# Promela Formalization

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**Abstract**

We present an executable formalization of the language Promela, the description language for models of the model checker SPIN. This formalization is part of the work for a completely verified model checker (CAVA), but also serves as a useful (and executable!) description of the semantics of the language itself, something that is currently missing. The formalization uses three steps: It takes an abstract syntax tree generated from an SML parser, removes syntactic sugar and enriches it with type information. This further gets translated into a transition system, on which the semantic engine (read: successor function) operates.

# Contents

# 1 Introduction

Promela [1] is a modeling language, mainly used in the model checker SPIN [2]. It offers a C-like syntax and allows to define processes to be run concurrently. Those processes can communicate via shared global variables or by message-passing via channels. Inside a process, constructs exist for non-deterministic choice, starting other processes and enforcing atomicity. It furthermore allows different means for specifying properties: LTL formulae, assertions in the code, never claims (i. e. an automata that explicitly specifies unwanted behavior) and others.

Some constructs found in Promela models, like `#include` and `#define`, are not part of the language Promela itself, but belong to the language of the C preprocessor. SPIN does not process those, but calls the C compiler internally to process them. We do not deal with them here, but also expect the sources to be preprocessed.

Observing the output of SPIN and examining the generated graphs often is the only way of determining the semantics of a certain construct. This is complicated further by SPIN unconditionally applying optimizations. For the current formalization we chose to copy the semantics of SPIN, including the aforementioned optimizations. For some constructs, we had to restrict the semantics, i. e. some models are accepted by SPIN, but not by this formalization. Those deviations are:

- `run` is a statement instead of an expression. SPIN here has a complicated set of restrictions unto where `run` can occur inside an expression. The sole use of it is to be able to get the ID of a spawned process. We omitted this feature to guarantee expressions to be free of side-effects.

- Variable declarations which got jumped over are seen as not existing. In SPIN, such constructs show surprising behavior:
  `int i; goto L; i = 5; L: printf("%d", i)` yields 0, while
  `goto L; int i = 5; L: printf("%d", i)` yields 5.
  The latter is forbidden in our formalization (it will get rejected with "unknown variable i"), while the first behaves as in SPIN.

- Violating an `assert` does not abort, but instead sets the variable `__assert__` to true. This needs to be checked explicitly in the LTL formula. We plan on adding this check in an automatic manner.

- Types are bounded. Except for well-defined types like booleans, overflow is not allowed and will result in an error. The same holds for assigning a value that is outside the bounds. SPIN does not specify any explicit semantics here, but solely refers to the underlying C-compiler and its semantics. This might result in two models behaving differently on different systems when run with SPIN, while this formalization, due to the explicit bounds in the semantics, is not affected.

Additionally, some constructs are currently not supported, and the compilation will abort if they are encountered: `d_step`[1], `typedef`, remote references, bit-operations, `unsigned`, and property specifications except `ltl` and `assert`. Other constructs are accepted but ignored, because they do not change the behavior of a model: advanced variable scoping, `xr`, `xs`, `print*`, priorities, and visibility of variables.

Nonetheless, for models not using those unsupported constructs, we generate the very same number of states as SPIN does. An exception applies for large `goto` chains and when simultaneous termination of multiple processes is involved, as SPIN's semantics is too vague here.

# 2 Abstract Syntax Tree

**theory** *PromelaAST*
**imports** *Main*
**begin**

The abstract syntax tree is generated from the handwritten SML parser. This theory only mirrors the data structures from the SML level to make them available in Isabelle.

**context**
**begin**

⟨*ML*⟩

**datatype** *binOp* =
*BinOpAdd*
| *BinOpSub*
| *BinOpMul*
| *BinOpDiv*
| *BinOpMod*
| *BinOpBitAnd*
| *BinOpBitXor*
| *BinOpBitOr*
| *BinOpGr*
| *BinOpLe*
| *BinOpGEq*
| *BinOpLEq*
| *BinOpEq*
| *BinOpNEq*
| *BinOpShiftL*
| *BinOpShiftR*
| *BinOpAnd*
| *BinOpOr*

[1] This can be safely replaced by `atomic`, though larger models will be produced then.

```
datatype unOp =
              UnOpComp
            | UnOpMinus
            | UnOpNeg

datatype expr =
              ExprBinOp binOp  expr  expr
            | ExprUnOp unOp expr
            | ExprCond  expr  expr  expr
            | ExprLen varRef
            | ExprPoll varRef recvArg list
            | ExprRndPoll varRef recvArg list
            | ExprVarRef varRef
            | ExprConst integer
            | ExprTimeOut
            | ExprNP
            | ExprEnabled expr
            | ExprPC expr
            | ExprRemoteRef  String.literal
                             expr option
                             String.literal
            | ExprGetPrio expr
            | ExprSetPrio  expr  expr
            | ExprFull varRef
            | ExprEmpty varRef
            | ExprNFull varRef
            | ExprNEmpty varRef

  and varRef = VarRef  String.literal
                       expr option
                       varRef option

  and recvArg = RecvArgVar varRef
              | RecvArgEval expr
              | RecvArgConst integer

datatype range =
              RangeFromTo  varRef
                           expr
                             expr
            | RangeIn  varRef  varRef

datatype varType =
                VarTypeBit
              | VarTypeBool
              | VarTypeByte
              | VarTypePid
              | VarTypeShort
```

```
        | VarTypeInt
        | VarTypeMType
        | VarTypeChan
        | VarTypeUnsigned
        | VarTypeCustom String.literal

datatype varDecl =
          VarDeclNum   String.literal
                       integer option
                       expr option
        | VarDeclChan  String.literal
                       integer option
                       (integer * varType list) option
        | VarDeclUnsigned  String.literal
                           integer
                           expr option
        | VarDeclMType  String.literal
                        integer option
                        String.literal option


datatype decl =
          Decl  bool option
                varType
                varDecl list


datatype stmnt =
          StmntIf (step list) list
        | StmntDo (step list) list
        | StmntFor range step list
        | StmntAtomic step list
        | StmntDStep step list
        | StmntSelect range
        | StmntSeq step list
        | StmntSend varRef expr list
        | StmntSortSend varRef expr list
        | StmntRecv varRef recvArg list
        | StmntRndRecv varRef recvArg list
        | StmntRecvX varRef recvArg list
        | StmntRndRecvX varRef recvArg list
        | StmntAssign varRef expr
        | StmntIncr varRef
        | StmntDecr varRef
        | StmntElse
        | StmntBreak
        | StmntGoTo String.literal
        | StmntLabeled String.literal stmnt
        | StmntPrintF String.literal expr list
```

```
        | StmntPrintM String.literal
        | StmntRun  String.literal
                    expr list
                    integer option
        | StmntAssert expr
        | StmntCond expr
        | StmntCall String.literal varRef list

  and step = StepStmnt stmnt  stmnt option
         | StepDecl decl
         | StepXR varRef list
         | StepXS varRef list

datatype module =
              ProcType  (integer option) option
                        String.literal
                       decl list
                        integer option
                        expr option
                         step list
             | DProcType  (integer option) option
                         String.literal
                        decl list
                         integer option
                         expr option
                          step list
             | Init  integer option step list
             | Never step list
             | Trace step list
             | NoTrace step list
             | Inline String.literal String.literal list step list
             | TypeDef String.literal decl list
             | MType String.literal list
             | ModuDecl decl
             | Ltl  String.literal  String.literal

end
end
```

# 3 Data structures as used in Promela

```
theory PromelaDatastructures
imports
  CAVA-Base.CAVA-Base
  CAVA-Base.Lexord-List
  PromelaAST
  HOL−Library.IArray
  Deriving.Compare-Instances
  CAVA-Base.CAVA-Code-Target
```

**begin**

## 3.1 Abstract Syntax Tree *after* preprocessing

From the plain AST stemming from the parser, we'd like to have one containing more information while also removing duplicated constructs. This is achieved in the preprocessing step.

The additional information contains:

- variable type (including whether it represents a channel or not)
- global vs local variable

Also certain constructs are expanded (like for-loops) or different nodes in the AST are collapsed into one parametrized node (e.g. the different send-operations).

This preprocessing phase also tries to detect certain static errors and will bail out with an exception if such is encountered.

**datatype** *binOp* = *BinOpAdd*
| *BinOpSub*
| *BinOpMul*
| *BinOpDiv*
| *BinOpMod*
| *BinOpGr*
| *BinOpLe*
| *BinOpGEq*
| *BinOpLEq*
| *BinOpEq*
| *BinOpNEq*
| *BinOpAnd*
| *BinOpOr*

**datatype** *unOp* = *UnOpMinus*
| *UnOpNeg*

**datatype** *expr* = *ExprBinOp binOp expr expr*
| *ExprUnOp unOp expr*
| *ExprCond expr expr expr*
| *ExprLen chanRef*
| *ExprVarRef varRef*
| *ExprConst integer*
| *ExprMConst integer String.literal*
| *ExprTimeOut*
| *ExprFull chanRef*
| *ExprEmpty chanRef*
| *ExprPoll chanRef recvArg list bool*

**and** *varRef* = *VarRef bool*

```
                    String.literal
                    expr option
  and chanRef = ChanRef varRef — explicit type for channels
  and recvArg = RecvArgVar varRef
              | RecvArgEval expr
              | RecvArgConst integer
              | RecvArgMConst integer String.literal

datatype varType = VTBounded integer integer
                 | VTChan
```

Variable declarations at the beginning of a proctype or at global level.

```
datatype varDecl = VarDeclNum  integer integer
                               String.literal
                               integer option
                               expr option
                 | VarDeclChan  String.literal
                                integer option
                                (integer * varType list) option
```

Variable declarations during a proctype.

```
datatype procVarDecl = ProcVarDeclNum   integer integer
                                        String.literal
                                        integer option
                                        expr option
                     | ProcVarDeclChan  String.literal
                                        integer option

datatype procArg = ProcArg varType String.literal

datatype stmnt = StmntIf (step list) list
               | StmntDo (step list) list
               | StmntAtomic step list
               | StmntSeq step list
               | StmntSend chanRef expr list bool
               | StmntRecv chanRef recvArg list bool  bool
               | StmntAssign varRef expr
               | StmntElse
               | StmntBreak
               | StmntSkip
               | StmntGoTo String.literal
               | StmntLabeled String.literal stmnt
               | StmntRun  String.literal
                           expr list
               | StmntCond expr
               | StmntAssert expr

  and step = StepStmnt stmnt  stmnt option
           | StepDecl procVarDecl list
```

```
        | StepSkip

datatype proc = ProcType  (integer option) option
                          String.literal
                          procArg list
                          varDecl list
                          step list
              | Init varDecl list step list

type-synonym ltl = — name: String.literal × — formula: String.literal
type-synonym promela = varDecl list × proc list × ltl list
```

## 3.2 Preprocess the AST of the parser into our variant

We setup some functionality for printing warning or even errors.

