and** $P{=}P$], *auto*)
by(*subst alphaRes*[**where** $x{=}x$ **and** $y{=}y$ **and** $P{=}M(\!|\lambda * xvec\ N|\!).P$]) (*auto simp add*: *inputChainFresh eqvts*)
qed

lemma *bisimCasePushRes*:
fixes x :: *name*
and Ψ :: $'b$
and Cs :: $('c \times ('a,\ 'b,\ 'c)\ psi)\ list$

assumes $x \sharp (map\ fst\ Cs)$

shows $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \sim Cases(map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu x|\!)P))\ Cs)$
proof −
{
fix x::*name* **and** Cs::$('c \times ('a,\ 'b,\ 'c)\ psi)\ list$
assume $x \sharp \Psi$ **and** $x \sharp (map\ fst\ Cs)$
let *?X1* = $\{(\Psi,\ (\!|\nu x|\!)(Cases\ Cs),\ Cases(map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu x|\!)P))\ Cs)) \mid \Psi\ x\ Cs.\ x \sharp \Psi \wedge x \sharp (map\ fst\ Cs)\}$
let *?X2* = $\{(\Psi,\ Cases(map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu x|\!)P))\ Cs),\ (\!|\nu x|\!)(Cases\ Cs)) \mid \Psi\ x\ Cs.\ x \sharp \Psi \wedge x \sharp (map\ fst\ Cs)\}$
let *?X* = *?X1* ∪ *?X2*

have *eqvt ?X* **apply**(*rule-tac eqvtUnion*)
apply(*auto simp add*: *eqvt-def eqvts*)
apply(*rule-tac* $x{=}p \cdot x$ **in** *exI*)
apply(*rule-tac* $x{=}p \cdot Cs$ **in** *exI*)
apply(*perm-extend-simp*)
apply(*auto simp add*: *eqvts*)
apply(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi*=*p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi*=*p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *eqvts*)
apply(*perm-extend-simp*)
apply(*simp add*: *eqvts*)
apply(*rule-tac* $x{=}p \cdot x$ **in** *exI*)
apply(*rule-tac* $x{=}p \cdot Cs$ **in** *exI*)
apply *auto*
apply(*perm-extend-simp*)
apply(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi*=*p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*drule-tac pi*=*p* **in** *pt-fresh-bij1*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *eqvts*)
apply(*perm-extend-simp*)
by(*simp add*: *eqvts*)

from ‹$x \sharp \Psi$› ‹$x \sharp$ *map fst Cs*› **have** (Ψ, (νx)(*Cases Cs*), *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νx)P)) *Cs*)) $\in$ *?X* **by** *auto*
hence $\Psi \rhd$ (νx)(*Cases Cs*) $\sim$ *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νx)P)) *Cs*)
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* Ψ Q R)
thus *?case* **using** *freshComp* **by**(*force intro*: *frameResFresh FrameStatEqSym*)
next
case(*cSim* Ψ Q R)
thus *?case* **using** ‹*eqvt ?X*›
by(*fastforce intro*: *casePushResLeft casePushResRight bisimReflexive*)
next
case(*cExt* Ψ Q R Ψ')
show *?case*
proof(*case-tac* (Ψ, Q, R) $\in$ *?X1*)
assume (Ψ, Q, R) $\in$ *?X1*
then obtain *x Cs* **where** *Qeq*: Q = (νx)(*Cases Cs*) **and** *Req*: R = *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νx)P)) *Cs*)
and $x \sharp \Psi$ **and** $x \sharp$ (*map fst Cs*) **by** *blast*
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp \Psi'$ **and** $y \sharp$ *Cs*
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
from ‹$y \sharp$ *Cs*› ‹$x \sharp$ (*map fst Cs*)› **have** $y \sharp$ *map fst* ([(x, y)] $\cdot$ *Cs*) **by**(*induct Cs*) (*auto simp add*: *fresh-list-cons fresh-list-nil*)

moreover with ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** ($\Psi \otimes \Psi'$, (νy)(*Cases* ([(x, y)] $\cdot$ *Cs*)), *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νy)P)) ([(x, y)] $\cdot$ *Cs*))) $\in$ *?X*
by *auto*
moreover from *Qeq* ‹$y \sharp$ *Cs*› **have** Q = (νy)(*Cases*([(x, y)] $\cdot$ *Cs*))
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts*)
moreover from *Req* ‹$y \sharp$ *Cs*› ‹$x \sharp$ (*map fst Cs*)› **have** R = *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νy)P)) ([(x, y)] $\cdot$ *Cs*))
by(*induct Cs arbitrary*: R) (*auto simp add*: *fresh-list-cons fresh-prod alphaRes*)
ultimately show *?case* **by** *blast*
next
assume (Ψ, Q, R) $\notin$ *?X1*
with ‹(Ψ, Q, R) $\in$ *?X*› **have** (Ψ, Q, R) $\in$ *?X2* **by** *blast*
then obtain *x Cs* **where** *Req*: R = (νx)(*Cases Cs*) **and** *Qeq*: Q = *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νx)P)) *Cs*)
and $x \sharp \Psi$ **and** $x \sharp$ (*map fst Cs*) **by** *blast*
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp \Psi'$ **and** $y \sharp$ *Cs*
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
from ‹$y \sharp$ *Cs*› ‹$x \sharp$ (*map fst Cs*)› **have** $y \sharp$ *map fst* ([(x, y)] $\cdot$ *Cs*) **by**(*induct Cs*) (*auto simp add*: *fresh-list-cons fresh-list-nil*)

moreover with ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** ($\Psi \otimes \Psi'$, (νy)(*Cases* ([(x, y)] $\cdot$ *Cs*)), *Cases*(*map* ($\lambda(\varphi, P)$. (φ, (νy)P)) ([(x, y)] $\cdot$ *Cs*))) $\in$ *?X*
by *auto*
moreover from *Req* ‹$y \sharp$ *Cs*› **have** R = (νy)(*Cases*([(x, y)] $\cdot$ *Cs*))
apply *auto* **by**(*subst alphaRes*[*of y*]) (*auto simp add*: *eqvts*)

moreover from *Qeq* ‹$y \sharp Cs$› ‹$x \sharp (map\ fst\ Cs)$› **have** $Q = Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu y|\!)P))\ ([(x, y)] \cdot Cs))$
by(*induct Cs arbitrary*: *Q*) (*auto simp add*: *fresh-list-cons fresh-prod alphaRes*)
ultimately show *?case* **by** *blast*
qed
next
case(*cSym* Ψ *R Q*)
thus *?case* **by** *blast*
qed
}
moreover obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp Cs$ **by**(*generate-fresh name*) *auto*
moreover from ‹$x \sharp map\ fst\ Cs$› **have** $y \sharp map\ fst([(x, y)] \cdot Cs)$
by(*induct Cs*) (*auto simp add*: *fresh-left calc-atm*)
ultimately have $\Psi \rhd (\!|\nu y|\!)(Cases\ ([(x, y)] \cdot Cs)) \sim Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu y|\!)P))\ ([(x, y)] \cdot Cs))$
by *auto*
moreover from ‹$y \sharp Cs$› **have** $(\!|\nu y|\!)(Cases\ ([(x, y)] \cdot Cs)) = (\!|\nu x|\!)(Cases\ Cs)$
by(*simp add*: *alphaRes eqvts*)
moreover from ‹$x \sharp map\ fst\ Cs$› ‹$y \sharp Cs$› **have** $Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu y|\!)P))\ ([(x, y)] \cdot Cs)) = Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
by(*induct Cs*) (*auto simp add*: *alphaRes*)
ultimately show *?thesis* **by** *auto*
qed

lemma *bangExt*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

assumes *guarded P*

shows $\Psi \rhd !P \sim P \parallel !P$
proof −
let *?X* = $\{(\Psi, !P, P \parallel !P) \mid \Psi\ P.\ guarded\ P\} \cup \{(\Psi, P \parallel !P, !P) \mid \Psi\ P.\ guarded\ P\}$
from ‹*guarded P*› **have** $(\Psi, !P, P \parallel !P) \in ?X$ **by** *auto*
thus *?thesis*
proof(*coinduct rule*: *bisimCoinduct*)
case(*cStatEq* Ψ *Q R*)
from ‹$(\Psi, Q, R) \in ?X$› **obtain** *P* **where** *Eq*: $(Q = !P \wedge R = P \parallel !P) \vee (Q = P \parallel !P \wedge R = !P)$ **and** *guarded P*
by *auto*
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame P* = $\langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **by**(*rule freshFrame*)
from *FrP* ‹*guarded P*› **have** $\Psi_P \simeq SBottom'$ **by**(*blast dest*: *guardedStatEq*)
from ‹$\Psi_P \simeq SBottom'$› **have** $\Psi \otimes SBottom' \simeq \Psi \otimes \Psi_P \otimes SBottom'$ **by**(*metis Identity Composition AssertionStatEqTrans Commutativity AssertionStatEqSym*)
hence $\langle A_P, \Psi \otimes SBottom'\rangle \simeq_F \langle A_P, \Psi \otimes \Psi_P \otimes SBottom'\rangle$

```
    by(force intro: frameResChainPres)
   moreover from ‹A_P ♯* Ψ› have ⟨ε, Ψ ⊗ SBottom'⟩ ≃_F ⟨A_P, Ψ ⊗ SBottom'⟩
    by(rule-tac FrameStatEqSym) (fastforce intro: frameResFreshChain)
   ultimately show ?case using Eq ‹A_P ♯* Ψ› FrP
    by auto (blast dest: FrameStatEqTrans FrameStatEqSym)+
  next
   case(cSim Ψ Q R)
   thus ?case by(auto intro: bangExtLeft bangExtRight bisimReflexive)
  next
   case(cExt Ψ Q R)
   thus ?case by auto
  next
   case(cSym Ψ Q R)
   thus ?case by auto
  qed
qed

lemma bisimParPresSym:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   R :: ('a, 'b, 'c) psi

  assumes Ψ ▷ P ∼ Q

  shows Ψ ▷ R ∥ P ∼ R ∥ Q
using assms
by(metis bisimParComm bisimParPres bisimTransitive)

lemma bisimScopeExtSym:
  fixes x :: name
  and   Q :: ('a, 'b, 'c) psi
  and   P :: ('a, 'b, 'c) psi

  assumes x ♯ Ψ
  and     x ♯ Q

  shows Ψ ▷ (⦇νx⦈)(P ∥ Q) ∼ ((⦇νx⦈)P) ∥ Q
using assms
by(metis bisimScopeExt bisimTransitive bisimParComm bisimSymmetric bisimResPres)

lemma bisimScopeExtChainSym:
  fixes xvec :: name list
  and   Q    :: ('a, 'b, 'c) psi
  and   P    :: ('a, 'b, 'c) psi

  assumes xvec ♯* Ψ
  and     xvec ♯* Q
```

shows $\Psi \rhd (\!|\nu * xvec|\!)(P \parallel Q) \sim ((\!|\nu * xvec|\!)P) \parallel Q$
using *assms*
by(*induct xvec*) (*auto intro*: *bisimScopeExtSym bisimReflexive bisimTransitive bisimResPres*)

lemma *bisimParPresAuxSym*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \otimes \Psi_R \rhd P \sim Q$
and *extractFrame* $R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$

shows $\Psi \rhd R \parallel P \sim R \parallel Q$
using *assms*
by(*metis bisimParComm bisimParPresAux bisimTransitive*)

lemma *bangDerivative*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ action$
and $P' :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd !P \longmapsto \alpha \prec P'$
and $\Psi \rhd P \sim Q$
and $bn\ \alpha \sharp* \Psi$
and $bn\ \alpha \sharp* P$
and $bn\ \alpha \sharp* Q$
and $bn\ \alpha \sharp* subject\ \alpha$
and *guarded* Q

obtains $Q'\ R\ T$ **where** $\Psi \rhd !Q \longmapsto \alpha \prec Q'$ **and** $\Psi \rhd P' \sim R \parallel !P$ **and** $\Psi \rhd Q' \sim T \parallel !Q$ **and** $\Psi \rhd R \sim T$
and $((supp\ R)::name\ set) \subseteq supp\ P'$ **and** $((supp\ T)::name\ set) \subseteq supp\ Q'$
proof −
from $\langle \Psi \rhd !P \longmapsto \alpha \prec P' \rangle$ **have** *guarded* P **apply** − **by**(*ind-cases* $\Psi \rhd !P \longmapsto \alpha \prec P'$) (*auto simp add*: *psi.inject*)
assume $\bigwedge Q'\ R\ T.\ [\![\Psi \rhd !Q \longmapsto \alpha \prec Q';\ \Psi \rhd P' \sim R \parallel !P;\ \Psi \rhd Q' \sim T \parallel !Q;\ \Psi \rhd R \sim T;\ ((supp\ R)::name\ set) \subseteq supp\ P';$
$((supp\ T)::name\ set) \subseteq supp\ Q']\!] \Longrightarrow thesis$
moreover from $\langle \Psi \rhd !P \longmapsto \alpha \prec P' \rangle$ $\langle bn\ \alpha \sharp* subject\ \alpha \rangle$ $\langle bn\ \alpha \sharp* \Psi \rangle$ $\langle bn\ \alpha \sharp* P \rangle$ $\langle bn\ \alpha \sharp* Q \rangle$ $\langle \Psi \rhd P \sim Q \rangle$ $\langle guarded\ Q \rangle$
have $\exists\, Q'\ T\ R\ .\ \Psi \rhd !Q \longmapsto \alpha\ \prec Q' \wedge \Psi \rhd P' \sim R \parallel !P \wedge \Psi \rhd Q' \sim T \parallel !Q$

$\wedge\ \Psi \rhd R \sim T \wedge$
((*supp* R)::*name set*) $\subseteq$ *supp* $P' \wedge$ ((*supp* T)::*name set*) $\subseteq$ *supp* Q'
proof(*nominal-induct avoiding*: Q *rule*: *bangInduct'*)
case(*cAlpha* α P' p Q)
then obtain Q' T R **where** *QTrans*: $\Psi \rhd\ !Q \longmapsto \alpha \prec Q'$ **and** $\Psi \rhd P' \sim R \parallel (P \parallel\ !P)$ **and** $\Psi \rhd Q' \sim T \parallel\ !Q$ **and** $\Psi \rhd R \sim T$
and *suppR*: ((*supp* R)::*name set*) $\subseteq$ *supp* P' **and** *suppT*: ((*supp* T)::*name set*) $\subseteq$ *supp* Q'
by *blast*
from *QTrans* **have** *distinct*(*bn* α) **by**(*rule boundOutputDistinct*)
have *S*: *set* $p \subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$)) **by** *fact*
from *QTrans* ‹*bn*($p \cdot \alpha$) $\sharp*$ Q› ‹*bn*($p \cdot \alpha$) $\sharp*$ α› ‹*bn* α $\sharp*$ *subject* α› ‹*distinct*(*bn* α)› **have** *bn*($p \cdot \alpha$) $\sharp*$ Q'
by(*drule-tac freeFreshChainDerivative*) *simp*+
with *QTrans* ‹*bn*($p \cdot \alpha$) $\sharp*$ α› S ‹*bn* α $\sharp*$ *subject* α› **have** $\Psi \rhd\ !Q \longmapsto (p \cdot \alpha) \prec (p \cdot Q')$
by(*force simp add*: *residualAlpha*)
moreover from ‹$\Psi \rhd P' \sim R \parallel (P \parallel\ !P)$› **have** $(p \cdot \Psi) \rhd (p \cdot P') \sim (p \cdot (R \parallel (P \parallel\ !P)))$
by(*rule bisimClosed*)
with ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ P› ‹*bn*($p \cdot \alpha$) $\sharp*$ Ψ› ‹*bn*($p \cdot \alpha$) $\sharp*$ P› S **have** $\Psi \rhd (p \cdot P') \sim (p \cdot R) \parallel (P \parallel\ !P)$
by(*simp add*: *eqvts*)
moreover from ‹$\Psi \rhd Q' \sim T \parallel\ !Q$› **have** $(p \cdot \Psi) \rhd (p \cdot Q') \sim (p \cdot (T \parallel\ !Q))$
by(*rule bisimClosed*)
with ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ Q› ‹*bn*($p \cdot \alpha$) $\sharp*$ Ψ› ‹*bn*($p \cdot \alpha$) $\sharp*$ Q› S **have** $\Psi \rhd (p \cdot Q') \sim (p \cdot T) \parallel\ !Q$
by(*simp add*: *eqvts*)
moreover from ‹$\Psi \rhd R \sim T$› **have** $(p \cdot \Psi) \rhd (p \cdot R) \sim (p \cdot T)$
by(*rule bisimClosed*)
with ‹*bn* α $\sharp*$ Ψ› ‹*bn*($p \cdot \alpha$) $\sharp*$ Ψ› S **have** $\Psi \rhd (p \cdot R) \sim (p \cdot T)$
by(*simp add*: *eqvts*)
moreover from *suppR* **have** ((*supp*($p \cdot R$))::*name set*) $\subseteq$ *supp*($p \cdot P'$)
apply(*erule-tac rev-mp*)
by(*subst subsetClosed*[*of p*, *symmetric*]) (*simp add*: *eqvts*)
moreover from *suppT* **have** ((*supp*($p \cdot T$))::*name set*) $\subseteq$ *supp*($p \cdot Q'$)
apply(*erule-tac rev-mp*)
by(*subst subsetClosed*[*of p*, *symmetric*]) (*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
case(*cPar1* α P' Q)
from ‹$\Psi \rhd P \sim Q$› ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹*bn* α $\sharp*$ Ψ› ‹*bn* α $\sharp*$ Q›
obtain Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto \alpha \prec Q'$ **and** $\Psi \rhd P' \sim Q'$
by(*blast dest*: *bisimE simE*)
from *QTrans* **have** $\Psi \otimes SBottom' \rhd Q \longmapsto \alpha \prec Q'$ **by**(*metis statEqTransition Identity AssertionStatEqSym*)
hence $\Psi \rhd Q \parallel\ !Q \longmapsto \alpha \prec (Q' \parallel\ !Q)$ **using** ‹*bn* α $\sharp*$ Q› **by**(*rule-tac Par1*) (*assumption* | *simp*)+
hence $\Psi \rhd\ !Q \longmapsto \alpha \prec (Q' \parallel\ !Q)$ **using** ‹*guarded* Q› **by**(*rule Bang*)

moreover from ‹*guarded P*› **have** $\Psi \rhd P' \parallel !P \sim P' \parallel (P \parallel !P)$ **by**(*metis bangExt bisimParPresSym*)
moreover have $\Psi \rhd Q' \parallel !Q \sim Q' \parallel !Q$ **by**(*rule bisimReflexive*)
ultimately show *?case* **using** ‹$\Psi \rhd P' \sim Q'$› **by**(*force simp add*: *psi.supp*)
next
case(*cPar2* α P' Q)
then obtain Q' T R **where** *QTrans*: $\Psi \rhd !Q \longmapsto \alpha \prec Q'$ **and** $\Psi \rhd P' \sim R \parallel !P$ **and** $\Psi \rhd Q' \sim T \parallel !Q$ **and** $\Psi \rhd R \sim T$
and *suppR*: ((*supp* R)::*name set*) $\subseteq$ *supp* P' **and** *suppT*: ((*supp* T)::*name set*) $\subseteq$ *supp* Q'
by *blast*
note *QTrans*
from ‹$\Psi \rhd P' \sim R \parallel !P$› **have** $\Psi \rhd P \parallel P' \sim R \parallel (P \parallel !P)$
by(*metis bisimParPresSym bisimParComm bisimTransitive bisimParAssoc*)
with *QTrans* **show** *?case* **using** ‹$\Psi \rhd Q' \sim T \parallel !Q$› ‹$\Psi \rhd R \sim T$› *suppR* *suppT*
by(*force simp add*: *psi.supp*)
next
case(*cComm1 M N P' K xvec P'' Q*)
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \rhd Q \rightsquigarrow [bisim]\ P$ **by**(*metis bisimE*)
with ‹$\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$› **obtain** Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto M(\!|N|\!) \prec Q'$ **and** $\Psi \rhd Q' \sim P'$
by(*force dest*: *simE*)
from *QTrans* **have** $\Psi \otimes SBottom' \rhd Q \longmapsto M(\!|N|\!) \prec Q'$ **by**(*metis statEqTransition Identity AssertionStatEqSym*)
moreover obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* Q$ **and** $A_Q \sharp* M$
by(*rule-tac* C=(Ψ, Q, M) **in** *freshFrame*) *auto*
note *FrQ*
moreover from *FrQ* ‹*guarded Q*› **have** $\Psi_Q \simeq SBottom'$ **by**(*blast dest*: *guardedStatEq*)
from ‹$\Psi \rhd !P \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec P''$› ‹*xvec* $\sharp*$ K› ‹$\Psi \rhd P \sim Q$› ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ P› ‹*xvec* $\sharp*$ Q› ‹*guarded Q*›
obtain Q'' T R **where** *QTrans'*: $\Psi \rhd !Q \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec Q''$ **and** $\Psi \rhd P'' \sim R \parallel !P$ **and** $\Psi \rhd Q'' \sim T \parallel !Q$ **and** $\Psi \rhd R \sim T$
and *suppR*: ((*supp* R)::*name set*) $\subseteq$ *supp* P'' **and** *suppT*: ((*supp* T)::*name set*) $\subseteq$ *supp* Q'' **using** *cComm1*
by *fastforce*
from *QTrans'* ‹$\Psi_Q \simeq SBottom'$› **have** $\Psi \otimes \Psi_Q \rhd !Q \longmapsto K(\!|\nu*xvec|\!)\langle N \rangle \prec Q''$
by(*metis statEqTransition Identity compositionSym AssertionStatEqSym*)
moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹$\Psi_Q \simeq SBottom'$› **have** $\Psi \otimes \Psi_Q \otimes SBottom' \vdash M \leftrightarrow K$ **by**(*metis statEqEnt Identity compositionSym AssertionStatEqSym*)
ultimately have $\Psi \rhd Q \parallel !Q \longmapsto \tau \prec ((\!|\nu*xvec|\!)(Q' \parallel Q''))$ **using** ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* M$› ‹*xvec* $\sharp*$ Q›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
hence $\Psi \rhd !Q \longmapsto \tau \prec ((\!|\nu*xvec|\!)(Q' \parallel Q''))$ **using** ‹*guarded Q*› **by**(*rule Bang*)
moreover from ‹$\Psi \rhd P'' \sim R \parallel !P$› ‹*guarded P*› ‹*xvec* $\sharp*$ Ψ› **have** $\Psi \rhd (\!|\nu*xvec|\!)(P' \parallel P'') \sim (\!|\nu*xvec|\!)((P' \parallel R) \parallel (P \parallel !P))$

by(*metis bisimParPresSym bangExt bisimTransitive bisimParAssoc bisimSymmetric bisimResChainPres*)
with ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *P*› **have** Ψ ▷ (⦇ν*xvec⦈)(*P*′ ∥ *P*″) ∼ ((⦇ν*xvec⦈)(*P*′ ∥ *R*)) ∥ (*P* ∥ !*P*)
by(*metis bisimScopeExtChainSym bisimTransitive psiFreshVec*)
moreover from ‹Ψ ▷ *Q*″ ∼ *T* ∥ !*Q*› ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *Q*› **have** Ψ ▷ (⦇ν*xvec⦈)(*Q*′ ∥ *Q*″) ∼ ((⦇ν*xvec⦈)(*Q*′ ∥ *T*)) ∥ !*Q*
by(*metis bisimParPresSym bisimTransitive bisimParAssoc bisimSymmetric bisimResChainPres bisimScopeExtChainSym psiFreshVec*)
moreover from ‹Ψ ▷ *R* ∼ *T*› ‹Ψ ▷ *Q*′ ∼ *P*′› ‹*xvec* ♯* Ψ› **have** Ψ ▷ (⦇ν*xvec⦈)(*P*′ ∥ *R*) ∼ (⦇ν*xvec⦈)(*Q*′ ∥ *T*)
by(*metis bisimParPresSym bisimTransitive bisimResChainPres bisimParComm bisimE*(4))
moreover from *suppR* **have** ((*supp*((⦇ν*xvec⦈)(*P*′ ∥ *R*)))::*name set*) ⊆ *supp*(((⦇ν*xvec⦈)(*P*′ ∥ *P*″)))
by(*auto simp add*: *psi.supp resChainSupp*)
moreover from *suppT* **have** ((*supp*((⦇ν*xvec⦈)(*Q*′ ∥ *T*)))::*name set*) ⊆ *supp*(((⦇ν*xvec⦈)(*Q*′ ∥ *Q*″)))
by(*auto simp add*: *psi.supp resChainSupp*)
ultimately show *?case* **by** *blast*
next
case(*cComm2 M xvec N P*′ *K P*″ *Q*)
from ‹Ψ ▷ *P* ∼ *Q*› ‹Ψ ▷ *P* ⟼*M*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *P*′› ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *Q*›
obtain *Q*′ **where** *QTrans*: Ψ ▷ *Q* ⟼*M*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *Q*′ **and** Ψ ▷ *P*′ ∼ *Q*′
by(*metis bisimE simE bn.simps*)
from *QTrans* **have** Ψ ⊗ *SBottom*′ ▷ *Q* ⟼*M*(⦇ν*xvec⦈)⟨*N*⟩ ≺ *Q*′ **by**(*metis statEqTransition Identity AssertionStatEqSym*)
moreover obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame Q* = ⟨A_Q, Ψ_Q⟩ **and** A_Q ♯* Ψ **and** A_Q ♯* *Q* **and** A_Q ♯* *M*
by(*rule-tac C*=(Ψ, *Q*, *M*) **in** *freshFrame*) *auto*
note *FrQ*
moreover from *FrQ* ‹*guarded Q*› **have** Ψ_Q ≃ *SBottom*′ **by**(*blast dest*: *guardedStatEq*)
from ‹Ψ ▷ !*P* ⟼*K*(⦇*N*⦈) ≺ *P*″› ‹Ψ ▷ *P* ∼ *Q*› ‹*guarded Q*›
obtain *Q*″ *T R* **where** *QTrans*′: Ψ ▷ !*Q* ⟼*K*(⦇*N*⦈) ≺ *Q*″ **and** Ψ ▷ *P*″ ∼ *R* ∥ !*P* **and** Ψ ▷ *Q*″ ∼ *T* ∥ !*Q* **and** Ψ ▷ *R* ∼ *T*
and *suppR*: ((*supp R*)::*name set*) ⊆ *supp P*″ **and** *suppT*: ((*supp T*)::*name set*) ⊆ *supp Q*″ **using** *cComm2*
by *fastforce*
from *QTrans*′ ‹Ψ_Q ≃ *SBottom*′› **have** Ψ ⊗ Ψ_Q ▷ !*Q* ⟼*K*(⦇*N*⦈) ≺ *Q*″
by(*metis statEqTransition Identity compositionSym AssertionStatEqSym*)
moreover from ‹Ψ ⊢ *M* ↔ *K*› ‹Ψ_Q ≃ *SBottom*′› **have** Ψ ⊗ Ψ_Q ⊗ *SBottom*′ ⊢ *M* ↔ *K* **by**(*metis statEqEnt Identity compositionSym AssertionStatEqSym*)
ultimately have Ψ ▷ *Q* ∥ !*Q* ⟼τ ≺ ((⦇ν*xvec⦈)(*Q*′ ∥ *Q*″)) **using** ‹A_Q ♯* Ψ› ‹A_Q ♯* *Q*› ‹A_Q ♯* *M*› ‹*xvec* ♯* *Q*›
by(*rule-tac Comm2*) (*assumption* | *simp*)+
hence Ψ ▷ !*Q* ⟼τ ≺ ((⦇ν*xvec⦈)(*Q*′ ∥ *Q*″)) **using** ‹*guarded Q*› **by**(*rule Bang*)
moreover from ‹Ψ ▷ *P*″ ∼ *R* ∥ !*P*› ‹*guarded P*› ‹*xvec* ♯* Ψ› **have** Ψ ▷ (⦇ν*xvec⦈)(*P*′ ∥ *P*″) ∼ (⦇ν*xvec⦈)((*P*′ ∥ *R*) ∥ (*P* ∥ !*P*))

by(*metis bisimParPresSym bangExt bisimTransitive bisimParAssoc bisimSymmetric bisimResChainPres*)
with ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *P*› **have** Ψ ⊳ (⦇ν*xvec⦈)(*P'* ∥ *P''*) ∼ ((⦇ν*xvec⦈)(*P'* ∥ *R*)) ∥ (*P* ∥ !*P*)
by(*metis bisimScopeExtChainSym bisimTransitive psiFreshVec*)
moreover from ‹Ψ ⊳ *Q''* ∼ *T* ∥ !*Q*› ‹*xvec* ♯* Ψ› ‹*xvec* ♯* *Q*› **have** Ψ ⊳ (⦇ν*xvec⦈)(*Q'* ∥ *Q''*) ∼ ((⦇ν*xvec⦈)(*Q'* ∥ *T*)) ∥ !*Q*
by(*metis bisimParPresSym bisimTransitive bisimParAssoc bisimSymmetric bisimResChainPres bisimScopeExtChainSym psiFreshVec*)
moreover from ‹Ψ ⊳ *R* ∼ *T*› ‹Ψ ⊳ *P'* ∼ *Q'*› ‹*xvec* ♯* Ψ› **have** Ψ ⊳ (⦇ν*xvec⦈)(*P'* ∥ *R*) ∼ (⦇ν*xvec⦈)(*Q'* ∥ *T*)
by(*metis bisimParPresSym bisimTransitive bisimResChainPres bisimParComm*)
moreover from *suppR* **have** ((*supp*((⦇ν*xvec⦈)(*P'* ∥ *R*)))::*name set*) ⊆ *supp*((⦇ν*xvec⦈)(*P'* ∥ *P''*)))
by(*auto simp add*: *psi.supp resChainSupp*)
moreover from *suppT* **have** ((*supp*((⦇ν*xvec⦈)(*Q'* ∥ *T*)))::*name set*) ⊆ *supp*((⦇ν*xvec⦈)(*Q'* ∥ *Q''*)))
by(*auto simp add*: *psi.supp resChainSupp*)
ultimately show *?case* **by** *blast*
next
case(*cBang* α *P'* *Q*)
then obtain *Q'* *T* *R* **where** *QTrans*: Ψ ⊳ !*Q* ⟼α ≺ *Q'* **and** Ψ ⊳ *P'* ∼ *R* ∥ (*P* ∥ !*P*) **and** Ψ ⊳ *Q'* ∼ *T* ∥ !*Q* **and** Ψ ⊳ *R* ∼ *T*
and *suppR*: ((*supp* *R*)::*name set*) ⊆ *supp* *P'* **and** *suppT*: ((*supp* *T*)::*name set*) ⊆ *supp* *Q'*
by *blast*
from ‹Ψ ⊳ *P'* ∼ *R* ∥ (*P* ∥ !*P*)› ‹*guarded* *P*› **have** Ψ ⊳ *P'* ∼ *R* ∥ !*P* **by**(*metis bangExt bisimParPresSym bisimTransitive bisimSymmetric*)
with *QTrans* **show** *?case* **using** ‹Ψ ⊳ *Q'* ∼ *T* ∥ !*Q*› ‹Ψ ⊳ *R* ∼ *T*› *suppR* *suppT*
by *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *structCongBisim*:
fixes *P* :: ('*a*, '*b*, '*c*) *psi*
and *Q* :: ('*a*, '*b*, '*c*) *psi*

assumes *P* ≡*s* *Q*

shows *P* ∼ *Q*
using *assms*
by(*induct rule*: *structCong.induct*)
(*auto intro*: *bisimReflexive bisimSymmetric bisimTransitive bisimParComm bisimParAssoc bisimParNil bisimResNil bisimResComm bisimScopeExt bisimCasePushRes bisimInputPushRes bisimOutputPushRes bangExt*)

lemma *bisimBangPres*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \sim Q$
and *guarded P*
and *guarded Q*

shows $\Psi \rhd !P \sim !Q$
proof −
let *?X* = $\{(\Psi, R \parallel !P, R \parallel !Q) \mid \Psi\ P\ Q\ R.\ \Psi \rhd P \sim Q \wedge guarded\ P \wedge guarded\ Q\}$
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ P'\ Q'\ Q.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in ?X \wedge \Psi \rhd Q' \sim Q\}$
from *assms* **have** $(\Psi, \mathbf{0} \parallel !P, \mathbf{0} \parallel !Q) \in ?X$ **by**(*blast intro*: *bisimReflexive*)

moreover have *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*drule-tac p=p* **in** *bisimClosed*)
by *fastforce*
ultimately have $\Psi \rhd \mathbf{0} \parallel !P \sim \mathbf{0} \parallel !Q$
proof(*coinduct rule*: *weakTransitiveCoinduct*)
case(*cStatEq* Ψ *P Q*)
thus *?case* **by** *auto*
next
case(*cSim* Ψ *RP RQ*)
from ‹$(\Psi, RP, RQ) \in ?X$› **obtain** *P Q R* **where** $\Psi \rhd P \sim Q$ **and** *guarded P* **and** *guarded Q*
and $RP = R \parallel !P$ **and** $RQ = R \parallel !Q$
by *auto*
note ‹$\Psi \rhd P \sim Q$›
moreover from ‹*eqvt ?X*› **have** *eqvt ?Y* **by** *blast*
moreover note ‹*guarded P*› ‹*guarded Q*› *bisimE*(*2*) *bisimE*(*3*) *bisimE*(*4*) *statEqBisim bisimClosed bisimParAssoc*[*THEN bisimSymmetric*]
bisimParPres bisimParPresAuxSym bisimResChainPres bisimScopeExtChainSym bisimTransitive
moreover have $\bigwedge \Psi\ P\ Q\ R\ T.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in ?Y; \Psi \rhd R \sim T]\!] \Longrightarrow (\Psi, P, T) \in ?Y$
by *auto* (*metis bisimTransitive*)
moreover have $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \sim Q; guarded\ P; guarded\ Q]\!] \Longrightarrow (\Psi, R \parallel !P, R \parallel !Q) \in ?Y$ **by**(*blast intro*: *bisimReflexive*)
moreover have $\bigwedge \Psi\ P\ \alpha\ P'\ Q.\ [\![\Psi \rhd !P \longmapsto \alpha \prec P'; \Psi \rhd P \sim Q; bn\ \alpha \sharp* \Psi; bn\ \alpha \sharp* P;\ bn\ \alpha \sharp* Q; guarded\ Q; bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow$
$\exists Q'\ R\ T.\ \Psi \rhd !Q \longmapsto \alpha \prec Q' \wedge \Psi \rhd P' \sim R \parallel !P \wedge\ \Psi \rhd Q' \sim T \parallel !Q \wedge$
$\Psi \rhd R \sim T \wedge ((supp\ R)::name\ set) \subseteq supp\ P' \wedge$
$((supp\ T)::name\ set) \subseteq supp\ Q'$

by(*blast elim*: *bangDerivative*)
ultimately have $\Psi \rhd R \parallel !P \rightsquigarrow[?Y]\ R \parallel !Q$
by(*rule bangPres*)
with ‹$RP = R \parallel !P$› ‹$RQ = R \parallel !Q$› **show** *?case*
by *blast*
next
case(*cExt* Ψ *RP RQ* Ψ')
thus *?case* **by**(*blast dest*: *bisimE*)
next
case(*cSym* Ψ *RP RQ*)
thus *?case* **by**(*blast dest*: *bisimE*)
qed
thus *?thesis*
by(*metis bisimTransitive bisimParNil bisimSymmetric bisimParComm*)
qed

end

end

theory *Weak-Bisimulation*
imports *Weak-Simulation Weak-Stat-Imp Bisim-Struct-Cong*
begin

context *env* **begin**

lemma *monoCoinduct*: $\bigwedge x\ y\ xa\ xb\ xc\ P\ Q\ \Psi$.
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \rightsquigarrow<\{(xc, xb, xa).\ x\ xc\ xb\ xa\}>\ P) \longrightarrow$
$(\Psi \rhd Q \rightsquigarrow<\{(xb, xa, xc).\ y\ xb\ xa\ xc\}>\ P)$
apply *auto*
apply(*rule weakSimMonotonic*)
by(*auto dest*: *le-funE*)

lemma *monoCoinduct2*: $\bigwedge x\ y\ xa\ xb\ xc\ P\ Q\ \Psi$.
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \lessapprox<\{(xc, xb, xa).\ x\ xc\ xb\ xa\}>\ P) \longrightarrow$
$(\Psi \rhd Q \lessapprox<\{(xb, xa, xc).\ y\ xb\ xa\ xc\}>\ P)$
apply *auto*
apply(*rule weakStatImpMonotonic*)
by(*auto dest*: *le-funE*)

coinductive-set *weakBisim* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
where
step: $[\![\Psi \rhd P \lessapprox<weakBisim> Q;\ \Psi \rhd P \rightsquigarrow<weakBisim> Q;$
$\forall \Psi'.\ (\Psi \otimes \Psi',\ P,\ Q) \in weakBisim;\ (\Psi,\ Q,\ P) \in weakBisim]\!] \Longrightarrow (\Psi,\ P,$
$Q) \in weakBisim$
monos *monoCoinduct monoCoinduct2*

abbreviation
weakBisimJudge (- ▷ - ≈ - [*70*, *70*, *70*] *65*) **where** $\Psi \rhd P \approx Q \equiv (\Psi, P, Q) \in$ *weakBisim*
abbreviation
weakBisimNilJudge (- ≈ - [*70*, *70*] *65*) **where** $P \approx Q \equiv \mathbf{1} \rhd P \approx Q$

lemma *weakBisimCoinductAux*[*consumes 1*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \rhd P \lessapprox <(X \cup weakBisim)> Q) \wedge$
$(\Psi \rhd P \rightsquigarrow <(X \cup weakBisim)> Q) \wedge$
$(\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X \vee (\Psi \otimes \Psi', P, Q) \in weakBisim) \wedge$
$((\Psi, Q, P) \in X \vee (\Psi, Q, P) \in weakBisim)$

shows $(\Psi, P, Q) \in weakBisim$
proof −
have $X \cup weakBisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in weakBisim\}$
by *auto*
with *assms* **show** *?thesis*
by *coinduct* (*simp add*: *rtrancl-def*)
qed

lemma *weakBisimCoinduct*[*consumes 1*, *case-names cStatImp cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \lessapprox <(X \cup weakBisim)> S$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \rightsquigarrow <(X \cup weakBisim)> S$
and $\bigwedge \Psi'\ R\ S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee \Psi' \otimes \Psi'' \rhd R \approx S$
and $\bigwedge \Psi'\ R\ S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee \Psi' \rhd S \approx R$

shows $\Psi \rhd P \approx Q$
proof −
have $X \cup weakBisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in weakBisim\}$
by *auto*
with *assms* **show** *?thesis*
by *coinduct* (*simp add*: *rtrancl-def*)
qed

lemma *weakBisimWeakCoinductAux*[*consumes 1*]:

fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lessapprox_{<X>} Q \wedge \Psi \rhd P \rightsquigarrow_{<X>} Q \wedge (\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X) \wedge (\Psi, Q, P) \in X$

shows $\Psi \rhd P \approx Q$
using *assms*
by(*coinduct rule*: *weakBisimCoinductAux*) (*blast intro*: *weakSimMonotonic weakStatImpMonotonic*)

lemma *weakBisimWeakCoinduct*[*consumes 1*, *case-names cStatImp cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lessapprox_{<X>} Q$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow_{<X>} Q$
and $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

shows $(\Psi, P, Q) \in weakBisim$
proof −
have $X \cup weakBisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in weakBisim\}$
by *auto*
with *assms* **show** *?thesis*
by(*coinduct rule*: *weakBisimWeakCoinductAux*) *blast*
qed

lemma *weakBisimE*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi :: 'b$
and $\Psi' :: 'b$

assumes $\Psi \rhd P \approx Q$

shows $\Psi \rhd P \lessapprox_{<weakBisim>} Q$
and $\Psi \rhd P \rightsquigarrow_{<weakBisim>} Q$
and $\Psi \otimes \Psi' \rhd P \approx Q$
and $\Psi \rhd Q \approx P$
using *assms*
by(*auto intro*: *weakBisim.cases simp add*: *rtrancl-def*)

lemma *weakBisimI*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi :: 'b$

assumes $\Psi \rhd P \lesssim_{<weakBisim>} Q$
and $\Psi \rhd P \rightsquigarrow_{<weakBisim>} Q$
and $\forall \Psi'.\ \Psi \otimes \Psi' \rhd P \approx Q$
and $\Psi \rhd Q \approx P$

shows $\Psi \rhd P \approx Q$
using *assms*
by(*rule-tac weakBisim.step*) (*auto simp add*: *rtrancl-def*)

lemma *weakBisimReflexive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \approx P$
proof −
let *?X* = $\{(\Psi, P, P) \mid \Psi\ P.\ True\}$
have $(\Psi, P, P) \in$ *?X* **by** *simp*
thus *?thesis*
by(*coinduct rule*: *weakBisimWeakCoinduct*, *auto intro*: *weakSimReflexive weakStatImpReflexive*)
qed

lemma *weakBisimClosed*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $p :: name\ prm$

assumes $\Psi \rhd P \approx Q$

shows $(p \cdot \Psi) \rhd\ (p \cdot P) \approx (p \cdot Q)$
proof −
let *?X* = $\{(p \cdot \Psi, p \cdot P, p \cdot Q) \mid (p::name\ prm)\ \Psi\ \ P\ Q.\ \Psi \rhd P \approx Q\}$
have *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*rule-tac x*=*pa*@*p* **in** *exI*)
by(*auto simp add*: *pt2*[*OF pt-name-inst*])
from ‹$\Psi \rhd P \approx Q$› **have** $(p \cdot \Psi, p \cdot P, p \cdot Q) \in$ *?X* **by** *blast*
thus *?thesis*
proof(*coinduct rule*: *weakBisimWeakCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
{
fix $\Psi\ P\ Q\ p$

```
      assume Ψ ▷ P ≈ (Q::('a, 'b, 'c) psi)
      hence Ψ ▷ P ≲<weakBisim> Q by(rule weakBisimE)
      hence Ψ ▷ P ≲<?X> Q
        apply(rule-tac A=weakBisim in weakStatImpMonotonic, auto)
        by(rule-tac x=[]::name prm in exI) auto
      with ‹eqvt ?X› have ((p::name prm) · Ψ) ▷ (p · P) ≲<?X> (p · Q)
        by(rule weakStatImpClosed)
    }
    with ‹(Ψ, P, Q) ∈ ?X› show ?case by blast
  next
    case(cSim Ψ P Q)
    {
      fix p :: name prm
      fix Ψ P Q
      assume Ψ ▷ P ⇝<weakBisim> Q
      hence Ψ ▷ P ⇝<?X> Q
        apply(rule-tac A=weakBisim in weakSimMonotonic, auto)
        by(rule-tac x=[]::name prm in exI) auto
      with ‹eqvt ?X› have ((p::name prm) · Ψ) ▷ (p · P) ⇝<?X> (p · Q)
        by(rule-tac weakSimClosed)
    }
    with ‹(Ψ, P, Q) ∈ ?X› show ?case
      by(blast dest: weakBisimE)
  next
    case(cExt Ψ P Q Ψ')
    {
      fix p :: name prm
      fix Ψ P Q Ψ'
      assume ∀Ψ'. (Ψ ⊗ Ψ', P, Q) ∈ weakBisim
      hence ((p · Ψ) ⊗ Ψ', p · P, p · Q) ∈ ?X
        apply(auto, rule-tac x=p in exI)
        apply(rule-tac x=Ψ ⊗ (rev p · Ψ') in exI)
        by(auto simp add: eqvts)
    }
    with ‹(Ψ, P, Q) ∈ ?X› show ?case
      by(blast dest: weakBisimE)
  next
    case(cSym Ψ P Q)
    thus ?case
      by(blast dest: weakBisimE)
  qed
qed

lemma weakBisimEqvt[simp]:
  shows eqvt weakBisim
by(auto simp add: eqvt-def weakBisimClosed)

lemma statEqWeakBisim:
  fixes Ψ :: 'b
```

```
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   Ψ' :: 'b

  assumes Ψ ▷ P ≈ Q
  and     Ψ ≃ Ψ'

  shows Ψ' ▷ P ≈ Q
proof −
  let ?X = {(Ψ', P, Q) | Ψ P Q Ψ'. Ψ ▷ P ≈ Q ∧ Ψ ≃ Ψ'}
  from ‹Ψ ▷ P ≈ Q› ‹Ψ ≃ Ψ'› have (Ψ', P, Q) ∈ ?X by auto
  thus ?thesis
  proof(coinduct rule: weakBisimCoinduct)
    case(cStatImp Ψ' P Q)
    from ‹(Ψ', P, Q) ∈ ?X› obtain Ψ where Ψ ▷ P ≈ Q and Ψ ≃ Ψ'
      by auto
    from ‹Ψ ▷ P ≈ Q› have Ψ ▷ P ≲<weakBisim> Q by(rule weakBisimE)
    moreover note ‹Ψ ≃ Ψ'›
    moreover have ⋀Ψ P Q Ψ'. ⟦Ψ ▷ P ≈ Q; Ψ ≃ Ψ'⟧ ⟹ (Ψ', P, Q) ∈ ?X ∪
weakBisim
      by auto
    ultimately show ?case by(rule weakStatImpStatEq)
  next
    case(cSim Ψ' P Q)
    from ‹(Ψ', P, Q) ∈ ?X› obtain Ψ where Ψ ▷ P ≈ Q and Ψ ≃ Ψ'
      by auto
   from ‹Ψ ▷ P ≈ Q› have Ψ ▷ P ⇝<weakBisim> Q by(blast dest: weakBisimE)
    moreover have eqvt ?X
      by(auto simp add: eqvt-def) (metis weakBisimClosed AssertionStatEqClosed)
    hence eqvt(?X ∪ weakBisim) by auto
    moreover note ‹Ψ ≃ Ψ'›
    moreover have ⋀Ψ P Q Ψ'. ⟦Ψ ▷ P ≈ Q; Ψ ≃ Ψ'⟧ ⟹ (Ψ', P, Q) ∈ ?X ∪
weakBisim
      by auto
    ultimately show ?case by(rule weakSimStatEq)
  next
    case(cExt Ψ' P Q Ψ'')
    from ‹(Ψ', P, Q) ∈ ?X› obtain Ψ where Ψ ▷ P ≈ Q and Ψ ≃ Ψ'
      by auto
    from ‹Ψ ▷ P ≈ Q› have Ψ ⊗ Ψ'' ▷ P ≈ Q by(rule weakBisimE)
    moreover from ‹Ψ ≃ Ψ'› have Ψ ⊗ Ψ'' ≃ Ψ' ⊗ Ψ'' by(rule Composition)
    ultimately show ?case by blast
  next
    case(cSym Ψ' P Q)
    from ‹(Ψ', P, Q) ∈ ?X› obtain Ψ where Ψ ▷ P ≈ Q and Ψ ≃ Ψ'
      by auto
    from ‹Ψ ▷ P ≈ Q› have Ψ ▷ Q ≈ P by(rule weakBisimE)
    thus ?case using ‹Ψ ≃ Ψ'› by auto
  qed
```

qed

lemma *weakBisimTransitive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes *PQ*: $\Psi \rhd P \approx Q$
and *QR*: $\Psi \rhd Q \approx R$

shows $\Psi \rhd P \approx R$
proof −
let *?X* = $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ \Psi \rhd P \approx Q \wedge \Psi \rhd Q \approx R\}$
from *PQ QR* **have** $(\Psi, P, R) \in$ *?X* **by** *auto*
thus *?thesis*
proof(*coinduct rule*: *weakBisimCoinduct*)
case(*cStatImp* $\Psi\ P\ R$)
from ‹$(\Psi, P, R) \in$ *?X*› **obtain** Q **where** $\Psi \rhd P \approx Q$ **and** $\Psi \rhd Q \approx R$ **by** *blast*
from ‹$\Psi \rhd P \approx Q$› **have** $\Psi \rhd P \lessapprox_{<weakBisim>} Q$ **by**(*rule weakBisimE*)
moreover note ‹$\Psi \rhd Q \approx R$›
moreover have *?X* $\subseteq$ *?X* $\cup$ *weakBisim* **by** *auto*
moreover note *weakBisimE*(*1*)
moreover from *weakBisimE*(*2*) **have** $\bigwedge \Psi\ P\ Q\ P'.\ [\![\Psi \rhd P \approx Q; \Psi \rhd P \Longrightarrow\hat{}_{\tau}\ P']\!] \Longrightarrow \exists Q'.\ \Psi \rhd Q \Longrightarrow\hat{}_{\tau}\ Q' \wedge \Psi \rhd P' \approx Q'$
by(*metis weakBisimE*(*4*) *weakSimTauChain*)
ultimately show *?case* **by**(*rule weakStatImpTransitive*)
next
case(*cSim* $\Psi\ P\ R$)
{
fix $\Psi\ P\ Q\ R$
assume $\Psi \rhd P \approx Q$ **and** $\Psi \rhd Q \rightsquigarrow_{<weakBisim>} R$
moreover have *eqvt ?X*
by(*force simp add*: *eqvt-def dest*: *weakBisimClosed*)
with *weakBisimEqvt* **have** *eqvt* (*?X* $\cup$ *weakBisim*) **by** *blast*
moreover have *?X* $\subseteq$ *?X* $\cup$ *weakBisim* **by** *auto*
moreover note *weakBisimE*(*2*)
ultimately have $\Psi \rhd P \rightsquigarrow_{<(?X \cup weakBisim)>} R$
by(*rule-tac weakSimTransitive*) *auto*
}
with ‹$(\Psi, P, R) \in$ *?X*› **show** *?case*
by(*blast dest*: *weakBisimE*)
next
case(*cExt* $\Psi\ P\ R\ \Psi'$)
thus *?case* **by**(*blast dest*: *weakBisimE*)
next
case(*cSym* $\Psi\ P\ R$)
thus *?case* **by**(*blast dest*: *weakBisimE*)

```
  qed
qed

lemma strongBisimWeakBisim:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi

  assumes Ψ ▷ P ∼ Q

  shows Ψ ▷ P ≈ Q
proof −
  from ‹Ψ ▷ P ∼ Q›
  show ?thesis
  proof(coinduct rule: weakBisimWeakCoinduct)
    case(cStatImp Ψ P Q)
    from ‹Ψ ▷ P ∼ Q› have insertAssertion(extractFrame P) Ψ ↪F insertAssertion(extractFrame Q) Ψ
      by(metis bisimE FrameStatEq-def)
    moreover from ‹Ψ ▷ P ∼ Q› have ⋀Ψ'. Ψ ⊗ Ψ' ▷ P ∼ Q by(rule bisimE)
    ultimately show ?case by(rule statImpWeakStatImp)
  next
    case(cSim Ψ P Q)
    note ‹Ψ ▷ P ∼ Q›
    moreover have ⋀Ψ P Q. Ψ ▷ P ∼ Q ⟹ insertAssertion(extractFrame Q) Ψ ↪F insertAssertion(extractFrame P) Ψ
      by(drule-tac bisimE) (simp add: FrameStatEq-def)
    ultimately show ?case using bisimE(2) bisimE(3)
      by(rule strongSimWeakSim)
  next
    case(cExt Ψ P Q Ψ')
    from ‹Ψ ▷ P ∼ Q› show ?case
      by(rule bisimE)
  next
    case(cSym Ψ P Q)
    from ‹Ψ ▷ P ∼ Q› show ?case
      by(rule bisimE)
  qed
qed

lemma structCongWeakBisim:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi

  assumes P ≡s Q

  shows P ≈ Q
using assms
```

by(*metis structCongBisim strongBisimWeakBisim*)

lemma *simTauChain*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and Q' :: (*'a*, *'b*, *'c*) *psi*

assumes $(\Psi, P, Q) \in Rel$
and $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q'$
and *Sim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in Rel \Longrightarrow \Psi \rhd P \rightsquigarrow[Rel]\ Q$

obtains P' **where** $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** $(\Psi, P', Q') \in Rel$
proof −
assume *A*: $\bigwedge P'.\ [\![\Psi \rhd P \Longrightarrow\hat{}_\tau\ P';\ (\Psi, P', Q') \in Rel]\!] \Longrightarrow thesis$
from ‹$\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q'$› ‹$(\Psi, P, Q) \in Rel$› *A* **show** *?thesis*
proof(*induct arbitrary*: *P thesis rule*: *tauChainInduct*)
case(*TauBase Q P*)
moreover have $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P$ **by** *simp*
ultimately show *?case* **by** *blast*
next
case(*TauStep Q Q' Q'' P*)
from ‹$(\Psi, P, Q) \in Rel$› **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** $(\Psi, P', Q') \in Rel$
by(*rule TauStep*)
from ‹$(\Psi, P', Q') \in Rel$› **have** $\Psi \rhd P' \rightsquigarrow[Rel]\ Q'$ **by**(*rule Sim*)
then obtain P'' **where** *P'Chain*: $\Psi \rhd P' \longmapsto\tau \prec P''$ **and** $(\Psi, P'', Q'') \in Rel$
using ‹$\Psi \rhd Q' \longmapsto\tau \prec Q''$› **by**(*drule-tac simE*) *auto*
from *PChain P'Chain* **have** $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''$ **by**(*auto dest*: *tauActTauChain*)
thus *?case* **using** ‹$(\Psi, P'', Q'') \in Rel$› **by**(*rule TauStep*)
qed
qed

lemma *quietBisimNil*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *quiet P*

shows $\Psi \rhd P \approx \mathbf{0}$
proof −
let *?X* = $\{(\Psi, \mathbf{0}, P) \mid \Psi\ P.\ quiet\ P\} \cup \{(\Psi, P, \mathbf{0}) \mid \Psi\ P.\ quiet\ P\}$

from ‹*quiet P*› **have** $(\Psi, P, \mathbf{0}) \in ?X$ **by** *auto*
thus *?thesis*
proof(*coinduct rule*: *weakBisimWeakCoinduct*)
case(*cStatImp* Ψ *P Q*)
thus *?case*
apply(*simp add*: *weakStatImp-def*)

apply(*rule allI*)
apply(*rule-tac x*=*Q* **in** *exI*)
apply *auto*
apply(*drule-tac* Ψ=Ψ **in** *quietFrame*)
apply(*rule-tac G*=⟨ε, Ψ⟩ **in** *FrameStatImpTrans*)
using *Identity*
apply(*simp add*: *AssertionStatEq-def*)
apply(*simp add*: *FrameStatEq-def*)
apply(*drule-tac* Ψ=Ψ **in** *quietFrame*)
apply(*rule-tac G*=⟨ε, Ψ⟩ **in** *FrameStatImpTrans*)
apply *auto*
defer
using *Identity*
apply(*simp add*: *AssertionStatEq-def*)
apply(*simp add*: *FrameStatEq-def*)
done
next
case(*cSim* Ψ *P Q*)
moreover have *eqvt ?X* **by**(*auto simp add*: *eqvt-def intro*: *quietEqvt*)
ultimately show *?case*
apply *auto*
apply(*rule quietSim*)
apply *auto*
apply(*auto simp add*: *weakSimulation-def*)
done
next
case(*cExt* Ψ *P Q* Ψ′)
thus *?case* **by** *blast*
next
case(*cSym* Ψ *P Q*)
thus *?case* **by** *blast*
qed
qed

lemma *weakTransitiveWeakCoinduct*[*case-names cStatImp cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes Ψ :: *′b*
and *P* :: (*′a*, *′b*, *′c*) *psi*
and *Q* :: (*′a*, *′b*, *′c*) *psi*
and *X* :: (*′b* × (*′a*, *′b*, *′c*) *psi* × (*′a*, *′b*, *′c*) *psi*) *set*

assumes *p*: (Ψ, *P*, *Q*) ∈ *X*
and *Eqvt*: *eqvt X*
and *rStatImp*: ⋀Ψ *P Q*. (Ψ, *P*, *Q*) ∈ *X* ⟹ Ψ ⊳ *P* ≲<*X*> *Q*
and *rSim*: ⋀Ψ *P Q*. (Ψ, *P*, *Q*) ∈ *X* ⟹ Ψ ⊳ *P* ⇝<({(Ψ, *P*, *Q*) | Ψ *P Q*. ∃ *P′* *Q′*. Ψ ⊳ *P* ∼ *P′* ∧ (Ψ, *P′*, *Q′*) ∈ *X* ∧ Ψ ⊳ *Q′* ∼ *Q*})> *Q*
and *rExt*: ⋀Ψ *P Q* Ψ′. (Ψ, *P*, *Q*) ∈ *X* ⟹ (Ψ ⊗ Ψ′, *P*, *Q*) ∈ *X*

and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in X$

shows $\Psi \rhd P \approx Q$
proof −
let *?X* = $\{(\Psi,\ P,\ Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
from *p* **have** $(\Psi,\ P,\ Q) \in ?X$
by(*blast intro*: *bisimReflexive*)
thus *?thesis*
proof(*coinduct rule*: *weakBisimWeakCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
{
fix Ψ'
from ‹$(\Psi\ ,\ P,\ Q) \in ?X$› **obtain** $P'\ Q'$ **where** $\Psi \rhd P \sim P'$ **and** $(\Psi,\ P',\ Q') \in X$ **and** $\Psi \rhd Q' \sim Q$ **by** *auto*
from ‹$(\Psi,\ P',\ Q') \in X$› **obtain** $Q''\ Q'''$ **where** *Q'Chain*: $\Psi \rhd Q' \Longrightarrow\!\!\hat{}_{\tau}\ Q''$ **and** *PImpQ*: *insertAssertion* (*extractFrame* P') $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'') Ψ **and** *Q''Chain*: $\Psi \otimes \Psi' \rhd Q'' \Longrightarrow\!\!\hat{}_{\tau}\ Q'''$ **and** $(\Psi \otimes \Psi',\ P',\ Q''') \in X$
apply(*drule-tac rStatImp*) **by**(*auto simp add*: *weakStatImp-def*) *blast*
from ‹$\Psi \rhd Q' \sim Q$› **have** $\Psi \rhd Q \sim Q'$ **by**(*rule bisimE*)
then obtain Q'''' **where** $\Psi \rhd Q \Longrightarrow\!\!\hat{}_{\tau}\ Q''''$ **and** $\Psi \rhd Q'''' \sim Q''$ **using** ‹$\Psi \rhd Q' \Longrightarrow\!\!\hat{}_{\tau}\ Q''$› *bisimE*(*2*)
by(*rule simTauChain*)
note ‹$\Psi \rhd Q \Longrightarrow\!\!\hat{}_{\tau}\ Q''''$›
moreover have *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'''') Ψ
proof −
from ‹$\Psi \rhd P \sim P'$› **have** *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P') Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)
moreover from ‹$\Psi \rhd Q'''' \sim Q''$› **have** *insertAssertion* (*extractFrame* Q'') $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'''') Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)
ultimately show *?thesis* **using** *PImpQ* **by**(*blast intro*: *FrameStatImpTrans*)
qed
moreover from ‹$\Psi \rhd Q'''' \sim Q''$› **have** $\Psi \otimes \Psi' \rhd Q'''' \sim Q''$ **by**(*metis bisimE*)
then obtain Q''''' **where** *Q''''Chain*: $\Psi \otimes \Psi' \rhd Q'''' \Longrightarrow\!\!\hat{}_{\tau}\ Q'''''$ **and** $\Psi \otimes \Psi' \rhd Q''''' \sim Q'''$ **using** *Q''Chain bisimE*(*2*)
by(*rule simTauChain*)
moreover from ‹$\Psi \rhd P \sim P'$› ‹$(\Psi \otimes \Psi'\ ,\ P',\ Q''') \in X$› ‹$\Psi \otimes \Psi' \rhd Q''''' \sim Q'''$› **have** $(\Psi \otimes \Psi',\ P,\ Q''''') \in ?X$ **by**(*auto dest*: *bisimE*)
ultimately have $\exists\ Q'\ Q''.\ \Psi \rhd Q \Longrightarrow\!\!\hat{}_{\tau}\ Q'' \wedge$ *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'') $\Psi \wedge \Psi \otimes \Psi' \rhd Q'' \Longrightarrow\!\!\hat{}_{\tau}\ Q' \wedge (\Psi \otimes \Psi',\ P,\ Q') \in ?X$ **by** *blast*
}
with ‹$(\Psi,\ P,\ Q) \in ?X$› **show** *?case* **by**(*simp add*: *weakStatImp-def*) *blast*

next
case(*cSim* Ψ P Q)
from ‹(Ψ, P, Q) ∈ *?X*› **obtain** P' Q' **where** $\Psi \rhd P \sim P'$ **and** (Ψ, P', Q') ∈ X **and** $\Psi \rhd Q' \sim Q$ **by** *auto*
from ‹(Ψ, P', Q') ∈ X› **have** $\Psi \rhd P' \rightsquigarrow <?X> Q'$
by(*rule rSim*)
moreover from ‹$\Psi \rhd Q' \sim Q$› **have** $\Psi \rhd Q' \rightsquigarrow [bisim]\ Q$ **by**(*rule bisimE*)
moreover from ‹*eqvt X*› **have** *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*drule-tac p*=*p* **in** *bisimClosed*)
apply(*drule-tac p*=*p* **in** *bisimClosed*)
apply(*rule-tac x*=*p* ∙ P' **in** *exI*, *simp*)
by(*rule-tac x*=*p* ∙ Q' **in** *exI*, *auto*)
moreover from ‹$\Psi \rhd Q' \sim Q$› **have** *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q'*) Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)
moreover have {(Ψ, P, R) | Ψ P R. ∃ Q. (Ψ, P, Q) ∈ *?X* ∧ $\Psi \rhd Q \sim R$} ⊆ *?X*
by(*blast intro*: *bisimTransitive*)
moreover note *bisimE*(*3*)
ultimately have $\Psi \rhd P' \rightsquigarrow <?X> Q$ **by**(*rule strongAppend*)
moreover have {(Ψ, P, R) | Ψ P R. ∃ Q. $\Psi \rhd P \sim Q$ ∧ (Ψ, Q, R) ∈ *?X*} ⊆ *?X*
by(*blast intro*: *bisimTransitive*)
moreover {
fix Ψ P Q

assume $\Psi \rhd P \sim Q$
moreover have $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow$ *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P*) Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)
ultimately have $\Psi \rhd P \rightsquigarrow <bisim> Q$ **using** *bisimE*(*2*) *bisimE*(*3*)
by(*rule strongSimWeakSim*)
}
ultimately show *?case* **using** ‹$\Psi \rhd P \sim P'$› ‹*eqvt ?X*›
by(*rule-tac weakSimTransitive*)
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*blast dest*: *bisimE intro*: *rExt*)
next
case(*cSym* Ψ P Q)
thus *?case* **by**(*blast dest*: *bisimE intro*: *rSym*)
qed
qed

lemma *weakTransitiveCoinduct*[*case-names cStatImp cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes p: $(\Psi, P, Q) \in X$
and *Eqvt*: $eqvt\ X$
and *rStatImp*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lesssim <(X \cup weakBisim)> Q$
and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow <(\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in (X \cup weakBisim) \wedge \Psi \rhd Q' \sim Q\})> Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X \cup weakBisim$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X \cup weakBisim$

shows $\Psi \rhd P \approx Q$
proof −
from p **have** $(\Psi, P, Q) \in X \cup weakBisim$ **by** *auto*
moreover from ‹*eqvt X*› **have** $eqvt(X \cup weakBisim)$ **by** *auto*
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveWeakCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
thus *?case* **by**(*blast dest*: *rStatImp weakBisimE*(*1*) *weakStatImpMonotonic*)
next
case(*cSim* $\Psi\ P\ Q$)
thus *?case* **by**(*fastforce intro*: *rSim weakBisimE*(*2*) *weakSimMonotonic bisimReflexive*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*blast dest*: *weakBisimE rExt*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by**(*blast dest*: *weakBisimE rSym*)
qed
qed

lemma *weakTransitiveCoinduct2*[*case-names cStatImp cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes p: $(\Psi, P, Q) \in X$
and *Eqvt*: $eqvt\ X$
and *rStatImp*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lesssim <X> Q$
and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow <(\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \approx P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})> Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

shows $\Psi \rhd P \approx Q$
proof −
let *?X* = $\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \approx P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \approx P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \approx Q\}$

from ‹*eqvt X*› **have** *eqvt ?X*
apply(*auto simp add*: *eqvt-def*)
apply(*drule-tac p*=*p* **in** *bisimClosed*)
apply(*drule-tac p*=*p* **in** *weakBisimClosed*)
apply(*rule-tac x*=*p* ∙ *P′* **in** *exI*, *simp*)
by(*rule-tac x*=*p* ∙ *Q′* **in** *exI*, *auto*)
from ‹*eqvt X*› **have** *eqvt ?Y*
apply(*auto simp add*: *eqvt-def*)
apply(*drule-tac p*=*p* **in** *weakBisimClosed*)
apply(*drule-tac p*=*p* **in** *weakBisimClosed*)
apply(*rule-tac x*=*p* ∙ *P′* **in** *exI*, *simp*)
by(*rule-tac x*=*p* ∙ *Q′* **in** *exI*, *auto*)

{
fix $\Psi\ P\ Q$
assume $(\Psi, P, Q) \in$ *?X*
then obtain $P'\ Q'$ **where** $\Psi \rhd P \approx P'$ **and** $(\Psi, P', Q') \in X$ **and** $\Psi \rhd Q' \sim Q$
by *auto*
note ‹$\Psi \rhd P \approx P'$›
moreover from ‹$(\Psi, P', Q') \in X$› **have** $\Psi \rhd P' \rightsquigarrow <?X> Q'$ **by**(*rule rSim*)
moreover note ‹*eqvt ?X*›
moreover have $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ \Psi \rhd P \approx Q \wedge (\Psi, Q, R) \in ?X\} \subseteq ?X$
by(*blast intro*: *weakBisimTransitive*)
ultimately have $\Psi \rhd P \rightsquigarrow <?X> Q'$ **using** *weakBisimE*(*2*) **by**(*rule weakSimTransitive*)
moreover from ‹$\Psi \rhd Q' \sim Q$› **have** $\Psi \rhd Q' \rightsquigarrow [bisim]\ Q$ **by**(*rule bisimE*)
moreover note ‹*eqvt ?X*›
moreover from ‹$\Psi \rhd Q' \sim Q$› **have** *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q′*) Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)
moreover have $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in ?X \wedge \Psi \rhd Q \sim R\} \subseteq ?X$
by(*blast dest*: *bisimTransitive*)
moreover note *bisimE*(*3*)
ultimately have $\Psi \rhd P \rightsquigarrow <?X> Q$ **by**(*rule-tac strongAppend*)
}
note *Goal* = *this*

from *p* **have** $(\Psi, P, Q) \in$ *?Y* **by**(*blast intro*: *weakBisimReflexive bisimReflexive*

rSym)
thus *?thesis*
proof(*coinduct rule*: *weakBisimWeakCoinduct*)
next
case(*cStatImp* Ψ P Q)
{
fix Ψ'

from ‹(Ψ, P, Q) $\in$ *?Y*› **obtain** R S **where** $\Psi \rhd P \approx R$ **and** (Ψ, R, S) $\in X$ **and** $\Psi \rhd S \approx Q$ **by** *auto*
from ‹$\Psi \rhd P \approx R$› **obtain** R'' R'
where *RChain*: $\Psi \rhd R \Longrightarrow\!\hat{}_\tau\ R''$
and *PImpR''*: *insertAssertion* (*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* R'') Ψ
and *R''Chain*: $\Psi \otimes \Psi' \rhd R'' \Longrightarrow\!\hat{}_\tau\ R'$
and $\Psi \otimes \Psi' \rhd P \approx R'$
apply(*drule-tac weakBisimE*) **by**(*simp add*: *weakStatImp-def*) *blast*

from ‹(Ψ, R, S) $\in X$› **have** (Ψ, S, R) $\in$ *?X* **by**(*blast intro*: *weakBisimReflexive bisimReflexive rSym*)
with *RChain* **obtain** S'' **where** *SChain*: $\Psi \rhd S \Longrightarrow\!\hat{}_\tau\ S''$ **and** (Ψ, S'', R'') $\in$ *?X* **using** *Goal*
by(*rule weakSimTauChain*)

from ‹(Ψ, S'', R'') $\in$ *?X*› **obtain** T U **where** $\Psi \rhd S'' \approx T$ **and** (Ψ, T, U) $\in X$ **and** $\Psi \rhd U \sim R''$
by *auto*
from ‹$\Psi \rhd U \sim R''$› **have** *R''ImpU*: *insertAssertion* (*extractFrame* R'') Ψ $\hookrightarrow_F$ *insertAssertion* (*extractFrame* U) Ψ
by(*drule-tac bisimE*) (*simp add*: *FrameStatEq-def*)

from ‹(Ψ, T, U) $\in X$› *weakStatImp-def*
obtain T'' T' **where** *TChain*: $\Psi \rhd T \Longrightarrow\!\hat{}_\tau\ T''$
and *UImpT''*: *insertAssertion* (*extractFrame* U) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* T'') Ψ
and *T''Chain*: $\Psi \otimes \Psi' \rhd T'' \Longrightarrow\!\hat{}_\tau\ T'$
and ($\Psi \otimes \Psi'$, U, T') $\in X$
by(*blast dest*: *rStatImp rSym*)

from *TChain* ‹$\Psi \rhd S'' \approx T$› *weakBisimE*(*2*) **obtain** S''' **where** *S''Chain*: $\Psi \rhd S'' \Longrightarrow\!\hat{}_\tau\ S'''$ **and** $\Psi \rhd S''' \approx T''$
by(*rule weakSimTauChain*)

from ‹$\Psi \rhd S''' \approx T''$› *weakStatImp-def*
obtain S'''' S''''' **where** *S'''Chain*: $\Psi \rhd S''' \Longrightarrow\!\hat{}_\tau\ S''''$
and *T''ImpS''''*: *insertAssertion* (*extractFrame* T'') $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* S'''') Ψ
and *S''''Chain*: $\Psi \otimes \Psi' \rhd S'''' \Longrightarrow\!\hat{}_\tau\ S'''''$
and $\Psi \otimes \Psi' \rhd T'' \approx S''''$

by(*metis weakBisimE*)

from *SChain S''Chain S'''Chain* **have** $\Psi \rhd S \Longrightarrow\hat{}_\tau\ S'''''$ **by** *auto*
moreover from $\langle \Psi \rhd S \approx Q \rangle$ **have** $\Psi \rhd Q \approx S$ **by**(*rule weakBisimE*)
ultimately obtain Q''' **where** *QChain*: $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q'''$ **and** $\Psi \rhd Q''' \approx S'''''$ **using** *weakBisimE*(*2*)
by(*rule weakSimTauChain*)
from $\langle \Psi \rhd Q''' \approx S''''' \rangle$ **have** $\Psi \rhd S''''' \approx Q'''$ **by**(*rule weakBisimE*)
then obtain Q'' Q' **where** *Q'''Chain*: $\Psi \rhd Q''' \Longrightarrow\hat{}_\tau\ Q''$
and *S'''''ImpQ''*: *insertAssertion* (*extractFrame S'''''*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q''*) Ψ
and *Q''Chain*: $\Psi \otimes \Psi' \rhd Q'' \Longrightarrow\hat{}_\tau\ Q'$
and $\Psi \otimes \Psi' \rhd S''''' \approx Q'$ **using** *weakStatImp-def*
by(*metis weakBisimE*)

from *QChain Q'''Chain* **have** $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q''$ **by** *auto*
moreover from *PImpR'' R''ImpU UImpT'' T''ImpS''''' S'''''ImpQ''*
have *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame Q''*) Ψ
by(*blast dest*: *FrameStatImpTrans*)

moreover from $\langle \Psi \rhd U \sim R'' \rangle$ **have** $\Psi \otimes \Psi' \rhd U \approx R''$ **by**(*metis weakBisimE strongBisimWeakBisim*)
with *R''Chain* **obtain** U' **where** *UChain*: $\Psi \otimes \Psi' \rhd U \Longrightarrow\hat{}_\tau\ U'$ **and** $\Psi \otimes \Psi' \rhd U' \approx R'$ **using** *weakBisimE*(*2*)
by(*rule weakSimTauChain*)
from $\langle \Psi \otimes \Psi' \rhd U' \approx R' \rangle$ **have** $\Psi \otimes \Psi' \rhd R' \approx U'$ **by**(*rule weakBisimE*)
from $\langle (\Psi \otimes \Psi',\ U,\ T') \in X \rangle$ **have** $(\Psi \otimes \Psi',\ T',\ U) \in ?X$ **by**(*blast intro*: *rSym weakBisimReflexive bisimReflexive*)
with *UChain* **obtain** T''' **where** *T'Chain*: $\Psi \otimes \Psi' \rhd T' \Longrightarrow\hat{}_\tau\ T'''$ **and** $(\Psi \otimes \Psi',\ T''',\ U') \in ?X$ **using** *Goal*
by(*rule weakSimTauChain*)
from $\langle (\Psi \otimes \Psi',\ T''',\ U') \in ?X \rangle$ **have** $(\Psi \otimes \Psi',\ U',\ T''') \in ?Y$
by(*blast dest*: *weakBisimE rSym strongBisimWeakBisim*)
from *T''Chain T'Chain* **have** $\Psi \otimes \Psi' \rhd T'' \Longrightarrow\hat{}_\tau\ T'''$ **by** *auto*
then obtain S'''''' **where** *S''''Chain*: $\Psi \otimes \Psi' \rhd S'''' \Longrightarrow\hat{}_\tau\ S''''''$ **and** $\Psi \otimes \Psi' \rhd T''' \approx S''''''$
using $\langle \Psi \otimes \Psi' \rhd T'' \approx S'''' \rangle$ *weakBisimE*(*2*)
apply(*drule-tac weakBisimE*(*4*))
by(*rule weakSimTauChain*) (*auto dest*: *weakBisimE*(*4*))
from *S'''''Chain S''''Chain* **have** $\Psi \otimes \Psi' \rhd S''''' \Longrightarrow\hat{}_\tau\ S''''''$ **by** *auto*

with $\langle \Psi \otimes \Psi' \rhd S''''' \approx Q' \rangle$
obtain Q'''' **where** *Q'Chain*: $\Psi \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''''$ **and** $\Psi \otimes \Psi' \rhd S'''''' \approx Q''''$ **using** *weakBisimE*(*2*)
apply(*drule-tac weakBisimE*(*4*))
by(*rule weakSimTauChain*) (*auto dest*: *weakBisimE*(*4*))

from *Q''Chain Q'Chain* **have** $\Psi \otimes \Psi' \rhd Q'' \Longrightarrow\hat{}_\tau\ Q''''$ **by** *auto*

moreover from ‹$\Psi \otimes \Psi' \rhd P \approx R'$› ‹$\Psi \otimes \Psi' \rhd R' \approx U'$› ‹$(\Psi \otimes \Psi', U', T''') \in ?Y$› ‹$\Psi \otimes \Psi' \rhd T''' \approx S''''''$›
‹$\Psi \otimes \Psi' \rhd S'''''' \approx Q''''$›
have $(\Psi \otimes \Psi', P, Q'''') \in ?Y$
by *auto* (*blast dest*: *weakBisimTransitive*)
ultimately have $\exists Q''\ Q'.\ \Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'' \wedge$ *insertAssertion* (*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'') $\Psi \wedge \Psi \otimes \Psi' \rhd Q'' \Longrightarrow^{\hat{}}_{\tau} Q' \wedge (\Psi \otimes \Psi', P, Q') \in ?Y$
by *blast*
}
thus *?case* **by**(*simp add*: *weakStatImp-def*)
next
case(*cSim* Ψ P Q)
moreover {
fix Ψ P P' Q' Q
assume $\Psi \rhd P \approx P'$ **and** $(\Psi, P', Q') \in X$ **and** $\Psi \rhd Q' \approx Q$
from ‹$(\Psi, P', Q') \in X$› **have** $(\Psi, P', Q') \in ?X$
by(*blast intro*: *weakBisimReflexive bisimReflexive*)
moreover from ‹$\Psi \rhd Q' \approx Q$› **have** $\Psi \rhd Q' \rightsquigarrow$<*weakBisim*>$ Q$ **by**(*rule weakBisimE*)
moreover note ‹*eqvt ?Y*›
moreover have $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in ?X \wedge \Psi \rhd Q \approx R\} \subseteq ?Y$
by(*blast dest*: *weakBisimTransitive strongBisimWeakBisim*)
ultimately have $\Psi \rhd P' \rightsquigarrow$<*?Y*>$ Q$ **using** *Goal*
by(*rule weakSimTransitive*)
note ‹$\Psi \rhd P \approx P'$› *this* ‹*eqvt ?Y*›
moreover have $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ \Psi \rhd P \approx Q \wedge (\Psi, Q, R) \in ?Y\} \subseteq ?Y$
by(*blast dest*: *weakBisimTransitive*)
ultimately have $\Psi \rhd P \rightsquigarrow$<*?Y*>$ Q$ **using** *weakBisimE*(*2*)
by(*rule weakSimTransitive*)
}
ultimately show *?case* **by** *auto*
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*blast dest*: *bisimE weakBisimE intro*: *rExt*)
next
case(*cSym* Ψ P Q)
thus *?case* **by**(*blast dest*: *bisimE*(*4*) *weakBisimE*(*4*) *rSym*)
qed
qed

end

end

theory *Weak-Sim-Pres*
imports *Sim-Pres Weak-Simulation Weak-Stat-Imp*

begin

context *env* **begin**

lemma *weakInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes *PRelQ*: $\bigwedge Tvec\ \Psi'$. *length xvec* = *length Tvec* $\Longrightarrow (\Psi \otimes \Psi', P[xvec::=Tvec], Q[xvec::=Tvec]) \in Rel$

shows $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \rightsquigarrow <Rel> M(\!|\lambda * xvec\ N|\!).Q$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ Q'$)
from ‹$\Psi \rhd M(\!|\lambda * xvec\ N|\!).Q \longmapsto \alpha \prec Q'$› **show** *?case*
proof(*induct rule*: *inputCases*)
case(*cInput K Tvec*)
from ‹$\Psi \vdash M \leftrightarrow K$› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec* = *length Tvec*› ‹*distinct xvec*›
have $\Psi : (M(\!|\lambda * xvec\ N|\!).Q) \rhd M(\!|\lambda * xvec\ N|\!).P \Longrightarrow K(\!|(N[xvec::=Tvec])|\!) \prec (P[xvec::=Tvec])$
by(*rule-tac weakInput*) *auto*
moreover have $\Psi \otimes \Psi' \rhd P[xvec::=Tvec] \Longrightarrow\!\!\hat{}_{\tau}\ \ P[xvec::=Tvec]$ **by** *simp*
moreover from ‹*length xvec* = *length Tvec*› **have** $(\Psi \otimes \Psi',\ P[xvec::=Tvec], Q[xvec::=Tvec]) \in Rel$
by(*rule PRelQ*)
ultimately show *?case* **by** *blast*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd M(\!|\lambda * xvec\ N|\!).Q \longmapsto \tau \prec Q'$› **have** *False* **by** *auto*
thus *?case* **by** *simp*
qed

lemma *weakOutputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$

assumes *PRelQ*: $\bigwedge \Psi'.\ (\Psi \otimes \Psi', P, Q) \in Rel$

shows $\Psi \rhd M\langle N\rangle.P \rightsquigarrow <Rel> M\langle N\rangle.Q$
proof(*induct rule*: *weakSimI2*)
case(*cAct* Ψ' α Q')
from ‹$\Psi \rhd M\langle N\rangle.Q \longmapsto \alpha \prec Q'$› **show** *?case*
proof(*induct rule*: *outputCases*)
case(*cOutput* K)
from ‹$\Psi \vdash M \leftrightarrow K$›
have $\Psi : (M\langle N\rangle.Q) \rhd M\langle N\rangle.P \Longrightarrow K\langle N\rangle \prec P$ **by**(*rule weakOutput*) *auto*
moreover have $\Psi \otimes \Psi' \rhd P \Longrightarrow\hat{}_{\tau} \ P$ **by** *simp*
moreover have $(\Psi \otimes \Psi',\ P,\ Q) \in Rel$ **by**(*rule PRelQ*)
ultimately show *?case* **by** *blast*
qed
next
case(*cTau* Q')
from ‹$\Psi \rhd M\langle N\rangle.Q \longmapsto \tau \prec Q'$› **have** *False* **by** *auto*
thus *?case* **by** *simp*
qed

lemma *resTauCases*[*consumes 3*, *case-names cRes*]:
fixes Ψ :: $'b$
and x :: *name*
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*
and C :: $'d$::*fs-name*

assumes *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto \tau \prec P'$
and $x \sharp \Psi$
and $x \sharp P'$
and *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto \tau \prec P']\!] \Longrightarrow Prop\ ((\!|\nu x|\!)P')$

shows $Prop\ P'$
proof −
note *Trans* ‹$x \sharp \Psi$›
moreover have $x \sharp (\tau)$ **by** *simp*
moreover note ‹$x \sharp P'$›
moreover have $bn(\tau) \sharp* \Psi$ **and** $bn(\tau) \sharp* P$ **and** $bn(\tau) \sharp* subject(\tau)$ **and** $bn(\tau) = []$ **by** *simp*+
ultimately show *?thesis*
by(*induct rule*: *resCases*) (*auto intro*: *rScope*)
qed

lemma *weakResPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)$ *psi*
and x :: *name*
and *Rel'* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *eqvt Rel'*
and $x \sharp \Psi$
and $Rel \subseteq Rel'$
and *C1*: $\bigwedge \Psi' R S y.\ [\![(\Psi', R, S) \in Rel;\ y \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)R, (\!|\nu y|\!)S) \in Rel'$

shows $\Psi \rhd (\!|\nu x|\!)P \rightsquigarrow <Rel'> (\!|\nu x|\!)Q$
proof −
note ‹*eqvt Rel'*› ‹$x \sharp \Psi$›
moreover have $x \sharp (\!|\nu x|\!)P$ **and** $x \sharp (\!|\nu x|\!)Q$ **by**(*simp add*: *abs-fresh*)+
ultimately show *?thesis*
proof(*induct rule*: *weakSimIFresh*[*of* - - - - - ()])
case(*cAct* α Q' Ψ')
from ‹*bn* $\alpha \sharp* (\!|\nu x|\!)P$› ‹*bn* $\alpha \sharp* (\!|\nu x|\!)Q$› ‹$x \sharp \alpha$› **have** *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* Q$ **by** *simp*+
from ‹$\Psi \rhd (\!|\nu x|\!)Q \longmapsto \alpha \prec Q'$› ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› ‹$x \sharp Q'$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*bn* $\alpha \sharp* P$› ‹$x \sharp \alpha$›
show *?case*
proof(*induct rule*: *resCases*)
case(*cOpen M xvec1 xvec2 y N Q'*)
from ‹$bn(M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle) \sharp* P$› **have** *xvec1* $\sharp*$ *P* **and** *xvec2* $\sharp*$ *P* **and** $y \sharp P$ **by** *simp*+
from ‹$x \sharp (M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle)$› **have** $x \sharp$ *xvec1* **and** $x \neq y$ **and** $x \sharp$ *xvec2* **and** $x \sharp M$ **by** *simp*+
from *PSimQ* ‹$\Psi \rhd Q \longmapsto M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec ([(x,\ y)] \cdot Q')$› ‹*xvec1* $\sharp*$ Ψ› ‹*xvec2* $\sharp*$ Ψ› ‹*xvec1* $\sharp*$ *P*› ‹*xvec2* $\sharp*$ *P*› ‹$\alpha \neq \tau$›
obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle \prec P''$
and *P''Chain*: $\Psi \otimes ([(x,\ y)] \cdot \Psi') \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: $(\Psi \otimes ([(x,\ y)] \cdot \Psi'), P', ([(x,\ y)] \cdot Q')) \in Rel$
by (*fastforce dest*: *weakSimE*)
from *PTrans* **have** $([(x,\ y)] \cdot \Psi) : ([(x,\ y)] \cdot Q) \rhd ([(x,\ y)] \cdot P) \Longrightarrow([(x,\ y)] \cdot (M(\!|\nu*(xvec1@xvec2)|\!)\langle([(x,\ y)] \cdot N)\rangle)) \prec ([(x,\ y)] \cdot P'')$
by(*rule eqvts*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› ‹$x \sharp M$› ‹$y \sharp M$› ‹$x \sharp$ *xvec1*› ‹$y \sharp$ *xvec1*› ‹$x \sharp$ *xvec2*› ‹$y \sharp$ *xvec2*›
have $\Psi : ([(x,\ y)] \cdot Q) \rhd ([(x,\ y)] \cdot P) \Longrightarrow M(\!|\nu*(xvec1@xvec2)|\!)\langle N\rangle \prec ([(x,\ y)] \cdot P'')$
by(*simp add*: *eqvts*)
hence $\Psi : (\!|\nu y|\!)([(x,\ y)] \cdot Q) \rhd (\!|\nu y|\!)([(x,\ y)] \cdot P) \Longrightarrow M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec ([(x,\ y)] \cdot P'')$
using ‹$y \in$ *supp N*› ‹$y \sharp \Psi$› ‹$y \sharp M$› ‹$y \sharp$ *xvec1*› ‹$y \sharp$ *xvec2*›
by(*rule weakOpen*)
with ‹$y \sharp P$› ‹$y \sharp Q$› **have** $\Psi : (\!|\nu x|\!)Q \rhd (\!|\nu x|\!)P \Longrightarrow M(\!|\nu*(xvec1@y\#xvec2)|\!)\langle N\rangle \prec ([(x,\ y)] \cdot P'')$
by(*simp add*: *alphaRes*)
moreover from *P''Chain* **have** $([(x,\ y)] \cdot (\Psi \otimes ([(x,\ y)] \cdot \Psi'))) \rhd ([(x,\ y)] \cdot P'') \Longrightarrow\hat{}_\tau\ ([(x,\ y)] \cdot P')$

by(*rule eqvts*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \otimes \Psi' \rhd ([(x, y)] \cdot P'') \Longrightarrow\hat{}_\tau ([(x, y)] \cdot P')$ **by**(*simp add*: *eqvts*)
moreover from *P'RelQ'* ‹*Rel* $\subseteq$ *Rel'*› **have** $(\Psi \otimes ([(x, y)] \cdot \Psi'), P', ([(x, y)] \cdot Q')) \in Rel'$ **by** *auto*
with ‹*eqvt Rel'*› **have** $([(x, y)] \cdot (\Psi \otimes ([(x, y)] \cdot \Psi'), P', ([(x, y)] \cdot Q'))) \in Rel'$ **by**(*rule eqvtI*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $(\Psi \otimes \Psi', [(x, y)] \cdot P', Q') \in Rel'$ **by**(*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
case(*cRes Q'*)
from *PSimQ* ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$› ‹$\alpha \neq \tau$›
obtain *P'' P'* **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$
and *P''Chain*: $\Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau P'$ **and** *P'RelQ'*: $(\Psi \otimes \Psi', P', Q') \in Rel$
by(*blast dest*: *weakSimE*)
from *PTrans* ‹$x \sharp \Psi$› ‹$x \sharp \alpha$› **have** $\Psi : (\!|\nu x|\!) Q \rhd (\!|\nu x|\!) P \Longrightarrow \alpha \prec (\!|\nu x|\!) P''$
by(*rule-tac weakScope*)
moreover from *P''Chain* ‹$x \sharp \Psi$› ‹$x \sharp \Psi'$› **have** $\Psi \otimes \Psi' \rhd (\!|\nu x|\!) P'' \Longrightarrow\hat{}_\tau (\!|\nu x|\!) P'$
by(*rule-tac tauChainResPres*) *auto*
moreover from *P'RelQ'* ‹$x \sharp \Psi$› ‹$x \sharp \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu x|\!) P', (\!|\nu x|\!) Q') \in Rel'$
apply(*rule-tac C1*) **by**(*auto simp add*: *fresh-left calc-atm*)
ultimately show *?case* **by** *blast*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd (\!|\nu x|\!) Q \longmapsto \tau \prec Q'$› ‹$x \sharp \Psi$› ‹$x \sharp Q'$›
show *?case*
proof(*induct rule*: *resTauCases*)
case(*cRes Q'*)
from *PSimQ* ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$› **obtain** *P'* **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$ **and** *P'RelQ'*: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)
from *PChain* ‹$x \sharp \Psi$› **have** $\Psi \rhd (\!|\nu x|\!) P \Longrightarrow\hat{}_\tau (\!|\nu x|\!) P'$ **by**(*rule tauChainResPres*)
moreover from *P'RelQ'* ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu x|\!) P', (\!|\nu x|\!) Q') \in Rel'$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed
qed
qed

lemma *weakResChainPres*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

and Q :: $('a, 'b, 'c)$ *psi*
and *xvec* :: *name list*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *eqvt Rel*
and $xvec \sharp* \Psi$
and *C1*: $\bigwedge \Psi' R S yvec. [\![(\Psi', R, S) \in Rel; yvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * yvec|\!) R, (\!|\nu * yvec|\!) S) \in Rel$

shows $\Psi \rhd (\!|\nu * xvec|\!) P \rightsquigarrow <Rel> (\!|\nu * xvec|\!) Q$
using ‹*xvec* $\sharp*$ Ψ›
proof(*induct xvec*)
case *Nil*
from *PSimQ* **show** *?case* **by** *simp*
next
case(*Cons x xvec*)
from ‹$(x\#xvec) \sharp* \Psi$› **have** $x \sharp \Psi$ **and** $xvec \sharp* \Psi$ **by** *simp+*
from ‹$xvec \sharp* \Psi \Longrightarrow \Psi \rhd (\!|\nu * xvec|\!) P \rightsquigarrow <Rel> (\!|\nu * xvec|\!) Q$› ‹$xvec \sharp* \Psi$›
have $\Psi \rhd (\!|\nu * xvec|\!) P \rightsquigarrow <Rel> (\!|\nu * xvec|\!) Q$ **by** *simp*
moreover note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover have $Rel \subseteq Rel$ **by** *simp*
moreover have $\bigwedge \Psi\ P\ Q\ x. [\![(\Psi, P, Q) \in Rel; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu * [x]|\!) P, (\!|\nu * [x]|\!) Q) \in Rel$
by(*rule-tac yvec*=$[x]$ **in** *C1*) *auto*
hence $\bigwedge \Psi\ P\ Q\ x. [\![(\Psi, P, Q) \in Rel; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!) P, (\!|\nu x|\!) Q) \in Rel$
by *simp*
ultimately have $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!) P) \rightsquigarrow <Rel> (\!|\nu x|\!)((\!|\nu * xvec|\!) Q)$
by(*rule weakResPres*)
thus *?case* **by** *simp*
qed

lemma *parTauCases*[*consumes 1*, *case-names cPar1 cPar2 cComm1 cComm2*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*
and C :: $'d$::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto \tau \prec R$
and *rPar1*: $\bigwedge P' A_Q \Psi_Q. [\![\Psi \otimes \Psi_Q \rhd P \longmapsto \tau \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* C]\!] \Longrightarrow Prop\ (P' \parallel Q)$
and *rPar2*: $\bigwedge Q' A_P \Psi_P. [\![\Psi \otimes \Psi_P \rhd Q \longmapsto \tau \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* C]\!] \Longrightarrow Prop\ (P \parallel Q')$
and *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P \rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q \rangle;$
$distinct\ A_Q;$

$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C\rrbracket \Longrightarrow$
Prop $((\!|\nu*xvec|\!)(P' \parallel Q'))$

and *rComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q$.
$\llbracket \Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu*xvec|\!)\langle N\rangle \prec P'$; *extractFrame* $P = \langle A_P, \Psi_P\rangle$; *distinct* A_P;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$; *distinct* A_Q;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; *distinct xvec*;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C\rrbracket \Longrightarrow$
Prop $((\!|\nu*xvec|\!)(P' \parallel Q'))$

shows *Prop R*
proof −
from *Trans* **obtain** α **where** $\Psi \rhd P \parallel Q \longmapsto \alpha \prec R$ **and** *bn* $\alpha \sharp* \Psi$ **and** *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* Q$ **and** *bn* $\alpha \sharp*$ *subject* α **and** $\alpha = \tau$ **by** *auto*
thus *?thesis* **using** *rPar1 rPar2 rComm1 rComm2*
by(*induct rule*: *parCases*[**where** *C*=*C*]) (*auto simp add*: *residualInject*)
qed

lemma *weakParPres*:
fixes Ψ :: *'b*
and *P* :: *('a, 'b, 'c) psi*
and *Rel* :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*
and *Q* :: *('a, 'b, 'c) psi*
and *R* :: *('a, 'b, 'c) psi*
and *Rel'* :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*

assumes *PRelQ*: $\bigwedge A_R\ \Psi_R$. $\llbracket$*extractFrame* $R = \langle A_R, \Psi_R\rangle$; $A_R \sharp* \Psi$; $A_R \sharp* P$; $A_R \sharp* Q\rrbracket \Longrightarrow (\Psi \otimes \Psi_R, P, Q) \in Rel$
and *Eqvt*: *eqvt Rel*
and *Eqvt'*: *eqvt Rel'*

and *Sim*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow <Rel> T$
and *Sym*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', T, S) \in Rel$
and *Ext*: $\bigwedge \Psi'\ S\ T\ \Psi'.\ \llbracket(\Psi', S, T) \in Rel\rrbracket \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$
and *StatImp*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \lessapprox <Rel> T$
and *C1*: $\bigwedge \Psi'\ S\ T\ A_U\ \Psi_U\ U.\ \llbracket(\Psi' \otimes \Psi_U, S, T) \in Rel$; *extractFrame* $U = \langle A_U, \Psi_U\rangle$; $A_U \sharp* \Psi'$; $A_U \sharp* S$; $A_U \sharp* T\rrbracket \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel'$
and *C2*: $\bigwedge \Psi'\ S\ T\ xvec.\ \llbracket(\Psi', S, T) \in Rel'$; $xvec \sharp* \Psi'\rrbracket \Longrightarrow (\Psi', (\!|\nu*xvec|\!)S,$

$(\!|\nu{*}xvec|\!)\, T) \in Rel'$
and *C3*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'', S, T) \in Rel$

shows $\Psi \rhd P \parallel R \rightsquigarrow <Rel'> Q \parallel R$
proof −
from *Eqvt'* **show** *?thesis*
proof(*induct rule*: *weakSimI*[**where** *C*=()])
case(*cAct* $\Psi'\ \alpha\ QR$)
from ‹*bn* $\alpha \sharp* (P \parallel R)$› ‹*bn* $\alpha \sharp* (Q \parallel R)$›
have *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* Q$ **and** *bn* $\alpha \sharp* R$
by *simp*+
from ‹$\Psi \rhd Q \parallel R \longmapsto \alpha \prec QR$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* Q$› ‹*bn* $\alpha \sharp* R$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$\alpha \neq \tau$› **show** *?case*
proof(*induct rule*: *parCases*[**where** *C*=(P, R, Ψ')])
case(*cPar1* $Q'\ A_R\ \Psi_R$)
from ‹$A_R \sharp* (P, R, \Psi')$› **have** $A_R \sharp* P$ **and** $A_R \sharp* \Psi'$ **by** *simp*+
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$A_R \sharp* \alpha$› ‹*bn* $\alpha \sharp* R$› *FrR*
have *bn* $\alpha \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*
have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto \alpha \prec Q'$ **by** *fact*
from *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow <Rel> Q$
by(*blast intro*: *PRelQ Sim*)
then obtain $P''\ P'$ **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow \alpha \prec P''$
and *P''Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: $((\Psi \otimes \Psi_R) \otimes \Psi', P', Q') \in Rel$
using *QTrans* ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* \Psi_R$› ‹*bn* $\alpha \sharp* P$› ‹*bn* $\alpha \sharp* R$› ‹$A_R \sharp* \alpha$› ‹$\alpha \neq \tau$›
by(*drule-tac* Ψ'=Ψ' **in** *weakSimE*(*1*)) *auto*

from *PTrans QTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* \alpha$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹*distinct*(*bn* α)›
have $A_R \sharp* P''$ **and** $A_R \sharp* Q'$
by(*fastforce dest*: *weakFreshChainDerivative freeFreshChainDerivative*)+
with *P''Chain* **have** $A_R \sharp* P'$ **by**(*force dest*: *tauChainFreshChain*)

from *PTrans FrR* ‹*bn* $\alpha \sharp* R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* \alpha$› ‹$A_R \sharp* Q$›
have $\Psi : Q \parallel R \rhd P \parallel R \Longrightarrow \alpha \prec (P'' \parallel R)$
by(*rule-tac weakPar1*)
moreover from *P''Chain* **have** $(\Psi \otimes \Psi') \otimes \Psi_R \rhd P'' \Longrightarrow\hat{}_\tau\ P'$
by(*metis tauChainStatEq Associativity Commutativity Composition*)
with *FrR* **have** $\Psi \otimes \Psi' \rhd P'' \parallel R \Longrightarrow\hat{}_\tau\ P' \parallel R$ **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi'$› ‹$A_R \sharp* P''$›
by(*rule-tac tauChainPar1*) *auto*

moreover from *P'RelQ'* **have** $((\Psi \otimes \Psi') \otimes \Psi_R, P', Q') \in Rel$
by(*metis C3 compositionSym Associativity Commutativity*)
hence $(\Psi \otimes \Psi', P' \parallel R, Q' \parallel R) \in Rel'$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi'$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$›
by(*rule-tac C1*) *auto*

ultimately show *?case* **by** *blast*
next
case(*cPar2* R' A_Q Ψ_Q)
from ‹A_Q ♯* (P, R, Ψ')› **have** A_Q ♯* P **and** A_Q ♯* R **and** A_Q ♯* Ψ' **by** *simp+*
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** A_P ♯* (Ψ, A_Q, Ψ_Q, α, R)
by(*rule freshFrame*)
hence A_P ♯* Ψ **and** A_P ♯* A_Q **and** A_P ♯* Ψ_Q **and** A_P ♯* α **and** A_P ♯* R
by *simp+*

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from ‹A_Q ♯* P› *FrP* ‹A_P ♯* A_Q› **have** A_Q ♯* Ψ_P
by(*drule-tac extractFrameFreshChain*) *auto*

from *FrP FrQ* ‹*bn* α ♯* P› ‹*bn* α ♯* Q› ‹A_P ♯* α› ‹A_Q ♯* α›
have *bn* α ♯* Ψ_P **and** *bn* α ♯* Ψ_Q
by(*force dest*: *extractFrameFreshChain*)+

obtain A_R Ψ_R **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** A_R ♯* (Ψ, P, Q, A_Q, A_P, Ψ_Q, Ψ_P, α, R, Ψ')
and *distinct* A_R
by(*rule freshFrame*)
then have A_R ♯* Ψ **and** A_R ♯* P **and** A_R ♯* Q **and** A_R ♯* A_Q **and** A_R ♯* A_P **and** A_R ♯* Ψ_Q **and** A_R ♯* Ψ_P
and A_R ♯* α **and** A_R ♯* R **and** A_R ♯* Ψ'
by *simp+*

from ‹A_Q ♯* R› *FrR* ‹A_R ♯* A_Q› **have** A_Q ♯* Ψ_R **by**(*drule-tac extractFrameFreshChain*) *auto*
from ‹A_P ♯* R› ‹A_R ♯* A_P› *FrR* **have** A_P ♯* Ψ_R **by**(*drule-tac extractFrameFreshChain*) *auto*

moreover from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$› *FrR* ‹*distinct* A_R› ‹A_R ♯* Ψ› ‹A_R ♯* P› ‹A_R ♯* Q› ‹A_R ♯* R› ‹A_R ♯* α› ‹A_R ♯* Ψ'›
‹*bn* α ♯* Ψ› ‹*bn* α ♯* P› ‹*bn* α ♯* Q› ‹*bn* α ♯* R› ‹*bn* α ♯* *subject* α› ‹*distinct*(*bn* α)›
obtain p Ψ'' A_R' Ψ_R' **where** S: *set* $p \subseteq$ *set*(*bn* α) $\times$ *set*(*bn*($p \cdot \alpha$)) **and** ($p \cdot \Psi_R$) $\otimes$ $\Psi'' \simeq \Psi_R'$
and *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$
and *bn*($p \cdot \alpha$) ♯* Ψ **and** *bn*($p \cdot \alpha$) ♯* P **and** *bn*($p \cdot \alpha$) ♯* Q
and A_R' ♯* Ψ **and** A_R' ♯* P **and** A_R' ♯* Q **and** A_R' ♯* Ψ' **and** *distinctPerm* p
by(*rule-tac* C=(Ψ, Ψ', P, Q) **and** C'=(Ψ, P, Q) **in** *expandFrame*) (*assumption* | *simp*)+

from *FrR* ‹A_R ♯* Ψ› ‹A_R ♯* P› ‹A_R ♯* Q›
have ($\Psi \otimes \Psi_R$, P, Q) $\in$ *Rel* **by**(*rule PRelQ*)
hence ($\Psi \otimes \Psi_R$, Q, P) $\in$ *Rel* **by**(*rule Sym*)

hence $\Psi \otimes \Psi_R \rhd Q \lesssim_{<Rel>} P$ **by**(*rule StatImp*)
then obtain P' P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau P'$
and *QimpP'*: *insertAssertion*(*extractFrame* Q) $(\Psi \otimes \Psi_R) \hookrightarrow_F$ *insertAssertion*(*extractFrame* P') $(\Psi \otimes \Psi_R)$
and *P'Chain*: $(\Psi \otimes \Psi_R) \otimes ((p \cdot \Psi'') \otimes (p \cdot \Psi')) \rhd P' \Longrightarrow\hat{}_\tau P''$
and *P'RelQ*: $((\Psi \otimes \Psi_R) \otimes ((p \cdot \Psi'') \otimes (p \cdot \Psi')),\ Q,\ P'') \in Rel$
by(*rule weakStatImpE*)
obtain A_P' Ψ_P' **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* \Psi_R$ **and** $A_P' \sharp* \Psi_Q$
and $A_P' \sharp* A_Q$ **and** $A_P' \sharp* R$ **and** $A_P' \sharp* A_R$ **and** $A_P' \sharp* \alpha$
by(*rule-tac* C=(Ψ, Ψ_R, Ψ_Q, A_Q, R, A_R, α) **in** *freshFrame*) *auto*

from *PChain* $\langle A_R \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle bn\ \alpha \sharp* P\rangle$ **have** $A_Q \sharp* P'$ **and** $A_R \sharp* P'$ **and** $bn\ \alpha \sharp* P'$
by(*force intro*: *tauChainFreshChain*)+
from $\langle A_R \sharp* P'\rangle$ $\langle A_P' \sharp* A_R\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_P' \sharp* A_Q\rangle$ *FrP'* **have** $A_Q \sharp* \Psi_P'$ **and** $A_R \sharp* \Psi_P'$
by(*force dest*: *extractFrameFreshChain*)+

from *PChain FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau P' \parallel R$ **by**(*rule tauChainPar1*)
moreover have *insertAssertion* (*extractFrame*($Q \parallel R$)) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame*($P' \parallel R$)) Ψ
proof −
have $\langle A_Q, \Psi \otimes \Psi_Q \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
moreover from *QimpP' FrQ FrP'* $\langle A_Q \sharp* \Psi_R\rangle$ $\langle A_P' \sharp* \Psi_R\rangle$ $\langle A_P' \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle$ **using** *freshCompChain* **by** *simp*
moreover have $\langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle \simeq_F \langle A_P', \Psi \otimes \Psi_P' \otimes \Psi_R\rangle$
by(*metis Associativity Composition Commutativity AssertionStatEqTrans AssertionStatEqSym frameNilStatEq frameResChainPres*)
ultimately have $\langle A_Q, \Psi \otimes \Psi_Q \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P', \Psi \otimes \Psi_P' \otimes \Psi_R\rangle$
by(*rule FrameStatEqImpCompose*)
hence $\langle (A_R @ A_Q), \Psi \otimes \Psi_Q \otimes \Psi_R\rangle \hookrightarrow_F \langle (A_R @ A_P'), \Psi \otimes \Psi_P' \otimes \Psi_R\rangle$
by(*force intro*: *frameImpResChainPres simp add*: *frameChainAppend*)
with *FrQ FrR FrP'* $\langle A_R \sharp* A_Q\rangle$ $\langle A_P' \sharp* A_R\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$ $\langle A_R \sharp* \Psi_P'\rangle$ $\langle A_P' \sharp* \Psi_R\rangle$
$\langle A_P' \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi\rangle$
show *?thesis*
by(*simp add*: *frameChainAppend*) (*metis frameImpChainComm FrameStatImpTrans*)
qed
moreover have *RTrans*: $\Psi \otimes \Psi_P' \rhd R \longmapsto \alpha \prec R'$
proof −

have $\Psi \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$ **by** *fact*
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_P') \otimes \Psi_R\rangle$
proof −
have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym FrameStatEqSym*)
moreover with *FrP' FrQ QimpP'* $\langle A_P' \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_P' \sharp* \Psi_R\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle \simeq_F \langle A_P', (\Psi \otimes \Psi_P') \otimes \Psi_R\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym frameIntAssociativity*[*THEN FrameStatEqSym*])
ultimately show *?thesis*
by(*rule FrameStatEqImpCompose*)
qed
ultimately show *?thesis*
using $\langle A_P' \sharp* \Psi\rangle$ $\langle A_P' \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P'\rangle$ $\langle A_P' \sharp* R\rangle$ $\langle A_Q \sharp* R\rangle$ $\langle A_P' \sharp* \alpha\rangle$ $\langle A_Q \sharp* \alpha\rangle$
$\langle A_P' \sharp* A_R\rangle$ $\langle A_R \sharp* A_Q\rangle$ $\langle A_R \sharp* \Psi_P'\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P'\rangle$ $\langle A_Q \sharp* \Psi_P'\rangle$ *FrR* $\langle$*distinct* $A_R\rangle$
by(*force intro*: *transferFrame*)
qed
hence $\Psi \rhd P' \parallel R \longmapsto \alpha \prec (P' \parallel R')$ **using** *FrP'* $\langle bn\ \alpha \sharp* P'\rangle$ $\langle A_P' \sharp* \Psi\rangle$ $\langle A_P' \sharp* R\rangle$ $\langle A_P' \sharp* \alpha\rangle$ $\langle A_P' \sharp* \alpha\rangle$
by(*rule-tac Par2*)
ultimately have $\Psi : (Q \parallel R) \rhd P \parallel R \Longrightarrow \alpha \prec (P' \parallel R')$
by(*rule-tac weakTransitionI*)
moreover from *PChain P'Chain* $\langle bn\ \alpha \sharp* P'\rangle$ $\langle bn(p \cdot \alpha) \sharp* P\rangle$ $\langle A_R' \sharp* P\rangle$
have $bn\ \alpha \sharp* P''$ **and** $bn(p \cdot \alpha) \sharp* P'$ **and** $bn(p \cdot \alpha) \sharp* P''$ **and** $A_R' \sharp* P'$ **and** $A_R' \sharp* P''$
by(*metis tauChainFreshChain*)+
from *P'Chain* **have** $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'') \otimes (p \cdot \Psi'))) \rhd (p \cdot P') \Longrightarrow\hat{}_\tau (p \cdot P'')$
by(*rule eqvts*)
with $\langle bn\ \alpha \sharp* \Psi\rangle$ $\langle bn(p \cdot \alpha) \sharp* \Psi\rangle$ $\langle bn\ \alpha \sharp* P'\rangle$ $\langle bn(p \cdot \alpha) \sharp* P'\rangle$ $\langle bn\ \alpha \sharp* P''\rangle$ $\langle bn(p \cdot \alpha) \sharp* P''\rangle$
S $\langle$*distinctPerm p*$\rangle$
have $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi'' \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau P''$
by(*simp add*: *eqvts*)
hence $(\Psi \otimes \Psi') \otimes (p \cdot \Psi_R) \otimes \Psi'' \rhd P' \Longrightarrow\hat{}_\tau P''$
by(*rule tauChainStatEq*) (*metis Associativity Commutativity Composition AssertionStatEqTrans*)
with $\langle (p \cdot \Psi_R) \otimes \Psi'' \simeq \Psi_R'\rangle$ **have** $(\Psi \otimes \Psi') \otimes \Psi_R' \rhd P' \Longrightarrow\hat{}_\tau P''$ **by**(*metis tauChainStatEq compositionSym*)
hence $\Psi \otimes \Psi' \rhd P' \parallel R' \Longrightarrow\hat{}_\tau P'' \parallel R'$ **using** *FrR'* $\langle A_R' \sharp* \Psi\rangle$ $\langle A_R' \sharp* \Psi'\rangle$ $\langle A_R' \sharp* P'\rangle$ **by**(*rule-tac tauChainPar1*) *auto*
moreover from *P'RelQ* **have** $((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'') \otimes (p \cdot \Psi'),\ P'',\ Q) \in$

Rel **by**(*rule Sym*)
with ‹*eqvt Rel*› **have** $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'') \otimes (p \cdot \Psi'),\ P'',\ Q)) \in Rel$ **by**(*rule eqvtI*)
with ‹$bn\ \alpha \sharp* \Psi$› ‹$bn(p \cdot \alpha) \sharp* \Psi$› ‹$bn\ \alpha \sharp* P''$› ‹$bn(p \cdot \alpha) \sharp* P''$› ‹$bn\ \alpha \sharp* Q$› ‹$bn(p \cdot \alpha) \sharp* Q$› S ‹*distinctPerm p*›
have $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi'' \otimes \Psi',\ P'',\ Q) \in Rel$ **by**(*simp add*: *eqvts*)
with ‹$(p \cdot \Psi_R) \otimes \Psi'' \simeq {\Psi_R}'$› **have** $((\Psi \otimes \Psi') \otimes {\Psi_R}',\ P'',\ Q) \in Rel$
by(*rule-tac C3*, *auto*) (*metis Associativity Commutativity Composition AssertionStatEqTrans*)
with *FrR'* ‹${A_R}' \sharp* \Psi$› ‹${A_R}' \sharp* \Psi'$› ‹${A_R}' \sharp* P''$› ‹${A_R}' \sharp* Q$›
have $(\Psi \otimes \Psi',\ P'' \parallel R',\ Q \parallel R') \in Rel'$ **by**(*rule-tac C1*) *auto*
ultimately show *?case* **by** *blast*
next
case *cComm1*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
next
case *cComm2*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
qed
next
case(*cTau QR*)
from ‹$\Psi \rhd Q \parallel R \longmapsto \tau \prec QR$› **show** *?case*
proof(*induct rule*: *parTauCases*[**where** $C=(P,\ R)$])
case(*cPar1 Q' A_R Ψ_R*)
from ‹$A_R \sharp* (P,\ R)$› **have** $A_R \sharp* P$
by *simp+*
have *FrR*: *extractFrame* $R = \langle A_R,\ \Psi_R\rangle$ **by** *fact*
with ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow <Rel> Q$
by(*blast intro*: *PRelQ Sim*)
moreover have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto \tau \prec Q'$ **by** *fact*
ultimately obtain P' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** $P'RelQ'$: $(\Psi \otimes \Psi_R,\ P',\ Q') \in Rel$
by(*force dest*: *weakSimE*)
from *PChain QTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $A_R \sharp* P'$ **and** $A_R \sharp* Q'$
by(*force dest*: *freeFreshChainDerivative tauChainFreshChain*)+
from *PChain FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau\ (P' \parallel R)$
by(*rule tauChainPar1*)
moreover from *P'RelQ' FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$› **have** $(\Psi,\ P' \parallel R,\ Q' \parallel R) \in Rel'$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 R' A_Q Ψ_Q*)
from ‹$A_Q \sharp* (P,\ R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp+*
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame* $P = \langle A_P,\ \Psi_P\rangle$ **and** $A_P \sharp* (\Psi,\ A_Q,\ \Psi_Q,\ R)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* R$

by *simp*+

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from $\langle A_Q \sharp* P\rangle$ *FrP* $\langle A_P \sharp* A_Q\rangle$ **have** $A_Q \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*

obtain $A_R\ \Psi_R$ **where** *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* (\Psi, P, Q, A_Q, A_P, \Psi_Q, \Psi_P, R)$ **and** *distinct* A_R
by(*rule freshFrame*)
then have $A_R \sharp* \Psi$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* A_Q$ **and** $A_R \sharp* A_P$ **and** $A_R \sharp* \Psi_Q$ **and** $A_R \sharp* \Psi_P$
and $A_R \sharp* R$
by *simp*+

from $\langle A_Q \sharp* R\rangle$ *FrR* $\langle A_R \sharp* A_Q\rangle$ **have** $A_Q \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from $\langle A_P \sharp* R\rangle$ $\langle A_R \sharp* A_P\rangle$ *FrR* **have** $A_P \sharp* \Psi_R$ **by**(*drule-tac extractFrameFreshChain*) *auto*

moreover from $\langle \Psi \otimes \Psi_Q \rhd R \longmapsto \tau \prec R'\rangle$ *FrR* $\langle$*distinct* $A_R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$ $\langle A_R \sharp* R\rangle$
obtain $\Psi'\ A_R'\ \Psi_R'$ **where** $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$
and $A_R' \sharp* \Psi$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q$
by(*rule-tac* C=(Ψ, P, Q, R) **in** *expandTauFrame*) (*assumption* | *simp*)+

from *FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$
obtain $P'\ P''$ **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau\ P'$
and *QimpP'*: *insertAssertion*(*extractFrame* Q) $(\Psi \otimes \Psi_R) \hookrightarrow_F$ *insertAssertion*(*extractFrame* P') $(\Psi \otimes \Psi_R)$
and *P'Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ P''$
and *P'RelQ*: $((\Psi \otimes \Psi_R) \otimes \Psi', Q, P'') \in Rel$
by(*metis StatImp PRelQ Sym weakStatImpE*)
obtain $A_P'\ \Psi_P'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P', \Psi_P'\rangle$ **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* \Psi_R$ **and** $A_P' \sharp* \Psi_Q$
and $A_P' \sharp* A_Q$ **and** $A_P' \sharp* R$ **and** $A_P' \sharp* A_R$
by(*rule-tac* C=$(\Psi, \Psi_R, \Psi_Q, A_Q, R, A_R)$ **in** *freshFrame*) *auto*

from *PChain P'Chain* $\langle A_R \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle A_R' \sharp* P\rangle$ **have** $A_Q \sharp* P'$ **and** $A_R \sharp* P'$ **and** $A_R' \sharp* P'$ **and** $A_R' \sharp* P''$
by(*force intro*: *tauChainFreshChain*)+
from $\langle A_R \sharp* P'\rangle$ $\langle A_P' \sharp* A_R\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle A_P' \sharp* A_Q\rangle$ *FrP'* **have** $A_Q \sharp* \Psi_P'$ **and** $A_R \sharp* \Psi_P'$
by(*force dest*: *extractFrameFreshChain*)+

from *PChain FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau\ P' \parallel R$ **by**(*rule tauChainPar1*)
moreover have *RTrans*: $\Psi \otimes \Psi_P' \rhd R \longmapsto \tau \prec R'$
proof −

have $\Psi \otimes \Psi_Q \rhd R \longmapsto\tau \prec R'$ **by** *fact*
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_P') \otimes \Psi_R\rangle$
proof −
have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym FrameStatEqSym*)
moreover with *FrP' FrQ QimpP'* ‹$A_P' \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P' \sharp* \Psi_R$› ‹$A_Q \sharp* \Psi_R$›
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle \simeq_F \langle A_P', (\Psi \otimes \Psi_P') \otimes \Psi_R\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym frameIntAssociativity*[*THEN FrameStatEqSym*])
ultimately show *?thesis*
by(*rule FrameStatEqImpCompose*)
qed
ultimately show *?thesis*
using ‹$A_P' \sharp* \Psi$› ‹$A_P' \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_P'$› ‹$A_P' \sharp* R$› ‹$A_Q \sharp* R$›
‹$A_P' \sharp* A_R$› ‹$A_R \sharp* A_Q$› ‹$A_R \sharp* \Psi_P'$› ‹$A_R \sharp* \Psi_Q$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_P'$› ‹$A_Q \sharp* \Psi_P'$› *FrR* ‹*distinct* A_R›
by(*force intro*: *transferTauFrame*)
qed
hence $\Psi \rhd P' \parallel R \longmapsto\tau \prec (P' \parallel R')$ **using** *FrP'* ‹$A_P' \sharp* \Psi$› ‹$A_P' \sharp* R$›
by(*rule-tac Par2*) *auto*
moreover from *P'Chain* **have** $\Psi \otimes \Psi_R \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau P''$
by(*rule tauChainStatEq*) (*metis Associativity*)
with ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$› **have** $\Psi \otimes \Psi_R' \rhd P' \Longrightarrow\hat{}_\tau P''$ **by**(*metis tauChainStatEq compositionSym*)
hence $\Psi \rhd P' \parallel R' \Longrightarrow\hat{}_\tau P'' \parallel R'$ **using** *FrR'* ‹$A_R' \sharp* \Psi$› ‹$A_R' \sharp* P'$› **by**(*rule-tac tauChainPar1*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau (P'' \parallel R')$
by(*fastforce dest*: *rtrancl-into-rtrancl*)

moreover from *P'RelQ* ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes \Psi_R', P'', Q) \in Rel$ **by**(*blast intro*: *C3 Associativity compositionSym Sym*)
with *FrR'* ‹$A_R' \sharp* \Psi$› ‹$A_R' \sharp* P''$› ‹$A_R' \sharp* Q$› **have** $(\Psi, P'' \parallel R', Q \parallel R') \in Rel'$ **by**(*rule-tac C1*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* Ψ_R *M N Q'* A_Q Ψ_Q *K xvec R'* A_R)
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from ‹$A_Q \sharp* (P, R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp*+

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$A_R \sharp* (P, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* R$ **by** *simp*+
from ‹*xvec* $\sharp* (P, R)$› **have** *xvec* $\sharp* P$ **and** *xvec* $\sharp* R$ **by** *simp*+

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|N|\!) \prec Q'$ **and** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec R'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$ **by** *fact+*

from *RTrans FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* xvec$› ‹$A_R \sharp* N$› ‹$xvec \sharp* R$› ‹$xvec \sharp* Q$› ‹$xvec \sharp* \Psi$› ‹$xvec \sharp* \Psi_Q$› ‹$A_R \sharp* Q$›
‹$A_R \sharp* \Psi$› ‹$A_R \sharp* \Psi_Q$› ‹$xvec \sharp* K$› ‹$A_R \sharp* K$› ‹$A_R \sharp* R$› ‹$xvec \sharp* R$› ‹$A_R \sharp* P$› ‹$xvec \sharp* P$›
‹$A_Q \sharp* A_R$› ‹$A_Q \sharp* xvec$› ‹$xvec \sharp* K$› ‹*distinct xvec*› ‹$A_R \sharp* N$›
obtain $p\ \Psi'\ A_R'\ \Psi_R'$ **where** *S*: *set* $p \subseteq$ *set* $xvec \times set(p \cdot xvec)$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$
and $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* Q$ **and** $A_R' \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi_Q$ **and** $(p \cdot xvec) \sharp* K$ **and** $(p \cdot xvec) \sharp* R$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* A_Q$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* N$
by(*rule-tac* C=(Ψ, Q, Ψ_Q, K, R, P, A_Q) **and** C'=(Ψ, Q, Ψ_Q, K, R, P, A_Q) **in** *expandFrame*) (*assumption* | *simp*)+

from ‹$A_R \sharp* \Psi$› **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› *S* **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
from ‹$A_R \sharp* P$› **have** $(p \cdot A_R) \sharp* (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* P$› ‹$(p \cdot xvec) \sharp* P$› *S* **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
from ‹$A_R \sharp* Q$› **have** $(p \cdot A_R) \sharp* (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* Q$› ‹$(p \cdot xvec) \sharp* Q$› *S* **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
from ‹$A_R \sharp* R$› **have** $(p \cdot A_R) \sharp* (p \cdot R)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* R$› ‹$(p \cdot xvec) \sharp* R$› *S* **have** $(p \cdot A_R) \sharp* R$ **by** *simp*
from ‹$A_R \sharp* K$› **have** $(p \cdot A_R) \sharp* (p \cdot K)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$xvec \sharp* K$› ‹$(p \cdot xvec) \sharp* K$› *S* **have** $(p \cdot A_R) \sharp* K$ **by** *simp*

from ‹$A_Q \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_Q$› ‹$A_Q \sharp* M$› *S* **have** $A_Q \sharp* (p \cdot M)$ **by**(*simp add*: *freshChainSimps*)
from ‹$A_Q \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_Q$› ‹$A_Q \sharp* A_R$› *S* **have** $A_Q \sharp* (p \cdot A_R)$ **by**(*simp add*: *freshChainSimps*)

from *QTrans S* ‹$xvec \sharp* Q$› ‹$(p \cdot xvec) \sharp* Q$› **have** $(p \cdot (\Psi \otimes \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*rule-tac inputPermFrameSubject*) (*assumption* | *auto simp add*: *fresh-star-def*)+
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› *S* **have** *QTrans*: $(\Psi \otimes (p \cdot \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*simp add*: *eqvts*)
from *FrR* **have** $(p \cdot extractFrame\ R) = p \cdot \langle A_R, \Psi_R\rangle$ **by** *simp*
with ‹$xvec \sharp* R$› ‹$(p \cdot xvec) \sharp* R$› *S* **have** *FrR*: *extractFrame* $R = \langle (p \cdot A_R),$

$(p \cdot \Psi_R)\rangle$
by(*simp add*: *eqvts*)

from *MeqK* **have** $(p \cdot (\Psi \otimes \Psi_Q \otimes \Psi_R)) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$ **by**(*rule chanEqClosed*)
with ‹*xvec* ♯* Ψ› ‹$(p \cdot xvec)$ ♯* Ψ› ‹*xvec* ♯* Ψ_Q› ‹$(p \cdot xvec)$ ♯* Ψ_Q› ‹*xvec* ♯* K› ‹$(p \cdot xvec)$ ♯* K› *S*
have *MeqK*: $\Psi \otimes \Psi_Q \otimes (p \cdot \Psi_R) \vdash (p \cdot M) \leftrightarrow K$ **by**(*simp add*: *eqvts*)

from *FrR* ‹$(p \cdot A_R)$ ♯* Ψ› ‹$(p \cdot A_R)$ ♯* P› ‹$(p \cdot A_R)$ ♯* Q›
have $\Psi \otimes (p \cdot \Psi_R) \rhd P \rightsquigarrow <Rel> Q$ **by**(*force intro*: *Sim PRelQ*)

with *QTrans* **obtain** P' P'' **where** *PTrans*: $\Psi \otimes (p \cdot \Psi_R) : Q \rhd P \Longrightarrow (p \cdot M)(\!|N|\!) \prec P''$
and *P''Chain*: $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau P'$
and *P'RelQ'*: $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', P', Q') \in Rel$
by(*fastforce dest*: *weakSimE*)
from *PTrans QTrans* ‹A_R' ♯* P› ‹A_R' ♯* Q› ‹A_R' ♯* N› **have** A_R' ♯* P'' **and** A_R' ♯* Q'
by(*auto dest*: *weakInputFreshChainDerivative inputFreshChainDerivative*)

from *PTrans FrQ RTrans FrR MeqK* ‹A_Q ♯* Ψ› ‹A_Q ♯* P› ‹A_Q ♯* Q› ‹A_Q ♯* R› ‹A_Q ♯* $(p \cdot M)$› ‹A_Q ♯* $(p \cdot A_R)$›
‹$(p \cdot A_R)$ ♯* Ψ› ‹$(p \cdot A_R)$ ♯* P› ‹$(p \cdot A_R)$ ♯* Q› ‹$(p \cdot A_R)$ ♯* R› ‹$(p \cdot A_R)$ ♯* K› ‹*xvec* ♯* P› ‹*distinct* A_R›
have $\Psi \rhd P \parallel R \Longrightarrow_\tau (\!|\nu * xvec|\!)(P'' \parallel R')$ **apply**(*rule-tac weakComm1*)
by(*assumption* | *simp*)+

moreover from *P''Chain* ‹A_R' ♯* P''› **have** A_R' ♯* P' **by**(*rule tauChainFreshChain*)
from ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \simeq \Psi \otimes \Psi_R'$
by(*metis Associativity AssertionStatEqTrans AssertionStatEqSym compositionSym*)
with *P''Chain* **have** $\Psi \otimes \Psi_R' \rhd P'' \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauChainStatEq*)
hence $\Psi \rhd P'' \parallel R' \Longrightarrow\hat{}_\tau P' \parallel R'$ **using** *FrR'* ‹A_R' ♯* Ψ› ‹A_R' ♯* P''› **by**(*rule tauChainPar1*)
hence $\Psi \rhd (\!|\nu * xvec|\!)(P'' \parallel R') \Longrightarrow\hat{}_\tau (\!|\nu * xvec|\!)(P' \parallel R')$ **using** ‹*xvec* ♯* Ψ› **by**(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau (\!|\nu * xvec|\!)(P' \parallel R')$
by *auto*
moreover from *P'RelQ'* ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes \Psi_R', P', Q') \in Rel$ **by**(*metis C3 Associativity compositionSym*)
with *FrR'* ‹A_R' ♯* P'› ‹A_R' ♯* Q'› ‹A_R' ♯* Ψ› **have** $(\Psi, P' \parallel R', Q' \parallel R') \in Rel'$ **by**(*rule-tac C1*)
with ‹*xvec* ♯* Ψ› **have** $(\Psi, (\!|\nu * xvec|\!)(P' \parallel R'), (\!|\nu * xvec|\!)(Q' \parallel R')) \in Rel'$
by(*rule-tac C2*)
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_R *M xvec N Q'* A_Q Ψ_Q *K R'* A_R)

have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **by** *fact*
from ‹$A_Q \sharp* (P, R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp+*

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact*
from ‹$A_R \sharp* (P, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* R$ **by** *simp+*
from ‹*xvec* $\sharp* (P, R)$› **have** *xvec* $\sharp* P$ **and** *xvec* $\sharp* R$ **by** *simp+*

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$ **and** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$ **by** *fact+*

from *RTrans FrR* ‹*distinct* A_R› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* Q'$› ‹$A_R \sharp* N$› ‹$A_R \sharp* P$› ‹$A_R \sharp*$ *xvec*› ‹$A_R \sharp* K$›
obtain Ψ' A_R' Ψ_R' **where** *ReqR'*: $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R' \rangle$
and $A_R' \sharp* \Psi$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q'$ **and** $A_R' \sharp* N$ **and** $A_R' \sharp*$ *xvec*
by(*rule-tac* C=(Ψ, P, Q', N, *xvec*) **and** C'=(Ψ, P, Q', N, *xvec*) **in** *expandFrame*) (*assumption* | *simp*)+

from *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow <Rel> Q$ **by**(*force intro*: *Sim PRelQ*)

with *QTrans* ‹*xvec* $\sharp* \Psi$› ‹*xvec* $\sharp* \Psi_R$› ‹*xvec* $\sharp* P$›
obtain P'' P' **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N \rangle \prec P''$
and *P''Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau P'$
and *P'RelQ'*: $((\Psi \otimes \Psi_R) \otimes \Psi', P', Q') \in Rel$
by(*fastforce dest*: *weakSimE*)
from *PTrans* **obtain** P''' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau P'''$
and *QimpP'''*: *insertAssertion* (*extractFrame* Q) ($\Psi \otimes \Psi_R$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame* P''') ($\Psi \otimes \Psi_R$)
and *P'''Trans*: $\Psi \otimes \Psi_R \rhd P''' \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P''$
by(*rule weakTransitionE*)

from *PChain* ‹$A_R \sharp* P$› **have** $A_R \sharp* P'''$ **by**(*rule tauChainFreshChain*)

obtain A_P''' Ψ_P''' **where** *FrP'''*: *extractFrame* $P''' = \langle A_P''', \Psi_P''' \rangle$ **and** $A_P''' \sharp* (\Psi, A_Q, \Psi_Q, A_R, \Psi_R, M, N, K, R, P''',$ *xvec*) **and** *distinct* A_P'''
by(*rule freshFrame*)
hence $A_P''' \sharp* \Psi$ **and** $A_P''' \sharp* A_Q$ **and** $A_P''' \sharp* \Psi_Q$ **and** $A_P''' \sharp* M$ **and** $A_P''' \sharp* R$
and $A_P''' \sharp* N$ **and** $A_P''' \sharp* K$ **and** $A_P''' \sharp* A_R$ **and** $A_P''' \sharp* P'''$ **and** $A_P''' \sharp*$ *xvec* **and** $A_P''' \sharp* \Psi_R$
by *simp+*

have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R \rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q \rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover with *QimpP''' FrP''' FrQ* ‹$A_P''' \sharp* \Psi$› ‹$A_Q \sharp* \Psi$› ‹$A_P''' \sharp* \Psi_R$›

$\langle A_Q \sharp* \Psi_R\rangle$
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle {A_P}''', (\Psi \otimes \Psi_R) \otimes {\Psi_P}'''\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle {A_P}''', (\Psi \otimes \Psi_R) \otimes {\Psi_P}'''\rangle \simeq_F \langle {A_P}''', (\Psi \otimes {\Psi_P}''') \otimes \Psi_R\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately have *QImpP'''*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle {A_P}''', (\Psi \otimes {\Psi_P}''') \otimes \Psi_R\rangle$
by(*rule FrameStatEqImpCompose*)

from *PChain FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau P''' \parallel R$ **by**(*rule tauChainPar1*)
moreover from *RTrans FrR P'''Trans MeqK QImpP''' FrP''' FrQ* $\langle$*distinct* ${A_P}'''\rangle$ $\langle$*distinct* $A_R\rangle$ $\langle {A_P}''' \sharp* A_R\rangle$ $\langle A_Q \sharp* A_R\rangle$
$\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P'''\rangle$ $\langle A_R \sharp* Q\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* K\rangle$ $\langle {A_P}''' \sharp* \Psi\rangle$ $\langle {A_P}''' \sharp* R\rangle$
$\langle {A_P}''' \sharp* P'''\rangle$ $\langle {A_P}''' \sharp* M\rangle$ $\langle A_Q \sharp* R\rangle$ $\langle A_Q \sharp* M\rangle$ $\langle A_R \sharp* xvec\rangle$ $\langle xvec \sharp* M\rangle$
obtain K' **where** $\Psi \otimes {\Psi_P}''' \rhd R \longmapsto K'(\!|N|\!) \prec R'$ **and** $\Psi \otimes {\Psi_P}''' \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
by(*rule-tac comm2Aux*) (*assumption* | *simp*)+

with *P'''Trans FrP'''* **have** $\Psi \rhd P''' \parallel R \longmapsto \tau \prec (\!|\nu* xvec|\!)(P'' \parallel R')$ **using** *FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P'''\rangle$ $\langle A_R \sharp* R\rangle$
$\langle xvec \sharp* R\rangle$ $\langle {A_P}''' \sharp* \Psi\rangle$ $\langle {A_P}''' \sharp* P'''\rangle$ $\langle {A_P}''' \sharp* R\rangle$ $\langle {A_P}''' \sharp* M\rangle$ $\langle A_R \sharp* K'\rangle$ $\langle {A_P}''' \sharp* A_R\rangle$
by(*rule-tac Comm2*)
moreover from *P'''Trans* $\langle A_R \sharp* P'''\rangle$ $\langle A_R \sharp* xvec\rangle$ $\langle xvec \sharp* M\rangle$ $\langle$*distinct* $xvec\rangle$ **have** $A_R \sharp* P''$
by(*rule-tac outputFreshChainDerivative*) *auto*

from *PChain* $\langle {A_R}' \sharp* P\rangle$ **have** ${A_R}' \sharp* P'''$ **by**(*rule tauChainFreshChain*)
with *P'''Trans* **have** ${A_R}' \sharp* P''$ **using** $\langle {A_R}' \sharp* xvec\rangle$ $\langle xvec \sharp* M\rangle$ $\langle$*distinct* $xvec\rangle$
by(*rule-tac outputFreshChainDerivative*) *auto*

with *P''Chain* **have** ${A_R}' \sharp* P'$ **by**(*rule tauChainFreshChain*)
from $\langle \Psi_R \otimes \Psi' \simeq {\Psi_R}'\rangle$ **have** $(\Psi \otimes \Psi_R) \otimes \Psi' \simeq \Psi \otimes {\Psi_R}'$
by(*metis Associativity AssertionStatEqTrans AssertionStatEqSym compositionSym*)
with *P''Chain* **have** $\Psi \otimes {\Psi_R}' \rhd P'' \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauChainStatEq*)
hence $\Psi \rhd P'' \parallel R' \Longrightarrow\hat{}_\tau P' \parallel R'$ **using** *FrR'* $\langle {A_R}' \sharp* \Psi\rangle$ $\langle {A_R}' \sharp* P''\rangle$
by(*rule tauChainPar1*)
hence $\Psi \rhd (\!|\nu* xvec|\!)(P'' \parallel R') \Longrightarrow\hat{}_\tau (\!|\nu* xvec|\!)(P' \parallel R')$
using $\langle xvec \sharp* \Psi\rangle$ **by**(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau (\!|\nu* xvec|\!)(P' \parallel R')$ **by**(*drule-tac tauActTauChain*) *auto*
moreover from *P'RelQ'* $\langle \Psi_R \otimes \Psi' \simeq {\Psi_R}'\rangle$ **have** $(\Psi \otimes {\Psi_R}', P', Q') \in Rel$

by(*metis C3 Associativity compositionSym*)
with *FrR'* ‹A_R' ♯* P'› ‹A_R' ♯* Q'› ‹A_R' ♯* Ψ› **have** (Ψ, P' ∥ R', Q' ∥ R') ∈ *Rel'* **by**(*rule-tac C1*)
with ‹*xvec* ♯* Ψ› **have** (Ψ, (|ν*xvec|)(P' ∥ R'), (|ν*xvec|)(Q' ∥ R')) ∈ *Rel'*
by(*rule-tac C2*)
ultimately show *?case* **by** *blast*
qed
qed
qed
no-notation *relcomp* (**infixr** *O* 75)
lemma *weakSimBangPres*:
fixes Ψ :: 'b
and P :: ('a, 'b, 'c) *psi*
and Q :: ('a, 'b, 'c) *psi*
and *Rel* :: ('b × ('a, 'b, 'c) *psi* × ('a, 'b, 'c) *psi*) *set*
and *Rel'* :: ('b × ('a, 'b, 'c) *psi* × ('a, 'b, 'c) *psi*) *set*
and *Rel''* :: ('b × ('a, 'b, 'c) *psi* × ('a, 'b, 'c) *psi*) *set*

assumes (Ψ, P, Q) ∈ *Rel*
and *eqvt Rel''*
and *guarded P*
and *guarded Q*
and *Rel'Rel*: *Rel'* ⊆ *Rel*

and *FrameParPres*: ⋀Ψ' Ψ_U S T U A_U. ⟦(Ψ' ⊗ Ψ_U, S, T) ∈ *Rel*; *extractFrame* U = ⟨A_U, Ψ_U⟩; A_U ♯* Ψ'; A_U ♯* S; A_U ♯* T⟧ ⟹ (Ψ', U ∥ S, U ∥ T) ∈ *Rel*
and *C1*: ⋀Ψ' S T U. ⟦(Ψ', S, T) ∈ *Rel*; *guarded S*; *guarded T*⟧ ⟹ (Ψ', U ∥ !S, U ∥ !T) ∈ *Rel''*
and *ResPres*: ⋀Ψ' S T *xvec*. ⟦(Ψ', S, T) ∈ *Rel*; *xvec* ♯* Ψ'⟧ ⟹ (Ψ', (|ν*xvec|)S, (|ν*xvec|)T) ∈ *Rel*
and *ResPres'*: ⋀Ψ' S T *xvec*. ⟦(Ψ', S, T) ∈ *Rel'*; *xvec* ♯* Ψ'⟧ ⟹ (Ψ', (|ν*xvec|)S, (|ν*xvec|)T) ∈ *Rel'*

and *Closed*: ⋀Ψ' S T p. (Ψ', S, T) ∈ *Rel* ⟹ ((p::*name prm*) ∙ Ψ', p ∙ S, p ∙ T) ∈ *Rel*
and *Closed'*: ⋀Ψ' S T p. (Ψ', S, T) ∈ *Rel'* ⟹ ((p::*name prm*) ∙ Ψ', p ∙ S, p ∙ T) ∈ *Rel'*
and *StatEq*: ⋀Ψ' S T Ψ''. ⟦(Ψ', S, T) ∈ *Rel*; Ψ' ≃ Ψ''⟧ ⟹ (Ψ'', S, T) ∈ *Rel*
and *StatEq'*: ⋀Ψ' S T Ψ''. ⟦(Ψ', S, T) ∈ *Rel'*; Ψ' ≃ Ψ''⟧ ⟹ (Ψ'', S, T) ∈ *Rel'*
and *Trans*: ⋀Ψ' S T U. ⟦(Ψ', S, T) ∈ *Rel*; (Ψ', T, U) ∈ *Rel*⟧ ⟹ (Ψ', S, U) ∈ *Rel*
and *Trans'*: ⋀Ψ' S T U. ⟦(Ψ', S, T) ∈ *Rel'*; (Ψ', T, U) ∈ *Rel'*⟧ ⟹ (Ψ', S, U) ∈ *Rel'*

and *cSim*: ⋀Ψ' S T. (Ψ', S, T) ∈ *Rel* ⟹ Ψ' ▷ S ⇝<*Rel*> T
and *cExt*: ⋀Ψ' S T Ψ''. (Ψ', S, T) ∈ *Rel* ⟹ (Ψ' ⊗ Ψ'', S, T) ∈ *Rel*

and *cExt'*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ (\Psi',\ S,\ T) \in Rel' \Longrightarrow (\Psi' \otimes \Psi'',\ S,\ T) \in Rel'$
and *cSym*: $\bigwedge \Psi'\ S\ T.\ (\Psi',\ S,\ T) \in Rel \Longrightarrow (\Psi',\ T,\ S) \in Rel$
and *cSym'*: $\bigwedge \Psi'\ S\ T.\ (\Psi',\ S,\ T) \in Rel' \Longrightarrow (\Psi',\ T,\ S) \in Rel'$

and *ParPres*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi',\ S,\ T) \in Rel \Longrightarrow (\Psi',\ S \parallel U,\ T \parallel U) \in Rel$
and *ParPres2*: $\bigwedge \Psi'\ S\ T.\ (\Psi',\ S,\ T) \in Rel \Longrightarrow (\Psi',\ S \parallel S,\ T \parallel T) \in Rel$
and *ParPres'*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi',\ S,\ T) \in Rel' \Longrightarrow (\Psi',\ U \parallel S,\ U \parallel T) \in Rel'$

and *Assoc*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi',\ S \parallel (T \parallel U),\ (S \parallel T) \parallel U) \in Rel$
and *Assoc'*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi',\ S \parallel (T \parallel U),\ (S \parallel T) \parallel U) \in Rel'$
and *ScopeExt*: $\bigwedge xvec\ \Psi'\ T\ S.\ [\![xvec \sharp* \Psi';\ xvec \sharp* T]\!] \Longrightarrow (\Psi',\ (\!|\nu * xvec|\!)(S \parallel T),\ ((\!|\nu * xvec|\!)S) \parallel T) \in Rel$
and *ScopeExt'*: $\bigwedge xvec\ \Psi'\ T\ S.\ [\![xvec \sharp* \Psi';\ xvec \sharp* T]\!] \Longrightarrow (\Psi',\ (\!|\nu * xvec|\!)(S \parallel T),\ ((\!|\nu * xvec|\!)S) \parallel T) \in Rel'$

and *Compose*: $\bigwedge \Psi'\ S\ T\ U\ O.\ [\![(\Psi',\ S,\ T) \in Rel;\ (\Psi',\ T,\ U) \in Rel'';\ (\Psi',\ U,\ O) \in Rel']\!] \Longrightarrow (\Psi',\ S,\ O) \in Rel''$

and *rBangActE*: $\bigwedge \Psi'\ S\ \alpha\ S'.\ [\![\Psi' \rhd\ !S \longmapsto \alpha \prec S';\ guarded\ S;\ bn\ \alpha \sharp* S;\ \alpha \neq \tau;\ bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow \exists\ T.\ \Psi' \rhd S \longmapsto \alpha \prec T \wedge (\mathbf{1},\ S',\ T \parallel\ !S) \in Rel'$
and *rBangTauE*: $\bigwedge \Psi'\ S\ S'.\ [\![\Psi' \rhd\ !S \longmapsto \tau \prec S';\ guarded\ S]\!] \Longrightarrow \exists\ T.\ \Psi' \rhd S \parallel S \longmapsto \tau \prec T \wedge (\mathbf{1},\ S',\ T \parallel\ !S) \in Rel'$
and *rBangTauI*: $\bigwedge \Psi'\ S\ S'.\ [\![\Psi' \rhd S \parallel S \Longrightarrow\hat{}_\tau\ S';\ guarded\ S]\!] \Longrightarrow \exists\ T.\ \Psi' \rhd\ !S \Longrightarrow\hat{}_\tau\ T \wedge (\Psi',\ T,\ S' \parallel\ !S) \in Rel'$
shows $\Psi \rhd R \parallel\ !P \rightsquigarrow <Rel''>\ R \parallel\ !Q$
using ‹*eqvt Rel''*›
proof(*induct rule*: *weakSimI*[**where** *C*=()])
case(*cAct* $\Psi'\ \alpha\ RQ'$)
from ‹*bn* $\alpha \sharp* (R \parallel\ !P)$› ‹*bn* $\alpha \sharp* (R \parallel\ !Q)$› **have** *bn* $\alpha \sharp* P$ **and** *bn* $\alpha \sharp* (!Q)$ **and** *bn* $\alpha \sharp* Q$ **and** *bn* $\alpha \sharp* R$
by *simp+*
from ‹$\Psi \rhd R \parallel\ !Q \longmapsto \alpha \prec RQ'$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* R$› ‹*bn* $\alpha \sharp*\ !Q$› ‹*bn* $\alpha \sharp*$ *subject* α› ‹$\alpha \neq \tau$› **show** *?case*
proof(*induct rule*: *parCases*[**where** $C=(\Psi',\ P,\ Q,\ R)$])
case(*cPar1* $R'\ A_Q\ \Psi_Q$)
from ‹*extractFrame* $(!Q) = \langle A_Q,\ \Psi_Q\rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp+*
with ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto \alpha \prec R'$› ‹$\Psi_Q = SBottom'$› ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$›
have $\Psi \rhd R \parallel\ !P \longmapsto \alpha \prec (R' \parallel\ !P)$ **by**(*rule-tac Par1*) (*assumption* | *simp*)+
hence $\Psi : R \parallel\ !Q \rhd R \parallel\ !P \Longrightarrow \alpha \prec R' \parallel\ !P$ **by**(*rule-tac transitionWeakTransition*) *auto*
moreover have $\Psi \otimes \Psi' \rhd R' \parallel\ !P \Longrightarrow\hat{}_\tau\ R' \parallel\ !P$ **by** *auto*
moreover from ‹$(\Psi,\ P,\ Q) \in Rel$›
have $(\Psi \otimes \Psi',\ P,\ Q) \in Rel$ **by**(*rule cExt*)
hence $(\Psi \otimes \Psi',\ R' \parallel\ !P,\ R' \parallel\ !Q) \in Rel''$ **using** ‹*guarded P*› ‹*guarded Q*›
by(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2* $Q'\ A_R\ \Psi_R$)
from ‹$A_R \sharp* (\Psi',\ P,\ Q,\ R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* \Psi'$ **and**

$A_R \sharp* R$ **by** *simp+*
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
with ‹*bn* $\alpha \sharp* R$› ‹$A_R \sharp* \alpha$› **have** *bn* $\alpha \sharp* \Psi_R$ **by**(*auto dest*: *extractFrameFreshChain*)

obtain A_Q Ψ_Q **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* \Psi_R$ **and** $A_Q \sharp* A_R$
by(*rule-tac* C=(Ψ, Ψ_R, A_R) **in** *freshFrame*) *auto*
from *FrQ* ‹*guarded Q*› **have** $\Psi_Q \simeq \mathbf{1}$ **and** *supp* $\Psi_Q = (\{\}::name\ set)$ **by**(*blast dest*: *guardedStatEq*)+
hence $A_R \sharp* \Psi_Q$ **and** $A_Q \sharp* \Psi_Q$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from ‹$\Psi \otimes \Psi_R \rhd\ !Q \longmapsto \alpha \prec Q'$› ‹*guarded Q*› ‹*bn* $\alpha \sharp* Q$› ‹$\alpha \neq \tau$› ‹*bn* $\alpha \sharp*$ *subject* α›
obtain T **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto \alpha \prec T$ **and** $(\mathbf{1}, Q', T \parallel\ !Q) \in Rel'$
by(*blast dest*: *rBangActE*)

from ‹$(\Psi, P, Q) \in Rel$›
have $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)
with *QTrans* ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* \Psi_R$› ‹*bn* $\alpha \sharp* P$› ‹$\alpha \neq \tau$›
obtain P'' S **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow \alpha \prec P''$
and *P''Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ S$
and *SRelT*: $((\Psi \otimes \Psi_R) \otimes \Psi', S, T) \in Rel$
by(*blast dest*: *cSim weakSimE*)
from *PTrans* **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} : Q \rhd P \Longrightarrow \alpha \prec P''$
by(*metis weakTransitionStatEq Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R : Q \parallel\ !P \rhd P \parallel\ !P \Longrightarrow \alpha \prec P'' \parallel\ !P$ **using** ‹*bn* $\alpha \sharp* P$›
by(*force intro*: *weakPar1*)
hence $\Psi \otimes \Psi_R : Q \parallel\ !P \rhd\ !P \Longrightarrow \alpha \prec P'' \parallel\ !P$ **using** ‹*guarded P*›
by(*rule weakBang*)
hence $\Psi : R \parallel (Q \parallel\ !P) \rhd R \parallel\ !P \Longrightarrow \alpha \prec R \parallel (P'' \parallel\ !P)$
using *FrR* ‹*bn* $\alpha \sharp* R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* Q$›‹$A_R \sharp* P$› ‹$A_R \sharp* \alpha$› ‹$A_R \sharp* \alpha$›
by(*rule-tac weakPar2*) *auto*
moreover have *insertAssertion* (*extractFrame*($R \parallel\ !Q$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($R \parallel (Q \parallel\ !P)$)) Ψ
proof −
have *insertAssertion* (*extractFrame*($R \parallel\ !P$)) $\Psi = \langle A_R, \Psi \otimes \Psi_R \otimes \mathbf{1}\rangle$ **using** *FrR* ‹$A_R \sharp* \Psi$›
by *auto*
moreover from ‹$\Psi_Q \simeq \mathbf{1}$› **have** $\langle A_R, \Psi \otimes \Psi_R \otimes \mathbf{1}\rangle \simeq_F \langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1}\rangle$
by(*rule-tac frameResChainPres*, *auto*) (*metis Identity compositionSym AssertionStatEqTrans AssertionStatEqSym*)
moreover have $\langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1}\rangle \simeq_F (\!|\nu* A_Q|\!)(\langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1}\rangle)$ **using** ‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi_Q$› ‹$A_Q \sharp* \Psi_R$› ‹$A_Q \sharp* A_R$› *freshCompChain*
by(*subst frameResFreshChain*[**where** *xvec*=A_Q, *THEN FrameStatEqSym*]) *auto*
moreover have $(\!|\nu* A_Q|\!)(\langle A_R, \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1}\rangle) \simeq_F (\!|\nu* A_R|\!)(\langle A_Q, \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1}\rangle)$

by(*rule frameResChainComm*)
moreover have *insertAssertion* (*extractFrame*($R \parallel (Q \parallel !P)$)) $\Psi = \langle (A_R @ A_Q), \Psi \otimes \Psi_R \otimes \Psi_Q \otimes \mathbf{1} \rangle$
using *FrR FrQ* ‹$A_R \sharp^* \Psi$› ‹$A_Q \sharp^* \Psi$› ‹$A_Q \sharp^* A_R$› ‹$A_Q \sharp^* \Psi_R$› ‹$A_R \sharp^* \Psi_Q$› *freshCompChain*
by *auto*
ultimately have *insertAssertion* (*extractFrame*($R \parallel !P$)) $\Psi \simeq_F$ *insertAssertion* (*extractFrame*($R \parallel (Q \parallel !P)$)) Ψ
by(*auto simp add*: *frameChainAppend*) (*blast dest*: *FrameStatEqTrans*)
thus *?thesis* **by**(*simp add*: *FrameStatEq-def*)
qed
ultimately have $\Psi : R \parallel !Q \rhd R \parallel !P \Longrightarrow \alpha \prec R \parallel (P'' \parallel !P)$
by(*rule weakTransitionFrameImp*)

moreover from *PTrans* ‹$A_R \sharp^* P$› ‹$A_R \sharp^* \alpha$› ‹*bn* $\alpha \sharp^*$ *subject* α› ‹*distinct*(*bn* α)›
have $A_R \sharp^* P''$ **by**(*force dest*: *weakFreshChainDerivative*)
with *P''Chain* **have** $A_R \sharp^* S$ **by**(*force dest*: *tauChainFreshChain*)
from ‹$\Psi \otimes \Psi_R \rhd Q \longmapsto \alpha \prec T$› ‹$A_R \sharp^* Q$› ‹$A_R \sharp^* \alpha$› ‹*bn* $\alpha \sharp^*$ *subject* α› ‹*distinct*(*bn* α)› **have** $A_R \sharp^* T$
by(*force dest*: *freeFreshChainDerivative*)

from *P''Chain* **have** $((\Psi \otimes \Psi') \otimes \Psi_R) \otimes \mathbf{1} \rhd P'' \Longrightarrow\hat{}_\tau S$
by(*rule tauChainStatEq*) (*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym Identity*)
hence $(\Psi \otimes \Psi') \otimes \Psi_R \rhd P'' \parallel !P \Longrightarrow\hat{}_\tau S \parallel !P$
by(*rule-tac tauChainPar1*) *auto*
hence $\Psi \otimes \Psi' \rhd R \parallel (P'' \parallel !P) \Longrightarrow\hat{}_\tau R \parallel (S \parallel !P)$ **using** *FrR* ‹$A_R \sharp^* \Psi$› ‹$A_R \sharp^* \Psi'$› ‹$A_R \sharp^* P''$› ‹$A_R \sharp^* P$›
by(*rule-tac tauChainPar2*) *auto*
moreover have $(\Psi \otimes \Psi', R \parallel (S \parallel !P), R \parallel Q') \in Rel''$
proof −
from ‹$((\Psi \otimes \Psi_R) \otimes \Psi', S, T) \in Rel$› **have** $((\Psi \otimes \Psi') \otimes \Psi_R, S, T) \in Rel$
by(*rule StatEq*) (*metis Associativity Commutativity Composition Assertion-StatEqTrans AssertionStatEqSym*)
with *FrR* ‹$A_R \sharp^* \Psi$› ‹$A_R \sharp^* \Psi'$› ‹$A_R \sharp^* S$› ‹$A_R \sharp^* T$› **have** $(\Psi \otimes \Psi', R \parallel S, R \parallel T) \in Rel$
by(*rule-tac FrameParPres*) *auto*
hence $(\Psi \otimes \Psi', R \parallel T, R \parallel S) \in Rel$ **by**(*rule cSym*)
hence $(\Psi \otimes \Psi', (R \parallel T) \parallel !P, (R \parallel S) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi \otimes \Psi', (R \parallel S) \parallel !P, (R \parallel T) \parallel !P) \in Rel$ **by**(*rule cSym*)
hence $(\Psi \otimes \Psi', R \parallel (S \parallel !P), (R \parallel T) \parallel !P) \in Rel$ **by**(*metis Trans Assoc*)
moreover from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi', P, Q) \in Rel$ **by**(*rule cExt*)
hence $(\Psi \otimes \Psi', (R \parallel T) \parallel !P, (R \parallel T) \parallel !Q) \in Rel''$ **using** ‹*guarded P*› ‹*guarded Q*› **by**(*rule C1*)
moreover from ‹$(\mathbf{1}, Q', T \parallel !Q) \in Rel'$› **have** $(\mathbf{1} \otimes \Psi \otimes \Psi', Q', T \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi \otimes \Psi', Q', T \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity Asser-*

tionStatEqTrans)
hence $(\Psi \otimes \Psi', R \parallel Q', R \parallel (T \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi \otimes \Psi', R \parallel Q', (R \parallel T) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi \otimes \Psi', (R \parallel T) \parallel !Q, R \parallel Q') \in Rel'$ **by**(*rule cSym'*)
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
next
case *cComm1*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
next
case *cComm2*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
qed
next
case(*cTau RQ'*)
from ‹$\Psi \rhd R \parallel !Q \longmapsto \tau \prec RQ'$› **show** *?case*
proof(*induct rule*: *parTauCases*[**where** $C=(P, Q, R)$])
case(*cPar1 R' A_Q Ψ_Q*)
from ‹*extractFrame* $(!Q) = \langle A_Q, \Psi_Q\rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp+*
with ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto \tau \prec R'$› ‹$\Psi_Q = SBottom'$›
have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (R' \parallel !P)$ **by**(*rule-tac Par1*) (*assumption* | *simp*)+
hence $\Psi \rhd R \parallel !P \Longrightarrow\hat{}_\tau\ R' \parallel !P$ **by** *auto*
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹*guarded P*› ‹*guarded Q*› **have** $(\Psi, R' \parallel !P, R' \parallel !Q) \in Rel''$
by(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 Q' A_R Ψ_R*)
from ‹$A_R \sharp* (P, Q, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **by** *simp+*
have *FrR*: *extractFrame R* $= \langle A_R, \Psi_R\rangle$ **by** *fact*

obtain $A_Q\ \Psi_Q$ **where** *FrQ*: *extractFrame Q* $= \langle A_Q, \Psi_Q\rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* \Psi_R$ **and** $A_Q \sharp* A_R$
by(*rule-tac* $C=(\Psi, \Psi_R, A_R)$ **in** *freshFrame*) *auto*
from *FrQ* ‹*guarded Q*› **have** $\Psi_Q \simeq \mathbf{1}$ **and** *supp* $\Psi_Q = (\{\}::name\ set)$ **by**(*blast dest*: *guardedStatEq*)+
hence $A_R \sharp* \Psi_Q$ **and** $A_Q \sharp* \Psi_Q$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from ‹$\Psi \otimes \Psi_R \rhd !Q \longmapsto \tau \prec Q'$› ‹*guarded Q*›
obtain T **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \parallel Q \longmapsto \tau \prec T$ **and** $(\mathbf{1}, Q', T \parallel !Q) \in Rel'$
by(*blast dest*: *rBangTauE*)

from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)
hence $(\Psi \otimes \Psi_R, P \parallel P, Q \parallel Q) \in Rel$ **by**(*rule ParPres2*)

with *QTrans*
obtain S **where** *PTrans*: $\Psi \otimes \Psi_R \rhd P \parallel P \Longrightarrow\hat{}_\tau\ S$ **and** *SRelT*: $(\Psi \otimes \Psi_R,\ S,\ T) \in Rel$
by(*blast dest*: *cSim weakSimE*)
from *PTrans* ‹*guarded P*› **obtain** U **where** *PChain*: $\Psi \otimes \Psi_R \rhd !P \Longrightarrow\hat{}_\tau\ U$ **and** $(\Psi \otimes \Psi_R,\ U,\ S \parallel !P) \in Rel'$
by(*blast dest*: *rBangTauI*)
from *PChain* ‹$A_R \sharp* P$› **have** $A_R \sharp* U$ **by**(*force dest*: *tauChainFreshChain*)
from ‹$\Psi \otimes \Psi_R \rhd !P \Longrightarrow\hat{}_\tau\ U$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› **have** $\Psi \rhd R \parallel !P \Longrightarrow\hat{}_\tau\ R \parallel U$
by(*rule-tac tauChainPar2*) *auto*
moreover from *PTrans* ‹$A_R \sharp* P$› **have** $A_R \sharp* S$ **by**(*force dest*: *tauChainFreshChain*)
from *QTrans* ‹$A_R \sharp* Q$› **have** $A_R \sharp* T$ **by**(*force dest*: *tauFreshChainDerivative*)
have $(\Psi,\ R \parallel U,\ R \parallel Q') \in Rel''$
proof −
from ‹$(\Psi \otimes \Psi_R,\ U,\ S \parallel !P) \in Rel'$› *Rel'Rel* **have** $(\Psi \otimes \Psi_R,\ U,\ S \parallel !P) \in Rel$
by *auto*
hence $(\Psi,\ R \parallel U,\ R \parallel (S \parallel !P)) \in Rel$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* U$› ‹$A_R \sharp* S$› ‹$A_R \sharp* P$›
by(*rule-tac FrameParPres*) *auto*

moreover from ‹$(\Psi \otimes \Psi_R,\ S,\ T) \in Rel$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* S$› ‹$A_R \sharp* T$› **have** $(\Psi,\ R \parallel S,\ R \parallel T) \in Rel$
by(*rule-tac FrameParPres*) *auto*
hence $(\Psi,\ R \parallel T,\ R \parallel S) \in Rel$ **by**(*rule cSym*)
hence $(\Psi,\ (R \parallel T) \parallel !P,\ (R \parallel S) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi,\ (R \parallel S) \parallel !P,\ (R \parallel T) \parallel !P) \in Rel$ **by**(*rule cSym*)
hence $(\Psi,\ R \parallel (S \parallel !P),\ (R \parallel T) \parallel !P) \in Rel$ **by**(*metis Trans Assoc*)
ultimately have $(\Psi,\ R \parallel U,\ (R \parallel T) \parallel !P) \in Rel$ **by**(*rule Trans*)
moreover from ‹$(\Psi,\ P,\ Q) \in Rel$› **have** $(\Psi,\ (R \parallel T) \parallel !P,\ (R \parallel T) \parallel !Q) \in Rel''$ **using** ‹*guarded P*› ‹*guarded Q*› **by**(*rule C1*)
moreover from ‹$(\mathbf{1},\ Q',\ T \parallel !Q) \in Rel'$› **have** $(\mathbf{1} \otimes \Psi,\ Q',\ T \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi,\ Q',\ T \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence $(\Psi,\ R \parallel Q',\ R \parallel (T \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi,\ R \parallel Q',\ (R \parallel T) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi,\ (R \parallel T) \parallel !Q,\ R \parallel Q') \in Rel'$ **by**(*rule cSym'*)
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
next
case(*cComm1* $\Psi_Q\ M\ N\ R'\ A_R\ \Psi_R\ K\ xvec\ Q'\ A_Q$)
from ‹$A_R \sharp* (P,\ Q,\ R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **by** *simp+*
from ‹$xvec \sharp* (P,\ Q,\ R)$› **have** $xvec \sharp* P$ **and** $xvec \sharp* Q$ **and** $xvec \sharp* R$ **by** *simp+*

have *FrQ*: *extractFrame*$(!Q) = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$› *FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* N$› ‹$A_R \sharp* xvec$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* M$›
obtain ${A_R}' {\Psi_R}' \Psi'$ **where** *FrR*′: *extractFrame* $R' = \langle {A_R}', {\Psi_R}'\rangle$ **and** $\Psi_R \otimes \Psi' \simeq {\Psi_R}'$ **and** ${A_R}' \sharp* xvec$ **and** ${A_R}' \sharp* P$ **and** ${A_R}' \sharp* Q$ **and** ${A_R}' \sharp* \Psi$
by(*rule-tac* C=(Ψ, *xvec*, P, Q) **and** C'=(Ψ, *xvec*, P, Q) **in** *expandFrame*) *auto*
from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)
moreover from ‹$\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec Q'$› ‹*guarded* Q› ‹*xvec* $\sharp*$ Q› ‹*xvec* $\sharp*$ K›
obtain S **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec S$ **and** $(\mathbf{1}, Q', S \parallel !Q) \in Rel'$
by(*fastforce dest*: *rBangActE*)
ultimately obtain P' T **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow K(\!|\nu*xvec|\!)\langle N\rangle \prec P'$ **and** *P′Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ T$ **and** $((\Psi \otimes \Psi_R) \otimes \Psi', T, S) \in Rel$
using ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ Ψ_R› ‹*xvec* $\sharp*$ P›
by(*fastforce dest*: *cSim weakSimE*)

from *PTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* xvec$› ‹${A_R}' \sharp* P$› ‹${A_R}' \sharp* xvec$› ‹*xvec* $\sharp*$ K› ‹*distinct xvec*›
have $A_R \sharp* P'$ **and** ${A_R}' \sharp* P'$
by(*force dest*: *weakOutputFreshChainDerivative*)+
with *P′Chain* **have** ${A_R}' \sharp* T$ **by**(*force dest*: *tauChainFreshChain*)+
from *QTrans* ‹${A_R}' \sharp* Q$› ‹${A_R}' \sharp* xvec$› ‹*xvec* $\sharp*$ K› ‹*distinct xvec*›
have ${A_R}' \sharp* S$ **by**(*force dest*: *outputFreshChainDerivative*)

obtain ${A_Q}' {\Psi_Q}'$ **where** *FrQ*′: *extractFrame* $Q = \langle {A_Q}', {\Psi_Q}'\rangle$ **and** ${A_Q}' \sharp* \Psi$ **and** ${A_Q}' \sharp* \Psi_R$ **and** ${A_Q}' \sharp* A_R$ **and** ${A_Q}' \sharp* M$ **and** ${A_Q}' \sharp* R$ **and** ${A_Q}' \sharp* K$
by(*rule-tac* C=(Ψ, Ψ_R, A_R, K, M, R) **in** *freshFrame*) *auto*
from *FrQ*′ ‹*guarded* Q› **have** ${\Psi_Q}' \simeq \mathbf{1}$ **and** *supp* ${\Psi_Q}' = (\{\}::name\ set)$ **by**(*blast dest*: *guardedStatEq*)+
hence ${A_Q}' \sharp* {\Psi_Q}'$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau\ P''$
and *NilImpP″*: $\langle {A_Q}', (\Psi \otimes \Psi_R) \otimes {\Psi_Q}'\rangle \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') $(\Psi \otimes \Psi_R)$
and *P″Trans*: $\Psi \otimes \Psi_R \rhd P'' \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle \prec P'$
using *FrQ*′ ‹${A_Q}' \sharp* \Psi$› ‹${A_Q}' \sharp* \Psi_R$› *freshCompChain*
by(*drule-tac weakTransitionE*) *auto*

from *PChain* **have** $\Psi \rhd R \parallel !P \Longrightarrow\hat{}_\tau\ (\!|\nu*xvec|\!)(R' \parallel (P' \parallel !P))$
proof(*induct rule*: *tauChainCases*)
case *TauBase*
from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$› *FrQ* **have** $\Psi \otimes \mathbf{1} \rhd R \longmapsto M(\!|N|\!) \prec R'$ **by** *simp*
moreover note *FrR*
moreover from *P″Trans* ‹$P = P''$› **have** $\Psi \otimes \Psi_R \rhd P \longmapsto K(\!|\nu*xvec|\!)\langle N\rangle$