All those constants are logically *undefined*, but replaced by the parser for something meaningful.

```
consts
  warn :: String.literal ⇒ unit

abbreviation with-warn msg e ≡ let - = warn msg in e
abbreviation the-warn opt msg ≡ case opt of None ⇒ () | - ⇒ warn msg

usc: "Unsupported Construct"

definition [code del]: usc (c :: String.literal) ≡ undefined

definition  [code del]: err (e :: String.literal) = undefined
abbreviation errv e v ≡ err (e + v)

definition [simp, code del]: abort (msg :: String.literal) f = f ()
abbreviation abortv msg v f ≡ abort (msg + v) f

code-printing
  code-module PromelaUtils ⇀ (SML) ‹
    structure PromelaUtils = struct
      exception UnsupportedConstruct of string
      exception StaticError of string
      exception RuntimeError of string
      fun warn msg = TextIO.print (Warning: ^ msg ^ \n)
      fun usc  c   = raise (UnsupportedConstruct c)
      fun err  e   = raise (StaticError e)
      fun abort msg - = raise (RuntimeError msg)
    end›
| constant warn ⇀ (SML) PromelaUtils.warn
| constant usc ⇀ (SML) PromelaUtils.usc
| constant err ⇀ (SML) PromelaUtils.err
| constant abort ⇀ (SML) PromelaUtils.abort
code-reserved (SML) PromelaUtils
```

⟨*ML*⟩

The preprocessing is done for each type on its own.

```
primrec ppBinOp :: AST.binOp ⇒ binOp
where
  ppBinOp AST.BinOpAdd = BinOpAdd
| ppBinOp AST.BinOpSub = BinOpSub
| ppBinOp AST.BinOpMul = BinOpMul
| ppBinOp AST.BinOpDiv = BinOpDiv
| ppBinOp AST.BinOpMod = BinOpMod
| ppBinOp AST.BinOpGr = BinOpGr
| ppBinOp AST.BinOpLe = BinOpLe
| ppBinOp AST.BinOpGEq = BinOpGEq
| ppBinOp AST.BinOpLEq = BinOpLEq
| ppBinOp AST.BinOpEq = BinOpEq
| ppBinOp AST.BinOpNEq = BinOpNEq
| ppBinOp AST.BinOpAnd = BinOpAnd
| ppBinOp AST.BinOpOr = BinOpOr
| ppBinOp AST.BinOpBitAnd = usc STR ''BinOpBitAnd''
| ppBinOp AST.BinOpBitXor = usc STR ''BinOpBitXor''
| ppBinOp AST.BinOpBitOr = usc STR ''BinOpBitOr''
| ppBinOp AST.BinOpShiftL = usc STR ''BinOpShiftL''
| ppBinOp AST.BinOpShiftR = usc STR ''BinOpShiftR''

primrec ppUnOp :: AST.unOp ⇒ unOp
where
  ppUnOp AST.UnOpMinus = UnOpMinus
| ppUnOp AST.UnOpNeg = UnOpNeg
| ppUnOp AST.UnOpComp = usc STR ''UnOpComp''
```

The data structure holding all information on variables we found so far.

```
type-synonym var-data =
    (String.literal, (integer option × bool)) lm — channels
    × (String.literal, (integer option × bool)) lm — variables
    × (String.literal, integer) lm — mtypes
    × (String.literal, varRef) lm — aliases (used for inlines)

definition dealWithVar
  :: var-data ⇒ String.literal
     ⇒ (String.literal ⇒ integer option × bool ⇒ expr option ⇒ 'a)
     ⇒ (String.literal ⇒ integer option × bool ⇒ expr option ⇒ 'a)
     ⇒ (integer ⇒ 'a) ⇒ 'a
where
  dealWithVar cvm n fC fV fM ≡ (
    let (c,v,m,a) = cvm in
    let (n, idx) = case lm.lookup n a of
                     None ⇒ (n, None)
                   | Some (VarRef - name idx) ⇒ (name, idx)
```

```
    in
    case lm.lookup n m of
      Some i ⇒ (case idx of None ⇒ fM i
                          | - ⇒ err STR ''Array subscript used on MType (via alias).'')
    | None ⇒ (case lm.lookup n v of
                Some g ⇒ fV n g idx
              | None ⇒ (case lm.lookup n c of
                          Some g ⇒ fC n g idx
                        | None ⇒ err (STR ''Unknown variable referenced: '' + n))))

primrec enforceChan :: varRef + chanRef ⇒ chanRef where
  enforceChan (Inl -) = err STR ''Channel expected. Got normal variable.''
| enforceChan (Inr c) = c

fun liftChan :: varRef + chanRef ⇒ varRef where
  liftChan (Inl v) = v
| liftChan (Inr (ChanRef v)) = v

fun resolveIdx :: expr option ⇒ expr option ⇒ expr option
where
  resolveIdx None None = None
| resolveIdx idx  None = idx
| resolveIdx None aliasIdx = aliasIdx
| resolveIdx -    -      = err STR ''Array subscript used twice (via alias).''

fun ppExpr :: var-data ⇒ AST.expr ⇒ expr
and ppVarRef :: var-data ⇒ AST.varRef ⇒ varRef + chanRef
and ppRecvArg :: var-data ⇒ AST.recvArg ⇒ recvArg
where
  ppVarRef cvm (AST.VarRef name idx None) = dealWithVar cvm name
                  (λname (-,g) aIdx. let idx = map-option (ppExpr cvm) idx in
                      Inr (ChanRef (VarRef g name (resolveIdx idx aIdx))))
                  (λname (-,g) aIdx. let idx = map-option (ppExpr cvm) idx in
                      Inl (VarRef g name (resolveIdx idx aIdx)))
                  (λ-. err STR ''Variable expected. Got MType.'')
| ppVarRef cvm (AST.VarRef - - (Some -)) =
    usc STR ''next operation on variables''

| ppExpr cvm AST.ExprTimeOut = ExprTimeOut
| ppExpr cvm (AST.ExprConst c) = ExprConst c

| ppExpr cvm (AST.ExprBinOp bo l r) =
    ExprBinOp (ppBinOp bo) (ppExpr cvm l) (ppExpr cvm r)
| ppExpr cvm (AST.ExprUnOp uo e) =
    ExprUnOp (ppUnOp uo) (ppExpr cvm e)
| ppExpr cvm (AST.ExprCond c t f) =
    ExprCond (ppExpr cvm c) (ppExpr cvm t) (ppExpr cvm f)

| ppExpr cvm (AST.ExprLen v) =
```

```
    ExprLen (enforceChan (ppVarRef cvm v))
| ppExpr cvm (AST.ExprFull v) =
    ExprFull (enforceChan (ppVarRef cvm v))
| ppExpr cvm (AST.ExprEmpty v) =
    ExprEmpty (enforceChan (ppVarRef cvm v))

| ppExpr cvm (AST.ExprNFull v) =
    ExprUnOp UnOpNeg (ExprFull (enforceChan (ppVarRef cvm v)))
| ppExpr cvm (AST.ExprNEmpty v) =
    ExprUnOp UnOpNeg (ExprEmpty (enforceChan (ppVarRef cvm v)))

| ppExpr cvm (AST.ExprVarRef v) = (
        let to-exp = λ-. ExprVarRef (liftChan (ppVarRef cvm v)) in
        case v of
            AST.VarRef name None None ⇒
              dealWithVar cvm name
                  (λ- - -. to-exp())
                  (λ- - -. to-exp())
                  (λi. ExprMConst i name)
          | - ⇒ to-exp())

| ppExpr cvm (AST.ExprPoll v es) =
    ExprPoll (enforceChan (ppVarRef cvm v)) (map (ppRecvArg cvm) es) False
| ppExpr cvm (AST.ExprRndPoll v es) =
    ExprPoll (enforceChan (ppVarRef cvm v)) (map (ppRecvArg cvm) es) True

| ppExpr cvm AST.ExprNP = usc STR ''ExprNP''
| ppExpr cvm (AST.ExprEnabled -) = usc STR ''ExprEnabled''
| ppExpr cvm (AST.ExprPC -) = usc STR ''ExprPC''
| ppExpr cvm (AST.ExprRemoteRef - - -) = usc STR ''ExprRemoteRef''
| ppExpr cvm (AST.ExprGetPrio -) = usc STR ''ExprGetPrio''
| ppExpr cvm (AST.ExprSetPrio - -) = usc STR ''ExprSetPrio''

| ppRecvArg cvm (AST.RecvArgVar v) = (
        let to-ra = λ-. RecvArgVar (liftChan (ppVarRef cvm v)) in
        case v of
            AST.VarRef name None None ⇒
              dealWithVar cvm name
                  (λ- - -. to-ra())
                  (λ- - -. to-ra())
                  (λi. RecvArgMConst i name)
          | - ⇒ to-ra())
| ppRecvArg cvm (AST.RecvArgEval e) = RecvArgEval (ppExpr cvm e)
| ppRecvArg cvm (AST.RecvArgConst c) = RecvArgConst c

primrec ppVarType :: AST.varType ⇒ varType where
  ppVarType AST.VarTypeBit = VTBounded 0 1
| ppVarType AST.VarTypeBool = VTBounded 0 1
| ppVarType AST.VarTypeByte = VTBounded 0 255
```

```
| ppVarType AST.VarTypePid = VTBounded 0 255
| ppVarType AST.VarTypeShort = VTBounded (−(2^15)) ((2^15) − 1)
| ppVarType AST.VarTypeInt = VTBounded (−(2^31)) ((2^31) − 1)
| ppVarType AST.VarTypeMType = VTBounded 1 255
| ppVarType AST.VarTypeChan = VTChan
| ppVarType AST.VarTypeUnsigned = usc STR ''VarTypeUnsigned''
| ppVarType (AST.VarTypeCustom -) = usc STR ''VarTypeCustom''

fun ppVarDecl
  :: var-data ⇒ varType ⇒ bool ⇒ AST.varDecl ⇒ var-data × varDecl
where
  ppVarDecl (c,v,m,a) (VTBounded l h) g
                    (AST.VarDeclNum name sze init) = (
      case lm.lookup name v of
        Some - ⇒ errv STR ''Duplicate variable '' name
      | - ⇒ (case lm.lookup name a of
              Some - ⇒ errv
                         STR ''Variable name clashes with alias: '' name
            | - ⇒ ((c, lm.update name (sze,g) v, m, a),
                     VarDeclNum l h name sze
                       (map-option (ppExpr (c,v,m,a)) init))))
| ppVarDecl - - g (AST.VarDeclNum name sze init) =
      err STR ''Assiging num to non−num''

| ppVarDecl (c,v,m,a) VTChan g
                    (AST.VarDeclChan name sze cap) = (
      let cap' = map-option (apsnd (map ppVarType)) cap in
      case lm.lookup name c of
        Some - ⇒ errv STR ''Duplicate variable '' name
      | - ⇒ (case lm.lookup name a of
              Some - ⇒ errv
                         STR ''Variable name clashes with alias: '' name
            | - ⇒ ((lm.update name (sze, g) c, v, m, a),
                     VarDeclChan name sze cap')))
| ppVarDecl - - g (AST.VarDeclChan name sze init) =
      err STR ''Assiging chan to non−chan''

| ppVarDecl (c,v,m,a) (VTBounded l h) g
                    (AST.VarDeclMType name sze init) = (
      let init = map-option (λmty.
      case lm.lookup mty m of
        None ⇒ errv STR ''Unknown MType '' mty
      | Some mval ⇒ ExprMConst mval mty) init in
          case lm.lookup name c of
            Some - ⇒ errv STR ''Duplicate variable '' name
          | - ⇒ (case lm.lookup name a of Some -
                    ⇒ errv STR ''Variable name clashes with alias: '' name
          | - ⇒ ((c, lm.update name (sze,g) v, m, a),
                   VarDeclNum l h name sze init)))
```

```
| ppVarDecl - - g (AST.VarDeclMType name sze init) =
    err STR ''Assiging num to non−num''

| ppVarDecl - - - (AST.VarDeclUnsigned - - -) =
    usc STR ''VarDeclUnsigned''

definition ppProcVarDecl
  :: var-data ⇒ varType ⇒ bool ⇒ AST.varDecl ⇒ var-data × procVarDecl
where
  ppProcVarDecl cvm ty g v = (case ppVarDecl cvm ty g v of
      (cvm, VarDeclNum l h name sze init) ⇒ (cvm, ProcVarDeclNum l h name
sze init)
    | (cvm, VarDeclChan name sze None) ⇒ (cvm, ProcVarDeclChan name sze)
    | - ⇒ err STR ''Channel initilizations only allowed at the beginning of proc-
types.'')

fun ppProcArg
  :: var-data ⇒ varType ⇒ bool ⇒ AST.varDecl ⇒ var-data × procArg
where
  ppProcArg (c,v,m,a) (VTBounded l h) g
                    (AST.VarDeclNum name None None) = (
    case lm.lookup name v of
      Some - ⇒ errv STR ''Duplicate variable '' name
    | - ⇒ (case lm.lookup name a of
            Some - ⇒ errv
                       STR ''Variable name clashes with alias: '' name
          | - ⇒ ((c, lm.update name (None, g) v, m, a),
                 ProcArg (VTBounded l h) name)))
| ppProcArg - - - (AST.VarDeclNum - - -) =
    err STR ''Invalid proctype arguments''

| ppProcArg (c,v,m,a) VTChan g
                    (AST.VarDeclChan name None None) = (
    case lm.lookup name c of
      Some - ⇒ errv STR ''Duplicate variable '' name
    | - ⇒ (case lm.lookup name a of
            Some - ⇒ errv
                       STR ''Variable name clashes with alias: '' name
          | - ⇒ ((lm.update name (None, g) c, v, m, a), ProcArg VTChan name)))
| ppProcArg - - - (AST.VarDeclChan - - -) =
    err STR ''Invalid proctype arguments''

| ppProcArg (c,v,m,a) (VTBounded l h) g
                    (AST.VarDeclMType name None None) = (
    case lm.lookup name v of
      Some - ⇒ errv STR ''Duplicate variable '' name
    | - ⇒ (case lm.lookup name a of
            Some - ⇒ errv
```

```
                    STR ''Variable name clashes with alias: '' name
          | - ⇒ ((c, lm.update name (None, g) v, m, a),
                 ProcArg (VTBounded l h) name)))
| ppProcArg - - - (AST.VarDeclMType - - -) =
    err STR ''Invalid proctype arguments''

| ppProcArg - - - (AST.VarDeclUnsigned - - -) = usc STR ''VarDeclUnsigned''
```