$\prec P'$ **by** *simp*
hence $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by**(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec (P' \parallel !P)$ **using** ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ Ψ_R› ‹*xvec* $\sharp*$ P›
by(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_R \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec (P' \parallel !P)$ **using** ‹*guarded* P› **by**(*rule Bang*)
moreover from ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› *FrQ* **have** $\Psi \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K$ **by** *simp*
ultimately have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (\!|\nu * xvec|\!)(R' \parallel (P' \parallel !P))$ **using** ‹A_R $\sharp*$ Ψ› ‹A_R $\sharp*$ R› ‹A_R $\sharp*$ P› ‹A_R $\sharp*$ M› ‹*xvec* $\sharp*$ R›
by(*force intro*: *Comm1*)
thus *?case* **by**(*rule tauActTauChain*)
next
case *TauStep*
obtain $A_P{''}$ $\Psi_P{''}$ **where** *FrP''*: *extractFrame* $P'' = \langle A_P{''}, \Psi_P{''}\rangle$ **and** $A_P{''}$ $\sharp*$ Ψ **and** $A_P{''}$ $\sharp*$ K **and** $A_P{''}$ $\sharp*$ Ψ_R **and** $A_P{''}$ $\sharp*$ R **and** $A_P{''}$ $\sharp*$ P'' **and** $A_P{''}$ $\sharp*$ P
and $A_P{''}$ $\sharp*$ A_R **and** *distinct* $A_P{''}$
by(*rule-tac* C=(Ψ, K, A_R, Ψ_R, R, P'', P) **in** *freshFrame*) *auto*
from *PChain* ‹A_R $\sharp*$ P› **have** A_R $\sharp*$ P'' **by**(*drule-tac tauChainFreshChain*) *auto*
with *FrP''* ‹$A_P{''}$ $\sharp*$ A_R› **have** A_R $\sharp*$ $\Psi_P{''}$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from ‹$\Psi \otimes \Psi_R \rhd P \Longrightarrow_\tau P''$› **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \Longrightarrow_\tau P''$ **by**(*rule tauStepChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \Longrightarrow_\tau P'' \parallel !P$ **by**(*rule-tac tauStepChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \rhd !P \Longrightarrow_\tau P'' \parallel !P$ **using** ‹*guarded* P› **by**(*rule tauStepChainBang*)
hence $\Psi \rhd R \parallel !P \Longrightarrow_\tau R \parallel (P'' \parallel !P)$ **using** *FrR* ‹A_R $\sharp*$ Ψ› ‹A_R $\sharp*$ P›
by(*rule-tac tauStepChainPar2*) *auto*
moreover have $\Psi \rhd R \parallel (P'' \parallel !P) \longmapsto \tau \prec (\!|\nu * xvec|\!)(R' \parallel (P' \parallel !P))$
proof −
from *FrQ* ‹$\Psi_Q{'} \simeq \mathbf{1}$› ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$› **have** $\Psi \otimes \Psi_Q{'} \rhd R \longmapsto M(\!|N|\!) \prec R'$
by *simp* (*metis statEqTransition AssertionStatEqSym compositionSym*)
moreover from *P''Trans* **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P'' \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence *P''PTrans*: $\Psi \otimes \Psi_R \rhd P'' \parallel !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec (P' \parallel !P)$ **using** ‹*xvec* $\sharp*$ P›
by(*rule-tac Par1*) *auto*
moreover from *FrP''* **have** *FrP''P*: *extractFrame*$(P'' \parallel !P) = \langle A_P{''}, \Psi_P{''} \otimes \mathbf{1}\rangle$
by *auto*
moreover from *FrQ* ‹$\Psi_Q{'} \simeq \mathbf{1}$› ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_Q{'} \otimes \Psi_R \vdash M \leftrightarrow K$
by *simp* (*metis statEqEnt Composition AssertionStatEqSym Commutativity*)

hence $\Psi \otimes \Psi_Q{}' \otimes \Psi_R \vdash K \leftrightarrow M$ **by**(*rule chanEqSym*)
moreover have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1})) \otimes \Psi_R\rangle$
proof −
have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \simeq_F \langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle$
by(*rule-tac frameResChainPres*, *simp*)
(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
moreover from *NilImpP'' FrQ FrP''* ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* \Psi_R$› *freshCompChain* **have** $\langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle$
by *auto*
moreover have $\langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle \simeq_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R\rangle$
by(*rule frameResChainPres*, *simp*)
(*metis Identity AssertionStatEqSym Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
ultimately obtain M' **where** *RTrans*: $\Psi \otimes \Psi_P{}'' \otimes \mathbf{1} \rhd R \longmapsto M'\langle\!|N|\!\rangle \prec R'$ **and** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$ **and** $A_R \sharp* M'$
using *FrR FrQ'* ‹*distinct* A_R› ‹*distinct* $A_P{}''$› ‹$A_P{}'' \sharp* A_R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_Q{}' \sharp* R$› ‹$A_Q{}' \sharp* K$› ‹$A_Q{}' \sharp* A_R$› ‹$A_R \sharp* P$› ‹$A_P{}'' \sharp* P$› ‹$A_R \sharp* xvec$›
‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* K$› ‹$xvec \sharp* K$› ‹*distinct xvec*›
by(*rule-tac* A_Q=$A_Q{}'$ **and** Q=Q **in** *comm2Aux*) (*assumption* | *simp*)+

note *RTrans FrR P''PTrans FrP''P*
moreover from ‹$\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$› **have** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash M' \leftrightarrow K$ **by**(*rule chanEqSym*)
hence $\Psi \otimes \Psi_R \otimes \Psi_P{}'' \otimes \mathbf{1} \vdash M' \leftrightarrow K$ **by**(*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
ultimately show *?thesis* **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* P$› ‹$A_R \sharp* M'$› ‹$A_P{}'' \sharp* A_R$› ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* P$› ‹$A_P{}'' \sharp* K$› ‹$xvec \sharp* R$›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
qed
ultimately show *?thesis*
by(*drule-tac tauActTauChain*) *auto*
qed

moreover from *P'Chain* **have** $((\Psi \otimes \Psi_R) \otimes \Psi') \otimes \mathbf{1} \rhd P' \Longrightarrow\hat{}_\tau\ T$
by(*rule tauChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$
by(*rule-tac tauChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \otimes \Psi' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$
by(*rule tauChainStatEq*) (*metis Associativity*)
hence $\Psi \otimes \Psi_R{}' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$ **using** ‹$\Psi_R \otimes \Psi' \simeq \Psi_R{}'$›
by(*rule-tac tauChainStatEq*) (*auto intro*: *compositionSym*)

hence $\Psi \rhd R' \parallel (P' \parallel !P) \Longrightarrow\hat{}_\tau\ R' \parallel (T \parallel !P)$ **using** *FrR'* ‹$A_R{}' \sharp* \Psi$› ‹$A_R{}' \sharp* P$› ‹$A_R{}' \sharp* P'$›
by(*rule-tac tauChainPar2*) *auto*
hence $\Psi \rhd (\!|\nu * xvec|\!)(R' \parallel (P' \parallel !P)) \Longrightarrow\hat{}_\tau\ (\!|\nu * xvec|\!)(R' \parallel (T \parallel !P))$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd R \parallel !P \Longrightarrow\hat{}_\tau\ (\!|\nu * xvec|\!)(R' \parallel (T \parallel !P))$
by *auto*
moreover have $(\Psi, (\!|\nu * xvec|\!)(R' \parallel (T \parallel !P)), (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel''$
proof −
from ‹$((\Psi \otimes \Psi_R) \otimes \Psi', T, S) \in Rel$› **have** $(\Psi \otimes \Psi_R \otimes \Psi', T, S) \in Rel$
by(*rule StatEq*) (*metis Associativity*)
hence $(\Psi \otimes \Psi_R{}', T, S) \in Rel$ **using** ‹$\Psi_R \otimes \Psi' \simeq \Psi_R{}'$›
by(*rule-tac StatEq*) (*auto dest*: *compositionSym*)

with *FrR'* ‹$A_R{}' \sharp* \Psi$› ‹$A_R{}' \sharp* S$› ‹$A_R{}' \sharp* T$› **have** $(\Psi, R' \parallel T, R' \parallel S) \in Rel$
by(*rule-tac FrameParPres*) *auto*
hence $(\Psi, (R' \parallel T) \parallel !P, (R' \parallel S) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel T) \parallel !P), (\!|\nu * xvec|\!)((R' \parallel S) \parallel !P)) \in Rel$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*rule ResPres*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel T) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel S)) \parallel !P) \in Rel$ **using** ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *P*›
by(*force intro*: *Trans ScopeExt*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel (T \parallel !P)), ((\!|\nu * xvec|\!)(R' \parallel S)) \parallel !P) \in Rel$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*force intro*: *Trans ResPres Assoc*)

moreover from ‹$(\Psi, P, Q) \in Rel$› ‹*guarded P*› ‹*guarded Q*› **have** $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel S)) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel S)) \parallel !Q) \in Rel''$
by(*rule C1*)
moreover from ‹$(\mathbf{1}, Q', S \parallel !Q) \in Rel'$› **have** $(\mathbf{1} \otimes \Psi, Q', S \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi, Q', S \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence $(\Psi, R' \parallel Q', R' \parallel (S \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi, R' \parallel Q', (R' \parallel S) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi, (R' \parallel S) \parallel !Q, R' \parallel Q') \in Rel'$ **by**(*rule cSym'*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel S) \parallel !Q), (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*rule ResPres'*)
hence $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel S)) \parallel !Q, (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ *Q*›
by(*force intro*: *Trans' ScopeExt'*[*THEN cSym'*])
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
next

case(*cComm2* Ψ_Q M *xvec* N R' A_R Ψ_R K Q' A_Q)
from ‹$A_R \sharp* (P, Q, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **by** *simp*+
from ‹*xvec* $\sharp*$ (P, Q, R)› **have** *xvec* $\sharp*$ P **and** *xvec* $\sharp*$ Q **and** *xvec* $\sharp*$ R **by** *simp*+
have *FrQ*: *extractFrame*$(!Q) = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec R'$› *FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* N$› ‹$A_R \sharp* xvec$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* \Psi$› ‹*xvec* $\sharp*$ R› ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ P› ‹*xvec* $\sharp*$ Q› ‹*xvec* $\sharp*$ M› ‹*distinct xvec*› ‹$A_R \sharp* M$›
obtain p A_R' Ψ_R' Ψ' **where** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R'\rangle$ **and** $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* xvec$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q$ **and** $A_R' \sharp* \Psi$ **and** S: *set* $p \subseteq$ *set xvec* $\times$ *set*$(p \cdot xvec)$ **and** *distinctPerm* p **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* R'$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* \Psi$ **and** $A_R' \sharp* N$ **and** $A_R' \sharp* xvec$ **and** $A_R' \sharp* (p \cdot xvec)$
by(*rule-tac* C=(Ψ, P, Q) **and** C'=(Ψ, P, Q) **in** *expandFrame*) (*assumption* | *simp*)+

from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec R'$› S ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* R'$›
have *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu * (p \cdot xvec)|\!)\langle (p \cdot N)\rangle \prec (p \cdot R')$
by(*simp add*: *boundOutputChainAlpha''* *residualInject*)
from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)
moreover from ‹$\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|N|\!) \prec Q'$› S ‹$(p \cdot xvec) \sharp* Q$› ‹*xvec* $\sharp*$ Q› ‹*distinctPerm p*›
have $\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|(p \cdot N)|\!) \prec (p \cdot Q')$
by(*rule-tac inputAlpha*) *auto*
then obtain S **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|(p \cdot N)|\!) \prec S$ **and** $(\mathbf{1}, (p \cdot Q'), S \parallel !Q) \in Rel'$
using ‹*guarded Q*›
by(*fastforce dest*: *rBangActE*)
ultimately obtain P' T **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow K(\!|(p \cdot N)|\!) \prec P'$
and *P'Chain*: $(\Psi \otimes \Psi_R) \otimes (p \cdot \Psi') \rhd P' \Longrightarrow\hat{}_\tau\ T$
and $((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'), T, S) \in Rel$
by(*fastforce dest*: *cSim weakSimE*)

from ‹$A_R' \sharp* N$› ‹$A_R' \sharp* xvec$› ‹$A_R' \sharp* (p \cdot xvec)$› S **have** $A_R' \sharp* (p \cdot N)$
by(*simp add*: *freshChainSimps*)
with *PTrans* ‹$A_R' \sharp* P$› **have** $A_R' \sharp* P'$ **by**(*force dest*: *weakInputFreshChainDerivative*)
with *P'Chain* **have** $A_R' \sharp* T$ **by**(*force dest*: *tauChainFreshChain*)+
from *QTrans* ‹$A_R' \sharp* Q$› ‹$A_R' \sharp* (p \cdot N)$› **have** $A_R' \sharp* S$ **by**(*force dest*: *inputFreshChainDerivative*)

obtain A_Q' Ψ_Q' **where** *FrQ'*: *extractFrame* $Q = \langle A_Q', \Psi_Q'\rangle$ **and** $A_Q' \sharp* \Psi$ **and** $A_Q' \sharp* \Psi_R$ **and** $A_Q' \sharp* A_R$ **and** $A_Q' \sharp* M$ **and** $A_Q' \sharp* R$ **and** $A_Q' \sharp* K$
by(*rule-tac* C=$(\Psi, \Psi_R, A_R, K, M, R)$ **in** *freshFrame*) *auto*
from *FrQ'* ‹*guarded Q*› **have** $\Psi_Q' \simeq \mathbf{1}$ **and** *supp* $\Psi_Q' = (\{\}$::*name set*) **by**(*blast dest*: *guardedStatEq*)+

hence $A_Q' \sharp* \Psi_Q'$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau P''$
and *NilImpP''*: $\langle A_Q', (\Psi \otimes \Psi_R) \otimes \Psi_Q' \rangle \hookrightarrow_F insertAssertion\ (extractFrame\ P'')\ (\Psi \otimes \Psi_R)$
and *P''Trans*: $\Psi \otimes \Psi_R \rhd P'' \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
using *FrQ'* ‹$A_Q' \sharp* \Psi$› ‹$A_Q' \sharp* \Psi_R$› *freshCompChain*
by(*drule-tac weakTransitionE*) *auto*

from ‹$(p \cdot xvec) \sharp* P$› ‹$xvec \sharp* P$› *PChain* **have** $(p \cdot xvec) \sharp* P''$ **and** $xvec \sharp* P''$
by(*force dest*: *tauChainFreshChain*)+
from ‹$(p \cdot xvec) \sharp* N$› ‹*distinctPerm p*› **have** $xvec \sharp* (p \cdot N)$
by(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=*p*, *symmetric*]) *simp*
with *P''Trans* ‹$xvec \sharp* P''$› **have** $xvec \sharp* P'$ **by**(*force dest*: *inputFreshChainDerivative*)
hence $(p \cdot xvec) \sharp* (p \cdot P')$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

from *PChain* **have** $\Psi \rhd R \parallel !P \Longrightarrow\hat{}_\tau (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$
proof(*induct rule*: *tauChainCases*)
case *TauBase*
from *RTrans FrQ* **have** $\Psi \otimes \mathbf{1} \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$ **by** *simp*
moreover note *FrR*
moreover from *P''Trans* ‹$P = P''$› **have** $\Psi \otimes \Psi_R \rhd P \longmapsto K(\!|(p \cdot N)|\!) \prec P'$ **by** *simp*
hence $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$ **by**(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_R \rhd !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover from ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› *FrQ* **have** $\Psi \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K$ **by** *simp*
ultimately have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$ **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P$› ‹$A_R \sharp* M$› ‹$(p \cdot xvec) \sharp* P$›
by(*force intro*: *Comm2*)
thus *?case* **by**(*rule tauActTauChain*)
next
case *TauStep*
obtain A_P'' Ψ_P'' **where** *FrP''*: $extractFrame\ P'' = \langle A_P'', \Psi_P'' \rangle$ **and** $A_P'' \sharp* \Psi$ **and** $A_P'' \sharp* K$ **and** $A_P'' \sharp* \Psi_R$ **and** $A_P'' \sharp* R$ **and** $A_P'' \sharp* P''$ **and** $A_P'' \sharp* P$
and $A_P'' \sharp* A_R$ **and** *distinct* A_P''
by(*rule-tac C*=(Ψ, K, A_R, Ψ_R, R, P'', P) **in** *freshFrame*) *auto*
from *PChain* ‹$A_R \sharp* P$› **have** $A_R \sharp* P''$ **by**(*drule-tac tauChainFreshChain*) *auto*
with *FrP''* ‹$A_P'' \sharp* A_R$› **have** $A_R \sharp* \Psi_P''$ **by**(*drule-tac extractFrame-*

FreshChain) *auto*
from ‹$\Psi \otimes \Psi_R \rhd P \Longrightarrow_\tau P''$› **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \Longrightarrow_\tau P''$ **by**(*rule tauStepChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \Longrightarrow_\tau P'' \parallel !P$ **by**(*rule-tac tauStepChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \rhd !P \Longrightarrow_\tau P'' \parallel !P$ **using** ‹*guarded P*› **by**(*rule tauStepChainBang*)
hence $\Psi \rhd R \parallel !P \Longrightarrow_\tau R \parallel (P'' \parallel !P)$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$›
by(*rule-tac tauStepChainPar2*) *auto*
moreover have $\Psi \rhd R \parallel (P'' \parallel !P) \longmapsto\tau \prec (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$
proof −
from *FrQ* ‹$\Psi_Q{}' \simeq \mathbf{1}$› *RTrans* **have** $\Psi \otimes \Psi_Q{}' \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$
by *simp* (*metis statEqTransition AssertionStatEqSym compositionSym*)
moreover from P''*Trans* **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P'' \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence P''*PTrans*: $\Psi \otimes \Psi_R \rhd P'' \parallel !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$
by(*rule-tac Par1*) *auto*
moreover from *FrP''* **have** *FrP''P*: *extractFrame*$(P'' \parallel !P) = \langle A_P{}'', \Psi_P{}'' \otimes \mathbf{1}\rangle$
by *auto*
moreover from *FrQ* ‹$\Psi_Q{}' \simeq \mathbf{1}$› ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_Q{}' \otimes \Psi_R \vdash M \leftrightarrow K$
by *simp* (*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
hence $\Psi \otimes \Psi_Q{}' \otimes \Psi_R \vdash K \leftrightarrow M$ **by**(*rule chanEqSym*)
moreover have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1})) \otimes \Psi_R\rangle$
proof −
have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \simeq_F \langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle$
by(*rule-tac frameResChainPres*, *simp*)
(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
moreover from *NilImpP''* *FrQ* *FrP''* ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* \Psi_R$› *freshCompChain* **have** $\langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle$
by *auto*
moreover have $\langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle \simeq_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R\rangle$
by(*rule frameResChainPres*, *simp*)
(*metis Identity AssertionStatEqSym Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
ultimately obtain M' **where** *RTrans*: $\Psi \otimes \Psi_P{}'' \otimes \mathbf{1} \rhd R \longmapsto M'(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$ **and** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$ **and** $A_R \sharp* M'$
using *FrR* *FrQ'* ‹*distinct* A_R› ‹*distinct* $A_P{}''$› ‹$A_P{}'' \sharp* A_R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_Q{}' \sharp* R$› ‹$A_Q{}' \sharp* K$› ‹$A_Q{}' \sharp* A_R$› ‹$A_R \sharp* P$› ‹$A_P{}'' \sharp* P$›
‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* K$›
by(*rule-tac* $A_Q{=}A_Q{}'$ **and** $Q{=}Q$ **in** *comm1Aux*) (*assumption* | *simp*)+

note *RTrans FrR P''PTrans FrP''P*

moreover from ‹$\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$› **have** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash M' \leftrightarrow K$ **by**(*rule chanEqSym*)

hence $\Psi \otimes \Psi_R \otimes \Psi_P{}'' \otimes \mathbf{1} \vdash M' \leftrightarrow K$ **by**(*metis statEqEnt Composition AssertionStatEqSym Commutativity*)

ultimately show *?thesis* **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* P$› ‹$A_R \sharp* M'$› ‹$A_P{}'' \sharp* A_R$› ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* P$› ‹$A_P{}'' \sharp* K$› ‹$(p \cdot xvec) \sharp* P''$› ‹$(p \cdot xvec) \sharp* P$›

by(*rule-tac Comm2*) (*assumption* | *simp*)+

qed

ultimately show *?thesis*

by(*drule-tac tauActTauChain*) *auto*

qed

hence $\Psi \rhd R \parallel !P \Longrightarrow^{\hat{}}_{\tau} (\!|\nu* xvec|\!)(R' \parallel ((p \cdot P') \parallel !P))$

using ‹$xvec \sharp* P$› ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* (p \cdot P')$› ‹$(p \cdot xvec) \sharp* R'$› S ‹*distinctPerm p*›

by(*subst resChainAlpha*[**where** *p*=*p*]) *auto*

moreover from *P'Chain* **have** $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'))) \rhd (p \cdot P') \Longrightarrow^{\hat{}}_{\tau} (p \cdot T)$

by(*rule tauChainEqvt*)

with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S ‹*distinctPerm p*›

have $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd (p \cdot P') \Longrightarrow^{\hat{}}_{\tau} (p \cdot T)$ **by**(*simp add*: *eqvts*)

hence $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi') \otimes \mathbf{1} \rhd (p \cdot P') \Longrightarrow^{\hat{}}_{\tau} (p \cdot T)$

by(*rule tauChainStatEq*) (*metis Identity AssertionStatEqSym*)

hence $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd (p \cdot P') \parallel !P \Longrightarrow^{\hat{}}_{\tau} (p \cdot T) \parallel !P$

by(*rule-tac tauChainPar1*) *auto*

hence $\Psi \otimes (p \cdot \Psi_R) \otimes \Psi' \rhd (p \cdot P') \parallel !P \Longrightarrow^{\hat{}}_{\tau} (p \cdot T) \parallel !P$

by(*rule tauChainStatEq*) (*metis Associativity*)

hence $\Psi \otimes \Psi_R{}' \rhd (p \cdot P') \parallel !P \Longrightarrow^{\hat{}}_{\tau} (p \cdot T) \parallel !P$ **using** ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R{}'$›

by(*rule-tac tauChainStatEq*) (*auto intro*: *compositionSym*)

hence $\Psi \rhd R' \parallel ((p \cdot P') \parallel !P) \Longrightarrow^{\hat{}}_{\tau} R' \parallel ((p \cdot T) \parallel !P)$

using *FrR'* ‹$A_R{}' \sharp* \Psi$› ‹$A_R{}' \sharp* P$› ‹$A_R{}' \sharp* P'$› ‹$A_R{}' \sharp* xvec$› ‹$A_R{}' \sharp* (p \cdot xvec)$› S

by(*rule-tac tauChainPar2*) (*auto simp add*: *freshChainSimps*)

hence $\Psi \rhd (\!|\nu* xvec|\!)(R' \parallel ((p \cdot P') \parallel !P)) \Longrightarrow^{\hat{}}_{\tau} (\!|\nu* xvec|\!)(R' \parallel ((p \cdot T) \parallel !P))$ **using** ‹$xvec \sharp* \Psi$›

by(*rule tauChainResChainPres*)

ultimately have $\Psi \rhd R \parallel !P \Longrightarrow^{\hat{}}_{\tau} (\!|\nu* xvec|\!)(R' \parallel ((p \cdot T) \parallel !P))$

by *auto*

moreover have $(\Psi, (\!|\nu* xvec|\!)(R' \parallel ((p \cdot T) \parallel !P)), (\!|\nu* xvec|\!)(R' \parallel Q')) \in Rel''$

proof −

from ‹$((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'), T, S) \in Rel$›

have $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi')), (p \cdot T), (p \cdot S)) \in Rel$

by(*rule Closed*)

with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› ‹*distinctPerm p*› S

have $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', p \cdot T, p \cdot S) \in Rel$

by(*simp add*: *eqvts*)

hence $(\Psi \otimes (p \cdot \Psi_R) \otimes \Psi', p \cdot T, p \cdot S) \in Rel$
by(*rule StatEq*) (*metis Associativity*)
hence $(\Psi \otimes \Psi_R', p \cdot T, p \cdot S) \in Rel$ **using** $\langle (p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R' \rangle$
by(*rule-tac StatEq*) (*auto dest*: *compositionSym*)
moreover from $\langle A_R' \sharp* S \rangle$ $\langle A_R' \sharp* T \rangle$ $\langle A_R' \sharp* xvec \rangle$ $\langle A_R' \sharp* (p \cdot xvec) \rangle$ *S*
have $A_R' \sharp* (p \cdot S)$ **and** $A_R' \sharp* (p \cdot T)$
by(*simp add*: *freshChainSimps*)+
ultimately have $(\Psi, R' \parallel (p \cdot T), R' \parallel (p \cdot S)) \in Rel$ **using** *FrR'* $\langle A_R' \sharp* \Psi \rangle$
by(*rule-tac FrameParPres*) *auto*
hence $(\Psi, (R' \parallel (p \cdot T)) \parallel !P, (R' \parallel (p \cdot S)) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel (p \cdot T)) \parallel !P), (\!|\nu * xvec|\!)((R' \parallel (p \cdot S)) \parallel !P)) \in Rel$
using $\langle xvec \sharp* \Psi \rangle$
by(*rule ResPres*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel (p \cdot T)) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P) \in Rel$
using $\langle xvec \sharp* \Psi \rangle$ $\langle xvec \sharp* P \rangle$
by(*force intro*: *Trans ScopeExt*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel ((p \cdot T) \parallel !P)), ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P) \in Rel$
using $\langle xvec \sharp* \Psi \rangle$
by(*force intro*: *Trans ResPres Assoc*)
moreover from $\langle (\Psi, P, Q) \in Rel \rangle$ $\langle guarded\ P \rangle$ $\langle guarded\ Q \rangle$
have $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !Q) \in Rel''$
by(*rule C1*)
moreover from $\langle (\mathbf{1}, (p \cdot Q'), S \parallel !Q) \in Rel' \rangle$
have $(p \cdot \mathbf{1}, p \cdot p \cdot Q', p \cdot (S \parallel !Q)) \in Rel'$ **by**(*rule Closed'*)
with $\langle xvec \sharp* Q \rangle$ $\langle (p \cdot xvec) \sharp* Q \rangle$ *S* $\langle distinctPerm\ p \rangle$
have $(\mathbf{1}, Q', (p \cdot S) \parallel !Q) \in Rel'$ **by**(*simp add*: *eqvts*)
hence $(\mathbf{1} \otimes \Psi, Q', (p \cdot S) \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi, Q', (p \cdot S) \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence $(\Psi, R' \parallel Q', R' \parallel ((p \cdot S) \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi, R' \parallel Q', (R' \parallel (p \cdot S)) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi, (R' \parallel (p \cdot S)) \parallel !Q, R' \parallel Q') \in Rel'$ **by**(*rule cSym'*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel (p \cdot S)) \parallel !Q), (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** $\langle xvec \sharp* \Psi \rangle$
by(*rule ResPres'*)
hence $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !Q, (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** $\langle xvec \sharp* \Psi \rangle$ $\langle xvec \sharp* Q \rangle$
by(*force intro*: *Trans' ScopeExt'*[*THEN cSym'*])
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
qed
qed
notation *relcomp* (**infixr** *O 75*)
end

end

theory *Weak-Stat-Imp-Pres*
imports *Weak-Stat-Imp*
begin

context *env* **begin**

lemma *weakStatImpInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes *PRelQ*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', M(\!|\lambda * xvec\ N|\!).P, M(\!|\lambda * xvec\ N|\!).Q) \in Rel$

shows $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \lessapprox <Rel> M(\!|\lambda * xvec\ N|\!).Q$
using *assms*
by(*fastforce simp add*: *weakStatImp-def*)

lemma *weakStatImpOutputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$

assumes *PRelQ*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', M\langle N\rangle.P, M\langle N\rangle.Q) \in Rel$

shows $\Psi \rhd M\langle N\rangle.P \lessapprox <Rel> M\langle N\rangle.Q$
using *assms*
by(*fastforce simp add*: *weakStatImp-def*)

lemma *weakStatImpResPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and x :: *name*
and *Rel'* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*

assumes *PSimQ*: $\Psi \rhd P \lessapprox <Rel> Q$
and *eqvt Rel*
and $x \sharp \Psi$

and *C1*: $\bigwedge \Psi'\ R\ S\ y.\ [\![(\Psi', R, S) \in Rel;\ y \sharp \Psi']\!] \Longrightarrow (\Psi', (\!|\nu y|\!)R, (\!|\nu y|\!)S) \in Rel'$

shows $\Psi \rhd (\!|\nu x|\!)P \lessapprox_{<Rel'>} (\!|\nu x|\!)Q$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp \Psi'$ **and** $y \sharp P$ **and** $y \sharp Q$ **by**(*generate-fresh name*) *auto*
from ‹*eqvt Rel*› ‹$\Psi \rhd P \lessapprox_{<Rel>} Q$› **have** $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \lessapprox_{<Rel>} ([(x, y)] \cdot Q)$ **by**(*rule weakStatImpClosed*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \rhd ([(x, y)] \cdot P) \lessapprox_{<Rel>} ([(x, y)] \cdot Q)$ **by** *simp*
then obtain Q' Q'' **where** *QChain*: $\Psi \rhd ([(x, y)] \cdot Q) \Longrightarrow\hat{}_\tau\ Q'$
and *PimpQ'*: *insertAssertion* (*extractFrame* ($[(x, y)] \cdot P$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q') Ψ
and *Q'Chain*: $\Psi \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''$ **and** $(\Psi \otimes \Psi', ([(x, y)] \cdot P), Q'') \in Rel$
by(*rule weakStatImpE*)
from *QChain* ‹$y \sharp \Psi$› **have** $\Psi \rhd (\!|\nu y|\!)([(x, y)] \cdot Q) \Longrightarrow\hat{}_\tau\ (\!|\nu y|\!)Q'$ **by**(*rule tauChainResPres*)
with ‹$y \sharp Q$› **have** $\Psi \rhd (\!|\nu x|\!)Q \Longrightarrow\hat{}_\tau\ (\!|\nu y|\!)Q'$ **by**(*simp add*: *alphaRes*)
moreover from *PimpQ'* ‹$y \sharp \Psi$› **have** *insertAssertion* (*extractFrame*($(\!|\nu y|\!)([(x, y)] \cdot P)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($(\!|\nu y|\!)Q'$)) Ψ
by(*force intro*: *frameImpResPres*)
with ‹$y \sharp P$› **have** *insertAssertion* (*extractFrame*($(\!|\nu x|\!)P$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($(\!|\nu y|\!)Q'$)) Ψ
by(*simp add*: *alphaRes*)
moreover from *Q'Chain* ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** $\Psi \otimes \Psi' \rhd (\!|\nu y|\!)Q' \Longrightarrow\hat{}_\tau\ (\!|\nu y|\!)Q''$ **by**(*rule-tac tauChainResPres*) *auto*
moreover from ‹$(\Psi \otimes \Psi', ([(x, y)] \cdot P), Q'') \in Rel$› ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu y|\!)([(x, y)] \cdot P), (\!|\nu y|\!)Q'') \in Rel'$
by(*blast intro*: *C1*)
with ‹$y \sharp P$› **have** $(\Psi \otimes \Psi', (\!|\nu x|\!)P, (\!|\nu y|\!)Q'') \in Rel'$ **by**(*simp add*: *alphaRes*)
ultimately show *?case*
by *blast*
qed

lemma *weakStatImpParPres*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *R* :: (*'a*, *'b*, *'c*) *psi*
and *Rel'* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

assumes *PStatImpQ*: $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R\rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q]\!] \Longrightarrow \Psi \otimes \Psi_R \rhd P \lessapprox_{<Rel>} Q$
and *xvec* $\sharp* \Psi$
and *Eqvt*: *eqvt Rel*

and *C1*: $\bigwedge \Psi'\ S\ T\ A_U\ \Psi_U\ U.\ [\![(\Psi' \otimes \Psi_U,\ S,\ T) \in Rel;\ extractFrame\ U = \langle A_U,\ \Psi_U\rangle;\ A_U \sharp* \Psi';\ A_U \sharp* S;\ A_U \sharp* T]\!] \Longrightarrow (\Psi',\ S \parallel U,\ T \parallel U) \in Rel'$
and *C2*: $\bigwedge \Psi'\ S\ T\ yvec.\ [\![(\Psi',\ S,\ T) \in Rel';\ yvec \sharp* \Psi']\!] \Longrightarrow (\Psi',\ (\!|\nu* yvec|\!) S,\ (\!|\nu* yvec|\!)\ T) \in Rel'$
and *C3*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi',\ S,\ T) \in Rel;\ \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'',\ S,\ T) \in Rel$

shows $\Psi \rhd (\!|\nu* xvec|\!)(P \parallel R) \lessapprox_{<Rel'>} (\!|\nu* xvec|\!)(Q \parallel R)$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
obtain $A_R\ \Psi_R$ **where** *FrR*: $extractFrame\ R = \langle A_R,\ \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* \Psi'$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **and** $A_R \sharp* xvec$
by(*rule-tac* F=*extractFrame* R **and** C=($xvec$, Ψ, Ψ', P, Q, R, $xvec$) **in** *freshFrame*) *auto*

hence $\Psi \otimes \Psi_R \rhd P \lessapprox_{<Rel>} Q$ **by**(*rule-tac PStatImpQ*)

obtain p::*name prm* **where** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* \Psi'$ **and** $(p \cdot xvec) \sharp* \Psi_R$
and $(p \cdot xvec) \sharp* A_R$ **and** $(p \cdot xvec) \sharp* R$ **and** $(p \cdot xvec) \sharp* P$
and *distinctPerm p* **and** *S*: *set* $p \subseteq$ *set* $xvec \times$ *set* $(p \cdot xvec)$
by(*rule-tac* c=(P, Q, R, Ψ, Ψ', A_R, Ψ_R, P) **in** *name-list-avoiding*) *auto*

from *FrR* **have** $(p \cdot extractFrame\ R) = (p \cdot \langle A_R,\ \Psi_R\rangle)$ **by**(*rule pt-bij3*)
with ‹$A_R \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_R$› S **have** *FrpR*: $extractFrame(p \cdot R) = \langle A_R,\ p \cdot \Psi_R\rangle$ **by**(*simp add*: *eqvts*)
from ‹*eqvt Rel*› ‹$\Psi \otimes \Psi_R \rhd P \lessapprox_{<Rel>} Q$› **have** $(p \cdot (\Psi \otimes \Psi_R)) \rhd (p \cdot P) \lessapprox_{<Rel>} (p \cdot Q)$ **by**(*rule weakStatImpClosed*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S **have** $\Psi \otimes (p \cdot \Psi_R) \rhd (p \cdot P) \lessapprox_{<Rel>} (p \cdot Q)$ **by**(*simp add*: *eqvts*)
then obtain $Q'\ Q''$ **where** *QChain*: $\Psi \otimes (p \cdot \Psi_R) \rhd (p \cdot Q) \Longrightarrow\hat{}_\tau\ Q'$
and *PimpQ'*: $insertAssertion\ (extractFrame\ (p \cdot P))\ (\Psi \otimes (p \cdot \Psi_R)) \hookrightarrow_F insertAssertion\ (extractFrame\ Q')\ (\Psi \otimes (p \cdot \Psi_R))$
and *Q'Chain*: $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''$ **and** $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi',\ (p \cdot P),\ Q'') \in Rel$
by(*rule weakStatImpE*)

from ‹$A_R \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_R$› ‹$A_R \sharp* Q$› S **have** $A_R \sharp* (p \cdot Q)$ **by**(*simp add*: *freshChainSimps*)
moreover from ‹$(p \cdot xvec) \sharp* Q$› **have** $(p \cdot p \cdot xvec) \sharp* (p \cdot Q)$
by(*simp only*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
hence $xvec \sharp* (p \cdot Q)$ **using** ‹*distinctPerm p*› **by** *simp*
ultimately have $A_R \sharp* Q'$ **and** $A_R \sharp* Q''$ **and** $xvec \sharp* Q'$ **and** $xvec \sharp* Q''$ **using** *QChain Q'Chain*
by(*metis tauChainFreshChain*)+

obtain $A_P\ \Psi_P$ **where** *FrP*: $extractFrame(p \cdot P) = \langle A_P,\ \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ $A_P \sharp* \Psi'$ **and** $A_P \sharp* A_R$ **and** $A_P \sharp* (p \cdot \Psi_R)$
by(*rule-tac* C=(Ψ, Ψ', A_R, $p \cdot \Psi_R$) **in** *freshFrame*) *auto*
obtain $A_Q'\ \Psi_Q'$ **where** *FrQ'*: $extractFrame\ Q' = \langle A_Q',\ \Psi_Q'\rangle$ **and** $A_Q' \sharp* \Psi$**and**

$A_Q{}' \sharp* \Psi'$ **and** $A_Q{}' \sharp* A_R$ **and** $A_Q{}' \sharp* (p \cdot \Psi_R)$
by(*rule-tac* C=(Ψ, Ψ', A_R, $p \cdot \Psi_R$) **in** *freshFrame*) *auto*
from ‹$A_R \sharp* xvec$› ‹$(p \cdot xvec) \sharp* A_R$› ‹$A_R \sharp* P$› S **have** $A_R \sharp* (p \cdot P)$ **by**(*simp add*: *freshChainSimps*)
with ‹$A_R \sharp* Q'$› ‹$A_P \sharp* A_R$› ‹$A_Q{}' \sharp* A_R$› *FrP FrQ'* **have** $A_R \sharp* \Psi_P$ **and** $A_R \sharp* \Psi_Q{}'$
by(*force dest*: *extractFrameFreshChain*)+

from *QChain FrpR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* (p \cdot Q)$› **have** $\Psi \rhd (p \cdot Q) \parallel (p \cdot R) \Longrightarrow\hat{}_\tau\ Q' \parallel (p \cdot R)$ **by**(*rule tauChainPar1*)
hence $(p \cdot \Psi) \rhd (p \cdot ((p \cdot Q) \parallel (p \cdot R))) \Longrightarrow\hat{}_\tau\ p \cdot (Q' \parallel (p \cdot R))$ **by**(*rule eqvts*)
with ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S ‹*distinctPerm p*› **have** $\Psi \rhd Q \parallel R \Longrightarrow\hat{}_\tau (p \cdot Q') \parallel R$ **by**(*simp add*: *eqvts*)
hence $\Psi \rhd (\!|\nu*xvec|\!)(Q \parallel R) \Longrightarrow\hat{}_\tau (\!|\nu*xvec|\!)((p \cdot Q') \parallel R)$ **using** ‹*xvec* $\sharp* \Psi$› **by**(*rule tauChainResChainPres*)
moreover have $\langle (A_P @ A_R), \Psi \otimes \Psi_P \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle (A_Q{}' @ A_R), \Psi \otimes \Psi_Q{}' \otimes (p \cdot \Psi_R)\rangle$
proof −
have $\langle A_P, \Psi \otimes \Psi_P \otimes (p \cdot \Psi_R)\rangle \simeq_F \langle A_P, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P\rangle$
by(*metis frameResChainPres frameNilStatEq Associativity Commutativity AssertionStatEqTrans Composition*)
moreover with *FrP FrQ' PimpQ'* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* (p \cdot \Psi_R)$› ‹$A_Q{}' \sharp* \Psi$› ‹$A_Q{}' \sharp* (p \cdot \Psi_R)$›
have $\langle A_P, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P\rangle \hookrightarrow_F \langle A_Q{}', (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q{}'\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle A_Q{}', (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q{}'\rangle \simeq_F \langle A_Q{}', \Psi \otimes \Psi_Q{}' \otimes (p \cdot \Psi_R)\rangle$
by(*metis frameResChainPres frameNilStatEq Associativity Commutativity AssertionStatEqTrans Composition*)
ultimately have $\langle A_P, \Psi \otimes \Psi_P \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle A_Q{}', \Psi \otimes \Psi_Q{}' \otimes (p \cdot \Psi_R)\rangle$
by(*rule FrameStatEqImpCompose*)
hence $\langle (A_R @ A_P), \Psi \otimes \Psi_P \otimes (p \cdot \Psi_R)\rangle \hookrightarrow_F \langle (A_R @ A_Q{}'), \Psi \otimes \Psi_Q{}' \otimes (p \cdot \Psi_R)\rangle$
by(*drule-tac frameImpResChainPres*) (*simp add*: *frameChainAppend*)
thus *?thesis*
apply(*simp add*: *frameChainAppend*)
by(*metis frameImpChainComm FrameStatImpTrans*)
qed
with *FrP FrpR FrQ'* ‹$A_P \sharp* A_R$› ‹$A_P \sharp* (p \cdot \Psi_R)$› ‹$A_Q{}' \sharp* A_R$› ‹$A_Q{}' \sharp* (p \cdot \Psi_R)$› ‹$A_R \sharp* \Psi_P$› ‹$A_R \sharp* \Psi_Q{}'$›
‹$A_P \sharp* \Psi$› ‹$A_Q{}' \sharp* \Psi$› ‹$A_R \sharp* \Psi$›
have *insertAssertion*(*extractFrame*($(p \cdot P) \parallel (p \cdot R)$)) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame*($Q' \parallel (p \cdot R)$)) Ψ
by *simp*
hence ($p \cdot$ *insertAssertion*(*extractFrame*($(p \cdot P) \parallel (p \cdot R)$)) Ψ) $\hookrightarrow_F$ ($p \cdot$ *insertAssertion*(*extractFrame*($Q' \parallel (p \cdot R)$)) Ψ)
by(*rule FrameStatImpClosed*)
with ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S ‹*distinctPerm p*›
have *insertAssertion*(*extractFrame*($P \parallel R$)) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame*((*p*

$\cdot$ Q') $\parallel$ R)) Ψ
by(*simp add*: *eqvts*)
with ‹*xvec* $\sharp*$ Ψ› **have** *insertAssertion*(*extractFrame*($\langle\!|\nu*xvec|\!\rangle$($P$ $\parallel$ R))) Ψ $\hookrightarrow_F$ *insertAssertion*(*extractFrame*($\langle\!|\nu*xvec|\!\rangle$(($p$ $\cdot$ Q') $\parallel$ R))) Ψ
by(*force intro*: *frameImpResChainPres*)
moreover from *Q'Chain* **have** (Ψ $\otimes$ Ψ') $\otimes$ (p $\cdot$ Ψ_R) $\rhd$ Q' $\Longrightarrow\hat{}_\tau$ Q''
by(*rule tauChainStatEq*) (*metis Associativity AssertionStatEqTrans Commutativity Composition*)
hence Ψ $\otimes$ Ψ' $\rhd$ Q' $\parallel$ (p $\cdot$ R) $\Longrightarrow\hat{}_\tau$ Q'' $\parallel$ (p $\cdot$ R) **using** *FrpR* ‹A_R $\sharp*$ Ψ› ‹A_R $\sharp*$ Ψ'› ‹A_R $\sharp*$ Q'› ‹A_R $\sharp*$ *xvec*› ‹(p $\cdot$ *xvec*) $\sharp*$ A_R› S
by(*force intro*: *tauChainPar1 simp add*: *freshChainSimps*)
hence Ψ $\otimes$ Ψ' $\rhd$ $\langle\!|\nu*(p$ $\cdot$ *xvec*)$|\!\rangle$(Q' $\parallel$ (p $\cdot$ R)) $\Longrightarrow\hat{}_\tau$ $\langle\!|\nu*(p$ $\cdot$ *xvec*)$|\!\rangle$(Q'' $\parallel$ (p $\cdot$ R)) **using** ‹(p $\cdot$ *xvec*) $\sharp*$ Ψ› ‹(p $\cdot$ *xvec*) $\sharp*$ Ψ'›
by(*rule-tac tauChainResChainPres*) *auto*
hence Ψ $\otimes$ Ψ' $\rhd$ $\langle\!|\nu*xvec|\!\rangle$(($p$ $\cdot$ Q') $\parallel$ R) $\Longrightarrow\hat{}_\tau$ $\langle\!|\nu*xvec|\!\rangle$(($p$ $\cdot$ Q'') $\parallel$ R) **using** ‹*xvec* $\sharp*$ Q'› ‹*xvec* $\sharp*$ Q''› ‹(p $\cdot$ *xvec*) $\sharp*$ R› S ‹*distinctPerm p*›
apply(*subst resChainAlpha*) **apply**(*auto simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
by(*subst resChainAlpha*[*of* - *xvec*]) (*auto simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
moreover from ‹((Ψ $\otimes$ (p $\cdot$ Ψ_R)) $\otimes$ Ψ', (p $\cdot$ P), Q'') $\in$ *Rel*› **have** ((Ψ $\otimes$ Ψ') $\otimes$ (p $\cdot$ Ψ_R), (p $\cdot$ P), Q'') $\in$ *Rel*
by(*rule C3*) (*metis Associativity AssertionStatEqTrans Commutativity Composition*)
hence (Ψ $\otimes$ Ψ', (p $\cdot$ P) $\parallel$ (p $\cdot$ R), Q'' $\parallel$ (p $\cdot$ R)) $\in$ *Rel'*
using *FrpR* ‹A_R $\sharp*$ Ψ› ‹A_R $\sharp*$ Ψ'› ‹A_R $\sharp*$ Q''› ‹A_R $\sharp*$ P› ‹A_R $\sharp*$ *xvec*› ‹(p $\cdot$ *xvec*) $\sharp*$ A_R› S
by(*rule-tac C1*) (*auto simp add*: *freshChainSimps*)
hence (Ψ $\otimes$ Ψ', $\langle\!|\nu*(p$ $\cdot$ *xvec*)$|\!\rangle$((p $\cdot$ P) $\parallel$ (p $\cdot$ R)), $\langle\!|\nu*(p$ $\cdot$ *xvec*)$|\!\rangle$(Q'' $\parallel$ (p $\cdot$ R))) $\in$ *Rel'* **using** ‹(p $\cdot$ *xvec*) $\sharp*$ Ψ› ‹(p $\cdot$ *xvec*) $\sharp*$ Ψ'›
by(*rule-tac C2*) *auto*
hence (Ψ $\otimes$ Ψ', $\langle\!|\nu*xvec|\!\rangle$($P$ $\parallel$ R), $\langle\!|\nu*xvec|\!\rangle$(($p$ $\cdot$ Q'') $\parallel$ R)) $\in$ *Rel'* **using** ‹(p $\cdot$ *xvec*) $\sharp*$ P› ‹*xvec* $\sharp*$ Q''› ‹(p $\cdot$ *xvec*) $\sharp*$ R› S ‹*distinctPerm p*›
apply(*subst resChainAlpha*[**where** *p*=*p*])
apply *simp*
apply *simp*
apply(*subst resChainAlpha*[**where** *xvec*=*xvec* **and** *p*=*p*])
by(*auto simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
ultimately show *?case*
by *blast*
qed

end

end

theory *Weak-Bisim-Pres*
imports *Weak-Bisimulation Weak-Sim-Pres Weak-Stat-Imp-Pres*
begin

context *env* **begin**

lemma *weakBisimInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $\bigwedge Tvec.$ *length xvec* = *length Tvec* $\Longrightarrow \Psi \rhd P[xvec::=Tvec] \approx Q[xvec::=Tvec]$

shows $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \approx M(\!|\lambda * xvec\ N|\!).Q$
proof −
let *?X* = $\{(\Psi, M(\!|\lambda * xvec\ N|\!).P, M(\!|\lambda * xvec\ N|\!).Q) \mid \Psi\ M\ xvec\ N\ P\ Q.\ \forall\ Tvec.$ *length xvec* = *length Tvec* $\longrightarrow \Psi \rhd P[xvec::=Tvec] \approx Q[xvec::=Tvec]\}$

from *assms* **have** $(\Psi, M(\!|\lambda * xvec\ N|\!).P, M(\!|\lambda * xvec\ N|\!).Q) \in ?X$ **by** *blast*
thus *?thesis*
proof(*coinduct rule*: *weakBisimCoinduct*)
case(*cStatImp* Ψ P Q)
thus *?case* **by**(*fastforce intro*: *weakStatImpInputPres dest*: *weakBisimE*(*3*))
next
case(*cSim* Ψ P Q)
thus *?case*
by *auto* (*blast intro*: *weakInputPres dest*: *weakBisimE*)
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*blast dest*: *weakBisimE*)
next
case(*cSym* Ψ P Q)
thus *?case* **by**(*blast dest*: *weakBisimE*)
qed
qed

lemma *weakBisimOutputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $\Psi \rhd P \approx Q$

shows $\Psi \rhd M\langle N\rangle.P \approx M\langle N\rangle.Q$
proof −
let *?X* = $\{(\Psi, M\langle N\rangle.P, M\langle N\rangle.Q) \mid \Psi\ M\ N\ P\ Q.\ \Psi \rhd P \approx Q\}$

```
  from assms have (Ψ, M⟨N⟩.P, M⟨N⟩.Q) ∈ ?X by blast
  thus ?thesis
  proof(coinduct rule: weakBisimCoinduct)
    case(cStatImp Ψ P Q)
    thus ?case by auto (blast intro: weakStatImpOutputPres dest: weakBisimE(3))
  next
    case(cSim Ψ P Q)
    thus ?case
      by(auto intro: weakOutputPres dest: weakBisimE)
  next
    case(cExt Ψ P Q Ψ')
    thus ?case by(blast dest: weakBisimE)
  next
    case(cSym Ψ P Q)
    thus ?case by(blast dest: weakBisimE)
  qed
qed

lemma weakBisimResPres:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   x :: name

  assumes Ψ ▷ P ≈ Q
  and     x ♯ Ψ

  shows Ψ ▷ (|νx|)P ≈ (|νx|)Q
proof −
  let ?X = {(Ψ, (|νx|)P, (|νx|)Q) | Ψ x P Q. Ψ ▷ P ≈ Q ∧ x ♯ Ψ}

  from assms have (Ψ, (|νx|)P, (|νx|)Q) ∈ ?X by auto
  thus ?thesis
  proof(coinduct rule: weakBisimCoinduct)
    case(cStatImp Ψ xP xQ)
    {
      fix Ψ P Q x
      assume Ψ ▷ P ≈ Q
      hence Ψ ▷ P ≲<weakBisim> Q by(rule weakBisimE)
      moreover have eqvt weakBisim by auto
      moreover assume (x::name) ♯ Ψ
     moreover have ⋀Ψ P Q x. ⟦(Ψ, P, Q) ∈ weakBisim; x ♯ Ψ⟧ ⟹ (Ψ, (|νx|)P,
(|νx|)Q) ∈ ?X ∪ weakBisim
        by auto
      ultimately have Ψ ▷ (|νx|)P ≲<(?X ∪ weakBisim)> (|νx|)Q
        by(rule weakStatImpResPres)
    }
    with ‹(Ψ, xP, xQ) ∈ ?X› show ?case by auto
```

next
case(*cSim* Ψ *xP xQ*)
from ‹(Ψ, *xP*, *xQ*) ∈ *?X*› **obtain** *x P Q* **where** $\Psi \rhd P \approx Q$ **and** $x \sharp \Psi$ **and** $xP = (\!|\nu x|\!)P$ **and** $xQ = (\!|\nu x|\!)Q$
by *auto*
from ‹$\Psi \rhd P \approx Q$› **have** $\Psi \rhd P \rightsquigarrow$<*weakBisim*> *Q* **by**(*rule weakBisimE*)
moreover have *eqvt ?X*
by(*force simp add*: *eqvt-def weakBisimClosed pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
hence *eqvt*(*?X* ∪ *weakBisim*) **by** *auto*
moreover note ‹$x \sharp \Psi$›
moreover have *weakBisim* ⊆ *?X* ∪ *weakBisim* **by** *auto*
moreover have $\bigwedge\Psi\ P\ Q\ x.\ [\![(\Psi, P, Q) \in weakBisim; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X \cup weakBisim$
by *auto*
ultimately have $\Psi \rhd (\!|\nu x|\!)P \rightsquigarrow$<(*?X* ∪ *weakBisim*)> $(\!|\nu x|\!)Q$
by(*rule weakResPres*)
with ‹$xP = (\!|\nu x|\!)P$› ‹$xQ = (\!|\nu x|\!)Q$› **show** *?case*
by *simp*
next
case(*cExt* Ψ *xP xQ* Ψ')
from ‹(Ψ, *xP*, *xQ*) ∈ *?X*› **obtain** *x P Q* **where** $\Psi \rhd P \approx Q$ **and** $x \sharp \Psi$ **and** $xP = (\!|\nu x|\!)P$ **and** $xQ = (\!|\nu x|\!)Q$
by *auto*
obtain *y*::*name* **where** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp \Psi$ **and** $y \sharp \Psi'$
by(*generate-fresh name*, *auto simp add*: *fresh-prod*)
from ‹$\Psi \rhd P \approx Q$› **have** $\Psi \otimes ([(x, y)] \cdot \Psi') \rhd P \approx Q$
by(*rule weakBisimE*)
hence $([(x, y)] \cdot (\Psi \otimes ([(x, y)] \cdot \Psi'))) \rhd ([(x, y)] \cdot P) \approx ([(x, y)] \cdot Q)$
by(*rule weakBisimClosed*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \otimes \Psi' \rhd ([(x, y)] \cdot P) \approx ([(x, y)] \cdot Q)$
by(*simp add*: *eqvts*)
with ‹$y \sharp \Psi$› ‹$y \sharp \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu y|\!)([(x, y)] \cdot P), (\!|\nu y|\!)([(x, y)] \cdot Q)) \in ?X$
by *auto*
moreover from ‹$y \sharp P$› ‹$y \sharp Q$› **have** $(\!|\nu x|\!)P = (\!|\nu y|\!)([(x, y)] \cdot P)$ **and** $(\!|\nu x|\!)Q = (\!|\nu y|\!)([(x, y)] \cdot Q)$
by(*simp add*: *alphaRes*)+
ultimately show *?case* **using** ‹$xP = (\!|\nu x|\!)P$› ‹$xQ = (\!|\nu x|\!)Q$› **by** *simp*
next
case(*cSym* Ψ *P Q*)
thus *?case* **by**(*blast dest*: *weakBisimE*)
qed
qed

lemma *weakBisimResChainPres*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

and *xvec* :: *name list*

assumes $\Psi \rhd P \approx Q$
and $xvec \sharp* \Psi$

shows $\Psi \rhd (\!|\nu * xvec|\!) P \approx (\!|\nu * xvec|\!) Q$
using *assms*
by(*induct xvec*) (*auto intro*: *weakBisimResPres*)

lemma *weakBisimParPresAux*:
fixes Ψ :: $'b$
and Ψ_R :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*
and A_R :: *name list*

assumes $\Psi \otimes \Psi_R \rhd P \approx Q$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$

shows $\Psi \rhd P \parallel R \approx Q \parallel R$
proof −
let *?X* = $\{(\Psi, (\!|\nu * xvec|\!)(P \parallel R), (\!|\nu * xvec|\!)(Q \parallel R)) \mid xvec\ \Psi\ P\ Q\ R.\ xvec \sharp* \Psi \wedge (\forall A_R\ \Psi_R.\ (extractFrame\ R = \langle A_R, \Psi_R \rangle \wedge A_R \sharp* \Psi \wedge A_R \sharp* P \wedge A_R \sharp* Q) \longrightarrow \Psi \otimes \Psi_R \rhd P \approx Q)\}$
{
fix *xvec* :: *name list*
and Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*

assume $xvec \sharp* \Psi$
and $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R \rangle; A_R \sharp* \Psi; A_R \sharp* P; A_R \sharp* Q]\!] \Longrightarrow \Psi \otimes \Psi_R \rhd P \approx Q$

hence $(\Psi, (\!|\nu * xvec|\!)(P \parallel R), (\!|\nu * xvec|\!)(Q \parallel R)) \in$ *?X*
by *blast*
}
note *XI* = *this*

{
fix *xvec* :: *name list*
and Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*

and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*
and C :: $'d$::*fs-name*

assume *xvec* $\sharp*$ Ψ
and A: $\bigwedge A_R\ \Psi_R.\ [\![\mathit{extractFrame}\ R = \langle A_R, \Psi_R\rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q;\ A_R \sharp* C]\!] \Longrightarrow \Psi \otimes \Psi_R \rhd P \approx Q$

from ‹*xvec* $\sharp*$ Ψ› **have** $(\Psi, (\!|\nu*\mathit{xvec}|\!)(P \parallel R), (\!|\nu*\mathit{xvec}|\!)(Q \parallel R)) \in ?X$
proof(*rule XI*)
fix $A_R\ \Psi_R$
assume *FrR*: $\mathit{extractFrame}\ R = \langle A_R, \Psi_R\rangle$
obtain p::*name prm* **where** $(p \cdot A_R) \sharp* \Psi$ **and** $(p \cdot A_R) \sharp* P$ **and** $(p \cdot A_R) \sharp* Q$ **and** $(p \cdot A_R) \sharp* R$ **and** $(p \cdot A_R) \sharp* C$
and $(p \cdot A_R) \sharp* \Psi_R$ **and** S: $(\mathit{set}\ p) \subseteq (\mathit{set}\ A_R) \times (\mathit{set}(p \cdot A_R))$ **and** *distinctPerm p*
by(*rule-tac c*=$(\Psi, P, Q, R, \Psi_R, C)$ **in** *name-list-avoiding*) *auto*
from *FrR* ‹$(p \cdot A_R) \sharp* \Psi_R$› S **have** $\mathit{extractFrame}\ R = \langle (p \cdot A_R), p \cdot \Psi_R\rangle$ **by**(*simp add*: *frameChainAlpha′*)

moreover assume $A_R \sharp* \Psi$
hence $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* \Psi$› S **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
moreover assume $A_R \sharp* P$
hence $(p \cdot A_R) \sharp* (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* P$› ‹$(p \cdot A_R) \sharp* P$› S **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
moreover assume $A_R \sharp* Q$
hence $(p \cdot A_R) \sharp* (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$A_R \sharp* Q$› ‹$(p \cdot A_R) \sharp* Q$› S **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
ultimately have $\Psi \otimes (p \cdot \Psi_R) \rhd P \approx Q$ **using** ‹$(p \cdot A_R) \sharp* C$› A **by** *blast*
hence $(p \cdot (\Psi \otimes (p \cdot \Psi_R))) \rhd (p \cdot P) \approx (p \cdot Q)$ **by**(*rule weakBisimClosed*)
with ‹$A_R \sharp* \Psi$› ‹$(p \cdot A_R) \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$(p \cdot A_R) \sharp* P$› ‹$A_R \sharp* Q$› ‹$(p \cdot A_R) \sharp* Q$› S ‹*distinctPerm p*›
show $\Psi \otimes \Psi_R \rhd P \approx Q$ **by**(*simp add*: *eqvts*)
qed
}
note $XI' = this$

have *eqvt* $?X$
apply(*auto simp add*: *eqvt-def*)
apply(*rule-tac x*=$p \cdot \mathit{xvec}$ **in** *exI*)
apply(*rule-tac x*=$p \cdot P$ **in** *exI*)
apply(*rule-tac x*=$p \cdot Q$ **in** *exI*)
apply(*rule-tac x*=$p \cdot R$ **in** *exI*)
apply(*simp add*: *eqvts*)
apply(*simp add*: *fresh-star-bij*)

apply(*clarify*)
apply(*erule-tac x=*(*rev p*) ∙ A_R **in** *allE*)
apply(*erule-tac x=*(*rev p*) ∙ Ψ_R **in** *allE*)
apply(*drule mp*)
apply(*rule conjI*)
apply(*rule-tac pi=p* **in** *pt-bij4*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *eqvts*)
defer
apply(*drule-tac p=p* **in** *weakBisimClosed*)
apply(*simp add*: *eqvts*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
apply *simp*
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
apply *simp*
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of p*, *THEN sym*])
by *simp*

moreover have *Res*: $\bigwedge \Psi\ P\ Q\ x.\ [\![(\Psi, P, Q) \in ?X \cup weakBisim;\ x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X \cup weakBisim$
proof −
fix $\Psi\ P\ Q\ x$
assume $(\Psi, P, Q) \in ?X \cup weakBisim$ **and** $(x::name) \sharp \Psi$
show $(\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X \cup weakBisim$
proof(*case-tac* $(\Psi, P, Q) \in ?X$)
assume $(\Psi, P, Q) \in ?X$
with ‹$x \sharp \Psi$› **have** $(\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in ?X$
apply *auto*
by(*rule-tac x=x#xvec* **in** *exI*) *auto*
thus *?thesis* **by** *simp*
next
assume $\neg(\Psi, P, Q) \in ?X$
with ‹$(\Psi, P, Q) \in ?X \cup weakBisim$› **have** $\Psi \rhd P \approx Q$
by *blast*
hence $\Psi \rhd (\!|\nu x|\!)P \approx (\!|\nu x|\!)Q$ **using** ‹$x \sharp \Psi$›
by(*rule weakBisimResPres*)
thus *?thesis*
by *simp*
qed
qed

{
fix Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and Ψ' :: *'b*

assume $\Psi \rhd P \approx Q$

hence $\Psi \rhd Q \approx P$ **by**(*rule weakBisimE*)
then obtain P' P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$
and *QimpP'*: *insertAssertion*(*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* P') Ψ
and *P'Chain*: $\Psi \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ P''$
and $\Psi \otimes \Psi' \rhd Q \approx P''$ **using** *weakStatImp-def*
by(*blast dest*: *weakBisimE*)
note *PChain QimpP' P'Chain*
moreover from ‹$\Psi \otimes \Psi' \rhd Q \approx P''$› **have** $\Psi \otimes \Psi' \rhd P'' \approx Q$ **by**(*rule weakBisimE*)
ultimately have $\exists P'\ P''.\ \Psi \rhd P \Longrightarrow\hat{}_\tau\ P' \wedge$ *insertAssertion*(*extractFrame* Q) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* P') $\Psi \wedge$
$\Psi \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ P'' \wedge \Psi \otimes \Psi' \rhd P'' \approx Q$
by *blast*
}
moreover
{
fix Ψ P Q A_R Ψ_R R
assume *PSimQ*: $\Psi \otimes \Psi_R \rhd P \approx Q$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$
hence $(\Psi, P \parallel R, Q \parallel R) \in ?X$
proof −
have $P \parallel R = (\!|\nu*[]|\!)(P \parallel R)$ **by** *simp*
moreover have $Q \parallel R = (\!|\nu*[]|\!)(Q \parallel R)$ **by** *simp*
moreover have ([]::*name list*) $\sharp* \Psi$ **by** *simp*
moreover
{
fix A_R' Ψ_R'

assume *FrR'*: *extractFrame* $R = \langle A_R', \Psi_R' \rangle$
and $A_R' \sharp* \Psi$
and $A_R' \sharp* P$
and $A_R' \sharp* Q$
obtain p **where** $(p \cdot A_R') \sharp* A_R$
and $(p \cdot A_R') \sharp* \Psi_R'$
and $(p \cdot A_R') \sharp* \Psi$
and $(p \cdot A_R') \sharp* P$
and $(p \cdot A_R') \sharp* Q$
and S: $(set\ p) \subseteq (set\ A_R') \times (set(p \cdot A_R'))$ **and** *distinctPerm* p
by(*rule-tac c*=$(A_R, \Psi, \Psi_R', P, Q)$ **in** *name-list-avoiding*) *auto*

from ‹$(p \cdot A_R') \sharp* \Psi_R'$› S **have** $\langle A_R', \Psi_R' \rangle = \langle p \cdot A_R', p \cdot \Psi_R' \rangle$
by(*simp add*: *frameChainAlpha*)

with *FrR'* **have** *FrR''*: *extractFrame R* = ⟨*p* ∙ *A*$_R$', *p* ∙ Ψ$_R$'⟩ **by** *simp*
with *FrR* ‹(*p* ∙ *A*$_R$') ♯* *A*$_R$›
obtain *q* **where** *p* ∙ Ψ$_R$' = (*q*::*name prm*) ∙ Ψ$_R$ **and** *S'*: *set q* ⊆ (*set A*$_R$) × *set*(*p* ∙ *A*$_R$') **and** *distinctPerm q*
apply *auto*
apply(*drule-tac sym*) **apply** *simp*
by(*drule-tac frameChainEq*) *auto*
from *PSimQ* **have** (*q* ∙ (Ψ ⊗ Ψ$_R$)) ▷ (*q* ∙ *P*) ≈ (*q* ∙ *Q*)
by(*rule weakBisimClosed*)
with ‹*A*$_R$ ♯* Ψ› ‹*A*$_R$ ♯* *P*› ‹*A*$_R$ ♯* *Q*› ‹(*p* ∙ *A*$_R$') ♯* Ψ› ‹(*p* ∙ *A*$_R$') ♯* *P*› ‹(*p* ∙ *A*$_R$') ♯* *Q*› *S'*
have Ψ ⊗ (*q* ∙ Ψ$_R$) ▷ *P* ≈ *Q* **by**(*simp add*: *eqvts*)
hence (*p* ∙ (Ψ ⊗ (*q* ∙ Ψ$_R$))) ▷ (*p* ∙ *P*) ≈ (*p* ∙ *Q*) **by**(*rule weakBisimClosed*)
with ‹*A*$_R$' ♯* Ψ› ‹*A*$_R$' ♯* *P*› ‹*A*$_R$' ♯* *Q*› ‹(*p* ∙ *A*$_R$') ♯* Ψ› ‹(*p* ∙ *A*$_R$') ♯* *P*› ‹(*p* ∙ *A*$_R$') ♯* *Q*› *S* ‹*distinctPerm p*› ‹(*p* ∙ Ψ$_R$') = *q* ∙ Ψ$_R$›
have Ψ ⊗ Ψ$_R$' ▷ *P* ≈ *Q*
by(*drule-tac sym*) (*simp add*: *eqvts*)
}
ultimately show *?thesis*
by *blast*
qed
hence (Ψ, *P* ∥ *R*, *Q* ∥ *R*) ∈ *?X* ∪ *weakBisim*
by *simp*
}
note *C1* = *this*

have *C2*: ⋀Ψ *P Q xvec*. ⟦(Ψ, *P*, *Q*) ∈ *?X* ∪ *weakBisim*; (*xvec*::*name list*) ♯* Ψ⟧ ⟹ (Ψ, (⦇ν*xvec⦈)*P*, (⦇ν*xvec⦈)*Q*) ∈ *?X* ∪ *weakBisim*
proof −
fix Ψ *P Q xvec*
assume (Ψ, *P*, *Q*) ∈ *?X* ∪ *weakBisim*
assume (*xvec*::*name list*) ♯* Ψ
thus (Ψ, (⦇ν*xvec⦈)*P*, (⦇ν*xvec⦈)*Q*) ∈ *?X* ∪ *weakBisim*
proof(*induct xvec*)
case *Nil*
thus *?case* **using** ‹(Ψ, *P*, *Q*) ∈ *?X* ∪ *weakBisim*› **by** *simp*
next
case(*Cons x xvec*)
thus *?case* **by**(*simp only*: *resChain.simps*) (*rule-tac Res*, *auto*)
qed
qed

{
fix Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *R* :: (*'a*, *'b*, *'c*) *psi*
and *A*$_R$ :: *name list*

and $\Psi_R :: 'b$

assume $\Psi \otimes \Psi_R \rhd P \approx Q$
and *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$

have $(\Psi, P \parallel R, Q \parallel R) \in ?X$
proof −
{
fix $A_R' :: name\ list$
and $\Psi_R' :: 'b$

assume *FrR'*: *extractFrame* $R = \langle A_R', \Psi_R' \rangle$
and $A_R' \sharp* \Psi$
and $A_R' \sharp* P$
and $A_R' \sharp* Q$

obtain p **where** $(p \cdot A_R') \sharp* A_R$ **and** $(p \cdot A_R') \sharp* \Psi_R'$ **and** $(p \cdot A_R') \sharp* \Psi$ **and** $(p \cdot A_R') \sharp* P$ **and** $(p \cdot A_R') \sharp* Q$
and *Sp*: $(set\ p) \subseteq (set\ A_R') \times (set(p \cdot A_R'))$ **and** *distinctPerm p*
by(*rule-tac c*=$(A_R, \Psi, \Psi_R', P, Q)$ **in** *name-list-avoiding*) *auto*

from *FrR'* ‹$(p \cdot A_R') \sharp* \Psi_R'$› *Sp* **have** *extractFrame* $R = \langle (p \cdot A_R'), p \cdot \Psi_R' \rangle$
by(*simp add*: *frameChainAlpha eqvts*)
with *FrR* ‹$(p \cdot A_R') \sharp* A_R$› **obtain** q::*name prm*
where *Sq*: *set* $q \subseteq set(p \cdot A_R') \times set\ A_R$ **and** *distinctPerm q* **and** $\Psi_R = q \cdot p \cdot \Psi_R'$
by(*force elim*: *frameChainEq*)

from ‹$\Psi \otimes \Psi_R \rhd P \approx Q$› ‹$\Psi_R = q \cdot p \cdot \Psi_R'$› **have** $\Psi \otimes (q \cdot p \cdot \Psi_R') \rhd P \approx Q$ **by** *simp*
hence $(q \cdot (\Psi \otimes (q \cdot p \cdot \Psi_R'))) \rhd (q \cdot P) \approx (q \cdot Q)$ **by**(*rule weakBisimClosed*)
with *Sq* ‹$A_R \sharp* \Psi$› ‹$(p \cdot A_R') \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$(p \cdot A_R') \sharp* P$› ‹$A_R \sharp* Q$› ‹$(p \cdot A_R') \sharp* Q$› ‹*distinctPerm q*›
have $\Psi \otimes (p \cdot \Psi_R') \rhd P \approx Q$ **by**(*simp add*: *eqvts*)
hence $(p \cdot (\Psi \otimes (p \cdot \Psi_R'))) \rhd (p \cdot P) \approx (p \cdot Q)$ **by**(*rule weakBisimClosed*)
with *Sp* ‹$A_R' \sharp* \Psi$› ‹$(p \cdot A_R') \sharp* \Psi$› ‹$A_R' \sharp* P$› ‹$(p \cdot A_R') \sharp* P$› ‹$A_R' \sharp* Q$› ‹$(p \cdot A_R') \sharp* Q$› ‹*distinctPerm p*›
have $\Psi \otimes \Psi_R' \rhd P \approx Q$ **by**(*simp add*: *eqvts*)
}
thus *?thesis*
apply *auto*
apply(*rule-tac x*=[] **in** *exI*)
by *auto blast*
qed

}
note *Goal* = *this*
with *assms* **have** $(\Psi,\ P \parallel R,\ Q \parallel R) \in ?X$ **by** *blast*
thus *?thesis*
proof(*coinduct rule*: *weakBisimCoinduct*)
case(*cStatImp* Ψ *PR QR*)
{
fix *xvec* :: *name list*
fix P Q R
assume A: $\forall A_R\ \Psi_R.$ *extractFrame* $R = \langle A_R, \Psi_R\rangle \wedge A_R \sharp* \Psi \wedge A_R \sharp* P \wedge A_R \sharp* Q \longrightarrow \Psi \otimes \Psi_R \rhd P \approx Q$
{
fix $A_R\ \Psi_R$
assume *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$
with A **have** $\Psi \otimes \Psi_R \rhd P \lessapprox <weakBisim> Q$ **by**(*auto dest*: *weakBisimE*)
}
moreover assume *xvec* $\sharp* \Psi$
moreover have *eqvt weakBisim* **by** *auto*
moreover note *C1 C2 statEqWeakBisim*
ultimately have $\Psi \rhd (\!|\nu* xvec|\!)(P \parallel R) \lessapprox <(?X \cup weakBisim)> (\!|\nu* xvec|\!)(Q \parallel R)$
by(*rule weakStatImpParPres*)
}
with ‹$(\Psi,\ PR,\ QR) \in ?X$› **show** *?case* **by** *auto*
next
case(*cSim* Ψ *PR QR*)
from ‹$(\Psi,\ PR,\ QR) \in ?X$›
obtain *xvec* P Q R A_R Ψ_R **where** *PFrR*: $PR = (\!|\nu* xvec|\!)(P \parallel R)$ **and** *QFrR*: $QR = (\!|\nu* xvec|\!)(Q \parallel R)$
and *xvec* $\sharp* \Psi$
by *auto*
with ‹$(\Psi,\ PR,\ QR) \in ?X$› **have** $(\Psi, (\!|\nu* xvec|\!)(P \parallel R), (\!|\nu* xvec|\!)(Q \parallel R)) \in ?X$ **by** *simp*
hence $\Psi \rhd (\!|\nu* xvec|\!)(P \parallel R) \rightsquigarrow <(?X \cup weakBisim)> (\!|\nu* xvec|\!)(Q \parallel R)$ **using** ‹*xvec* $\sharp* \Psi$›
proof(*induct xvec*)
case *Nil*
from ‹$(\Psi, (\!|\nu* []|\!)(P \parallel R), (\!|\nu* []|\!)(Q \parallel R)) \in ?X$› **have** *PRQR*: $(\Psi,\ P \parallel R,\ Q \parallel R) \in ?X$ **by** *simp*
from *PRQR* **have** $\bigwedge A_R\ \Psi_R.\ [\![extractFrame\ R = \langle A_R, \Psi_R\rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q]\!] \Longrightarrow (\Psi \otimes \Psi_R,\ P,\ Q) \in weakBisim$
by *auto*
moreover note *weakBisimEqvt*
moreover from ‹*eqvt ?X*› **have** *eqvt*(*?X* $\cup$ *weakBisim*) **by** *auto*
moreover note *weakBisimE*(*2*) *weakBisimE*(*4*) *weakBisimE*(*3*) *weakBisimE*(*1*)
moreover note *C1 C2*
ultimately have $\Psi \rhd P \parallel R \rightsquigarrow <(?X \cup weakBisim)> Q \parallel R$ **using** *statEqWeakBisim*

by(*rule weakParPres*)
thus *?case* **by** *simp*
next
case(*Cons x xvec′*)
from ‹(*x*#*xvec′*) ♯* Ψ› **have** *x* ♯ Ψ **and** *xvec′* ♯* Ψ **by** *simp*+
with ‹(Ψ, ⦇*ν*∗(*x*#*xvec′*)⦈*P* ∥ *R*, ⦇*ν*∗(*x*#*xvec′*)⦈*Q* ∥ *R*) ∈ *?X*›
have (Ψ, ⦇*ν*∗(*xvec′*)⦈*P* ∥ *R*, ⦇*ν*∗(*xvec′*)⦈*Q* ∥ *R*) ∈ *?X*
apply *auto*
apply(*subgoal-tac* ∃ *y yvec*. *xvec*=*y*#*yvec*)
apply(*clarify*)
apply *simp*
apply(*simp add*: *psi.inject alpha*)
apply(*clarify*)
apply(*erule disjE*)
apply(*erule disjE*)
apply(*clarify*)
apply *blast*
apply(*clarify*)
apply(*clarify*)
apply(*simp add*: *eqvts*)
apply(*rule-tac x*=[(*x*, *y*)] ∙ *yvec* **in** *exI*)
apply(*rule-tac x*=[(*x*, *y*)] ∙ *P* **in** *exI*)
apply(*rule-tac x*=[(*x*, *y*)] ∙ *Q* **in** *exI*)
apply(*rule-tac x*=[(*x*, *y*)] ∙ *R* **in** *exI*)
apply(*clarsimp*)
apply(*rule conjI*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of* [(*x*, *y*)], *THEN sym*])
apply *simp*
apply(*clarify*)
apply(*erule-tac x*=[(*x*, *y*)] ∙ *A*$_R$ **in** *allE*)
apply(*erule-tac x*=[(*x*, *y*)] ∙ Ψ$_R$ **in** *allE*)
apply(*drule mp*)
apply(*rule conjI*)
apply(*rule-tac pi*=[(*x*, *y*)] **in** *pt-bij4*[*OF pt-name-inst*, *OF at-name-inst*])
apply(*simp add*: *eqvts*)
apply(*rule conjI*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of* [(*x*, *y*)], *THEN sym*])
apply *simp*
apply(*rule conjI*)
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of* [(*x*, *y*)], *THEN sym*])
apply *simp*
apply(*subst pt-fresh-star-bij*[*OF pt-name-inst*,*OF at-name-inst*, *of* [(*x*, *y*)], *THEN sym*])
apply *simp*
apply(*drule-tac p*=[(*x*, *y*)] **in** *weakBisimClosed*)
apply(*simp add*: *eqvts*)

by(*case-tac xvec*) *auto*

with ‹$[\![(\Psi, (\!|\nu{*}xvec'|\!)(P \parallel R), (\!|\nu{*}xvec'|\!)(Q \parallel R)) \in ?X;\ xvec' \sharp* \Psi]\!] \Longrightarrow \Psi \rhd (\!|\nu{*}xvec'|\!)(P \parallel R) \rightsquigarrow<(?X \cup weakBisim)> (\!|\nu{*}xvec'|\!)(Q \parallel R)$› ‹$xvec' \sharp* \Psi$›
have $\Psi \rhd (\!|\nu{*}xvec'|\!)(P \parallel R) \rightsquigarrow<(?X \cup weakBisim)> (\!|\nu{*}xvec'|\!)(Q \parallel R)$ **by** *blast*
moreover note ‹*eqvt ?X*›
moreover from ‹*eqvt ?X*› **have** *eqvt*$(?X \cup weakBisim)$ **by** *auto*
moreover note ‹$x \sharp \Psi$›
moreover have $?X \cup weakBisim \subseteq ?X \cup weakBisim$ **by** *simp*
ultimately have $\Psi \rhd (\!|\nu x|\!)((\!|\nu{*}xvec'|\!)(P \parallel R)) \rightsquigarrow<(?X \cup weakBisim)> (\!|\nu x|\!)((\!|\nu{*}xvec'|\!)(Q \parallel R))$ **using** *Res*
by(*rule-tac weakResPres*)
thus *?case*
by *simp*
qed
with *PFrR QFrR* **show** *?case*
by *simp*
next
case(*cExt* Ψ *PR QR* Ψ')

from ‹$(\Psi, PR, QR) \in ?X$›
obtain *xvec P Q R* A_R Ψ_R **where** *PFrR*: $PR = (\!|\nu{*}xvec|\!)(P \parallel R)$ **and** *QFrR*: $QR = (\!|\nu{*}xvec|\!)(Q \parallel R)$
and *xvec* $\sharp*$ Ψ **and** *A*: $\forall A_R\ \Psi_R.\ (extractFrame\ R = \langle A_R, \Psi_R\rangle \wedge A_R \sharp* \Psi \wedge A_R \sharp* P \wedge A_R \sharp* Q) \longrightarrow \Psi \otimes \Psi_R \rhd P \approx Q$
by *auto*

obtain *p* **where** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* Q$
and $(p \cdot xvec) \sharp* R$
and $(p \cdot xvec) \sharp* \Psi'$
and *S*: $(set\ p) \subseteq (set\ xvec) \times (set(p \cdot xvec))$ **and** *distinctPerm p*
by(*rule-tac c*=(Ψ, P, Q, R, Ψ') **in** *name-list-avoiding*) *auto*

from ‹$(p \cdot xvec) \sharp* P$› ‹$(p \cdot xvec) \sharp* R$› *S* **have** $(\!|\nu{*}xvec|\!)(P \parallel R) = (\!|\nu{*}(p \cdot xvec)|\!)(p \cdot (P \parallel R))$
by(*subst resChainAlpha*) *auto*
hence *PRAlpha*: $(\!|\nu{*}xvec|\!)(P \parallel R) = (\!|\nu{*}(p \cdot xvec)|\!)((p \cdot P) \parallel (p \cdot R))$
by(*simp add*: *eqvts*)

from ‹$(p \cdot xvec) \sharp* Q$› ‹$(p \cdot xvec) \sharp* R$› *S* **have** $(\!|\nu{*}xvec|\!)(Q \parallel R) = (\!|\nu{*}(p \cdot xvec)|\!)(p \cdot (Q \parallel R))$
by(*subst resChainAlpha*) *auto*
hence *QRAlpha*: $(\!|\nu{*}xvec|\!)(Q \parallel R) = (\!|\nu{*}(p \cdot xvec)|\!)((p \cdot Q) \parallel (p \cdot R))$
by(*simp add*: *eqvts*)

from ‹$(p \cdot xvec) \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi'$› **have** $(\Psi \otimes \Psi', (\!|\nu{*}(p \cdot xvec)|\!)((p \cdot P)$

$\| (p \cdot R)), (\!|\nu*(p \cdot xvec)|\!)((p \cdot Q) \| (p \cdot R))) \in ?X$
proof(*rule-tac C2*=(Ψ, $(p \cdot P)$, $(p \cdot Q)$, R, Ψ', *xvec*, $p \cdot xvec$) **in** *XI'*, *auto*)
fix A_R Ψ_R
assume *FrR*: *extractFrame* $(p \cdot R) = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* \Psi'$ **and** $A_R \sharp* (p \cdot P)$ **and** $A_R \sharp* (p \cdot Q)$
from *FrR* **have** $(p \cdot (extractFrame\ (p \cdot R))) = (p \cdot \langle A_R, \Psi_R\rangle)$ **by** *simp*
with ‹*distinctPerm p*› **have** *extractFrame* $R = \langle p \cdot A_R, p \cdot \Psi_R\rangle$ **by**(*simp add*: *eqvts*)
moreover from ‹$A_R \sharp* \Psi$› **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› *S* **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
moreover from ‹$A_R \sharp* (p \cdot P)$› **have** $(p \cdot A_R) \sharp* (p \cdot p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
moreover from ‹$A_R \sharp* (p \cdot Q)$› **have** $(p \cdot A_R) \sharp* (p \cdot p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹*distinctPerm p*› **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
ultimately have $\Psi \otimes (p \cdot \Psi_R) \rhd P \approx Q$ **using** *A* **by** *blast*
hence $(\Psi \otimes (p \cdot \Psi_R)) \otimes (p \cdot \Psi') \rhd P \approx Q$ **by**(*rule weakBisimE*)
moreover have $(\Psi \otimes (p \cdot \Psi_R)) \otimes (p \cdot \Psi') \simeq (\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R)$
by(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
ultimately have $(\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R) \rhd P \approx Q$
by(*rule statEqWeakBisim*)
hence $(p \cdot ((\Psi \otimes (p \cdot \Psi')) \otimes (p \cdot \Psi_R))) \rhd (p \cdot P) \approx (p \cdot Q)$
by(*rule weakBisimClosed*)
with ‹*distinctPerm p*› ‹*xvec* $\sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› *S* **show** $(\Psi \otimes \Psi') \otimes \Psi_R \rhd (p \cdot P) \approx (p \cdot Q)$
by(*simp add*: *eqvts*)
qed
with *PFrR QFrR PRAlpha QRAlpha* **show** *?case* **by** *simp*
next
case(*cSym* Ψ *PR QR*)
thus *?case* **by**(*blast dest*: *weakBisimE*)
qed
qed

lemma *weakBisimParPres*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *R* :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \approx Q$

shows $\Psi \rhd P \| R \approx Q \| R$
proof −
obtain A_R Ψ_R **where** *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **and** $A_R \sharp* \Psi$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$

by(*rule-tac* $C=(\Psi, P, Q)$ **in** *freshFrame*) *auto*
moreover from ‹$\Psi \rhd P \approx Q$›
have $\Psi \otimes \Psi_R \rhd P \approx Q$ **by**(*rule weakBisimE*)
ultimately show *?thesis* **by**(*rule-tac weakBisimParPresAux*)
qed

end

end

theory *Weak-Bisim-Struct-Cong*
imports *Weak-Bisim-Pres Bisim-Struct-Cong*
begin

context *env* **begin**

lemma *weakBisimParComm*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \parallel Q \approx Q \parallel P$
by(*metis bisimParComm strongBisimWeakBisim*)

lemma *weakBisimResComm*:
fixes $x :: name$
and $\Psi :: 'b$
and $y :: name$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $y \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \approx (\!|\nu y|\!)((\!|\nu x|\!)P)$
using *assms*
by(*metis bisimResComm strongBisimWeakBisim*)

lemma *weakBisimResComm'*:
fixes $x :: name$
and $\Psi :: 'b$
and $xvec :: name\ list$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $xvec \sharp* \Psi$

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!)P) \approx (\!|\nu * xvec|\!)((\!|\nu x|\!)P)$
using *assms*
by(*metis bisimResComm' strongBisimWeakBisim*)

lemma *weakBisimScopeExt*:
  fixes $x :: name$
  and $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$

  assumes $x \sharp \Psi$
  and $x \sharp P$

  shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \approx P \parallel (\!|\nu x|\!) Q$
using *assms*
by(*metis bisimScopeExt strongBisimWeakBisim*)

lemma *weakBisimScopeExtChain*:
  fixes $xvec :: name\ list$
  and $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$

  assumes $xvec \sharp* \Psi$
  and $xvec \sharp* P$

  shows $\Psi \rhd (\!|\nu* xvec|\!)(P \parallel Q) \approx P \parallel ((\!|\nu* xvec|\!) Q)$
using *assms*
by(*metis bisimScopeExtChain strongBisimWeakBisim*)

lemma *weakBisimParAssoc*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$
  and $R :: ('a, 'b, 'c)\ psi$

  shows $\Psi \rhd (P \parallel Q) \parallel R \approx P \parallel (Q \parallel R)$
by(*metis bisimParAssoc strongBisimWeakBisim*)

lemma *weakBisimParNil*:
  fixes $P :: ('a, 'b, 'c)\ psi$

  shows $\Psi \rhd P \parallel \mathbf{0} \approx P$
by(*metis bisimParNil strongBisimWeakBisim*)

lemma *weakBisimResNil*:
  fixes $x :: name$
  and $\Psi :: 'b$

  assumes $x \sharp \Psi$

  shows $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \approx \mathbf{0}$
using *assms*

by(*metis bisimResNil strongBisimWeakBisim*)

lemma *weakBisimOutputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $N :: 'a$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp M$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \approx M\langle N\rangle.(\!|\nu x|\!)P$
using *assms*
by(*metis bisimOutputPushRes strongBisimWeakBisim*)

lemma *weakBisimInputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $xvec :: name\ list$
and $N :: 'a$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp M$
and $x \sharp xvec$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \approx M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
using *assms*
by(*metis bisimInputPushRes strongBisimWeakBisim*)

lemma *weakBisimCasePushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $Cs :: ('c \times ('a, 'b, 'c)\ psi)\ list$

assumes $x \sharp \Psi$
and $x \sharp (map\ fst\ Cs)$

shows $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \approx Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
using *assms*
by(*metis bisimCasePushRes strongBisimWeakBisim*)

lemma *weakBangExt*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

assumes *guarded P*

shows $\Psi \rhd !P \approx P \parallel !P$
using *assms*
by(*metis bangExt strongBisimWeakBisim*)

lemma *weakBisimParSym*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \approx Q$

shows $\Psi \rhd R \parallel P \approx R \parallel Q$
using *assms*
by(*metis weakBisimParComm weakBisimParPres weakBisimTransitive*)

lemma *weakBisimScopeExtSym*:
fixes $x :: name$
and $Q :: ('a, 'b, 'c)\ psi$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp Q$

shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \approx ((\!|\nu x|\!)P) \parallel Q$
using *assms*
by(*metis weakBisimScopeExt weakBisimTransitive weakBisimParComm weakBisimE weakBisimResPres*)

lemma *weakBisimScopeExtChainSym*:
fixes $xvec :: name\ list$
and $Q :: ('a, 'b, 'c)\ psi$
and $P :: ('a, 'b, 'c)\ psi$

assumes $xvec \sharp* \Psi$
and $xvec \sharp* Q$

shows $\Psi \rhd (\!|\nu * xvec|\!)(P \parallel Q) \approx ((\!|\nu * xvec|\!)P) \parallel Q$
using *assms*
by(*induct xvec*) (*auto intro*: *weakBisimScopeExtSym weakBisimReflexive weakBisimTransitive weakBisimResPres*)

lemma *weakBisimParPresAuxSym*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \otimes \Psi_R \rhd P \approx Q$
and *extractFrame* $R = \langle A_R, \Psi_R \rangle$
and $A_R \sharp* \Psi$
and $A_R \sharp* P$
and $A_R \sharp* Q$

shows $\Psi \rhd R \parallel P \approx R \parallel Q$
using *assms*
by(*metis weakBisimParComm weakBisimParPresAux weakBisimTransitive*)

lemma *weakBisimParPresSym*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \approx Q$

shows $\Psi \rhd R \parallel P \approx R \parallel Q$
using *assms*
by(*metis weakBisimParComm weakBisimParPres weakBisimTransitive*)

lemma *guardedFrameStatEq*:
fixes $P :: ('a, 'b, 'c)\ psi$

assumes *guarded P*

shows *extractFrame* $P \simeq_F \langle \varepsilon, \mathbf{1} \rangle$
proof −
obtain $A_P\ \Psi_P$ **where** *FrR*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$
by(*rule freshFrame*)

with ‹*guarded P*› **have** $\Psi_P \simeq \mathbf{1}$ **and** ((*supp* Ψ_P)::*name set*) = {}
by(*metis guardedStatEq*)+
from ‹*supp* Ψ_P = {}› **have** $A_P \sharp* \Psi_P$ **by**(*auto simp add*: *fresh-star-def fresh-def*)
hence $\langle A_P, \Psi_P \rangle \simeq_F \langle [], \Psi_P \rangle$ **by**(*force intro*: *frameResFreshChain*)
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle [], \Psi_P \rangle \simeq_F \langle [], \mathbf{1} \rangle$
by(*simp add*: *FrameStatEq-def FrameStatImp-def AssertionStatEq-def AssertionStatImp-def*)
ultimately show *?thesis* **using** *FrR* **by**(*rule-tac FrameStatEqTrans*) *auto*
qed

lemma *guardedInsertAssertion*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $\Psi :: 'b$

assumes *guarded P*

shows *insertAssertion* (*extractFrame P*) $\Psi \simeq_F \langle \varepsilon, \Psi \rangle$
proof −
obtain A_P Ψ_P **where** *FrR*: *extractFrame P* $= \langle A_P, \Psi_P \rangle$ **and** $A_P \sharp* \Psi$
by(*rule freshFrame*)

with ‹*guarded P*› **have** $\Psi_P \simeq \mathbf{1}$ **and** ((*supp* Ψ_P)::*name set*) = {}
by(*metis guardedStatEq*)+
from ‹*supp* $\Psi_P = \{\}$› **have** $A_P \sharp* \Psi_P$ **by**(*auto simp add*: *fresh-star-def fresh-def*)
hence $\langle A_P, \Psi \otimes \Psi_P \rangle \simeq_F \langle [], \Psi \otimes \Psi_P \rangle$ **using** ‹$A_P \sharp* \Psi$› **by**(*force intro*: *frameResFreshChain*)
moreover from ‹$\Psi_P \simeq \mathbf{1}$› **have** $\langle \varepsilon, \Psi \otimes \Psi_P \rangle \simeq_F \langle \varepsilon, \Psi \otimes \mathbf{1} \rangle$ **by**(*force intro*: *compositionSym*)
moreover have $\langle \varepsilon, \Psi \otimes \mathbf{1} \rangle \simeq_F \langle \varepsilon, \Psi \rangle$ **by**(*force intro*: *Identity*)
ultimately show *?thesis* **using** *FrR* ‹$A_P \sharp* \Psi$›
by(*force intro*: *FrameStatEqTrans AssertionStatEqTrans*)
qed

lemma *insertDoubleAssertionStatEq′*:
fixes F :: *'b frame*
and Ψ :: *'b*
and Ψ' :: *'b*

shows *insertAssertion*(*insertAssertion F* Ψ) $\Psi' \simeq_F$ (*insertAssertion F*) ($\Psi' \otimes \Psi$)
proof −
obtain A_F Ψ_F **where** $F = \langle A_F, \Psi_F \rangle$ **and** $A_F \sharp* \Psi$ **and** $A_F \sharp* \Psi'$ **and** $A_F \sharp* (\Psi' \otimes \Psi)$
by(*rule-tac C*=(Ψ, Ψ') **in** *freshFrame*) *auto*
thus *?thesis*
by *auto* (*metis frameIntAssociativity FrameStatEqSym*)
qed

lemma *bangActE*:
assumes $\Psi \rhd !P \longmapsto \alpha \prec P'$
and *bn* $\alpha \sharp* subject\ \alpha$
and *guarded P*
and $\alpha \neq \tau$
and *bn* $\alpha \sharp* P$

obtains Q **where** $\Psi \rhd P \longmapsto \alpha \prec Q$ **and** $P' \sim Q \parallel !P$
proof −
assume $\bigwedge Q.\ [\![\Psi \rhd P \longmapsto \alpha \prec Q;\ P' \sim Q \parallel !P]\!] \Longrightarrow thesis$
moreover from ‹$\Psi \rhd !P \longmapsto \alpha \prec P'$› ‹*bn* $\alpha \sharp* subject\ \alpha$› ‹$\alpha \neq \tau$› ‹*bn* $\alpha \sharp* P$›
have $\exists Q.\ \Psi \rhd P \longmapsto \alpha \prec Q \wedge P' \sim Q \parallel !P$
proof(*nominal-induct rule*: *bangInduct′*)
case(*cAlpha* α P' *p*)
then obtain Q **where** $\Psi \rhd P \longmapsto \alpha \prec Q$ **and** $P' \sim Q \parallel (P \parallel !P)$ **by** *fastforce*
from ‹$\Psi \rhd P \longmapsto \alpha \prec Q$› **have** *distinct*(*bn* α) **by**(*rule boundOutputDistinct*)

have S: *set* $p \subseteq$ *set*$(bn\ \alpha) \times$ *set*$(bn(p \cdot \alpha))$ **by** *fact*
from ‹$\Psi \rhd P \longmapsto \alpha \prec Q$› ‹$bn(p \cdot \alpha) \sharp* \alpha$› ‹$bn(p \cdot \alpha) \sharp* P$› ‹$bn\ \alpha \sharp*$ *subject* α› ‹*distinct*$(bn\ \alpha)$›
have $bn(p \cdot \alpha) \sharp* Q$ **by**(*force dest*: *freeFreshChainDerivative*)
with ‹$\Psi \rhd P \longmapsto \alpha \prec Q$› ‹$bn(p \cdot \alpha) \sharp* \alpha$› S ‹$bn\ \alpha \sharp*$ *subject* α› ‹*distinct*$(bn\ \alpha)$› **have** $\Psi \rhd P \longmapsto (p \cdot \alpha) \prec (p \cdot Q)$
by(*fastforce simp add*: *residualAlpha*)
moreover from ‹$P' \sim Q \parallel (P \parallel\ !P)$› **have** $(p \cdot \mathbf{1}) \rhd (p \cdot P') \sim (p \cdot (Q \parallel (P \parallel\ !P)))$
by(*rule bisimClosed*)
with ‹$(bn\ \alpha) \sharp* P$› ‹$bn(p \cdot \alpha) \sharp* P$› S **have** $(p \cdot P') \sim (p \cdot Q) \parallel (P \parallel\ !P)$
by(*simp add*: *eqvts*)
ultimately show *?case* **by** *blast*
next
case(*cPar1* $\alpha\ P'$)
from ‹*guarded* P› **have** $P' \parallel\ !P \sim P' \parallel (P \parallel\ !P)$ **by**(*metis bangExt bisimParPresSym*)
with ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› **show** *?case* **by** *blast*
next
case(*cPar2* $\alpha\ P'$)
then obtain Q **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec Q$ **and** $P' \sim Q \parallel\ !P$ **by** *blast*
from ‹$P' \sim Q \parallel\ !P$› **have** $P \parallel P' \sim Q \parallel (P \parallel\ !P)$
by(*metis bisimParPresSym bisimParAssoc bisimTransitive bisimParComm*)
with *PTrans* **show** *?case* **by** *blast*
next
case *cComm1*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
next
case *cComm2*
from ‹$\tau \neq \tau$› **have** *False* **by** *simp*
thus *?case* **by** *simp*
next
case(*cBang* $\alpha\ P'$)
then obtain Q **where** *PTrans*: $\Psi \rhd P \longmapsto \alpha \prec Q$ **and** $P' \sim Q \parallel (P \parallel\ !P)$ **by** *blast*
from ‹$P' \sim Q \parallel (P \parallel\ !P)$› ‹*guarded* P› **have** $P' \sim Q \parallel\ !P$ **by**(*metis bisimTransitive bisimParPresSym bangExt bisimSymmetric*)
with *PTrans* **show** *?case* **by** *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *bangTauE*:
assumes $\Psi \rhd\ !P \longmapsto \tau \prec P'$
and *guarded* P

obtains Q **where** $\Psi \rhd P \parallel P \longmapsto \tau \prec Q$ **and** $P' \sim Q \parallel\ !P$
using *assms*

proof −
assume $\bigwedge Q.\ [\![\Psi \rhd P \parallel P \longmapsto \tau \prec Q;\ P' \sim Q \parallel !P]\!] \Longrightarrow \mathit{thesis}$
moreover from *assms* **have** $\exists\, Q.\ \Psi \rhd P \parallel P \longmapsto \tau \prec Q \wedge P' \sim Q \parallel !P$
proof(*nominal-induct rule*: *bangTauInduct*)
case(*cPar1 P'*)
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* P$
by(*rule-tac C*=(Ψ, P) **in** *freshFrame*) *auto*
from ‹*guarded P*› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **by**(*drule-tac guardedStatEq*) *auto*
with ‹$\Psi \rhd P \longmapsto \tau \prec P'$› **have** $\Psi \otimes \Psi_P \rhd P \longmapsto \tau \prec P'$
by(*rule-tac statEqTransition*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)
hence $\Psi \rhd P \parallel P \longmapsto \tau \prec P' \parallel P$ **using** *FrP* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› **by**(*rule-tac Par1*) *auto*
moreover from ‹*guarded P*› **have** $P' \parallel !P \sim (P' \parallel P) \parallel (P \parallel !P)$
by(*metis bisimParPresSym bisimParAssoc bisimTransitive bisimParComm bangExt*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 P'*)
then obtain *n Q* **where** *PTrans*: $\Psi \rhd P \parallel P \longmapsto \tau \prec Q$ **and** $P' \sim Q \parallel !P$ **by** *blast*
from ‹$P' \sim Q \parallel !P$› **have** $P \parallel P' \sim Q \parallel (P \parallel !P)$
by(*metis bisimParPresSym bisimParAssoc bisimTransitive bisimParComm*)
with *PTrans* **show** *?case* **by** *blast*
next
case(*cComm1 M N P' K xvec P''*)
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P\rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* P$ **and** $A_P \sharp* M$
by(*rule-tac C*=(Ψ, P, M) **in** *freshFrame*) *auto*
from ‹*guarded P*› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **by**(*drule-tac guardedStatEq*) *auto*
obtain $A_P'\ \Psi_P'$ **where** *FrP'*: *extractFrame* $P = \langle A_P', \Psi_P'\rangle$ **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* P$ **and** $A_P' \sharp* K$ **and** $A_P' \sharp* A_P$
by(*rule-tac C*=(Ψ, P, K, A_P) **in** *freshFrame*) *auto*
from ‹*guarded P*› *FrP'* **have** $\Psi_P' \simeq \mathbf{1}$ **by**(*drule-tac guardedStatEq*) *auto*
with ‹$\Psi \rhd P \longmapsto M(\!|N|\!) \prec P'$› **have** $\Psi \otimes \Psi_P' \rhd P \longmapsto M(\!|N|\!) \prec P'$
by(*rule-tac statEqTransition*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)
moreover from ‹$\Psi \rhd !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P''$› ‹*guarded P*› ‹*xvec* $\sharp*$ *P*› ‹*xvec* $\sharp*$ *K*›
obtain *Q* **where** *PTrans*: $\Psi \rhd P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q$ **and** $P'' \sim Q \parallel !P$
by(*drule-tac bangActE*) *auto*
from *PTrans* ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \rhd P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec Q$
by(*rule-tac statEqTransition*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)
moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› ‹$\Psi_P' \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \Psi_P' \vdash M \leftrightarrow K$
by(*rule-tac statEqEnt*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)

ultimately have $\Psi \rhd P \parallel P \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q)$ **using** *FrP FrP'* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P' \sharp* \Psi$› ‹$A_P' \sharp* P$› ‹$A_P' \sharp* K$› ‹$A_P' \sharp* A_P$› ‹*xvec* $\sharp* P$›

by(*rule-tac Comm1*) (*assumption* | *simp*)+

moreover from ‹$P'' \sim Q \parallel !P$› ‹*guarded P*› **have** $P' \parallel P'' \sim (P' \parallel Q) \parallel (P \parallel !P)$

by(*metis bisimTransitive bangExt bisimParPresSym bisimParAssoc bisimSymmetric*)

hence $(\!|\nu * xvec|\!)(P' \parallel P'') \sim (\!|\nu * xvec|\!)((P' \parallel Q) \parallel (P \parallel !P))$ **by**(*rule-tac bisimResChainPres*) *auto*

with ‹*xvec* $\sharp* P$› ‹*xvec* $\sharp* \Psi$› **have** $(\!|\nu * xvec|\!)(P' \parallel P'') \sim ((\!|\nu * xvec|\!)(P' \parallel Q)) \parallel (P \parallel !P)$

by(*force intro*: *bisimTransitive bisimScopeExtChainSym*)

ultimately show *?case* **by** *blast*

next

case(*cComm2 M N P' K xvec P''*)

obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **and** $A_P \sharp* \Psi$ **and** $A_P \sharp* P$ **and** $A_P \sharp* M$

by(*rule-tac* C=(Ψ, P, M) **in** *freshFrame*) *auto*

from ‹*guarded P*› *FrP* **have** $\Psi_P \simeq \mathbf{1}$ **by**(*drule-tac guardedStatEq*) *auto*

obtain A_P' Ψ_P' **where** *FrP'*: *extractFrame* $P = \langle A_P', \Psi_P' \rangle$ **and** $A_P' \sharp* \Psi$ **and** $A_P' \sharp* P$ **and** $A_P' \sharp* K$ **and** $A_P' \sharp* A_P$

by(*rule-tac* C=(Ψ, P, K, A_P) **in** *freshFrame*) *auto*

from ‹*guarded P*› *FrP'* **have** $\Psi_P' \simeq \mathbf{1}$ **by**(*drule-tac guardedStatEq*) *auto*

with ‹$\Psi \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P'$› **have** $\Psi \otimes \Psi_P' \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N \rangle \prec P'$

by(*rule-tac statEqTransition*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)

moreover from ‹$\Psi \rhd !P \longmapsto K(\!|N|\!) \prec P''$› ‹*guarded P*›

obtain Q **where** *PTrans*: $\Psi \rhd P \longmapsto K(\!|N|\!) \prec Q$ **and** $P'' \sim Q \parallel !P$ **by**(*rule-tac bangActE*) *auto*

from *PTrans* ‹$\Psi_P \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \rhd P \longmapsto K(\!|N|\!) \prec Q$

by(*rule-tac statEqTransition*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)

moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹$\Psi_P \simeq \mathbf{1}$› ‹$\Psi_P' \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \Psi_P' \vdash M \leftrightarrow K$

by(*rule-tac statEqEnt*, *auto*) (*metis Identity AssertionStatEqSym compositionSym AssertionStatEqTrans*)

ultimately have $\Psi \rhd P \parallel P \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel Q)$ **using** *FrP FrP'* ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P' \sharp* \Psi$› ‹$A_P' \sharp* P$› ‹$A_P' \sharp* K$› ‹$A_P' \sharp* A_P$› ‹*xvec* $\sharp* P$›

by(*rule-tac Comm2*) (*assumption* | *simp*)+

moreover from ‹$P'' \sim Q \parallel !P$› ‹*guarded P*› **have** $P' \parallel P'' \sim (P' \parallel Q) \parallel (P \parallel !P)$

by(*metis bisimTransitive bangExt bisimParPresSym bisimParAssoc bisimSymmetric*)

hence $(\!|\nu * xvec|\!)(P' \parallel P'') \sim (\!|\nu * xvec|\!)((P' \parallel Q) \parallel (P \parallel !P))$ **by**(*rule-tac bisimResChainPres*) *auto*

with ‹*xvec* $\sharp* P$› ‹*xvec* $\sharp* \Psi$› **have** $(\!|\nu * xvec|\!)(P' \parallel P'') \sim ((\!|\nu * xvec|\!)(P' \parallel Q)) \parallel$

$(P \parallel !P)$
by(*force intro*: *bisimTransitive bisimScopeExtChainSym*)
ultimately show *?case* **by** *blast*
next
case(*cBang P'*)
then obtain Q **where** *PTrans*: $\Psi \rhd P \parallel P \longmapsto\tau \prec Q$ **and** $P' \sim Q \parallel (P \parallel !P)$
by *blast*
from ‹$P' \sim Q \parallel (P \parallel !P)$› ‹*guarded P*› **have** $P' \sim Q \parallel !P$ **by**(*metis bisimTransitive bisimParPresSym bangExt bisimSymmetric*)
with *PTrans* **show** *?case* **by** *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *tauBangI*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \parallel P \longmapsto\tau \prec P'$
and *guarded P*

obtains Q **where** $\Psi \rhd !P \longmapsto\tau \prec Q$ **and** $Q \sim P' \parallel !P$
proof −
assume $\bigwedge Q.\ [\![\Psi \rhd !P \longmapsto\tau \prec Q;\ Q \sim P' \parallel !P]\!] \Longrightarrow$ *thesis*
moreover from ‹$\Psi \rhd P \parallel P \longmapsto\tau \prec P'$› **have** $\exists Q.\ \Psi \rhd !P \longmapsto\tau \prec Q \wedge Q \sim P' \parallel !P$
proof(*induct rule*: *parTauCases*[**where** *C*=()])
case(*cPar1 P' A*$_P$ Ψ_P)
from ‹$\Psi \otimes \Psi_P \rhd P \longmapsto\tau \prec P'$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \rhd P \longmapsto\tau \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_P \rhd P \parallel !P \longmapsto\tau \prec P' \parallel !P$ **by**(*rule-tac Par1*) *auto*
hence $\Psi \otimes \Psi_P \rhd !P \longmapsto\tau \prec P' \parallel !P$ **using** ‹*guarded P*› **by**(*rule Bang*)
hence $\Psi \rhd P \parallel !P \longmapsto\tau \prec P \parallel (P' \parallel !P)$ **using** ‹*extractFrame P* = $\langle A_P, \Psi_P\rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$›
by(*rule-tac Par2*) *auto*
hence $\Psi \rhd !P \longmapsto\tau \prec P \parallel (P' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover have $P \parallel (P' \parallel !P) \sim P' \parallel P \parallel !P$
by(*metis bisimParAssoc bisimParComm bisimTransitive bisimSymmetric bisimParPres*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 P' A*$_P$ Ψ_P)
from ‹$\Psi \otimes \Psi_P \rhd P \longmapsto\tau \prec P'$› **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \rhd P \longmapsto\tau \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_P \rhd P \parallel !P \longmapsto\tau \prec P' \parallel !P$ **by**(*rule-tac Par1*) *auto*
hence $\Psi \otimes \Psi_P \rhd !P \longmapsto\tau \prec P' \parallel !P$ **using** ‹*guarded P*› **by**(*rule Bang*)
hence $\Psi \rhd P \parallel !P \longmapsto\tau \prec P \parallel (P' \parallel !P)$ **using** ‹*extractFrame P* = $\langle A_P, \Psi_P\rangle$› ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$›

by(*rule-tac Par2*) *auto*
hence $\Psi \rhd !P \longmapsto\tau \prec P \parallel (P' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover have $P \parallel (P' \parallel !P) \sim P \parallel P' \parallel !P$
by(*metis bisimParAssoc bisimSymmetric*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* $\Psi_P{'}$ M N P' A_P Ψ_P K *xvec* P'' $A_P{'}$)
from ‹*extractFrame* $P = \langle A_P', \Psi_P{'}\rangle$› ‹*guarded P*› **have** $\Psi_P{'} \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
with $\langle\Psi \otimes \Psi_P{'} \rhd P \longmapsto M(\!|N|\!) \prec P'\rangle$ **have** $\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|N|\!) \prec P'$
by(*rule-tac statEqTransition*, *auto*) (*metis compositionSym AssertionStatEqSym*)
moreover note ‹*extractFrame P* = $\langle A_P, \Psi_P\rangle$›
moreover from $\langle\Psi \otimes \Psi_P \rhd P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P''\rangle$ **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \rhd P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P''$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_P \rhd P \parallel !P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P'' \parallel !P)$ **using** ‹*xvec* $\sharp*$ *P*› **by**(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_P \rhd !P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P'' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover from $\langle\Psi \otimes \Psi_P \otimes \Psi_P{'} \vdash M \leftrightarrow K\rangle$ $\langle\Psi_P{'} \simeq \mathbf{1}\rangle$ **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K$
by(*rule-tac statEqEnt*, *auto*) (*metis compositionSym AssertionStatEqSym*)
ultimately have $\Psi \rhd P \parallel !P \longmapsto\tau \prec (\!|\nu{*}xvec|\!)(P' \parallel (P'' \parallel !P))$
using $\langle A_P \sharp* \Psi\rangle$ $\langle A_P \sharp* P\rangle$ $\langle A_P \sharp* M\rangle$ $\langle A_P \sharp* A_P{'}\rangle$ $\langle A_P{'} \sharp* \Psi\rangle$ $\langle A_P{'} \sharp* P\rangle$ $\langle A_P{'} \sharp* K\rangle$ ‹*xvec* $\sharp*$ *P*›
by(*force intro*: *Comm1*)
hence $\Psi \rhd !P \longmapsto\tau \prec (\!|\nu{*}xvec|\!)(P' \parallel (P'' \parallel !P))$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover have $(\!|\nu{*}xvec|\!)(P' \parallel (P'' \parallel !P)) \sim ((\!|\nu{*}xvec|\!)(P' \parallel P'')) \parallel !P$
proof −
have $(\!|\nu{*}xvec|\!)(P' \parallel (P'' \parallel !P)) \sim (\!|\nu{*}xvec|\!)((P' \parallel P'') \parallel !P)$
by(*force intro*: *bisimResChainPres bisimParAssoc*[*THEN bisimSymmetric*])
moreover have $(\!|\nu{*}xvec|\!)((P' \parallel P'') \parallel !P) \sim ((\!|\nu{*}xvec|\!)(P' \parallel P'')) \parallel !P$ **using** ‹*xvec* $\sharp*$ *P*›
by(*rule-tac bisimScopeExtChainSym*) *auto*
ultimately show *?thesis* **by**(*rule bisimTransitive*)
qed
ultimately show *?case* **by** *blast*
next
case(*cComm2* $\Psi_P{'}$ M *xvec* N P' A_P Ψ_P K P'' $A_P{'}$)
from ‹*extractFrame* $P = \langle A_P', \Psi_P{'}\rangle$› ‹*guarded P*› **have** $\Psi_P{'} \simeq \mathbf{1}$ **by**(*blast dest*: *guardedStatEq*)
with $\langle\Psi \otimes \Psi_P{'} \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'\rangle$ **have** $\Psi \otimes \mathbf{1} \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
by(*rule-tac statEqTransition*, *auto*) (*metis compositionSym AssertionStatEqSym*)
moreover note ‹*extractFrame P* = $\langle A_P, \Psi_P\rangle$›
moreover from $\langle\Psi \otimes \Psi_P \rhd P \longmapsto K(\!|N|\!) \prec P''\rangle$ **have** $(\Psi \otimes \Psi_P) \otimes \mathbf{1} \rhd P$

$\longmapsto K(\!|N|\!) \prec P''$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_P \rhd P \parallel !P \longmapsto K(\!|N|\!) \prec (P'' \parallel !P)$ **by**(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_P \rhd !P \longmapsto K(\!|N|\!) \prec (P'' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover from ‹$\Psi \otimes \Psi_P \otimes {\Psi_P}' \vdash M \leftrightarrow K$› ‹${\Psi_P}' \simeq \mathbf{1}$› **have** $\Psi \otimes \Psi_P \otimes \mathbf{1} \vdash M \leftrightarrow K$
by(*rule-tac statEqEnt*, *auto*) (*metis compositionSym AssertionStatEqSym*)
ultimately have $\Psi \rhd P \parallel !P \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel (P'' \parallel !P))$
using ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* P$› ‹$A_P \sharp* M$› ‹$A_P \sharp* {A_P}'$› ‹${A_P}' \sharp* \Psi$› ‹${A_P}' \sharp* P$› ‹${A_P}' \sharp* K$› ‹$xvec \sharp* P$›
by(*force intro*: *Comm2*)
hence $\Psi \rhd !P \longmapsto \tau \prec (\!|\nu * xvec|\!)(P' \parallel (P'' \parallel !P))$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover have $(\!|\nu * xvec|\!)(P' \parallel (P'' \parallel !P)) \sim ((\!|\nu * xvec|\!)(P' \parallel P'')) \parallel !P$
proof −
have $(\!|\nu * xvec|\!)(P' \parallel (P'' \parallel !P)) \sim (\!|\nu * xvec|\!)((P' \parallel P'') \parallel !P)$
by(*force intro*: *bisimResChainPres bisimParAssoc*[*THEN bisimSymmetric*])
moreover have $(\!|\nu * xvec|\!)((P' \parallel P'') \parallel !P) \sim ((\!|\nu * xvec|\!)(P' \parallel P'')) \parallel !P$ **using** ‹$xvec \sharp* P$›
by(*rule-tac bisimScopeExtChainSym*) *auto*
ultimately show *?thesis* **by**(*rule bisimTransitive*)
qed
ultimately show *?case* **by** *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *tauChainBangI*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \parallel P \Longrightarrow\!\!\hat{}_\tau\ P'$
and *guarded P*

obtains Q **where** $\Psi \rhd !P \Longrightarrow\!\!\hat{}_\tau\ Q$ **and** $\Psi \rhd Q \sim P' \parallel !P$
proof −
assume $\bigwedge Q.\ [\![\Psi \rhd !P \Longrightarrow\!\!\hat{}_\tau\ Q;\ \Psi \rhd Q \sim P' \parallel !P]\!] \Longrightarrow thesis$
moreover from ‹$\Psi \rhd P \parallel P \Longrightarrow\!\!\hat{}_\tau\ P'$› **have** $\exists Q.\ \Psi \rhd !P \Longrightarrow\!\!\hat{}_\tau\ Q \wedge \Psi \rhd Q \sim P' \parallel !P$
proof(*induct x1*==$P \parallel P\ P'$ *rule*: *tauChainInduct*)
case *TauBase*
have $\Psi \rhd !P \Longrightarrow\!\!\hat{}_\tau\ !P$ **by** *simp*
moreover have $\Psi \rhd !P \sim (P \parallel P) \parallel !P$ **using** ‹*guarded P*›
by(*metis bisimParAssoc bisimTransitive bisimParPresSym bangExt bisimParComm*)
ultimately show *?case* **by** *blast*
next
case(*TauStep R' R''*)

then obtain Q **where** *PChain*: $\Psi \rhd !P \Longrightarrow\hat{}_{\tau}\ Q$ **and** $\Psi \rhd Q \sim R' \parallel !P$ **by** *auto*
from $\langle \Psi \rhd R' \longmapsto \tau \prec R'' \rangle$ **have** $\Psi \otimes \mathbf{1} \rhd R' \longmapsto \tau \prec R''$ **by**(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \rhd R' \parallel !P \longmapsto \tau \prec R'' \parallel !P$ **by**(*rule-tac Par1*) *auto*
with $\langle \Psi \rhd Q \sim R' \parallel !P \rangle$ **obtain** Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto \tau \prec Q'$ **and** $\Psi \rhd Q' \sim R'' \parallel !P$
by(*force dest*: *bisimE*(*2*) *simE*)
from *PChain QTrans* **have** $\Psi \rhd !P \Longrightarrow\hat{}_{\tau}\ Q'$ **by**(*auto dest*: *tauActTauChain*)
thus *?case* **using** $\langle \Psi \rhd Q' \sim R'' \parallel !P \rangle$ **by** *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *weakBisimBangPresAux*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \approx Q$
and *guarded P*
and *guarded Q*

shows $\Psi \rhd R \parallel !P \approx R \parallel !Q$
proof −
let *?X* = $\{(\Psi, R \parallel !P, R \parallel !Q) \mid \Psi\ R\ P\ Q.\ guarded\ P \wedge guarded\ Q \wedge \Psi \rhd P \approx Q\}$
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists R\ S.\ \Psi \rhd P \approx R \wedge (\Psi, R, S) \in ?X \wedge \Psi \rhd S \sim Q\}$

from *assms* **have** $(\Psi, R \parallel !P, R \parallel !Q) \in ?X$ **by** *auto*
moreover have *eqvt ?X*
by(*fastforce simp add*: *eqvt-def intro*: *weakBisimClosed*)
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct2*)
case(*cStatImp* Ψ P Q)
thus *?case* **by**(*force dest*: *weakBisimE*(*3*) *simp add*: *weakStatImp-def*)
next
case(*cSim* Ψ P Q)
moreover {
fix Ψ P Q R
assume $\Psi \rhd P \approx Q$
moreover have *eqvt ?Y*
apply(*auto simp add*: *eqvt-def*)
apply(*rule-tac x*=$p \cdot (Ra \parallel !P)$ **in** *exI*, *auto*)
apply(*fastforce dest*: *weakBisimClosed simp add*: *eqvts*)
apply(*rule-tac x*=$(p \cdot Ra) \parallel !(p \cdot Q)$ **in** *exI*, *auto*)
apply(*rule-tac x*=$p \cdot Ra$ **in** *exI*)

apply(*rule-tac x=p · P* **in** *exI*, *auto*)
apply(*rule-tac x=p · Q* **in** *exI*, *auto*)
apply(*blast intro*: *weakBisimClosed*)
by(*fastforce dest*: *bisimClosed simp add*: *eqvts*)
moreover assume *guarded P* **and** *guarded Q*
moreover note *weakBisimClosed bisimClosed weakBisimE*(*3*) *bisimE*(*3*) *weakBisimE*(*2*) *weakBisimE*(*4*) *bisimE*(*4*) *statEqWeakBisim statEqBisim weakBisimTransitive bisimTransitive weakBisimParAssoc*[*THEN weakBisimE*(*4*)] *bisimParAssoc*[*THEN bisimE*(*4*)] *weakBisimParPres*
moreover have $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \approx Q \Longrightarrow \Psi \rhd P \parallel P \approx Q \parallel Q$
by(*metis weakBisimParPres weakBisimParComm weakBisimE*(*4*) *weakBisimTransitive*)
moreover note *bisimParPresSym*
moreover have $bisim \subseteq weakBisim$ **by**(*auto dest*: *strongBisimWeakBisim*)
moreover have $\bigwedge \Psi\ \Psi_R\ P\ Q\ R\ A_R.\ [\![\Psi \otimes \Psi_R \rhd P \approx Q;\ extractFrame\ R = \langle A_R, \Psi_R\rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q]\!] \Longrightarrow \Psi \rhd R \parallel P \approx R \parallel Q$
by(*metis weakBisimParComm weakBisimTransitive weakBisimParPresAux*)
moreover note *weakBisimResChainPres bisimResChainPres weakBisimScopeExtChainSym bisimScopeExtChainSym*
moreover have $\bigwedge \Psi\ P\ R\ S\ Q.\ [\![\Psi \rhd P \approx R;\ (\Psi, R, S) \in ?Y;\ \Psi \rhd S \sim Q]\!] \Longrightarrow (\Psi, P, Q) \in ?Y$
by(*blast dest*: *weakBisimTransitive bisimTransitive*)
moreover have $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \approx Q;\ guarded\ P;\ guarded\ Q]\!] \Longrightarrow (\Psi, R \parallel\ !P, R \parallel\ !Q) \in ?Y$
by(*blast intro*: *bisimReflexive weakBisimReflexive*)
moreover from *bangActE* **have** $\bigwedge \Psi\ P\ \alpha\ P'.\ [\![\Psi \rhd\ !P \longmapsto \alpha \prec P';\ bn\ \alpha \sharp* P;\ guarded\ P;\ \alpha \neq \tau;\ bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow \exists Q.\ \Psi \rhd P \longmapsto \alpha \prec Q \wedge P' \sim Q \parallel\ !P$
by *blast*
moreover from *bangTauE* **have** $\bigwedge \Psi\ P\ P'.\ [\![\Psi \rhd\ !P \longmapsto \tau \prec P';\ guarded\ P]\!] \Longrightarrow \exists Q.\ \Psi \rhd P \parallel P \longmapsto \tau \prec Q \wedge P' \sim Q \parallel\ !P$
by *blast*
moreover from *tauChainBangI* **have** $\bigwedge \Psi\ P\ P'.\ [\![\Psi \rhd P \parallel P \Longrightarrow\hat{}_\tau\ P';\ guarded\ P]\!] \Longrightarrow \exists Q.\ \Psi \rhd\ !P \Longrightarrow\hat{}_\tau\ Q \wedge \Psi \rhd Q \sim P' \parallel\ !P$
by *blast*
ultimately have $\Psi \rhd R \parallel\ !P \rightsquigarrow <?Y>\ R \parallel\ !Q$
by(*rule-tac weakSimBangPres*)
}
ultimately show *?case* **by** *blast*
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*blast dest*: *weakBisimE*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by**(*blast dest*: *weakBisimE*)
qed
qed

lemma *weakBisimBangPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \approx Q$
and *guarded P*
and *guarded Q*

shows $\Psi \rhd !P \approx !Q$
proof −
from *assms* **have** $\Psi \rhd \mathbf{0} \parallel !P \approx \mathbf{0} \parallel !Q$ **by**(*rule weakBisimBangPresAux*)
thus *?thesis*
by(*metis weakBisimParNil weakBisimParComm weakBisimTransitive weakBisimE*(*4*))
qed

end

end

theory *Close-Subst*
imports *Agent*
begin

context *substPsi*
begin

definition *closeSubst* :: (*'b*::*fs-name* × (*'a*::*fs-name*, *'b*, *'c*::*fs-name*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set* ⇒ (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
where *closeSubst Rel* ≡ {(Ψ, *P*, *Q*) | Ψ *P Q*. (∀σ. *wellFormedSubst* σ ⟶ (Ψ, *P*[<σ>], *Q*[<σ>]) ∈ *Rel*)}

lemma *closeSubstI*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes ⋀σ. *wellFormedSubst* σ ⟹ (Ψ, *P*[<σ>], *Q*[<σ>]) ∈ *Rel*

shows (Ψ, *P*, *Q*) ∈ *closeSubst Rel*
using *assms*
by(*unfold closeSubst-def*) *auto*

lemma *closeSubstE*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and σ :: (*name list* × *'a list*) *list*

assumes $(\Psi, P, Q) \in closeSubst\ Rel$
and $wellFormedSubst\ \sigma$

shows $(\Psi, P[<\sigma>], Q[<\sigma>]) \in Rel$
using *assms*
by(*unfold closeSubst-def*) *auto*

lemma *closeSubstClosed*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $p :: name\ prm$

assumes *eqvt Rel*
and $(\Psi, P, Q) \in closeSubst\ Rel$

shows $(p \cdot \Psi, p \cdot P, p \cdot Q) \in closeSubst\ Rel$
proof(*rule closeSubstI*)
fix σ
assume $wellFormedSubst(\sigma::(name\ list \times 'a\ list)\ list)$
with $\langle(\Psi, P, Q) \in closeSubst\ Rel\rangle\ \langle wellFormedSubst\ \sigma\rangle$
have $(\Psi, P[<(rev\ p \cdot \sigma)>], Q[<(rev\ p \cdot \sigma)>]) \in Rel$
by(*rule-tac closeSubstE*) *auto*
hence $(p \cdot \Psi, p \cdot (P[<(rev\ p \cdot \sigma)>]), p \cdot (Q[<(rev\ p \cdot \sigma)>])) \in Rel$
by(*drule-tac p=p* **in** *eqvtI*[*OF* $\langle eqvt\ Rel\rangle$]) (*simp add*: *eqvts*)
thus $(p \cdot \Psi, (p \cdot P)[<\sigma>], (p \cdot Q)[<\sigma>]) \in Rel$
by(*simp del*: *seqSubs-def add*: *eqvts*)
qed

lemma *closeSubstEqvt*:
assumes *eqvt Rel*

shows *eqvt*(*closeSubst Rel*)
proof(*auto simp add*: *eqvt-def*)
fix $\Psi\ P\ Q\ p$
assume $(\Psi, P, Q) \in closeSubst\ Rel$
thus $((p::name\ prm) \cdot \Psi, p \cdot P, p \cdot Q) \in closeSubst\ Rel$
by(*drule-tac p=p* **in** *closeSubstClosed*[*OF* $\langle eqvt\ Rel\rangle$]) (*simp add*: *eqvts*)
qed

lemma *closeSubstUnfold*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\sigma :: (name\ list \times 'a\ list)\ list$

assumes $(\Psi, P, Q) \in closeSubst\ Rel$
and $wellFormedSubst\ \sigma$

shows $(\Psi, P[<\sigma>], Q[<\sigma>]) \in closeSubst\ Rel$
proof(*rule closeSubstI*)
fix σ'::(*name list* × $'a$ *list*) *list*
assume *wellFormedSubst* σ'
with ‹*wellFormedSubst* σ› **have** *wellFormedSubst*$(\sigma@\sigma')$ **by** *simp*
with ‹$(\Psi, P, Q) \in closeSubst\ Rel$› **have** $(\Psi, P[<(\sigma@\sigma')>], Q[<(\sigma@\sigma')>]) \in Rel$
by(*rule closeSubstE*)
thus $(\Psi, P[<\sigma>][<\sigma'>], Q[<\sigma>][<\sigma'>]) \in Rel$
by *simp*
qed

end

end

theory *Bisim-Subst*
imports *Bisim-Struct-Cong Close-Subst*
begin

context *env* **begin**

abbreviation
bisimSubstJudge (- ▷ - ∼$_s$ - [*70*, *70*, *70*] *65*) **where** $\Psi \rhd P \sim_s Q \equiv (\Psi, P, Q) \in closeSubst\ bisim$
abbreviation
bisimSubstNilJudge (- ∼$_s$ - [*70*, *70*] *65*) **where** $P \sim_s Q \equiv SBottom' \rhd P \sim_s Q$

lemmas *bisimSubstClosed*[*eqvt*] = *closeSubstClosed*[*OF bisimEqvt*]
lemmas *bisimSubstEqvt*[*simp*] = *closeSubstEqvt*[*OF bisimEqvt*]

lemma *bisimSubstOutputPres*:
fixes Ψ :: $'b$
and P :: ($'a$, $'b$, $'c$) *psi*
and Q :: ($'a$, $'b$, $'c$) *psi*
and M :: $'a$
and N :: $'a$

assumes $\Psi \rhd P \sim_s Q$

shows $\Psi \rhd M\langle N\rangle.P \sim_s M\langle N\rangle.Q$
using *assms*
by(*fastforce intro*: *closeSubstI closeSubstE bisimOutputPres*)

lemma *seqSubstInputChain*[*simp*]:
fixes *xvec* :: *name list*
and N :: $'a$
and P :: ($'a$, $'b$, $'c$) *psi*

and σ :: $(name\ list \times 'a\ list)\ list$

assumes $xvec \sharp* \sigma$

shows $seqSubs'\ (inputChain\ xvec\ N\ P)\ \sigma = inputChain\ xvec\ (substTerm.seqSubst\ N\ \sigma)\ (seqSubs\ P\ \sigma)$
using *assms*
by(*induct xvec*) *auto*

lemma *bisimSubstInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and Q :: $('a, 'b, 'c)\ psi$
and M :: $'a$
and $xvec$ :: $name\ list$
and N :: $'a$

assumes $\Psi \rhd P \sim_s Q$
and $xvec \sharp* \Psi$
and *distinct xvec*

shows $\Psi \rhd M(\!|\lambda* xvec\ N|\!).P \sim_s M(\!|\lambda* xvec\ N|\!).Q$
proof(*rule-tac closeSubstI*)
fix σ
assume *wellFormedSubst*(σ::$(name\ list \times 'a\ list)\ list$)
obtain p **where** $(p \cdot xvec) \sharp* \sigma$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* N$
and S: $set\ p \subseteq set\ xvec \times set\ (p \cdot xvec)$
by(*rule-tac* c=(σ, P, Q, Ψ, N) **in** *name-list-avoiding*) *auto*

from ‹$\Psi \rhd P \sim_s Q$› **have** $(p \cdot \Psi) \rhd (p \cdot P) \sim_s (p \cdot Q)$
by(*rule bisimSubstClosed*)
with ‹$xvec \sharp* \Psi$› ‹$(p \cdot xvec) \sharp* \Psi$› S **have** $\Psi \rhd (p \cdot P) \sim_s (p \cdot Q)$
by *simp*

{
fix $Tvec$ :: $'a\ list$
from ‹$\Psi \rhd (p \cdot P) \sim_s (p \cdot Q)$› ‹*wellFormedSubst* σ› **have** $\Psi \rhd (p \cdot P)[<\sigma>] \sim_s (p \cdot Q)[<\sigma>]$
by(*rule closeSubstUnfold*)
moreover assume *length xvec* = *length Tvec* **and** *distinct xvec*
ultimately have $\Psi \rhd ((p \cdot P)[<\sigma>])[(p \cdot xvec)::=Tvec] \sim ((p \cdot Q)[<\sigma>])[(p \cdot xvec)::=Tvec]$
by(*drule-tac closeSubstE*[**where** σ=[((p · xvec), Tvec)]]) *auto*
}

with ‹$(p \cdot xvec) \sharp* \sigma$› ‹*distinct xvec*›
have $\Psi \rhd (M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P))[<\sigma>] \sim (M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p$

$\cdot$ Q))[<σ>]
  by(*force intro*: *bisimInputPres*)
 moreover from ‹(p $\cdot$ *xvec*) $\sharp*$ N› ‹(p $\cdot$ *xvec*) $\sharp*$ P› S **have** $M\langle\!\langle\lambda*(p \cdot xvec)\ (p \cdot N)\rangle\!\rangle.(p \cdot P) = M\langle\!\langle\lambda*xvec\ N\rangle\!\rangle.P$
  apply(*simp add*: *psi.inject*) **by**(*rule inputChainAlpha*[*symmetric*]) *auto*
 moreover from ‹(p $\cdot$ *xvec*) $\sharp*$ N› ‹(p $\cdot$ *xvec*) $\sharp*$ Q› S **have** $M\langle\!\langle\lambda*(p \cdot xvec)\ (p \cdot N)\rangle\!\rangle.(p \cdot Q) = M\langle\!\langle\lambda*xvec\ N\rangle\!\rangle.Q$
  apply(*simp add*: *psi.inject*) **by**(*rule inputChainAlpha*[*symmetric*]) *auto*
 ultimately show $\Psi \rhd (M\langle\!\langle\lambda*xvec\ N\rangle\!\rangle.P)[<\sigma>] \sim (M\langle\!\langle\lambda*xvec\ N\rangle\!\rangle.Q)[<\sigma>]$
  by *force*
qed

lemma *bisimSubstCasePresAux*:
 fixes Ψ :: $'b$
 and *CsP* :: ($'c \times$ ($'a$, $'b$, $'c$) *psi*) *list*
 and *CsQ* :: ($'c \times$ ($'a$, $'b$, $'c$) *psi*) *list*

 assumes *C1*: $\bigwedge\varphi\ P.\ (\varphi, P)$ *mem CsP* $\Longrightarrow \exists\, Q.\ (\varphi, Q)$ *mem CsQ* $\wedge$ *guarded* Q $\wedge\ \Psi \rhd P \sim_s Q$
 and *C2*: $\bigwedge\varphi\ Q.\ (\varphi, Q)$ *mem CsQ* $\Longrightarrow \exists\, P.\ (\varphi, P)$ *mem CsP* $\wedge$ *guarded* P $\wedge$ $\Psi \rhd P \sim_s Q$

 shows $\Psi \rhd$ *Cases CsP* $\sim_s$ *Cases CsQ*
proof $-$
 {
  fix σ :: (*name list* $\times$ $'a$ *list*) *list*

  assume *wellFormedSubst* σ

  have $\Psi \rhd$ *Cases*(*caseListSeqSubst CsP* σ) $\sim$ *Cases*(*caseListSeqSubst CsQ* σ)
  proof(*rule bisimCasePres*)
   fix φ P
   assume (φ, P) *mem* (*caseListSeqSubst CsP* σ)
   then obtain φ' P' **where** (φ', P') *mem CsP* **and** φ = *substCond.seqSubst* φ' σ **and** *PeqP'*: $P = (P'[<\sigma>])$
    by(*induct CsP*) *force*+
   from ‹(φ', P') *mem CsP*› **obtain** Q' **where** (φ', Q') *mem CsQ* **and** *guarded* Q' **and** $\Psi \rhd P' \sim_s Q'$ **by**(*blast dest*: *C1*)
   from ‹(φ', Q') *mem CsQ*› ‹φ = *substCond.seqSubst* φ' σ› **obtain** Q **where** (φ, Q) *mem* (*caseListSeqSubst CsQ* σ) **and** $Q = Q'[<\sigma>]$
    by(*induct CsQ*) *auto*
   with *PeqP'* ‹*guarded* Q'› ‹$\Psi \rhd P' \sim_s Q'$› ‹*wellFormedSubst* σ› **show** $\exists\, Q.$ (φ, Q) *mem* (*caseListSeqSubst CsQ* σ) $\wedge$ *guarded* $Q \wedge \Psi \rhd P \sim Q$
    by(*blast dest*: *closeSubstE guardedSeqSubst*)
  next
   fix φ Q
   assume (φ, Q) *mem* (*caseListSeqSubst CsQ* σ)
   then obtain φ' Q' **where** (φ', Q') *mem CsQ* **and** φ = *substCond.seqSubst* φ' σ **and** *QeqQ'*: $Q = Q'[<\sigma>]$

by(*induct CsQ*) *force+*
from ‹(φ', Q') *mem CsQ*› **obtain** P' **where** (φ', P') *mem CsP* **and** *guarded* P' **and** $\Psi \rhd P' \sim_s Q'$ **by**(*blast dest*: *C2*)
from ‹(φ', P') *mem CsP*› ‹φ = *substCond.seqSubst* φ' σ› **obtain** P **where** (φ, P) *mem* (*caseListSeqSubst CsP* σ) **and** $P = P'[<\sigma>]$
by(*induct CsP*) *auto*
with *QeqQ'* ‹*guarded* P'› ‹$\Psi \rhd P' \sim_s Q'$› ‹*wellFormedSubst* σ› **show** $\exists P.$ (φ, P) *mem* (*caseListSeqSubst CsP* σ) $\wedge$ *guarded* $P \wedge \Psi \rhd P \sim Q$
by(*blast dest*: *closeSubstE guardedSeqSubst*)
qed
}
thus *?thesis*
by(*rule-tac closeSubstI*) *auto*
qed

lemma *bisimSubstReflexive*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

shows $\Psi \rhd P \sim_s P$
by(*auto intro*: *closeSubstI bisimReflexive*)

lemma *bisimSubstTransitive*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and R :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \sim_s Q$
and $\Psi \rhd Q \sim_s R$

shows $\Psi \rhd P \sim_s R$
using *assms*
by(*auto intro*: *closeSubstI closeSubstE bisimTransitive*)

lemma *bisimSubstSymmetric*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \sim_s Q$

shows $\Psi \rhd Q \sim_s P$
using *assms*
by(*auto intro*: *closeSubstI closeSubstE bisimE*)

lemma *bisimSubstParPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \sim_s Q$

shows $\Psi \rhd P \parallel R \sim_s Q \parallel R$
using *assms*
by(*fastforce intro*: *closeSubstI closeSubstE bisimParPres*)

lemma *bisimSubstResPres*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $x :: name$

assumes $\Psi \rhd P \sim_s Q$
and $x \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!) P \sim_s (\!|\nu x|\!) Q$
proof(*rule-tac closeSubstI*)
fix $\sigma :: (name\ list \times 'a\ list)\ list$

assume *wellFormedSubst* σ
obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp \sigma$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)

from ‹$\Psi \rhd P \sim_s Q$› **have** $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \sim_s ([(x, y)] \cdot Q)$
by(*rule bisimSubstClosed*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \rhd ([(x, y)] \cdot P) \sim_s ([(x, y)] \cdot Q)$
by *simp*
hence $\Psi \rhd ([(x, y)] \cdot P)[<\sigma>] \sim ([(x, y)] \cdot Q)[<\sigma>]$ **using** ‹*wellFormedSubst* σ›
by(*rule closeSubstE*)
hence $\Psi \rhd (\!|\nu y|\!)(([(x, y)] \cdot P)[<\sigma>]) \sim (\!|\nu y|\!)(([(x, y)] \cdot Q)[<\sigma>])$ **using** ‹$y \sharp \Psi$›
by(*rule bisimResPres*)
with ‹$y \sharp P$› ‹$y \sharp Q$› ‹$y \sharp \sigma$›
show $\Psi \rhd ((\!|\nu x|\!) P)[<\sigma>] \sim ((\!|\nu x|\!) Q)[<\sigma>]$
by(*simp add*: *alphaRes*)
qed

lemma *bisimSubstBangPres*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \sim_s Q$
and *guarded P*
and *guarded Q*

shows $\Psi \rhd !P \sim_s !Q$
using *assms*
by(*fastforce intro*: *closeSubstI closeSubstE bisimBangPres guardedSeqSubst*)

lemma *substNil*[*simp*]:
fixes *xvec* :: *name list*
and *Tvec* :: *'a list*

assumes *wellFormedSubst* σ
and *distinct xvec*

shows $(\mathbf{0}[<\sigma>]) = \mathbf{0}$
using *assms*
by *simp*

lemma *bisimSubstParNil*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*

shows $\Psi \rhd P \parallel \mathbf{0} \sim_s P$
by(*fastforce intro*: *closeSubstI bisimParNil*)

lemma *bisimSubstParComm*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

shows $\Psi \rhd P \parallel Q \sim_s Q \parallel P$
apply(*rule closeSubstI*)
by(*fastforce intro*: *closeSubstI bisimParComm*)

lemma *bisimSubstParAssoc*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *R* :: (*'a*, *'b*, *'c*) *psi*

shows $\Psi \rhd (P \parallel Q) \parallel R \sim_s P \parallel (Q \parallel R)$
apply(*rule closeSubstI*)
by(*fastforce intro*: *closeSubstI bisimParAssoc*)

lemma *bisimSubstResNil*:
fixes Ψ :: *'b*
and *x* :: *name*

shows $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \sim_s \mathbf{0}$
proof(*rule closeSubstI*)
fix σ:: (*name list* × *'a list*) *list*

assume *wellFormedSubst* σ
obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp \sigma$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
have $\Psi \rhd (\!|\nu y|\!)\mathbf{0} \sim \mathbf{0}$ **by**(*rule bisimResNil*)
with ‹$y \sharp \sigma$› ‹*wellFormedSubst* σ› **show** $\Psi \rhd ((\!|\nu x|\!)\mathbf{0})[<\sigma>] \sim \mathbf{0}[<\sigma>]$
by(*subst alphaRes*[*of y*]) *auto*
qed

lemma *seqSubst2*:
fixes x :: *name*
and P :: ('*a*, '*b*, '*c*) *psi*

assumes *wellFormedSubst* σ
and $x \sharp \sigma$
and $x \sharp P$

shows $x \sharp P[<\sigma>]$
using *assms*
by(*induct* σ *arbitrary*: P, *auto*) (*blast dest*: *subst2*)

notation *substTerm.seqSubst* (-[<->] [*100*, *100*] *100*)

lemma *bisimSubstScopeExt*:
fixes Ψ :: '*b*
and x :: *name*
and P :: ('*a*, '*b*, '*c*) *psi*
and Q :: ('*a*, '*b*, '*c*) *psi*

assumes $x \sharp P$

shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \sim_s P \parallel (\!|\nu x|\!)Q$
proof(*rule closeSubstI*)
fix σ:: (*name list* × '*a list*) *list*

assume *wellFormedSubst* σ
obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp \sigma$ **and** $y \sharp P$ **and** $y \sharp Q$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
moreover from ‹*wellFormedSubst* σ› ‹$y \sharp \sigma$› ‹$y \sharp P$› **have** $y \sharp P[<\sigma>]$
by(*rule seqSubst2*)
hence $\Psi \rhd (\!|\nu y|\!)((P[<\sigma>]) \parallel (([(x, y)] \cdot Q)[<\sigma>])) \sim (P[<\sigma>]) \parallel (\!|\nu y|\!)(([(x, y)] \cdot Q)[<\sigma>])$
by(*rule bisimScopeExt*)
with ‹$x \sharp P$› ‹$y \sharp P$› ‹$y \sharp Q$› ‹$y \sharp \sigma$› **show** $\Psi \rhd ((\!|\nu x|\!)(P \parallel Q))[<\sigma>] \sim (P \parallel (\!|\nu x|\!)Q)[<\sigma>]$
apply(*subst alphaRes*[*of y*], *simp*)
apply(*subst alphaRes*[*of y Q*], *simp*)
by(*simp add*: *eqvts*)
qed

lemma *bisimSubstCasePushRes*:
fixes x :: *name*
and Ψ :: $'b$
and Cs :: $('c \times ('a, 'b, 'c)\ psi)\ list$

assumes $x \sharp map\ fst\ Cs$

shows $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \sim_s Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs$
proof(*rule closeSubstI*)
fix σ:: $(name\ list \times 'a\ list)\ list$

assume *wellFormedSubst* σ
obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp \sigma$ **and** $y \sharp Cs$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)

{
fix x :: *name*
and Cs :: $('c \times ('a, 'b, 'c)\ psi)\ list$
and σ :: $(name\ list \times 'a\ list)\ list$

assume $x \sharp \sigma$

hence (*Cases map* $(\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)[<\sigma>] = Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ (caseListSeqSubst\ Cs\ \sigma)$
by(*induct Cs*) *auto*
}
note *C1* = *this*

{
fix x :: *name*
and y :: *name*
and Cs :: $('c \times ('a, 'b, 'c)\ psi)\ list$

assume $x \sharp map\ fst\ Cs$
and $y \sharp map\ fst\ Cs$
and $y \sharp Cs$

hence $(Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs) = Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu y|\!)P))\ ([(x, y)] \cdot Cs)$
by(*induct Cs*) (*auto simp add*: *fresh-list-cons alphaRes*)
}
note *C2* = *this*

from ‹$y \sharp Cs$› **have** $y \sharp map\ fst\ Cs$ **by**(*induct Cs*) (*auto simp add*: *fresh-list-cons fresh-list-nil*)
from ‹$y \sharp Cs$› ‹$y \sharp \sigma$› ‹$x \sharp map\ fst\ Cs$› ‹*wellFormedSubst* σ› **have** $y \sharp map\ fst\ (caseListSeqSubst\ ([(x, y)] \cdot Cs)\ \sigma)$
by(*induct Cs*) (*auto intro*: *substCond.seqSubst2 simp add*: *fresh-list-cons fresh-list-nil fresh-prod*)

hence $\Psi \rhd (\!|\nu y|\!)(Cases(caseListSeqSubst\ ([(x,\ y)] \cdot Cs)\ \sigma)) \sim Cases\ map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu y|\!)P))\ (caseListSeqSubst\ ([(x,\ y)] \cdot Cs)\ \sigma)$
by(*rule bisimCasePushRes*)

with ‹$y \sharp Cs$› ‹$x \sharp map\ fst\ Cs$› ‹$y \sharp map\ fst\ Cs$› ‹$y \sharp \sigma$› ‹*wellFormedSubst* σ›
show $\Psi \rhd ((\!|\nu x|\!)(Cases\ Cs))[<\sigma>] \sim (Cases\ map\ (\lambda(\varphi,\ P).\ (\varphi,\ (\!|\nu x|\!)P))\ Cs)[<\sigma>]$
apply(*subst C2*[*of x Cs y*])
apply *assumption*+
apply(*subst C1*)
apply *assumption*+
apply(*subst alphaRes*[*of y*], *simp*)
by(*simp add*: *eqvts*)
qed

lemma *bisimSubstOutputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $N :: 'a$
and $P :: ('a,\ 'b,\ 'c)\ psi$

assumes $x \sharp M$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \sim_s M\langle N\rangle.(\!|\nu x|\!)P$
proof(*rule closeSubstI*)
fix σ:: $(name\ list \times 'a\ list)\ list$

assume *wellFormedSubst* σ
obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp \sigma$ **and** $y \sharp P$ **and** $y \sharp M$ **and** $y \sharp N$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
from ‹*wellFormedSubst* σ› ‹$y \sharp M$› ‹$y \sharp \sigma$› **have** $y \sharp M[<\sigma>]$ **by** *auto*
moreover from ‹*wellFormedSubst* σ› ‹$y \sharp N$› ‹$y \sharp \sigma$› **have** $y \sharp N[<\sigma>]$ **by** *auto*
ultimately have $\Psi \rhd (\!|\nu y|\!)((M[<\sigma>])\langle(N[<\sigma>])\rangle.(([(x,\ y)] \cdot P)[<\sigma>])) \sim (M[<\sigma>])\langle(N[<\sigma>])\rangle.((\!|\nu y|\!)(([(x,\ y)] \cdot P)[<\sigma>]))$
by(*rule bisimOutputPushRes*)
with ‹$y \sharp M$› ‹$y \sharp N$› ‹$y \sharp P$› ‹$x \sharp M$› ‹$x \sharp N$› ‹$y \sharp \sigma$› ‹*wellFormedSubst* σ›
show $\Psi \rhd ((\!|\nu x|\!)(M\langle N\rangle.P))[<\sigma>] \sim (M\langle N\rangle.(\!|\nu x|\!)P)[<\sigma>]$
apply(*subst alphaRes*[*of y*], *simp*)
apply(*subst alphaRes*[*of y P*], *simp*)
by(*simp add*: *eqvts*)
qed

lemma *bisimSubstInputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $xvec :: name\ list$

and $N :: 'a$

assumes $x \sharp M$
and $x \sharp xvec$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \sim_s M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
proof(*rule closeSubstI*)
fix $\sigma :: (name\ list \times 'a\ list)\ list$

assume *wellFormedSubst* σ
obtain $y::name$ **where** $y \sharp \Psi$ **and** $y \sharp \sigma$ **and** $y \sharp P$ **and** $y \sharp M$ **and** $y \sharp xvec$ **and** $y \sharp N$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
obtain $p::name\ prm$ **where** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* P$ **and** $x \sharp (p \cdot xvec)$ **and** $y \sharp (p \cdot xvec)$ **and** $(p \cdot xvec) \sharp* \sigma$
and S: $set\ p \subseteq set\ xvec \times set(p \cdot xvec)$
by(*rule-tac* $c=(N, P, x, y, \sigma)$ **in** *name-list-avoiding*) *auto*

from ‹*wellFormedSubst* σ› ‹$y \sharp M$› ‹$y \sharp \sigma$ › **have** $y \sharp M[<\sigma>]$ **by** *auto*
moreover note ‹$y \sharp (p \cdot xvec)$›
moreover from ‹$y \sharp N$› **have** $(p \cdot y) \sharp (p \cdot N)$ **by**(*simp add*: *pt-fresh-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with ‹$y \sharp xvec$› ‹$y \sharp (p \cdot xvec)$› S **have** $y \sharp p \cdot N$ **by** *simp*
with ‹*wellFormedSubst* σ› **have** $y \sharp (p \cdot N)[<\sigma>]$ **using** ‹$y \sharp \sigma$› **by** *auto*
ultimately have $\Psi \rhd (\!|\nu y|\!)((M[<\sigma>])(\!|\lambda * (p \cdot xvec)\ ((p \cdot N)[<\sigma>])|\!).(([[(x, y)] \cdot (p \cdot P))[<\sigma>])) \sim (M[<\sigma>])(\!|\lambda * (p \cdot xvec)\ ((p \cdot N)[<\sigma>])|\!).((\!|\nu y|\!)(([(x, y)] \cdot p \cdot P)[<\sigma>]))$
by(*rule bisimInputPushRes*)
with ‹$y \sharp M$› ‹$y \sharp N$› ‹$y \sharp P$› ‹$x \sharp M$› ‹$x \sharp N$› ‹$y \sharp xvec$› ‹$x \sharp xvec$› ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* P$›
‹$x \sharp (p \cdot xvec)$› ‹$y \sharp (p \cdot xvec)$› ‹$y \sharp \sigma$› ‹$(p \cdot xvec) \sharp* \sigma$› S ‹*wellFormedSubst* σ›
show $\Psi \rhd ((\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P))[<\sigma>] \sim (M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P)[<\sigma>]$
apply(*subst inputChainAlpha'*)
apply *assumption*+
apply(*subst inputChainAlpha'*[*of p xvec*])
apply(*simp add*: *abs-fresh-star*)
apply *assumption*+
apply(*simp add*: *eqvts*)
apply(*subst alphaRes*[*of y*], *simp*)
apply(*simp add*: *inputChainFresh*)
apply(*simp add*: *freshChainSimps*)
apply(*subst alphaRes*[*of y* $(p \cdot P)$])
apply(*simp add*: *freshChainSimps*)
by(*simp add*: *freshChainSimps eqvts*)
qed

lemma *bisimSubstResComm*:

fixes x :: *name*
and y :: *name*

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \sim_s (\!|\nu y|\!)((\!|\nu x|\!)P)$
proof(*case-tac* $x = y$)
assume $x = y$
thus *?thesis* **by**(*force intro*: *bisimSubstReflexive*)
next
assume $x \neq y$
show *?thesis*
proof(*rule closeSubstI*)
fix σ:: (*name list* × *'a list*) *list*
assume *wellFormedSubst* σ

obtain x'::*name* **where** $x' \sharp \Psi$ **and** $x' \sharp \sigma$ **and** $x' \sharp P$ **and** $x \neq x'$ **and** $y \neq x'$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)
obtain y'::*name* **where** $y' \sharp \Psi$ **and** $y' \sharp \sigma$ **and** $y' \sharp P$ **and** $x \neq y'$ **and** $y \neq y'$ **and** $x' \neq y'$
by(*generate-fresh name*) (*auto simp add*: *fresh-prod*)

have $\Psi \rhd (\!|\nu x'|\!)((\!|\nu y'|\!)(([(x, x')] \cdot [(y, y')] \cdot P)[<\sigma>])) \sim (\!|\nu y'|\!)((\!|\nu x'|\!)(([(x, x')] \cdot [(y, y')] \cdot P)[<\sigma>]))$
by(*rule bisimResComm*)
moreover from ‹$x' \sharp P$› ‹$y' \sharp P$› ‹$x \neq y'$› ‹$x' \neq y'$› **have** $(\!|\nu x|\!)((\!|\nu y|\!)P) = (\!|\nu x'|\!)((\!|\nu y'|\!)(([(x, x')] \cdot [(y, y')] \cdot P)))$
apply(*subst alphaRes*[*of* y' P], *simp*)
by(*subst alphaRes*[*of* x']) (*auto simp add*: *abs-fresh fresh-left calc-atm eqvts*)
moreover from ‹$x' \sharp P$› ‹$y' \sharp P$› ‹$y \neq x'$› ‹$x \neq y'$› ‹$x' \neq y'$› ‹$x \neq x'$› ‹$x \neq y$› **have** $(\!|\nu y|\!)((\!|\nu x|\!)P) = (\!|\nu y'|\!)((\!|\nu x'|\!)(([(x, x')] \cdot [(y, y')] \cdot P)))$
apply(*subst alphaRes*[*of* x' P], *simp*)
apply(*subst alphaRes*[*of* y'], *simp add*: *abs-fresh fresh-left calc-atm*)
apply(*simp add*: *eqvts calc-atm*)
by(*subst perm-compose*) (*simp add*: *calc-atm*)

ultimately show $\Psi \rhd ((\!|\nu x|\!)((\!|\nu y|\!)P))[<\sigma>] \sim ((\!|\nu y|\!)((\!|\nu x|\!)P))[<\sigma>]$
using ‹*wellFormedSubst* σ› ‹$x' \sharp \sigma$› ‹$y' \sharp \sigma$›
by *simp*
qed
qed

lemma *bisimSubstExtBang*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *guarded* P

shows $\Psi \rhd !P \sim_s P \parallel !P$
using *assms*

by(*fastforce intro*: *closeSubstI bangExt guardedSeqSubst*)

lemma *structCongBisimSubst*:
  fixes P :: ('*a*, '*b*, '*c*) *psi*
  and Q :: ('*a*, '*b*, '*c*) *psi*

  assumes $P \equiv_s Q$

  shows $P \sim_s Q$
using *assms*
by(*induct rule*: *structCong.induct*)
  (*auto intro*: *bisimSubstReflexive bisimSubstSymmetric bisimSubstTransitive bisimSubstParComm bisimSubstParAssoc bisimSubstParNil bisimSubstResNil bisimSubstResComm bisimSubstScopeExt bisimSubstCasePushRes bisimSubstInputPushRes bisimSubstOutputPushRes bisimSubstExtBang*)

end

end

theory *Weak-Bisim-Subst*
  imports *Weak-Bisim-Struct-Cong Weak-Bisim-Pres Bisim-Subst*
begin

context *env* **begin**

abbreviation
  weakBisimSubstJudge (- ▷ - ≈$_s$ - [*70*, *70*, *70*] *65*) **where** $\Psi \rhd P \approx_s Q \equiv (\Psi, P, Q) \in$ *closeSubst weakBisim*
abbreviation
  weakBisimSubstNilJudge (- ≈$_s$ - [*70*, *70*] *65*) **where** $P \approx_s Q \equiv \mathbf{1} \rhd P \approx_s Q$

lemmas *weakBisimSubstClosed*[*eqvt*] = *closeSubstClosed*[*OF weakBisimEqvt*]
lemmas *weakBisimEqvt*[*simp*] = *closeSubstEqvt*[*OF weakBisimEqvt*]

lemma *strongBisimSubstWeakBisimSubst*:
  fixes Ψ :: '*b*
  and P :: ('*a*, '*b*, '*c*) *psi*
  and Q :: ('*a*, '*b*, '*c*) *psi*

  assumes $\Psi \rhd P \sim_s Q$

  shows $\Psi \rhd P \approx_s Q$
using *assms*
by(*metis closeSubstI closeSubstE strongBisimWeakBisim*)

lemma *weakBisimSubstOutputPres*:
  fixes Ψ :: '*b*
  and P :: ('*a*, '*b*, '*c*) *psi*

and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $N :: 'a$

assumes $\Psi \rhd P \approx_s Q$

shows $\Psi \rhd M\langle N\rangle.P \approx_s M\langle N\rangle.Q$
using *assms*
by(*fastforce intro*: *closeSubstI closeSubstE weakBisimOutputPres*)

lemma *bisimSubstInputPres*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and *xvec* :: *name list*
and $N :: 'a$

assumes $\Psi \rhd P \approx_s Q$
and *xvec* $\sharp*$ Ψ
and *distinct xvec*

shows $\Psi \rhd M(\!|\lambda*xvec\ N|\!).P \approx_s M(\!|\lambda*xvec\ N|\!).Q$
proof(*rule-tac closeSubstI*)
fix σ :: (*name list* × $'a$ *list*) *list*
assume *wellFormedSubst* σ
obtain p **where** $(p \cdot xvec) \sharp* \sigma$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* N$
and *S*: *set p* ⊆ *set xvec* × *set* $(p \cdot xvec)$
by(*rule-tac c*=(σ, P, Q, Ψ, N) **in** *name-list-avoiding*) *auto*

from ‹$\Psi \rhd P \approx_s Q$› **have** $(p \cdot \Psi) \rhd (p \cdot P) \approx_s (p \cdot Q)$
by(*rule weakBisimSubstClosed*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec) \sharp* \Psi$› *S* **have** $\Psi \rhd (p \cdot P) \approx_s (p \cdot Q)$
by *simp*

{
fix *Tvec′* :: $'a$ *list*
assume *length* $(p \cdot xvec)$ = *length Tvec′*
with ‹*wellFormedSubst* σ› ‹*distinct xvec*› **have** *wellFormedSubst* $(\sigma$ @ $[(p \cdot xvec, Tvec')])$
by *simp*
with ‹$\Psi \rhd (p \cdot P) \approx_s (p \cdot Q)$›
have $\Psi \rhd (p \cdot P)[<(\sigma$ @ $[(p \cdot xvec, Tvec')])>] \approx (p \cdot Q)[<(\sigma$ @ $[(p \cdot xvec, Tvec')])>]$
by (*rule closeSubstE*)
then have $\Psi \rhd ((p \cdot P)[<\sigma>])[(p \cdot xvec)::=Tvec'] \approx ((p \cdot Q)[<\sigma>])[(p \cdot xvec)::=Tvec']$
by (*metis seqSubsCons seqSubsNil seqSubsTermAppend*)

}

then have $\Psi \rhd M[<\sigma>](\!|\lambda*(p \cdot xvec)\ (p \cdot N)[<\sigma>]|\!).((p \cdot P)[<\sigma>]) \approx M[<\sigma>](\!|\lambda*(p \cdot xvec)\ (p \cdot N)[<\sigma>]|\!).((p \cdot Q)[<\sigma>])$
using *weakBisimInputPres* **by** *metis*
with $\langle(p \cdot xvec)\ \sharp*\ \sigma\rangle$ **have** $\Psi \rhd (M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P))[<\sigma>] \approx (M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot Q))[<\sigma>]$
by (*metis seqSubstInputChain seqSubstSimps*(3))
moreover from $\langle(p \cdot xvec)\ \sharp*\ N\rangle\ \langle(p \cdot xvec)\ \sharp*\ P\rangle\ S$ **have** $M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P) = M(\!|\lambda*xvec\ N|\!).P$
apply (*simp add*: *psi.inject*) **by** (*rule inputChainAlpha*[*symmetric*]) *auto*
moreover from $\langle(p \cdot xvec)\ \sharp*\ N\rangle\ \langle(p \cdot xvec)\ \sharp*\ Q\rangle\ S$ **have** $M(\!|\lambda*(p \cdot xvec)\ (p \cdot N)|\!).(p \cdot Q) = M(\!|\lambda*xvec\ N|\!).Q$
apply (*simp add*: *psi.inject*) **by** (*rule inputChainAlpha*[*symmetric*]) *auto*
ultimately show $\Psi \rhd (M(\!|\lambda*xvec\ N|\!).P)[<\sigma>] \approx (M(\!|\lambda*xvec\ N|\!).Q)[<\sigma>]$
by *force*
qed

lemma *weakBisimSubstReflexive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \approx_s P$
by(*auto intro*: *closeSubstI weakBisimReflexive*)

lemma *bisimSubstTransitive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \approx_s Q$
and $\Psi \rhd Q \approx_s R$

shows $\Psi \rhd P \approx_s R$
using *assms*
by(*auto intro*: *closeSubstI closeSubstE weakBisimTransitive*)

lemma *weakBisimSubstSymmetric*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \approx_s Q$

shows $\Psi \rhd Q \approx_s P$
using *assms*
by(*auto intro*: *closeSubstI closeSubstE weakBisimE*)

lemma *weakBisimSubstParPres*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$
  and $R :: ('a, 'b, 'c)\ psi$

  assumes $\Psi \rhd P \approx_s Q$

  shows $\Psi \rhd P \parallel R \approx_s Q \parallel R$
using *assms*
by(*fastforce intro*: *closeSubstI closeSubstE weakBisimParPres*)

lemma *weakBisimSubstResPres*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$
  and $Q :: ('a, 'b, 'c)\ psi$
  and $x :: name$

  assumes $\Psi \rhd P \approx_s Q$
  and $x \sharp \Psi$

  shows $\Psi \rhd (\!|\nu x|\!) P \approx_s (\!|\nu x|\!) Q$
proof(*rule closeSubstI*)
  fix $\sigma :: (name\ list \times 'a\ list)\ list$
  assume *wellFormedSubst* σ
  obtain y::*name* **where** $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp Q$ **and** $y \sharp \sigma$
    by (*generate-fresh name*) *auto*

  from $\langle \Psi \rhd P \approx_s Q \rangle$ **have** $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \approx_s ([(x, y)] \cdot Q)$
    by (*rule weakBisimSubstClosed*)
  with $\langle x \sharp \Psi \rangle$ $\langle y \sharp \Psi \rangle$ **have** $\Psi \rhd ([(x, y)] \cdot P) \approx_s ([(x, y)] \cdot Q)$
    by *simp*
  hence $\Psi \rhd ([(x, y)] \cdot P)[<\sigma>] \approx ([(x, y)] \cdot Q)[<\sigma>]$
    using $\langle$*wellFormedSubst* $\sigma\rangle$ **by** (*rule closeSubstE*)
  hence $\Psi \rhd (\!|\nu y|\!)(([(x, y)] \cdot P)[<\sigma>]) \approx (\!|\nu y|\!)(([(x, y)] \cdot Q)[<\sigma>])$
    using $\langle y \sharp \Psi \rangle$ **by** (*rule weakBisimResPres*)
  with $\langle y \sharp P \rangle$ $\langle y \sharp Q \rangle$ $\langle y \sharp \sigma \rangle$ **show** $\Psi \rhd ((\!|\nu x|\!) P)[<\sigma>] \approx ((\!|\nu x|\!) Q)[<\sigma>]$
    by (*simp add*: *alphaRes*)
qed

lemma *weakBisimSubstParNil*:
  fixes $\Psi :: 'b$
  and $P :: ('a, 'b, 'c)\ psi$

  shows $\Psi \rhd P \parallel \mathbf{0} \approx_s P$
by(*metis strongBisimSubstWeakBisimSubst bisimSubstParNil*)

lemma *weakBisimSubstParComm*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and Q :: $('a, 'b, 'c)$ *psi*

  shows $\Psi \rhd P \parallel Q \approx_s Q \parallel P$
by(*metis strongBisimSubstWeakBisimSubst bisimSubstParComm*)

lemma *weakBisimSubstParAssoc*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and Q :: $('a, 'b, 'c)$ *psi*
  and R :: $('a, 'b, 'c)$ *psi*

  shows $\Psi \rhd (P \parallel Q) \parallel R \approx_s P \parallel (Q \parallel R)$
by(*metis strongBisimSubstWeakBisimSubst bisimSubstParAssoc*)

lemma *weakBisimSubstResNil*:
  fixes Ψ :: $'b$
  and x :: *name*

  shows $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \sim_s \mathbf{0}$
by(*metis strongBisimSubstWeakBisimSubst bisimSubstResNil*)

lemma *weakBisimSubstScopeExt*:
  fixes Ψ :: $'b$
  and x :: *name*
  and P :: $('a, 'b, 'c)$ *psi*
  and Q :: $('a, 'b, 'c)$ *psi*

  assumes $x \sharp P$

  shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \approx_s P \parallel (\!|\nu x|\!)Q$
using *assms*
by(*metis strongBisimSubstWeakBisimSubst bisimSubstScopeExt*)

lemma *weakBisimSubstCasePushRes*:
  fixes x :: *name*
  and Ψ :: $'b$
  and *Cs* :: $('c \times ('a, 'b, 'c)$ *psi*$)$ *list*

  assumes $x \sharp$ *map fst Cs*

  shows $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \approx_s Cases\ map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs$
using *assms*
by(*metis strongBisimSubstWeakBisimSubst bisimSubstCasePushRes*)

lemma *weakBisimSubstOutputPushRes*:
  fixes x :: *name*

and $\Psi :: 'b$
and $M :: 'a$
and $N :: 'a$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp M$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \approx_s M\langle N\rangle.(\!|\nu x|\!)P$
using *assms*
by(*metis strongBisimSubstWeakBisimSubst bisimSubstOutputPushRes*)

lemma *weakBisimSubstInputPushRes*:
fixes x :: *name*
and Ψ :: $'b$
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $x \sharp M$
and $x \sharp xvec$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \approx_s M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
using *assms*
by(*metis strongBisimSubstWeakBisimSubst bisimSubstInputPushRes*)

lemma *weakBisimSubstResComm*:
fixes x :: *name*
and y :: *name*

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \approx_s (\!|\nu y|\!)((\!|\nu x|\!)P)$
by(*metis strongBisimSubstWeakBisimSubst bisimSubstResComm*)

lemma *weakBisimSubstExtBang*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

assumes *guarded P*

shows $\Psi \rhd !P \approx_s P \parallel !P$
using *assms*
by(*metis strongBisimSubstWeakBisimSubst bisimSubstExtBang*)

end

end

theory *Weakening*
imports *Weak-Bisimulation*
begin

locale *weak* = *env* +
assumes *weaken*: $\Psi \hookrightarrow \Psi \otimes \Psi'$
begin

lemma *entWeaken*:
fixes Ψ :: *'b*
and φ :: *'c*

assumes $\Psi \vdash \varphi$

shows $\Psi \otimes \Psi' \vdash \varphi$
using *assms weaken*
by(*auto simp add*: *AssertionStatImp-def*)

lemma *assertWeaken*:
fixes Ψ :: *'b*
and Ψ' :: *'b*

shows $\Psi \hookrightarrow \Psi \otimes \Psi'$
by(*auto simp add*: *AssertionStatImp-def entWeaken*)

lemma *frameWeaken*:
fixes *F* :: *'b frame*
and *G* :: *'b frame*

shows $F \hookrightarrow_F F \otimes_F G$
proof −
obtain A_F Ψ_F **where** *FrF*: $F = \langle A_F, \Psi_F\rangle$ **and** $A_F \sharp* F$ **and** $A_F \sharp* G$
by(*rule-tac F*=*F* **and** *C*=(*F*, *G*) **in** *freshFrame*) *auto*
obtain A_G Ψ_G **where** *FrG*: $G = \langle A_G, \Psi_G\rangle$ **and** $A_G \sharp* F$ **and** $A_G \sharp* G$ **and** $A_G \sharp* A_F$ **and** $A_G \sharp* \Psi_F$
by(*rule-tac F*=*G* **and** *C*=(*F*, *G*, A_F, Ψ_F) **in** *freshFrame*) *auto*
from *FrG* ‹$A_F \sharp* G$› ‹$A_G \sharp* A_F$› **have** $A_F \sharp* \Psi_G$ **by** *auto*
have $\Psi_F \hookrightarrow \Psi_F \otimes \Psi_G$ **by**(*rule weaken*)
hence $\langle A_G, \Psi_F\rangle \hookrightarrow_F \langle A_G, \Psi_F \otimes \Psi_G\rangle$ **by**(*rule-tac frameImpResChainPres*) *auto*
with ‹$A_G \sharp* \Psi_F$› **have** $\langle \varepsilon, \Psi_F\rangle \hookrightarrow_F \langle A_G, \Psi_F \otimes \Psi_G\rangle$ **using** *frameResFreshChain*
by(*rule-tac FrameStatImpTrans*) (*auto simp add*: *FrameStatEq-def*)
with *FrF FrG* ‹$A_G \sharp* A_F$› ‹$A_G \sharp* \Psi_F$› ‹$A_F \sharp* \Psi_G$› **show** *?thesis*
by(*force simp add*: *frameChainAppend intro*: *frameImpResChainPres*)
qed

lemma *unitAssertWeaken*:
fixes Ψ :: *'b*

shows $\mathbf{1} \hookrightarrow \Psi$

proof −
have $\mathbf{1} \hookrightarrow \mathbf{1} \otimes \Psi$ **by**(*rule assertWeaken*)
moreover have $\mathbf{1} \otimes \Psi \hookrightarrow \Psi$ **by**(*metis Identity AssertionStatEq-def Commutativity AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule AssertionStatImpTrans*)
qed

lemma *unitFrameWeaken*:
fixes $F :: 'b$ *frame*

shows $\langle \varepsilon, \mathbf{1} \rangle \hookrightarrow_F F$
proof −
have $\langle \varepsilon, \mathbf{1} \rangle \hookrightarrow_F ((\langle \varepsilon, \mathbf{1} \rangle) \otimes_F F)$ **by**(*rule frameWeaken*)
moreover obtain $A_F\ \Psi_F$ **where** *FrF*: $F = \langle A_F, \Psi_F \rangle$
by(*rule-tac* $F{=}F$ **and** $C{=}()$ **in** *freshFrame*) *auto*
hence $(\langle \varepsilon, \mathbf{1} \rangle) \otimes_F F \simeq_F F$
by *simp* (*metis frameIntIdentity frameIntCommutativity FrameStatEqTrans FrameStatEqSym*)
ultimately show *?thesis* **by**(*metis FrameStatImpTrans FrameStatEq-def*)
qed

lemma *insertAssertionWeaken*:
fixes $F :: 'b$ *frame*
and $\Psi :: 'b$

shows $\langle \varepsilon, \Psi \rangle \hookrightarrow_F$ *insertAssertion* $F\ \Psi$
proof −
have $\langle \varepsilon, \Psi \rangle \hookrightarrow_F (\langle \varepsilon, \Psi \rangle) \otimes_F F$ **by**(*rule frameWeaken*)
thus *?thesis* **by** *simp*
qed

lemma *frameImpStatEq*:
fixes $A_F :: name\ list$
and $\Psi :: 'b$
and $\Psi' :: 'b$
and $\varphi :: 'c$

assumes $(\langle A_F, \Psi \rangle) \vdash_F \varphi$
and $\Psi \simeq \Psi'$

shows $(\langle A_F, \Psi' \rangle) \vdash_F \varphi$
proof −
obtain p::*name prm* **where** $(p \cdot A_F) \sharp* \varphi$ **and** $(p \cdot A_F) \sharp* \Psi$ **and** $(p \cdot A_F) \sharp* \Psi'$
and *distinctPerm* p **and** S: $set\ p \subseteq set\ A_F \times set(p \cdot A_F)$
by(*rule-tac* $c{=}(\varphi, \Psi, \Psi')$ **in** *name-list-avoiding*) *auto*
from $\langle(\langle A_F, \Psi \rangle) \vdash_F \varphi\rangle$ $\langle(p \cdot A_F) \sharp* \Psi\rangle$ S **have** $(\langle (p \cdot A_F), p \cdot \Psi \rangle) \vdash_F \varphi$ **by**(*simp add*: *frameChainAlpha*)
hence $(p \cdot \Psi) \vdash \varphi$ **using** $\langle(p \cdot A_F) \sharp* \varphi\rangle$ **by**(*rule frameImpE*)

moreover from ‹$\Psi \simeq \Psi'$› **have** $(p \cdot \Psi) \simeq (p \cdot \Psi')$ **by**(*rule AssertionStatEq-Closed*)
ultimately have $(p \cdot \Psi') \vdash \varphi$ **by**(*simp add*: *AssertionStatEq-def AssertionStatImp-def*)
hence $(\langle (p \cdot A_F), p \cdot \Psi' \rangle) \vdash_F \varphi$ **using** ‹$(p \cdot A_F) \sharp* \varphi$›
by(*rule-tac frameImpI*) *auto*
with ‹$(p \cdot A_F) \sharp* \Psi'$› *S* **show** *?thesis* **by**(*simp add*: *frameChainAlpha*)
qed

lemma *statImpTauDerivative*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \longmapsto \tau \prec P'$

shows *insertAssertion* (*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P'*) Ψ
proof(*auto simp add*: *FrameStatImp-def*)
fix φ :: *'c*
obtain A_P Ψ_P **where** *FrP*: *extractFrame* $P = \langle A_P, \Psi_P \rangle$ **and** $A_P \sharp* P$ **and** $A_P \sharp* \varphi$ **and** $A_P \sharp* \Psi$ **and** *distinct* A_P
by(*rule-tac C*=(P, φ, Ψ) **in** *freshFrame*) *auto*
with ‹$\Psi \rhd P \longmapsto \tau \prec P'$› **obtain** Ψ' $A_P{}'$ $\Psi_P{}'$ **where** *FrP'*: *extractFrame* $P' = \langle A_P{}', \Psi_P{}' \rangle$ **and** $\Psi_P \otimes \Psi' \simeq \Psi_P{}'$
and $A_P{}' \sharp* P'$ **and** $A_P{}' \sharp* \varphi$ **and** $A_P{}' \sharp* \Psi$
by(*rule-tac C*=(Ψ, φ) **in** *expandTauFrame*) *auto*
assume *insertAssertion* (*extractFrame P*) $\Psi \vdash_F \varphi$
with *FrP* ‹$A_P \sharp* \varphi$› ‹$A_P \sharp* \Psi$› **have** $\Psi \otimes \Psi_P \vdash \varphi$ **by**(*auto dest*: *frameImpE*)
hence $(\Psi \otimes \Psi_P) \otimes \Psi' \vdash \varphi$ **by**(*rule entWeaken*)
hence $\Psi \otimes \Psi_P{}' \vdash \varphi$ **using** ‹$\Psi_P \otimes \Psi' \simeq \Psi_P{}'$›
by(*rule-tac statEqEnt*, *auto*) (*metis Associativity compositionSym AssertionStatEqTrans AssertionStatEqSym Commutativity*)
with ‹$A_P{}' \sharp* \varphi$› ‹$A_P{}' \sharp* \Psi$› *FrP'* **show** *insertAssertion* (*extractFrame P'*) $\Psi \vdash_F \varphi$
by(*force intro*: *frameImpI*)
qed

lemma *weakenTransition*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Rs :: (*'a*, *'b*, *'c*) *residual*
and Ψ' :: *'b*

assumes $\Psi \rhd P \longmapsto Rs$

shows $\Psi \otimes \Psi' \rhd P \longmapsto Rs$
using *assms*
proof(*nominal-induct avoiding*: Ψ' *rule*: *semantics.strong-induct*)

case(*cInput* Ψ M K *xvec* N *Tvec* P Ψ')
from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi' \vdash M \leftrightarrow K$ **by**(*rule entWeaken*)
thus *?case* **using** ‹*distinct xvec*› ‹*set xvec* $\subseteq$ (*supp* N)› ‹*length xvec* = *length Tvec*›
by(*rule Input*)
next
case(*Output* Ψ M K N P Ψ')
from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi' \vdash M \leftrightarrow K$ **by**(*rule entWeaken*)
thus *?case* **by**(*rule semantics.Output*)
next
case(*Case* Ψ P *Rs* φ *Cs* Ψ')
have $\Psi \otimes \Psi' \rhd P \longmapsto Rs$ **by**(*rule Case*)
moreover note ‹(φ, P) *mem Cs*›
moreover from ‹$\Psi \vdash \varphi$›
have $\Psi \otimes \Psi' \vdash \varphi$ **by**(*rule entWeaken*)
ultimately show *?case* **using** ‹*guarded P*›
by(*rule semantics.Case*)
next
case(*cPar1* Ψ Ψ_Q P α P' Q A_Q Ψ')
have $(\Psi \otimes \Psi_Q) \otimes \Psi' \rhd P \longmapsto \alpha \prec P'$ **by**(*rule cPar1*)
hence $(\Psi \otimes \Psi') \otimes \Psi_Q \rhd P \longmapsto \alpha \prec P'$
by(*metis statEqTransition Composition Associativity Commutativity Assertion-StatEqTrans*)
thus *?case* **using** ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$› ‹*bn* α $\sharp*$ Q› ‹A_Q $\sharp*$ Ψ› ‹A_Q $\sharp*$ Ψ'› ‹A_Q $\sharp*$ P› ‹A_Q $\sharp*$ α›
by(*rule-tac Par1*) *auto*
next
case(*cPar2* Ψ Ψ_P Q α Q' P A_P Ψ')
have $(\Psi \otimes \Psi_P) \otimes \Psi' \rhd Q \longmapsto \alpha \prec Q'$ **by**(*rule cPar2*)
hence $(\Psi \otimes \Psi') \otimes \Psi_P \rhd Q \longmapsto \alpha \prec Q'$
by(*metis statEqTransition Composition Associativity Commutativity Assertion-StatEqTrans*)
thus *?case* **using** ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$› ‹*bn* α $\sharp*$ P› ‹A_P $\sharp*$ Ψ› ‹A_P $\sharp*$ Ψ'› ‹A_P $\sharp*$ Q› ‹A_P $\sharp*$ α›
by(*rule-tac Par2*) *auto*
next
case(*cComm1* Ψ Ψ_Q P M N P' A_P Ψ_P Q K *xvec* Q' A_Q Ψ')
have $(\Psi \otimes \Psi_Q) \otimes \Psi' \rhd P \longmapsto M(\!|N|\!) \prec P'$ **by**(*rule cComm1*)
hence $(\Psi \otimes \Psi') \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P'$
by(*metis statEqTransition Composition Associativity Commutativity Assertion-StatEqTrans*)
moreover note ‹*extractFrame* $P = \langle A_P, \Psi_P \rangle$›
moreover have $(\Psi \otimes \Psi_P) \otimes \Psi' \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$ **by**(*rule cComm1*)
hence $(\Psi \otimes \Psi') \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu * xvec|\!)\langle N \rangle \prec Q'$
by(*metis statEqTransition Composition Associativity Commutativity Assertion-StatEqTrans*)
moreover note ‹*extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$›
moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_P \otimes \Psi_Q) \otimes \Psi' \vdash M \leftrightarrow K$ **by**(*rule entWeaken*)
hence $(\Psi \otimes \Psi') \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$ **by**(*metis statEqEnt Composition*