Some preprocessing functions enrich the *var-data* argument and hence return a new updated one. When chaining multiple calls to such functions after another, we need to make sure, the *var-data* is passed accordingly. *cvm-fold* does exactly that for such a function $g$ and a list of nodes *ss*.

```
definition cvm-fold where
  cvm-fold g cvm ss = foldl (λ(cvm,ss) s. apsnd (λs'. ss@[s']) (g cvm s))
                            (cvm, []) ss

lemma cvm-fold-cong[fundef-cong]:
  assumes cvm = cvm'
  and stepss = stepss'
  and ⋀x d. x ∈ set stepss ⟹ g d x = g' d x
  shows cvm-fold g cvm stepss = cvm-fold g' cvm' stepss'
⟨proof⟩

fun liftDecl where
  liftDecl f g cvm (AST.Decl vis t decls) = (
    let - = the-warn vis STR ''Visibility in declarations not supported. Ignored.'' in
    let t = ppVarType t in
    cvm-fold (λcvm. f cvm t g) cvm decls)

definition ppDecl
  :: bool ⇒ var-data ⇒ AST.decl ⇒ var-data × varDecl list
where
  ppDecl = liftDecl ppVarDecl

definition ppDeclProc
  :: var-data ⇒ AST.decl ⇒ var-data × procVarDecl list
where
  ppDeclProc = liftDecl ppProcVarDecl False

definition ppDeclProcArg
  :: var-data ⇒ AST.decl ⇒ var-data × procArg list
where
  ppDeclProcArg = liftDecl ppProcArg False


definition incr :: varRef ⇒ stmnt where
  incr v = StmntAssign v (ExprBinOp BinOpAdd (ExprVarRef v) (ExprConst 1))
```

**definition** *decr* :: *varRef* ⇒ *stmnt* **where**
*decr v* = *StmntAssign v* (*ExprBinOp BinOpSub* (*ExprVarRef v*) (*ExprConst 1*))

Transforms `for (i : lb .. ub) steps` into

```
{
  i = lb;
  do
    :: i =< ub -> steps; i++
    :: else -> break
  od
}
```

**definition** *forFromTo* :: *varRef* ⇒ *expr* ⇒ *expr* ⇒ *step list* ⇒ *stmnt* **where**
*forFromTo i lb ub steps* = (
  *let*
    — *i* = *lb*
    *loop-pre* = *StepStmnt* (*StmntAssign i lb*) *None*;
    — *i* ≤ *ub*
    *loop-cond* = *StepStmnt* (*StmntCond*
                  (*ExprBinOp BinOpLEq* (*ExprVarRef i*) *ub*))
                  *None*;
    — *i*++
    *loop-incr* = *StepStmnt* (*incr i*) *None*;
    — *i* ≤ *ub* −> ...; *i*++
    *loop-body* = *loop-cond* # *steps* @ [*loop-incr*];
    — *else* −> *break*
    *loop-abort* = [*StepStmnt StmntElse None*, *StepStmnt StmntBreak None*];
    — *do* :: *i* ≤ *ub* −> ... :: *else* −> *break od*
    *loop* = *StepStmnt* (*StmntDo* [*loop-body*, *loop-abort*]) *None*
  *in*
    *StmntSeq* [*loop-pre*, *loop*])

Transforms (where $a$ is an array with $N$ entries) `for (i in a) steps` into

```
{
  i = 0;
  do
    :: i < N -> steps; i++
    :: else -> break
  od
}
```

**definition** *forInArray* :: *varRef* ⇒ *integer* ⇒ *step list* ⇒ *stmnt* **where**
*forInArray i N steps* = (
  *let*
    — *i* = *0*
    *loop-pre* = *StepStmnt* (*StmntAssign i* (*ExprConst 0*)) *None*;

```
    — i < N
    loop-cond = StepStmnt (StmntCond
                          (ExprBinOp BinOpLe (ExprVarRef i)
                            (ExprConst N)))
                          None;
    — i++
    loop-incr = StepStmnt (incr i) None;
    — i < N -> ...; i++
    loop-body = loop-cond # steps @ [loop-incr];
    — else -> break
    loop-abort = [StepStmnt StmntElse None, StepStmnt StmntBreak None];
    — do :: i < N -> ... :: else -> break od
    loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
  in
    StmntSeq [loop-pre, loop])
```

Transforms (where $c$ is a channel) `for (msg in c) steps` into

```
{
   byte :tmp: = 0;
   do
     :: :tmp: < len(c) ->
          c?msg; c!msg;
          steps;
          :tmp:++
     :: else -> break
   od
}
```

```
definition forInChan :: varRef ⇒ chanRef ⇒ step list ⇒ stmnt where
  forInChan msg c steps = (
    let
      — byte :tmp: = 0
      tmpStr = STR ":tmp:";
      loop-pre = StepDecl
                 [ProcVarDeclNum 0 255 tmpStr None (Some (ExprConst 0))];
      tmp = VarRef False tmpStr None;
      — :tmp: < len(c)
      loop-cond = StepStmnt (StmntCond
                            (ExprBinOp BinOpLe (ExprVarRef tmp)
                              (ExprLen c)))
                          None;
      — :tmp:++
      loop-incr = StepStmnt (incr tmp) None;
      — c?msg
      recv = StepStmnt (StmntRecv c [RecvArgVar msg] False True) None;
      — c!msg
      send = StepStmnt (StmntSend c [ExprVarRef msg] False) None;
```

```
    — :tmp: < len(c) -> c?msg; c!msg; ...; :tmp:++
    loop-body = [loop-cond, recv, send] @ steps @ [loop-incr];
    — else -> break
    loop-abort = [StepStmnt StmntElse None, StepStmnt StmntBreak None];
    — do :: :tmp: < len(c) -> ... :: else -> break od
    loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
  in
    StmntSeq [loop-pre, loop])
```

Transforms `select (i : lb .. ub)` into

```
{
   i = lb;
   do
     :: i < ub -> i++
     :: break
   od
}
```

```
definition select :: varRef ⇒ expr ⇒ expr ⇒ stmnt where
  select i lb ub = (
    let
      — i = lb
      pre = StepStmnt (StmntAssign i lb) None;
      — i < ub
      cond = StepStmnt (StmntCond (ExprBinOp BinOpLe (ExprVarRef i) ub))
                       None;
      — i++
      incr = StepStmnt (incr i) None;
      — i < ub -> i++
      loop-body = [cond, incr];
      — break
      loop-abort = [StepStmnt StmntBreak None];
      — do :: i < ub -> ... :: break od
      loop = StepStmnt (StmntDo [loop-body, loop-abort]) None
    in
      StmntSeq [pre, loop])

type-synonym inlines =
  (String.literal, String.literal list × (var-data ⇒ var-data × step list)) lm
type-synonym stmnt-data =
   bool × varDecl list × inlines × var-data

fun ppStep :: stmnt-data ⇒ AST.step ⇒ stmnt-data ∗ step
and ppStmnt :: stmnt-data ⇒ AST.stmnt ⇒ stmnt-data ∗ stmnt
where
  ppStep cvm (AST.StepStmnt s u) = (
    let (cvm′, s′) = ppStmnt cvm s in
    case u of None ⇒ (cvm′, StepStmnt s′ None)
```

```
                | Some u ⇒ let (cvm'',u') = ppStmnt cvm' u in
                              (cvm'', StepStmnt s' (Some u')))
| ppStep (False, ps, i, cvm) (AST.StepDecl d) =
    map-prod (λcvm. (False, ps, i, cvm)) StepDecl (ppDeclProc cvm d)
| ppStep (True, ps, i, cvm) (AST.StepDecl d) = (
          let (cvm', ps') = ppDecl False cvm d
          in ((True, ps@ps', i, cvm'), StepSkip))
| ppStep (-,cvm) (AST.StepXR -) =
    with-warn STR ''StepXR not supported. Ignored.'' ((False,cvm), StepSkip)
| ppStep (-,cvm) (AST.StepXS -) =
    with-warn STR ''StepXS not supported. Ignored.'' ((False,cvm), StepSkip)

| ppStmnt (-,cvm) (AST.StmntBreak) = ((False,cvm), StmntBreak)
| ppStmnt (-,cvm) (AST.StmntElse) = ((False,cvm), StmntElse)
| ppStmnt (-,cvm) (AST.StmntGoTo l) = ((False,cvm), StmntGoTo l)
| ppStmnt (-,cvm) (AST.StmntLabeled l s) =
    apsnd (StmntLabeled l) (ppStmnt (False,cvm) s)
| ppStmnt (-,ps,i,cvm) (AST.StmntCond e) =
    ((False,ps,i,cvm), StmntCond (ppExpr cvm e))
| ppStmnt (-,ps,i,cvm) (AST.StmntAssert e) =
    ((False,ps,i,cvm), StmntAssert (ppExpr cvm e))
| ppStmnt (-,ps,i,cvm) (AST.StmntAssign v e) =
    ((False,ps,i,cvm), StmntAssign (liftChan (ppVarRef cvm v)) (ppExpr cvm e))
| ppStmnt (-,ps,i,cvm) (AST.StmntSend v es) =
    ((False,ps,i,cvm), StmntSend (enforceChan (ppVarRef cvm v))
                                 (map (ppExpr cvm) es) False)
| ppStmnt (-,ps,i,cvm) (AST.StmntSortSend v es) =
    ((False,ps,i,cvm), StmntSend (enforceChan (ppVarRef cvm v))
                                 (map (ppExpr cvm) es) True)
| ppStmnt (-,ps,i,cvm) (AST.StmntRecv v rs) =
    ((False,ps,i,cvm), StmntRecv (enforceChan (ppVarRef cvm v))
                                 (map (ppRecvArg cvm) rs) False True)
| ppStmnt (-,ps,i,cvm) (AST.StmntRecvX v rs) =
    ((False,ps,i,cvm), StmntRecv (enforceChan (ppVarRef cvm v))
                                 (map (ppRecvArg cvm) rs) False False)
| ppStmnt (-,ps,i,cvm) (AST.StmntRndRecv v rs) =
    ((False,ps,i,cvm), StmntRecv (enforceChan (ppVarRef cvm v))
                                 (map (ppRecvArg cvm) rs) True True)
| ppStmnt (-,ps,i,cvm) (AST.StmntRndRecvX v rs) =
    ((False,ps,i,cvm), StmntRecv (enforceChan (ppVarRef cvm v))
                                 (map (ppRecvArg cvm) rs) True False)
| ppStmnt (-,ps,i,cvm) (AST.StmntRun n es p) = (
    let - = the-warn p STR ''Priorities for 'run' not supported. Ignored.'' in
   ((False,ps,i,cvm), StmntRun n (map (ppExpr cvm) es)))
| ppStmnt (-,cvm) (AST.StmntSeq ss) =
    apsnd StmntSeq (cvm-fold ppStep (False,cvm) ss)
| ppStmnt (-,cvm) (AST.StmntAtomic ss) =
    apsnd StmntAtomic (cvm-fold ppStep (False,cvm) ss)
| ppStmnt (-,cvm) (AST.StmntIf sss) =
```

```
    apsnd StmntIf (cvm-fold (cvm-fold ppStep) (False,cvm) sss)
| ppStmnt (-,cvm) (AST.StmntDo sss) =
    apsnd StmntDo (cvm-fold (cvm-fold ppStep) (False,cvm) sss)


| ppStmnt (-,ps,i,cvm) (AST.StmntIncr v) =
    ((False,ps,i,cvm), incr (liftChan (ppVarRef cvm v)))
| ppStmnt (-,ps,i,cvm) (AST.StmntDecr v) =
    ((False,ps,i,cvm), decr (liftChan (ppVarRef cvm v)))

| ppStmnt (-,cvm) (AST.StmntPrintF - -) =
    with-warn STR "PrintF ignored" ((False,cvm), StmntSkip)
| ppStmnt (-,cvm) (AST.StmntPrintM -) =
    with-warn STR "PrintM ignored" ((False,cvm), StmntSkip)

| ppStmnt (-,ps,inl,cvm) (AST.StmntFor
                           (AST.RangeFromTo i lb ub)
                           steps) = (
    let
      i = liftChan (ppVarRef cvm i);
      (lb,ub) = (ppExpr cvm lb, ppExpr cvm ub)
    in
      apsnd (forFromTo i lb ub) (cvm-fold ppStep (False,ps,inl,cvm) steps))
| ppStmnt (-,ps,inl,cvm) (AST.StmntFor
                           (AST.RangeIn i v)
                           steps) = (
    let
      i = liftChan (ppVarRef cvm i);
      (cvm',steps) = cvm-fold ppStep (False,ps,inl,cvm) steps
    in
      case ppVarRef cvm v of
       Inr c ⇒ (cvm', forInChan i c steps)
     | Inl (VarRef - - (Some -)) ⇒ err STR "Iterating over array−member."
     | Inl (VarRef - name None) ⇒ (
          let (-,v,-) = cvm in
          case fst (the (lm.lookup name v)) of
           None ⇒ err STR "Iterating over non−array variable."
         | Some N ⇒ (cvm', forInArray i N steps)))

| ppStmnt (-,ps,inl,cvm) (AST.StmntSelect
                           (AST.RangeFromTo i lb ub)) = (
    let
      i = liftChan (ppVarRef cvm i);
      (lb, ub) = (ppExpr cvm lb, ppExpr cvm ub)
    in
      ((False,ps,inl,cvm), select i lb ub))
| ppStmnt (-,cvm) (AST.StmntSelect (AST.RangeIn - -)) =
    err STR "in not allowed in select"
```

```
| ppStmnt (-,ps,inl,cvm) (AST.StmntCall macro args) = (
    let
      args = map (liftChan ∘ ppVarRef cvm) args;
      (c,v,m,a) = cvm
    in
      case lm.lookup macro inl of
       None ⇒ errv STR ''Calling unknown macro '' macro
     | Some (names,sF) ⇒
        if length names ≠ length args then
          (err STR ''Called macro with wrong number of arguments.'')
        else
          let a' = foldl (λa (k,v). lm.update k v a) a (zip names args) in
          let ((c,v,m,-),steps) = sF (c,v,m,a') in
          ((False,ps,inl,c,v,m,a), StmntSeq steps))

| ppStmnt cvm (AST.StmntDStep -) = usc STR ''StmntDStep''

fun ppModule
  :: var-data × inlines ⇒ AST.module
     ⇒ var-data × inlines × (varDecl list + proc + ltl)
where
  ppModule (cvm, inl) (AST.ProcType act name args prio prov steps) = (
    let
      - = the-warn prio STR ''Priorities for procs not supported. Ignored.'';
      - = the-warn prov STR ''Priov (??) for procs not supported. Ignored.'';
      (cvm', args) = cvm-fold ppDeclProcArg cvm args;
      ((-, vars, -, -), steps) = cvm-fold ppStep (True,[],inl,cvm') steps
    in
      (cvm, inl, Inr (Inl (ProcType act name (concat args) vars steps))))

| ppModule (cvm,inl) (AST.Init prio steps) = (
    let - = the-warn prio STR ''Priorities for procs not supported. Ignored.'' in
    let ((-, vars, -, -), steps) = cvm-fold ppStep (True,[],inl,cvm) steps in
    (cvm, inl, Inr (Inl (Init vars steps))))

| ppModule (cvm,inl) (AST.Ltl name formula) =
    (cvm, inl, Inr (Inr (name, formula)))

| ppModule (cvm,inl) (AST.ModuDecl decl) =
    apsnd (λds. (inl,Inl ds)) (ppDecl True cvm decl)

| ppModule (cvm,inl) (AST.MType mtys) = (
    let (c,v,m,a) = cvm in
    let num = integer-of-nat (lm.size m) + 1 in
    let (m',-) = foldr (λmty (m,num).
                  let m' = lm.update mty num m
                  in (m',num+1)) mtys (m,num)
    in
       ((c,v,m',a), inl, Inl []))
```

```
| ppModule (cvm,inl) (AST.Inline name args steps) = (
    let stepF = (λcvm. let ((-,-,-,cvm),steps) =
                    cvm-fold ppStep (False,[],inl,cvm) steps
                  in (cvm,steps))
    in let inl = lm.update name (args, stepF) inl
    in (cvm,inl, Inl[]))

| ppModule cvm (AST.DProcType - - - - - -) = usc STR ''DProcType''
| ppModule cvm (AST.Never -) = usc STR ''Never''
| ppModule cvm (AST.Trace -) = usc STR ''Trace''
| ppModule cvm (AST.NoTrace -) = usc STR ''NoTrace''
| ppModule cvm (AST.TypeDef - -) = usc STR ''TypeDef''

definition preprocess :: AST.module list ⇒ promela where
  preprocess ms = (
    let
      dflt-vars = [(STR ''-pid'', (None, False)),
                   (STR ''--assert--'', (None, True)),
                   (STR ''-'', (None, True))];
      cvm = (lm.empty(), lm.to-map dflt-vars, lm.empty(), lm.empty());
      (-,-,pr) = (foldl (λ(cvm,inl,vs,ps,ls) m.
                      let (cvm', inl', m') = ppModule (cvm,inl) m in
                      case m' of
                        Inl v ⇒ (cvm',inl',vs@v,ps,ls)
                      | Inr (Inl p) ⇒ (cvm',inl',vs,ps@[p],ls)
                      | Inr (Inr l) ⇒ (cvm',inl',vs,ps,ls@[l]))
                    (cvm, lm.empty(),[],[],[]) ms)
      in
        pr)

fun extractLTL
  :: AST.module ⇒ ltl option
where
  extractLTL (AST.Ltl name formula) = Some (name, formula)
| extractLTL - = None

primrec extractLTLs
  :: AST.module list ⇒ (String.literal, String.literal) lm
where
  extractLTLs [] = lm.empty()
| extractLTLs (m#ms) = (case extractLTL m of
                      None ⇒ extractLTLs ms
                    | Some (n,f) ⇒ lm.update n f (extractLTLs ms))

definition lookupLTL
  :: AST.module list ⇒ String.literal ⇒ String.literal option
  where lookupLTL ast k = lm.lookup k (extractLTLs ast)
```