Associativity Commutativity AssertionStatEqTrans)
ultimately show *?case* **using** ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi'$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* A_Q$›
‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi'$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹*xvec* $\sharp* P$›
by(*rule-tac Comm1*) (*assumption* | *auto*)+
next
case(*cComm2* Ψ Ψ_Q P M *xvec* N P' A_P Ψ_P Q K Q' A_Q Ψ')
have $(\Psi \otimes \Psi_Q) \otimes \Psi' \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$ **by**(*rule cComm2*)
hence $(\Psi \otimes \Psi') \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*metis statEqTransition Composition Associativity Commutativity AssertionStatEqTrans*)
moreover note ‹*extractFrame* $P = \langle A_P, \Psi_P\rangle$›
moreover have $(\Psi \otimes \Psi_P) \otimes \Psi' \rhd Q \longmapsto K(\!|N|\!) \prec Q'$ **by**(*rule cComm2*)
hence $(\Psi \otimes \Psi') \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$
by(*metis statEqTransition Composition Associativity Commutativity AssertionStatEqTrans*)
moreover note ‹*extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$›
moreover from ‹$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $(\Psi \otimes \Psi_P \otimes \Psi_Q) \otimes \Psi' \vdash M \leftrightarrow K$ **by**(*rule entWeaken*)
hence $(\Psi \otimes \Psi') \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$ **by**(*metis statEqEnt Composition Associativity Commutativity AssertionStatEqTrans*)
ultimately show *?case* **using** ‹$A_P \sharp* \Psi$› ‹$A_P \sharp* \Psi'$› ‹$A_P \sharp* P$› ‹$A_P \sharp* Q$› ‹$A_P \sharp* M$› ‹$A_P \sharp* A_Q$›
‹$A_Q \sharp* \Psi$› ‹$A_Q \sharp* \Psi'$› ‹$A_Q \sharp* P$› ‹$A_Q \sharp* Q$› ‹$A_Q \sharp* K$› ‹*xvec* $\sharp* Q$›
by(*rule-tac Comm2*) (*assumption* | *auto*)+
next
case(*cOpen* Ψ P M *xvec* *yvec* N P' x Ψ')
have $\Psi \otimes \Psi' \rhd P \longmapsto M(\!|\nu * (xvec @ yvec)|\!)\langle N\rangle \prec P'$ **by**(*rule cOpen*)
thus *?case* **using** ‹$x \in supp\ N$› ‹$x \sharp \Psi$› ‹$x \sharp \Psi'$› ‹$x \sharp M$› ‹$x \sharp xvec$› ‹$x \sharp yvec$›
by(*rule-tac Open*) *auto*
next
case(*cScope* Ψ P α P' x Ψ')
have $\Psi \otimes \Psi' \rhd P \longmapsto \alpha \prec P'$ **by**(*rule cScope*)
thus *?case* **using** ‹$x \sharp \Psi$› ‹$x \sharp \Psi'$› ‹$x \sharp \alpha$› **by**(*rule-tac Scope*) *auto*
next
case(*Bang* Ψ P *Rs* Ψ')
have $\Psi \otimes \Psi' \rhd P \parallel\ !P \longmapsto Rs$ **by**(*rule Bang*)
thus *?case* **using** ‹*guarded* P› **by**(*rule semantics.Bang*)
qed

end

end

theory *Weaken-Transition*
imports *Weakening*
begin

context *weak*
begin

definition *weakenTransition* :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *'a action* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ▷ - ⟹- ≺ - [*80*, *80*, *80*, *80*] *80*)
where
  Ψ ▷ *P* ⟹α ≺ *P'* ≡ (∃ *P'''* *P''*. Ψ ▷ *P* ⟹^τ *P'''* ∧ Ψ ▷ *P'''* ⟼α ≺ *P''* ∧ Ψ ▷ *P''* ⟹^τ *P'*) ∨ (*P* = *P'* ∧ α = τ)

lemma *weakenTransitionCases*[*consumes 1*, *case-names cBase cStep*]:
  assumes Ψ ▷ *P* ⟹α ≺ *P'*
  and *Prop* (τ) *P*
  and ⋀*P'''* *P''*. ⟦Ψ ▷ *P* ⟹^τ *P'''*; Ψ ▷ *P'''* ⟼α ≺ *P''*; Ψ ▷ *P''* ⟹^τ *P'*⟧ ⟹ *Prop* α *P'*

  shows *Prop* α *P'*
using *assms*
by(*auto simp add*: *weakenTransition-def*)

lemma *statImpTauChainDerivative*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *P'* :: (*'a*, *'b*, *'c*) *psi*

  assumes Ψ ▷ *P* ⟹^τ *P'*

  shows *insertAssertion* (*extractFrame P*) Ψ ↪*F* *insertAssertion* (*extractFrame P'*) Ψ
using *assms*
by(*induct rule*: *tauChainInduct*) (*auto intro*: *statImpTauDerivative dest*: *FrameStatImpTrans*)

lemma *weakenTauChain*:
  fixes Ψ :: *'b*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and *P'* :: (*'a*, *'b*, *'c*) *psi*
  and Ψ' :: *'b*

  assumes Ψ ▷ *P* ⟹^τ *P'*
  shows Ψ ⊗ Ψ' ▷ *P* ⟹^τ *P'*
using *assms*
proof(*induct rule*: *tauChainInduct*)
  case(*TauBase P*)
  thus *?case* **by** *simp*
next
  case(*TauStep P P' P''*)
  note ‹Ψ ⊗ Ψ' ▷ *P* ⟹^τ *P'*›
  moreover from ‹Ψ ▷ *P'* ⟼τ ≺ *P''*› **have** Ψ ⊗ Ψ' ▷ *P'* ⟼τ ≺ *P''* **by**(*rule*

weakenTransition)
ultimately show *?case* **by**(*auto dest*: *tauActTauChain*)
qed

end

end

theory *Weaken-Stat-Imp*
imports *Weaken-Transition*
begin

context *weak* **begin**

definition
weakenStatImp :: $'b \Rightarrow ('a, 'b, 'c)\ psi \Rightarrow$
$('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set \Rightarrow$
$('a, 'b, 'c)\ psi \Rightarrow bool$ (- ▷ - $\lessapprox_w$<-> - [80, 80, 80, 80] 80)
where $\Psi \rhd P \lessapprox_w <Rel> Q \equiv \exists Q'.\ \Psi \rhd Q \Longrightarrow\!\!\hat{}_\tau\ Q' \wedge$ *insertAssertion*(*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q') $\Psi \wedge (\Psi, P, Q') \in Rel$

lemma *weakenStatImpMonotonic*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $A :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $B :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $\Psi \rhd P \lessapprox_w <A> Q$
and $A \subseteq B$

shows $\Psi \rhd P \lessapprox_w <B> Q$
using *assms*
by(*auto simp add*: *weakenStatImp-def*)

lemma *weakenStatImpI*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi' :: 'b$

assumes $\Psi \rhd Q \Longrightarrow\!\!\hat{}_\tau\ Q'$
and *insertAssertion*(*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q') Ψ
and $(\Psi, P, Q') \in Rel$

shows $\Psi \rhd P \lessapprox_w <Rel> Q$
using *assms*

by(*auto simp add*: *weakenStatImp-def*)

lemma *weakenStatImpE*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and Ψ' :: *'b*

assumes $\Psi \rhd P \lesssim_w <Rel> Q$

obtains Q' **where** $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$ **and** *insertAssertion*(*extractFrame* P) Ψ $\hookrightarrow_F$ *insertAssertion*(*extractFrame* Q') Ψ **and** $(\Psi, P, Q') \in Rel$
using *assms*
by(*auto simp add*: *weakenStatImp-def*)

lemma *weakStatImpWeakenStatImp*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes *cSim*: $\Psi \rhd P \lesssim <Rel> Q$
and *cStatEq*: $\bigwedge \Psi' R S \Psi''. [\![(\Psi', R, S) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', R, S) \in Rel$

shows $\Psi \rhd P \lesssim_w <Rel> Q$
proof −
from ‹$\Psi \rhd P \lesssim <Rel> Q$›
obtain Q' Q'' **where** *QChain*: $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$
and *PImpQ'*: *insertAssertion*(*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q') Ψ
and *Q'Chain*: $\Psi \otimes \mathbf{1} \rhd Q' \Longrightarrow^{\hat{}}_{\tau} Q''$ **and** $(\Psi \otimes \mathbf{1}, P, Q'') \in Rel$
by(*rule weakStatImpE*)
from *Q'Chain Identity* **have** *Q'Chain*: $\Psi \rhd Q' \Longrightarrow^{\hat{}}_{\tau} Q''$ **by**(*rule tauChainStatEq*)
with *QChain* **have** $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q''$ **by** *auto*
moreover from *Q'Chain* **have** *insertAssertion*(*extractFrame* Q') $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q'') Ψ
by(*rule statImpTauChainDerivative*)
with *PImpQ'* **have** *insertAssertion*(*extractFrame P*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q'') Ψ
by(*rule FrameStatImpTrans*)
moreover from ‹$(\Psi \otimes \mathbf{1}, P, Q'') \in Rel$› *Identity* **have** $(\Psi, P, Q'') \in Rel$ **by**(*rule cStatEq*)
ultimately show *?thesis* **by**(*rule weakenStatImpI*)
qed

lemma *weakenStatImpWeakStatImp*:

fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd P \lesssim_w <Rel> Q$
and *cExt*: $\bigwedge \Psi' R S \Psi''. (\Psi', R, S) \in Rel \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in Rel$

shows $\Psi \rhd P \lesssim <Rel> Q$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')

from ‹$\Psi \rhd P \lesssim_w <Rel> Q$›
obtain Q' **where** *QChain*: $\Psi \rhd Q \Longrightarrow^{\hat{}}_{\tau} Q'$
and *PImpQ'*: *insertAssertion*(*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame* Q') Ψ
and $(\Psi, P, Q') \in Rel$
by(*rule weakenStatImpE*)
note *QChain PImpQ'*
moreover have $\Psi \otimes \Psi' \rhd Q' \Longrightarrow^{\hat{}}_{\tau} Q'$ **by** *simp*
moreover from ‹$(\Psi, P, Q') \in Rel$› **have** $(\Psi \otimes \Psi', P, Q') \in Rel$ **by**(*rule cExt*)
ultimately show *?case* **by** *blast*
qed

end

end

theory *Weaken-Simulation*
imports *Weaken-Stat-Imp*
begin

context *weak*
begin

definition
weakenSimulation :: $'b \Rightarrow ('a, 'b, 'c)$ *psi* $\Rightarrow$
$('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set* $\Rightarrow$
$('a, 'b, 'c)$ *psi* $\Rightarrow$ *bool* (- $\rhd$ - $\rightsquigarrow_w$<-> - [80, 80, 80, 80] 80)
where
$\Psi \rhd P \rightsquigarrow_w <Rel> Q \equiv \forall \alpha\ Q'. \Psi \rhd Q \longmapsto \alpha \prec Q' \longrightarrow bn\ \alpha\ \sharp*\ \Psi \longrightarrow bn\ \alpha\ \sharp* P \longrightarrow (\exists P'. \Psi \rhd P \Longrightarrow \alpha \prec P' \wedge (\Psi, P', Q') \in Rel)$

lemma *weakenSimI*[*case-names cAct*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and C :: $'d$::*fs-name*

assumes *rOutput*: $\bigwedge\alpha\ Q'.\ [\![\Psi \rhd Q \longmapsto\alpha \prec Q';\ bn\ \alpha \sharp* \Psi;\ bn\ \alpha \sharp* P]\!] \Longrightarrow \exists P'.\ \Psi \rhd P \Longrightarrow\alpha \prec P' \wedge (\Psi, P', Q') \in Rel$

shows $\Psi \rhd P \rightsquigarrow_w <Rel> Q$
using *assms*
by(*auto simp add*: *weakenSimulation-def*)

lemma *weakenSimWeakSim*:
fixes Ψ :: *'b*
and P :: *('a, 'b, 'c) psi*
and Rel :: *('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set*
and Q :: *('a, 'b, 'c) psi*

assumes $(\Psi, P, Q) \in Rel$
and *cStatImp*: $\bigwedge\Psi'\ R\ S.\ (\Psi, R, S) \in Rel \Longrightarrow \Psi \rhd R \lessapprox_w <Rel> S$
and *cSim*: $\bigwedge\Psi'\ R\ S.\ (\Psi, R, S) \in Rel \Longrightarrow \Psi \rhd R \rightsquigarrow_w <Rel'> S$
and *cExt*: $\bigwedge\Psi'\ R\ S\ \Psi'.\ (\Psi, R, S) \in Rel' \Longrightarrow (\Psi \otimes \Psi', R, S) \in Rel'$
and *cSym*: $\bigwedge\Psi'\ R\ S.\ (\Psi, R, S) \in Rel \Longrightarrow (\Psi, S, R) \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel'> Q$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ Q'$)
from ‹$(\Psi, P, Q) \in Rel$› **obtain** P''''
where *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''''$
and *QImpP''''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'''') Ψ
and $(\Psi, P'''', Q) \in Rel$ **using** *weakenStatImp-def*
by(*metis cStatImp cSym*)

from ‹$(\Psi, P'''', Q) \in Rel$› **have** $\Psi \rhd P'''' \rightsquigarrow_w <Rel'> Q$ **by**(*rule cSim*)
moreover from *PChain* ‹*bn* $\alpha \sharp* P$› **have** *bn* $\alpha \sharp* P''''$ **by**(*rule tauChainFreshChain*)
ultimately obtain P' **where** *P''''Trans*: $\Psi \rhd P'''' \Longrightarrow\alpha \prec P'$ **and** $(\Psi, P', Q') \in Rel'$
using ‹$\Psi \rhd Q \longmapsto\alpha \prec Q'$› ‹*bn* $\alpha \sharp* \Psi$›
by(*unfold weakenSimulation-def*, *auto*)

from *P''''Trans* ‹$\alpha \neq \tau$› **obtain** P''' P'' **where** *P''''Chain*: $\Psi \rhd P'''' \Longrightarrow\hat{}_\tau\ P'''$ **and** *P'''Trans*: $\Psi \rhd P''' \longmapsto\alpha \prec P''$ **and** *P''Chain*: $\Psi \rhd P'' \Longrightarrow\hat{}_\tau\ P'$
by(*force simp add*: *weakenTransition-def*)
from *P''''Chain QImpP''''* **have** *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* P''') Ψ
by(*blast intro*: *statImpTauChainDerivative FrameStatImpTrans*)
with *PChain P''''Chain* **have** $\Psi : Q \rhd P \Longrightarrow\alpha \prec P''$ **using** *P'''Trans* **by**(*rule-tac weakTransitionI*) *auto*
moreover from *P''Chain* **have** $\Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **by**(*rule weakenTauChain*)

moreover from ‹$(\Psi, P', Q') \in Rel'$› **have** $(\Psi \otimes \Psi', P', Q') \in Rel'$ **by**(*rule*

cExt)
ultimately show *?case* **by** *blast*
next
case(*cTau* Q')
from ‹(Ψ, P, Q) $\in$ *Rel*› **have** $\Psi \rhd P \rightsquigarrow_w <Rel'> Q$ **by**(*rule cSim*)
then obtain P' **where** $\Psi \rhd P \Longrightarrow \tau \prec P'$ **and** (Ψ, P', Q') $\in$ *Rel'* **using** ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$›
by(*unfold weakenSimulation-def*, *fastforce*)
from ‹$\Psi \rhd P \Longrightarrow \tau \prec P'$› **have** $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$ **by**(*auto simp add*: *weakenTransition-def dest*: *tauActTauChain*)
with ‹(Ψ, P', Q') $\in$ *Rel'*› **show** *?case* **by** *blast*
qed

lemma *weakSimWeakenSim*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes *cSim*: $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *cStatEq*: $\bigwedge \Psi' R S \Psi''. [\![(\Psi', R, S) \in Rel; \Psi' \simeq \Psi']\!] \Longrightarrow (\Psi'', R, S) \in Rel$

shows $\Psi \rhd P \rightsquigarrow_w <Rel> Q$
proof(*induct rule*: *weakenSimI*)
case(*cAct* α Q')
show *?case*
proof(*cases* α=τ)
case *True*
from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹$\alpha = \tau$›
obtain P' **where** $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$ **and** (Ψ, P', Q') $\in$ *Rel*
by(*blast dest*: *weakSimE*)
from ‹$\Psi \rhd P \Longrightarrow\hat{}_\tau P'$› **have** $\Psi \rhd P \Longrightarrow \tau \prec P'$
by(*induct rule*: *tauChainInduct*) (*auto simp add*: *weakenTransition-def*)
thus *?thesis* **using** ‹(Ψ, P', Q') $\in$ *Rel*› ‹$\alpha = \tau$› **by** *blast*
next
case *False*
from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* α ♯* Ψ› ‹*bn* α ♯* P› ‹$\alpha \neq \tau$›
obtain P'' P' **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$ **and** *P''Chain*: $\Psi \otimes \mathbf{1} \rhd P'' \Longrightarrow\hat{}_\tau P'$ **and** ($\Psi \otimes \mathbf{1}$, P', Q') $\in$ *Rel*
by(*blast dest*: *weakSimE*)
from *PTrans* **have** $\Psi \rhd P \Longrightarrow \alpha \prec P''$ **by**(*auto simp add*: *weakTransition-def weakenTransition-def*)
moreover from *P''Chain* **have** $\Psi \rhd P'' \Longrightarrow\hat{}_\tau P'$ **by**(*metis tauChainStatEq Identity*)
moreover from ‹($\Psi \otimes \mathbf{1}$, P', Q') $\in$ *Rel*› **have** (Ψ, P', Q') $\in$ *Rel* **by**(*metis cStatEq Identity*)
ultimately show *?thesis*

```
    proof(induct rule: weakenTransitionCases)
      case cBase
      from ‹Ψ ▷ P ⟹^τ P'› have Ψ ▷ P ⟹τ ≺ P'
        by(induct rule: tauChainInduct) (auto simp add: weakenTransition-def)
      with ‹(Ψ, P', Q') ∈ Rel› show ?case by blast
    next
      case(cStep P'''' P''')
      thus ?case
        apply(unfold weakenTransition-def)
        by(rule-tac x=P' in exI) fastforce
    qed
  qed
qed

lemma weakenSimE:
  fixes F   :: 'b
  and   P   :: ('a, 'b, 'c) psi
  and   Rel :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set
  and   Q   :: ('a, 'b, 'c) psi

  assumes Ψ ▷ P ⇝w<Rel> Q

  shows ⋀α  Q'. [[Ψ ▷ Q ⟼α ≺ Q'; bn α ♯* Ψ; bn α ♯* P]] ⟹
                ∃P'. Ψ ▷ P ⟹α ≺ P' ∧ (Ψ, P', Q') ∈ Rel
using assms
by(auto simp add: weakenSimulation-def)

lemma weakenSimMonotonic:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   A :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set
  and   Q :: ('a, 'b, 'c) psi
  and   B :: ('b × ('a, 'b, 'c) psi × ('a, 'b, 'c) psi) set

  assumes Ψ ▷ P ⇝w<A> Q
  and     A ⊆ B

  shows Ψ ▷ P ⇝w<B> Q
using assms
by(simp (no-asm) add: weakenSimulation-def) (blast dest: weakenSimE)

end

end

theory Weaken-Bisimulation
  imports Weaken-Simulation Weaken-Stat-Imp
begin
```

context *weak*
begin

lemma *weakenMonoCoinduct*: $\bigwedge x\ y\ xa\ xb\ xc\ P\ Q\ \Psi.$
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \rightsquigarrow_w <\{(xc,\ xb,\ xa).\ x\ xc\ xb\ xa\}> P) \longrightarrow$
$(\Psi \rhd Q \rightsquigarrow_w <\{(xb,\ xa,\ xc).\ y\ xb\ xa\ xc\}> P)$
apply *auto*
apply(*rule weakenSimMonotonic*)
by(*auto dest*: *le-funE*)

lemma *weakenMonoCoinduct2*: $\bigwedge x\ y\ xa\ xb\ xc\ P\ Q\ \Psi.$
$x \leq y \Longrightarrow$
$(\Psi \rhd Q \lesssim_{\approx w} <\{(xc,\ xb,\ xa).\ x\ xc\ xb\ xa\}> P) \longrightarrow$
$(\Psi \rhd Q \lesssim_{\approx w} <\{(xb,\ xa,\ xc).\ y\ xb\ xa\ xc\}> P)$
apply *auto*
apply(*rule weakenStatImpMonotonic*)
by(*auto dest*: *le-funE*)

coinductive-set *weakenBisim* :: $('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$
where
step: $[\![\Psi \rhd P \lesssim_{\approx w} <weakenBisim> Q;\ \Psi \rhd P \rightsquigarrow_w <weakenBisim> Q;$
$\forall \Psi'.\ (\Psi \otimes \Psi',\ P,\ Q) \in weakenBisim;\ (\Psi,\ Q,\ P) \in weakenBisim]\!] \Longrightarrow (\Psi,$
$P,\ Q) \in weakenBisim$
monos *weakenMonoCoinduct weakenMonoCoinduct2*

abbreviation
weakenBisimJudge $(\text{-} \rhd \text{-} \approx_w \text{-}\ [70,\ 70,\ 70]\ 65)$ **where** $\Psi \rhd P \approx_w Q \equiv (\Psi,\ P,$
$Q) \in weakenBisim$
abbreviation
weakenBisimNilJudge $(\text{-} \approx_w \text{-}\ [70,\ 70]\ 65)$ **where** $P \approx_w Q \equiv \mathbf{1} \rhd P \approx_w Q$

lemma *weakenBisimCoinductAux*[*consumes 1*]:
fixes $F :: 'b$
and $P :: ('a,\ 'b,\ 'c)\ psi$
and $Q :: ('a,\ 'b,\ 'c)\ psi$
and $X :: ('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$

assumes $(\Psi,\ P,\ Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \rhd P \lesssim_{\approx w} <(X \cup weakenBisim)> Q) \wedge$
$(\Psi \rhd P \rightsquigarrow_w <(X \cup weakenBisim)> Q) \wedge$
$(\forall \Psi'.\ (\Psi \otimes \Psi',\ P,\ Q) \in X \vee (\Psi \otimes \Psi',\ P,\ Q) \in$
$weakenBisim) \wedge$
$((\Psi,\ Q,\ P) \in X \vee (\Psi,\ Q,\ P) \in weakenBisim)$

shows $(\Psi,\ P,\ Q) \in weakenBisim$
proof −
have $X \cup weakenBisim = \{(\Psi,\ P,\ Q).\ (\Psi,\ P,\ Q) \in X \vee (\Psi,\ P,\ Q) \in weaken$-
$Bisim\}$ **by** *auto*

with *assms* **show** *?thesis*
by *coinduct* (*simp add*: *rtrancl-def*)
qed

lemma *weakenBisimCoinduct*[*consumes 1*, *case-names cStatImp cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi' R S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \lesssim_w <(X \cup weakenBisim)> S$
and $\bigwedge \Psi' R S.\ (\Psi', R, S) \in X \Longrightarrow \Psi' \rhd R \leadsto_w <(X \cup weakenBisim)> S$
and $\bigwedge \Psi' R S\ \Psi''.\ (\Psi', R, S) \in X \Longrightarrow (\Psi' \otimes \Psi'', R, S) \in X \vee \Psi' \otimes \Psi'' \rhd R \approx_w S$
and $\bigwedge \Psi' R S.\ (\Psi', R, S) \in X \Longrightarrow (\Psi', S, R) \in X \vee \Psi' \rhd S \approx_w R$

shows $\Psi \rhd P \approx_w Q$
proof −
have $X \cup weakenBisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in weakenBisim\}$ **by** *auto*
with *assms* **show** *?thesis*
by *coinduct* (*simp add*: *rtrancl-def*)
qed

lemma *weakenBisimWeakCoinductAux*[*consumes 1*]:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lesssim_w <X> Q \wedge \Psi \rhd P \leadsto_w <X> Q \wedge (\forall \Psi'.\ (\Psi \otimes \Psi', P, Q) \in X) \wedge (\Psi, Q, P) \in X$

shows $\Psi \rhd P \approx_w Q$
using *assms*
by(*coinduct rule*: *weakenBisimCoinductAux*) (*blast intro*: *weakenSimMonotonic weakenStatImpMonotonic*)

lemma *weakenBisimE*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi :: 'b$
and $\Psi' :: 'b$

assumes $\Psi \rhd P \approx_w Q$

shows $\Psi \rhd P \lesssim_w <weakenBisim> Q$
and $\Psi \rhd P \rightsquigarrow_w <weakenBisim> Q$
and $\Psi \otimes \Psi' \rhd P \approx_w Q$
and $\Psi \rhd Q \approx_w P$
using *assms*
by(*auto intro*: *weakenBisim.cases simp add*: *rtrancl-def*)

lemma *weakenBisimWeakCoinduct*[*consumes 1*, *case-names cStatImp cSim cExt cSym*]:
fixes $F :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $X :: 'b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $(\Psi, P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lesssim_w <X> Q$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow_w <X> Q$
and $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
and $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

shows $(\Psi, P, Q) \in weakenBisim$
proof −
have $X \cup weakenBisim = \{(\Psi, P, Q).\ (\Psi, P, Q) \in X \vee (\Psi, P, Q) \in weakenBisim\}$ **by** *auto*
with *assms* **show** *?thesis*
by(*coinduct rule*: *weakenBisimWeakCoinductAux*) *blast*
qed

lemma *weakenBisimEqWeakBisim*[*simp*]: $weakenBisim = weakBisim$
proof *auto*
fix $\Psi\ P\ Q$
assume $\Psi \rhd P \approx_w Q$ **thus** $\Psi \rhd P \approx Q$
proof(*coinduct rule*: *weakBisimWeakCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
from $\langle \Psi \rhd P \approx_w Q \rangle$ **have** $\Psi \rhd P \lesssim_w <weakenBisim> Q$ **by**(*rule weakenBisimE*)
thus *?case* **using** *weakenBisimE*(*3*) **by**(*rule weakenStatImpWeakStatImp*)
next
case(*cSim* $\Psi\ P\ Q$)
from $\langle \Psi \rhd P \approx_w Q \rangle$ *weakenBisimE*
show *?case* **by**(*rule weakenSimWeakSim*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*rule weakenBisimE*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by**(*rule weakenBisimE*)
qed
next

fix Ψ P Q
assume $\Psi \rhd P \approx Q$ **thus** $\Psi \rhd P \approx_w Q$
proof(*coinduct rule*: *weakenBisimWeakCoinduct*)
case(*cStatImp* Ψ P Q)
from ‹$\Psi \rhd P \approx Q$› **have** $\Psi \rhd P \lessapprox_{<weakBisim>} Q$ **by**(*rule weakBisimE*)
thus *?case* **using** *statEqWeakBisim* **by**(*rule weakStatImpWeakenStatImp*)
next
case(*cSim* Ψ P Q)
from ‹$\Psi \rhd P \approx Q$› **have** $\Psi \rhd P \rightsquigarrow_{<weakBisim>} Q$ **by**(*rule weakBisimE*)
thus *?case* **using** *statEqWeakBisim* **by**(*rule weakSimWeakenSim*)
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by**(*rule weakBisimE*)
next
case(*cSym* Ψ P Q)
thus *?case* **by**(*rule weakBisimE*)
qed
qed

lemma *weakenTransitiveWeakCoinduct*[*case-names cStatImp cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and X :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *p*: $(\Psi, P, Q) \in X$
and *Eqvt*: *eqvt X*
and *rStatImp*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \lessapprox_w <X> Q$
and *rSim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow_w <(\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\})> Q$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi \otimes \Psi', P, Q) \in X$
and *rSym*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in X \Longrightarrow (\Psi, Q, P) \in X$

shows $\Psi \rhd P \approx_w Q$
proof −
from *p* ‹*eqvt X*› **have** $\Psi \rhd P \approx Q$
proof(*coinduct rule*: *weakTransitiveWeakCoinduct*)
case(*cStatImp* Ψ P Q)
from ‹$(\Psi, P, Q) \in X$› **have** $\Psi \rhd P \lessapprox_w <X> Q$ **by**(*rule rStatImp*)
thus *?case* **using** *rExt* **by**(*rule weakenStatImpWeakStatImp*)
next
case(*cSim* Ψ P Q)
let *?Y* = $\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in X \wedge \Psi \rhd Q' \sim Q\}$
note ‹$(\Psi, P, Q) \in X$›
moreover note *rStatImp rSim*

moreover have $\bigwedge\Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in ?Y \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in ?Y$
by(*blast dest*: *bisimE rExt*)
ultimately show *?case* **using** *rSym* **by**(*rule weakenSimWeakSim*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*rule rExt*)
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by**(*rule rSym*)
qed
thus *?thesis* **by**(*simp add*: *weakenBisimEqWeakBisim*)
qed

lemma *weakenTransitiveCoinduct*[*case-names cStatImp cSim cExt cSym*, *case-conclusion bisim step*, *consumes 2*]:
fixes $\Psi :: 'b$
and $P :: ('a,\ 'b,\ 'c)\ psi$
and $Q :: ('a,\ 'b,\ 'c)\ psi$
and $X :: ('b \times ('a,\ 'b,\ 'c)\ psi \times ('a,\ 'b,\ 'c)\ psi)\ set$

assumes *p*: $(\Psi,\ P,\ Q) \in X$
and *Eqvt*: *eqvt X*
and *rStatImp*: $\bigwedge\Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \lesssim_w <(X \cup weakenBisim)> Q$
and *rSim*: $\bigwedge\Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow \Psi \rhd P \rightsquigarrow_w <(\{(\Psi,\ P,\ Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in (X \cup weakenBisim) \wedge \Psi \rhd Q' \sim Q\})> Q$
and *rExt*: $\bigwedge\Psi\ P\ Q\ \Psi'.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi \otimes \Psi',\ P,\ Q) \in X \cup weakenBisim$
and *rSym*: $\bigwedge\Psi\ P\ Q.\ (\Psi,\ P,\ Q) \in X \Longrightarrow (\Psi,\ Q,\ P) \in X \cup weakenBisim$

shows $\Psi \rhd P \approx_w Q$
proof −
from *p* **have** $(\Psi,\ P,\ Q) \in X \cup weakenBisim$ **by** *auto*
moreover from ‹*eqvt X*› **have** $eqvt(X \cup weakenBisim)$ **by** *auto*
ultimately show *?thesis*
proof(*coinduct rule*: *weakenTransitiveWeakCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
thus *?case* **by**(*fastforce intro*: *rStatImp weakenBisimE*(*1*) *weakenStatImpMonotonic*)
next
case(*cSim* $\Psi\ P\ Q$)
thus *?case* **by**(*fastforce intro*: *rSim weakenBisimE*(*2*) *weakenSimMonotonic bisimReflexive*)
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by**(*blast dest*: *weakenBisimE rExt*)
next

case(*cSym* Ψ *P Q*)
thus *?case* **by**(*blast dest*: *weakenBisimE rSym*)
qed
qed

end

end

theory *Weak-Cong-Simulation*
imports *Weak-Simulation Tau-Chain*
begin

context *env* **begin**

definition
weakCongSimulation :: *'b* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒
(*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set* ⇒
(*'a*, *'b*, *'c*) *psi* ⇒ *bool* (- ▷ - ⇝«-» - [*80*, *80*, *80*, *80*] *80*)
where
Ψ ▷ *P* ⇝«*Rel*» *Q* ≡ ∀ *Q'*. Ψ ▷ *Q* ⟼τ ≺ *Q'* ⟶ (∃ *P'*. Ψ ▷ *P* ⟹τ *P'* ∧ (Ψ, *P'*, *Q'*) ∈ *Rel*)

abbreviation
weakCongSimulationNilJudge (- ⇝«-» - [*80*, *80*, *80*] *80*) **where** *P* ⇝«*Rel*» *Q* ≡ *SBottom'* ▷ *P* ⇝«*Rel*» *Q*

lemma *weakCongSimI*[*case-names cTau*]:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *C* :: *'d*::*fs-name*

assumes *rTau*: ⋀*Q'*. Ψ ▷ *Q* ⟼τ ≺ *Q'* ⟹ ∃ *P'*. Ψ ▷ *P* ⟹τ *P'* ∧ (Ψ, *P'*, *Q'*) ∈ *Rel*

shows Ψ ▷ *P* ⇝«*Rel*» *Q*
using *assms*
by(*auto simp add*: *weakCongSimulation-def*)

lemma *weakCongSimE*:
fixes *F* :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

assumes Ψ ▷ *P* ⇝«*Rel*» *Q*

and $\Psi \rhd Q \longmapsto \tau \prec Q'$

obtains P' **where** $\Psi \rhd P \Longrightarrow_\tau P'$ **and** $(\Psi, P', Q') \in Rel$
using *assms*
by(*auto simp add*: *weakCongSimulation-def*)

lemma *weakCongSimClosedAux*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)$ *psi*
and p :: *name prm*

assumes *EqvtRel*: *eqvt Rel*
and *PSimQ*: $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$

shows $(p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow\!\!\ll Rel \gg (p \cdot Q)$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from $\langle p \cdot \Psi \rhd p \cdot Q \longmapsto \tau \prec Q' \rangle$
have $(rev\ p \cdot p \cdot \Psi) \rhd (rev\ p \cdot p \cdot Q) \longmapsto (rev\ p \cdot (\tau \prec Q'))$
by(*blast dest*: *semantics.eqvt*)
hence $\Psi \rhd Q \longmapsto \tau \prec (rev\ p \cdot Q')$ **by**(*simp add*: *eqvts*)
with *PSimQ* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** $P'RelQ'$: $(\Psi, P', rev\ p \cdot Q') \in Rel$
by(*blast dest*: *weakCongSimE*)
from *PChain* **have** $(p \cdot \Psi) \rhd (p \cdot P) \Longrightarrow_\tau (p \cdot P')$ **by**(*rule tauStepChainEqvt*)
moreover from $P'RelQ'$ *EqvtRel* **have** $(p \cdot (\Psi, P', rev\ p \cdot Q')) \in Rel$
by(*simp only*: *eqvt-def*)
hence $(p \cdot \Psi, p \cdot P', Q') \in Rel$ **by**(*simp add*: *eqvts*)
ultimately show *?case*
by *blast*
qed

lemma *weakCongSimClosed*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)$ *psi*
and p :: *name prm*

assumes *EqvtRel*: *eqvt Rel*

shows $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q \Longrightarrow (p \cdot \Psi) \rhd (p \cdot P) \rightsquigarrow\!\!\ll Rel \gg (p \cdot Q)$
and $P \rightsquigarrow\!\!\ll Rel \gg Q \Longrightarrow (p \cdot P) \rightsquigarrow\!\!\ll Rel \gg (p \cdot Q)$
using *EqvtRel*
by(*force dest*: *weakCongSimClosedAux simp add*: *permBottom*)+

lemma *weakCongSimReflexive*:

fixes *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$

assumes $\{(\Psi, P, P) \mid \Psi\ P.\ True\} \subseteq Rel$

shows $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg P$
using *assms*
by(*auto simp add*: *weakCongSimulation-def dest*: *rtrancl-into-rtrancl*)

lemma *weakStepSimTauChain*:
fixes Ψ :: $'b$
and Q :: $('a, 'b, 'c)\ psi$
and Q' :: $('a, 'b, 'c)\ psi$
and P :: $('a, 'b, 'c)\ psi$
and *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes $\Psi \rhd Q \Longrightarrow_\tau Q'$
and $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
and *Sim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in Rel \Longrightarrow \Psi \rhd P \rightsquigarrow <Rel> Q$

obtains P' **where** $\Psi \rhd P \Longrightarrow_\tau P'$ **and** $(\Psi, P', Q') \in Rel$
proof −
assume *A*: $\bigwedge P'.\ [\![\Psi \rhd P \Longrightarrow_\tau P'; (\Psi, P', Q') \in Rel]\!] \Longrightarrow thesis$
from ‹$\Psi \rhd Q \Longrightarrow_\tau Q'$› ‹$\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$› *A* **show** *?thesis*
proof(*induct arbitrary*: *P thesis rule*: *tauStepChainInduct*)
case(*TauBase Q Q' P*)
with ‹$\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$› ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$› **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** *P'RelQ'*: $(\Psi, P', Q') \in Rel$
by(*rule-tac weakCongSimE*)
thus *?case* **by**(*rule TauBase*)
next
case(*TauStep Q Q' Q'' P*)
from ‹$\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$› **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** $(\Psi, P', Q') \in Rel$
by(*rule TauStep*)
from ‹$(\Psi, P', Q') \in Rel$› **have** $\Psi \rhd P' \rightsquigarrow <Rel> Q'$ **by**(*rule Sim*)
then obtain P'' **where** *P'Chain*: $\Psi \rhd P' \Longrightarrow\hat{}_\tau P''$ **and** $(\Psi, P'', Q'') \in Rel$
using ‹$\Psi \rhd Q' \longmapsto \tau \prec Q''$› **by**(*blast dest*: *weakSimE*)
from *PChain P'Chain* **have** $\Psi \rhd P \Longrightarrow_\tau P''$ **by** *simp*
thus *?case* **using** ‹$(\Psi, P'', Q'') \in Rel$› **by**(*rule TauStep*)
qed
qed

lemma *weakCongSimTransitive*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and *Rel* :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)\ psi$

and Rel' :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and T :: $('a, 'b, 'c)\ psi$
and Rel'' :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PRelQ*: $(\Psi, P, Q) \in Rel$
and *PSimQ*: $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
and *QSimR*: $\Psi \rhd Q \rightsquigarrow\!\!\ll Rel' \gg R$
and *Set*: $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ (\Psi, P, Q) \in Rel \wedge (\Psi, Q, R) \in Rel'\} \subseteq Rel''$
and *Sim*: $\bigwedge \Psi\ P\ Q.\ (\Psi, P, Q) \in Rel \Longrightarrow \Psi \rhd P \rightsquigarrow <Rel> Q$

shows $\Psi \rhd P \rightsquigarrow\!\!\ll Rel'' \gg R$
proof(*induct rule*: *weakCongSimI*)
case(*cTau R'*)
from *QSimR* $\langle \Psi \rhd R \longmapsto \tau \prec R' \rangle$ **obtain** Q' **where** *QChain*: $\Psi \rhd Q \Longrightarrow_\tau Q'$ **and** *Q'RelR'*: $(\Psi, Q', R') \in Rel'$
by(*blast dest*: *weakCongSimE*)
from *QChain PSimQ Sim* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** *P'RelQ'*: $(\Psi, P', Q') \in Rel$
by(*rule weakStepSimTauChain*)
moreover from *P'RelQ' Q'RelR' Set* **have** $(\Psi, P', R') \in Rel''$ **by** *blast*
ultimately show *?case* **by** *blast*
qed

lemma *weakCongSimStatEq*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)\ psi$
and Rel :: $('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and Q :: $('a, 'b, 'c)\ psi$
and Ψ' :: $'b$

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
and $\Psi \simeq \Psi'$
and *C1*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ [\![(\Psi, P, Q) \in Rel; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in Rel'$

shows $\Psi' \rhd P \rightsquigarrow\!\!\ll Rel' \gg Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau Q'*)
from $\langle \Psi' \rhd Q \longmapsto \tau \prec Q' \rangle$ $\langle \Psi \simeq \Psi' \rangle$
have $\Psi \rhd Q \longmapsto \tau \prec Q'$ **by**(*metis statEqTransition AssertionStatEqSym*)
with *PSimQ* **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** *P'RelQ'*: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakCongSimE*)

from *PChain* $\langle \Psi \simeq \Psi' \rangle$ **have** $\Psi' \rhd P \Longrightarrow_\tau P'$ **by**(*rule tauStepChainStatEq*)
moreover from $\langle (\Psi, P', Q') \in Rel \rangle$ $\langle \Psi \simeq \Psi' \rangle$ **have** $(\Psi', P', Q') \in Rel'$
by(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *weakCongSimMonotonic*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and A :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
  and Q :: $('a, 'b, 'c)$ *psi*
  and B :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*

  assumes $\Psi \rhd P \rightsquigarrow\!\!\!\!\ll A \gg Q$
  and $A \subseteq B$

  shows $\Psi \rhd P \rightsquigarrow\!\!\!\!\ll B \gg Q$
using *assms*
by(*simp* (*no-asm*) *add*: *weakCongSimulation-def*) (*blast dest*: *weakCongSimE*)+

lemma *strongSimWeakCongSim*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)$ *psi*
  and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
  and Q :: $('a, 'b, 'c)$ *psi*

  assumes $\Psi \rhd P \rightsquigarrow[Rel]\ Q$
  and $Rel \subseteq Rel'$

  shows $\Psi \rhd P \rightsquigarrow\!\!\!\!\ll Rel' \gg Q$
using *assms*
apply(*auto simp add*: *simulation-def weakCongSimulation-def*)
by(*erule-tac* $x{=}\tau$ **in** *allE*) *fastforce*

end

end

theory *Weak-Psi-Congruence*
  imports *Weak-Cong-Simulation Weak-Bisimulation*
begin

context *env* **begin**

definition *weakPsiCongruence* :: $'b \Rightarrow ('a, 'b, 'c)$ *psi* $\Rightarrow ('a, 'b, 'c)$ *psi* $\Rightarrow$ *bool* (- $\rhd$ - $\doteq$ - $[70, 70, 70]$ *65*)
where
  $\Psi \rhd P \doteq Q \equiv \Psi \rhd P \approx Q \wedge \Psi \rhd P \rightsquigarrow\!\!\!\!\ll weakBisim \gg Q \wedge \Psi \rhd Q \rightsquigarrow\!\!\!\!\ll weakBisim \gg P$

abbreviation
  weakPsiCongNilJudge (- $\doteq$ - $[70, 70]$ *65*) **where** $P \doteq Q \equiv \mathbf{1} \rhd P \doteq Q$

lemma *weakPsiCongSym*:

fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd P \doteq Q$

shows $\Psi \rhd Q \doteq P$
using *assms*
by(*auto simp add*: *weakPsiCongruence-def weakBisimE*)

lemma *weakPsiCongE*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and Ψ' :: $'b$

assumes $\Psi \rhd P \doteq Q$

shows $\Psi \rhd P \approx Q$
and $\Psi \rhd P \rightsquigarrow$«*weakBisim*» Q
and $\Psi \rhd Q \rightsquigarrow$«*weakBisim*» P
using *assms*
by(*auto simp add*: *weakPsiCongruence-def*)

lemma *weakPsiCongI*[*case-names cWeakBisim cSimLeft cSimRight*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and Ψ' :: $'b$

assumes $\Psi \rhd P \approx Q$
and $\Psi \rhd P \rightsquigarrow$«*weakBisim*» Q
and $\Psi \rhd Q \rightsquigarrow$«*weakBisim*» P

shows $\Psi \rhd P \doteq Q$
using *assms*
by(*auto simp add*: *weakPsiCongruence-def*)

lemma *weakPsiCongSymI*[*consumes 1*, *case-names cSym cWeakBisim cSim*]:
fixes Ψ :: $'b$
and P :: $'d$::*fs-name*
and Q :: $'d$
and Ψ' :: $'b$

assumes *Prop P Q*
and $\bigwedge P\ Q.\ Prop\ P\ Q \Longrightarrow Prop\ Q\ P$

and $\bigwedge P\ Q.\ Prop\ P\ Q \Longrightarrow \Psi \rhd (C\ P) \approx (C\ Q)$

and $\bigwedge P\ Q.\ Prop\ P\ Q \Longrightarrow \Psi \rhd (C\ P) \rightsquigarrow\!\!\!\!\ll weakBisim \gg (C\ Q)$

shows $\Psi \rhd (C\ P) \doteq (C\ Q)$
using *assms*
by(*rule-tac weakPsiCongI*) *auto*

lemma *weakPsiCongSym2*[*consumes 1*, *case-names cWeakBisim cSim*]:
fixes $\Psi :: 'b$
and $\Psi' :: 'b$

assumes $\Psi \rhd P \doteq Q$

and $\bigwedge P\ Q.\ \Psi \rhd P \doteq Q \Longrightarrow \Psi \rhd (C\ P) \approx (C\ Q)$

and $\bigwedge P\ Q.\ \Psi \rhd P \doteq Q \Longrightarrow \Psi \rhd (C\ P) \rightsquigarrow\!\!\!\!\ll weakBisim \gg (C\ Q)$

shows $\Psi \rhd (C\ P) \doteq (C\ Q)$
using *assms*
apply(*rule-tac weakPsiCongSymI*[**where** $C{=}C$])
apply *assumption*
by(*auto simp add*: *weakPsiCongruence-def dest*: *weakBisimE*)

lemma *statEqWeakCong*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\Psi' :: 'b$

assumes $\Psi \rhd P \doteq Q$
and $\Psi \simeq \Psi'$

shows $\Psi' \rhd P \doteq Q$
proof −
let *?Prop* $= \lambda P\ Q.\ \Psi \rhd P \doteq Q \wedge \Psi \simeq \Psi'$
from *assms* **have** *?Prop P Q* **by** *auto*
thus *?thesis*
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P Q*)
thus *?case* **by**(*blast dest*: *weakPsiCongSym*)
next
case(*cSim P Q*)
from $\langle \Psi \rhd P \doteq Q \wedge \Psi \simeq \Psi' \rangle$ **have** $\Psi \rhd P \doteq Q$ **and** $\Psi \simeq \Psi'$ **by** *simp+*
from $\langle \Psi \rhd P \doteq Q \rangle$ **have** $\Psi \rhd P \rightsquigarrow\!\!\!\!\ll weakBisim \gg Q$ **by**(*rule weakPsiCongE*)
with $\langle \Psi \simeq \Psi' \rangle$ **show** *?case* **using** *statEqWeakBisim*
by(*rule-tac weakCongSimStatEq*) *auto*
next
case(*cWeakBisim P Q*)
from $\langle \Psi \rhd P \doteq Q \wedge \Psi \simeq \Psi' \rangle$

```
    have Ψ ▷ P ≈ Q and Ψ ≃ Ψ' by(auto dest: weakPsiCongE)
    thus ?case by(rule statEqWeakBisim)
  qed
qed

lemma weakPsiCongReflexive:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi


  shows Ψ ▷ P ≐ P
by(fastforce intro: weakPsiCongI weakCongSimReflexive weakBisimReflexive)

lemma weakPsiCongClosed:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   p :: name prm

  assumes Ψ ▷ P ≐ Q

  shows (p • Ψ) ▷  (p • P) ≐ (p • Q)
using assms
proof(induct rule: weakPsiCongSymI)
  case(cSym P Q)
  thus ?case by(rule weakPsiCongSym)
next
  case(cSim P Q)
  from ‹Ψ ▷ P ≐ Q› have Ψ ▷ P ⇝«weakBisim» Q by(rule weakPsiCongE)
   thus ?case by(drule-tac p=p in weakCongSimClosed(1)[OF weakBisimEqvt])
(simp add: eqvts)
next
  case(cWeakBisim P Q)
  from ‹Ψ ▷ P ≐ Q› have Ψ ▷ P ≈ Q by(rule weakPsiCongE)
  thus ?case by(rule weakBisimClosed)
qed

lemma weakPsiCongTransitive:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   R :: ('a, 'b, 'c) psi

  assumes Ψ ▷ P ≐ Q
  and     Ψ ▷ Q ≐ R

  shows Ψ ▷ P ≐ R
proof −
  from assms have Ψ ▷ P ≐ Q ∧ Ψ ▷ Q ≐ R  by auto
```

thus *?thesis*
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P R*)
thus *?case* **by**(*auto dest*: *weakPsiCongSym*)
next
case(*cSim P R*)
hence $\Psi \rhd P \doteq Q$ **and** $\Psi \rhd Q \doteq R$ **by** *auto*
moreover from ‹$\Psi \rhd P \doteq Q$› **have** $\Psi \rhd P \approx Q$ **by**(*metis weakBisimE weakPsiCongE*)
moreover from ‹$\Psi \rhd P \doteq Q$› **have** $\Psi \rhd P \rightsquigarrow$«*weakBisim*» Q **by**(*rule weakPsiCongE*)
moreover from ‹$\Psi \rhd Q \doteq R$› **have** $\Psi \rhd Q \rightsquigarrow$«*weakBisim*» R **by**(*rule weakPsiCongE*)
moreover have $\{(\Psi, P, R) \mid \Psi\ P\ R.\ \exists Q.\ \Psi \rhd P \approx Q \wedge \Psi \rhd Q \approx R\} \subseteq$ *weakBisim*
by(*auto dest*: *weakBisimTransitive*)
ultimately show *?case* **using** *weakBisimE*(*2*) **by**(*rule-tac weakCongSimTransitive*)
next
case(*cWeakBisim P R*)
thus *?case* **by**(*auto dest*: *weakBisimTransitive weakPsiCongE*)
qed
qed

lemma *strongBisimWeakPsiCong*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \sim Q$

shows $\Psi \rhd P \doteq Q$
using *assms*
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P Q*)
from ‹$\Psi \rhd P \sim Q$› **show** *?case* **by**(*rule bisimE*)
next
case(*cSim P Q*)
from ‹$\Psi \rhd P \sim Q$› **have** $\Psi \rhd P \rightsquigarrow[bisim]\ Q$ **by**(*rule bisimE*)
thus $\Psi \rhd P \rightsquigarrow$«*weakBisim*» Q **using** *strongBisimWeakBisim*
by(*rule-tac strongSimWeakCongSim*) *auto*
next
case(*cWeakBisim P Q*)
thus *?case* **by**(*rule strongBisimWeakBisim*)
qed

lemma *structCongWeakPsiCong*:
fixes P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $P \equiv_s Q$

shows $P \doteq Q$
using *assms*
by(*metis structCongBisim strongBisimWeakPsiCong*)

end

end

theory *Weak-Cong-Sim-Pres*
imports *Weak-Sim-Pres Weak-Cong-Simulation*
begin

context *env* **begin**

lemma *caseWeakSimPres*:
fixes Ψ :: *'b*
and *CsP* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *CsQ* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *M* :: *'a*
and *N* :: *'a*

assumes *PRelQ*: $\bigwedge\varphi\ Q.\ (\varphi, Q)\ mem\ CsQ \Longrightarrow \exists P.\ (\varphi, P)\ mem\ CsP \wedge guarded\ P \wedge Eq\ \Psi\ P\ Q$
and *Sim*: $\bigwedge\Psi'\ P\ Q.\ (\Psi', P, Q) \in Rel \Longrightarrow \Psi' \rhd P \rightsquigarrow <Rel> Q$
and *EqRel*: $\bigwedge\Psi'\ P\ Q.\ Eq\ \Psi'\ P\ Q \Longrightarrow (\Psi', P, Q) \in Rel$
and *EqSim*: $\bigwedge\Psi'\ P\ Q.\ Eq\ \Psi'\ P\ Q \Longrightarrow \Psi' \rhd P \rightsquigarrow «Rel» Q$

shows $\Psi \rhd Cases\ CsP \rightsquigarrow <Rel> Cases\ CsQ$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ Q'$)
from ‹*bn* $\alpha \sharp* (Cases\ CsP)$› **have** *bn* $\alpha \sharp* CsP$ **by** *auto*
from ‹$\Psi \rhd Cases\ CsQ \longmapsto \alpha \prec Q'$›
show *?case*
proof(*induct rule*: *caseCases*)
case(*cCase* $\varphi\ Q$)
from ‹(φ, Q) *mem CsQ*› **obtain** P **where** (φ, P) *mem CsP* **and** *guarded P* **and** *Eq* $\Psi\ P\ Q$
by(*metis PRelQ*)
from ‹*Eq* $\Psi\ P\ Q$› **have** $\Psi \rhd P \rightsquigarrow <Rel> Q$ **by**(*metis EqRel Sim*)
moreover note ‹$\Psi \rhd Q \longmapsto \alpha \prec Q'$› ‹*bn* $\alpha \sharp* \Psi$›
moreover from ‹*bn* $\alpha \sharp* CsP$› ‹(φ, P) *mem CsP*› **have** *bn* $\alpha \sharp* P$ **by**(*auto dest*: *memFreshChain*)
ultimately obtain $P''\ P'$ **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$
and *P''Chain*: $\Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: $(\Psi \otimes \Psi', P', Q') \in Rel$

using ‹$\alpha \neq \tau$›
by(*blast dest*: *weakSimE*)
note *PTrans* ‹(*φ*, *P*) *mem CsP*› ‹Ψ ⊢ *φ*› ‹*guarded P*›
moreover from ‹*guarded Q*› **have** *insertAssertion* (*extractFrame Q*) Ψ ≃$_F$ ⟨*ε*, Ψ ⊗ **1**⟩
by(*rule insertGuardedAssertion*)
hence *insertAssertion* (*extractFrame*(*Cases CsQ*)) Ψ ↪$_F$ *insertAssertion* (*extractFrame Q*) Ψ
by(*auto simp add*: *FrameStatEq-def*)
moreover from *Identity* **have** *insertAssertion* (*extractFrame*(*Cases CsQ*)) Ψ ↪$_F$ ⟨*ε*, Ψ⟩
by(*auto simp add*: *AssertionStatEq-def*)
ultimately have Ψ : (*Cases CsQ*) ▷ *Cases CsP* ⟹*α* ≺ *P''*
by(*rule weakCase*)
with *P''Chain P'RelQ'* **show** *?case* **by** *blast*
qed
next
case(*cTau Q'*)
from ‹Ψ ▷ *Cases CsQ* ⟼*τ* ≺ *Q'*› **show** *?case*
proof(*induct rule*: *caseCases*)
case(*cCase φ Q*)
from ‹(*φ*, *Q*) *mem CsQ*› **obtain** *P* **where** (*φ*, *P*) *mem CsP* **and** *guarded P* **and** *Eq* Ψ *P Q*
by(*metis PRelQ*)
from ‹*Eq* Ψ *P Q*› ‹Ψ ▷ *Q* ⟼*τ* ≺ *Q'*›
obtain *P'* **where** *PChain*: Ψ ▷ *P* ⟹$_\tau$ *P'* **and** *P'RelQ'*: (Ψ, *P'*, *Q'*) ∈ *Rel*
by(*blast dest*: *EqSim weakCongSimE*)
from *PChain* ‹(*φ*, *P*) *mem CsP*› ‹Ψ ⊢ *φ*› ‹*guarded P*› **have** Ψ ▷ *Cases CsP* ⟹$_\tau$ *P'*
by(*rule tauStepChainCase*)
hence Ψ ▷ *Cases CsP* ⟹$\hat{}_\tau$ *P'* **by**(*simp add*: *trancl-into-rtrancl*)
with *P'RelQ'* **show** *?case* **by** *blast*
qed
qed

lemma *weakCongSimCasePres*:
fixes Ψ :: *'b*
and *CsP* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and *CsQ* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *M* :: *'a*
and *N* :: *'a*

assumes *PRelQ*: ⋀*φ Q*. (*φ*, *Q*) *mem CsQ* ⟹ ∃*P*. (*φ*, *P*) *mem CsP* ∧ *guarded P* ∧ *Eq* Ψ *P Q*
and *EqSim*: ⋀Ψ' *P Q*. *Eq* Ψ' *P Q* ⟹ Ψ' ▷ *P* ⇝«*Rel*» *Q*

shows Ψ ▷ *Cases CsP* ⇝«*Rel*» *Cases CsQ*
proof(*induct rule*: *weakCongSimI*)

case(*cTau* Q')
from ‹$\Psi \rhd$ *Cases CsQ* $\longmapsto\tau \prec Q'$› **show** *?case*
proof(*induct rule*: *caseCases*)
case(*cCase* φ Q)
from ‹(φ, Q) *mem CsQ*› **obtain** P **where** (φ, P) *mem CsP* **and** *guarded P* **and** *Eq* Ψ P Q
by(*metis PRelQ*)
from ‹*Eq* Ψ P Q› ‹$\Psi \rhd Q \longmapsto\tau \prec Q'$›
obtain P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** P'*RelQ'*: (Ψ, P', Q') $\in$ *Rel*
by(*blast dest*: *EqSim weakCongSimE*)
from *PChain* ‹(φ, P) *mem CsP*› ‹$\Psi \vdash \varphi$› ‹*guarded P*› **have** $\Psi \rhd$ *Cases CsP* $\Longrightarrow_\tau P'$
by(*rule tauStepChainCase*)
with P'*RelQ'* **show** *?case* **by** *blast*
qed
qed

lemma *weakCongSimResPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and *Rel* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*
and Q :: (*'a*, *'b*, *'c*) *psi*
and x :: *name*
and *Rel'* :: (*'b* × (*'a*, *'b*, *'c*) *psi* × (*'a*, *'b*, *'c*) *psi*) *set*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow$«*Rel*» Q
and *eqvt Rel'*
and $x \sharp \Psi$
and *Rel* ⊆ *Rel'*
and *C1*: $\bigwedge \Psi'\ R\ S\ x.\ [\![(\Psi', R, S) \in Rel; x \sharp \Psi]\!] \Longrightarrow (\Psi', (\!|\nu x|\!)R, (\!|\nu x|\!)S) \in Rel'$

shows $\Psi \rhd (\!|\nu x|\!)P \rightsquigarrow$«*Rel'*» $(\!|\nu x|\!)Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from ‹$\Psi \rhd (\!|\nu x|\!)Q \longmapsto\tau \prec Q'$› **have** $x \sharp Q'$ **by**(*auto dest*: *tauFreshDerivative simp add*: *abs-fresh*)
with ‹$\Psi \rhd (\!|\nu x|\!)Q \longmapsto\tau \prec Q'$› ‹$x \sharp \Psi$› **show** *?case*
proof(*induct rule*: *resTauCases*)
case(*cRes* Q')
from *PSimQ* ‹$\Psi \rhd Q \longmapsto\tau \prec Q'$› **obtain** P' **where** *PChain*: $\Psi \rhd P \Longrightarrow_\tau P'$ **and** P'*RelQ'*: (Ψ, P', Q') $\in$ *Rel*
by(*blast dest*: *weakCongSimE*)
from *PChain* ‹$x \sharp \Psi$› **have** $\Psi \rhd (\!|\nu x|\!)P \Longrightarrow_\tau (\!|\nu x|\!)P'$ **by**(*rule tauStepChainResPres*)
moreover from P'*RelQ'* ‹$x \sharp \Psi$› **have** (Ψ, $(\!|\nu x|\!)P'$, $(\!|\nu x|\!)Q'$) $\in$ *Rel'* **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

qed

lemma *weakCongSimResChainPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and *xvec* :: *name list*

assumes *PSimQ*: $\Psi \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
and *eqvt Rel*
and *xvec* $\sharp*$ Ψ
and *C1*: $\bigwedge \Psi' R S$ *xvec*. $[\![(\Psi', R, S) \in Rel; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu*xvec|\!)R,$
$(\!|\nu*xvec|\!)S) \in Rel$

shows $\Psi \rhd (\!|\nu*xvec|\!)P \rightsquigarrow\!\!\ll Rel \gg (\!|\nu*xvec|\!)Q$
using ‹*xvec* $\sharp*$ Ψ›
proof(*induct xvec*)
case *Nil*
from *PSimQ* **show** *?case* **by** *simp*
next
case(*Cons x xvec*)
from ‹$(x\#xvec) \sharp* \Psi$› **have** $x \sharp \Psi$ **and** *xvec* $\sharp*$ Ψ **by** *simp+*
from ‹*xvec* $\sharp*$ $\Psi \Longrightarrow \Psi \rhd (\!|\nu*xvec|\!)P \rightsquigarrow\!\!\ll Rel \gg (\!|\nu*xvec|\!)Q$› ‹*xvec* $\sharp*$ Ψ›
have $\Psi \rhd (\!|\nu*xvec|\!)P \rightsquigarrow\!\!\ll Rel \gg (\!|\nu*xvec|\!)Q$ **by** *simp*
moreover note ‹*eqvt Rel*› ‹$x \sharp \Psi$›
moreover have *Rel* $\subseteq$ *Rel* **by** *simp*
moreover have $\bigwedge \Psi\ P\ Q\ x. [\![(\Psi, P, Q) \in Rel; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu*[x]|\!)P, (\!|\nu*[x]|\!)Q)$
$\in Rel$
by(*rule-tac xvec*=$[x]$ **in** *C1*) *auto*
hence $\bigwedge \Psi\ P\ Q\ x. [\![(\Psi, P, Q) \in Rel; x \sharp \Psi]\!] \Longrightarrow (\Psi, (\!|\nu x|\!)P, (\!|\nu x|\!)Q) \in Rel$
by *simp*
ultimately have $\Psi \rhd (\!|\nu x|\!)((\!|\nu*xvec|\!)P) \rightsquigarrow\!\!\ll Rel \gg (\!|\nu x|\!)((\!|\nu*xvec|\!)Q)$
by(*rule weakCongSimResPres*)
thus *?case* **by** *simp*
qed

lemma *weakCongSimParPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and *Rel* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*
and *Rel'* :: $('b \times ('a, 'b, 'c)$ *psi* $\times ('a, 'b, 'c)$ *psi*$)$ *set*

assumes *PSimQ*: $\bigwedge \Psi'. \Psi' \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
and *PSimQ'*: $\bigwedge \Psi'. \Psi' \rhd P \rightsquigarrow\!\!< Rel > Q$
and *StatImp*: $\bigwedge \Psi'. \Psi' \rhd Q \lessapprox\!\!< Rel > P$

and *eqvt Rel*
and *eqvt Rel′*

and *Sym*: $\bigwedge \Psi'\ S\ T.\ [\![(\Psi',\ S,\ T) \in Rel]\!] \Longrightarrow (\Psi',\ T,\ S) \in Rel$
and *Ext*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi',\ S,\ T) \in Rel]\!] \Longrightarrow (\Psi' \otimes \Psi'',\ S,\ T) \in Rel$

and *C1*: $\bigwedge \Psi'\ S\ T\ A_U\ \Psi_U\ U.\ [\![(\Psi' \otimes \Psi_U,\ S,\ T) \in Rel;\ extractFrame\ U = \langle A_U,\ \Psi_U\rangle;\ A_U \sharp* \Psi';\ A_U \sharp* S;\ A_U \sharp* T]\!] \Longrightarrow (\Psi',\ S \parallel U,\ T \parallel U) \in Rel'$
and *C2*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi',\ S,\ T) \in Rel';\ xvec \sharp* \Psi']\!] \Longrightarrow (\Psi',\ (\!|\nu * xvec|\!)S,\ (\!|\nu * xvec|\!)\,T) \in Rel'$
and *C3*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi',\ S,\ T) \in Rel;\ \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'',\ S,\ T) \in Rel$

shows $\Psi \rhd P \parallel R \rightsquigarrow\!\!\ll Rel' \gg Q \parallel R$
proof(*induct rule*: *weakCongSimI*)
case(*cTau QR*)
from ‹$\Psi \rhd Q \parallel R \longmapsto \tau \prec QR$› **show** *?case*
proof(*induct rule*: *parTauCases*[**where** $C=(P,\ R)$])
case(*cPar1 Q′ A_R Ψ_R*)
from ‹$A_R \sharp* (P,\ R)$› **have** $A_R \sharp* P$
by *simp+*
have *FrR*: *extractFrame R* = $\langle A_R,\ \Psi_R\rangle$ **by** *fact*
with ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $\Psi \otimes \Psi_R \rhd P \rightsquigarrow\!\!\ll Rel \gg Q$
by(*rule-tac PSimQ*)
moreover have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto \tau \prec Q'$ **by** *fact*
ultimately obtain P' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow_\tau P'$ **and** *P′RelQ′*: $(\Psi \otimes \Psi_R,\ P',\ Q') \in Rel$
by(*rule weakCongSimE*)
from *PChain QTrans* ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› **have** $A_R \sharp* P'$ **and** $A_R \sharp* Q'$
by(*force dest*: *freeFreshChainDerivative tauStepChainFreshChain*)+
from *PChain FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› **have** $\Psi \rhd P \parallel R \Longrightarrow_\tau (P' \parallel R)$
by(*rule tauStepChainPar1*)
moreover from *P′RelQ′ FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'$› ‹$A_R \sharp* Q'$› **have** $(\Psi,\ P' \parallel R,\ Q' \parallel R) \in Rel'$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 R′ A_Q Ψ_Q*)
from ‹$A_Q \sharp* (P,\ R)$› **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp+*
obtain $A_P\ \Psi_P$ **where** *FrP*: *extractFrame P* = $\langle A_P,\ \Psi_P\rangle$ **and** $A_P \sharp* (\Psi,\ A_Q,\ \Psi_Q,\ R)$
by(*rule freshFrame*)
hence $A_P \sharp* \Psi$ **and** $A_P \sharp* A_Q$ **and** $A_P \sharp* \Psi_Q$ **and** $A_P \sharp* R$
by *simp+*

have *FrQ*: *extractFrame Q* = $\langle A_Q,\ \Psi_Q\rangle$ **by** *fact*
from ‹$A_Q \sharp* P$› *FrP* ‹$A_P \sharp* A_Q$› **have** $A_Q \sharp* \Psi_P$
by(*drule-tac extractFrameFreshChain*) *auto*

obtain $A_R\ \Psi_R$ **where** *FrR*: *extractFrame R* = $\langle A_R,\ \Psi_R\rangle$ **and** $A_R \sharp* (\Psi,\ P,\ Q,\ A_Q,\ A_P,\ \Psi_Q,\ \Psi_P,\ R)$ **and** *distinct* A_R

by(*rule freshFrame*)
then have $A_R \sharp* \Psi$ **and** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* A_Q$ **and** $A_R \sharp* A_P$ **and** $A_R \sharp* \Psi_Q$ **and** $A_R \sharp* \Psi_P$
and $A_R \sharp* R$
by *simp*+

from $\langle A_Q \sharp* R\rangle$ *FrR* $\langle A_R \sharp* A_Q\rangle$ **have** $A_Q \sharp* \Psi_R$ **by**(*drule-tac extractFrame-FreshChain*) *auto*
from $\langle A_P \sharp* R\rangle$ $\langle A_R \sharp* A_P\rangle$ *FrR* **have** $A_P \sharp* \Psi_R$ **by**(*drule-tac extractFrame-FreshChain*) *auto*

moreover from $\langle \Psi \otimes \Psi_Q \rhd R \longmapsto \tau \prec R'\rangle$ *FrR* $\langle$*distinct* $A_R\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$ $\langle A_R \sharp* R\rangle$
obtain Ψ' ${A_R}'$ ${\Psi_R}'$ **where** $\Psi_R \otimes \Psi' \simeq {\Psi_R}'$ **and** *FrR'*: *extractFrame* $R' = \langle {A_R}', {\Psi_R}'\rangle$
and ${A_R}' \sharp* \Psi$ **and** ${A_R}' \sharp* P$ **and** ${A_R}' \sharp* Q$
by(*rule-tac* $C=(\Psi, P, Q, R)$ **in** *expandTauFrame*) (*assumption* | *simp*)+

from *FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* Q\rangle$
obtain P' P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau P'$
and *QimpP'*: *insertAssertion*(*extractFrame* Q) $(\Psi \otimes \Psi_R) \hookrightarrow_F$ *insertAssertion*(*extractFrame* P') $(\Psi \otimes \Psi_R)$
and *P'Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau P''$
and *P'RelQ*: $((\Psi \otimes \Psi_R) \otimes \Psi', P'', Q) \in Rel$
by(*metis StatImp weakStatImp-def Sym*)
obtain ${A_P}'$ ${\Psi_P}'$ **where** *FrP'*: *extractFrame* $P' = \langle {A_P}', {\Psi_P}'\rangle$ **and** ${A_P}' \sharp* \Psi$ **and** ${A_P}' \sharp* \Psi_R$ **and** ${A_P}' \sharp* \Psi_Q$
and ${A_P}' \sharp* A_Q$ **and** ${A_P}' \sharp* R$ **and** ${A_P}' \sharp* A_R$
by(*rule-tac* $C=(\Psi, \Psi_R, \Psi_Q, A_Q, R, A_R)$ **in** *freshFrame*) *auto*

from *PChain P'Chain* $\langle A_R \sharp* P\rangle$ $\langle A_Q \sharp* P\rangle$ $\langle {A_R}' \sharp* P\rangle$ **have** $A_Q \sharp* P'$ **and** $A_R \sharp* P'$ **and** ${A_R}' \sharp* P'$ **and** ${A_R}' \sharp* P''$
by(*force intro*: *tauChainFreshChain*)+
from $\langle A_R \sharp* P'\rangle$ $\langle {A_P}' \sharp* A_R\rangle$ $\langle A_Q \sharp* P'\rangle$ $\langle {A_P}' \sharp* A_Q\rangle$ *FrP'* **have** $A_Q \sharp* {\Psi_P}'$ **and** $A_R \sharp* {\Psi_P}'$
by(*force dest*: *extractFrameFreshChain*)+

from *PChain FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$ **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau P' \parallel R$ **by**(*rule tauChainPar1*)
moreover have *RTrans*: $\Psi \otimes {\Psi_P}' \rhd R \longmapsto \tau \prec R'$
proof −
have $\Psi \otimes \Psi_Q \rhd R \longmapsto \tau \prec R'$ **by** *fact*
moreover have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle {A_P}', (\Psi \otimes {\Psi_P}') \otimes \Psi_R\rangle$
proof −
have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameInt-CompositionSym FrameStatEqSym*)
moreover with *FrP' FrQ QimpP'* $\langle {A_P}' \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle {A_P}' \sharp* \Psi_R\rangle$ $\langle A_Q \sharp* \Psi_R\rangle$

have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle A_P', (\Psi \otimes \Psi_R) \otimes \Psi_P'\rangle \simeq_F \langle A_P', (\Psi \otimes \Psi_P') \otimes \Psi_R\rangle$
by(*metis frameIntAssociativity Commutativity FrameStatEqTrans frameIntCompositionSym frameIntAssociativity*[*THEN FrameStatEqSym*])
ultimately show *?thesis*
by(*rule FrameStatEqImpCompose*)
qed
ultimately show *?thesis*
using $\langle A_P' \sharp* \Psi\rangle$ $\langle A_P' \sharp* \Psi_Q\rangle$ $\langle A_Q \sharp* \Psi\rangle$ $\langle A_Q \sharp* \Psi_P'\rangle$ $\langle A_P' \sharp* R\rangle$ $\langle A_Q \sharp* R\rangle$
$\langle A_P' \sharp* A_R\rangle$ $\langle A_R \sharp* A_Q\rangle$ $\langle A_R \sharp* \Psi_P'\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$ $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_P'\rangle$ $\langle A_Q \sharp* \Psi_P'\rangle$ *FrR* $\langle$*distinct* $A_R\rangle$
by(*force intro*: *transferTauFrame*)
qed
hence $\Psi \rhd P' \parallel R \longmapsto\tau \prec (P' \parallel R')$ **using** *FrP'* $\langle A_P' \sharp* \Psi\rangle$ $\langle A_P' \sharp* R\rangle$
by(*rule-tac Par2*) *auto*
moreover from *P'Chain* **have** $\Psi \otimes \Psi_R \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau P''$
by(*metis tauChainStatEq Associativity*)
with $\langle \Psi_R \otimes \Psi' \simeq \Psi_R'\rangle$ **have** $\Psi \otimes \Psi_R' \rhd P' \Longrightarrow\hat{}_\tau P''$
by(*rule-tac tauChainStatEq*, *auto*) (*metis compositionSym*)
hence $\Psi \rhd P' \parallel R' \Longrightarrow\hat{}_\tau P'' \parallel R'$ **using** *FrR'* $\langle A_R' \sharp* \Psi\rangle$ $\langle A_R' \sharp* P'\rangle$ **by**(*rule-tac tauChainPar1*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow_\tau (P'' \parallel R')$
by(*drule-tac tauActTauStepChain*) *auto*

moreover from *P'RelQ* $\langle \Psi_R \otimes \Psi' \simeq \Psi_R'\rangle$ **have** $(\Psi \otimes \Psi_R', P'', Q) \in Rel$ **by**(*blast intro*: *C3 Associativity compositionSym*)
with *FrR'* $\langle A_R' \sharp* \Psi\rangle$ $\langle A_R' \sharp* P''\rangle$ $\langle A_R' \sharp* Q\rangle$ **have** $(\Psi, P'' \parallel R', Q \parallel R') \in Rel'$ **by**(*rule-tac C1*)
ultimately show *?case* **by** *blast*
next
case(*cComm1* Ψ_R *M N* Q' A_Q Ψ_Q *K xvec* R' A_R)
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from $\langle A_Q \sharp* (P, R)\rangle$ **have** $A_Q \sharp* P$ **and** $A_Q \sharp* R$ **by** *simp+*

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from $\langle A_R \sharp* (P, R)\rangle$ **have** $A_R \sharp* P$ **and** $A_R \sharp* R$ **by** *simp+*
from $\langle xvec \sharp* (P, R)\rangle$ **have** *xvec* $\sharp* P$ **and** *xvec* $\sharp* R$ **by** *simp+*

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|N|\!) \prec Q'$ **and** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec R'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$ **by** *fact+*

from *RTrans FrR* $\langle$*distinct* $A_R\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* xvec\rangle$ $\langle xvec \sharp* R\rangle$ $\langle xvec \sharp* Q\rangle$ $\langle xvec \sharp* \Psi\rangle$ $\langle xvec \sharp* \Psi_Q\rangle$ $\langle A_R \sharp* Q\rangle$
$\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* \Psi_Q\rangle$ $\langle xvec \sharp* K\rangle$ $\langle A_R \sharp* K\rangle$ $\langle A_R \sharp* R\rangle$ $\langle xvec \sharp* R\rangle$ $\langle A_R \sharp* P\rangle$ $\langle xvec \sharp* P\rangle$

$\langle A_Q \sharp* A_R\rangle$ $\langle A_Q \sharp* xvec\rangle$ $\langle A_R \sharp* K\rangle$ $\langle A_R \sharp* N\rangle$ $\langle xvec \sharp* K\rangle$ $\langle distinct\ xvec\rangle$

obtain $p\ \Psi'\ A_R'\ \Psi_R'$ **where** S: $set\ p \subseteq set\ xvec \times set(p \cdot xvec)$ **and** FrR': $extractFrame\ R' = \langle A_R', \Psi_R'\rangle$
and $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* Q$ **and** $A_R' \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* \Psi$
and $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* \Psi_Q$ **and** $(p \cdot xvec) \sharp* K$ **and** $(p \cdot xvec) \sharp* R$
and $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* A_Q$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* N$
by(*rule-tac* C=(Ψ, Q, Ψ_Q, K, R, P, A_Q) **and** C'=(Ψ, Q, Ψ_Q, K, R, P, A_Q) **in** *expandFrame*) (*assumption* | *simp*)+

from $\langle A_R \sharp* \Psi\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot \Psi)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* \Psi\rangle$ $\langle (p \cdot xvec) \sharp* \Psi\rangle$ S **have** $(p \cdot A_R) \sharp* \Psi$ **by** *simp*
from $\langle A_R \sharp* P\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot P)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* P\rangle$ $\langle (p \cdot xvec) \sharp* P\rangle$ S **have** $(p \cdot A_R) \sharp* P$ **by** *simp*
from $\langle A_R \sharp* Q\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot Q)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* Q\rangle$ $\langle (p \cdot xvec) \sharp* Q\rangle$ S **have** $(p \cdot A_R) \sharp* Q$ **by** *simp*
from $\langle A_R \sharp* R\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot R)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* R\rangle$ $\langle (p \cdot xvec) \sharp* R\rangle$ S **have** $(p \cdot A_R) \sharp* R$ **by** *simp*
from $\langle A_R \sharp* K\rangle$ **have** $(p \cdot A_R) \sharp* (p \cdot K)$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])
with $\langle xvec \sharp* K\rangle$ $\langle (p \cdot xvec) \sharp* K\rangle$ S **have** $(p \cdot A_R) \sharp* K$ **by** *simp*

from $\langle A_Q \sharp* xvec\rangle$ $\langle (p \cdot xvec) \sharp* A_Q\rangle$ $\langle A_Q \sharp* M\rangle$ S **have** $A_Q \sharp* (p \cdot M)$ **by**(*simp add*: *freshChainSimps*)
from $\langle A_Q \sharp* xvec\rangle$ $\langle (p \cdot xvec) \sharp* A_Q\rangle$ $\langle A_Q \sharp* A_R\rangle$ S **have** $A_Q \sharp* (p \cdot A_R)$ **by**(*simp add*: *freshChainSimps*)

from *QTrans* S $\langle xvec \sharp* Q\rangle$ $\langle (p \cdot xvec) \sharp* Q\rangle$ **have** $(p \cdot (\Psi \otimes \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*rule-tac inputPermFrameSubject*) (*assumption* | *auto simp add*: *fresh-star-def*)+
with $\langle xvec \sharp* \Psi\rangle$ $\langle (p \cdot xvec) \sharp* \Psi\rangle$ S **have** *QTrans*: $(\Psi \otimes (p \cdot \Psi_R)) \rhd Q \longmapsto (p \cdot M)(\!|N|\!) \prec Q'$
by(*simp add*: *eqvts*)
from *FrR* **have** $(p \cdot extractFrame\ R) = p \cdot \langle A_R, \Psi_R\rangle$ **by** *simp*
with $\langle xvec \sharp* R\rangle$ $\langle (p \cdot xvec) \sharp* R\rangle$ S **have** *FrR*: $extractFrame\ R = \langle (p \cdot A_R), (p \cdot \Psi_R)\rangle$
by(*simp add*: *eqvts*)

from *MeqK* **have** $(p \cdot (\Psi \otimes \Psi_Q \otimes \Psi_R)) \vdash (p \cdot M) \leftrightarrow (p \cdot K)$ **by**(*rule chanEqClosed*)
with $\langle xvec \sharp* \Psi\rangle$ $\langle (p \cdot xvec) \sharp* \Psi\rangle$ $\langle xvec \sharp* \Psi_Q\rangle$ $\langle (p \cdot xvec) \sharp* \Psi_Q\rangle$ $\langle xvec \sharp* K\rangle$ $\langle (p \cdot xvec) \sharp* K\rangle$ S

have *MeqK*: $\Psi \otimes \Psi_Q \otimes (p \cdot \Psi_R) \vdash (p \cdot M) \leftrightarrow K$ **by**(*simp add*: *eqvts*)

have $\Psi \otimes (p \cdot \Psi_R) \rhd P \rightsquigarrow <Rel> Q$ **by**(*rule PSimQ'*)

with *QTrans* **obtain** P' P'' **where** *PTrans*: $\Psi \otimes (p \cdot \Psi_R) : Q \rhd P \Longrightarrow (p \cdot M)(\!|N|\!) \prec P''$
and *P''Chain*: $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau P'$
and *P'RelQ'*: $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', P', Q') \in Rel$
by(*fastforce dest*: *weakSimE*)
from *PTrans QTrans* ‹A_R' ♯* P› ‹A_R' ♯* Q› ‹A_R' ♯* N› **have** A_R' ♯* P'' **and** A_R' ♯* Q'
by(*blast dest*: *weakInputFreshChainDerivative inputFreshChainDerivative*)+

from *PTrans* **obtain** P''' **where** *PChain*: $\Psi \otimes (p \cdot \Psi_R) \rhd P \Longrightarrow\hat{}_\tau P'''$
and *QimpP'''*: *insertAssertion* (*extractFrame Q*) ($\Psi \otimes (p \cdot \Psi_R)$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame P'''*) ($\Psi \otimes (p \cdot \Psi_R)$)
and *P'''Trans*: $\Psi \otimes (p \cdot \Psi_R) \rhd P''' \longmapsto (p \cdot M)(\!|N|\!) \prec P''$
by(*rule weakTransitionE*)

from *PChain* ‹*xvec* ♯* P› ‹$(p \cdot A_R)$ ♯* P› ‹A_R' ♯* P› **have** *xvec* ♯* P''' **and** $(p \cdot A_R)$ ♯* P''' **and** A_R' ♯* P'''
by(*force intro*: *tauChainFreshChain*)+
from *P'''Trans* ‹A_R' ♯* P'''› ‹A_R' ♯* N› **have** A_R' ♯* P'' **by**(*force dest*: *inputFreshChainDerivative*)

obtain A_P''' Ψ_P''' **where** *FrP'''*: *extractFrame* $P''' = \langle A_P''', \Psi_P''' \rangle$ **and** A_P''' ♯* (Ψ, A_Q, Ψ_Q, $p \cdot A_R$, $p \cdot \Psi_R$, $p \cdot M$, N, K, R, P''', *xvec*) **and** *distinct* A_P'''
by(*rule freshFrame*)
hence A_P''' ♯* Ψ **and** A_P''' ♯* A_Q **and** A_P''' ♯* Ψ_Q **and** A_P''' ♯* $(p \cdot M)$ **and** A_P''' ♯* R
and A_P''' ♯* N **and** A_P''' ♯* K **and** A_P''' ♯* $(p \cdot A_R)$ **and** A_P''' ♯* P''' **and** A_P''' ♯* *xvec* **and** A_P''' ♯* $(p \cdot \Psi_R)$
by *simp*+

have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes (p \cdot \Psi_R) \rangle \simeq_F \langle A_Q, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q \rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover with *QimpP''' FrP''' FrQ* ‹A_P''' ♯* Ψ› ‹A_Q ♯* Ψ› ‹A_P''' ♯* $(p \cdot \Psi_R)$› ‹A_Q ♯* Ψ_R› ‹A_Q ♯* *xvec*› ‹$(p \cdot xvec)$ ♯* A_Q› *S*
have $\langle A_Q, (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_Q \rangle \hookrightarrow_F \langle A_P''', (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P''' \rangle$ **using** *freshCompChain*
by(*simp add*: *freshChainSimps*)
moreover have $\langle A_P''', (\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi_P''' \rangle \simeq_F \langle A_P''', (\Psi \otimes \Psi_P''') \otimes (p \cdot \Psi_R) \rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately have *QImpP'''*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes (p \cdot \Psi_R) \rangle \hookrightarrow_F \langle A_P''', (\Psi \otimes \Psi_P''') \otimes (p \cdot \Psi_R) \rangle$
by(*rule FrameStatEqImpCompose*)

from *PChain FrR* ‹$(p \cdot A_R) \sharp^* \Psi$› ‹$(p \cdot A_R) \sharp^* P$› **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau P''' \parallel R$ **by**(*rule tauChainPar1*)
moreover from *RTrans FrR P'''Trans MeqK QImpP''' FrP''' FrQ* ‹*distinct* A_P'''› ‹*distinct* A_R› ‹$A_P''' \sharp^* (p \cdot A_R)$› ‹$A_Q \sharp^* (p \cdot A_R)$›
‹$(p \cdot A_R) \sharp^* \Psi$› ‹$(p \cdot A_R) \sharp^* P'''$› ‹$(p \cdot A_R) \sharp^* Q$› ‹$(p \cdot A_R) \sharp^* R$› ‹$(p \cdot A_R) \sharp^* K$› ‹$A_P''' \sharp^* \Psi$› ‹$A_P''' \sharp^* R$›
‹$A_P''' \sharp^* P'''$› ‹$A_P''' \sharp^* (p \cdot M)$› ‹$A_Q \sharp^* R$› ‹$A_Q \sharp^* (p \cdot M)$›
obtain K' **where** $\Psi \otimes \Psi_P''' \rhd R \longmapsto K'(\!|\nu * xvec|\!)\langle N\rangle \prec R'$ **and** $\Psi \otimes \Psi_P''' \otimes (p \cdot \Psi_R) \vdash (p \cdot M) \leftrightarrow K'$ **and** $(p \cdot A_R) \sharp^* K'$
by(*rule-tac comm1Aux*) (*assumption* | *simp*)+

with *P'''Trans FrP'''* **have** $\Psi \rhd P''' \parallel R \longmapsto \tau \prec (\!|\nu * xvec|\!)(P'' \parallel R')$ **using** *FrR* ‹$(p \cdot A_R) \sharp^* \Psi$› ‹$(p \cdot A_R) \sharp^* P'''$› ‹$(p \cdot A_R) \sharp^* R$›
‹*xvec* $\sharp^* P'''$› ‹$A_P''' \sharp^* \Psi$› ‹$A_P''' \sharp^* P'''$› ‹$A_P''' \sharp^* R$› ‹$A_P''' \sharp^* (p \cdot M)$› ‹$(p \cdot A_R) \sharp^* K'$› ‹$A_P''' \sharp^* (p \cdot A_R)$›
by(*rule-tac Comm1*)

moreover from *P''Chain* ‹$A_R' \sharp^* P''$› **have** $A_R' \sharp^* P'$ **by**(*rule tauChainFreshChain*)
from ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \simeq \Psi \otimes \Psi_R'$
by(*metis Associativity AssertionStatEqTrans AssertionStatEqSym compositionSym*)
with *P''Chain* **have** $\Psi \otimes \Psi_R' \rhd P'' \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauChainStatEq*)
hence $\Psi \rhd P'' \parallel R' \Longrightarrow\hat{}_\tau P' \parallel R'$ **using** *FrR'* ‹$A_R' \sharp^* \Psi$› ‹$A_R' \sharp^* P''$› **by**(*rule tauChainPar1*)
hence $\Psi \rhd (\!|\nu * xvec|\!)(P'' \parallel R') \Longrightarrow\hat{}_\tau (\!|\nu * xvec|\!)(P' \parallel R')$ **using** ‹*xvec* $\sharp^* \Psi$› **by**(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow_\tau (\!|\nu * xvec|\!)(P' \parallel R')$
by(*drule-tac tauActTauStepChain*) *auto*
moreover from *P'RelQ'* ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes \Psi_R', P', Q') \in Rel$ **by**(*metis C3 Associativity compositionSym*)
with *FrR'* ‹$A_R' \sharp^* P'$› ‹$A_R' \sharp^* Q'$› ‹$A_R' \sharp^* \Psi$› **have** $(\Psi, P' \parallel R', Q' \parallel R') \in Rel'$ **by**(*rule-tac C1*)
with ‹*xvec* $\sharp^* \Psi$› **have** $(\Psi, (\!|\nu * xvec|\!)(P' \parallel R'), (\!|\nu * xvec|\!)(Q' \parallel R')) \in Rel'$
by(*rule-tac C2*)
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_R *M xvec N Q'* A_Q Ψ_Q *K R'* A_R)
have *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q\rangle$ **by** *fact*
from ‹$A_Q \sharp^* (P, R)$› **have** $A_Q \sharp^* P$ **and** $A_Q \sharp^* R$ **by** *simp*+

have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R\rangle$ **by** *fact*
from ‹$A_R \sharp^* (P, R)$› **have** $A_R \sharp^* P$ **and** $A_R \sharp^* R$ **by** *simp*+
from ‹*xvec* $\sharp^* (P, R)$› **have** *xvec* $\sharp^* P$ **and** *xvec* $\sharp^* R$ **by** *simp*+

have *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec Q'$ **and** *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto K(\!|N|\!) \prec R'$
and *MeqK*: $\Psi \otimes \Psi_Q \otimes \Psi_R \vdash M \leftrightarrow K$ **by** *fact*+

from *RTrans FrR* ‹*distinct* A_R› ‹A_R ♯* Ψ› ‹A_R ♯* R› ‹A_R ♯* Q'› ‹A_R ♯* N› ‹A_R ♯* P› ‹A_R ♯* *xvec*› ‹A_R ♯* K›
obtain Ψ' A_R' Ψ_R' **where** *ReqR'*: $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** *FrR'*: *extractFrame* $R' = \langle A_R', \Psi_R' \rangle$
and A_R' ♯* Ψ **and** A_R' ♯* P **and** A_R' ♯* Q' **and** A_R' ♯* N **and** A_R' ♯* *xvec*
by(*rule-tac* C=(Ψ, P, Q', N, *xvec*) **and** C'=(Ψ, P, Q', N, *xvec*) **in** *expandFrame*) *auto*

have $\Psi \otimes \Psi_R \rhd P \rightsquigarrow <Rel> Q$ **by**(*rule PSimQ'*)

with *QTrans* ‹*xvec* ♯* Ψ› ‹*xvec* ♯* Ψ_R› ‹*xvec* ♯* P›
obtain P'' P' **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow M(\!|\nu * xvec|\!)\langle N\rangle \prec P''$
and *P''Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_\tau P'$
and *P'RelQ'*: $((\Psi \otimes \Psi_R) \otimes \Psi', P', Q') \in Rel$
by(*fastforce dest*: *weakSimE*)

from *PTrans* **obtain** P''' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau P'''$
and *QimpP'''*: *insertAssertion* (*extractFrame* Q) ($\Psi \otimes \Psi_R$) $\hookrightarrow_F$ *insertAssertion* (*extractFrame* P''') ($\Psi \otimes \Psi_R$)
and *P'''Trans*: $\Psi \otimes \Psi_R \rhd P''' \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P''$
by(*rule weakTransitionE*)

from *PChain* ‹A_R ♯* P› **have** A_R ♯* P''' **by**(*rule tauChainFreshChain*)

obtain A_P''' Ψ_P''' **where** *FrP'''*: *extractFrame* $P''' = \langle A_P''', \Psi_P'''\rangle$ **and** A_P''' ♯* (Ψ, A_Q, Ψ_Q, A_R, Ψ_R, M, N, K, R, P''', *xvec*) **and** *distinct* A_P'''
by(*rule freshFrame*)
hence A_P''' ♯* Ψ **and** A_P''' ♯* A_Q **and** A_P''' ♯* Ψ_Q **and** A_P''' ♯* M **and** A_P''' ♯* R
and A_P''' ♯* N **and** A_P''' ♯* K **and** A_P''' ♯* A_R **and** A_P''' ♯* P''' **and** A_P''' ♯* *xvec* **and** A_P''' ♯* Ψ_R
by *simp+*

have $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \simeq_F \langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
moreover with *QimpP''' FrP''' FrQ* ‹A_P''' ♯* Ψ› ‹A_Q ♯* Ψ› ‹A_P''' ♯* Ψ_R› ‹A_Q ♯* Ψ_R›
have $\langle A_Q, (\Psi \otimes \Psi_R) \otimes \Psi_Q\rangle \hookrightarrow_F \langle A_P''', (\Psi \otimes \Psi_R) \otimes \Psi_P'''\rangle$ **using** *freshCompChain*
by *simp*
moreover have $\langle A_P''', (\Psi \otimes \Psi_R) \otimes \Psi_P'''\rangle \simeq_F \langle A_P''', (\Psi \otimes \Psi_P''') \otimes \Psi_R\rangle$
by(*metis frameResChainPres frameNilStatEq Commutativity AssertionStatEqTrans Composition Associativity*)
ultimately have *QImpP'''*: $\langle A_Q, (\Psi \otimes \Psi_Q) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P''', (\Psi \otimes \Psi_P''') \otimes \Psi_R\rangle$
by(*rule FrameStatEqImpCompose*)

from *PChain FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› **have** $\Psi \rhd P \parallel R \Longrightarrow\hat{}_\tau P''' \parallel R$ **by**(*rule tauChainPar1*)

moreover from *RTrans FrR P'''Trans MeqK QImpP''' FrP''' FrQ* ‹*distinct* A_P'''› ‹*distinct* A_R› ‹$A_P''' \sharp* A_R$› ‹$A_Q \sharp* A_R$›
‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'''$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* K$› ‹$A_P''' \sharp* \Psi$› ‹$A_P''' \sharp* R$›
‹$A_P''' \sharp* P'''$› ‹$A_P''' \sharp* M$› ‹$A_Q \sharp* R$› ‹$A_Q \sharp* M$› ‹$A_R \sharp* xvec$› ‹$xvec \sharp* M$›
obtain K' **where** $\Psi \otimes \Psi_P''' \rhd R \longmapsto K'\langle\!|N|\!\rangle \prec R'$ **and** $\Psi \otimes \Psi_P''' \otimes \Psi_R \vdash M \leftrightarrow K'$ **and** $A_R \sharp* K'$
by(*rule-tac comm2Aux*) (*assumption* | *simp*)+

with *P'''Trans FrP'''* **have** $\Psi \rhd P''' \parallel R \longmapsto\tau \prec (\!|\nu*xvec|\!)(P'' \parallel R')$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P'''$› ‹$A_R \sharp* R$›
‹$xvec \sharp* R$› ‹$A_P''' \sharp* \Psi$› ‹$A_P''' \sharp* P'''$› ‹$A_P''' \sharp* R$› ‹$A_P''' \sharp* M$› ‹$A_R \sharp* K'$› ‹$A_P''' \sharp* A_R$›
by(*rule-tac Comm2*)
moreover from *P'''Trans* ‹$A_R \sharp* P'''$› ‹$A_R \sharp* xvec$› ‹$xvec \sharp* M$› ‹*distinct xvec*› **have** $A_R \sharp* P''$
by(*rule-tac outputFreshChainDerivative*) *auto*

from *PChain* ‹$A_R' \sharp* P$› **have** $A_R' \sharp* P'''$ **by**(*rule tauChainFreshChain*)
with *P'''Trans* ‹$xvec \sharp* M$› ‹*distinct xvec*› **have** $A_R' \sharp* P''$ **using** ‹$A_R' \sharp* xvec$›
by(*rule-tac outputFreshChainDerivative*) *auto*

with *P''Chain* **have** $A_R' \sharp* P'$ **by**(*rule tauChainFreshChain*)
from ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes \Psi_R) \otimes \Psi' \simeq \Psi \otimes \Psi_R'$
by(*metis Associativity AssertionStatEqTrans AssertionStatEqSym compositionSym*)
with *P''Chain* **have** $\Psi \otimes \Psi_R' \rhd P'' \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauChainStatEq*)
hence $\Psi \rhd P'' \parallel R' \Longrightarrow\hat{}_\tau P' \parallel R'$ **using** *FrR'* ‹$A_R' \sharp* \Psi$› ‹$A_R' \sharp* P''$›
by(*rule tauChainPar1*)
hence $\Psi \rhd (\!|\nu*xvec|\!)(P'' \parallel R') \Longrightarrow\hat{}_\tau (\!|\nu*xvec|\!)(P' \parallel R')$
using ‹$xvec \sharp* \Psi$› **by**(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd P \parallel R \Longrightarrow_\tau (\!|\nu*xvec|\!)(P' \parallel R')$ **by**(*drule-tac tauActTauStepChain*) *auto*
moreover from *P'RelQ'* ‹$\Psi_R \otimes \Psi' \simeq \Psi_R'$› **have** $(\Psi \otimes \Psi_R', P', Q') \in Rel$ **by**(*metis C3 Associativity compositionSym*)
with *FrR'* ‹$A_R' \sharp* P'$› ‹$A_R' \sharp* Q'$› ‹$A_R' \sharp* \Psi$› **have** $(\Psi, P' \parallel R', Q' \parallel R') \in Rel'$ **by**(*rule-tac C1*)
with ‹$xvec \sharp* \Psi$› **have** $(\Psi, (\!|\nu*xvec|\!)(P' \parallel R'), (\!|\nu*xvec|\!)(Q' \parallel R')) \in Rel'$
by(*rule-tac C2*)
ultimately show *?case* **by** *blast*
qed
qed
no-notation *relcomp* (**infixr** *O 75*)

lemma *weakCongSimBangPres*:

fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $Rel :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Rel' :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$
and $Rel'' :: ('b \times ('a, 'b, 'c)\ psi \times ('a, 'b, 'c)\ psi)\ set$

assumes *PEqQ*: $Eq\ P\ Q$
and *PRelQ*: $(\Psi, P, Q) \in Rel$
and $guarded\ P$
and $guarded\ Q$
and *Rel'Rel*: $Rel' \subseteq Rel$
and *FrameParPres*: $\bigwedge \Psi'\ \Psi_U\ S\ T\ U\ A_U.\ [\![(\Psi' \otimes \Psi_U, S, T) \in Rel; extractFrame\ U = \langle A_U, \Psi_U \rangle; A_U \sharp* \Psi'; A_U \sharp* S; A_U \sharp* T]\!] \Longrightarrow (\Psi', U \parallel S, U \parallel T) \in Rel$
and *C1*: $\bigwedge \Psi'\ S\ T\ U.\ [\![(\Psi', S, T) \in Rel; guarded\ S; guarded\ T]\!] \Longrightarrow (\Psi', U \parallel !S, U \parallel !T) \in Rel''$
and *Closed*: $\bigwedge \Psi'\ S\ T\ p.\ (\Psi', S, T) \in Rel \Longrightarrow ((p::name\ prm) \cdot \Psi', p \cdot S, p \cdot T) \in Rel$
and *Closed'*: $\bigwedge \Psi'\ S\ T\ p.\ (\Psi', S, T) \in Rel' \Longrightarrow ((p::name\ prm) \cdot \Psi', p \cdot S, p \cdot T) \in Rel'$
and *StatEq*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel; \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'', S, T) \in Rel$
and *StatEq'*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ [\![(\Psi', S, T) \in Rel'; \Psi' \simeq \Psi'']\!] \Longrightarrow (\Psi'', S, T) \in Rel'$
and *Trans*: $\bigwedge \Psi'\ S\ T\ U.\ [\![(\Psi', S, T) \in Rel; (\Psi', T, U) \in Rel]\!] \Longrightarrow (\Psi', S, U) \in Rel$
and *Trans'*: $\bigwedge \Psi'\ S\ T\ U.\ [\![(\Psi', S, T) \in Rel'; (\Psi', T, U) \in Rel']\!] \Longrightarrow (\Psi', S, U) \in Rel'$
and *EqSim*: $\bigwedge \Psi'\ S\ T.\ Eq\ S\ T \Longrightarrow \Psi' \rhd S \rightsquigarrow «Rel»\ T$
and *cSim*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow \Psi' \rhd S \rightsquigarrow <Rel>\ T$
and *cSym*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', T, S) \in Rel$
and *cSym'*: $\bigwedge \Psi'\ S\ T.\ (\Psi', S, T) \in Rel' \Longrightarrow (\Psi', T, S) \in Rel'$
and *cExt*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel$
and *cExt'*: $\bigwedge \Psi'\ S\ T\ \Psi''.\ (\Psi', S, T) \in Rel' \Longrightarrow (\Psi' \otimes \Psi'', S, T) \in Rel'$
and *ParPres*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S, T) \in Rel \Longrightarrow (\Psi', S \parallel U, T \parallel U) \in Rel$
and *ParPres'*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S, T) \in Rel' \Longrightarrow (\Psi', U \parallel S, U \parallel T) \in Rel'$
and *ParPres2*: $\bigwedge \Psi'\ S\ T.\ Eq\ S\ T \Longrightarrow Eq\ (S \parallel S)\ (T \parallel T)$
and *ResPres*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * xvec|\!) S, (\!|\nu * xvec|\!) T) \in Rel$
and *ResPres'*: $\bigwedge \Psi'\ S\ T\ xvec.\ [\![(\Psi', S, T) \in Rel'; xvec \sharp* \Psi']\!] \Longrightarrow (\Psi', (\!|\nu * xvec|\!) S, (\!|\nu * xvec|\!) T) \in Rel'$
and *Assoc*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S \parallel (T \parallel U), (S \parallel T) \parallel U) \in Rel$
and *Assoc'*: $\bigwedge \Psi'\ S\ T\ U.\ (\Psi', S \parallel (T \parallel U), (S \parallel T) \parallel U) \in Rel'$
and *ScopeExt*: $\bigwedge xvec\ \Psi'\ T\ S.\ [\![xvec \sharp* \Psi'; xvec \sharp* T]\!] \Longrightarrow (\Psi', (\!|\nu * xvec|\!)(S \parallel T), ((\!|\nu * xvec|\!) S) \parallel T) \in Rel$
and *ScopeExt'*: $\bigwedge xvec\ \Psi'\ T\ S.\ [\![xvec \sharp* \Psi'; xvec \sharp* T]\!] \Longrightarrow (\Psi', (\!|\nu * xvec|\!)(S \parallel T), ((\!|\nu * xvec|\!) S) \parallel T) \in Rel'$
and *Compose*: $\bigwedge \Psi'\ S\ T\ U\ O.\ [\![(\Psi', S, T) \in Rel; (\Psi', T, U) \in Rel''; (\Psi', U,$

$O) \in Rel'[\!] \Longrightarrow (\Psi', S, O) \in Rel''$
and *rBangActE*: $\bigwedge \Psi' S \alpha S'.\ [\![\Psi' \rhd !S \longmapsto \alpha \prec S'; guarded\ S; bn\ \alpha \sharp* S; \alpha \neq \tau; bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow \exists T.\ \Psi' \rhd S \longmapsto \alpha \prec T \wedge (\mathbf{1}, S', T \parallel !S) \in Rel'$
and *rBangTauE*: $\bigwedge \Psi' S S'.\ [\![\Psi' \rhd !S \longmapsto \tau \prec S'; guarded\ S]\!] \Longrightarrow \exists T.\ \Psi' \rhd S \parallel S \longmapsto \tau \prec T \wedge (\mathbf{1}, S', T \parallel !S) \in Rel'$
and *rBangTauI*: $\bigwedge \Psi' S S'.\ [\![\Psi' \rhd S \parallel S \Longrightarrow_\tau S'; guarded\ S]\!] \Longrightarrow \exists T.\ \Psi' \rhd !S \Longrightarrow_\tau T \wedge (\Psi', T, S' \parallel !S) \in Rel'$
shows $\Psi \rhd R \parallel !P \rightsquigarrow\!\langle\!\langle Rel'' \rangle\!\rangle R \parallel !Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau RQ'*)
from ‹$\Psi \rhd R \parallel !Q \longmapsto \tau \prec RQ'$› **show** *?case*
proof(*induct rule*: *parTauCases*[**where** C=(P, Q, R)])
case(*cPar1 R' A_Q Ψ_Q*)
from ‹*extractFrame* $(!Q) = \langle A_Q, \Psi_Q \rangle$› **have** $A_Q = []$ **and** $\Psi_Q = SBottom'$ **by** *simp*+
with ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto \tau \prec R'$› ‹$\Psi_Q = SBottom'$›
have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (R' \parallel !P)$ **by**(*rule-tac Par1*) (*assumption* | *simp*)+
hence $\Psi \rhd R \parallel !P \Longrightarrow_\tau R' \parallel !P$ **by** *auto*
moreover from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi, R' \parallel !P, R' \parallel !Q) \in Rel''$ **using** ‹*guarded P*› ‹*guarded Q*›
by(*rule C1*)
ultimately show *?case* **by** *blast*
next
case(*cPar2 Q' A_R Ψ_R*)
from ‹$A_R \sharp* (P, Q, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **by** *simp*+
have *FrR*: *extractFrame* $R = \langle A_R, \Psi_R \rangle$ **by** *fact*

obtain $A_Q\ \Psi_Q$ **where** *FrQ*: *extractFrame* $Q = \langle A_Q, \Psi_Q \rangle$ **and** $A_Q \sharp* \Psi$ **and** $A_Q \sharp* \Psi_R$ **and** $A_Q \sharp* A_R$
by(*rule-tac* C=(Ψ, Ψ_R, A_R) **in** *freshFrame*) *auto*
from *FrQ* ‹*guarded Q*› **have** $\Psi_Q \simeq \mathbf{1}$ **and** *supp* Ψ_Q = ({}::*name set*) **by**(*blast dest*: *guardedStatEq*)+
hence $A_R \sharp* \Psi_Q$ **and** $A_Q \sharp* \Psi_Q$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from ‹$\Psi \otimes \Psi_R \rhd !Q \longmapsto \tau \prec Q'$› ‹*guarded Q*›
obtain T **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \parallel Q \longmapsto \tau \prec T$ **and** $(\mathbf{1}, Q', T \parallel !Q) \in Rel'$
by(*blast dest*: *rBangTauE*)

from ‹*Eq P Q*› **have** *Eq* $(P \parallel P)$ $(Q \parallel Q)$ **by**(*rule ParPres2*)
with *QTrans*
obtain S **where** *PTrans*: $\Psi \otimes \Psi_R \rhd P \parallel P \Longrightarrow_\tau S$ **and** *SRelT*: $(\Psi \otimes \Psi_R, S, T) \in Rel$
by(*blast dest*: *EqSim weakCongSimE*)
from *PTrans* ‹*guarded P*› **obtain** U **where** *PChain*: $\Psi \otimes \Psi_R \rhd !P \Longrightarrow_\tau U$ **and** $(\Psi \otimes \Psi_R, U, S \parallel !P) \in Rel'$
by(*blast dest*: *rBangTauI*)
from *PChain* ‹$A_R \sharp* P$› **have** $A_R \sharp* U$ **by**(*force dest*: *tauStepChainFreshChain*)
from ‹$\Psi \otimes \Psi_R \rhd !P \Longrightarrow_\tau U$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$› **have** $\Psi \rhd R \parallel !P$

$\Longrightarrow_\tau R \parallel U$
by(*rule-tac tauStepChainPar2*) *auto*
moreover from *PTrans* ‹$A_R \sharp* P$› **have** $A_R \sharp* S$ **by**(*force dest*: *tauStepChainFreshChain*)
from *QTrans* ‹$A_R \sharp* Q$› **have** $A_R \sharp* T$ **by**(*force dest*: *tauFreshChainDerivative*)
have $(\Psi, R \parallel U, R \parallel Q') \in Rel''$
proof −
from ‹$(\Psi \otimes \Psi_R, U, S \parallel !P) \in Rel'$› *Rel'Rel* **have** $(\Psi \otimes \Psi_R, U, S \parallel !P) \in Rel$
by *auto*
hence $(\Psi, R \parallel U, R \parallel (S \parallel !P)) \in Rel$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* U$› ‹$A_R \sharp* S$› ‹$A_R \sharp* P$›
by(*rule-tac FrameParPres*) *auto*

moreover from ‹$(\Psi \otimes \Psi_R, S, T) \in Rel$› *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* S$› ‹$A_R \sharp* T$› **have** $(\Psi, R \parallel S, R \parallel T) \in Rel$
by(*rule-tac FrameParPres*) *auto*
hence $(\Psi, R \parallel T, R \parallel S) \in Rel$ **by**(*rule cSym*)
hence $(\Psi, (R \parallel T) \parallel !P, (R \parallel S) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi, (R \parallel S) \parallel !P, (R \parallel T) \parallel !P) \in Rel$ **by**(*rule cSym*)
hence $(\Psi, R \parallel (S \parallel !P), (R \parallel T) \parallel !P) \in Rel$ **by**(*metis Trans Assoc*)
ultimately have $(\Psi, R \parallel U, (R \parallel T) \parallel !P) \in Rel$ **by**(*rule Trans*)
moreover from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi, (R \parallel T) \parallel !P, (R \parallel T) \parallel !Q) \in Rel''$ **using** ‹*guarded P*› ‹*guarded Q*› **by**(*rule C1*)
moreover from ‹$(\mathbf{1}, Q', T \parallel !Q) \in Rel'$› **have** $(\mathbf{1} \otimes \Psi, Q', T \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi, Q', T \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence $(\Psi, R \parallel Q', R \parallel (T \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi, R \parallel Q', (R \parallel T) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi, (R \parallel T) \parallel !Q, R \parallel Q') \in Rel'$ **by**(*rule cSym'*)
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
next
case(*cComm1* Ψ_Q *M N R'* A_R Ψ_R *K xvec Q'* A_Q)
from ‹$A_R \sharp* (P, Q, R)$› **have** $A_R \sharp* P$ **and** $A_R \sharp* Q$ **and** $A_R \sharp* R$ **by** *simp+*
from ‹$xvec \sharp* (P, Q, R)$› **have** $xvec \sharp* P$ **and** $xvec \sharp* Q$ **and** $xvec \sharp* R$ **by** *simp+*
have *FrQ*: $extractFrame(!Q) = \langle A_Q, \Psi_Q \rangle$ **by** *fact*
have *FrR*: $extractFrame\ R = \langle A_R, \Psi_R \rangle$ **by** *fact*
from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$› *FrR* ‹*distinct* A_R› ‹$A_R \sharp* R$› ‹$A_R \sharp* N$› ‹$A_R \sharp* xvec$› ‹$A_R \sharp* P$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* M$›
obtain $A_R'\ \Psi_R'\ \Psi'$ **where** *FrR'*: $extractFrame\ R' = \langle A_R', \Psi_R' \rangle$ **and** $\Psi_R \otimes \Psi' \simeq \Psi_R'$ **and** $A_R' \sharp* xvec$ **and** $A_R' \sharp* P$ **and** $A_R' \sharp* Q$ **and** $A_R' \sharp* \Psi$
by(*rule-tac* C=$(\Psi, xvec, P, Q)$ **and** C'=$(\Psi, xvec, P, Q)$ **in** *expandFrame*) *auto*
from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)

moreover from ‹$\Psi \otimes \Psi_R \rhd\ !Q \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec Q'$› ‹*guarded* Q› ‹*xvec* $\sharp{*}$ Q› ‹*xvec* $\sharp{*}$ K›
obtain S **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec S$ **and** $(\mathbf{1}, Q', S \parallel\ !Q) \in Rel'$
by(*fastforce dest*: *rBangActE*)
ultimately obtain P' T **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$ **and** *P'Chain*: $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \Longrightarrow\hat{}_\tau\ T$ **and** $((\Psi \otimes \Psi_R) \otimes \Psi', T, S) \in Rel$
using ‹*xvec* $\sharp{*}$ Ψ› ‹*xvec* $\sharp{*}$ Ψ_R› ‹*xvec* $\sharp{*}$ P›
by(*fastforce dest*: *cSim weakSimE*)

from *PTrans* ‹$A_R \sharp{*} P$› ‹$A_R \sharp{*}$ *xvec*› ‹$A_R' \sharp{*} P$› ‹$A_R' \sharp{*}$ *xvec*› ‹*xvec* $\sharp{*}$ K› ‹*distinct xvec*›
have $A_R \sharp{*} P'$ **and** $A_R' \sharp{*} P'$
by(*force dest*: *weakOutputFreshChainDerivative*)+
with *P'Chain* **have** $A_R' \sharp{*} T$ **by**(*force dest*: *tauChainFreshChain*)+
from *QTrans* ‹$A_R' \sharp{*} Q$› ‹$A_R' \sharp{*}$ *xvec*› ‹*xvec* $\sharp{*}$ K› ‹*distinct xvec*›
have $A_R' \sharp{*} S$ **by**(*force dest*: *outputFreshChainDerivative*)

obtain A_Q' Ψ_Q' **where** *FrQ'*: *extractFrame* $Q = \langle A_Q', \Psi_Q'\rangle$ **and** $A_Q' \sharp{*} \Psi$ **and** $A_Q' \sharp{*} \Psi_R$ **and** $A_Q' \sharp{*} A_R$ **and** $A_Q' \sharp{*} M$ **and** $A_Q' \sharp{*} R$ **and** $A_Q' \sharp{*} K$
by(*rule-tac* C=(Ψ, Ψ_R, A_R, K, M, R) **in** *freshFrame*) *auto*
from *FrQ'* ‹*guarded* Q› **have** $\Psi_Q' \simeq \mathbf{1}$ **and** *supp* $\Psi_Q' = (\{\}::name\ set)$ **by**(*blast dest*: *guardedStatEq*)+
hence $A_Q' \sharp{*} \Psi_Q'$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau\ P''$
and *NilImpP''*: $\langle A_Q', (\Psi \otimes \Psi_R) \otimes \Psi_Q'\rangle \hookrightarrow_F$ *insertAssertion* (*extractFrame* P'') ($\Psi \otimes \Psi_R$)
and *P''Trans*: $\Psi \otimes \Psi_R \rhd P'' \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$
using *FrQ'* ‹$A_Q' \sharp{*} \Psi$› ‹$A_Q' \sharp{*} \Psi_R$› *freshCompChain*
by(*drule-tac weakTransitionE*) *auto*

from *PChain* **have** $\Psi \rhd R \parallel\ !P \Longrightarrow_\tau (\!|\nu{*}xvec|\!)(R' \parallel (P' \parallel\ !P))$
proof(*induct rule*: *tauChainCases*)
case *TauBase*
from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'$› *FrQ* **have** $\Psi \otimes \mathbf{1} \rhd R \longmapsto M(\!|N|\!) \prec R'$ **by** *simp*
moreover note *FrR*
moreover from *P''Trans* ‹$P = P''$› **have** $\Psi \otimes \Psi_R \rhd P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$ **by** *simp*
hence $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$ **by**(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel\ !P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel\ !P)$ **using** ‹*xvec* $\sharp{*}$ Ψ› ‹*xvec* $\sharp{*}$ Ψ_R› ‹*xvec* $\sharp{*}$ P›
by(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_R \rhd\ !P \longmapsto K(\!|\nu{*}xvec|\!)\langle N\rangle \prec (P' \parallel\ !P)$ **using** ‹*guarded* P› **by**(*rule Bang*)
moreover from ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› *FrQ* **have** $\Psi \otimes \Psi_R \otimes \mathbf{1} \vdash M$

$\leftrightarrow K$ **by** *simp*
ultimately have $\Psi \rhd R \parallel !P \longmapsto\tau \prec (\!|\nu * xvec|\!)(R' \parallel (P' \parallel !P))$ **using** $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* R\rangle$ $\langle A_R \sharp* P\rangle$ $\langle A_R \sharp* M\rangle$ $\langle xvec \sharp* R\rangle$
by(*force intro*: *Comm1*)
thus *?case* **by**(*rule tauActTauStepChain*)
next
case *TauStep*
obtain $A_P{''}$ $\Psi_P{''}$ **where** *FrP''*: *extractFrame* $P'' = \langle A_P{''}, \Psi_P{''}\rangle$ **and** $A_P{''} \sharp* \Psi$ **and** $A_P{''} \sharp* K$ **and** $A_P{''} \sharp* \Psi_R$ **and** $A_P{''} \sharp* R$ **and** $A_P{''} \sharp* P''$ **and** $A_P{''} \sharp* P$
and $A_P{''} \sharp* A_R$ **and** *distinct* $A_P{''}$
by(*rule-tac* C=(Ψ, K, A_R, Ψ_R, R, P'', P) **in** *freshFrame*) *auto*
from *PChain* $\langle A_R \sharp* P\rangle$ **have** $A_R \sharp* P''$ **by**(*drule-tac tauChainFreshChain*) *auto*
with *FrP''* $\langle A_P{''} \sharp* A_R\rangle$ **have** $A_R \sharp* \Psi_P{''}$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from $\langle \Psi \otimes \Psi_R \rhd P \Longrightarrow\tau P''\rangle$ **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \Longrightarrow\tau P''$ **by**(*rule tauStepChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \Longrightarrow\tau P'' \parallel !P$ **by**(*rule-tac tauStepChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \rhd !P \Longrightarrow\tau P'' \parallel !P$ **using** $\langle guarded\ P\rangle$ **by**(*rule tauStepChainBang*)
hence $\Psi \rhd R \parallel !P \Longrightarrow\tau R \parallel (P'' \parallel !P)$ **using** *FrR* $\langle A_R \sharp* \Psi\rangle$ $\langle A_R \sharp* P\rangle$
by(*rule-tac tauStepChainPar2*) *auto*
moreover have $\Psi \rhd R \parallel (P'' \parallel !P) \longmapsto\tau \prec (\!|\nu * xvec|\!)(R' \parallel (P' \parallel !P))$
proof −
from *FrQ* $\langle \Psi_Q{'} \simeq \mathbf{1}\rangle$ $\langle \Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|N|\!) \prec R'\rangle$ **have** $\Psi \otimes \Psi_Q{'} \rhd R \longmapsto M(\!|N|\!) \prec R'$
by *simp* (*metis statEqTransition AssertionStatEqSym compositionSym*)
moreover from *P''Trans* **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P'' \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence *P''PTrans*: $\Psi \otimes \Psi_R \rhd P'' \parallel !P \longmapsto K(\!|\nu * xvec|\!)\langle N\rangle \prec (P' \parallel !P)$ **using** $\langle xvec \sharp* P\rangle$
by(*rule-tac Par1*) *auto*
moreover from *FrP''* **have** *FrP''P*: *extractFrame*$(P'' \parallel !P) = \langle A_P{''}, \Psi_P{''} \otimes \mathbf{1}\rangle$
by *auto*
moreover from *FrQ* $\langle \Psi_Q{'} \simeq \mathbf{1}\rangle$ $\langle \Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K\rangle$ **have** $\Psi \otimes \Psi_Q{'} \otimes \Psi_R \vdash M \leftrightarrow K$
by *simp* (*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
hence $\Psi \otimes \Psi_Q{'} \otimes \Psi_R \vdash K \leftrightarrow M$ **by**(*rule chanEqSym*)
moreover have $\langle A_Q{'}, (\Psi \otimes \Psi_Q{'}) \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P{''}, (\Psi \otimes (\Psi_P{''} \otimes \mathbf{1})) \otimes \Psi_R\rangle$
proof −
have $\langle A_Q{'}, (\Psi \otimes \Psi_Q{'}) \otimes \Psi_R\rangle \simeq_F \langle A_Q{'}, (\Psi \otimes \Psi_R) \otimes \Psi_Q{'}\rangle$
by(*rule-tac frameResChainPres*, *simp*)
(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
moreover from *NilImpP''* *FrQ* *FrP''* $\langle A_P{''} \sharp* \Psi\rangle$ $\langle A_P{''} \sharp* \Psi_R\rangle$ *fresh-*

CompChain **have** $\langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}' \rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}'' \rangle$
by *auto*
moreover have $\langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}'' \rangle \simeq_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \rangle$
by(*rule frameResChainPres*, *simp*)
(*metis Identity AssertionStatEqSym Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
ultimately obtain M' **where** *RTrans*: $\Psi \otimes \Psi_P{}'' \otimes \mathbf{1} \rhd R \longmapsto M'\langle\!|N|\!\rangle \prec R'$ **and** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$ **and** $A_R \sharp^* M'$
using *FrR FrQ'* ‹*distinct* A_R› ‹*distinct* $A_P{}''$› ‹$A_P{}'' \sharp^* A_R$› ‹$A_R \sharp^* \Psi$› ‹$A_R \sharp^* P''$› ‹$A_R \sharp^* Q$› ‹$A_R \sharp^* R$› ‹$A_R \sharp^* M$› ‹$A_Q{}' \sharp^* R$› ‹$A_Q{}' \sharp^* K$› ‹$A_Q{}' \sharp^* A_R$› ‹$A_R \sharp^* P$› ‹$A_P{}'' \sharp^* P$› ‹$A_R \sharp^* xvec$›
‹$A_P{}'' \sharp^* \Psi$› ‹$A_P{}'' \sharp^* R$› ‹$A_P{}'' \sharp^* P''$› ‹$A_P{}'' \sharp^* K$› ‹*xvec* $\sharp^*$ K› ‹*distinct xvec*›
by(*rule-tac* A_Q=$A_Q{}'$ **and** Q=Q **in** *comm2Aux*) (*assumption* | *simp*)+

note *RTrans FrR P''PTrans FrP''P*
moreover from ‹$\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$› **have** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash M' \leftrightarrow K$ **by**(*rule chanEqSym*)
hence $\Psi \otimes \Psi_R \otimes \Psi_P{}'' \otimes \mathbf{1} \vdash M' \leftrightarrow K$ **by**(*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
ultimately show *?thesis* **using** ‹$A_R \sharp^* \Psi$› ‹$A_R \sharp^* R$› ‹$A_R \sharp^* P''$› ‹$A_R \sharp^* P$› ‹$A_R \sharp^* M'$› ‹$A_P{}'' \sharp^* A_R$› ‹$A_P{}'' \sharp^* \Psi$› ‹$A_P{}'' \sharp^* R$› ‹$A_P{}'' \sharp^* P''$› ‹$A_P{}'' \sharp^* P$› ‹$A_P{}'' \sharp^* K$› ‹*xvec* $\sharp^*$ R›
by(*rule-tac Comm1*) (*assumption* | *simp*)+
qed
ultimately show *?thesis*
by(*drule-tac tauActTauStepChain*) *auto*
qed

moreover from *P'Chain* **have** $((\Psi \otimes \Psi_R) \otimes \Psi') \otimes \mathbf{1} \rhd P' \Longrightarrow\hat{}_\tau\ T$
by(*rule tauChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $(\Psi \otimes \Psi_R) \otimes \Psi' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$
by(*rule-tac tauChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \otimes \Psi' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$
by(*rule tauChainStatEq*) (*metis Associativity*)
hence $\Psi \otimes \Psi_R{}' \rhd P' \parallel !P \Longrightarrow\hat{}_\tau\ T \parallel !P$ **using** ‹$\Psi_R \otimes \Psi' \simeq \Psi_R{}'$›
by(*rule-tac tauChainStatEq*) (*auto intro*: *compositionSym*)
hence $\Psi \rhd R' \parallel (P' \parallel !P) \Longrightarrow\hat{}_\tau\ R' \parallel (T \parallel !P)$ **using** *FrR'* ‹$A_R{}' \sharp^* \Psi$› ‹$A_R{}' \sharp^* P$› ‹$A_R{}' \sharp^* P'$›
by(*rule-tac tauChainPar2*) *auto*
hence $\Psi \rhd \langle\!|\nu * xvec|\!\rangle(R' \parallel (P' \parallel !P)) \Longrightarrow\hat{}_\tau\ \langle\!|\nu * xvec|\!\rangle(R' \parallel (T \parallel !P))$ **using** ‹*xvec* $\sharp^*$ Ψ›
by(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd R \parallel !P \Longrightarrow_\tau \langle\!|\nu * xvec|\!\rangle(R' \parallel (T \parallel !P))$
by *auto*
moreover have $(\Psi, \langle\!|\nu * xvec|\!\rangle(R' \parallel (T \parallel !P)), \langle\!|\nu * xvec|\!\rangle(R' \parallel Q')) \in Rel''$

proof −
from ‹((Ψ ⊗ Ψ_R) ⊗ Ψ′, T, S) ∈ *Rel*› **have** (Ψ ⊗ Ψ_R ⊗ Ψ′, T, S) ∈ *Rel*
by(*rule StatEq*) (*metis Associativity*)
hence (Ψ ⊗ Ψ_R′, T, S) ∈ *Rel* **using** ‹Ψ_R ⊗ Ψ′ ≃ Ψ_R′›
by(*rule-tac StatEq*) (*auto dest*: *compositionSym*)

with *FrR*′ ‹A_R′ ♯* Ψ› ‹A_R′ ♯* S› ‹A_R′ ♯* T› **have** (Ψ, R′ ∥ T, R′ ∥ S) ∈ *Rel*
by(*rule-tac FrameParPres*) *auto*
hence (Ψ, (R′ ∥ T) ∥ !P, (R′ ∥ S) ∥ !P) ∈ *Rel* **by**(*rule ParPres*)
hence (Ψ, (|ν∗*xvec*|)((R′ ∥ T) ∥ !P), (|ν∗*xvec*|)((R′ ∥ S) ∥ !P)) ∈ *Rel* **using** ‹*xvec* ♯* Ψ›
by(*rule ResPres*)
hence (Ψ, (|ν∗*xvec*|)(R′ ∥ T) ∥ !P, ((|ν∗*xvec*|)(R′ ∥ S)) ∥ !P) ∈ *Rel* **using** ‹*xvec* ♯* Ψ› ‹*xvec* ♯* P›
by(*force intro*: *Trans ScopeExt*)
hence (Ψ, (|ν∗*xvec*|)(R′ ∥ (T ∥ !P)), ((|ν∗*xvec*|)(R′ ∥ S)) ∥ !P) ∈ *Rel* **using** ‹*xvec* ♯* Ψ›
by(*force intro*: *Trans ResPres Assoc*)

moreover from ‹(Ψ, P, Q) ∈ *Rel*› ‹*guarded* P› ‹*guarded* Q› **have** (Ψ, ((|ν∗*xvec*|)(R′ ∥ S)) ∥ !P, ((|ν∗*xvec*|)(R′ ∥ S)) ∥ !Q) ∈ *Rel*″
by(*rule C1*)
moreover from ‹(**1**, Q′, S ∥ !Q) ∈ *Rel*′› **have** (**1** ⊗ Ψ, Q′, S ∥ !Q) ∈ *Rel*′ **by**(*rule cExt*′)
hence (Ψ, Q′, S ∥ !Q) ∈ *Rel*′
by(*rule StatEq*′) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence (Ψ, R′ ∥ Q′, R′ ∥ (S ∥ !Q)) ∈ *Rel*′ **by**(*rule ParPres*′)
hence (Ψ, R′ ∥ Q′, (R′ ∥ S) ∥ !Q) ∈ *Rel*′ **by**(*metis Trans*′ *Assoc*′)
hence (Ψ, (R′ ∥ S) ∥ !Q, R′ ∥ Q′) ∈ *Rel*′ **by**(*rule cSym*′)
hence (Ψ, (|ν∗*xvec*|)((R′ ∥ S) ∥ !Q), (|ν∗*xvec*|)(R′ ∥ Q′)) ∈ *Rel*′ **using** ‹*xvec* ♯* Ψ›
by(*rule ResPres*′)
hence (Ψ, ((|ν∗*xvec*|)(R′ ∥ S)) ∥ !Q, (|ν∗*xvec*|)(R′ ∥ Q′)) ∈ *Rel*′ **using** ‹*xvec* ♯* Ψ› ‹*xvec* ♯* Q›
by(*force intro*: *Trans*′ *ScopeExt*′[*THEN cSym*′])
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
next
case(*cComm2* Ψ_Q M *xvec* N R′ A_R Ψ_R K Q′ A_Q)
from ‹A_R ♯* (P, Q, R)› **have** A_R ♯* P **and** A_R ♯* Q **and** A_R ♯* R **by** *simp*+
from ‹*xvec* ♯* (P, Q, R)› **have** *xvec* ♯* P **and** *xvec* ♯* Q **and** *xvec* ♯* R **by** *simp*+
have *FrQ*: *extractFrame*(!Q) = ⟨A_Q, Ψ_Q⟩ **by** *fact*
have *FrR*: *extractFrame* R = ⟨A_R, Ψ_R⟩ **by** *fact*
from ‹Ψ ⊗ Ψ_Q ▷ R ⟼M(|ν∗*xvec*|)⟨N⟩ ≺ R′› *FrR* ‹*distinct* A_R› ‹A_R ♯* R› ‹A_R ♯* N› ‹A_R ♯* *xvec*› ‹A_R ♯* P› ‹A_R ♯* Q› ‹A_R ♯* Ψ› ‹*xvec* ♯* R› ‹*xvec* ♯* Ψ› ‹*xvec* ♯* P› ‹*xvec* ♯* Q› ‹*xvec* ♯* M› ‹*distinct xvec*› ‹A_R ♯* M›

obtain p $A_R{}'$ $\Psi_R{}'$ Ψ' **where** *FrR'*: *extractFrame* $R' = \langle A_R{}', \Psi_R{}'\rangle$ **and** $(p \cdot \Psi_R) \otimes \Psi' \simeq \Psi_R{}'$ **and** $A_R{}' \sharp* xvec$ **and** $A_R{}' \sharp* P$ **and** $A_R{}' \sharp* Q$ **and** $A_R{}' \sharp* \Psi$ **and** S: *set* $p \subseteq$ *set* $xvec \times$ *set*$(p \cdot xvec)$ **and** *distinctPerm* p **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* Q$ **and** $(p \cdot xvec) \sharp* R'$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* \Psi$ **and** $A_R{}' \sharp* N$ **and** $A_R{}' \sharp* xvec$ **and** $A_R{}' \sharp* (p \cdot xvec)$
by(*rule-tac* C=(Ψ, P, Q) **and** C'=(Ψ, P, Q) **in** *expandFrame*) (*assumption* | *simp*)+

from ‹$\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu* xvec|\!)\langle N\rangle \prec R'$› S ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* R'$›
have *RTrans*: $\Psi \otimes \Psi_Q \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$
by(*simp add*: *boundOutputChainAlpha''* *residualInject*)
from ‹$(\Psi, P, Q) \in Rel$›
have $(\Psi \otimes \Psi_R, P, Q) \in Rel$ **by**(*rule cExt*)
moreover from ‹$\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|N|\!) \prec Q'$› S ‹$(p \cdot xvec) \sharp* Q$› ‹$xvec \sharp* Q$› ‹*distinctPerm p*›
have $\Psi \otimes \Psi_R \rhd !Q \longmapsto K(\!|(p \cdot N)|\!) \prec (p \cdot Q')$ **by**(*rule-tac inputAlpha*) *auto*
then obtain S **where** *QTrans*: $\Psi \otimes \Psi_R \rhd Q \longmapsto K(\!|(p \cdot N)|\!) \prec S$ **and** $(\mathbf{1}, (p \cdot Q'), S \parallel !Q) \in Rel'$
using ‹*guarded Q*›
by(*fastforce dest*: *rBangActE*)
ultimately obtain P' T **where** *PTrans*: $\Psi \otimes \Psi_R : Q \rhd P \Longrightarrow K(\!|(p \cdot N)|\!) \prec P'$
and *P'Chain*: $(\Psi \otimes \Psi_R) \otimes (p \cdot \Psi') \rhd P' \Longrightarrow\hat{}_\tau\ T$
and $((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'), T, S) \in Rel$
by(*fastforce dest*: *cSim weakSimE*)

from ‹$A_R{}' \sharp* N$› ‹$A_R{}' \sharp* xvec$› ‹$A_R{}' \sharp* (p \cdot xvec)$› S **have** $A_R{}' \sharp* (p \cdot N)$
by(*simp add*: *freshChainSimps*)
with *PTrans* ‹$A_R{}' \sharp* P$› **have** $A_R{}' \sharp* P'$ **by**(*force dest*: *weakInputFreshChainDerivative*)
with *P'Chain* **have** $A_R{}' \sharp* T$ **by**(*force dest*: *tauChainFreshChain*)+
from *QTrans* ‹$A_R{}' \sharp* Q$› ‹$A_R{}' \sharp* (p \cdot N)$› **have** $A_R{}' \sharp* S$ **by**(*force dest*: *inputFreshChainDerivative*)

obtain $A_Q{}'$ $\Psi_Q{}'$ **where** *FrQ'*: *extractFrame* $Q = \langle A_Q{}', \Psi_Q{}'\rangle$ **and** $A_Q{}' \sharp* \Psi$ **and** $A_Q{}' \sharp* \Psi_R$ **and** $A_Q{}' \sharp* A_R$ **and** $A_Q{}' \sharp* M$ **and** $A_Q{}' \sharp* R$ **and** $A_Q{}' \sharp* K$
by(*rule-tac* C=$(\Psi, \Psi_R, A_R, K, M, R)$ **in** *freshFrame*) *auto*
from *FrQ'* ‹*guarded Q*› **have** $\Psi_Q{}' \simeq \mathbf{1}$ **and** *supp* $\Psi_Q{}' = (\{\}::name\ set)$ **by**(*blast dest*: *guardedStatEq*)+
hence $A_Q{}' \sharp* \Psi_Q{}'$ **by**(*auto simp add*: *fresh-star-def fresh-def*)

from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \otimes \Psi_R \rhd P \Longrightarrow\hat{}_\tau\ P''$
and *NilImpP''*: $\langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle \hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) $(\Psi \otimes \Psi_R)$
and *P''Trans*: $\Psi \otimes \Psi_R \rhd P'' \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
using *FrQ'* ‹$A_Q{}' \sharp* \Psi$› ‹$A_Q{}' \sharp* \Psi_R$› *freshCompChain*
by(*drule-tac weakTransitionE*) *auto*

from ‹$(p \cdot xvec) \sharp* P$› ‹$xvec \sharp* P$› *PChain* **have** $(p \cdot xvec) \sharp* P''$ **and** $xvec \sharp*$

P''
by(*force dest*: *tauChainFreshChain*)+
from ‹$(p \cdot xvec) \sharp* N$› ‹*distinctPerm p*› **have** *xvec* $\sharp*$ $(p \cdot N)$
by(*subst pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*, **where** *pi*=*p*, *symmetric*]) *simp*
with *P''Trans* ‹*xvec* $\sharp*$ P''› **have** *xvec* $\sharp*$ P' **by**(*force dest*: *inputFreshChainDerivative*)
hence $(p \cdot xvec) \sharp* (p \cdot P')$ **by**(*simp add*: *pt-fresh-star-bij*[*OF pt-name-inst*, *OF at-name-inst*])

from *PChain* **have** $\Psi \rhd R \parallel !P \Longrightarrow_\tau (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$
proof(*induct rule*: *tauChainCases*)
case *TauBase*
from *RTrans FrQ* **have** $\Psi \otimes \mathbf{1} \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$ **by** *simp*
moreover note *FrR*
moreover from *P''Trans* ‹$P = P''$› **have** $\Psi \otimes \Psi_R \rhd P \longmapsto K(\!|(p \cdot N)|\!) \prec P'$ **by** *simp*
hence $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$ **by**(*force intro*: *Par1*)
hence $\Psi \otimes \Psi_R \rhd !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$ **using** ‹*guarded P*› **by**(*rule Bang*)
moreover from ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› *FrQ* **have** $\Psi \otimes \Psi_R \otimes \mathbf{1} \vdash M \leftrightarrow K$ **by** *simp*
ultimately have $\Psi \rhd R \parallel !P \longmapsto \tau \prec (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$ **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P$› ‹$A_R \sharp* M$› ‹$(p \cdot xvec) \sharp* P$›
by(*force intro*: *Comm2*)
thus *?case* **by**(*rule tauActTauStepChain*)
next
case *TauStep*
obtain A_P'' Ψ_P'' **where** *FrP''*: *extractFrame* $P'' = \langle A_P'', \Psi_P''\rangle$ **and** $A_P'' \sharp* \Psi$ **and** $A_P'' \sharp* K$ **and** $A_P'' \sharp* \Psi_R$ **and** $A_P'' \sharp* R$ **and** $A_P'' \sharp* P''$ **and** $A_P'' \sharp* P$
and $A_P'' \sharp* A_R$ **and** *distinct* A_P''
by(*rule-tac C*=(Ψ, K, A_R, Ψ_R, R, P'', P) **in** *freshFrame*) *auto*
from *PChain* ‹$A_R \sharp* P$› **have** $A_R \sharp* P''$ **by**(*drule-tac tauChainFreshChain*) *auto*
with *FrP''* ‹$A_P'' \sharp* A_R$› **have** $A_R \sharp* \Psi_P''$ **by**(*drule-tac extractFrameFreshChain*) *auto*
from ‹$\Psi \otimes \Psi_R \rhd P \Longrightarrow_\tau P''$› **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P \Longrightarrow_\tau P''$ **by**(*rule tauStepChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \otimes \Psi_R \rhd P \parallel !P \Longrightarrow_\tau P'' \parallel !P$ **by**(*rule-tac tauStepChainPar1*) *auto*
hence $\Psi \otimes \Psi_R \rhd !P \Longrightarrow_\tau P'' \parallel !P$ **using** ‹*guarded P*› **by**(*rule tauStepChainBang*)
hence $\Psi \rhd R \parallel !P \Longrightarrow_\tau R \parallel (P'' \parallel !P)$ **using** *FrR* ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P$›
by(*rule-tac tauStepChainPar2*) *auto*
moreover have $\Psi \rhd R \parallel (P'' \parallel !P) \longmapsto \tau \prec (\!|\nu*(p \cdot xvec)|\!)((p \cdot R') \parallel (P' \parallel !P))$

proof −
from *FrQ* ‹$\Psi_Q{}' \simeq \mathbf{1}$› *RTrans* **have** $\Psi \otimes \Psi_Q{}' \rhd R \longmapsto M(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$
by *simp* (*metis statEqTransition AssertionStatEqSym compositionSym*)
moreover from *P''Trans* **have** $(\Psi \otimes \Psi_R) \otimes \mathbf{1} \rhd P'' \longmapsto K(\!|(p \cdot N)|\!) \prec P'$
by(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence *P''PTrans*: $\Psi \otimes \Psi_R \rhd P'' \parallel !P \longmapsto K(\!|(p \cdot N)|\!) \prec (P' \parallel !P)$
by(*rule-tac Par1*) *auto*
moreover from *FrP''* **have** *FrP''P*: *extractFrame*$(P'' \parallel !P) = \langle A_P{}'', \Psi_P{}'' \otimes \mathbf{1}\rangle$
by *auto*
moreover from *FrQ* ‹$\Psi_Q{}' \simeq \mathbf{1}$› ‹$\Psi \otimes \Psi_R \otimes \Psi_Q \vdash M \leftrightarrow K$› **have** $\Psi \otimes \Psi_Q{}' \otimes \Psi_R \vdash M \leftrightarrow K$
by *simp* (*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
hence $\Psi \otimes \Psi_Q{}' \otimes \Psi_R \vdash K \leftrightarrow M$ **by**(*rule chanEqSym*)
moreover have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1})) \otimes \Psi_R\rangle$
proof −
have $\langle A_Q{}', (\Psi \otimes \Psi_Q{}') \otimes \Psi_R\rangle \simeq_F \langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle$
by(*rule-tac frameResChainPres*, *simp*)
(*metis Associativity Commutativity Composition AssertionStatEqTrans AssertionStatEqSym*)
moreover from *NilImpP''* *FrQ* *FrP''* ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* \Psi_R$› *freshCompChain* **have** $\langle A_Q{}', (\Psi \otimes \Psi_R) \otimes \Psi_Q{}'\rangle \hookrightarrow_F \langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle$
by *auto*
moreover have $\langle A_P{}'', (\Psi \otimes \Psi_R) \otimes \Psi_P{}''\rangle \simeq_F \langle A_P{}'', (\Psi \otimes \Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R\rangle$
by(*rule frameResChainPres*, *simp*)
(*metis Identity AssertionStatEqSym Associativity Commutativity Composition AssertionStatEqTrans*)
ultimately show *?thesis* **by**(*rule FrameStatEqImpCompose*)
qed
ultimately obtain M' **where** *RTrans*: $\Psi \otimes \Psi_P{}'' \otimes \mathbf{1} \rhd R \longmapsto M'(\!|\nu*(p \cdot xvec)|\!)\langle(p \cdot N)\rangle \prec (p \cdot R')$ **and** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$ **and** $A_R \sharp* M'$
using *FrR* *FrQ'* ‹*distinct* A_R› ‹*distinct* $A_P{}''$› ‹$A_P{}'' \sharp* A_R$› ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* Q$› ‹$A_R \sharp* R$› ‹$A_R \sharp* M$› ‹$A_Q{}' \sharp* R$› ‹$A_Q{}' \sharp* K$› ‹$A_Q{}' \sharp* A_R$› ‹$A_R \sharp* P$› ‹$A_P{}'' \sharp* P$›
‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* K$›
by(*rule-tac* A_Q=$A_Q{}'$ **and** Q=Q **in** *comm1Aux*) (*assumption* | *simp*)+

note *RTrans FrR P''PTrans FrP''P*
moreover from ‹$\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash K \leftrightarrow M'$› **have** $\Psi \otimes (\Psi_P{}'' \otimes \mathbf{1}) \otimes \Psi_R \vdash M' \leftrightarrow K$ **by**(*rule chanEqSym*)
hence $\Psi \otimes \Psi_R \otimes \Psi_P{}'' \otimes \mathbf{1} \vdash M' \leftrightarrow K$ **by**(*metis statEqEnt Composition AssertionStatEqSym Commutativity*)
ultimately show *?thesis* **using** ‹$A_R \sharp* \Psi$› ‹$A_R \sharp* R$› ‹$A_R \sharp* P''$› ‹$A_R \sharp* P$› ‹$A_R \sharp* M'$› ‹$A_P{}'' \sharp* A_R$› ‹$A_P{}'' \sharp* \Psi$› ‹$A_P{}'' \sharp* R$› ‹$A_P{}'' \sharp* P''$› ‹$A_P{}'' \sharp* P$› ‹$A_P{}'' \sharp* K$› ‹$(p \cdot xvec) \sharp* P''$› ‹$(p \cdot xvec) \sharp* P$›
by(*rule-tac Comm2*) (*assumption* | *simp*)+

qed
ultimately show *?thesis*
by(*drule-tac tauActTauStepChain*) *auto*
qed
hence $\Psi \rhd R \parallel !P \Longrightarrow_\tau (\!|\nu * xvec|\!)(R' \parallel ((p \cdot P') \parallel !P))$
using ‹*xvec* $\sharp*$ P› ‹$(p \cdot xvec)$ $\sharp*$ P› ‹$(p \cdot xvec)$ $\sharp*$ $(p \cdot P')$› ‹$(p \cdot xvec)$ $\sharp*$ R'› S ‹*distinctPerm p*›
by(*subst resChainAlpha*[**where** *p*=*p*]) *auto*
moreover from *P'Chain* **have** $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'))) \rhd (p \cdot P') \Longrightarrow\hat{}_\tau (p \cdot T)$
by(*rule tauChainEqvt*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec)$ $\sharp*$ Ψ› S ‹*distinctPerm p*›
have $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd (p \cdot P') \Longrightarrow\hat{}_\tau (p \cdot T)$ **by**(*simp add*: *eqvts*)
hence $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi') \otimes \mathbf{1} \rhd (p \cdot P') \Longrightarrow\hat{}_\tau (p \cdot T)$
by(*rule tauChainStatEq*) (*metis Identity AssertionStatEqSym*)
hence $(\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi' \rhd (p \cdot P') \parallel !P \Longrightarrow\hat{}_\tau (p \cdot T) \parallel !P$
by(*rule-tac tauChainPar1*) *auto*
hence $\Psi \otimes (p \cdot \Psi_R) \otimes \Psi' \rhd (p \cdot P') \parallel !P \Longrightarrow\hat{}_\tau (p \cdot T) \parallel !P$
by(*rule tauChainStatEq*) (*metis Associativity*)
hence $\Psi \otimes {\Psi_R}' \rhd (p \cdot P') \parallel !P \Longrightarrow\hat{}_\tau (p \cdot T) \parallel !P$ **using** ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq {\Psi_R}'$›
by(*rule-tac tauChainStatEq*) (*auto intro*: *compositionSym*)
hence $\Psi \rhd R' \parallel ((p \cdot P') \parallel !P) \Longrightarrow\hat{}_\tau R' \parallel ((p \cdot T) \parallel !P)$
using *FrR'* ‹${A_R}'$ $\sharp*$ Ψ› ‹${A_R}'$ $\sharp*$ P› ‹${A_R}'$ $\sharp*$ P'› ‹${A_R}'$ $\sharp*$ *xvec*› ‹${A_R}'$ $\sharp*$ $(p \cdot xvec)$› S
by(*rule-tac tauChainPar2*) (*auto simp add*: *freshChainSimps*)
hence $\Psi \rhd (\!|\nu * xvec|\!)(R' \parallel ((p \cdot P') \parallel !P)) \Longrightarrow\hat{}_\tau (\!|\nu * xvec|\!)(R' \parallel ((p \cdot T) \parallel !P))$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*rule tauChainResChainPres*)
ultimately have $\Psi \rhd R \parallel !P \Longrightarrow_\tau (\!|\nu * xvec|\!)(R' \parallel ((p \cdot T) \parallel !P))$
by *auto*
moreover have $(\Psi, (\!|\nu * xvec|\!)(R' \parallel ((p \cdot T) \parallel !P)), (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel''$
proof −
from ‹$((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi'), T, S) \in Rel$›
have $(p \cdot ((\Psi \otimes \Psi_R) \otimes (p \cdot \Psi')), (p \cdot T), (p \cdot S)) \in Rel$
by(*rule Closed*)
with ‹*xvec* $\sharp*$ Ψ› ‹$(p \cdot xvec)$ $\sharp*$ Ψ› ‹*distinctPerm p*› S
have $((\Psi \otimes (p \cdot \Psi_R)) \otimes \Psi', p \cdot T, p \cdot S) \in Rel$
by(*simp add*: *eqvts*)
hence $(\Psi \otimes (p \cdot \Psi_R) \otimes \Psi', p \cdot T, p \cdot S) \in Rel$
by(*rule StatEq*) (*metis Associativity*)
hence $(\Psi \otimes {\Psi_R}', p \cdot T, p \cdot S) \in Rel$ **using** ‹$(p \cdot \Psi_R) \otimes \Psi' \simeq {\Psi_R}'$›
by(*rule-tac StatEq*) (*auto dest*: *compositionSym*)
moreover from ‹${A_R}'$ $\sharp*$ S› ‹${A_R}'$ $\sharp*$ T› ‹${A_R}'$ $\sharp*$ *xvec*› ‹${A_R}'$ $\sharp*$ $(p \cdot xvec)$› S
have ${A_R}'$ $\sharp*$ $(p \cdot S)$ **and** ${A_R}'$ $\sharp*$ $(p \cdot T)$
by(*simp add*: *freshChainSimps*)+
ultimately have $(\Psi, R' \parallel (p \cdot T), R' \parallel (p \cdot S)) \in Rel$ **using** *FrR'* ‹${A_R}'$ $\sharp*$ Ψ›
by(*rule-tac FrameParPres*) *auto*

hence $(\Psi, (R' \parallel (p \cdot T)) \parallel !P, (R' \parallel (p \cdot S)) \parallel !P) \in Rel$ **by**(*rule ParPres*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel (p \cdot T)) \parallel !P), (\!|\nu * xvec|\!)((R' \parallel (p \cdot S)) \parallel !P)) \in Rel$
using ‹*xvec* $\sharp*$ Ψ›
by(*rule ResPres*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel (p \cdot T)) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P) \in Rel$
using ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ P›
by(*force intro*: *Trans ScopeExt*)
hence $(\Psi, (\!|\nu * xvec|\!)(R' \parallel ((p \cdot T) \parallel !P)), ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P) \in Rel$
using ‹*xvec* $\sharp*$ Ψ›
by(*force intro*: *Trans ResPres Assoc*)
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹*guarded* P› ‹*guarded* Q›
have $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !P, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !Q) \in Rel''$
by(*rule C1*)
moreover from ‹$(\mathbf{1}, (p \cdot Q'), S \parallel !Q) \in Rel'$›
have $(p \cdot \mathbf{1}, p \cdot p \cdot Q', p \cdot (S \parallel !Q)) \in Rel'$ **by**(*rule Closed'*)
with ‹*xvec* $\sharp*$ Q› ‹$(p \cdot xvec) \sharp* Q$› S ‹*distinctPerm* p›
have $(\mathbf{1}, Q', (p \cdot S) \parallel !Q) \in Rel'$ **by**(*simp add*: *eqvts*)
hence $(\mathbf{1} \otimes \Psi, Q', (p \cdot S) \parallel !Q) \in Rel'$ **by**(*rule cExt'*)
hence $(\Psi, Q', (p \cdot S) \parallel !Q) \in Rel'$
by(*rule StatEq'*) (*metis Identity AssertionStatEqSym Commutativity AssertionStatEqTrans*)
hence $(\Psi, R' \parallel Q', R' \parallel ((p \cdot S) \parallel !Q)) \in Rel'$ **by**(*rule ParPres'*)
hence $(\Psi, R' \parallel Q', (R' \parallel (p \cdot S)) \parallel !Q) \in Rel'$ **by**(*metis Trans' Assoc'*)
hence $(\Psi, (R' \parallel (p \cdot S)) \parallel !Q, R' \parallel Q') \in Rel'$ **by**(*rule cSym'*)
hence $(\Psi, (\!|\nu * xvec|\!)((R' \parallel (p \cdot S)) \parallel !Q), (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** ‹*xvec* $\sharp*$ Ψ›
by(*rule ResPres'*)
hence $(\Psi, ((\!|\nu * xvec|\!)(R' \parallel (p \cdot S))) \parallel !Q, (\!|\nu * xvec|\!)(R' \parallel Q')) \in Rel'$ **using** ‹*xvec* $\sharp*$ Ψ› ‹*xvec* $\sharp*$ Q›
by(*force intro*: *Trans' ScopeExt'*[*THEN cSym'*])
ultimately show *?thesis* **by**(*rule-tac Compose*)
qed
ultimately show *?case* **by** *blast*
qed
qed
notation *relcomp* (**infixr** *O* 75)

end

end

theory *Weak-Cong-Pres*
imports *Weak-Psi-Congruence Weak-Cong-Sim-Pres Weak-Bisim-Pres*
begin

context *env* **begin**

lemma *weakPsiCongInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $\bigwedge Tvec.\ length\ xvec = length\ Tvec \Longrightarrow \Psi \rhd P[xvec::=Tvec] \approx Q[xvec::=Tvec]$

shows $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \doteq M(\!|\lambda * xvec\ N|\!).Q$
proof −
from *assms* **have** $\forall\ Tvec.\ length\ xvec = length\ Tvec \longrightarrow \Psi \rhd P[xvec::=Tvec] \approx Q[xvec::=Tvec]$
by *simp*
thus *?thesis*
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P Q*)
thus *?case* **by**(*auto dest*: *weakBisimE*)
next
case(*cSim P Q*)
show *?case*
by(*induct rule*: *weakCongSimI*) *auto*
next
case(*cWeakBisim P Q*)
thus *?case* **by**(*rule-tac weakBisimInputPres*) *auto*
qed
qed

lemma *weakPsiCongOutputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $\Psi \rhd P \doteq Q$

shows $\Psi \rhd M\langle N\rangle.P \doteq M\langle N\rangle.Q$
using *assms*
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P Q*)
thus *?case* **by**(*rule weakPsiCongSym*)
next
case(*cSim P Q*)
show *?case* **by**(*induct rule*: *weakCongSimI*) *auto*
next
case(*cWeakBisim P Q*)

thus *?case* **by**(*metis weakPsiCongE weakBisimOutputPres*)
qed

lemma *weakBisimCasePres*:
fixes Ψ :: *'b*
and *CsP* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *CsQ* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*

assumes *A*: $\bigwedge\varphi$ *P*. (*φ*, *P*) *mem CsP* ⟹ ∃ *Q*. (*φ*, *Q*) *mem CsQ* ∧ *guarded Q* ∧ (∀ Ψ. Ψ ▷ *P* ≐ *Q*)
and *B*: $\bigwedge\varphi$ *Q*. (*φ*, *Q*) *mem CsQ* ⟹ ∃ *P*. (*φ*, *P*) *mem CsP* ∧ *guarded P* ∧ (∀ Ψ. Ψ ▷ *P* ≐ *Q*)

shows Ψ ▷ *Cases CsP* ≈ *Cases CsQ*
proof −
let *?X* = {(Ψ, *Cases CsP*, *Cases CsQ*) | Ψ *CsP CsQ*. (∀ *φ P*. (*φ*, *P*) *mem CsP* ⟶ (∃ *Q*. (*φ*, *Q*) *mem CsQ* ∧ *guarded Q* ∧ (∀ Ψ. Ψ ▷ *P* ≐ *Q*))) ∧ (∀ *φ Q*. (*φ*, *Q*) *mem CsQ* ⟶ (∃ *P*. (*φ*, *P*) *mem CsP* ∧ *guarded P* ∧ (∀ Ψ. Ψ ▷ *P* ≐ *Q*)))}
from *assms* **have** (Ψ, *Cases CsP*, *Cases CsQ*) ∈ *?X* **by** *auto*
thus *?thesis*
proof(*coinduct rule*: *weakBisimCoinduct*)
case(*cStatImp* Ψ *CasesP CasesQ*)
thus *?case*
apply(*auto simp add*: *weakStatImp-def*)
by(*rule-tac x*=*Cases CsQ* **in** *exI*, *auto*)
next
case(*cSim* Ψ *CasesP CasesQ*)
then obtain *CsP CsQ* **where** *C1*: $\bigwedge\varphi$ *Q*. (*φ*, *Q*) *mem CsQ* ⟹ ∃ *P*. (*φ*, *P*) *mem CsP* ∧ *guarded P* ∧ (∀ Ψ. Ψ ▷ *P* ≐ *Q*)
and *A*: *CasesP* = *Cases CsP* **and** *B*: *CasesQ* = *Cases CsQ*
by *auto*
note *C1*
moreover have $\bigwedge$Ψ *P Q*. Ψ ▷ *P* ≈ *Q* ⟹ Ψ ▷ *P* ⇝<*weakBisim*> *Q* **by**(*rule weakBisimE*)
moreover note *weakPsiCongE*(*1*) *weakPsiCongE*(*2*)
ultimately have Ψ ▷ *Cases CsP* ⇝<*weakBisim*> *Cases CsQ*
by(*rule-tac caseWeakSimPres*) (*assumption* | *blast*)+
hence Ψ ▷ *Cases CsP* ⇝<(*?X* ∪ *weakBisim*)> *Cases CsQ*
by(*rule-tac weakSimMonotonic*) *auto*
with *A B* **show** *?case* **by** *blast*
next
case(*cExt* Ψ *P Q* Ψ′)
thus *?case* **by** *auto*
next
case(*cSym* Ψ *P Q*)
thus *?case* **by** *auto* (*metis weakPsiCongSym*)+
qed
qed

lemma *weakPsiCongCasePres*:
fixes Ψ :: *'b*
and *CsP* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *CsQ* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*

assumes *A*: $\bigwedge\varphi\ P.\ (\varphi,\ P)\ mem\ CsP \Longrightarrow \exists\, Q.\ (\varphi,\ Q)\ mem\ CsQ \wedge guarded\ Q \wedge (\forall\, \Psi.\ \Psi \rhd P \doteq Q)$
and *B*: $\bigwedge\varphi\ Q.\ (\varphi,\ Q)\ mem\ CsQ \Longrightarrow \exists\, P.\ (\varphi,\ P)\ mem\ CsP \wedge guarded\ P \wedge (\forall\, \Psi.\ \Psi \rhd P \doteq Q)$

shows $\Psi \rhd Cases\ CsP \doteq Cases\ CsQ$
proof −
let *?Prop* = $\lambda CsP\ CsQ.\ \forall\, \varphi\ P.\ (\varphi,\ P)\ mem\ CsP \longrightarrow (\exists\, Q.\ (\varphi,\ Q)\ mem\ CsQ \wedge guarded\ Q \wedge (\forall\, \Psi.\ \Psi \rhd P \doteq Q))$
from *A B* **have** *?Prop CsP CsQ* ∧ *?Prop CsQ CsP* **by**(*metis weakPsiCongSym*)
thus *?thesis*
proof(*induct rule*: *weakPsiCongSymI*[**where** *C*=$\lambda P.\ Cases\ P$])
case(*cSym P Q*)
thus *?case* **by** *auto*
next
case(*cWeakBisim CsP CsQ*)
thus *?case*
by(*rule-tac weakBisimCasePres*) (*metis weakPsiCongSym*)+
next
case(*cSim CsP CsQ*)
thus *?case*
apply *auto*
apply(*rule-tac weakCongSimCasePres*, *auto*)
by(*blast dest*: *weakPsiCongE*)
qed
qed

lemma *weakPsiCongResPres*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*
and *x* :: *name*

assumes $\Psi \rhd P \doteq Q$
and $x \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!)P \doteq (\!|\nu x|\!)Q$
using ‹$\Psi \rhd P \doteq Q$›
proof(*induct rule*: *weakPsiCongSymI*)
case(*cSym P Q*)
thus *?case* **by**(*rule weakPsiCongSym*)
next
case(*cWeakBisim P Q*)

thus *?case* **using** ‹$x \sharp \Psi$› **by**(*metis weakPsiCongE weakBisimResPres*)
next
case(*cSim P Q*)
obtain *y*::*name* **where** $y \sharp \Psi$ **and** $y \sharp P$ **and** $y \sharp Q$
by(*generate-fresh name*) *auto*
from ‹$\Psi \rhd P \doteq Q$› **have** $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \doteq ([(x, y)] \cdot Q)$ **by**(*rule weakPsiCongClosed*)
hence $([(x, y)] \cdot \Psi) \rhd ([(x, y)] \cdot P) \rightsquigarrow\!\!\ll weakBisim \gg ([(x, y)] \cdot Q)$ **by**(*rule weakPsiCongE*)
with ‹$x \sharp \Psi$› ‹$y \sharp \Psi$› **have** $\Psi \rhd ([(x, y)] \cdot P) \rightsquigarrow\!\!\ll weakBisim \gg ([(x, y)] \cdot Q)$ **by** *simp*
moreover have *eqvt weakBisim* **by** *simp*
moreover note ‹$y \sharp \Psi$›
moreover have $weakBisim \subseteq weakBisim$ **by** *auto*
moreover note *weakBisimResPres*
ultimately have $\Psi \rhd (\!|\nu y|\!)([(x, y)] \cdot P) \rightsquigarrow\!\!\ll weakBisim \gg (\!|\nu y|\!)([(x, y)] \cdot Q)$ **by**(*rule weakCongSimResPres*)
with ‹$y \sharp P$› ‹$y \sharp Q$› **show** *?case* **by**(*simp add*: *alphaRes*)
qed

lemma *weakPsiCongResChainPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and *xvec* :: *name list*

assumes $\Psi \rhd P \doteq Q$
and *xvec* $\sharp*$ Ψ

shows $\Psi \rhd (\!|\nu * xvec|\!) P \doteq (\!|\nu * xvec|\!) Q$
using *assms*
by(*induct xvec*) (*auto intro*: *weakPsiCongResPres*)

lemma *weakPsiCongParPres*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and R :: (*'a*, *'b*, *'c*) *psi*

assumes $\forall \Psi.\ \Psi \rhd P \doteq Q$

shows $\Psi \rhd P \parallel R \doteq Q \parallel R$
using *assms*
proof(*induct rule*: *weakPsiCongSymI*[**where** $C{=}\lambda P.\ P \parallel R$])
case(*cSym P Q*)
thus *?case* **by**(*blast dest*: *weakPsiCongSym*)
next
case(*cWeakBisim P Q*)
thus *?case* **by**(*metis weakBisimParPres weakPsiCongE*)

next
case(*cSim P Q*)
{
fix Ψ
from ‹∀Ψ. Ψ ▷ *P* ≐ *Q*› **have** Ψ ▷ *P* ⇝«*weakBisim*» *Q* **by**(*auto dest*: *weakPsiCongE*)
}
moreover {
fix Ψ
from ‹∀Ψ. Ψ ▷ *P* ≐ *Q*› **have** Ψ ▷ *P* ⇝<*weakBisim*> *Q* **by**(*auto dest*: *weakPsiCongE weakBisimE*)
}
moreover {
fix Ψ
from ‹∀Ψ. Ψ ▷ *P* ≐ *Q*› **have** Ψ ▷ *P* ≈ *Q* **by**(*auto dest*: *weakPsiCongE*)
hence Ψ ▷ *Q* ≲<*weakBisim*> *P* **by**(*metis weakBisimE*)
}
ultimately show *?case* **using** *weakBisimEqvt weakBisimEqvt weakBisimE*(*4*) *weakBisimE*(*3*) *weakBisimParPresAux weakBisimResChainPres statEqWeakBisim*
by(*rule-tac weakCongSimParPres*)
qed