## 3.3 The transition system

The edges in our transition system consist of a condition (evaluated under the current environment) and an effect (modifying the current environment). Further they may be atomic, i. e. a whole row of such edges is taken before yielding a new state. Additionally, they carry a priority: the edges are checked from highest to lowest priority, and if one edge on a higher level can be taken, the lower levels are ignored.

The states of the system do not carry any information.

```
datatype edgeCond = ECElse
                  | ECTrue
                  | ECFalse
                  | ECExpr expr
                  | ECRun String.literal
                  | ECSend chanRef
                  | ECRecv chanRef recvArg list bool

datatype edgeEffect = EEEnd
                    | EEId
                    | EEGoto
                    | EEAssert expr
                    | EEAssign varRef expr
                    | EEDecl procVarDecl
                    | EERun String.literal expr list
                    | EESend chanRef expr list bool
                    | EERecv chanRef recvArg list bool  bool

datatype edgeIndex = Index nat | LabelJump String.literal nat option
datatype edgeAtomic = NonAtomic | Atomic | InAtomic

record edge =
  cond   :: edgeCond
  effect :: edgeEffect
  target :: edgeIndex
  prio   :: integer
  atomic :: edgeAtomic

definition isAtomic :: edge ⇒ bool where
  isAtomic e = (case atomic e of Atomic ⇒ True | - ⇒ False)

definition inAtomic :: edge ⇒ bool where
  inAtomic e = (case atomic e of NonAtomic ⇒ False | - ⇒ True)
```

## 3.4 State

```
datatype variable = Var varType integer
                  | VArray varType nat integer iarray
```

**datatype** *channel* = *Channel integer varType list integer list list*
| *HSChannel varType list*
| *InvChannel*

**type-synonym** *var-dict* = (*String.literal*, *variable*) *lm*
**type-synonym** *labels* = (*String.literal*, *nat*) *lm*
**type-synonym** *ltls* = (*String.literal*, *String.literal*) *lm*
**type-synonym** *states* = (— prio: *integer* × *edge list*) *iarray*
**type-synonym** *channels* = *channel list*

**type-synonym** *process* =
*nat* — offset
× *edgeIndex* — start
× *procArg list* — args
× *varDecl list* — top decls

**record** *program* =
*processes* :: *process iarray*
*labels* :: *labels iarray*
*states* :: *states iarray*
*proc-names* :: *String.literal iarray*
*proc-data* :: (*String.literal*, *nat*) *lm*

**record** *pState* = — State of a process
*pid* :: *nat* — Process identifier
*vars* :: *var-dict* — Dictionary of variables
*pc* :: *nat* — Program counter
*channels* :: *integer list* — List of channels created in the process. Used to close them on finalization.
*idx* :: *nat* — Offset into the arrays of *program*

**hide-const** (**open**) *idx*

**record** *gState* = — Global state
*vars* :: *var-dict* — Global variables
*channels* :: *channels* — Channels are by construction part of the global state, even when created in a process.
*timeout* :: *bool* — Set to True if no process can take a transition.
*procs* :: *pState list* — List of all running processes. A process is removed from it, when there is no running one with a higher index.

**record** $gState_I$ = *gState* + — Additional internal infos
*handshake* :: *nat*
*hsdata* :: *integer list* — Data transferred via a handshake.
*exclusive* :: *nat* — Set to the PID of the process, which is in an exclusive (= atomic) state.
*else* :: *bool* — Set to True for each process, if it can not take a transition. Used before timeout.

## 3.5 Printing

**primrec** *printBinOp* :: *binOp* ⇒ *string* **where**
  *printBinOp BinOpAdd* = ″+″
| *printBinOp BinOpSub* = ″−″
| *printBinOp BinOpMul* = ″∗″
| *printBinOp BinOpDiv* = ″/″
| *printBinOp BinOpMod* = ″*mod*″
| *printBinOp BinOpGr* = ″>″
| *printBinOp BinOpLe* = ″<″
| *printBinOp BinOpGEq* = ″>=″
| *printBinOp BinOpLEq* = ″=<″
| *printBinOp BinOpEq* = ″==″
| *printBinOp BinOpNEq* = ″!=″
| *printBinOp BinOpAnd* = ″&&″
| *printBinOp BinOpOr* = ″||″

**primrec** *printUnOp* :: *unOp* ⇒ *string* **where**
  *printUnOp UnOpMinus* = ″−″
| *printUnOp UnOpNeg* = ″!″

**definition** *printList* :: (′*a* ⇒ *string*) ⇒ ′*a list* ⇒ *string* ⇒ *string* ⇒ *string* ⇒ *string*
**where**
  *printList f xs l r sep* = (
                *let f′* = (λ*str x*. *if str* = [] *then f x*
                                *else str* @ *sep* @ *f x*)
                *in l* @ (*foldl f′* [] *xs*) @ *r*)

**lemma** *printList-cong* [*fundef-cong*]:
  **assumes** *xs* = *xs′*
  **and** *l* = *l′*
  **and** *r* = *r′*
  **and** *sep* = *sep′*
  **and** ⋀*x*. *x* ∈ *set xs* ⟹ *f x* = *f′ x*
  **shows** *printList f xs l r sep* = *printList f′ xs′ l′ r′ sep′*
  ⟨*proof*⟩