end

end

theory *Weak-Cong-Struct-Cong*
imports *Weak-Cong-Pres Weak-Bisim-Struct-Cong*
begin

context *env* **begin**

lemma *weakPsiCongParComm*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*
and *Q* :: (*'a*, *'b*, *'c*) *psi*

shows Ψ ▷ *P* ∥ *Q* ≐ *Q* ∥ *P*
by(*metis bisimParComm strongBisimWeakPsiCong*)

lemma *weakPsiCongResComm*:
fixes *x* :: *name*
and Ψ :: *'b*
and *y* :: *name*
and *P* :: (*'a*, *'b*, *'c*) *psi*

assumes *x* ♯ Ψ
and *y* ♯ Ψ

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu y|\!)P) \doteq (\!|\nu y|\!)((\!|\nu x|\!)P)$
using *assms*
by(*metis bisimResComm strongBisimWeakPsiCong*)

lemma *weakPsiCongResComm'*:
fixes x :: *name*
and Ψ :: $'b$
and *xvec* :: *name list*
and P :: $('a, 'b, 'c)$ *psi*

assumes $x \sharp \Psi$
and *xvec* $\sharp*$ Ψ

shows $\Psi \rhd (\!|\nu x|\!)((\!|\nu * xvec|\!)P) \doteq (\!|\nu * xvec|\!)((\!|\nu x|\!)P)$
using *assms*
by(*metis bisimResComm' strongBisimWeakPsiCong*)

lemma *weakPsiCongScopeExt*:
fixes x :: *name*
and Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $x \sharp \Psi$
and $x \sharp P$

shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \doteq P \parallel (\!|\nu x|\!)Q$
using *assms*
by(*metis bisimScopeExt strongBisimWeakPsiCong*)

lemma *weakPsiCongScopeExtChain*:
fixes *xvec* :: *name list*
and Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes *xvec* $\sharp*$ Ψ
and *xvec* $\sharp*$ P

shows $\Psi \rhd (\!|\nu * xvec|\!)(P \parallel Q) \doteq P \parallel ((\!|\nu * xvec|\!)Q)$
using *assms*
by(*metis bisimScopeExtChain strongBisimWeakPsiCong*)

lemma *weakPsiCongParAssoc*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and R :: $('a, 'b, 'c)$ *psi*

shows $\Psi \rhd (P \parallel Q) \parallel R \doteq P \parallel (Q \parallel R)$
by(*metis bisimParAssoc strongBisimWeakPsiCong*)

lemma *weakPsiCongParNil*:
fixes $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \parallel \mathbf{0} \doteq P$
by(*metis bisimParNil strongBisimWeakPsiCong*)

lemma *weakPsiCongResNil*:
fixes $x :: name$
and $\Psi :: 'b$

assumes $x \sharp \Psi$

shows $\Psi \rhd (\!|\nu x|\!)\mathbf{0} \doteq \mathbf{0}$
using *assms*
by(*metis bisimResNil strongBisimWeakPsiCong*)

lemma *weakPsiCongOutputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $N :: 'a$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp M$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M\langle N\rangle.P) \doteq M\langle N\rangle.(\!|\nu x|\!)P$
using *assms*
by(*metis bisimOutputPushRes strongBisimWeakPsiCong*)

lemma *weakPsiCongInputPushRes*:
fixes $x :: name$
and $\Psi :: 'b$
and $M :: 'a$
and $xvec :: name\ list$
and $N :: 'a$
and $P :: ('a, 'b, 'c)\ psi$

assumes $x \sharp \Psi$
and $x \sharp M$
and $x \sharp xvec$
and $x \sharp N$

shows $\Psi \rhd (\!|\nu x|\!)(M(\!|\lambda * xvec\ N|\!).P) \doteq M(\!|\lambda * xvec\ N|\!).(\!|\nu x|\!)P$
using *assms*

by(*metis bisimInputPushRes strongBisimWeakPsiCong*)

lemma *weakPsiCongCasePushRes*:
  fixes x :: *name*
  and Ψ :: $'b$
  and Cs :: $('c \times ('a, 'b, 'c)\ psi)\ list$

  assumes $x \sharp \Psi$
  and $x \sharp (map\ fst\ Cs)$

  shows $\Psi \rhd (\!|\nu x|\!)(Cases\ Cs) \doteq Cases(map\ (\lambda(\varphi, P).\ (\varphi, (\!|\nu x|\!)P))\ Cs)$
using *assms*
by(*metis bisimCasePushRes strongBisimWeakPsiCong*)

lemma *weakBangExt*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)\ psi$

  assumes *guarded P*

  shows $\Psi \rhd !P \doteq P \parallel !P$
using *assms*
by(*metis bangExt strongBisimWeakPsiCong*)

lemma *weakPsiCongParSym*:
  fixes Ψ :: $'b$
  and P :: $('a, 'b, 'c)\ psi$
  and Q :: $('a, 'b, 'c)\ psi$
  and R :: $('a, 'b, 'c)\ psi$

  assumes $\forall \Psi.\ \Psi \rhd P \doteq Q$

  shows $\Psi \rhd R \parallel P \doteq R \parallel Q$
using *assms*
by(*metis weakPsiCongParComm weakPsiCongParPres weakPsiCongTransitive*)

lemma *weakPsiCongScopeExtSym*:
  fixes x :: *name*
  and Q :: $('a, 'b, 'c)\ psi$
  and P :: $('a, 'b, 'c)\ psi$

  assumes $x \sharp \Psi$
  and $x \sharp Q$

  shows $\Psi \rhd (\!|\nu x|\!)(P \parallel Q) \doteq ((\!|\nu x|\!)P) \parallel Q$
using *assms*
by(*metis weakPsiCongScopeExt weakPsiCongTransitive weakPsiCongParComm weakPsiCongE weakPsiCongResPres*)

lemma *weakPsiCongScopeExtChainSym*:
  fixes *xvec* :: *name list*
  and *Q* :: ('*a*, '*b*, '*c*) *psi*
  and *P* :: ('*a*, '*b*, '*c*) *psi*

  assumes *xvec* ♯* Ψ
  and *xvec* ♯* *Q*

  shows Ψ ⊳ (⦇ν*xvec⦈)(*P* ∥ *Q*) ≐ ((⦇ν*xvec⦈)*P*) ∥ *Q*
using *assms*
by(*induct xvec*) (*auto intro*: *weakPsiCongScopeExtSym weakPsiCongReflexive weakPsiCongTransitive weakPsiCongResPres*)

lemma *weakPsiCongParPresSym*:
  fixes Ψ :: '*b*
  and *P* :: ('*a*, '*b*, '*c*) *psi*
  and *Q* :: ('*a*, '*b*, '*c*) *psi*
  and *R* :: ('*a*, '*b*, '*c*) *psi*

  assumes ⋀Ψ. Ψ ⊳ *P* ≐ *Q*

  shows Ψ ⊳ *R* ∥ *P* ≐ *R* ∥ *Q*
using *assms*
by(*metis weakPsiCongParComm weakPsiCongParPres weakPsiCongTransitive*)

lemma *tauCongChainBangI*:
  fixes Ψ :: '*b*
  and *P* :: ('*a*, '*b*, '*c*) *psi*
  and *P'* :: ('*a*, '*b*, '*c*) *psi*

  assumes Ψ ⊳ *P* ∥ *P* ⟹$_\tau$ *P'*
  and *guarded P*

  obtains *Q* **where** Ψ ⊳ !*P* ⟹$_\tau$ *Q* **and** Ψ ⊳ *Q* ∼ *P'* ∥ !*P*
proof −
  assume ⋀*Q*. ⟦Ψ ⊳ !*P* ⟹$_\tau$ *Q*; Ψ ⊳ *Q* ∼ *P'* ∥ !*P*⟧ ⟹ *thesis*
  moreover from ‹Ψ ⊳ *P* ∥ *P* ⟹$_\tau$ *P'*› **have** ∃ *Q*. Ψ ⊳ !*P* ⟹$_\tau$ *Q* ∧ Ψ ⊳ *Q* ∼ *P'* ∥ !*P*
  proof(*induct x1*==*P* ∥ *P P'* *rule*: *tauStepChainInduct*)
    case(*TauBase R'*)
    from ‹Ψ ⊳ *P* ∥ *P* ⟼τ ≺ *R'*›
    obtain *Q* **where** Ψ ⊳ !*P* ⟼τ ≺ *Q* **and** *Q* ∼ *R'* ∥ !*P* **using** ‹*guarded P*›
      by(*rule tauBangI*)
    from ‹Ψ ⊳ !*P* ⟼τ ≺ *Q*› **have** Ψ ⊳ !*P* ⟹$_\tau$ *Q* **by** *auto*
    moreover from ‹*Q* ∼ *R'* ∥ !*P*› **have** Ψ ⊳ *Q* ∼ *R'* ∥ !*P*
      apply(*drule-tac bisimE*(*3*)[**where** Ψ'=Ψ])
        by(*rule-tac statEqBisim*, *assumption*) (*metis Identity AssertionStatEqSym AssertionStatEqTrans Commutativity*)
    ultimately show *?case* **by** *blast*

next
case(*TauStep* R' R'')
then obtain Q **where** *PChain*: $\Psi \rhd !P \Longrightarrow_\tau Q$ **and** $\Psi \rhd Q \sim R' \parallel !P$ **by** *auto*
from $\langle \Psi \rhd R' \longmapsto \tau \prec R'' \rangle$ **have** $\Psi \otimes \mathbf{1} \rhd R' \longmapsto \tau \prec R''$ **by**(*rule statEqTransition*) (*metis Identity AssertionStatEqSym*)
hence $\Psi \rhd R' \parallel !P \longmapsto \tau \prec R'' \parallel !P$ **by**(*rule-tac Par1*) *auto*
with $\langle \Psi \rhd Q \sim R' \parallel !P \rangle$ **obtain** Q' **where** *QTrans*: $\Psi \rhd Q \longmapsto \tau \prec Q'$ **and** $\Psi \rhd Q' \sim R'' \parallel !P$
by(*force dest*: *bisimE*(*2*) *simE*)
from *PChain QTrans* **have** $\Psi \rhd !P \Longrightarrow_\tau Q'$ **by**(*auto dest*: *tauActTauStepChain*)
thus *?case* **using** $\langle \Psi \rhd Q' \sim R'' \parallel !P \rangle$ **by** *blast*
qed
ultimately show *?thesis* **by** *blast*
qed

lemma *weakPsiCongBangPres*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes *PeqQ*: $\forall \Psi.\ \Psi \rhd P \doteq Q$
and *guarded P*
and *guarded Q*

shows $\Psi \rhd !P \doteq !Q$
proof −
from *assms* **have** $(\forall \Psi.\ \Psi \rhd P \doteq Q) \wedge$ *guarded P* $\wedge$ *guarded Q* **by** *auto*
hence $\Psi \rhd \mathbf{0} \parallel !P \doteq \mathbf{0} \parallel !Q$
proof(*induct rule*: *weakPsiCongSymI*[**where** $C{=}\lambda P.\ \mathbf{0} \parallel !P$])
case(*cSym P Q*)
thus *?case* **by**(*auto dest*: *weakPsiCongSym*)
next
case(*cWeakBisim P Q*)
thus *?case* **by**(*metis weakPsiCongE weakBisimBangPresAux*)
next
case(*cSim P Q*)
then have $\forall \Psi.\ \Psi \rhd P \doteq Q$ **and** *guarded P* **and** *guarded Q* **by** *auto*
moreover hence $\Psi \rhd P \approx Q$ **by**(*metis weakPsiCongE weakBisimE*)
moreover have $\bigwedge \Psi\ P\ Q.\ (\forall \Psi.\ (\Psi \rhd P \doteq Q)) \Longrightarrow \Psi \rhd P \rightsquigarrow \langle\!\langle \mathit{weakBisim} \rangle\!\rangle\ Q$
by(*blast dest*: *weakPsiCongE*)
moreover note *weakBisimClosed bisimClosed weakBisimE*(*3*) *bisimE*(*3*) *weakBisimE*(*2*)
weakBisimE(*4*) *bisimE*(*4*) *statEqWeakBisim statEqBisim weakBisimTransitive bisimTransitive weakBisimParAssoc*[*THEN weakBisimE*(*4*)]
bisimParAssoc[*THEN bisimE*(*4*)] *weakBisimParPres*
moreover have $\bigwedge P\ Q.\ \forall \Psi.\ \Psi \rhd P \doteq Q \Longrightarrow \forall \Psi.\ \Psi \rhd P \parallel P \doteq Q \parallel Q$
by(*metis weakPsiCongParPres weakPsiCongParComm weakPsiCongSym weakPsiCongTransitive*)

moreover note *bisimParPresSym*
moreover from *strongBisimWeakBisim* **have** *bisim* $\subseteq$ *weakBisim* **by** *auto*
moreover have $\bigwedge\Psi\ \Psi_R\ P\ Q\ R\ A_R.\ [\![\Psi \otimes \Psi_R \rhd P \approx Q;\ extractFrame\ R = \langle A_R,\ \Psi_R\rangle;\ A_R \sharp* \Psi;\ A_R \sharp* P;\ A_R \sharp* Q]\!] \Longrightarrow \Psi \rhd R \parallel P \approx R \parallel Q$
by(*metis weakBisimParComm weakBisimTransitive weakBisimParPresAux*)
moreover note *weakBisimResChainPres bisimResChainPres weakBisimScopeExtChainSym bisimScopeExtChainSym*
moreover have $\bigwedge\Psi\ P\ R\ S\ Q.\ [\![\Psi \rhd P \approx R;\ \Psi \rhd R \approx S;\ \Psi \rhd S \sim Q]\!] \Longrightarrow \Psi \rhd P \approx Q$
by(*blast dest*: *weakBisimTransitive strongBisimWeakBisim*)
moreover note *weakBisimBangPresAux*
moreover from *bangActE* **have** $\bigwedge\Psi\ P\ \alpha\ P'.\ [\![\Psi \rhd\ !P \longmapsto \alpha \prec P';\ bn\ \alpha \sharp* P;\ guarded\ P;\ \alpha \neq \tau;\ bn\ \alpha \sharp* subject\ \alpha]\!] \Longrightarrow \exists\, Q.\ \Psi \rhd P \longmapsto \alpha \prec Q \wedge P' \sim Q \parallel\ !P$
by *blast*
moreover from *bangTauE* **have** $\bigwedge\Psi\ P\ P'.\ [\![\Psi \rhd\ !P \longmapsto \tau \prec P';\ guarded\ P]\!] \Longrightarrow \exists\, Q.\ \Psi \rhd P \parallel P \longmapsto \tau \prec Q \wedge P' \sim Q \parallel\ !P$
by *blast*
moreover from *tauCongChainBangI* **have** $\bigwedge\Psi\ P\ P'.\ [\![\Psi \rhd P \parallel P \Longrightarrow_\tau P';\ guarded\ P]\!] \Longrightarrow \exists\, Q.\ \Psi \rhd\ !P \Longrightarrow_\tau Q \wedge \Psi \rhd Q \sim P' \parallel\ !P$
by *blast*
ultimately show *?case*
by(*rule-tac weakCongSimBangPres*[**where** *Rel*=*weakBisim* **and** *Rel'*=*bisim* **and** *Rel''*=*weakBisim* **and** *Eq*=$\lambda P\ Q.\ \forall\Psi.\ \Psi \rhd P \doteq Q$])
qed
thus *?thesis*
by(*metis weakPsiCongParNil weakPsiCongParComm weakPsiCongTransitive weakPsiCongSym*)
qed

end

end

theory *Weak-Congruence*
imports *Weak-Cong-Struct-Cong Bisim-Subst*
begin

context *env* **begin**

definition *weakCongruence* :: ('*a*, '*b*, '*c*) *psi* $\Rightarrow$ ('*a*, '*b*, '*c*) *psi* $\Rightarrow$ *bool* (- $\doteq_c$ - [*70*, *70*] *65*)
where
$P \doteq_c Q \equiv \forall\Psi\ \sigma.\ wellFormedSubst\ \sigma \longrightarrow \Psi \rhd P[<\sigma>] \doteq Q[<\sigma>]$

lemma *weakCongE*:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Q* :: ('*a*, '*b*, '*c*) *psi*
and σ :: (*name list* $\times$ '*a list*) *list*

assumes $P \doteq_c Q$
wellFormedSubst σ

shows $\Psi \rhd P[<\sigma>] \doteq Q[<\sigma>]$
using *assms*
by(*auto simp add*: *weakCongruence-def*)

lemma *weakCongI*[*case-names cWeakPsiCong*]:
fixes P :: ('*a*, '*b*, '*c*) *psi*
and Q :: ('*a*, '*b*, '*c*) *psi*

assumes $\bigwedge \Psi\ \sigma.$ *wellFormedSubst* $\sigma \Longrightarrow \Psi \rhd P[<\sigma>] \doteq Q[<\sigma>]$

shows $P \doteq_c Q$
using *assms*
by(*auto simp add*: *weakCongruence-def*)

lemma *weakCongClosed*:
fixes Ψ :: '*b*
and P :: ('*a*, '*b*, '*c*) *psi*
and Q :: ('*a*, '*b*, '*c*) *psi*
and p :: *name prm*

assumes $P \doteq_c Q$

shows $(p \cdot P) \doteq_c (p \cdot Q)$
proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* Ψ σ)
note ‹$P \doteq_c Q$›
moreover from ‹*wellFormedSubst* σ› **have** *wellFormedSubst* (*rev p* $\cdot$ σ) **by** *simp*
ultimately have ((*rev p*) $\cdot$ Ψ) $\rhd$ $P[<(rev\ p \cdot \sigma)>] \doteq Q[<(rev\ p \cdot \sigma)>]$
by(*rule weakCongE*)
thus *?case* **by**(*drule-tac p*=*p* **in** *weakPsiCongClosed*) (*simp add*: *eqvts*)
qed

lemma *weakCongReflexive*:
fixes Ψ :: '*b*
and P :: ('*a*, '*b*, '*c*) *psi*

shows $P \doteq_c P$
by(*auto intro*: *weakCongI weakPsiCongReflexive*)

lemma *weakCongSym*:
fixes Ψ :: '*b*
and P :: ('*a*, '*b*, '*c*) *psi*
and Q :: ('*a*, '*b*, '*c*) *psi*

assumes $P \doteq_c Q$

shows $Q \doteq_c P$
using *assms*
by(*auto simp add*: *weakCongruence-def weakPsiCongSym*)

lemma *weakCongTransitive*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $R :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \doteq Q$
and $\Psi \rhd Q \doteq R$

shows $\Psi \rhd P \doteq R$
using *assms*
by(*auto intro*: *weakCongI weakPsiCongTransitive dest*: *weakCongE*)

lemma *weakCongWeakBisim*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $P \doteq_c Q$

shows $\Psi \rhd P \approx Q$
using *assms*
apply(*drule-tac* σ=[] **in** *weakCongE*)
by(*auto dest*: *weakPsiCongE*)

lemma *weakCongWeakPsiCong*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $P \doteq_c Q$

shows $\Psi \rhd P \doteq Q$
using *assms*
by(*drule-tac weakCongE*[**where** Ψ=Ψ **and** σ=[]]) *auto*

lemma *strongBisimWeakCong*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $P \sim_s Q$

shows $P \doteq_c Q$

proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* Ψ σ)
from *assms* ‹*wellFormedSubst* σ› **have** $P[<\sigma>] \sim Q[<\sigma>]$
by(*rule closeSubstE*)
hence $\Psi \rhd P[<\sigma>] \sim Q[<\sigma>]$ **by**(*metis bisimE*(*3*) *statEqBisim Identity Commutativity*)
thus *?case* **by**(*rule strongBisimWeakPsiCong*)
qed

lemma *structCongWeakCong*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $P \equiv_s Q$

shows $P \doteq_c Q$
using *assms*
by(*metis strongBisimWeakCong structCongBisimSubst*)

lemma *weakCongUnfold*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and σ :: (*name list* × $'a$ *list*) *list*

assumes $P \doteq_c Q$
and *wellFormedSubst* σ

shows $P[<\sigma>] \doteq_c Q[<\sigma>]$
proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* Ψ σ')
with ‹*wellFormedSubst* σ› ‹*wellFormedSubst* σ'› **have** *wellFormedSubst*$(\sigma @ \sigma')$
by *simp*
with ‹$P \doteq_c Q$› **have** $\Psi \rhd P[<(\sigma @ \sigma')>] \doteq Q[<(\sigma @ \sigma')>]$
by(*rule weakCongE*)
thus $\Psi \rhd P[<\sigma>][<\sigma'>] \doteq Q[<\sigma>][<\sigma'>]$
by *simp*
qed

lemma *weakCongOutputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and N :: $'a$

assumes $P \doteq_c Q$

shows $M\langle N\rangle.P \doteq_c M\langle N\rangle.Q$
using *assms*
by(*fastforce intro*: *weakCongI weakCongE weakPsiCongOutputPres*)

lemma *weakCongInputPres*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$

assumes $P \doteq_c Q$
and *distinct xvec*

shows $M(\!|\lambda * xvec\ N|\!).P \doteq_c M(\!|\lambda * xvec\ N|\!).Q$
proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* Ψ σ)
obtain p **where** $(p \cdot xvec) \sharp * \sigma$
and $(p \cdot xvec) \sharp * P$ **and** $(p \cdot xvec) \sharp * Q$ **and** $(p \cdot xvec) \sharp * \Psi$ **and** $(p \cdot xvec) \sharp * N$
and S: *set* $p \subseteq$ *set xvec* $\times$ *set* $(p \cdot xvec)$
by(*rule-tac* $c=(\sigma, P, Q, \Psi, N)$ **in** *name-list-avoiding*) *auto*

from ‹$P \doteq_c Q$› **have** $(p \cdot P) \doteq_c (p \cdot Q)$
by(*rule weakCongClosed*)
{
fix *Tvec* :: $'a$ *list*
from ‹$(p \cdot P) \doteq_c (p \cdot Q)$› ‹*wellFormedSubst* σ› **have** $(p \cdot P)[<\sigma>] \doteq_c (p \cdot Q)[<\sigma>]$
by(*rule weakCongUnfold*)
moreover assume *length xvec* = *length Tvec* **and** *distinct xvec*
ultimately have $\Psi \rhd ((p \cdot P)[<\sigma>])[(p \cdot xvec)::=Tvec] \doteq ((p \cdot Q)[<\sigma>])[(p \cdot xvec)::=Tvec]$
by(*drule-tac weakCongE*[**where** σ=[((*p* · *xvec*), *Tvec*)]]) *auto*
hence $\Psi \rhd ((p \cdot P)[<\sigma>])[(p \cdot xvec)::=Tvec] \approx ((p \cdot Q)[<\sigma>])[(p \cdot xvec)::=Tvec]$
by(*rule weakPsiCongE*)
}

with ‹$(p \cdot xvec) \sharp * \sigma$› ‹*distinct xvec*›
have $\Psi \rhd (M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P))[<\sigma>] \doteq (M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot Q))[<\sigma>]$
by(*force intro*: *weakPsiCongInputPres*)
moreover from ‹$(p \cdot xvec) \sharp * N$› ‹$(p \cdot xvec) \sharp * P$› *S* **have** $M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P) = M(\!|\lambda * xvec\ N|\!).P$
apply(*simp add*: *psi.inject*) **by**(*rule inputChainAlpha*[*symmetric*]) *auto*
moreover from ‹$(p \cdot xvec) \sharp * N$› ‹$(p \cdot xvec) \sharp * Q$› *S* **have** $M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot Q) = M(\!|\lambda * xvec\ N|\!).Q$
apply(*simp add*: *psi.inject*) **by**(*rule inputChainAlpha*[*symmetric*]) *auto*

ultimately show *?case* **by** *force*
qed

lemma *weakCongCasePresAux*:
fixes Ψ :: *'b*
and *CsP* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*
and *CsQ* :: (*'c* × (*'a*, *'b*, *'c*) *psi*) *list*

assumes *C1*: $\bigwedge\varphi\ P.\ (\varphi, P)\ mem\ CsP \Longrightarrow \exists\, Q.\ (\varphi, Q)\ mem\ CsQ \wedge guarded\ Q \wedge P \doteq_c Q$
and *C2*: $\bigwedge\varphi\ Q.\ (\varphi, Q)\ mem\ CsQ \Longrightarrow \exists\, P.\ (\varphi, P)\ mem\ CsP \wedge guarded\ P \wedge P \doteq_c Q$

shows *Cases CsP* $\doteq_c$ *Cases CsQ*
proof −
{
fix Ψ :: *'b*
fix σ :: (*name list* × *'a list*) *list*

assume *wellFormedSubst* σ
have $\Psi \rhd$ *Cases*(*caseListSeqSubst CsP* σ) $\doteq$ *Cases*(*caseListSeqSubst CsQ* σ)
proof(*rule weakPsiCongCasePres*)
fix φ *P*
assume (φ, *P*) *mem* (*caseListSeqSubst CsP* σ)
then obtain φ' *P'* **where** (φ', *P'*) *mem CsP* **and** φ = *substCond.seqSubst* φ' σ **and** *PeqP'*: *P* = (*P'*[<σ>])
by(*induct CsP*) *force*+
from ‹(φ', *P'*) *mem CsP*› **obtain** *Q'* **where** (φ', *Q'*) *mem CsQ* **and** *guarded Q'* **and** *P'* $\doteq_c$ *Q'* **by**(*blast dest*: *C1*)
from ‹(φ', *Q'*) *mem CsQ*› ‹φ = *substCond.seqSubst* φ' σ› **obtain** *Q* **where** (φ, *Q*) *mem* (*caseListSeqSubst CsQ* σ) **and** *Q* = *Q'*[<σ>]
by(*induct CsQ*) *auto*
with *PeqP'* ‹*guarded Q'*› ‹*P'* $\doteq_c$ *Q'*› ‹*wellFormedSubst* σ› **show** ∃ *Q*. (φ, *Q*) *mem* (*caseListSeqSubst CsQ* σ) ∧ *guarded Q* ∧ (∀ Ψ. $\Psi \rhd P \doteq Q$)
by(*blast dest*: *weakCongE guardedSeqSubst*)
next
fix φ *Q*
assume (φ, *Q*) *mem* (*caseListSeqSubst CsQ* σ)
then obtain φ' *Q'* **where** (φ', *Q'*) *mem CsQ* **and** φ = *substCond.seqSubst* φ' σ **and** *QeqQ'*: *Q* = *Q'*[<σ>]
by(*induct CsQ*) *force*+
from ‹(φ', *Q'*) *mem CsQ*› **obtain** *P'* **where** (φ', *P'*) *mem CsP* **and** *guarded P'* **and** *P'* $\doteq_c$ *Q'* **by**(*blast dest*: *C2*)
from ‹(φ', *P'*) *mem CsP*› ‹φ = *substCond.seqSubst* φ' σ› **obtain** *P* **where** (φ, *P*) *mem* (*caseListSeqSubst CsP* σ) **and** *P* = *P'*[<σ>]
by(*induct CsP*) *auto*
with *QeqQ'* ‹*guarded P'*› ‹*P'* $\doteq_c$ *Q'*› ‹*wellFormedSubst* σ›
show ∃ *P*. (φ, *P*) *mem* (*caseListSeqSubst CsP* σ) ∧ *guarded P* ∧ (∀ Ψ. $\Psi \rhd P \doteq Q$)

```
      by(blast dest: weakCongE guardedSeqSubst)
    qed
  }
  thus ?thesis
    by(rule-tac weakCongI) auto
qed

lemma weakCongParPres:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   R :: ('a, 'b, 'c) psi

  assumes P ≐_c Q

  shows P ∥ R ≐_c Q ∥ R
using assms
by(fastforce intro: weakCongI weakCongE weakPsiCongParPres)

lemma weakCongResPres:
  fixes P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   x :: name

  assumes P ≐_c Q

  shows (⦇νx⦈)P ≐_c (⦇νx⦈)Q
proof(induct rule: weakCongI)
  case(cWeakPsiCong Ψ σ)
  obtain y::name where y ♯ (Ψ::'b) and y ♯ P and y ♯ Q and y ♯ σ
    by(generate-fresh name) (auto simp add: fresh-prod)

  from ‹P ≐_c Q› have ([(x, y)] • P) ≐_c ([(x, y)] • Q) by(rule weakCongClosed)
  hence Ψ ▷ ([(x, y)] • P)[<σ>] ≐ ([(x, y)] • Q)[<σ>] using ‹wellFormedSubst
σ›
    by(rule weakCongE)
  hence Ψ ▷ (⦇νy⦈)(([(x, y)] • P)[<σ>]) ≐ (⦇νy⦈)(([(x, y)] • Q)[<σ>]) using ‹y ♯ Ψ›
    by(rule weakPsiCongResPres)
  with ‹y ♯ P› ‹y ♯ Q› ‹y ♯ σ›
  show Ψ ▷ ((⦇νx⦈)P)[<σ>] ≐ ((⦇νx⦈)Q)[<σ>]
    by(simp add: alphaRes)
qed

lemma weakCongBangPres:
  fixes P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi

  assumes P ≐_c Q
  and     guarded P
```

and *guarded Q*

shows $!P \doteq_c !Q$
using *assms*
by(*fastforce intro*: *weakCongI weakCongE weakPsiCongBangPres guardedSeqSubst*)

end

end

theory *Tau*
imports *Weak-Congruence Bisim-Struct-Cong*
begin

locale *tau* = *env* +
fixes *nameTerm* :: *name* ⇒ *'a*

assumes *ntEqvt*[*eqvt*]: (*p*::*name prm*) ∙ (*nameTerm x*) = *nameTerm*(*p* ∙ *x*)
and *ntSupp*: *supp*(*nameTerm x*) = {*x*}
and *ntEq*: Ψ ⊢ (*nameTerm x*) ↔ *M* = (*M* = *nameTerm x*)
and *subst4*: ⟦*length xvec* = *length Tvec*; *distinct xvec*; *xvec* ♯* (*M*::*'a*)⟧ ⟹ *M*[*xvec*::=*Tvec*] = *M*
begin

lemma *ntChanEq*[*simp*]:
fixes Ψ :: *'b*
and *x* :: *name*

shows Ψ ⊢ (*nameTerm x*) ↔ (*nameTerm x*)
using *ntEq*
by *auto*

lemma *nameTermFresh*[*simp*]:
fixes *x* :: *name*
and *y* :: *name*

shows *x* ♯ (*nameTerm y*) = (*x* ≠ *y*)
using *ntSupp*
by(*auto simp add*: *fresh-def*)

lemma *nameTermFreshChain*[*simp*]:
fixes *xvec* :: *name list*
and *x* :: *name*

shows *xvec* ♯* (*nameTerm x*) = *x* ♯ *xvec*
by(*induct xvec*) *auto*

definition *tauPrefix* :: (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* (*τ*.- [*85*] *85*)
where *tauPrefix P* ≡ *THE P'*. ∃*x*::*name*. *x* ♯ *P* ∧ *P'* = ⦇*νx*⦈((*nameTerm*

$x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$

lemma *tauActionUnfold*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $C :: 'd::fs\text{-}name$

obtains $x::name$ **where** $x \sharp P$ **and** $x \sharp C$ **and** $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
proof −
obtain $x::name$ **where** $x \sharp P$ **and** $x \sharp C$ **by**(*generate-fresh name*) *auto*
hence $\exists x::name.\ x \sharp P \wedge x \sharp C \wedge \tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
apply(*simp add*: *tauPrefix-def*)
apply(*subst the-equality*)
apply(*rule-tac* $x{=}x$ **in** *exI*)
apply *simp*
apply(*clarify*)
apply(*simp add*: *psi.inject alpha*)
by(*auto simp add*: *calc-atm eqvts abs-fresh*)
moreover assume $\bigwedge x.\ [\![x \sharp P;\ x \sharp C;$
$\tau.(P) =$
$(\!|\nu x|\!)((nameTerm\ x(\!|\lambda*[]\ nameTerm\ x|\!).\mathbf{0})\ \|\ nameTerm\ x\langle nameTerm\ x\rangle.P)]\!] \Longrightarrow thesis$
ultimately show *?thesis* **by** *blast*
qed

lemma *tauActionI*:
fixes $P :: ('a, 'b, 'c)\ psi$

shows $\exists P'.\ \Psi \rhd \tau.(P) \longmapsto \tau \prec P' \wedge \Psi \rhd P \sim P'$
proof −
obtain $x::name$ **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule tauActionUnfold*)

then have $\Psi \rhd \tau.(P) \longmapsto \tau \prec (\!|\nu x|\!)(\mathbf{0}\ \|\ P)$
apply *simp*
apply(*rule Scope*)
apply *auto*
apply(*subgoal-tac* $\Psi \rhd ((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P) \longmapsto \tau \prec (\!|\nu*[]|\!)(\mathbf{0}\ \|\ P)$)
apply *simp*
apply(*rule-tac* $M{=}(nameTerm\ x)$ **and** $N{=}(nameTerm\ x)$ **and** $K{=}(nameTerm\ x)$ **in** *Comm1*)
apply(*auto intro*: *ntChanEq Output Input*)
apply(*subgoal-tac* $\Psi \otimes \mathbf{1} \rhd (nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0} \longmapsto (nameTerm\ x)(\!|((nameTerm\ x)[([])::=[]])|\!) \prec (\mathbf{0}[[]::=[]])$)
apply(*simp add*: *subst4*)
by(*rule-tac Input*) *auto*

moreover from ‹$x \sharp \Psi$› ‹$x \sharp P$› **have** $\Psi \rhd P \sim (\!|\nu x|\!)(\mathbf{0} \parallel P)$
by(*metis bisimTransitive bisimParNil bisimScopeExtSym bisimResNil bisimParPresSym bisimParComm bisimE*(*4*))
ultimately show *?thesis* **by** *blast*
qed

lemma *outputEmpy*[*dest*]:
assumes $\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|\nu * xvec|\!)\langle N'\rangle \prec P'$

shows *xvec* = []
using *assms*
apply −
by(*ind-cases* $\Psi \rhd M\langle N\rangle.P \longmapsto K(\!|\nu * xvec|\!)\langle N'\rangle \prec P'$) (*auto simp add*: *psi.inject residualInject*)

lemma *tauActionE*:
fixes *P* :: ('*a*, '*b*, '*c*) *psi*

assumes $\Psi \rhd \tau.(P) \longmapsto \tau \prec P'$

shows $\Psi \rhd P \sim P'$ **and** *supp* P' = ((*supp P*)::*name set*)
proof −
obtain *x*:: *name* **where** $x \sharp (\Psi, P')$ **and** $x \sharp P$ **and** $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda * ([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule tauActionUnfold*)
with *assms* **have** $\Psi \rhd P \sim P' \wedge$ *supp* P' = ((*supp P*)::*name set*)
apply *simp*
apply(*erule-tac resTauCases*)
apply *simp*+
apply(*erule-tac C*=*x* **in** *parCases*)
apply *simp*+
apply(*drule-tac nilTrans*(*3*)[**where** *xvec*=[], *simplified*])
apply *simp*
apply *force*
apply *simp*
apply(*subgoal-tac* $\Psi \otimes \mathbf{1} \rhd (nameTerm\ x)(\!|\lambda * []\ (nameTerm\ x)|\!).\mathbf{0} \longmapsto M(\!|N|\!) \prec P'a$)
apply(*erule-tac inputCases*)
apply *simp*
apply(*subgoal-tac xvec* = [])
apply(*erule-tac outputCases*)
apply *simp*
apply(*clarsimp*)
apply(*rule conjI*)
apply(*metis bisimTransitive bisimParNil bisimScopeExtSym bisimResNil bisimParPresSym bisimParComm bisimE*(*4*))
apply(*auto simp add*: *psi.supp abs-supp suppBottom*)
by(*simp add*: *fresh-def*)
thus $\Psi \rhd P \sim P'$ **and** *supp* P' = ((*supp P*)::*name set*)

by *auto*
qed

lemma *tauActionEqvt*[*eqvt*]:
fixes $P :: ('a, 'b, 'c)\ psi$
and $p :: name\ prm$

shows $(p \cdot \tau.(P)) = \tau.(p \cdot P)$
proof −
obtain $x::name$ **where** $x \sharp P$ **and** $x \sharp p$ **and** $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule tauActionUnfold*)
moreover obtain $y::name$ **where** $y \sharp p \cdot P$ **and** $y \sharp (p, x)$ **and** $\tau.(p \cdot P) = (\!|\nu y|\!)(((nameTerm\ y)(\!|\lambda*([])\ (nameTerm\ y)|\!).\mathbf{0}) \parallel ((nameTerm\ y)\langle(nameTerm\ y)\rangle.(p \cdot P)))$
by(*rule tauActionUnfold*)
ultimately show *?thesis*
by(*simp add*: *eqvts calc-atm at-prm-fresh*[*OF at-name-inst*] *psi.inject alpha pt-fresh-perm-app*[*OF pt-name-inst*, *OF at-name-inst*])
qed

lemma *resCases'*[*consumes* 7, *case-names cOpen cRes*]:
fixes Ψ :: $'b$
and x :: *name*
and P :: $('a, 'b, 'c)\ psi$
and α :: $'a\ action$
and P' :: $('a, 'b, 'c)\ psi$
and C :: $'d::fs\text{-}name$

assumes *Trans*: $\Psi \rhd (\!|\nu x|\!)P \longmapsto x\alpha \prec xP'$
and $x \sharp \Psi$
and $x \sharp x\alpha$
and $x \sharp xP'$
and $bn\ x\alpha \sharp* \Psi$
and $bn\ x\alpha \sharp* P$
and $bn\ x\alpha \sharp* subject\ x\alpha$
and *rOpen*: $\bigwedge M\ xvec\ yvec\ y\ N\ P'.\ [\![\Psi \rhd P \longmapsto M(\!|\nu*(xvec@yvec)|\!)\langle([(x, y)] \cdot N)\rangle \prec ([(x, y)] \cdot P'); y \in supp\ N;$
$x \sharp N; x \sharp P'; x \neq y; y \sharp xvec; y \sharp yvec; y \sharp M;$ *distinct xvec*; *distinct yvec*;
$xvec \sharp* \Psi; y \sharp \Psi; yvec \sharp* \Psi; xvec \sharp* P; y \sharp P;$ $yvec \sharp* P; xvec \sharp* M; y \sharp M;$
$yvec \sharp* M; xvec \sharp* yvec; x\alpha = M(\!|\nu*(xvec@y\#yvec)|\!)\langle N\rangle;$ $xP' = P']\!] \Longrightarrow$
$Prop$
and *rScope*: $\bigwedge P'.\ [\![\Psi \rhd P \longmapsto x\alpha \prec P'; xP' = (\!|\nu x|\!)P']\!] \Longrightarrow Prop$

shows *Prop*
proof −

from *Trans* **have** *distinct*(*bn* $x\alpha$) **by**(*auto dest*: *boundOutputDistinct*)
have *length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xP'$) **by** *simp*
note *Trans*
moreover have *length* [] = *inputLength*($(\!|\nu x|\!)P$) **and** *distinct* []
by(*auto simp add*: *inputLength-inputLength'-inputLength''.simps*)
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xP'$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xP'$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xP'$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xP'$)› ‹*distinct*(*bn* $x\alpha$)›
ultimately show *?thesis* **using** ‹*bn* $x\alpha \sharp* \Psi$› ‹*bn* $x\alpha \sharp* P$› ‹*bn* $x\alpha \sharp* subject\ x\alpha$› ‹$x \sharp \Psi$› ‹$x \sharp x\alpha$› ‹$x \sharp xP'$› ‹*distinct*(*bn* $x\alpha$)› *rScope rOpen*
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - - - x x])
apply(*auto simp add*: *psi.inject alpha abs-fresh residualInject boundOutputApp boundOutput.inject eqvts*)
apply(*subgoal-tac* $y \in supp\ Na$)
apply(*auto simp add*: *residualInject boundOutputApp*)
apply(*drule-tac pi*=[(x, y)] **in** *pt-set-bij2*[*OF pt-name-inst*, *OF at-name-inst*])
by(*simp add*: *calc-atm eqvts*)
qed

lemma *parCases'*[*consumes 5*, *case-names cPar1 cPar2 cComm1 cComm2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and T :: (*'a*, *'b*, *'c*) *psi*
and C :: *'d*::*fs-name*

assumes *Trans*: $\Psi \rhd P \parallel Q \longmapsto x\alpha \prec xT$
and *bn* $x\alpha \sharp* \Psi$
and *bn* $x\alpha \sharp* P$
and *bn* $x\alpha \sharp* Q$
and *bn* $x\alpha \sharp* subject\ x\alpha$
and *rPar1*: $\bigwedge P'\ A_Q\ \Psi_Q.\ [\![\Psi \otimes \Psi_Q \rhd P \longmapsto x\alpha \prec P';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$A_Q \sharp* \Psi;\ A_Q \sharp* P;\ A_Q \sharp* Q;\ A_Q \sharp* x\alpha;\ A_Q \sharp* P';\ A_Q \sharp* C;\ xT = P' \parallel Q]\!] \Longrightarrow Prop$
and *rPar2*: $\bigwedge Q'\ A_P\ \Psi_P.\ [\![\Psi \otimes \Psi_P \rhd Q \longmapsto x\alpha \prec Q';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$A_P \sharp* \Psi;\ A_P \sharp* P;\ A_P \sharp* Q;\ A_P \sharp* x\alpha;\ A_P \sharp* Q';\ A_P \sharp* C;\ xT = P \parallel Q']\!] \Longrightarrow Prop$
and *rComm1*: $\bigwedge \Psi_Q\ M\ N\ P'\ A_P\ \Psi_P\ K\ xvec\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|N|\!) \prec P';\ extractFrame\ P = \langle A_P, \Psi_P\rangle;\ distinct\ A_P;$
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|\nu* xvec|\!)\langle N\rangle \prec Q';\ extractFrame\ Q = \langle A_Q, \Psi_Q\rangle;\ distinct\ A_Q;$
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K;\ distinct\ xvec;$
$A_P \sharp* \Psi;\ A_P \sharp* \Psi_Q;\ A_P \sharp* P;\ A_P \sharp* M;\ A_P \sharp* N;\ A_P \sharp* P';\ A_P \sharp* Q;\ A_P \sharp* xvec;\ A_P \sharp* Q';\ A_P \sharp* A_Q;\ A_P \sharp* C;$
$A_Q \sharp* \Psi;\ A_Q \sharp* \Psi_P;\ A_Q \sharp* P;\ A_Q \sharp* K;\ A_Q \sharp* N;\ A_Q \sharp* P';\ A_Q \sharp*$

Q; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C$; $x\alpha{=}\tau$; $xT = (\!|\nu{*}xvec|\!)(P' \parallel Q')]\!] \Longrightarrow Prop$
and *rComm2*: $\bigwedge \Psi_Q\ M\ xvec\ N\ P'\ A_P\ \Psi_P\ K\ Q'\ A_Q.$
$[\![\Psi \otimes \Psi_Q \rhd P \longmapsto M(\!|\nu{*}xvec|\!)\langle N\rangle \prec P'$; $extractFrame\ P = \langle A_P, \Psi_P\rangle$; $distinct\ A_P$;
$\Psi \otimes \Psi_P \rhd Q \longmapsto K(\!|N|\!) \prec Q'$; $extractFrame\ Q = \langle A_Q, \Psi_Q\rangle$; $distinct\ A_Q$;
$\Psi \otimes \Psi_P \otimes \Psi_Q \vdash M \leftrightarrow K$; $distinct\ xvec$;
$A_P \sharp* \Psi$; $A_P \sharp* \Psi_Q$; $A_P \sharp* P$; $A_P \sharp* M$; $A_P \sharp* N$; $A_P \sharp* P'$; $A_P \sharp* Q$; $A_P \sharp* xvec$; $A_P \sharp* Q'$; $A_P \sharp* A_Q$; $A_P \sharp* C$;
$A_Q \sharp* \Psi$; $A_Q \sharp* \Psi_P$; $A_Q \sharp* P$; $A_Q \sharp* K$; $A_Q \sharp* N$; $A_Q \sharp* P'$; $A_Q \sharp* Q$; $A_Q \sharp* xvec$; $A_Q \sharp* Q'$; $A_Q \sharp* C$;
$xvec \sharp* \Psi$; $xvec \sharp* \Psi_P$; $xvec \sharp* P$; $xvec \sharp* M$; $xvec \sharp* K$; $xvec \sharp* Q$; $xvec \sharp* \Psi_Q$; $xvec \sharp* C$; $x\alpha{=}\tau$; $xT = (\!|\nu{*}xvec|\!)(P' \parallel Q')]\!] \Longrightarrow Prop$

shows *Prop*
proof −
from *Trans* **have** *distinct*(*bn* $x\alpha$) **by**(*auto dest*: *boundOutputDistinct*)
have *length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xT$) **by** *simp*
note *Trans*
moreover have *length* [] = *inputLength*($P \parallel Q$) **and** *distinct* []
by(*auto simp add*: *inputLength-inputLength′-inputLength″.simps*)
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xT$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xT$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xT$)› ‹*distinct*(*bn* $x\alpha$)›
moreover note ‹*length*(*bn* $x\alpha$) = *residualLength*($x\alpha \prec xT$)› ‹*distinct*(*bn* $x\alpha$)›
moreover obtain *x*::*name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp Q$ **and** $x \sharp x\alpha$ **and** $x \sharp xT$
by(*generate-fresh name*) *auto*
ultimately show *?thesis* **using** ‹*bn* $x\alpha \sharp* \Psi$› ‹*bn* $x\alpha \sharp* P$› ‹*bn* $x\alpha \sharp* Q$› ‹*bn* $x\alpha \sharp* subject\ x\alpha$› **using** *rPar1 rPar2 rComm1 rComm2*
by(*cases rule*: *semanticsCases*[*of* - - - - - - - - - *C x x*]) (*auto simp add*: *psi.inject residualInject residualInject′*)
qed

lemma *inputCases′*[*consumes 1*, *case-names cInput*]:
fixes Ψ :: *′b*
and M :: *′a*
and *xvec* :: *name list*
and N :: *′a*
and P :: (*′a*, *′b*, *′c*) *psi*
and α :: *′a action*
and P' :: (*′a*, *′b*, *′c*) *psi*

assumes *Trans*: $\Psi \rhd M(\!|\lambda{*}xvec\ N|\!).P \longmapsto \alpha \prec P'$
and *rInput*: $\bigwedge K\ Tvec.\ [\![\Psi \vdash M \leftrightarrow K$; *set xvec* $\subseteq$ *supp N*; *length xvec* = *length Tvec*; *distinct xvec*; $\alpha{=}K(\!|N[xvec::=Tvec]|\!)$; $P' = P[xvec::=Tvec]]\!] \Longrightarrow Prop$ $(K(\!|N[xvec::=Tvec]|\!))$ $(P[xvec::=Tvec])$

shows *Prop* α P'
proof −
{
fix *xvec N P*
assume *Trans*: $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P'$
and $xvec \sharp* \Psi$ **and** $xvec \sharp* M$ **and** $xvec \sharp* \alpha$ **and** $xvec \sharp* P'$ **and** *distinct xvec*
and *rInput*: $\bigwedge K\ Tvec.\ [\![\Psi \vdash M \leftrightarrow K;\ set\ xvec \subseteq supp\ N;\ length\ xvec = length\ Tvec;\ distinct\ xvec;\ \alpha = K(\!|N[xvec ::= Tvec]|\!);\ P' = P[xvec ::= Tvec]]\!] \Longrightarrow Prop\ (K(\!|N[xvec ::= Tvec]|\!))\ (P[xvec ::= Tvec])$

from *Trans* **have** *bn* $\alpha = []$
apply −
by(*ind-cases* $\Psi \rhd M(\!|\lambda * xvec\ N|\!).P \longmapsto \alpha \prec P'$) (*auto simp add*: *residualInject*)
from *Trans* **have** *distinct*(*bn* α) **by**(*auto dest*: *boundOutputDistinct*)
have *length*(*bn* α) = *residualLength*($\alpha \prec P'$) **by** *simp*
note *Trans*
moreover have *length xvec* = *inputLength*($M(\!|\lambda * xvec\ N|\!).P$) **by** *auto*
moreover note ‹*distinct xvec*›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover note ‹*length*(*bn* α) = *residualLength*($\alpha \prec P'$)› ‹*distinct*(*bn* α)›
moreover obtain *x*::*name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp M$ **and** $x \sharp xvec$ **and** $x \sharp \alpha$ **and** $x \sharp P'$ **and** $x \sharp N$
by(*generate-fresh name*) *auto*
ultimately have *Prop* α P' **using** ‹*bn* $\alpha = []$› ‹$xvec \sharp* \Psi$›‹$xvec \sharp* M$› ‹$xvec \sharp* \alpha$› ‹$xvec \sharp* P'$› *rInput*
apply(*cases rule*: *semanticsCases*[*of* - - - - - - - - - - *C x*])
by(*fastforce simp add*: *residualInject psi.inject inputChainFresh*)+
}
note *Goal* = *this*
moreover obtain *p* :: *name prm* **where** $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* M$ **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* P$
and $(p \cdot xvec) \sharp* \alpha$ **and** $(p \cdot xvec) \sharp* P'$ **and** *S*: *set p* $\subseteq$ *set xvec* $\times$ *set*($p \cdot xvec$)
and *distinctPerm p*
by(*rule-tac xvec*=*xvec* **and** *c*=(Ψ, *M*, *N*, *P*, α, P') **in** *name-list-avoiding*) *auto*
from *Trans* ‹$(p \cdot xvec) \sharp* N$› ‹$(p \cdot xvec) \sharp* P$› *S* **have** $\Psi \rhd M(\!|\lambda * (p \cdot xvec)\ (p \cdot N)|\!).(p \cdot P) \longmapsto \alpha \prec P'$
by(*simp add*: *inputChainAlpha'*)
moreover {
fix *K Tvec*
assume $\Psi \vdash M \leftrightarrow K$
moreover assume *set*($p \cdot xvec$) $\subseteq$ *supp*($p \cdot N$)
hence ($p \cdot$ *set*($p \cdot xvec$)) $\subseteq$ ($p \cdot$ *supp*($p \cdot N$)) **by** *simp*
with ‹*distinctPerm p*› **have** *set xvec* $\subseteq$ *supp N* **by**(*simp add*: *eqvts*)
moreover assume *length*($p \cdot xvec$) = *length*(*Tvec*::'*a list*)

hence *length xvec = length Tvec* **by** *simp*
moreover assume *distinct xvec*
moreover assume α=$K\langle\!|(p \cdot N)[(p \cdot xvec)::=Tvec]|\!\rangle$ **and** $P' = (p \cdot P)[(p \cdot xvec)::=Tvec]$
with ‹$(p \cdot xvec)$ $\sharp*$ P› ‹$(p \cdot xvec)$ $\sharp*$ N› ‹*distinctPerm p*› ‹*length xvec = length Tvec*› S
have α=$K\langle\!|(N[xvec::=Tvec])|\!\rangle$ **and** $P' = P[xvec::=Tvec]$
by(*simp add*: *renaming substTerm.renaming*)+
ultimately have *Prop* $(K\langle\!|N[xvec::=Tvec]|\!\rangle)$ $(P[xvec::=Tvec])$
by(*rule rInput*)
with ‹*length xvec = length Tvec*› S ‹*distinctPerm p*› ‹$(p \cdot xvec)$ $\sharp*$ N› ‹$(p \cdot xvec)$ $\sharp*$ P›
have *Prop* $(K\langle\!|(p \cdot N)[(p \cdot xvec)::=Tvec]|\!\rangle)$ $((p \cdot P)[(p \cdot xvec)::=Tvec])$
by(*simp add*: *renaming substTerm.renaming*)
}
moreover from *Trans* **have** *distinct xvec* **by**(*rule inputDistinct*)
hence *distinct*$(p \cdot xvec)$ **by** *simp*
ultimately show *?thesis* **using** ‹$(p \cdot xvec)$ $\sharp*$ Ψ› ‹$(p \cdot xvec)$ $\sharp*$ M› ‹$(p \cdot xvec)$ $\sharp*$ α› ‹$(p \cdot xvec)$ $\sharp*$ P'› ‹*distinct xvec*›
by(*rule-tac Goal*) *assumption*+
qed

lemma *outputCases'*[*consumes 1*, *case-names cOutput*]:
fixes Ψ :: $'b$
and M :: $'a$
and N :: $'a$
and P :: $('a, 'b, 'c)$ *psi*
and α :: $'a$ *action*
and P' :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd M\langle N\rangle.P \longmapsto \alpha \prec P'$
and $\bigwedge K.\ [\![\Psi \vdash M \leftrightarrow K;\ subject\ \alpha = Some\ K]\!] \Longrightarrow Prop\ (K\langle N\rangle)\ P$

shows *Prop* α P'
using *assms*
by(*cases rule*: *semantics.cases*) (*auto simp add*: *residualInject psi.inject*)

lemma *tauOutput*[*dest*]:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and M :: $'a$
and *xvec* :: *name list*
and N :: $'a$
and P' :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd \tau.(P) \longmapsto M\langle\!|\nu * xvec|\!\rangle\langle N\rangle \prec P'$

shows *False*
proof −

{
fix *xvec N P′*

assume $\Psi \rhd \tau.(P) \longmapsto M(\!|\nu * xvec|\!)\langle N\rangle \prec P'$
and $xvec \sharp* \Psi$
and $xvec \sharp* P$
and $xvec \sharp* M$

moreover obtain *x*:: *name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp xvec$ **and** $x \sharp M$ **and** $x \sharp N$ **and** $x \sharp P'$
and $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule-tac C*=(Ψ, *M*, *xvec*, *N*, *P′*) **in** *tauActionUnfold*) *auto*
ultimately have *False*
apply *simp*
apply(*erule-tac resCases′*)
apply *simp*+
apply(*erule-tac parOutputCases*)
apply(*simp add*: *action.inject psi.inject*)+
apply(*drule-tac nilTrans*(*2*)[**where** *xvec*=[], *simplified*])
apply *simp*+
apply(*simp add*: *action.inject*)
apply(*clarify*)
apply(*erule outputCases′*)
apply(*auto simp add*: *ntEq*)
apply(*erule-tac parCases′*)
apply(*auto simp add*: *abs-fresh*)
apply(*drule-tac nilTrans*(*2*)[**where** *xvec*=[], *simplified*])
apply *simp*
apply(*erule-tac outputCases′*)
apply *auto*
by(*simp add*: *ntEq*)
}
moreover obtain *p* **where** *set p* $\subseteq$ *set xvec* $\times$ *set*($p \cdot xvec$) **and** $(p \cdot xvec) \sharp* N$ **and** $(p \cdot xvec) \sharp* P'$
and $(p \cdot xvec) \sharp* \Psi$ **and** $(p \cdot xvec) \sharp* P$ **and** $(p \cdot xvec) \sharp* M$
by(*rule-tac c*=(*N*, *P′*, Ψ, *P*, *M*) **and** *xvec*=*xvec* **in** *name-list-avoiding*) *auto*
ultimately show *False* **using** *assms* **by**(*simp add*: *boundOutputChainAlpha″ residualInject*) *auto*
qed

lemma *tauInput*[*dest*]:
fixes Ψ :: *′b*
and *P* :: (*′a*, *′b*, *′c*) *psi*
and *M* :: *′a*
and *N* :: *′a*
and *P′* :: (*′a*, *′b*, *′c*) *psi*

assumes $\Psi \rhd \tau.(P) \longmapsto M(\!|N|\!) \prec P'$

shows *False*
proof −
obtain x:: *name* **where** $x \sharp \Psi$ **and** $x \sharp P$ **and** $x \sharp M$ **and** $x \sharp N$ **and** $x \sharp P'$
and $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule-tac* C=(Ψ, M, N, P') **in** *tauActionUnfold*) *auto*
with *assms* **show** *False*
apply *simp*
apply(*erule-tac resCases'*)
apply *auto*
apply(*drule-tac parCases'*)
apply(*auto simp add*: *abs-fresh*)
apply(*subgoal-tac* $\Psi \otimes \mathbf{1} \rhd (nameTerm\ x)(\!|\lambda*[]\ (nameTerm\ x)|\!).\mathbf{0} \longmapsto M(\!|N|\!) \prec P'a$)
apply(*erule-tac inputCases'*)
by(*auto simp add*: *action.inject subst4*)
qed

lemma *tauPrefixFrame*:
fixes P :: ($'a$, $'b$, $'c$) *psi*

shows $extractFrame(\tau.(P)) \simeq_F \langle\varepsilon, \mathbf{1}\rangle$
proof −
obtain x:: *name* **where** $x \sharp P$
and $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule-tac* C=() **in** *tauActionUnfold*) *auto*
thus *?thesis*
by(*force intro*: *frameResFresh Identity FrameStatEqTrans*)
qed

lemma *insertTauAssertion*:
fixes P :: ($'a$, $'b$, $'c$) *psi*
and Ψ :: $'b$

shows $insertAssertion\ (extractFrame(\tau.(P)))\ \Psi \simeq_F \langle\varepsilon, \Psi\rangle$
proof −
obtain x:: *name* **where** $x \sharp P$ **and** $x \sharp \Psi$
and $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule-tac* C=Ψ **in** *tauActionUnfold*) *auto*
thus *?thesis*
apply *auto*
apply(*rule-tac* G=$\langle\varepsilon, \Psi \otimes \mathbf{1} \otimes \mathbf{1}\rangle$ **in** *FrameStatEqTrans*)
apply(*rule-tac frameResFresh*) **apply** *auto*
apply(*subgoal-tac* $x \sharp \mathbf{1} \otimes \mathbf{1}$)
apply *auto*
by(*metis Identity AssertionStatEqTrans AssertionStatEqSym Associativity*)

qed

lemma *seqSubst4*:
assumes $y \sharp \sigma$
and *wellFormedSubst* σ

shows *substTerm.seqSubst* (*nameTerm* y) σ = *nameTerm* y
using *assms*
by(*induct* σ) (*auto simp add*: *subst4*)

lemma *tauSeqSubst*[*simp*]:
fixes P :: ('*a*, '*b*, '*c*) *psi*
and σ :: (*name list* × '*a list*) *list*

assumes *wellFormedSubst* σ

shows $(\tau.(P))[<\sigma>] = \tau.(P[<\sigma>])$
proof −
obtain x:: *name* **where** $x \sharp P[<\sigma>]$ **and** $x \sharp \sigma$ **and** $x \sharp P$
and $\tau.(P[<\sigma>]) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.(P[<\sigma>])))$
by(*rule-tac* C=(σ, P) **in** *tauActionUnfold*) *auto*
moreover obtain y:: *name* **where** $y \sharp P[<\sigma>]$ **and** $y \sharp \sigma$ **and** $y \sharp P$ **and** $x \neq y$
and $\tau.(P) = (\!|\nu y|\!)(((nameTerm\ y)(\!|\lambda*([])\ (nameTerm\ y)|\!).\mathbf{0}) \parallel ((nameTerm\ y)\langle(nameTerm\ y)\rangle.(P)))$
by(*rule-tac* C=(σ, $P[<\sigma>]$, x) **in** *tauActionUnfold*) *auto*
ultimately show *?thesis* **using** *assms*
by(*auto simp add*: *psi.inject alpha eqvts calc-atm psi.inject input.inject seqSubst4*)
qed

lemma *tauSubst*[*simp*]:
fixes P :: ('*a*, '*b*, '*c*) *psi*
and *xvec* :: *name list*
and *Tvec* :: '*a list*

assumes *distinct xvec*
and *length xvec* = *length Tvec*

shows $(\tau.(P))[xvec::=Tvec] = \tau.(P[xvec::=Tvec])$
proof −
obtain x:: *name* **where** $x \sharp P[xvec::=Tvec]$ **and** $x \sharp xvec$ **and** $x \sharp Tvec$ **and** $x \sharp P$
and $\tau.(P[xvec::=Tvec]) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0}) \parallel ((nameTerm\ x)\langle(nameTerm\ x)\rangle.(P[xvec::=Tvec])))$
by(*rule-tac* C=(*xvec*, *Tvec*, P) **in** *tauActionUnfold*) *auto*
moreover obtain y:: *name* **where** $y \sharp P[xvec::=Tvec]$ **and** $y \sharp xvec$ **and** $y \sharp Tvec$ **and** $y \sharp P$ **and** $x \neq y$

and $\tau.(P) = (\!|\nu y|\!)(((nameTerm\ y)(\!|\lambda*([])\ (nameTerm\ y)|\!).\mathbf{0})\ \|\ ((nameTerm\ y)\langle(nameTerm\ y)\rangle.(P)))$
by(*rule-tac* C=($xvec$, $Tvec$, P[$xvec$::=$Tvec$], x) **in** *tauActionUnfold*) *auto*
ultimately show *?thesis* **using** *assms*
by(*auto simp add*: *psi.inject alpha eqvts calc-atm psi.inject input.inject subst4*)
qed

lemma *tauFresh*[*simp*]:
fixes $P :: ('a, 'b, 'c)\ psi$
and $x :: name$

shows $x \sharp \tau.(P) = x \sharp P$
proof −
obtain y::*name* **where** $y \neq x$ **and** $y \sharp P$ **and** $\tau.(P) = (\!|\nu y|\!)(((nameTerm\ y)(\!|\lambda*([])\ (nameTerm\ y)|\!).\mathbf{0})\ \|\ ((nameTerm\ y)\langle(nameTerm\ y)\rangle.P))$
by(*rule-tac* C=x **in** *tauActionUnfold*) *auto*

thus *?thesis* **by**(*auto simp add*: *abs-fresh*)
qed

lemma *tauFreshChain*[*simp*]:
fixes $P :: ('a, 'b, 'c)\ psi$
and $xvec :: name\ list$

shows $xvec \sharp* (\tau.(P)) = (xvec \sharp* P)$
by(*induct xvec*) *auto*

lemma *guardedTau*:
fixes $P :: ('a, 'b, 'c)\ psi$

shows *guarded*$(\tau.(P))$
proof −
obtain x::*name* **where** $x \sharp P$ **and** $\tau.(P) = (\!|\nu x|\!)(((nameTerm\ x)(\!|\lambda*([])\ (nameTerm\ x)|\!).\mathbf{0})\ \|\ ((nameTerm\ x)\langle(nameTerm\ x)\rangle.P))$
by(*rule-tac* C=() **in** *tauActionUnfold*) *auto*

thus *?thesis* **by** *auto*
qed

lemma *tauChainBisim*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $P' :: ('a, 'b, 'c)\ psi$
and $P'' :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \Longrightarrow\!\hat{}_{\tau}\ P'$
and $\Psi \rhd P \sim P''$

obtains P''' **where** $\Psi \rhd P'' \Longrightarrow\!\hat{}_{\tau}\ P'''$ **and** $\Psi \rhd P' \sim P'''$

using *assms*
proof(*induct arbitrary*: *thesis rule*: *tauChainInduct*)
case(*TauBase P*)
thus *?case* **by** *auto*
next
case(*TauStep P P' P'''*)
from ‹$\Psi \rhd P \sim P''$› **obtain** P'''' **where** $P''Chain$: $\Psi \rhd P'' \Longrightarrow\hat{}_\tau P''''$ **and** $\Psi \rhd P' \sim P''''$
by(*rule-tac TauStep*) *auto*
from ‹$\Psi \rhd P' \sim P''''$› ‹$\Psi \rhd P' \longmapsto\tau \prec P'''$›
obtain P''''' **where** $P''''Trans$: $\Psi \rhd P'''' \longmapsto\tau \prec P'''''$ **and** $\Psi \rhd P''' \sim P'''''$
apply(*drule-tac bisimE*(*4*))
apply(*drule-tac bisimE*(*2*))
apply(*drule-tac simE*)
apply *auto*
apply(*drule-tac bisimE*(*4*))
by *blast*
from $P''Chain$ $P''''Trans$ **have** $\Psi \rhd P'' \Longrightarrow\hat{}_\tau P'''''$ **by**(*drule-tac tauActTauChain*) *auto*
with ‹$\Psi \rhd P''' \sim P'''''$› **show** *?case*
by(*metis TauStep*)
qed

lemma *tauChainStepCons*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$

obtains P'' **where** $\Psi \rhd \tau.(P) \Longrightarrow_\tau P''$ **and** $\Psi \rhd P' \sim P''$
proof −
assume *Goal*: $\bigwedge P''.\ [\![\Psi \rhd \tau.(P) \Longrightarrow_\tau P''; \Psi \rhd P' \sim P'']\!] \Longrightarrow thesis$
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto\tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *PChain* ‹$\Psi \rhd P \sim P''$› **obtain** P''' **where** $P''Chain$: $\Psi \rhd P'' \Longrightarrow\hat{}_\tau P'''$ **and** $\Psi \rhd P' \sim P'''$
by(*rule tauChainBisim*)
from *PTrans* $P''Chain$ **have** $\Psi \rhd \tau.(P) \Longrightarrow_\tau P'''$ **by**(*drule-tac tauActTauStepChain*) *auto*
thus *?thesis* **using** ‹$\Psi \rhd P' \sim P'''$›
by(*rule Goal*)
qed

lemma *tauChainCons*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$

obtains P'' **where** $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ P''$ **and** $\Psi \rhd P' \sim P''$
proof −
assume *Goal*: $\bigwedge P''.\ [\![\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ P''; \Psi \rhd P' \sim P'']\!] \Longrightarrow$ *thesis*
from *assms* **obtain** P'' **where** $\Psi \rhd \tau.(P) \Longrightarrow_\tau P''$ **and** $\Psi \rhd P' \sim P''$
by(*rule tauChainStepCons*)
thus *?thesis* **by**(*rule-tac Goal*) *auto*
qed

lemma *weakTransitionTau*:
fixes Ψ :: *'b*
and Q :: (*'a*, *'b*, *'c*) *psi*
and P :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a action*
and P' :: (*'a*, *'b*, *'c*) *psi*

assumes *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P'$
and $bn\ \alpha \sharp* \Psi$
and $bn\ \alpha \sharp* P$

obtains P'' **where** $\Psi : Q \rhd \tau.(P) \Longrightarrow \alpha \prec P''$ **and** $\Psi \rhd P' \sim P''$
proof −
assume *Goal*: $\bigwedge P''.\ [\![\Psi : Q \rhd \tau.(P) \Longrightarrow \alpha \prec P''; \Psi \rhd P' \sim P'']\!] \Longrightarrow$ *thesis*
from *PTrans* **obtain** P'' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P''$
and *QimpP''*: *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame P''*) Ψ
and *P''Trans*: $\Psi \rhd P'' \longmapsto \alpha \prec P'$
by(*blast dest*: *weakTransitionE*)

from *PChain* **obtain** P''' **where** *tPChain*: $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ P'''$ **and** $\Psi \rhd P'' \sim P'''$ **by**(*rule tauChainCons*)
moreover from *QimpP''* ‹$\Psi \rhd P'' \sim P'''$› **have** *insertAssertion* (*extractFrame Q*) $\Psi \hookrightarrow_F$ *insertAssertion*(*extractFrame P'''*) Ψ
by(*metis bisimE FrameStatEq-def FrameStatImpTrans*)
moreover from *tPChain* ‹$bn\ \alpha \sharp* P$› **have** $bn\ \alpha \sharp* P'''$ **by**(*force intro*: *tauChainFreshChain*)
with ‹$\Psi \rhd P'' \sim P'''$› *P''Trans* ‹$bn\ \alpha \sharp* \Psi$› **obtain** P'''' **where** $\Psi \rhd P''' \longmapsto \alpha \prec P''''$ **and** $\Psi \rhd P' \sim P''''$
by(*metis bisimE simE*)
ultimately show *?thesis*
by(*rule-tac Goal*) (*blast intro*: *weakTransitionI*)
qed

end

end

theory *Sum*
imports *Semantics Close-Subst*
begin

context *env*
begin

abbreviation *sumAssertJudge* (- $\oplus_{-}$ - [*150*, *50*, *50*] *150*)
where (*P*::('*a*, '*b*, '*c*) *psi*) $\oplus_{\varphi}$ *Q* $\equiv$ *Cases* [(φ, *P*), (φ, *Q*)]

lemma *SumAssert1*:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Q* :: ('*a*, '*b*, '*c*) *psi*

assumes $\Psi \rhd P \longmapsto Rs$
and $\Psi \vdash \varphi$
and *guarded P*

shows $\Psi \rhd P \oplus_{\varphi} Q \longmapsto Rs$
proof −
note ‹$\Psi \rhd P \longmapsto Rs$›
moreover have (φ, *P*) *mem* [(φ, *P*), (φ, *Q*)] **by** *simp*
ultimately show *?thesis* **using** ‹$\Psi \vdash \varphi$› ‹*guarded P*›
by(*rule Case*)
qed

lemma *SumAssert2*:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Q* :: ('*a*, '*b*, '*c*) *psi*

assumes $\Psi \rhd Q \longmapsto Rs$
and $\Psi \vdash \varphi$
and *guarded Q*

shows $\Psi \rhd P \oplus_{\varphi} Q \longmapsto Rs$
proof −
note ‹$\Psi \rhd Q \longmapsto Rs$›
moreover have (φ, *Q*) *mem* [(φ, *P*), (φ, *Q*)] **by** *simp*
ultimately show *?thesis* **using** ‹$\Psi \vdash \varphi$› ‹*guarded Q*›
by(*rule Case*)
qed

lemma *sumAssertCases*[*consumes 2*, *case-names cSum1 cSum2*]:
fixes Ψ :: '*b*
and *P* :: ('*a*, '*b*, '*c*) *psi*
and *Q* :: ('*a*, '*b*, '*c*) *psi*
and φ :: '*c*

```
  assumes Ψ ▷ P ⊕_φ Q ⟼ Rs
  and     Ψ ⊢ φ
  and     rSum1: ⟦Ψ ▷ P ⟼ Rs; guarded P⟧ ⟹ Prop
  and     rSum2: ⟦Ψ ▷ Q ⟼ Rs; guarded Q⟧ ⟹ Prop

  shows Prop
proof −
  from ‹Ψ ▷ P ⊕_φ Q ⟼ Rs› show ?thesis
  proof(induct rule: caseCases)
    case(cCase φ' P')
    from ‹(φ', P') mem [(φ, P), (φ, Q)]›
    have φ = φ' and D: P = P' ∨ Q = P' by auto
    from D show ?thesis
    proof(rule disjE)
      assume P = P'
      with ‹Ψ ▷ P' ⟼ Rs› ‹guarded P'› show ?case by(rule-tac rSum1) auto
    next
      assume Q = P'
      with ‹Ψ ▷ P' ⟼ Rs› ‹guarded P'› show ?case by(rule-tac rSum2) auto
    qed
  qed
qed

lemma sumElim[dest]:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi
  and   Q :: ('a, 'b, 'c) psi
  and   φ :: 'c

  assumes Ψ ▷ P ⊕_φ Q ⟼ Rs
  and     ¬(Ψ ⊢ φ)

  shows False
using assms
by(induct rule: caseCases) auto

end

locale sum = env +
  fixes T :: 'c

  assumes Top: Ψ ⊢ T
  and     TopEqvt[eqvt]: ((p::name prm) • T) = T
  and     TopSubst[simp]: substCond T xvec Tvec = T
begin

abbreviation topJudge (⊤ 150) where ⊤ ≡ T
abbreviation sumJudge (infixr ⊕ 80) where P ⊕ Q ≡ P ⊕_⊤ Q
```

lemma *topSeqSubst*[*simp*]:
  shows (*substCond.seqSubst T σ*) = *T*
by(*induct σ*) *auto*

lemma *suppTop*:
  shows ((*supp*(⊤))::*name set*) = {}
by(*auto simp add*: *supp-def eqvts*)

lemma *freshTop*[*simp*]:
  fixes *x*   :: *name*
  and *xvec* :: *name list*
  and *Xs*   :: *name set*

  shows *x* ♯ ⊤ **and** *xvec* ♯* ⊤ **and** *Xs* ♯* ⊤
by(*auto simp add*: *fresh-def fresh-star-def suppTop*)

lemma *sumSubst*[*simp*]:
  fixes *P*   :: (′*a*, ′*b*, ′*c*) *psi*
  and *Q*   :: (′*a*, ′*b*, ′*c*) *psi*
  and *xvec* :: *name list*
  and *Tvec* :: ′*a list*

  assumes *length xvec* = *length Tvec*
  and   *distinct xvec*

  shows (*P* ⊕ *Q*)[*xvec*::=*Tvec*] = (*P*[*xvec*::=*Tvec*] ⊕ *Q*[*xvec*::=*Tvec*])
by *auto*

lemma *sumSeqSubst*[*simp*]:
  fixes *P* :: (′*a*, ′*b*, ′*c*) *psi*
  and *Q* :: (′*a*, ′*b*, ′*c*) *psi*
  and *σ* :: (*name list* × ′*a list*) *list*

  assumes *wellFormedSubst σ*

  shows (*P* ⊕ *Q*)[<*σ*>] = ((*P*[<*σ*>]) ⊕ (*Q*[<*σ*>]))
using *assms*
by(*induct σ arbitrary*: *P Q*) *auto*

lemma *Sum1*:
  fixes Ψ :: ′*b*
  and *P* :: (′*a*, ′*b*, ′*c*) *psi*
  and *Q* :: (′*a*, ′*b*, ′*c*) *psi*

  assumes Ψ ▷ *P* ⟼ *Rs*
  and   *guarded P*

  shows Ψ ▷ *P* ⊕ *Q* ⟼ *Rs*

proof −
have $\Psi \vdash \top$ **by**(*rule Top*)
with ‹$\Psi \rhd P \longmapsto Rs$› **show** *?thesis* **using** ‹*guarded P*›
by(*rule SumAssert1*)
qed

lemma *Sum2*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd Q \longmapsto Rs$
and *guarded Q*

shows $\Psi \rhd P \oplus Q \longmapsto Rs$
proof −
have $\Psi \vdash \top$ **by**(*rule Top*)
with ‹$\Psi \rhd Q \longmapsto Rs$› **show** *?thesis* **using** ‹*guarded Q*›
by(*rule SumAssert2*)
qed

lemma *sumCases*[*consumes 1*, *case-names cSum1 cSum2*]:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \oplus Q \longmapsto Rs$
and *rSum1*: $[\![\Psi \rhd P \longmapsto Rs; \textit{guarded } P]\!] \Longrightarrow \textit{Prop}$
and *rSum2*: $[\![\Psi \rhd Q \longmapsto Rs; \textit{guarded } Q]\!] \Longrightarrow \textit{Prop}$

shows *Prop*
proof −
have $\Psi \vdash \top$ **by**(*rule Top*)
with ‹$\Psi \rhd P \oplus Q \longmapsto Rs$› **show** *?thesis*
proof(*induct rule*: *sumAssertCases*)
case *cSum1*
thus *?case* **by** (*rule rSum1*)
next
case *cSum2*
thus *?case* **by**(*rule rSum2*)
qed
qed

end

end

theory *Tau-Sim*
imports *Tau Sum*

begin

nominal-datatype *'a prefix* =
  pInput 'a::fs-name name list 'a
| *pOutput 'a 'a*
| *pTau*

context *tau*
begin

nominal-primrec *bindPrefix* :: *'a prefix* ⇒ (*'a*, *'b*, *'c*) *psi* ⇒ (*'a*, *'b*, *'c*) *psi* (-·- [*100*, *100*] *100*)
where
  bindPrefix (*pInput M xvec N*) *P* = *M*(⦇λ∗*xvec N*⦈).*P*
| *bindPrefix* (*pOutput M N*) *P* = *M*⟨*N*⟩.*P*
| *bindPrefix* (*pTau*) *P* = τ.(*P*)
by(*rule TrueI*)+

lemma *bindPrefixEqvt*[*eqvt*]:
  fixes *p* :: *name prm*
  and α :: *'a prefix*
  and *P* :: (*'a*, *'b*, *'c*) *psi*

  shows (*p* ∙ (α·*P*)) = (*p* ∙ α)·(*p* ∙ *P*)
by(*nominal-induct* α *rule*: *prefix.strong-inducts*) (*auto simp add*: *eqvts*)

lemma *prefixCases*[*consumes 1*, *case-names cInput cOutput cTau*]:
  fixes Ψ :: *'b*
  and α :: *'a prefix*
  and *P* :: (*'a*, *'b*, *'c*) *psi*
  and β :: *'a action*
  and *P'* :: (*'a*, *'b*, *'c*) *psi*

  assumes Ψ ▷ α·*P* ⟼β ≺ *P'*
  and *rInput*: ⋀*M xvec N K Tvec*. ⟦Ψ ⊢ *M* ↔ *K*; *set xvec* ⊆ *supp N*; *length xvec* = *length Tvec*; *distinct xvec*⟧ ⟹
                Prop (*pInput M xvec N*) (*K*(⦇*N*[*xvec*::=*Tvec*]⦈)) (*P*[*xvec*::=*Tvec*])
  and *rOutput*: ⋀*M N K*. Ψ ⊢ *M* ↔ *K* ⟹ *Prop* (*pOutput M N*) (*K*⟨*N*⟩) *P*
  and *rTau*: Ψ ▷ *P* ∼ *P'* ⟹ *Prop* (*pTau*) (τ) *P'*

  shows *Prop* α β *P'*
using ‹Ψ ▷ α·*P* ⟼β ≺ *P'*›
proof(*nominal-induct rule*: *prefix.strong-induct*)
  case(*pInput M xvec N*)
  from ‹Ψ ▷ (*pInput M xvec N*)·*P* ⟼β ≺ *P'*› **have** Ψ ▷ *M*(⦇λ∗*xvec N*⦈).*P* ⟼β ≺ *P'* **by** *simp*
  thus *?case* **by**(*auto elim*: *inputCases intro*: *rInput*)
next

case(*pOutput M N*)
from ‹$\Psi \rhd (pOutput\ M\ N)\cdot P \longmapsto \beta \prec P'$› **have** $\Psi \rhd M\langle N\rangle.P \longmapsto \beta \prec P'$ **by** *simp*
thus *?case* **by**(*auto elim*: *outputCases intro*: *rOutput*)
next
case *pTau*
from ‹$\Psi \rhd (pTau)\cdot P \longmapsto \beta \prec P'$› **have** $\Psi \rhd \tau.(P) \longmapsto \beta \prec P'$ **by** *simp*
thus *?case*
by(*nominal-induct rule*: *action.strong-induct*) (*auto dest*: *tauActionE intro*: *rTau*)
qed

lemma *prefixTauCases*[*consumes 1*, *case-names cTau*]:
fixes α :: $'a$ *prefix*
and P :: $('a, 'b, 'c)$ *psi*
and P' :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd \alpha\cdot P \longmapsto \tau \prec P'$
and *rTau*: $\Psi \rhd P \sim P' \Longrightarrow Prop\ (pTau)\ P'$

shows $Prop\ \alpha\ P'$
proof −
from ‹$\Psi \rhd \alpha\cdot P \longmapsto \tau \prec P'$› **obtain** β **where** $\Psi \rhd \alpha\cdot P \longmapsto \beta \prec P'$ **and** $\beta = \tau$
by *auto*
thus *?thesis*
by(*induct rule*: *prefixCases*) (*auto intro*: *rTau*)
qed

lemma *hennessySim1*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *C1*: $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel]\!] \Longrightarrow (\Psi, P, R) \in Rel$

shows $\Psi \rhd \tau.(P) \rightsquigarrow «Rel» Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau Q'*)
from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$›
obtain P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau\ P'$ **and** *P'RelQ'*: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from *PChain* **obtain** P'' **where** $\Psi \rhd \tau.(P) \Longrightarrow_\tau P''$ **and** $\Psi \rhd P' \sim P''$
by(*rule tauChainStepCons*)
thus *?case* **using** *P'RelQ'* **by**(*metis bisimE C1*)
qed

lemma *hennessySim2*:

fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes *PTrans*: $\Psi \rhd P \longmapsto \tau \prec P'$
and $P'RelQ$: $(\Psi, P', Q) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R.\ [\![(\Psi, P, Q) \in Rel; \Psi \rhd Q \sim R]\!] \Longrightarrow (\Psi, P, R) \in Rel$

shows $\Psi \rhd P \rightsquigarrow «Rel»\ \tau.(Q)$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from $\langle \Psi \rhd \tau.(Q) \longmapsto \tau \prec Q' \rangle$ **have** $\Psi \rhd Q \sim Q'$ **by**(*blast dest*: *tauActionE*)
with *PTrans* $P'RelQ$ **show** *?case* **by**(*metis C1 tauActTauStepChain*)
qed

lemma *hennessySim3*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *C1*: $\bigwedge Q'.\ \Psi \rhd Q \longmapsto \tau \prec Q' \Longrightarrow (\Psi, P, Q') \notin Rel$

shows $\Psi \rhd P \rightsquigarrow «Rel»\ Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from $\langle \Psi \rhd P \rightsquigarrow <Rel> Q \rangle$ $\langle \Psi \rhd Q \longmapsto \tau \prec Q' \rangle$
obtain P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_{\tau}\ P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)
with *C1* $\langle \Psi \rhd Q \longmapsto \tau \prec Q' \rangle$ **show** *?case*
by(*force simp add*: *rtrancl-eq-or-trancl*)
qed

lemma *tauLaw1SimLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes $\Psi \rhd P \rightsquigarrow <Rel> Q$
and *eqvt Rel*
and *C1*: $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel]\!] \Longrightarrow (\Psi, P, R) \in Rel$

shows $\Psi \rhd \tau.(P) \rightsquigarrow <Rel> Q$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ Q'$)
from $\langle bn\ \alpha \sharp* (\tau.(P)) \rangle$ **have** $bn\ \alpha \sharp* P$ **by** *simp*
with $\langle \Psi \rhd P \rightsquigarrow <Rel> Q \rangle$ $\langle \Psi \rhd Q \longmapsto \alpha \prec Q' \rangle$ $\langle bn\ \alpha \sharp* \Psi \rangle$ $\langle \alpha \neq \tau \rangle$
obtain $P''\ P'$ **where** *PTrans*: $\Psi : Q \rhd P \Longrightarrow \alpha \prec P''$ **and** $P''Chain$: $\Psi \otimes \Psi' \rhd P'' \Longrightarrow\hat{}_{\tau}\ P'$

and $P'RelQ'$: $(\Psi \otimes \Psi', P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from *PTrans* ‹*bn* $\alpha \sharp* \Psi$› ‹*bn* $\alpha \sharp* P$› **obtain** P''' **where** $\Psi : Q \rhd \tau.(P) \Longrightarrow \alpha \prec P'''$ **and** $\Psi \rhd P'' \sim P'''$
by(*rule weakTransitionTau*)
moreover from ‹$\Psi \rhd P'' \sim P'''$› **have** $\Psi \otimes \Psi' \rhd P'' \sim P'''$ **by**(*rule bisimE*)
with *P''Chain* **obtain** P'''' **where** $\Psi \otimes \Psi' \rhd P''' \Longrightarrow\hat{}_\tau P''''$ **and** $\Psi \otimes \Psi' \rhd P' \sim P''''$
by(*rule tauChainBisim*)
ultimately show *?case* **using** ‹$(\Psi \otimes \Psi', P', Q') \in Rel$› **by**(*metis bisimE C1*)
next
case(*cTau* Q')
from ‹$\Psi \rhd P \rightsquigarrow <Rel> Q$› ‹$\Psi \rhd Q \longmapsto \tau \prec Q'$›
obtain P' **where** *PChain*: $\Psi \rhd P \Longrightarrow\hat{}_\tau P'$ **and** $P'RelQ'$: $(\Psi, P', Q') \in Rel$
by(*blast dest*: *weakSimE*)

from *PChain* **obtain** P'' **where** $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau P''$ **and** $\Psi \rhd P' \sim P''$
by(*rule tauChainCons*)
thus *?case* **using** $P'RelQ'$ **by**(*metis bisimE C1*)
qed

lemma *tauLaw1SimRight*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)$ *psi*
and $Q :: ('a, 'b, 'c)$ *psi*

assumes *eqvt Rel*
and $(\Psi, P, Q) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R.\ [\![(\Psi, P, Q) \in Rel; \Psi \rhd Q \sim R]\!] \Longrightarrow (\Psi, P, R) \in Rel$

shows $\Psi \rhd P \rightsquigarrow <Rel> \tau.(Q)$
using ‹*eqvt Rel*›
proof(*induct rule*: *weakSimI*[**where** $C=Q$])
case(*cAct* $\Psi'\ \alpha\ Q'$)
hence *False* **by**(*cases rule*: *actionCases*[**where** $\alpha=\alpha$]) *auto*
thus *?case* **by** *simp*
next
case(*cTau* Q')
have $\Psi \rhd Q \Longrightarrow\hat{}_\tau Q$ **by** *simp*
moreover from ‹$\Psi \rhd \tau.(Q) \longmapsto \tau \prec Q'$› **have** $\Psi \rhd Q \sim Q'$ **by**(*rule tauActionE*)
with ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi, P, Q') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw3SimLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)$ *psi*
and $Q :: ('a, 'b, 'c)$ *psi*

and $\alpha :: 'a\ prefix$

assumes *eqvt Rel*
and $(\Psi, P, Q) \in Rel$
and *Subst*: $\bigwedge xvec\ Tvec.\ length\ xvec = length\ Tvec \Longrightarrow (\Psi, P[xvec::=Tvec], Q[xvec::=Tvec]) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi, P, S) \in Rel$
and *rExt*: $\bigwedge \Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in Rel \Longrightarrow (\Psi \otimes \Psi', P, Q) \in Rel$

shows $\Psi \rhd \alpha{\cdot}(\tau.(P)) \rightsquigarrow <Rel> \alpha{\cdot}Q$
using ‹*eqvt Rel*›
proof(*induct rule*: *weakSimI*[**where** $C=Q$])
case(*cAct* $\Psi'\ \beta\ Q'$)
from ‹$\Psi \rhd \alpha{\cdot}Q \longmapsto \beta \prec Q'$› ‹$\beta \neq \tau$› **show** *?case*
proof(*induct rule*: *prefixCases*)
case(*cInput M xvec N K Tvec*)
note ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec = length Tvec*›
moreover have *insertAssertion* (*extractFrame*($M(\!|\lambda{*}xvec\ N|\!).Q$)) $\Psi \hookrightarrow_F \langle \varepsilon, \Psi \otimes \mathbf{1}\rangle$ **by** *auto*
ultimately have $\Psi : (M(\!|\lambda{*}xvec\ N|\!).Q) \rhd (M(\!|\lambda{*}xvec\ N|\!).(\tau.(P))) \Longrightarrow K(\!|(N[xvec::=Tvec])|\!) \prec (\tau.(P))[xvec::=Tvec]$
by(*rule weakInput*)
hence $\Psi : (M(\!|\lambda{*}xvec\ N|\!).Q) \rhd (M(\!|\lambda{*}xvec\ N|\!).(\tau.(P))) \Longrightarrow K(\!|(N[xvec::=Tvec])|\!) \prec \tau.(P[xvec::=Tvec])$
using ‹*length xvec = length Tvec*› ‹*distinct xvec*›
by *simp*

moreover obtain P' **where** *PTrans*: $\Psi \otimes \Psi' \rhd \tau.(P[xvec::=Tvec]) \longmapsto \tau \prec P'$ **and** $\Psi \otimes \Psi' \rhd (P[xvec::=Tvec]) \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \otimes \Psi' \rhd \tau.(P[xvec::=Tvec]) \Longrightarrow\hat{}_\tau\ P'$ **by**(*rule tauActTauChain*)
moreover from ‹*length xvec = length Tvec*› **have** $(\Psi, P[xvec::=Tvec], Q[xvec::=Tvec]) \in Rel$ **by**(*rule Subst*)
hence $(\Psi \otimes \Psi', P[xvec::=Tvec], Q[xvec::=Tvec]) \in Rel$ **by**(*rule rExt*)
with ‹$\Psi \otimes \Psi' \rhd P[xvec::=Tvec] \sim P'$› **have** $(\Psi \otimes \Psi', P', Q[xvec::=Tvec]) \in Rel$ **by**(*blast intro*: *bisimE C1 bisimReflexive*)
ultimately show *?case* **by** *fastforce*
next
case(*cOutput M N K*)
note ‹$\Psi \vdash M \leftrightarrow K$›
moreover have *insertAssertion* (*extractFrame* ($M\langle N\rangle.Q$)) $\Psi \hookrightarrow_F \langle \varepsilon, \Psi \otimes \mathbf{1}\rangle$ **by** *auto*
ultimately have $\Psi : M\langle N\rangle.Q \rhd M\langle N\rangle.(\tau.(P)) \Longrightarrow K\langle N\rangle \prec \tau.(P)$ **by**(*rule weakOutput*)
moreover obtain P' **where** *PTrans*: $\Psi \otimes \Psi' \rhd \tau.(P) \longmapsto \tau \prec P'$ **and** $\Psi \otimes \Psi' \rhd P \sim P'$ **using** *tauActionI*

by *auto*
from *PTrans* **have** $\Psi \otimes \Psi' \rhd \tau.(P) \Longrightarrow\hat{}_{\tau}\ P'$ **by**(*rule tauActTauChain*)
moreover from ‹$(\Psi, P, Q) \in Rel$› **have** $(\Psi \otimes \Psi', P, Q) \in Rel$ **by**(*rule rExt*)
with ‹$\Psi \otimes \Psi' \rhd P \sim P'$› **have** $(\Psi \otimes \Psi', P', Q) \in Rel$ **by**(*blast intro*: *bisimE C1 bisimReflexive*)
ultimately show *?case* **by** *fastforce*
next
case *cTau*
with ‹$\tau \neq \tau$› **show** *?case*
by *simp*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd \alpha{\cdot}Q \longmapsto \tau \prec Q'$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *tPTrans*: $\Psi \rhd \tau.(\tau.(P)) \longmapsto \tau \prec P'$ **and** $\Psi \rhd \tau.(P) \sim P'$ **using** *tauActionI*
by *auto*
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *PTrans* ‹$\Psi \rhd \tau.(P) \sim P'$› **obtain** P''' **where** *P'Trans*: $\Psi \rhd P' \longmapsto \tau \prec P'''$ **and** $\Psi \rhd P'' \sim P'''$
apply(*drule-tac bisimE*(*4*))
apply(*drule-tac bisimE*(*2*))
apply(*drule-tac simE*, *auto*)
by(*metis bisimE*)
from *tPTrans P'Trans* **have** $\Psi \rhd \tau.(\tau.(P)) \Longrightarrow\hat{}_{\tau}\ P'''$ **by**(*fastforce dest*: *tauActTauChain*)
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹$\Psi \rhd P \sim P''$› ‹$\Psi \rhd P'' \sim P'''$› ‹$\Psi \rhd Q \sim Q'$› **have** $(\Psi, P''', Q') \in Rel$
by(*metis bisimTransitive C1 bisimSymmetric*)
ultimately show *?case* **by** *fastforce*
qed
qed

lemma *tauLaw3SimRight*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes *eqvt Rel*
and *Subst*: $\bigwedge \Psi\ xvec\ Tvec.\ length\ xvec = length\ Tvec \Longrightarrow (\Psi, P[xvec::=Tvec], \tau.(Q[xvec::=Tvec])) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi, P, S) \in Rel$
and $\bigwedge \Psi.\ (\Psi, P, \tau.(Q)) \in Rel$

shows $\Psi \rhd \alpha{\cdot}P \rightsquigarrow <Rel> \alpha{\cdot}(\tau.(Q))$
using ‹*eqvt Rel*›
proof(*induct rule*: *weakSimI*[**where** $C{=}Q$])
case(*cAct* Ψ' β Q')
from ‹$\Psi \rhd \alpha{\cdot}(\tau.(Q)) \longmapsto \beta \prec Q'$› ‹$\beta \neq \tau$›
show *?case*
proof(*induct rule*: *prefixCases*)
case(*cInput M xvec N K Tvec*)
note ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec=length Tvec*›
moreover have *insertAssertion* (*extractFrame* ($M(\!|\lambda{*}xvec\ N|\!).(\tau.(Q))$)) $\Psi \hookrightarrow_F \langle \varepsilon, \Psi \otimes \mathbf{1}\rangle$ **by** *auto*
ultimately have $\Psi : (M(\!|\lambda{*}xvec\ N|\!).(\tau.(Q))) \rhd M(\!|\lambda{*}xvec\ N|\!).P \Longrightarrow K(\!|(N[xvec::=Tvec])|\!) \prec P[xvec::=Tvec]$
by(*rule weakInput*)
moreover have $\Psi \otimes \Psi' \rhd P[xvec::=Tvec] \Longrightarrow\hat{}_\tau\ P[xvec::=Tvec]$ **by** *auto*
ultimately show *?case* **using** *Subst* ‹*length xvec=length Tvec*› ‹*distinct xvec*›
by *fastforce*
next
case(*cOutput M N K*)
note ‹$\Psi \vdash M \leftrightarrow K$›
moreover have *insertAssertion* (*extractFrame* ($M\langle N\rangle.(\tau.(Q))$)) $\Psi \hookrightarrow_F \langle \varepsilon, \Psi \otimes \mathbf{1}\rangle$ **by** *auto*
ultimately have $\Psi : M\langle N\rangle.(\tau.(Q)) \rhd M\langle N\rangle.P \Longrightarrow K\langle N\rangle \prec P$ **by**(*rule weakOutput*)
moreover have $\Psi \otimes \Psi' \rhd P \Longrightarrow\hat{}_\tau\ P$ **by** *auto*
ultimately show *?case* **using** ‹$(\Psi \otimes \Psi', P, \tau.(Q)) \in Rel$› **by** *fastforce*
next
case *cTau*
with ‹$\tau \neq \tau$› **show** *?case* **by** *simp*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd \alpha{\cdot}(\tau.(Q)) \longmapsto \tau \prec Q'$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P'$ **and** $\Psi \rhd P \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ P'$ **by**(*rule tauActTauChain*)
moreover from ‹$\Psi \rhd P \sim P'$› ‹$\Psi \rhd \tau.(Q) \sim Q'$› ‹$(\Psi, P, \tau.(Q)) \in Rel$›
have $(\Psi, P', Q') \in Rel$ **by**(*metis bisimTransitive bisimSymmetric C1*)
ultimately show *?case* **by** *fastforce*
qed
qed

lemma *tauLaw3CongSimLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

and Q :: $('a, 'b, 'c)$ *psi*

assumes $(\Psi, P, Q) \in Rel$
and *C1*: $\bigwedge\Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi, P, S) \in Rel$
and *rExt*: $\bigwedge\Psi\ P\ Q\ \Psi'.\ (\Psi, P, Q) \in Rel \Longrightarrow (\Psi \otimes \Psi', P, Q) \in Rel$

shows $\Psi \rhd \alpha{\cdot}(\tau.(P)) \leadsto«Rel»\ \alpha{\cdot}Q$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from ‹$\Psi \rhd \alpha{\cdot}Q \longmapsto\tau \prec Q'$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *tPTrans*: $\Psi \rhd \tau.(\tau.(P)) \longmapsto\tau \prec P'$ **and** $\Psi \rhd \tau.(P) \sim P'$ **using** *tauActionI*
by *auto*
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto\tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *PTrans* ‹$\Psi \rhd \tau.(P) \sim P'$› **obtain** P''' **where** *P'Trans*: $\Psi \rhd P' \longmapsto\tau \prec P'''$ **and** $\Psi \rhd P'' \sim P'''$
apply(*drule-tac bisimE(4)*)
apply(*drule-tac bisimE(2)*)
apply(*drule-tac simE*, *auto*)
by(*metis bisimE*)
from *tPTrans P'Trans* **have** $\Psi \rhd \tau.(\tau.(P)) \Longrightarrow_\tau P'''$ **by**(*fastforce dest*: *tauActTauStepChain*)
moreover from ‹$(\Psi, P, Q) \in Rel$› ‹$\Psi \rhd P \sim P''$› ‹$\Psi \rhd P'' \sim P'''$› ‹$\Psi \rhd Q \sim Q'$› **have** $(\Psi, P''', Q') \in Rel$
by(*metis bisimTransitive C1 bisimSymmetric*)
ultimately show *?case* **by** *fastforce*
qed
qed

lemma *tauLaw3CongSimRight*:
fixes Ψ :: $'b$
and P :: $('a, 'b, 'c)$ *psi*
and Q :: $('a, 'b, 'c)$ *psi*

assumes $(\Psi, P, Q) \in Rel$
and *C1*: $\bigwedge\Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi, P, S) \in Rel$
and $\bigwedge\Psi.\ (\Psi, P, \tau.(Q)) \in Rel$

shows $\Psi \rhd \alpha{\cdot}P \leadsto«Rel»\ \alpha{\cdot}(\tau.(Q))$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from ‹$\Psi \rhd \alpha{\cdot}(\tau.(Q)) \longmapsto\tau \prec Q'$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)

case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto\tau \prec P'$ **and** $\Psi \rhd P \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd \tau.(P) \Longrightarrow_\tau P'$ **by**(*rule tauActTauStepChain*)
moreover from $\langle \Psi \rhd P \sim P' \rangle$ $\langle \Psi \rhd \tau.(Q) \sim Q' \rangle$ $\langle (\Psi, P, \tau.(Q)) \in Rel \rangle$
have $(\Psi, P', Q') \in Rel$ **by**(*metis bisimTransitive bisimSymmetric C1*)
ultimately show *?case* **by** *fastforce*
qed
qed

end

locale *tauSum* = *tau* + *sum*
begin

lemma *tauLaw2SimLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *Id*: $\bigwedge \Psi\ P.\ (\Psi, P, P) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi, P, S) \in Rel$

shows $\Psi \rhd P \oplus \tau.(P) \rightsquigarrow <Rel> \tau.(P)$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \alpha\ P'$)
thus *?case* **by**(*nominal-induct* α *rule*: *action.strong-inducts*) *auto*
next
case(*cTau* P')
from $\langle \Psi \rhd \tau.(P) \longmapsto\tau \prec P' \rangle$ **have** $\Psi \rhd P \sim P'$ **by**(*rule tauActionE*)
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto\tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
have *guarded*($\tau.(P)$) **by**(*rule guardedTau*)
with *PTrans* **have** $\Psi \rhd P \oplus \tau.(P) \longmapsto\tau \prec P''$ **by**(*rule Sum2*)
hence $\Psi \rhd P \oplus \tau.(P) \Longrightarrow\hat{}_\tau P''$ **by**(*rule tauActTauChain*)
moreover from $\langle \Psi \rhd P \sim P'' \rangle$ $\langle (\Psi, P, P) \in Rel \rangle$ $\langle \Psi \rhd P \sim P' \rangle$ **have** $(\Psi, P'', P') \in Rel$
by(*metis C1 bisimE*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw2CongSimLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *Id*: $\bigwedge \Psi\ P.\ (\Psi, P, P) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in Rel; \Psi \rhd R \sim S]\!] \Longrightarrow (\Psi,$

P, S) $\in$ *Rel*

shows $\Psi \rhd P \oplus \tau.(P) \rightsquigarrow$«*Rel*» $\tau.(P)$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* P')
from ‹$\Psi \rhd \tau.(P) \longmapsto \tau \prec P'$› **have** $\Psi \rhd P \sim P'$ **by**(*rule tauActionE*)
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
have *guarded*($\tau.(P)$) **by**(*rule guardedTau*)
with *PTrans* **have** $\Psi \rhd P \oplus \tau.(P) \longmapsto \tau \prec P''$ **by**(*rule Sum2*)
hence $\Psi \rhd P \oplus \tau.(P) \Longrightarrow_\tau P''$ **by**(*rule tauActTauStepChain*)
moreover from ‹$\Psi \rhd P \sim P''$› ‹(Ψ, P, P) $\in$ *Rel*› ‹$\Psi \rhd P \sim P'$› **have** (Ψ, P'', P') $\in$ *Rel*
by(*metis C1 bisimE*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw2SimRight*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *C1*: $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi, P, Q) \in Rel$

shows $\Psi \rhd \tau.(P) \rightsquigarrow$<*Rel*> $P \oplus \tau.(P)$
proof(*induct rule*: *weakSimI2*)
case(*cAct* Ψ' α P')
from ‹$\Psi \rhd P \oplus \tau.(P) \longmapsto \alpha \prec P'$›
show *?case*
proof(*induct rule*: *sumCases*)
case *cSum1*
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau P''$ **by**(*rule tauActTauChain*)
moreover from ‹*guarded P*› **have** *insertAssertion* (*extractFrame P*) $\Psi \simeq_F \langle \varepsilon, \Psi \otimes \mathbf{1} \rangle$
by(*rule insertGuardedAssertion*)
with ‹$\Psi \rhd P \sim P''$› **have** *insertAssertion* (*extractFrame P''*) $\Psi \simeq_F \langle \varepsilon, \Psi \otimes \mathbf{1} \rangle$
by(*metis bisimE FrameStatEqTrans FrameStatEqSym*)
hence *insertAssertion* (*extractFrame*($P \oplus \tau.(P)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame P''*) Ψ
by(*simp add*: *FrameStatEq-def*)
moreover from *PTrans* ‹*bn* α $\sharp*$ ($\tau.(P)$)› **have** *bn* α $\sharp*$ P'' **by**(*rule tauFreshChainDerivative*)
with ‹$\Psi \rhd P \sim P''$› ‹$\Psi \rhd P \longmapsto \alpha \prec P'$› ‹*bn* α $\sharp*$ Ψ›
obtain P''' **where** P''*Trans*: $\Psi \rhd P'' \longmapsto \alpha \prec P'''$ **and** $\Psi \rhd P''' \sim P'$
by(*metis bisimE simE*)

```
    ultimately have Ψ : (P ⊕ τ.(P)) ▷ τ.(P) ⟹α ≺ P'''
      by(rule-tac weakTransitionI)
    moreover have Ψ ⊗ Ψ' ▷ P''' ⟹^τ P''' by auto
    moreover from ‹Ψ ▷ P''' ∼ P'› have Ψ ⊗ Ψ' ▷ P''' ∼ P' by(rule bisimE)
    hence (Ψ ⊗ Ψ', P''', P') ∈ Rel by(rule C1)
    ultimately show ?case by blast
  next
    case cSum2
    thus ?case using ‹α ≠ τ›
      by(nominal-induct α rule: action.strong-inducts) auto
  qed
next
  case(cTau P')
  from ‹Ψ ▷ P ⊕ τ.(P) ⟼τ ≺ P'›
  show ?case
  proof(induct rule: sumCases)
    case cSum1
    obtain P'' where PTrans: Ψ ▷ τ.(P) ⟼τ ≺ P'' and Ψ ▷ P ∼ P'' using
tauActionI
      by auto
    moreover from ‹Ψ ▷ P ∼ P''› ‹Ψ ▷ P ⟼τ ≺ P'›
    obtain P''' where P''Trans: Ψ ▷ P'' ⟼τ ≺ P''' and Ψ ▷ P''' ∼ P'
      apply(drule-tac bisimE(4))
      apply(drule-tac bisimE(2))
      by(drule-tac simE, auto)
   ultimately have Ψ ▷ τ.(P) ⟹^τ P''' by(auto dest: tauActTauChain rtrancl-into-rtrancl)
    moreover from ‹Ψ ▷ P''' ∼ P'› have (Ψ, P''', P') ∈ Rel by(rule C1)
    ultimately show ?case by blast
  next
    case cSum2
    from ‹Ψ ▷ τ.(P) ⟼τ ≺ P'› have Ψ ▷ P ∼ P' by(rule tauActionE)
    obtain P'' where PTrans: Ψ ▷ τ.(P) ⟼τ ≺ P'' and Ψ ▷ P ∼ P'' using
tauActionI
      by auto
    hence Ψ ▷ τ.(P) ⟹^τ P'' by(rule-tac tauActTauChain)
    moreover from ‹Ψ ▷ P ∼ P''› ‹Ψ ▷ P ∼ P'› have (Ψ, P'', P') ∈ Rel
      by(metis C1 bisimTransitive bisimE)
    ultimately show ?case by blast
  qed
qed

lemma tauLaw2CongSimRight:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi

  assumes C1: ⋀Ψ P Q. Ψ ▷ P ∼ Q ⟹ (Ψ, P, Q) ∈ Rel

  shows Ψ ▷ τ.(P) ⇝«Rel» P ⊕ τ.(P)
proof(induct rule: weakCongSimI)
```

case(*cTau* P')
from ‹$\Psi \rhd P \oplus \tau.(P) \longmapsto \tau \prec P'$›
show *?case*
proof(*induct rule*: *sumCases*)
case *cSum1*
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
moreover from ‹$\Psi \rhd P \sim P''$› ‹$\Psi \rhd P \longmapsto \tau \prec P'$›
obtain P''' **where** P''*Trans*: $\Psi \rhd P'' \longmapsto \tau \prec P'''$ **and** $\Psi \rhd P''' \sim P'$
apply(*drule-tac bisimE*(*4*))
apply(*drule-tac bisimE*(*2*))
by(*drule-tac simE*) *auto*
ultimately have $\Psi \rhd \tau.(P) \Longrightarrow_\tau P'''$ **by**(*auto dest*: *tauActTauStepChain trancl-into-trancl*)
moreover from ‹$\Psi \rhd P''' \sim P'$› **have** $(\Psi, P''', P') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
next
case *cSum2*
from ‹$\Psi \rhd \tau.(P) \longmapsto \tau \prec P'$› **have** $\Psi \rhd P \sim P'$ **by**(*rule tauActionE*)
obtain P'' **where** *PTrans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
hence $\Psi \rhd \tau.(P) \Longrightarrow_\tau P''$ **by**(*rule-tac tauActTauStepChain*)
moreover from ‹$\Psi \rhd P \sim P''$› ‹$\Psi \rhd P \sim P'$› **have** $(\Psi, P'', P') \in Rel$
by(*metis C1 bisimTransitive bisimE*)
ultimately show *?case* **by** *blast*
qed
qed

lemma *tauLaw4SimLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $N :: 'a$

assumes $\bigwedge \Psi\ P.\ (\Psi, P, P) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi, P, Q) \in Rel$

shows $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow <Rel> \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \beta\ PQ$)
from ‹$\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \longmapsto \beta \prec PQ$› ‹$\beta \neq \tau$›
show *?case*
proof(*induct rule*: *prefixCases*)
case(*cInput M xvec N K Tvec*)
have $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\!\hat{}_\tau\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\alpha{\cdot}(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *inser-*

tAssertion (*extractFrame*($\alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$)) Ψ
using *insertTauAssertion Identity*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*, *auto*)
(*rule FrameStatImpTrans*[**where** $G{=}\langle\varepsilon, \Psi\rangle$], *auto simp add*: *FrameStatEq-def AssertionStatEq-def*)
moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec*=*length Tvec*›
have $\Psi \rhd M(\!|\lambda{*}xvec\ N|\!).(\tau.(P) \oplus Q) \longmapsto K(\!|(N[xvec::=Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec::=Tvec]$
by(*rule Input*)
hence $\Psi \rhd (M(\!|\lambda{*}xvec\ N|\!).P) \oplus M(\!|\lambda{*}xvec\ N|\!).(\tau.(P) \oplus Q) \longmapsto K(\!|(N[xvec::=Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec::=Tvec]$
by(*rule-tac Sum2*) *auto*
ultimately have $\Psi : (M(\!|\lambda{*}xvec\ N|\!).(\tau.(P) \oplus Q)) \rhd (M(\!|\lambda{*}xvec\ N|\!).P) \oplus M(\!|\lambda{*}xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow K(\!|(N[xvec::=Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec::=Tvec]$
by(*rule-tac weakTransitionI*) *auto*
moreover have $\Psi \otimes \Psi' \rhd (\tau.(P) \oplus Q)[xvec::=Tvec] \Longrightarrow\hat{}_\tau\ (\tau.(P) \oplus Q)[xvec::=Tvec]$ **by** *auto*
ultimately show *?case* **using** ‹$(\Psi \otimes \Psi', (\tau.(P) \oplus Q)[xvec::=Tvec], (\tau.(P) \oplus Q)[xvec::=Tvec]) \in Rel$› **by** *fastforce*
next
case(*cOutput M N K*)
have $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\hat{}_\tau\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\alpha{\cdot}(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$)) Ψ
using *insertTauAssertion Identity*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*, *auto*)
(*rule FrameStatImpTrans*[**where** $G{=}\langle\varepsilon, \Psi\rangle$], *auto simp add*: *FrameStatEq-def AssertionStatEq-def*)

moreover from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \longmapsto K\langle N\rangle \prec (\tau.(P) \oplus Q)$
by(*rule Output*)
hence $\Psi \rhd M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q) \longmapsto K\langle N\rangle \prec (\tau.(P) \oplus Q)$ **by**(*rule-tac Sum2*) *auto*
ultimately have $\Psi : (M\langle N\rangle.(\tau.(P) \oplus Q)) \rhd M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q) \Longrightarrow K\langle N\rangle \prec (\tau.(P) \oplus Q)$
by(*rule-tac weakTransitionI*) *auto*
moreover have $\Psi \otimes \Psi' \rhd \tau.(P) \oplus Q \Longrightarrow\hat{}_\tau\ \tau.(P) \oplus Q$ **by** *auto*
ultimately show *?case* **using** ‹$(\Psi \otimes \Psi', \tau.(P) \oplus Q, \tau.(P) \oplus Q) \in Rel$› **by** *fastforce*
next
case *cTau*
from ‹$\tau \neq \tau$› **show** *?case*
by *simp*
qed
next
case(*cTau Q'*)
from ‹$\Psi \rhd \alpha{\cdot}(\tau.(P))\ \oplus_\top\ Q \longmapsto \tau \prec Q'$› **show** *?case*

proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.(\tau.(P) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd \tau.(P) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd (\tau.(P)) \oplus \tau.(\tau.(P) \oplus Q) \longmapsto \tau \prec P'$ **by**(*rule-tac Sum2*) (*auto intro*: *guardedTau*)
hence $\Psi \rhd (\tau.(P)) \oplus \tau.(\tau.(P) \oplus Q) \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauActTauChain*)
moreover from $\langle \Psi \rhd \tau.(P) \oplus Q \sim P' \rangle$ $\langle \Psi \rhd (\tau.(P)) \oplus Q \sim Q' \rangle$ **have** $\Psi \rhd P' \sim Q'$ **by**(*metis bisimSymmetric bisimTransitive*)
hence $(\Psi, P', Q') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
qed

lemma *tauLaw4CongSimLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $M :: 'a$
and $N :: 'a$

assumes $\bigwedge \Psi\ P.\ (\Psi, P, P) \in Rel$
and *C1*: $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi, P, Q) \in Rel$

shows $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow\langle\!\langle Rel \rangle\!\rangle\ \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakCongSimI*)
case(*cTau* Q')
from $\langle \Psi \rhd \alpha{\cdot}(\tau.(P)) \oplus_\top Q \longmapsto \tau \prec Q' \rangle$ **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.(\tau.(P) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd \tau.(P) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd (\tau.(P)) \oplus \tau.(\tau.(P) \oplus Q) \longmapsto \tau \prec P'$ **by**(*rule-tac Sum2*) (*auto intro*: *guardedTau*)
hence $\Psi \rhd (\tau.(P)) \oplus \tau.(\tau.(P) \oplus Q) \Longrightarrow_\tau P'$ **by**(*rule tauActTauStepChain*)
moreover from $\langle \Psi \rhd \tau.(P) \oplus Q \sim P' \rangle$ $\langle \Psi \rhd (\tau.(P)) \oplus Q \sim Q' \rangle$ **have** $\Psi \rhd P' \sim Q'$ **by**(*metis bisimSymmetric bisimTransitive*)
hence $(\Psi, P', Q') \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
qed

lemma *tauLaw4SimRight*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$

assumes *C1*: $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi, P, Q) \in Rel$
and $\bigwedge \Psi\ P.\ (\Psi, P, P) \in Rel$

shows $\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow <Rel> \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakSimI2*)
case(*cAct* $\Psi'\ \beta\ PQ$)
from ‹$\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \longmapsto \beta \prec PQ$› **show** *?case*
proof(*induct rule*: *sumCases*)
case *cSum1*
from ‹$\Psi \rhd \alpha{\cdot}P \longmapsto \beta \prec PQ$› ‹$\beta \neq \tau$› **show** *?case*
proof(*induct rule*: *prefixCases*)
case(*cInput M xvec N K Tvec*)
have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow\!\!\hat{}_{\tau}\ M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*((*M*$(\!|\lambda * xvec\ N|\!).P) \oplus M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)$)) Ψ
by *auto*
moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec* = *length Tvec*›
have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \longmapsto K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec ::= Tvec]$ **by**(*rule Input*)
ultimately have $\Psi : ((M(\!|\lambda * xvec\ N|\!).P) \oplus M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)) \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec ::= Tvec]$
by(*rule-tac weakTransitionI*) *auto*
with ‹*length xvec* = *length Tvec*› ‹*distinct xvec*›
have $\Psi : ((M(\!|\lambda * xvec\ N|\!).P) \oplus M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)) \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P[xvec ::= Tvec]) \oplus Q[xvec ::= Tvec])$
by *auto*
moreover obtain P' **where** *PTrans*: $\Psi \otimes \Psi' \rhd \tau.(P[xvec ::= Tvec]) \longmapsto \tau \prec P'$ **and** $\Psi \otimes \Psi' \rhd P[xvec ::= Tvec] \sim P'$ **using** *tauActionI*
by *auto*
have *guarded*($\tau.(P[xvec ::= Tvec])$) **by**(*rule guardedTau*)
with *PTrans* **have** $\Psi \otimes \Psi' \rhd (\tau.(P[xvec ::= Tvec])) \oplus (Q[xvec ::= Tvec]) \longmapsto \tau \prec P'$ **by**(*rule Sum1*)
hence $\Psi \otimes \Psi' \rhd (\tau.(P[xvec ::= Tvec])) \oplus (Q[xvec ::= Tvec]) \Longrightarrow\!\!\hat{}_{\tau}\ P'$ **by**(*rule tauActTauChain*)
moreover from ‹$\Psi \otimes \Psi' \rhd P[xvec ::= Tvec] \sim P'$› **have** ($\Psi \otimes \Psi'$, P', $P[xvec ::= Tvec]$) $\in$ *Rel* **by**(*metis bisimE C1*)
ultimately show *?case* **by** *fastforce*
next
case(*cOutput M N K*)
have $\Psi \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \Longrightarrow\!\!\hat{}_{\tau}\ M\langle N\rangle.(\tau.(P) \oplus Q)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M\langle N\rangle.(\tau.(P) \oplus Q)$)) Ψ
by *auto*
moreover from ‹$\Psi \vdash M \leftrightarrow K$› **have** $\Psi \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \longmapsto K\langle N\rangle \prec (\tau.(P) \oplus Q)$ **by**(*rule Output*)

ultimately have $\Psi : (M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q)) \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \Longrightarrow K\langle N\rangle \prec (\tau.(P) \oplus Q)$

by(*rule-tac weakTransitionI*) *auto*

moreover obtain P' **where** *PTrans*: $\Psi \otimes \Psi' \rhd \tau.(P) \longmapsto \tau \prec P'$ **and** $\Psi \otimes \Psi' \rhd P \sim P'$ **using** *tauActionI*

by *auto*

have *guarded*$(\tau.(P))$ **by**(*rule guardedTau*)

with *PTrans* **have** $\Psi \otimes \Psi' \rhd (\tau.(P)) \oplus Q \longmapsto \tau \prec P'$ **by**(*rule Sum1*)

hence $\Psi \otimes \Psi' \rhd (\tau.(P)) \oplus Q \Longrightarrow\hat{}_\tau P'$ **by**(*rule tauActTauChain*)

moreover from ‹$\Psi \otimes \Psi' \rhd P \sim P'$› **have** $(\Psi \otimes \Psi', P', P) \in Rel$ **by**(*metis bisimE C1*)

ultimately show *?case* **by** *fastforce*

next

case *cTau*

from ‹$\tau \neq \tau$› **show** *?case* **by** *simp*

qed

next

case *cSum2*

from ‹$\Psi \rhd \alpha.(\tau.(P) \oplus Q) \longmapsto \beta \prec PQ$› ‹$\beta \neq \tau$› **show** *?case*

proof(*induct rule*: *prefixCases*)

case(*cInput M xvec N K Tvec*)

have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow\hat{}_\tau M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)$ **by** *auto*

moreover have *insertAssertion* (*extractFrame*$((M(\!|\lambda * xvec\ N|\!).P) \oplus M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)))\ \Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*$(M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)))\ \Psi$

by *auto*

moreover from ‹$\Psi \vdash M \leftrightarrow K$› ‹*distinct xvec*› ‹*set xvec* $\subseteq$ *supp N*› ‹*length xvec* = *length Tvec*›

have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \longmapsto K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P) \oplus Q)[xvec ::= Tvec]$

by(*rule Input*)

with ‹*length xvec* = *length Tvec*› ‹*distinct xvec*›

have $\Psi \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \longmapsto K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P[xvec ::= Tvec]) \oplus Q[xvec ::= Tvec])$

by *simp*

ultimately have $\Psi : ((M(\!|\lambda * xvec\ N|\!).P) \oplus M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q)) \rhd M(\!|\lambda * xvec\ N|\!).(\tau.(P) \oplus Q) \Longrightarrow K(\!|(N[xvec ::= Tvec])|\!) \prec (\tau.(P[xvec ::= Tvec]) \oplus Q[xvec ::= Tvec])$

by(*rule-tac weakTransitionI*) *auto*

moreover have $\Psi \otimes \Psi' \rhd \tau.(P[xvec ::= Tvec]) \oplus (Q[xvec ::= Tvec]) \Longrightarrow\hat{}_\tau \tau.(P[xvec ::= Tvec]) \oplus (Q[xvec ::= Tvec])$ **by** *auto*

ultimately show *?case* **using** ‹$(\Psi \otimes \Psi', \tau.(P[xvec ::= Tvec]) \oplus (Q[xvec ::= Tvec]), \tau.(P[xvec ::= Tvec]) \oplus (Q[xvec ::= Tvec])) \in Rel$› ‹*length xvec* = *length Tvec*› ‹*distinct xvec*›

by *fastforce*

next

case(*cOutput M N K*)

have $\Psi \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \Longrightarrow\hat{}_\tau M\langle N\rangle.(\tau.(P) \oplus Q)$ **by** *auto*

moreover have *insertAssertion* (*extractFrame*($M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($M\langle N\rangle.(\tau.(P) \oplus Q)$)) Ψ
by *auto*
moreover from $\langle \Psi \vdash M \leftrightarrow K \rangle$ **have** $\Psi \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \longmapsto K\langle N\rangle \prec (\tau.(P) \oplus Q)$
by(*rule Output*)
ultimately have $\Psi : (M\langle N\rangle.P \oplus M\langle N\rangle.(\tau.(P) \oplus Q)) \rhd M\langle N\rangle.(\tau.(P) \oplus Q) \Longrightarrow K\langle N\rangle \prec (\tau.(P) \oplus Q)$
by(*rule-tac weakTransitionI*) *auto*
moreover have $\Psi \otimes \Psi' \rhd \tau.(P) \oplus Q \Longrightarrow\hat{}_\tau\ \tau.(P) \oplus Q$ **by** *auto*
ultimately show *?case* **using** $\langle(\Psi \otimes \Psi', \tau.(P) \oplus Q, \tau.(P) \oplus Q) \in Rel\rangle$ **by** *fastforce*
next
case *cTau*
from $\langle \tau \neq \tau \rangle$ **show** *?case* **by** *simp*
qed
qed
next
case(*cTau PQ*)
from $\langle \Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \longmapsto \tau \prec PQ\rangle$ **show** *?case*
proof(*induct rule*: *sumCases*)
case *cSum1*
from $\langle \Psi \rhd \alpha{\cdot}P \longmapsto \tau \prec PQ\rangle$ **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd (\tau.(P)) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
obtain P'' **where** *P'Trans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *P'Trans* **have** $\Psi \rhd \tau.(P) \oplus Q \longmapsto \tau \prec P''$ **by**(*rule-tac Sum1*) (*auto intro*: *guardedTau*)
with $\langle \Psi \rhd (\tau.(P)) \oplus Q \sim P' \rangle$ **obtain** P''' **where** *P''Trans*: $\Psi \rhd P' \longmapsto \tau \prec P'''$ **and** $\Psi \rhd P'' \sim P'''$
apply(*drule-tac bisimE*(*4*))
apply(*drule-tac bisimE*(*2*))
apply(*drule-tac simE*)
by(*auto dest*: *bisimE*)
from *PTrans P''Trans* **have** $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \Longrightarrow\hat{}_\tau\ P'''$ **by**(*fastforce dest*: *tauActTauChain*)
moreover from $\langle \Psi \rhd P \sim PQ\rangle$ $\langle \Psi \rhd P'' \sim P'''\rangle$ $\langle \Psi \rhd P \sim P''\rangle$ **have** $\Psi \rhd P''' \sim PQ$
by(*metis bisimSymmetric bisimTransitive*)
hence (Ψ, P''', PQ) $\in$ *Rel* **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
next
case *cSum2*

from ‹$\Psi \rhd \alpha{\cdot}((\tau.(P)) \oplus Q) \longmapsto \tau \prec PQ$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd (\tau.(P)) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \Longrightarrow\hat{}_{\tau} P'$ **by**(*rule tauActTauChain*)
moreover from ‹$\Psi \rhd (\tau.(P)) \oplus Q \sim P'$› ‹$\Psi \rhd \tau.(P) \oplus Q \sim PQ$› **have** $\Psi \rhd P' \sim PQ$
by(*metis bisimSymmetric bisimTransitive*)
hence (Ψ, P', PQ) $\in$ *Rel* **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
qed
qed

lemma *tauLaw4CongSimRight*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and α :: *'a prefix*

assumes *C1*: $\bigwedge \Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi, P, Q) \in Rel$

shows $\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow\langle\langle Rel \rangle\rangle\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakCongSimI*)
case(*cTau PQ*)
from ‹$\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \longmapsto \tau \prec PQ$› **show** *?case*
proof(*induct rule*: *sumCases*)
case *cSum1*
from ‹$\Psi \rhd \alpha{\cdot}P \longmapsto \tau \prec PQ$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd (\tau.(P)) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
obtain P'' **where** *P'Trans*: $\Psi \rhd \tau.(P) \longmapsto \tau \prec P''$ **and** $\Psi \rhd P \sim P''$ **using** *tauActionI*
by *auto*
from *P'Trans* **have** $\Psi \rhd \tau.(P) \oplus Q \longmapsto \tau \prec P''$ **by**(*rule-tac Sum1*) (*auto intro*: *guardedTau*)
with ‹$\Psi \rhd (\tau.(P)) \oplus Q \sim P'$› **obtain** P''' **where** *P''Trans*: $\Psi \rhd P' \longmapsto \tau \prec P'''$ **and** $\Psi \rhd P'' \sim P'''$
apply(*drule-tac bisimE*(*4*))
apply(*drule-tac bisimE*(*2*))
apply(*drule-tac simE*)
by(*auto dest*: *bisimE*)
from *PTrans P''Trans* **have** $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \Longrightarrow_{\tau} P'''$ **by**(*fastforce dest*: *tauActTauStepChain*)

moreover from ‹$\Psi \rhd P \sim PQ$› ‹$\Psi \rhd P'' \sim P'''$› ‹$\Psi \rhd P \sim P''$› **have** $\Psi \rhd P''' \sim PQ$
by(*metis bisimSymmetric bisimTransitive*)
hence (Ψ, P''', PQ) $\in$ *Rel* **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
next
case *cSum2*
from ‹$\Psi \rhd \alpha\cdot((\tau.(P)) \oplus Q) \longmapsto \tau \prec PQ$› **show** *?case*
proof(*induct rule*: *prefixTauCases*)
case *cTau*
obtain P' **where** *PTrans*: $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \longmapsto \tau \prec P'$ **and** $\Psi \rhd (\tau.(P)) \oplus Q \sim P'$ **using** *tauActionI*
by *auto*
from *PTrans* **have** $\Psi \rhd \tau.((\tau.(P)) \oplus Q) \Longrightarrow_\tau P'$ **by**(*rule tauActTauStepChain*)
moreover from ‹$\Psi \rhd (\tau.(P)) \oplus Q \sim P'$› ‹$\Psi \rhd \tau.(P) \oplus Q \sim PQ$› **have** $\Psi \rhd P' \sim PQ$
by(*metis bisimSymmetric bisimTransitive*)
hence (Ψ, P', PQ) $\in$ *Rel* **by**(*rule C1*)
ultimately show *?case* **by** *fastforce*
qed
qed
qed

end

end

theory *Tau-Stat-Imp*
imports *Tau-Sim Weaken-Stat-Imp*
begin

locale *weakTauLaws* = *weak* + *tau*
begin

lemma *tauLaw1StatImpLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*

assumes $\Psi \rhd P \lesssim_w <Rel> Q$
and (Ψ, $\tau.(P)$, Q) $\in$ *Rel*

shows $\Psi \rhd \tau.(P) \lesssim_w <Rel> Q$
proof −
have $\Psi \rhd Q \Longrightarrow\hat{}_\tau Q$ **by** *simp*
moreover have *insertAssertion* (*extractFrame*($\tau.(P)$)) $\Psi \simeq_F \langle \varepsilon, \Psi \rangle$ **by**(*rule insertTauAssertion*)
hence *insertAssertion* (*extractFrame*($\tau.(P)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*

Q) Ψ
  by(*metis FrameStatImpTrans FrameStatEq-def insertAssertionWeaken*)
 ultimately show *?thesis* **using** ‹(Ψ, τ.(P), Q) ∈ *Rel*›
  by(*rule weakenStatImpI*)
qed

lemma *tauLaw1StatImpRight*:
 fixes Ψ :: *'b*
 and P :: (*'a*, *'b*, *'c*) *psi*
 and Q :: (*'a*, *'b*, *'c*) *psi*

 assumes $\Psi \rhd P \lessapprox <Rel> Q$
 and *C1*: $\bigwedge \Psi\ P\ Q\ R.\ [\![(\Psi, P, Q) \in Rel; \Psi \rhd Q \sim R]\!] \Longrightarrow (\Psi, P, R) \in Rel'$

 shows $\Psi \rhd P \lessapprox <Rel'> \tau.(Q)$
proof(*induct rule*: *weakStatImpI*)
 case(*cStatImp* Ψ')
 from ‹$\Psi \rhd P \lessapprox <Rel> Q$› **obtain** Q' Q''
  where *QChain*: $\Psi \rhd Q \Longrightarrow\hat{}_\tau\ Q'$ **and** *PImpQ'*: *insertAssertion* (*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q') Ψ
   and *Q'Chain*: $\Psi \otimes \Psi' \rhd Q' \Longrightarrow\hat{}_\tau\ Q''$ **and** *PRelQ''*: ($\Psi \otimes \Psi'$, P, Q'') ∈ *Rel*
  by(*rule weakStatImpE*)

 obtain Q''' **where** *QTrans*: $\Psi \rhd \tau.(Q) \longmapsto \tau \prec Q'''$ **and** $\Psi \rhd Q \sim Q'''$ **using** *tauActionI* **by** *auto*

 from ‹$\Psi \rhd Q \sim Q'''$› *QChain bisimE*(*2*) **obtain** Q'''' **where** *Q'''Chain*: $\Psi \rhd Q''' \Longrightarrow\hat{}_\tau\ Q''''$ **and** $\Psi \rhd Q' \sim Q''''$
  by(*metis bisimE*(*4*) *simTauChain*)

 from *QTrans Q'''Chain* **have** $\Psi \rhd \tau.(Q) \Longrightarrow\hat{}_\tau\ Q''''$ **by**(*drule-tac tauActTauChain*) *auto*
 moreover from ‹$\Psi \rhd Q' \sim Q''''$› **have** *insertAssertion* (*extractFrame* Q') Ψ $\hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'''') Ψ
  by(*metis bisimE FrameStatEq-def*)
 with *PImpQ'* **have** *insertAssertion* (*extractFrame* P) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame* Q'''') Ψ
  by(*rule FrameStatImpTrans*)
 moreover from ‹$\Psi \rhd Q' \sim Q''''$› **have** $\Psi \otimes \Psi' \rhd Q' \sim Q''''$ **by**(*rule bisimE*)
 then obtain Q''''' **where** *Q''''Chain*: $\Psi \otimes \Psi' \rhd Q'''' \Longrightarrow\hat{}_\tau\ Q'''''$ **and** $\Psi \otimes \Psi' \rhd Q'' \sim Q'''''$ **using** *Q'Chain bisimE*(*2*)
  by(*metis bisimE*(*4*) *simTauChain*)
 note *Q''''Chain*
 moreover from ‹($\Psi \otimes \Psi'$, P, Q'') ∈ *Rel*› ‹$\Psi \otimes \Psi' \rhd Q'' \sim Q'''''$› **have** ($\Psi \otimes \Psi'$, P, Q''''') ∈ *Rel'*
  by(*rule C1*)
 ultimately show *?case* **by** *blast*
qed

end

context *tau* **begin**

lemma *tauLaw3StatImpLeft*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$

assumes *C1*: $\bigwedge \Psi'.\ (\Psi \otimes \Psi', \alpha \cdot (\tau.(P)), \alpha \cdot Q) \in Rel$

shows $\Psi \rhd \alpha \cdot (\tau.(P)) \lessapprox_{<Rel>} \alpha \cdot Q$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd \alpha \cdot Q \Longrightarrow\!\!\hat{}_{\tau}\ \alpha \cdot Q$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\alpha \cdot (\tau.(P))$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\alpha \cdot Q$)) Ψ **using** *insertTauAssertion*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*) (*auto simp add*: *FrameStatEq-def intro*: *FrameStatImpTrans*)
moreover have $\Psi \otimes \Psi' \rhd \alpha \cdot Q \Longrightarrow\!\!\hat{}_{\tau}\ \alpha \cdot Q$ **by** *auto*
moreover have $(\Psi \otimes \Psi', \alpha \cdot (\tau.(P)), \alpha \cdot Q) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw3StatImpRight*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$

assumes *C1*: $\bigwedge \Psi'.\ (\Psi \otimes \Psi', \alpha \cdot P, \alpha \cdot (\tau.(Q))) \in Rel$

shows $\Psi \rhd \alpha \cdot P \lessapprox_{<Rel>} \alpha \cdot (\tau.(Q))$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd\ \alpha \cdot (\tau.(Q)) \Longrightarrow\!\!\hat{}_{\tau}\ \alpha \cdot (\tau.(Q))$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\alpha \cdot P$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\alpha \cdot (\tau.(Q))$)) Ψ **using** *insertTauAssertion*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*) (*auto simp add*: *FrameStatEq-def intro*: *FrameStatImpTrans*)
moreover have $\Psi \otimes \Psi' \rhd\ \alpha \cdot (\tau.(Q)) \Longrightarrow\!\!\hat{}_{\tau}\ \alpha \cdot (\tau.(Q))$ **by** *auto*
moreover have $(\Psi \otimes \Psi', \alpha \cdot P, \alpha \cdot (\tau.(Q))) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

end

context *tauSum* **begin**

lemma *tauLaw2StatImpLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *C1*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', P \oplus \tau.(P), \tau.(P)) \in Rel$

shows $\Psi \rhd P \oplus \tau.(P) \lessapprox_{<Rel>} \tau.(P)$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ \tau.(P)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\tau.(P)$)) $\Psi \simeq_F \langle\varepsilon, \Psi\rangle$ **by**(*rule insertTauAssertion*)
hence *insertAssertion* (*extractFrame*($P \oplus \tau.(P)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\tau.(P)$)) Ψ **using** *Identity*
by(*rule-tac FrameStatImpTrans*) (*auto simp add*: *FrameStatEq-def AssertionStatEq-def*)
moreover have $\Psi \otimes \Psi' \rhd \tau.(P) \Longrightarrow\hat{}_\tau\ \tau.(P)$ **by** *auto*
moreover have $(\Psi \otimes \Psi', P \oplus \tau.(P), \tau.(P)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw2StatImpRight*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*

assumes *C1*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \tau.(P), P \oplus \tau.(P)) \in Rel$

shows $\Psi \rhd \tau.(P) \lessapprox_{<Rel>} P \oplus \tau.(P)$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd P \oplus \tau.(P) \Longrightarrow\hat{}_\tau\ P \oplus \tau.(P)$ **by** *auto*
moreover have *insertAssertion* (*extractFrame*($\tau.(P)$)) $\Psi \simeq_F \langle\varepsilon, \Psi\rangle$ **by**(*rule insertTauAssertion*)
hence *insertAssertion* (*extractFrame*($\tau.(P)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($P \oplus \tau.(P)$)) Ψ **using** *Identity*
by(*rule-tac FrameStatImpTrans*) (*auto simp add*: *FrameStatEq-def AssertionStatEq-def*)
moreover have $\Psi \otimes \Psi' \rhd P \oplus \tau.(P) \Longrightarrow\hat{}_\tau\ P \oplus \tau.(P)$ **by** *auto*
moreover have $(\Psi \otimes \Psi', \tau.(P), P \oplus \tau.(P)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw4StatImpLeft*:
fixes Ψ :: *'b*
and P :: (*'a*, *'b*, *'c*) *psi*
and Q :: (*'a*, *'b*, *'c*) *psi*
and M :: *'a*
and N :: *'a*

assumes *C1*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}(\tau.(P) \oplus Q)) \in Rel$

shows $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \lessapprox_{<Rel>} \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\hat{}_{\tau}\ \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
hence *insertAssertion* (*extractFrame*($\alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\alpha{\cdot}(\tau.(P) \oplus Q)$)) Ψ
using *insertTauAssertion Identity*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*, *auto*)
(*rule FrameStatImpTrans*[**where** $G{=}\langle\varepsilon, \Psi\rangle$], *auto simp add*: *FrameStatEq-def AssertionStatEq-def*)
moreover have $\Psi \otimes \Psi' \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\hat{}_{\tau}\ \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
moreover have $(\Psi \otimes \Psi', \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}(\tau.(P) \oplus Q)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

lemma *tauLaw4StatImpRight*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $Q :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$

assumes *C1*: $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)) \in Rel$

shows $\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \lessapprox_{<Rel>} \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakStatImpI*)
case(*cStatImp* Ψ')
have $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\hat{}_{\tau}\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
hence *insertAssertion* (*extractFrame*($\alpha{\cdot}(\tau.(P) \oplus Q)$)) $\Psi \hookrightarrow_F$ *insertAssertion* (*extractFrame*($\alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$)) Ψ
using *insertTauAssertion Identity*
by(*nominal-induct* α *rule*: *prefix.strong-inducts*, *auto*)
(*rule FrameStatImpTrans*[**where** $G{=}\langle\varepsilon, \Psi\rangle$], *auto simp add*: *FrameStatEq-def AssertionStatEq-def*)
moreover have $\Psi \otimes \Psi' \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \Longrightarrow\hat{}_{\tau}\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$ **by** *auto*
moreover have $(\Psi \otimes \Psi', \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)) \in Rel$ **by**(*rule C1*)
ultimately show *?case* **by** *blast*
qed

end

end

theory *Tau-Laws-Weak*
imports *Weaken-Bisimulation Weak-Congruence Tau-Sim Tau-Stat-Imp*

begin

context *weakTauLaws* **begin**

lemma *tauLaw1*:
  fixes Ψ :: *'b*
  and P :: (*'a*, *'b*, *'c*) *psi*

  shows $\Psi \rhd \tau.(P) \approx P$
proof −
  let *?X* = $\{(\Psi, \tau.(P), P) \mid \Psi\ P.\ True\}$ **let** *?Y* = $\{(\Psi, P, \tau.(P)) \mid \Psi\ P.\ True\}$
  have $(\Psi, \tau.(P), P) \in$ *?X* $\cup$ *?Y* **by** *auto*
  moreover have *eqvt*(*?X* $\cup$ *?Y*) **by**(*auto simp add*: *eqvt-def simp add*: *eqvts*)
  ultimately have $\Psi \rhd \tau.(P) \approx_w P$
  proof(*coinduct rule*: *weakenTransitiveCoinduct*)
    case(*cStatImp* Ψ *P Q*)
    show *?case*
    proof(*cases* $(\Psi, P, Q) \in$ *?X*)
      case *True*
      {
        fix Ψ *P*
        have $\Psi \rhd P \lessapprox_w <(?X \cup ?Y \cup weakBisim)> P$ **by**(*auto simp add*: *weakenStatImp-def intro*: *weakBisimReflexive*)
        moreover have $(\Psi, \tau.(P), P) \in ?X \cup ?Y \cup weakBisim$ **by** *auto*
        ultimately have $\Psi \rhd \tau.(P) \lessapprox_w <(?X \cup ?Y \cup weakBisim)> P$
          by(*rule tauLaw1StatImpLeft*)
      }
      with ‹$(\Psi, P, Q) \in$ *?X*› **show** *?thesis* **by** *auto*
    next
      case *False*
      from ‹$(\Psi, P, Q) \notin$ *?X*› ‹$(\Psi, P, Q) \in ?X \cup ?Y$› **have** $(\Psi, P, Q) \in$ *?Y* **by** *auto*
      {
        fix Ψ *P*
        have $\Psi \rhd P \lessapprox <weakBisim> P$ **using** *weakBisimReflexive*
          by(*rule weakBisimE*)
        moreover have $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \approx Q; \Psi \rhd Q \sim R]\!] \Longrightarrow (\Psi, P, R) \in ?X \cup ?Y \cup weakBisim$
          by(*fastforce intro*: *weakBisimTransitive strongBisimWeakBisim*)
        ultimately have $\Psi \rhd P \lessapprox <(?X \cup ?Y \cup weakBisim)> \tau.(P)$ **by**(*rule tauLaw1StatImpRight*)
        moreover have $\bigwedge \Psi\ P\ Q\ \Psi'.\ [\![(\Psi, P, Q) \in ?X \cup ?Y \cup weakBisim; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in ?X \cup ?Y \cup weakBisim$
          by(*auto dest*: *statEqWeakBisim*)
        ultimately have $\Psi \rhd P \lessapprox_w <(?X \cup ?Y \cup weakBisim)> \tau.(P)$ **by**(*rule weakStatImpWeakenStatImp*)
      }
      with ‹$(\Psi, P, Q) \in$ *?Y*› **show** *?thesis* **by** *auto*
    qed

next
case(*cSim* Ψ P Q)
let *?Z* = $\{(\Psi, P, Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi, P', Q') \in ?X \cup ?Y \cup weakenBisim \wedge \Psi \rhd Q' \sim Q\}$
have *eqvt ?Z*
apply *auto*
apply(*auto simp add*: *eqvt-def eqvts*)
apply(*rule-tac* x=$(p \cdot (\tau.(Q')))$ **in** *exI*)
apply(*auto intro*: *bisimClosed*)
apply(*simp add*: *eqvts*)
apply(*blast intro*: *bisimClosed weakBisimClosed*)
apply(*rule-tac* x=$(p \cdot P')$ **in** *exI*)
apply(*auto intro*: *bisimClosed*)
apply(*rule-tac* x=$\tau.(p \cdot P')$ **in** *exI*)
apply *auto*
apply(*drule-tac* p=p **and** Q=b **in** *bisimClosed*)
apply(*simp add*: *eqvts*)
apply(*rule-tac* x=$(p \cdot P')$ **in** *exI*)
apply(*auto intro*: *bisimClosed*)
apply(*rule-tac* x=$p \cdot Q'$ **in** *exI*)
apply *auto*
by(*blast intro*: *bisimClosed dest*: *weakBisimClosed*)+
show *?case*
proof(*cases* $(\Psi, P, Q) \in ?X$)
case *True*
{
fix P
have $\Psi \rhd P \rightsquigarrow <?Z> P$ **using** *weakenBisimEqWeakBisim* **by**(*blast intro*: *weakSimReflexive weakBisimReflexive bisimReflexive*)
moreover note ‹*eqvt ?Z*›
moreover have $\bigwedge \Psi\ P\ Q\ R.\ [\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in ?Z]\!] \Longrightarrow (\Psi, P, R) \in ?Z$
by(*blast intro*: *bisimTransitive*)
ultimately have $\Psi \rhd \tau.(P) \rightsquigarrow <?Z> P$
by(*rule tauLaw1SimLeft*)
moreover have $\bigwedge \Psi\ P\ Q\ \Psi'.\ [\![(\Psi, P, Q) \in ?Z; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in ?Z$
by *simp* (*blast intro*: *statEqWeakBisim statEqBisim*)
ultimately have $\Psi \rhd \tau.(P) \rightsquigarrow_w <?Z> P$ **by**(*rule weakSimWeakenSim*)
}
with ‹$(\Psi, P, Q) \in ?X$› **show** *?thesis* **by** *auto*
next
case *False*
from ‹$(\Psi, P, Q) \notin ?X$› ‹$(\Psi, P, Q) \in ?X \cup ?Y$› **have** $(\Psi, P, Q) \in ?Y$ **by** *auto*
moreover {
fix P
note ‹*eqvt ?Z*›
moreover have $(\Psi, P, P) \in ?Z$ **by** *simp* (*blast intro*: *weakBisimReflexive*

bisimReflexive)
moreover have $\bigwedge\Psi\ P\ Q\ R.\ [\![(\Psi, P, Q) \in ?Z; \Psi \rhd Q \sim R]\!] \Longrightarrow (\Psi, P, R) \in ?Z$
by(*blast intro*: *bisimTransitive*)
ultimately have $\Psi \rhd P \rightsquigarrow <?Z> \tau.(P)$ **by**(*rule tauLaw1SimRight*)
moreover have $\bigwedge\Psi\ P\ Q\ \Psi'.\ [\![(\Psi, P, Q) \in ?Z; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in ?Z$
by *simp* (*blast intro*: *statEqWeakBisim statEqBisim*)
ultimately have $\Psi \rhd P \rightsquigarrow_w <?Z> \tau.(P)$ **by**(*rule weakSimWeakenSim*)
}
ultimately show *?thesis* **by** *auto*
qed
next
case(*cExt* $\Psi\ P\ Q\ \Psi'$)
thus *?case* **by** *auto*
next
case(*cSym* $\Psi\ P\ Q$)
thus *?case* **by** *auto*
qed
thus *?thesis* **by** *simp*
qed

lemma *tauLaw3*:
fixes $\Psi :: 'b$
and $\alpha :: 'a\ prefix$
and $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd \alpha\cdot(\tau.(P)) \approx \alpha\cdot P$
proof −
let $?X = (\{(\Psi, \alpha\cdot(\tau.(P)), \alpha\cdot P) \mid \Psi\ \alpha\ P.\ True\})$
let $?Y = (\{(\Psi, \alpha\cdot P, \alpha\cdot(\tau.(P))) \mid \Psi\ \alpha\ P.\ True\})$

have $(\Psi, \alpha\cdot(\tau.(P)), \alpha\cdot P) \in ?X \cup ?Y$ **by** *blast*
moreover have *eqvt*$(?X \cup ?Y)$ **by**(*fastforce simp add*: *eqvt-def simp add*: *eqvts*)
ultimately have $\Psi \rhd \alpha\cdot(\tau.(P)) \approx_w \alpha\cdot P$
proof(*coinduct rule*: *weakenTransitiveCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
show *?case*
proof(*cases* $(\Psi, P, Q) \in ?X$)
case *True*
{
fix $\Psi\ \alpha\ P$
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \alpha\cdot(\tau.(P)), \alpha\cdot P) \in ?X \cup ?Y \cup weakenBisim$ **by** *auto*
hence $\Psi \rhd \alpha\cdot(\tau.(P)) \lessapprox <(?X \cup ?Y \cup weakenBisim)> \alpha\cdot P$ **by**(*rule tauLaw3StatImpLeft*)
moreover have $\bigwedge\Psi\ P\ Q\ \Psi'.\ [\![(\Psi, P, Q) \in ?X \cup ?Y \cup weakenBisim; \Psi \simeq \Psi']\!] \Longrightarrow (\Psi', P, Q) \in ?X \cup ?Y \cup weakenBisim$
by(*fastforce intro*: *statEqWeakBisim*)
ultimately have $\Psi \rhd \alpha\cdot(\tau.(P)) \lessapprox_w <(?X \cup ?Y \cup weakenBisim)> \alpha\cdot P$

by(*rule weakStatImpWeakenStatImp*)
}
with ‹$(\Psi,\ P,\ Q) \in ?X$› **show** *?thesis* **by** *blast*
next
case *False*
{
fix $\Psi\ \alpha\ P$
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi',\ \alpha{\cdot}P,\ \alpha{\cdot}(\tau.(P))) \in ?X \cup ?Y \cup weakenBisim$ **by** *auto*
hence $\Psi \rhd \alpha{\cdot}P \lessapprox <(?X \cup ?Y \cup weakenBisim)> \alpha{\cdot}(\tau.(P))$ **by**(*rule tauLaw3StatImpRight*)
moreover have $\bigwedge\Psi\ P\ Q\ \Psi'.\ [\![(\Psi,\ P,\ Q) \in ?X \cup ?Y \cup weakenBisim;\ \Psi \simeq \Psi']\!] \Longrightarrow (\Psi',\ P,\ Q) \in ?X \cup ?Y \cup weakenBisim$
by(*fastforce intro*: *statEqWeakBisim*)
ultimately have $\Psi \rhd \alpha{\cdot}P \lessapprox_w <(?X \cup ?Y \cup weakenBisim)> \alpha{\cdot}(\tau.(P))$ **by**(*rule weakStatImpWeakenStatImp*)
}
moreover from ‹$(\Psi,\ P,\ Q) \notin ?X$› ‹$(\Psi,\ P,\ Q) \in ?X \cup ?Y$› **have** $(\Psi,\ P,\ Q) \in ?Y$ **by** *blast*
ultimately show *?thesis* **by** *auto*
qed
next
case(*cSim* $\Psi\ P\ Q$)
let *?Z* $= \{(\Psi,\ P,\ Q) \mid \Psi\ P\ Q.\ \exists P'\ Q'.\ \Psi \rhd P \sim P' \wedge (\Psi,\ P',\ Q') \in ?X \cup ?Y \cup weakenBisim \wedge \Psi \rhd Q' \sim Q\}$
have *eqvt ?Z*
apply(*clarsimp simp add*: *eqvt-def*)
apply(*elim disjE*)
apply(*rule-tac x*=$p \cdot P'$ **in** *exI*)
apply(*clarsimp simp add*: *bisimClosed eqvts*)
apply(*blast intro*: *bisimClosed eqvts*)
apply(*rule-tac x*=$p \cdot P'$ **in** *exI*)
apply(*clarsimp simp add*: *bisimClosed eqvts*)
apply(*rule-tac x*=$p \cdot (\alpha{\cdot}(\tau.(P)))$ **in** *exI*)
apply(*clarsimp simp add*: *eqvts*)
apply(*rule conjI*)
apply(*rule disjI2*)
apply(*blast intro*: *bisimClosed eqvts*)
apply(*drule-tac p*=p **in** *bisimClosed*)
apply(*drule-tac p*=p **in** *bisimClosed*)
apply(*simp add*: *eqvts*)
by(*blast dest*: *bisimClosed weakBisimClosed*)
show *?case*
proof(*cases* $(\Psi,\ P,\ Q) \in ?X$)
case *True*
note ‹$(\Psi,\ P,\ Q) \in ?X$›
moreover {
fix $\Psi\ P\ \alpha$
note ‹*eqvt ?Z*›
moreover have $(\Psi,\ P,\ P) \in ?Z$ **using** *weakenBisimEqWeakBisim* **by**(*blast*

intro: *weakBisimReflexive bisimReflexive*)
moreover have $\bigwedge$*xvec Tvec. length xvec = length Tvec* $\Longrightarrow$ (Ψ, *P*[*xvec*::=*Tvec*], *P*[*xvec*::=*Tvec*]) $\in$ *?Z*
using *weakenBisimEqWeakBisim* **by**(*blast intro*: *weakBisimReflexive bisimReflexive*)
moreover have $\bigwedge\Psi$ *P Q R S*. $[\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in ?Z; \Psi \rhd R \sim S]\!]$ $\Longrightarrow$ (Ψ, *P*, *S*) $\in$ *?Z* **by**(*blast intro*: *bisimTransitive*)
moreover have $\bigwedge\Psi$ *P Q* Ψ'. (Ψ, *P*, *Q*) $\in$ *?Z* $\Longrightarrow$ ($\Psi \otimes \Psi'$, *P*, *Q*) $\in$ *?Z* **by**(*blast dest*: *weakenBisimE*(*3*) *bisimE*(*3*))
ultimately have $\Psi \rhd \alpha{\cdot}(\tau.(P)) \rightsquigarrow$<*?Z*> $\alpha{\cdot}P$ **by**(*rule tauLaw3SimLeft*)
moreover have $\bigwedge\Psi$ *P Q* Ψ'. $[\![(\Psi, P, Q) \in ?Z; \Psi \simeq \Psi']\!]$ $\Longrightarrow$ (Ψ', *P*, *Q*) $\in$ *?Z* **by** *simp* (*blast dest*: *statEqWeakBisim statEqBisim*)
ultimately have $\Psi \rhd \alpha{\cdot}(\tau.(P)) \rightsquigarrow_w$<*?Z*> $\alpha{\cdot}P$ **by**(*rule weakSimWeakenSim*)
}
ultimately show *?thesis* **by** *auto*
next
case *False*
from ‹(Ψ, *P*, *Q*) $\notin$ *?X*› ‹(Ψ, *P*, *Q*) $\in$ *?X* $\cup$ *?Y*› **have** (Ψ, *P*, *Q*) $\in$ *?Y* **by** *blast*
moreover {
fix Ψ *P* α
note ‹*eqvt ?Z*›
moreover have $\bigwedge\Psi$ *xvec Tvec. length xvec=length Tvec* $\Longrightarrow$ (Ψ, *P*[*xvec*::=*Tvec*], τ.(*P*[*xvec*::=*Tvec*])) $\in$ *?Z*
by *simp* (*blast intro*: *weakBisimE*(*4*) *bisimReflexive tauLaw1*)
moreover have $\bigwedge\Psi$ *P Q R S*. $[\![\Psi \rhd P \sim Q; (\Psi, Q, R) \in ?Z; \Psi \rhd R \sim S]\!]$ $\Longrightarrow$ (Ψ, *P*, *S*) $\in$ *?Z* **by**(*blast intro*: *bisimTransitive*)
moreover have $\bigwedge\Psi$. (Ψ, *P*, τ.(*P*)) $\in$ *?Z* **by** *simp* (*blast intro*: *weakBisimE*(*4*) *bisimReflexive tauLaw1*)
ultimately have $\Psi \rhd \alpha{\cdot}P \rightsquigarrow$<*?Z*> $\alpha{\cdot}(\tau.(P))$ **by**(*rule tauLaw3SimRight*)
moreover have $\bigwedge\Psi$ *P Q* Ψ'. $[\![(\Psi, P, Q) \in ?Z; \Psi \simeq \Psi']\!]$ $\Longrightarrow$ (Ψ', *P*, *Q*) $\in$ *?Z* **by** *simp* (*blast dest*: *statEqWeakBisim statEqBisim*)
ultimately have $\Psi \rhd \alpha{\cdot}P \rightsquigarrow_w$<*?Z*> $\alpha{\cdot}(\tau.(P))$ **by**(*rule weakSimWeakenSim*)
}
ultimately show *?thesis* **by** *auto*
qed
next
case(*cExt* Ψ *P Q* Ψ')
thus *?case* **by** *auto*
next
case(*cSym* Ψ *P Q*)
thus *?case* **by** *blast*
qed
thus *?thesis* **by** *simp*
qed

lemma *tauLaw3PsiCong*:
fixes Ψ :: *'b*
and *P* :: (*'a*, *'b*, *'c*) *psi*

shows $\Psi \rhd \alpha{\cdot}(\tau.(P)) \doteq \alpha{\cdot}P$
proof(*induct rule*: *weakPsiCongI*)
case *cWeakBisim*
show *?case* **by**(*rule tauLaw3*)
next
case *cSimLeft*
have $\Psi \rhd P \approx P$ **by**(*rule weakBisimReflexive*)
moreover have $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q;\ \Psi \rhd Q \approx R;\ \Psi \rhd R \sim S]\!] \Longrightarrow \Psi \rhd P \approx S$
by(*blast intro*: *weakBisimTransitive strongBisimWeakBisim*)
ultimately show *?case* **using** *weakBisimE*(*3*) **by**(*rule tauLaw3CongSimLeft*)
next
case *cSimRight*
have $\Psi \rhd P \approx P$ **by**(*rule weakBisimReflexive*)
moreover have $\bigwedge \Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q;\ \Psi \rhd Q \approx R;\ \Psi \rhd R \sim S]\!] \Longrightarrow \Psi \rhd P \approx S$
by(*blast intro*: *weakBisimTransitive strongBisimWeakBisim*)
ultimately show *?case* **using** *tauLaw1*[*THEN weakBisimE*(*4*)]
by(*rule tauLaw3CongSimRight*)
qed

lemma *tauLaw3Cong*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

shows $\alpha{\cdot}(\tau.(P)) \doteq_c \alpha{\cdot}P$
proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* $\Psi\ \sigma$)
show *?case*
proof(*nominal-induct* α *rule*: *prefix.strong-inducts*)
next
case(*pInput M yvec N*)
obtain *p* **where** *set p* $\subseteq$ *set yvec* $\times$ *set*(*p* $\cdot$ *yvec*) **and** (*p* $\cdot$ *yvec*) $\sharp*$ *N* **and** (*p* $\cdot$ *yvec*) $\sharp*$ *P* **and** (*p* $\cdot$ *yvec*) $\sharp*$ σ
by(*rule-tac xvec*=*yvec* **and** *c*=(*N*, *P*, σ) **in** *name-list-avoiding*) *auto*
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw3PsiCong*[**where** α=*pInput* (*substTerm.seqSubst M* σ) (*p* $\cdot$ *yvec*) (*substTerm.seqSubst* (*p* $\cdot$ *N*) σ)]
by(*simp add*: *inputChainAlpha' eqvts*)
next
case(*pOutput M N*)
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw3PsiCong*[**where** α=*pOutput* (*substTerm.seqSubst M* σ) (*substTerm.seqSubst N* σ)]
by *simp*
next
case *pTau*
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw3PsiCong*[**where** α=*pTau*]
by *simp*
qed

qed

end

end

theory *Tau-Laws-No-Weak*
imports *Tau-Sim Tau-Stat-Imp*
begin

context *tauSum*
begin

lemma *tauLaw2*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd P \oplus \tau.(P) \approx \tau.(P)$
proof −
let *?X* = $\{(\Psi, P \oplus \tau.(P), \tau.(P)) \mid \Psi\ P.\ True\}$
let *?Y* = $\{(\Psi, \tau.(P), P \oplus \tau.(P)) \mid \Psi\ P.\ True\}$

have $(\Psi, P \oplus \tau.(P), \tau.(P)) \in ?X \cup ?Y$ **by** *auto*
moreover have *eqvt*(*?X* ∪ *?Y*) **by**(*auto simp add*: *eqvt-def eqvts*)
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct*)
case(*cStatImp* Ψ *P Q*)
show *?case*
proof(*cases* $(\Psi, P, Q) \in ?X$)
case *True*
note ‹$(\Psi, P, Q) \in ?X$›
moreover {
fix Ψ *P*
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi', P \oplus \tau.(P), \tau.(P)) \in ?X \cup ?Y \cup weakBisim$ **by** *auto*
hence $\Psi \rhd P \oplus \tau.(P) \lessapprox <(?X \cup ?Y \cup weakBisim)> \tau.(P)$ **by**(*rule tauLaw2StatImpLeft*)
}
ultimately show *?thesis* **by** *blast*
next
case *False*
from ‹$(\Psi, P, Q) \notin ?X$› ‹$(\Psi, P, Q) \in ?X \cup ?Y$› **have** $(\Psi, P, Q) \in ?Y$ **by** *auto*
moreover {
fix Ψ *P*
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \tau.(P), P \oplus \tau.(P)) \in ?X \cup ?Y \cup weakBisim$ **by** *auto*
hence $\Psi \rhd \tau.(P) \lessapprox <(?X \cup ?Y \cup weakBisim)> P \oplus \tau.(P)$ **by**(*rule tauLaw2StatImpRight*)
}
ultimately show *?thesis* **by** *blast*

```
    qed
  next
    case(cSim Ψ P Q)
    let ?Z = {(Ψ, P, S) | Ψ P S. ∃ Q R. Ψ ▷ P ∼ Q ∧ (Ψ, Q, R) ∈ ?X ∪ ?Y ∪
weakBisim ∧ Ψ ▷ R ∼ S}
    show ?case

    proof(cases (Ψ, P, Q) ∈ ?X)
      case True
      note ‹(Ψ, P, Q) ∈ ?X›
      moreover {
        fix Ψ P
        have ⋀Ψ P. (Ψ, P, P) ∈ ?Z by(blast intro: weakBisimReflexive bisimRe-
flexive)
        moreover have ⋀Ψ P Q R S. ⟦Ψ ▷ P ∼ Q; (Ψ, Q, R) ∈ ?Z; Ψ ▷ R ∼ S⟧
⟹ (Ψ, P, S) ∈ ?Z
          by(blast intro: bisimTransitive dest: bisimE(4))
        ultimately have Ψ ▷ P ⊕ τ.(P) ⇝<?Z> τ.(P) by(rule tauLaw2SimLeft)
      }
      ultimately show ?thesis by blast
    next
      case False
      from ‹(Ψ, P, Q) ∉ ?X› ‹(Ψ, P, Q) ∈ ?X ∪ ?Y› have (Ψ, P, Q) ∈ ?Y by
auto
      moreover {
        fix Ψ P
        have ⋀Ψ P Q. Ψ ▷ P ∼ Q ⟹ (Ψ, P, Q) ∈ ?Z by(blast intro: weakBisim-
Reflexive bisimReflexive)
        hence Ψ ▷ τ.(P) ⇝<?Z> P ⊕ τ.(P) by(rule tauLaw2SimRight)
      }
      ultimately show ?thesis by blast
    qed
  next
    case(cExt Ψ P Q Ψ')
    thus ?case by auto
  next
    case(cSym Ψ P Q)
    thus ?case by auto
  qed
qed

lemma tauLawPsiCong2:
  fixes Ψ :: 'b
  and   P :: ('a, 'b, 'c) psi

  shows Ψ ▷ P ⊕ τ.(P) ≐ τ.(P)
proof(induct rule: weakPsiCongI)
  case cWeakBisim
  thus ?case by(rule tauLaw2)
```

next
case *cSimLeft*
note *weakBisimReflexive*
moreover have $\bigwedge\Psi\ P\ Q\ R\ S.\ [\![\Psi \rhd P \sim Q;\ \Psi \rhd Q \approx R;\ \Psi \rhd R \sim S]\!] \Longrightarrow \Psi \rhd P \approx S$
by(*metis weakBisimTransitive strongBisimWeakBisim*)
ultimately show *?case* **by**(*rule tauLaw2CongSimLeft*)
next
case *cSimRight*
from *strongBisimWeakBisim* **show** *?case* **by**(*rule tauLaw2CongSimRight*)
qed

lemma *tauLawCong2*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$

shows $P \oplus \tau.(P) \doteq_c \tau.(P)$
using *tauLawPsiCong2*
by(*rule-tac weakCongI*) *auto*

lemma *tauLaw4*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$

shows $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \approx \alpha{\cdot}(\tau.(P) \oplus Q)$
proof −
let $?X = \{(\Psi, \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}(\tau.(P) \oplus Q)) \mid \Psi\ P\ \alpha\ Q.\ True\}$
let $?Y = \{(\Psi, \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)) \mid \Psi\ P\ \alpha\ Q.\ True\}$
have $(\Psi, \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}(\tau.(P) \oplus Q)) \in ?X \cup ?Y$ **by** *auto*
moreover have *eqvt*$(?X \cup ?Y)$ **by**(*fastforce simp add*: *eqvt-def eqvts*)
ultimately show *?thesis*
proof(*coinduct rule*: *weakTransitiveCoinduct*)
case(*cStatImp* $\Psi\ P\ Q$)
show *?case*
proof(*cases* $(\Psi, P, Q) \in ?X$)
case *True*
note $\langle(\Psi, P, Q) \in ?X\rangle$
moreover {
fix $\Psi\ \alpha\ P\ Q$
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi', \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q), \alpha{\cdot}(\tau.(P) \oplus Q)) \in ?X \cup ?Y \cup weakBisim$ **by** *blast*
hence $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \lessapprox_{<(?X \cup ?Y \cup weakBisim)>} \alpha{\cdot}(\tau.(P) \oplus Q)$ **by**(*rule tauLaw4StatImpLeft*)
}
ultimately show *?thesis* **by** *blast*
next
case *False*
from $\langle(\Psi, P, Q) \notin ?X\rangle\ \langle(\Psi, P, Q) \in ?X \cup ?Y\rangle$ **have** $(\Psi, P, Q) \in ?Y$ **by**

blast
moreover {
fix Ψ α P Q
have $\bigwedge\Psi'.\ (\Psi \otimes \Psi',\ \alpha{\cdot}(\tau.(P) \oplus Q),\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)) \in ?X \cup ?Y \cup$ *weakBisim* **by** *auto*
hence $\Psi \rhd\ \alpha{\cdot}(\tau.(P) \oplus Q) \lessapprox <(?X \cup ?Y \cup weakBisim)>\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P)$ $\oplus\ Q)$ **by**(*rule tauLaw4StatImpRight*)
}
ultimately show *?thesis* **by** *blast*
qed
next
case(*cSim* Ψ P Q)
let $?Z = \{(\Psi,\ P,\ S) \mid \Psi\ P\ S.\ \exists\, Q\ R.\ \Psi \rhd P \sim Q \wedge (\Psi,\ Q,\ R) \in ?X \cup ?Y \cup$ $weakBisim \wedge \Psi \rhd R \sim S\}$
show *?case*
proof(*cases* $(\Psi,\ P,\ Q) \in ?X$)
case *True*
note $\langle(\Psi,\ P,\ Q) \in ?X\rangle$
moreover {
fix Ψ P α Q
have $\bigwedge\Psi\ P.\ (\Psi,\ P,\ P) \in ?Z$ **by**(*blast intro*: *weakBisimReflexive bisimReflexive*)
moreover have $\bigwedge\Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi,\ P,\ Q) \in ?Z$ **by**(*blast intro*: *weakBisimReflexive bisimReflexive*)
ultimately have $\Psi \rhd\ \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow <?Z>\ \alpha{\cdot}(\tau.(P) \oplus Q)$ **by**(*rule tauLaw4SimLeft*)
}
ultimately show *?thesis* **by** *blast*
next
case *False*
from $\langle(\Psi,\ P,\ Q) \notin ?X\rangle\ \langle(\Psi,\ P,\ Q) \in ?X \cup ?Y\rangle$ **have** $(\Psi,\ P,\ Q) \in ?Y$ **by** *blast*
moreover {
fix Ψ P α Q
have $\bigwedge\Psi\ P\ Q.\ \Psi \rhd P \sim Q \Longrightarrow (\Psi,\ P,\ Q) \in ?Z$ **by**(*blast intro*: *weakBisimReflexive bisimReflexive*)
moreover have $\bigwedge\Psi\ P.\ (\Psi,\ P,\ P) \in ?Z$ **by**(*blast intro*: *weakBisimReflexive bisimReflexive*)
ultimately have $\Psi \rhd \alpha{\cdot}(\tau.(P) \oplus Q) \rightsquigarrow <?Z> \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q)$ **by**(*rule tauLaw4SimRight*)
}
ultimately show *?thesis* **by** *blast*
qed
next
case(*cExt* Ψ P Q Ψ')
thus *?case* **by** *auto*
next
case(*cSym* Ψ P Q)
thus *?case* **by** *blast*

qed
qed

lemma *tauLaw4PsiCong*:
fixes $\Psi :: 'b$
and $P :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$
and $Q :: ('a, 'b, 'c)\ psi$

shows $\Psi \rhd \alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \doteq \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakPsiCongI*)
case *cWeakBisim*
show *?case* **by**(*rule tauLaw4*)
next
case *cSimLeft*
from *weakBisimReflexive strongBisimWeakBisim* **show** *?case* **by**(*rule tauLaw4CongSimLeft*)
next
case *cSimRight*
from *strongBisimWeakBisim* **show** *?case* **by**(*rule tauLaw4CongSimRight*)
qed

lemma *tauLaw4Cong*:
fixes $P :: ('a, 'b, 'c)\ psi$
and $\alpha :: 'a\ prefix$
and $Q :: ('a, 'b, 'c)\ psi$

shows $\alpha{\cdot}P \oplus \alpha{\cdot}(\tau.(P) \oplus Q) \doteq_c \alpha{\cdot}(\tau.(P) \oplus Q)$
proof(*induct rule*: *weakCongI*)
case(*cWeakPsiCong* $\Psi\ \sigma$)
show *?case*
proof(*nominal-induct* α *rule*: *prefix.strong-inducts*)
next
case(*pInput M yvec N*)
obtain p **where** *set* $p \subseteq$ *set yvec* $\times$ *set*$(p \cdot yvec)$ **and** $(p \cdot yvec) \sharp* N$ **and** $(p \cdot yvec) \sharp* P$ **and** $(p \cdot yvec) \sharp* Q$ **and** $(p \cdot yvec) \sharp* \sigma$
by(*rule-tac xvec*=*yvec* **and** c=(N, P, Q, σ) **in** *name-list-avoiding*) *auto*
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw4PsiCong*[**where** α=*pInput* (*substTerm.seqSubst M* σ) $(p \cdot yvec)$ (*substTerm.seqSubst* $(p \cdot N)$ σ)]
by(*simp add*: *inputChainAlpha' eqvts*)
next
case(*pOutput M N*)
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw4PsiCong*[**where** α=*pOutput* (*substTerm.seqSubst M* σ) (*substTerm.seqSubst N* σ)]
by *simp*
next
case *pTau*
thus *?case* **using** ‹*wellFormedSubst* σ› *tauLaw4PsiCong*[**where** α=*pTau*]
by *simp*

qed
qed

end

end

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