**fun** *printExpr* :: (*integer* ⇒ *string*) ⇒ *expr* ⇒ *string*
**and** *printFun* :: (*integer* ⇒ *string*) ⇒ *string* ⇒ *chanRef* ⇒ *string*
**and** *printVarRef* :: (*integer* ⇒ *string*) ⇒ *varRef* ⇒ *string*
**and** *printChanRef* :: (*integer* ⇒ *string*) ⇒ *chanRef* ⇒ *string*
**and** *printRecvArg* :: (*integer* ⇒ *string*) ⇒ *recvArg* ⇒ *string* **where**
  *printExpr f ExprTimeOut* = ″*timeout*″
| *printExpr f* (*ExprBinOp binOp left right*) =
    *printExpr f left* @ ″ ″ @ *printBinOp binOp* @ ″ ″ @ *printExpr f right*
| *printExpr f* (*ExprUnOp unOp e*) = *printUnOp unOp* @ *printExpr f e*
| *printExpr f* (*ExprVarRef varRef*) = *printVarRef f varRef*
| *printExpr f* (*ExprConst i*) = *f i*
| *printExpr f* (*ExprMConst i m*) = *String.explode m*

```
| printExpr f (ExprCond c l r) =
    ''( (( '' @ printExpr f c @ '' )) -> ''
    @ printExpr f l @ '' : ''
    @ printExpr f r @ '' )''
| printExpr f (ExprLen chan) = printFun f ''len'' chan
| printExpr f (ExprEmpty chan) = printFun f ''empty'' chan
| printExpr f (ExprFull chan) = printFun f ''full'' chan
| printExpr f (ExprPoll chan es srt) = (
    let p = if srt then ''??'' else ''?'' in
    printChanRef f chan @ p
    @ printList (printRecvArg f) es ''['' '']'' '', '')

| printVarRef - (VarRef - name None) = String.explode name
| printVarRef f (VarRef - name (Some indx)) =
    String.explode name @ ''['' @ printExpr f indx @ '']''

| printChanRef f (ChanRef v) = printVarRef f v

| printFun f fun var = fun @ ''('' @ printChanRef f var @ '')''

| printRecvArg f (RecvArgVar v) = printVarRef f v
| printRecvArg f (RecvArgConst c) = f c
| printRecvArg f (RecvArgMConst - m) = String.explode m
| printRecvArg f (RecvArgEval e) = ''eval('' @ printExpr f e @ '')''

fun printVarDecl :: (integer ⇒ string) ⇒ procVarDecl ⇒ string where
  printVarDecl f (ProcVarDeclNum - - n None None) =
    String.explode n @ '' = 0''
| printVarDecl f (ProcVarDeclNum - - n None (Some e)) =
    String.explode n @ '' = '' @ printExpr f e
| printVarDecl f (ProcVarDeclNum - - n (Some i) None) =
    String.explode n @ ''['' @ f i @ '']'' = 0''
| printVarDecl f (ProcVarDeclNum - - n (Some i) (Some e)) =
    String.explode n @ ''['' @ f i @ '']'' = '' @ printExpr f e
| printVarDecl f (ProcVarDeclChan n None) =
    ''chan '' @ String.explode n
| printVarDecl f (ProcVarDeclChan n (Some i)) =
    ''chan '' @ String.explode n @ ''['' @ f i @ '']''

primrec printCond :: (integer ⇒ string) ⇒ edgeCond ⇒ string where
  printCond f ECElse = ''else''
| printCond f ECTrue = ''true''
| printCond f ECFalse = ''false''
| printCond f (ECRun n) = ''run '' @ String.explode n @ ''(...)''
| printCond f (ECExpr e) = printExpr f e
| printCond f (ECSend c) = printChanRef f c @ ''! ...''
| printCond f (ECRecv c - -) = printChanRef f c @ ''? ...''

primrec printEffect :: (integer ⇒ string) ⇒ edgeEffect ⇒ string where
```

```
  printEffect f EEEnd = ''-- end --''
| printEffect f EEId = ''ID''
| printEffect f EEGoto = ''goto''
| printEffect f (EEAssert e) = ''assert('' @ printExpr f e @'')''
| printEffect f (EERun n -) = ''run '' @ String.explode n @ ''(...)''
| printEffect f (EEAssign v expr) =
    printVarRef f v @ '' = '' @ printExpr f expr
| printEffect f (EEDecl d) = printVarDecl f d
| printEffect f (EESend v es srt) = (
    let s = if srt then ''!!'' else ''!'' in
    printChanRef f v @ s @ printList (printExpr f) es ''('' '')'' '', '')
| printEffect f (EERecv v rs srt rem) = (
    let p = if srt then ''??'' else ''?'' in
    let (l,r) = if rem then (''('', '')'') else (''<'', ''>'') in
    printChanRef f v @ p @ printList (printRecvArg f) rs l r '', '')

primrec printIndex :: (integer ⇒ string) ⇒ edgeIndex ⇒ string where
  printIndex f (Index pos) = f (integer-of-nat pos)
| printIndex - (LabelJump l -) = String.explode l

definition printEdge :: (integer ⇒ string) ⇒ nat ⇒ edge ⇒ string where
  printEdge f indx e = (
    let
      tStr = printIndex f (target e);
      pStr = if prio e < 0 then '' Prio: '' @ f (prio e) else [];
      atom = if isAtomic e then λx. x @ '' {A}'' else id;
      pEff = λ-. atom (printEffect f (effect e));
      contStr = case (cond e) of
                  ECTrue ⇒ pEff ()
                | ECFalse ⇒ pEff ()
                | ECSend - ⇒ pEff()
                | ECRecv - - -⇒ pEff()
                | - ⇒  atom (''(( '' @ printCond f (cond e) @ '' ))'')
    in
      f (integer-of-nat indx) @ '' ---> '' @ tStr @ '' => '' @ contStr @ pStr)

definition printEdges :: (integer ⇒ string) ⇒ states ⇒ string list where
  printEdges f es = concat (map (λn. map (printEdge f n) (snd (es !! n)))
                                (rev [0..<IArray.length es]))

definition printLabels :: (integer ⇒ string) ⇒ labels ⇒ string list where
  printLabels f ls = lm.iterate ls (λ(k,l) res.
                                      (''Label '' @ String.explode k @ '': ''
                                       @ f (integer-of-nat l)) # res) []

fun printProcesses :: (integer ⇒ string) ⇒ program ⇒ string list where
  printProcesses f prog = lm.iterate (proc-data prog)
    (λ(k,idx) res.
        let (-,start,-,-) = processes prog !! idx in
```

[] # (''Process '' @ String.explode k) # [] # printEdges f (states prog !!
idx)
@ [''START ---> '' @ printIndex f start, []]
@ printLabels f (labels prog !! idx) @ res) []
⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨p

# 4 Invariants for Promela data structures

**theory** *PromelaInvariants*
**imports** *PromelaDatastructures*
**begin**

The different data structures used in the Promela implementation require different invariants, which are specified in this file. As there is no (useful) way of specifying *correctness* of the implementation, those invariants are tailored towards proving the finitness of the generated state-space.

⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩⟨proof⟩

## 4.1 Bounds

Finiteness requires that possible variable ranges are finite, as is the maximium number of processes. Currently, they are supplied here as constants. In a perfect world, they should be able to be set dynamically.

**definition** *min-var-value* :: *integer* **where**
*min-var-value* = −(2^31)
**definition** *max-var-value* :: *integer* **where**
*max-var-value* = (2^31) − 1

**lemma** *min-max-var-value-simps* [*simp*, *intro*!]:
*min-var-value* < *max-var-value*
*min-var-value* < 0
*min-var-value* ≤ 0
*max-var-value* > 0
*max-var-value* ≥ 0
⟨*proof*⟩

**definition** *max-procs* ≡ 255
**definition** *max-channels* ≡ 65535
**definition** *max-array-size* = 65535

## 4.2 Variables and similar

**fun** *varType-inv* :: *varType* ⇒ *bool* **where**
*varType-inv* (*VTBounded l h*)
⟷ *l* ≥ *min-var-value* ∧ *h* ≤ *max-var-value* ∧ *l* < *h*
| *varType-inv VTChan* ⟷ *True*

```
fun variable-inv :: variable ⇒ bool where
  variable-inv (Var t val)
  ⟷ varType-inv t ∧ val ∈ {min-var-value..max-var-value}
| variable-inv (VArray t sz ar)
  ⟷ varType-inv t
    ∧ sz ≤ max-array-size
    ∧ IArray.length ar = sz
    ∧ set (IArray.list-of ar) ⊆ {min-var-value..max-var-value}

fun channel-inv :: channel ⇒ bool where
  channel-inv (Channel cap ts q)
  ⟷ cap ≤ max-array-size
    ∧ cap ≥ 0
    ∧ set ts ⊆ Collect varType-inv
    ∧ length ts ≤ max-array-size
    ∧ length q ≤ max-array-size
    ∧ (∀ x ∈ set q. length x = length ts
    ∧ set x ⊆ {min-var-value..max-var-value})
| channel-inv (HSChannel ts)
  ⟷ set ts ⊆ Collect varType-inv ∧ length ts ≤ max-array-size
| channel-inv InvChannel ⟷ True

lemma varTypes-finite:
  finite (Collect varType-inv)
⟨proof⟩

lemma variables-finite:
  finite (Collect variable-inv)
⟨proof⟩

lemma channels-finite:
  finite (Collect channel-inv)
⟨proof⟩
```

To give an upper bound of variable names, we need a way to calculate it.

```
primrec procArgName :: procArg ⇒ String.literal where
  procArgName (ProcArg - name) = name

primrec varDeclName :: varDecl ⇒ String.literal where
  varDeclName (VarDeclNum - - name - -) = name
| varDeclName (VarDeclChan name - -) = name

primrec procVarDeclName :: procVarDecl ⇒ String.literal where
  procVarDeclName (ProcVarDeclNum - - name - -) = name
| procVarDeclName (ProcVarDeclChan name -) = name

definition edgeDecls :: edge ⇒ procVarDecl set where
  edgeDecls e = (
    case effect e of
```

```
    EEDecl p ⇒ {p}
  | - ⇒ {})

lemma edgeDecls-finite:
  finite (edgeDecls e)
⟨proof⟩

definition edgeSet :: states ⇒ edge set where
  edgeSet s = set (concat (map snd (IArray.list-of s)))

lemma edgeSet-finite:
  finite (edgeSet s)
⟨proof⟩

definition statesDecls :: states ⇒ procVarDecl set where
  statesDecls s = ⋃(edgeDecls ` (edgeSet s))

definition statesNames :: states ⇒ String.literal set where
  statesNames s = procVarDeclName ` statesDecls s

lemma statesNames-finite:
  finite (statesNames s)
⟨proof⟩


fun process-names :: states ⇒ process ⇒ String.literal set where
  process-names ss (-, -, args, decls) =
      statesNames ss
    ∪ procArgName ` set args
    ∪ varDeclName ` set decls
    ∪ {STR ''-'', STR ''--assert--'', STR ''-pid''}

lemma process-names-finite:
  finite (process-names ss p)
⟨proof⟩

definition vardict-inv :: states ⇒ process ⇒ var-dict ⇒ bool where
  vardict-inv ss p vs
   ⟷ lm.ball vs (λ(k,v). k ∈ process-names ss p ∧ variable-inv v)

lemma vardicts-finite:
  finite (Collect (vardict-inv ss p))
⟨proof⟩

lemma lm-to-map-vardict-inv:
  assumes ∀(k,v) ∈ set xs. k ∈ process-names ss proc ∧ variable-inv v
  shows vardict-inv ss proc (lm.to-map xs)
⟨proof⟩
```

## 4.3 Invariants of a process

**definition** *pState-inv* :: *program* ⇒ *pState* ⇒ *bool* **where**
*pState-inv prog p*
⟷ *pid p* ≤ *max-procs*
∧ *pState.idx p* < *IArray.length* (*states prog*)
∧ *IArray.length* (*states prog*) = *IArray.length* (*processes prog*)
∧ *pc p* < *IArray.length* ((*states prog*) !! *pState.idx p*)
∧ *set* (*pState.channels p*) ⊆ {−1..<*integer-of-nat max-channels*}
∧ *length* (*pState.channels p*) ≤ *max-channels*
∧ *vardict-inv* ((*states prog*) !! *pState.idx p*)
((*processes prog*) !! *pState.idx p*)
(*pState.vars p*)

**lemma** *pStates-finite*:
*finite* (*Collect* (*pState-inv prog*))
⟨*proof*⟩

Throughout the calculation of the semantic engine, a modified process is not necessarily part of *procs g*. Hence we need to establish an additional constraint for the relation between a global and a process state.

**definition** *cl-inv* :: ('*a gState-scheme* ∗ *pState*) ⇒ *bool* **where**
*cl-inv gp* = (*case gp of* (*g*,*p*) ⇒
*length* (*pState.channels p*) ≤ *length* (*gState.channels g*))

**lemma** *cl-inv-lengthD*:
*cl-inv* (*g*,*p*) ⟹ *length* (*pState.channels p*) ≤ *length* (*gState.channels g*)
⟨*proof*⟩

**lemma** *cl-invI*:
*length* (*pState.channels p*) ≤ *length* (*gState.channels g*) ⟹ *cl-inv* (*g*,*p*)
⟨*proof*⟩

**lemma** *cl-inv-trans*:
*length* (*channels g*) ≤ *length* (*channels g*′) ⟹ *cl-inv* (*g*,*p*) ⟹ *cl-inv* (*g*′,*p*)
⟨*proof*⟩

**lemma** *cl-inv-vars-update*[*intro*!]:
*cl-inv* (*g*,*p*) ⟹ *cl-inv* (*g*, *pState.vars-update vs p*)
*cl-inv* (*g*,*p*) ⟹ *cl-inv* (*gState.vars-update vs g*, *p*)
⟨*proof*⟩

**lemma** *cl-inv-handshake-update*[*intro*!]:
*cl-inv* (*g*,*p*) ⟹ *cl-inv* (*g*(|*handshake* := *h*|),*p*)
⟨*proof*⟩

**lemma** *cl-inv-hsdata-update*[*intro*!]:
*cl-inv* (*g*,*p*) ⟹ *cl-inv* (*g*(|*hsdata* := *h*|),*p*)
⟨*proof*⟩

**lemma** *cl-inv-procs-update*[*intro*!]:
*cl-inv* (*g*,*p*) ⟹ *cl-inv* (*g*(|*procs* := *ps*|),*p*)
⟨*proof*⟩

**lemma** *cl-inv-channels-update*:
**assumes** *cl-inv* (*g*,*p*)
**shows** *cl-inv* (*gState.channels-update* (λ*cs*. *cs*[*i*:=*c*]) *g*, *p*)
⟨*proof*⟩

## 4.4 Invariants of the global state

Note that *gState-inv* must be defined in a way to be applicable to both *gState* and $gState_I$.

**definition** *gState-inv* :: *program* ⇒ *'a gState-scheme* ⇒ *bool* **where**
*gState-inv prog g*
⟷ *length* (*procs g*) ≤ *max-procs*
∧ (∀ *p* ∈ *set* (*procs g*). *pState-inv prog p* ∧ *cl-inv* (*g*,*p*))
∧ *length* (*channels g*) ≤ *max-channels*
∧ *set* (*channels g*) ⊆ *Collect channel-inv*
∧ *lm.ball* (*vars g*) (λ(*k*,*v*). *variable-inv v*)

The set of global states adhering to the terms of *gState-inv* is not finite. But the set of all global states that can be constructed by the semantic engine from one starting state is. Thus we establish a progress relation, i. e. all successors of a state *g* relate to *g* under this specification.

**definition** *gState-progress-rel* :: *program* ⇒ (*'a gState-scheme*) *rel* **where**
*gState-progress-rel p* = {(*g*,*g'*). *gState-inv p g* ∧ *gState-inv p g'*
∧ *length* (*channels g*) ≤ *length* (*channels g'*)
∧ *dom* (*lm.α* (*vars g*)) = *dom* (*lm.α* (*vars g'*))}

**lemma** *gState-progress-rel-gState-invI1*[*intro*]:
(*g*,*g'*) ∈ *gState-progress-rel prog* ⟹ *gState-inv prog g*
⟨*proof*⟩

**lemma** *gState-progress-rel-gState-invI2*[*intro*]:
(*g*,*g'*) ∈ *gState-progress-rel prog* ⟹ *gState-inv prog g'*
⟨*proof*⟩

**lemma** *gState-progress-relI*:
**assumes** *gState-inv prog g*
**and** *gState-inv prog g'*
**and** *length* (*channels g*) ≤ *length* (*channels g'*)
**and** *dom* (*lm.α* (*vars g*)) = *dom* (*lm.α* (*vars g'*))
**shows** (*g*,*g'*) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *gState-progress-refl*[*simp*,*intro*!]:

*gState-inv prog g* $\Longrightarrow$ *(g,g)* $\in$ *(gState-progress-rel prog)*
⟨*proof*⟩

**lemma** *refl-on-gState-progress-rel*:
*refl-on (Collect (gState-inv prog)) (gState-progress-rel prog)*
⟨*proof*⟩

**lemma** *trans-gState-progress-rel*[*simp*]:
*trans (gState-progress-rel prog)*
⟨*proof*⟩

**lemmas** *gState-progress-rel-trans* [*trans*] = *trans-gState-progress-rel*[*THEN transD*]

**lemma** *gState-progress-rel-trancl-id*[*simp*]:
*(gState-progress-rel prog)*$^+$ = *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *gState-progress-rel-rtrancl-absorb*:
**assumes** *gState-inv prog g*
**shows** *(gState-progress-rel prog)*$^*$ `` {*g*} = *gState-progress-rel prog* `` {*g*}
⟨*proof*⟩

The main theorem: The set of all global states reachable from an initial state, is finite.

**lemma** *gStates-finite*:
**fixes** *g* :: *gState*
**shows** *finite ((gState-progress-rel prog)*$^*$ `` {*g*})
⟨*proof*⟩

**lemma** *gState-progress-rel-channels-update*:
**assumes** *gState-inv prog g*
**and** *channel-inv c*
**and** *i* < *length (channels g)*
**shows** *(g,gState.channels-update* (λ*cs. cs*[*i*:=*c*]) *g)* $\in$ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *gState-progress-rel-channels-update-step*:
**assumes** *gState-inv prog g*
**and** *step*: *(g,g′)* $\in$ *gState-progress-rel prog*
**and** *channel-inv c*
**and** *i* < *length (channels g′)*
**shows** *(g,gState.channels-update* (λ*cs. cs*[*i*:=*c*]) *g′)* $\in$ *gState-progress-rel prog*
⟨*proof*⟩

## 4.5 Invariants of the program

Naturally, we need our program to also adhere to certain invariants. Else we can't show, that the generated states are correct according to the invariants above.

**definition** *program-inv* **where**
*program-inv prog*
$\longleftrightarrow$ *IArray.length* (*states prog*) > *0*
$\wedge$ *IArray.length* (*states prog*) = *IArray.length* (*processes prog*)
$\wedge$ ($\forall$ *s* $\in$ *set* (*IArray.list-of* (*states prog*)). *IArray.length s* > *0*)
$\wedge$ *lm.ball* (*proc-data prog*)
($\lambda$(-,*sidx*).
*sidx* < *IArray.length* (*processes prog*)
$\wedge$ *fst* (*processes prog* !! *sidx*) = *sidx*)
$\wedge$ ($\forall$ (*sidx*,*start*,*procArgs*,*args*) $\in$ *set* (*IArray.list-of* (*processes prog*)).
($\exists$ *s*. *start* = *Index s* $\wedge$ *s* < *IArray.length* (*states prog* !! *sidx*)))

**lemma** *program-inv-length-states*:
**assumes** *program-inv prog*
**and** *n* < *IArray.length* (*states prog*)
**shows** *IArray.length* (*states prog* !! *n*) > *0*
⟨*proof*⟩

**lemma** *program-invI*:
**assumes** *0* < *IArray.length* (*states prog*)
**and** *IArray.length* (*states prog*) = *IArray.length* (*processes prog*)
**and** $\bigwedge$*s*. *s* $\in$ *set* (*IArray.list-of* (*states prog*))
$\Longrightarrow$ *0* < *IArray.length s*
**and** $\bigwedge$*sidx*. *sidx* $\in$ *ran* (*lm.α* (*proc-data prog*))
$\Longrightarrow$ *sidx* < *IArray.length* (*processes prog*)
$\wedge$ *fst* (*processes prog* !! *sidx*) = *sidx*
**and** $\bigwedge$*sidx start procArgs args*.
(*sidx*,*start*,*procArgs*,*args*) $\in$ *set* (*IArray.list-of* (*processes prog*))
$\Longrightarrow$ $\exists$ *s*. *start* = *Index s* $\wedge$ *s* < *IArray.length* (*states prog* !! *sidx*)
**shows** *program-inv prog*
⟨*proof*⟩

**end**

# 5 Formalization of Promela semantics

**theory** *Promela*
**imports**
*PromelaDatastructures*
*PromelaInvariants*
*PromelaStatistics*
**begin**

Auxiliary

**lemma** *mod-integer-le*:
‹*x mod* (*a* + *1*) ≤ *b*› **if** ‹*a* ≤ *b*› ‹*0* < *a*› **for** *a b x* :: *integer*
⟨*proof*⟩ **including** *integer.lifting* ⟨*proof*⟩

**lemma** *mod-integer-ge*:

‹$b \leq x \mod (a + 1)$› **if** ‹$b \leq 0$› ‹$0 < a$› **for** $a\ b\ x :: integer$
⟨*proof*⟩ **including** *integer.lifting* ⟨*proof*⟩

After having defined the datastructures, we present in this theory how to construct the transition system and how to generate the successors of a state, i. e. the real semantics of a Promela program. For the first task, we take the enriched AST as input, the second one operates on the transition system.

## 5.1 Misc Helpers

**definition** *add-label* :: *String.literal* ⇒ *labels* ⇒ *nat* ⇒ *labels* **where**
*add-label l lbls pos* = (
*case lm.lookup l lbls of*
*None* ⇒ *lm.update l pos lbls*
| *Some* - ⇒ *abortv STR ''Label given twice: '' l* (λ-. *lbls*))

**definition** *min-prio* :: *edge list* ⇒ *integer* ⇒ *integer* **where**
*min-prio es start* = *Min* ((*prio* ‘ *set es*) ∪ {*start*})

**lemma** *min-prio-code* [*code*]:
*min-prio es start* = *fold* (λ*e pri*. *if prio e* < *pri then prio e else pri*) *es start*
⟨*proof*⟩

**definition** *for-all* :: (*'a* ⇒ *bool*) ⇒ *'a list* ⇒ *bool* **where**
*for-all f xs* ⟷ (∀ *x* ∈ *set xs*. *f x*)

**lemma** *for-all-code*[*code*]:
*for-all f xs* ⟷ *foldli xs id* (λ*kv σ*. *f kv*) *True*
⟨*proof*⟩

**definition** *find-remove* :: (*'a* ⇒ *bool*) ⇒ *'a list* ⇒ *'a option* × *'a list* **where**
*find-remove P xs* = (*case List.find P xs of None* ⇒ (*None*, *xs*)
| *Some x* ⇒ (*Some x*, *List.remove1 x xs*))

**lemma** *find-remove-code* [*code*]:
*find-remove P* [] = (*None*, [])
*find-remove P* (*x*#*xs*) = (*if P x then* (*Some x*, *xs*)
*else apsnd* (*Cons x*) (*find-remove P xs*))
⟨*proof*⟩

**lemma** *find-remove-subset*:
*find-remove P xs* = (*res*, *xs'*) ⟹ *set xs'* ⊆ *set xs*
⟨*proof*⟩

**lemma** *find-remove-length*:
*find-remove P xs* = (*res*, *xs'*) ⟹ *length xs'* ≤ *length xs*
⟨*proof*⟩

## 5.2 Variable handling

Handling variables, with their different scopes (global vs. local), and their different types (array vs channel vs bounded) is one of the main challenges of the implementation.

```
fun lookupVar :: variable ⇒ integer option ⇒ integer where
  lookupVar (Var - val) None = val
| lookupVar (Var - -) (Some -) = abort STR ''Array used on var'' (λ-.0)
| lookupVar (VArray - - vals) None = vals !! 0
| lookupVar (VArray - siz vals) (Some idx) = vals !! nat-of-integer idx

primrec checkVarValue :: varType ⇒ integer ⇒ integer where
  checkVarValue (VTBounded lRange hRange) val = (
    if val ≤ hRange ∧ val ≥ lRange then val
    else — overflowing is well-defined and may actually be used (e.g. bool)
      if lRange = 0 ∧ val > 0
      then val mod (hRange + 1)
      else — we do not want to implement C-semantics (ie type casts)
        abort STR ''Value overflow'' (λ-. lRange))
| checkVarValue VTChan val = (
    if val < min-var-value ∨ val > max-var-value
    then abort STR ''Value overflow'' (λ-. 0)
    else val)

lemma [simp]:
  variable-inv (Var VTChan 0)
⟨proof⟩

context
  fixes type :: varType
  assumes varType-inv type
begin

lemma checkVarValue-bounded:
  checkVarValue type val ∈ {min-var-value..max-var-value}
  ⟨proof⟩

lemma checkVarValue-bounds:
  min-var-value ≤ checkVarValue type val
  checkVarValue type val ≤ max-var-value
  ⟨proof⟩

lemma checkVarValue-Var:
  variable-inv (Var type (checkVarValue type val))
  ⟨proof⟩

end

fun editVar :: variable ⇒ integer option ⇒ integer ⇒ variable where
```

```
  editVar (Var type - ) None val = Var type (checkVarValue type val)
| editVar (Var - -) (Some -) - = abort STR ''Array used on var'' (λ-. Var VTChan
0)
| editVar (VArray type siz vals) None val = (
    let lv = IArray.list-of vals in
    let v' = lv[0:=checkVarValue type val] in
    VArray type siz (IArray v'))
| editVar (VArray type siz vals) (Some idx) val = (
    let lv = IArray.list-of vals in
    let v' = lv[(nat-of-integer idx):=checkVarValue type val] in
    VArray type siz (IArray v'))

lemma editVar-variable-inv:
  assumes variable-inv v
  shows variable-inv (editVar v idx val)
⟨proof⟩

definition getVar'
  :: bool ⇒ String.literal ⇒ integer option
     ⇒ 'a gState-scheme ⇒ pState
     ⇒ integer option
where
  getVar' gl v idx g p = (
        let vars = if gl then gState.vars g else pState.vars p in
        map-option (λx. lookupVar x idx) (lm.lookup v vars))

definition setVar'
  :: bool ⇒ String.literal ⇒ integer option
     ⇒ integer
     ⇒ 'a gState-scheme ⇒ pState
     ⇒ 'a gState-scheme * pState
 where
  setVar' gl v idx val g p = (
    if gl then
      if v = STR ''-'' then (g,p) — ''-'' is a write-only scratch variable
      else case lm.lookup v (gState.vars g) of
             None ⇒ abortv STR ''Unknown global variable: '' v (λ-. (g,p))
           | Some x ⇒ (g(|gState.vars := lm.update v (editVar x idx val)
                                                   (gState.vars g)|)
                      , p)
    else
      case lm.lookup v (pState.vars p) of
        None ⇒ abortv STR ''Unknown proc variable: '' v (λ-. (g,p))
      | Some x ⇒ (g, p(|pState.vars := lm.update v (editVar x idx val)
                                                 (pState.vars p)|)))

lemma setVar'-gState-inv:
  assumes gState-inv prog g
  shows gState-inv prog (fst (setVar' gl v idx val g p))
```

⟨*proof*⟩

**lemma** *setVar'-gState-progress-rel*:
**assumes** *gState-inv prog g*
**shows** (*g*, *fst* (*setVar' gl v idx val g p*)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *vardict-inv-process-names*:
**assumes** *vardict-inv ss proc v*
**and** *lm.lookup k v* = *Some x*
**shows** *k* ∈ *process-names ss proc*
⟨*proof*⟩

**lemma** *vardict-inv-variable-inv*:
**assumes** *vardict-inv ss proc v*
**and** *lm.lookup k v* = *Some x*
**shows** *variable-inv x*
⟨*proof*⟩

**lemma** *vardict-inv-updateI*:
**assumes** *vardict-inv ss proc vs*
**and** *x* ∈ *process-names ss proc*
**and** *variable-inv v*
**shows** *vardict-inv ss proc* (*lm.update x v vs*)
⟨*proof*⟩

**lemma** *update-vardict-inv*:
**assumes** *vardict-inv ss proc v*
**and** *lm.lookup k v* = *Some x*
**and** *variable-inv x'*
**shows** *vardict-inv ss proc* (*lm.update k x' v*)
⟨*proof*⟩

**lemma** *setVar'-pState-inv*:
**assumes** *pState-inv prog p*
**shows** *pState-inv prog* (*snd* (*setVar' gl v idx val g p*))
⟨*proof*⟩

**lemma** *setVar'-cl-inv*:
**assumes** *cl-inv* (*g*,*p*)
**shows** *cl-inv* (*setVar' gl v idx val g p*)
⟨*proof*⟩

**definition** *withVar'*
:: *bool* ⇒ *String.literal* ⇒ *integer option*
⇒ (*integer* ⇒ *'x*)
⇒ *'a gState-scheme* ⇒ *pState*
⇒ *'x*
**where**

```
  withVar' gl v idx f g p = f (the (getVar' gl v idx g p))

definition withChannel'
  :: bool ⇒ String.literal ⇒ integer option
      ⇒ (nat ⇒ channel ⇒ 'x)
      ⇒ 'a gState-scheme ⇒ pState
      ⇒ 'x
where
  withChannel' gl v idx f g p = (
    let error = λ-. abortv STR ''Variable is not a channel: '' v
                          (λ-. f 0 InvChannel) in
    let abort = λ-. abortv STR ''Channel already closed / invalid: '' v
                          (λ-. f 0 InvChannel)
    in withVar' gl v idx (λi. let i = nat-of-integer i in
                          if i ≥ length (channels g) then error ()
                          else let c = channels g ! i in
                               case c of
                                 InvChannel ⇒ abort ()
                               | - ⇒ f i c) g p)
```

## 5.3 Expressions

Expressions are free of side-effects.

This is in difference to SPIN, where *run* is an expression with side-effect. We treat *run* as a statement.

```
abbreviation trivCond x ≡ if x then 1 else 0

fun exprArith :: 'a gState-scheme ⇒ pState ⇒ expr ⇒ integer
and pollCheck :: 'a gState-scheme ⇒ pState ⇒ channel ⇒ recvArg list ⇒ bool
                ⇒ bool
and recvArgsCheck :: 'a gState-scheme ⇒ pState ⇒ recvArg list ⇒ integer list
                    ⇒ bool
where
  exprArith g p (ExprConst x) = x
| exprArith g p (ExprMConst x -) = x

| exprArith g p ExprTimeOut = trivCond (timeout g)

| exprArith g p (ExprLen (ChanRef (VarRef gl name None))) =
    withChannel' gl name None (
      λ- c. case c of
              Channel - - q ⇒ integer-of-nat (length q)
            | HSChannel - ⇒ 0) g p

| exprArith g p (ExprLen (ChanRef (VarRef gl name (Some idx)))) =
    withChannel' gl name (Some (exprArith g p idx)) (
     λ- c. case c of
             Channel - - q ⇒ integer-of-nat (length q)
```

```
        | HSChannel - ⇒ 0) g p

| exprArith g p (ExprEmpty (ChanRef (VarRef gl name None))) =
    trivCond (withChannel' gl name None (
      λ- c. case c of Channel - - q ⇒ (q = [])
                    | HSChannel - ⇒ True) g p)

| exprArith g p (ExprEmpty (ChanRef (VarRef gl name (Some idx)))) =
    trivCond (withChannel' gl name (Some (exprArith g p idx)) (
      λ- c. case c of Channel - - q ⇒ (q = [])
                    | HSChannel - ⇒ True) g p)

| exprArith g p (ExprFull (ChanRef(VarRef gl name None))) =
    trivCond (withChannel' gl name None (
      λ- c. case c of
              Channel cap - q ⇒ integer-of-nat (length q) ≥ cap
            | HSChannel - ⇒ False) g p)

| exprArith g p (ExprFull (ChanRef(VarRef gl name (Some idx)))) =
    trivCond (withChannel' gl name (Some (exprArith g p idx)) (
      λ- c. case c of
              Channel cap - q ⇒ integer-of-nat (length q) ≥ cap
            | HSChannel - ⇒ False) g p)

| exprArith g p (ExprVarRef (VarRef gl name None)) =
    withVar' gl name None id g p

| exprArith g p (ExprVarRef (VarRef gl name (Some idx))) =
    withVar' gl name (Some (exprArith g p idx)) id g p

| exprArith g p (ExprUnOp UnOpMinus expr) = 0 − exprArith g p expr
| exprArith g p (ExprUnOp UnOpNeg expr) = ((exprArith g p expr) + 1) mod 2

| exprArith g p (ExprBinOp BinOpAdd lexpr rexpr) =
    (exprArith g p lexpr) + (exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpSub lexpr rexpr) =
    (exprArith g p lexpr) − (exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpMul lexpr rexpr) =
    (exprArith g p lexpr) ∗ (exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpDiv lexpr rexpr) =
    (exprArith g p lexpr) div (exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpMod lexpr rexpr) =
    (exprArith g p lexpr) mod (exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpGr lexpr rexpr) =
```

```
    trivCond (exprArith g p lexpr > exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpLe lexpr rexpr) =
    trivCond (exprArith g p lexpr < exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpGEq lexpr rexpr) =
    trivCond (exprArith g p lexpr ≥ exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpLEq lexpr rexpr) =
    trivCond (exprArith g p lexpr ≤ exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpEq lexpr rexpr) =
    trivCond (exprArith g p lexpr = exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpNEq lexpr rexpr) =
    trivCond (exprArith g p lexpr ≠ exprArith g p rexpr)

| exprArith g p (ExprBinOp BinOpAnd lexpr rexpr) =
    trivCond (exprArith g p lexpr ≠ 0 ∧ exprArith g p rexpr ≠ 0)

| exprArith g p (ExprBinOp BinOpOr lexpr rexpr) =
    trivCond (exprArith g p lexpr ≠ 0 ∨ exprArith g p rexpr ≠ 0)

| exprArith g p (ExprCond cexpr texpr fexpr) =
    (if exprArith g p cexpr ≠ 0 then exprArith g p texpr
     else exprArith g p fexpr)

| exprArith g p (ExprPoll (ChanRef (VarRef gl name None)) rs srt) =
    trivCond (withChannel' gl name None (
      λ- c. pollCheck g p c rs srt) g p)

| exprArith g p (ExprPoll (ChanRef (VarRef gl name (Some idx))) rs srt) =
    trivCond (withChannel' gl name (Some (exprArith g p idx)) (
      λ- c. pollCheck g p c rs srt) g p)

| pollCheck g p InvChannel - - =
    abort STR ''Channel already closed / invalid.'' (λ-. False)
| pollCheck g p (HSChannel -) - - = False
| pollCheck g p (Channel - - q) rs srt = (
    if q = [] then False
    else if ¬ srt then recvArgsCheck g p rs (hd q)
    else List.find (recvArgsCheck g p rs) q ≠ None)

| recvArgsCheck - - [] [] = True
| recvArgsCheck - - - [] =
    abort STR ''Length mismatch on receiving.'' (λ-. False)
| recvArgsCheck - - [] - =
    abort STR ''Length mismatch on receiving.'' (λ-. False)
| recvArgsCheck g p (r#rs) (v#vs) = ((
```

```
    case r of
      RecvArgConst c ⇒ c = v
    | RecvArgMConst c - ⇒ c = v
    | RecvArgVar var ⇒ True
    | RecvArgEval e ⇒ exprArith g p e = v ) ∧ recvArgsCheck g p rs vs)
```

*getVar′* etc. do operate on name, index, ... directly. Lift them to use *VarRef* instead.

```
fun liftVar where
  liftVar f (VarRef gl v idx) argm g p =
     f gl v (map-option (exprArith g p) idx) argm g p

definition getVar v = liftVar (λgl v idx arg. getVar′ gl v idx) v ()
definition setVar = liftVar setVar′
definition withVar = liftVar withVar′

primrec withChannel
  where withChannel (ChanRef v) = liftVar withChannel′ v

lemma setVar-gState-progress-rel:
  assumes gState-inv prog g
  shows (g, fst (setVar v val g p)) ∈ gState-progress-rel prog
⟨proof⟩

lemmas setVar-gState-inv =
  setVar-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma setVar-pState-inv:
  assumes pState-inv prog p
  shows pState-inv prog (snd (setVar v val g p))
⟨proof⟩

lemma setVar-cl-inv:
  assumes cl-inv (g,p)
  shows cl-inv (setVar v val g p)
⟨proof⟩
```

## 5.4 Variable declaration

```
lemma channel-inv-code [code]:
  channel-inv (Channel cap ts q)
  ⟷ cap ≤ max-array-size
    ∧ 0 ≤ cap
    ∧ for-all varType-inv ts
    ∧ length ts ≤ max-array-size
    ∧ length q ≤ max-array-size
    ∧ for-all (λx. length x = length ts
                 ∧ for-all (λy. y ≥ min-var-value
                              ∧ y ≤ max-var-value) x) q
```

```
  channel-inv (HSChannel ts)
  ⟷ for-all varType-inv ts ∧ length ts ≤ max-array-size
  ⟨proof⟩

primrec toVariable
  :: 'a gState-scheme ⇒ pState ⇒ varDecl ⇒ String.literal ∗ variable ∗ channels
where
  toVariable g p (VarDeclNum lb hb name siz init) = (
    let type = VTBounded lb hb in
     if ¬ varType-inv type then abortv STR ''Invalid var def (varType-inv failed):
'' name
                                  (λ-. (name, Var VTChan 0, []))
    else
      let
        init = checkVarValue type (case init of
                                     None ⇒ 0
                                   | Some e ⇒ exprArith g p e);
        v = (case siz of
               None ⇒ Var type init
             | Some s ⇒ if nat-of-integer s ≤ max-array-size
                        then VArray type (nat-of-integer s)
                                    (IArray.tabulate (s, λ-. init))
                        else abortv STR ''Invalid var def (array too large): '' name
                                    (λ-. Var VTChan 0))
       in
         (name, v, []))

| toVariable g p (VarDeclChan name siz types) = (
    let
      size = (case siz of None ⇒ 1 | Some s ⇒ nat-of-integer s);
      chans = (case types of
                 None ⇒ []
               | Some (cap, tys) ⇒
                   let C = (if cap = 0 then HSChannel tys
                            else Channel cap tys []) in
                   if ¬ channel-inv C
                   then abortv STR ''Invalid var def (channel-inv failed): ''
                                name (λ-. [])
                   else replicate size C);
      cidx = (case types of
                None ⇒ 0
              | Some - ⇒ integer-of-nat (length (channels g)));
      v = (case siz of
             None ⇒ Var VTChan cidx
           | Some s ⇒ if nat-of-integer s ≤ max-array-size
                      then VArray VTChan (nat-of-integer s)
                                  (IArray.tabulate (s,
                                    λi. if cidx = 0 then 0
                                        else i + cidx))
```

```
                else abortv STR ''Invalid var def (array too large): ''
                     name (λ-. Var VTChan 0))
    in
      (name, v, chans))

lemma toVariable-variable-inv:
  assumes gState-inv prog g
  shows variable-inv (fst (snd (toVariable g p v)))
⟨proof⟩
    including integer.lifting
    ⟨proof⟩

lemma toVariable-channels-inv:
  shows ∀ x ∈ set (snd (snd (toVariable g p v))). channel-inv x
⟨proof⟩

lemma toVariable-channels-inv':
  shows toVariable g p v = (a,b,c) ⟹ ∀ x ∈ set c. channel-inv x
⟨proof⟩

lemma toVariable-variable-inv':
  shows gState-inv prog g ⟹ toVariable g p v = (a,b,c) ⟹ variable-inv b
⟨proof⟩

definition mkChannels
  :: 'a gState-scheme ⇒ pState ⇒ channels ⇒ 'a gState-scheme * pState
 where
  mkChannels g p cs = (
    if cs = [] then (g,p) else
    let l = length (channels g) in
    if l + length cs > max-channels
    then abort STR ''Too much channels'' (λ-.  (g,p))
    else let
          cs_p = map integer-of-nat [l..<l + length cs];
          g' = g(|channels := channels g @ cs|);
          p' = p(|pState.channels := pState.channels p @ cs_p|)
         in
          (g', p'))

lemma mkChannels-gState-progress-rel:
  gState-inv prog g
   ⟹ set cs ⊆ Collect channel-inv
   ⟹ (g, fst (mkChannels g p cs)) ∈ gState-progress-rel prog
⟨proof⟩

lemmas mkChannels-gState-inv =
  mkChannels-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma mkChannels-pState-inv:
```

*pState-inv prog p*
⟹ *cl-inv (g,p)*
⟹ *pState-inv prog (snd (mkChannels g p cs))*
⟨*proof*⟩
**including** *integer.lifting*
⟨*proof*⟩

**lemma** *mkChannels-cl-inv*:
*cl-inv (g,p)* ⟹ *cl-inv (mkChannels g p cs)*
⟨*proof*⟩

**definition** *mkVarChannel*
:: *varDecl*
⇒ ((*var-dict* ⇒ *var-dict*) ⇒ *'a gState-scheme* ∗ *pState*
⇒ *'a gState-scheme* ∗ *pState*)
⇒ *'a gState-scheme* ⇒ *pState*
⇒ *'a gState-scheme* ∗ *pState*
**where**
*mkVarChannel v upd g p* = (
*let*
(*k*,*v*,*cs*) = *toVariable g p v*;
(*g'*,*p'*) = *upd* (*lm.update k v*) (*g*,*p*)
*in*
*mkChannels g' p' cs*)

**lemma** *mkVarChannel-gState-inv*:
**assumes** *gState-inv prog g*
**and** ⋀*k v' cs*. *toVariable g p v* = (*k*,*v'*,*cs*)
⟹ *gState-inv prog* (*fst* (*upd* (*lm.update k v'*) (*g*,*p*)))
**shows** *gState-inv prog* (*fst* (*mkVarChannel v upd g p*))
⟨*proof*⟩

**lemma** *mkVarChannel-gState-progress-rel*:
**assumes** *gState-inv prog g*
**and** ⋀*k v' cs*. *toVariable g p v* = (*k*,*v'*,*cs*)
⟹ (*g*, *fst* (*upd* (*lm.update k v'*) (*g*,*p*))) ∈ *gState-progress-rel prog*
**shows** (*g*, *fst* (*mkVarChannel v upd g p*)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *mkVarChannel-pState-inv*:
**assumes** *pState-inv prog p*
**and** *cl-inv (g,p)*
**and** ⋀*k v' cs*. *toVariable g p v* = (*k*,*v'*,*cs*)
⟹ *cl-inv* (*upd* (*lm.update k v'*) (*g*,*p*))
**and** ⋀*k v' cs*. *toVariable g p v* = (*k*,*v'*,*cs*)
⟹ *pState-inv prog* (*snd* (*upd* (*lm.update k v'*) (*g*,*p*)))
**shows** *pState-inv prog* (*snd* (*mkVarChannel v upd g p*))
⟨*proof*⟩

**lemma** *mkVarChannel-cl-inv*:
**assumes** *cl-inv* (*g*,*p*)
**and** $\bigwedge k\ v'\ cs.$ *toVariable g p v* = (*k*,*v'*,*cs*)
$\Longrightarrow$ *cl-inv* (*upd* (*lm.update k v'*) (*g*,*p*))
**shows** *cl-inv* (*mkVarChannel v upd g p*)
⟨*proof*⟩

**definition** *mkVarChannelProc*
:: *procVarDecl* ⇒ *'a gState-scheme* ⇒ *pState* ⇒ *'a gState-scheme* ∗ *pState*
**where**
*mkVarChannelProc v g p* = (
*let*
*v'* = *case v of*
*ProcVarDeclNum lb hb name siz init* ⇒
*VarDeclNum lb hb name siz init*
| *ProcVarDeclChan name siz* ⇒
*VarDeclChan name siz None*;
(*k*,*v*,*cs*) = *toVariable g p v'*
*in*
*mkVarChannel v'* (*apsnd* ∘ *pState.vars-update*) *g p*)

**lemma** *mkVarChannelProc-gState-progress-rel*:
**assumes** *gState-inv prog g*
**shows** (*g*, *fst* (*mkVarChannelProc v g p*)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemmas** *mkVarChannelProc-gState-inv* =
*mkVarChannelProc-gState-progress-rel*[*THEN gState-progress-rel-gState-invI2*]

**lemma** *toVariable-name*:
*toVariable g p* (*VarDeclNum lb hb name sz init*) = (*x*,*a*,*b*) $\Longrightarrow$ *x* = *name*
*toVariable g p* (*VarDeclChan name sz t*) = (*x*, *a*, *b*) $\Longrightarrow$ *x* = *name*
⟨*proof*⟩

**declare** *toVariable.simps*[*simp del*]

**lemma** *statesDecls-process-names*:
**assumes** *v* ∈ *statesDecls* (*states prog* !! (*pState.idx p*))
**shows** *procVarDeclName v* ∈ *process-names* (*states prog* !! (*pState.idx p*))
(*processes prog* !! (*pState.idx p*))
⟨*proof*⟩

**lemma** *mkVarChannelProc-pState-inv*:
**assumes** *pState-inv prog p*
**and** *gState-inv prog g*
**and** *cl-inv* (*g*, *p*)
**and** *decl*: *v* ∈ *statesDecls* (*states prog* !! (*pState.idx p*))
**shows** *pState-inv prog* (*snd* (*mkVarChannelProc v g p*))
⟨*proof*⟩

**lemma** *mkVarChannelProc-cl-inv*:
**assumes** *cl-inv* (*g*,*p*)
**shows** *cl-inv* (*mkVarChannelProc v g p*)
⟨*proof*⟩

## 5.5 Folding

Fold over lists (and lists of lists) of *step*/*stmnt*. The folding functions are doing a bit more than that, e.g. ensuring the offset into the program array is correct.

**definition** *step-fold'* **where**
*step-fold' g steps* (*lbls* :: *labels*) *pri pos*
(*nxt* :: *edgeIndex*) (*onxt* :: *edgeIndex option*) *iB* =
*foldr* (λ*step* (*pos*, *nxt*, *lbls*, *es*).
*let* (*e*,*enxt*,*lbls*) = *g step* (*lbls*, *pri*, *pos*, *nxt*, *onxt*, *iB*)
*in* (*pos* + *length e*, *enxt*, *lbls*, *es*@*e*)
) *steps* (*pos*, *nxt*, *lbls*, []))

**definition** *step-fold* **where**
*step-fold g steps lbls pri pos nxt onxt iB* = (
*let* (-,*nxt*,*lbls*,*s*) = *step-fold' g steps lbls pri pos nxt onxt iB*
*in* (*s*,*nxt*,*lbls*))

**lemma** *step-fold'-cong*:
**assumes** *lbls* = *lbls'*
**and** *pri* = *pri'*
**and** *pos* = *pos'*
**and** *steps* = *steps'*
**and** *nxt* = *nxt'*
**and** *onxt* = *onxt'*
**and** *iB* = *iB'*
**and** ⋀*x d*. *x* ∈ *set steps* ⟹ *g x d* = *g' x d*
**shows** *step-fold' g steps lbls pri pos nxt onxt iB* =
*step-fold' g' steps' lbls' pri' pos' nxt' onxt' iB'*
⟨*proof*⟩

**lemma** *step-fold-cong*[*fundef-cong*]:
**assumes** *lbls* = *lbls'*
**and** *pri* = *pri'*
**and** *pos* = *pos'*
**and** *steps* = *steps'*
**and** *nxt* = *nxt'*
**and** *onxt* = *onxt'*
**and** *iB* = *iB'*
**and** ⋀*x d*. *x* ∈ *set steps* ⟹ *g x d* = *g' x d*
**shows** *step-fold g steps lbls pri pos nxt onxt iB* =
*step-fold g' steps' lbls' pri' pos' nxt' onxt' iB'*
⟨*proof*⟩

**fun** *step-foldL-step* **where**
  *step-foldL-step - - - [] (pos, nxt, lbls, es, is) = (pos, nxt, lbls, es, is)*
| *step-foldL-step g pri onxt (s#steps) (pos, nxt, lbls, es, is) = (*
    *let (pos′, nxt′, lbls′, ss′) = step-fold′ g steps lbls pri pos nxt onxt False in*
    *let (s′, nxt′′, lbls′′) = g s (lbls′,pri,pos′,nxt′,onxt,True) in*
    *let rs = butlast s′; s′′ = last s′ in*
    *(pos′ + length rs, nxt, lbls′′, es@ss′@rs, s′′#is))*

**definition** *step-foldL* **where**
  *step-foldL g stepss lbls pri pos nxt onxt =*
    *foldr (step-foldL-step g pri onxt) stepss (pos,nxt,lbls,[],[])*

**lemma** *step-foldL-step-cong*:
  **assumes** *pri = pri′*
  **and** *onxt = onxt′*
  **and** *s = s′*
  **and** *d = d′*
  **and** $\bigwedge x\ d.\ x \in set\ s \Longrightarrow g\ x\ d = g′\ x\ d$
  **shows** *step-foldL-step g pri onxt s d = step-foldL-step g′ pri′ onxt′ s′ d′*
⟨*proof*⟩

**lemma** *step-foldL-cong*[*fundef-cong*]:
  **assumes** *lbls = lbls′*
  **and** *pri = pri′*
  **and** *pos = pos′*
  **and** *stepss = stepss′*
  **and** *nxt = nxt′*
  **and** *onxt = onxt′*
  **and** $\bigwedge x\ x′\ d.\ x \in set\ stepss \Longrightarrow x′ \in set\ x \Longrightarrow g\ x′\ d = g′\ x′\ d$
  **shows** *step-foldL g stepss lbls pri pos nxt onxt =*
      *step-foldL g′ stepss′ lbls′ pri′ pos′ nxt′ onxt′*
⟨*proof*⟩

## 5.6 Starting processes

**definition** *modProcArg*
  :: *(procArg ∗ integer) ⇒ String.literal ∗ variable*
**where**
  *modProcArg x = (*
    *case x of*
      *(ProcArg ty name, val) ⇒ if varType-inv ty*
                  *then let init = checkVarValue ty val*
                    *in (name, Var ty init)*
                  *else abortv STR ''Invalid proc arg (varType-inv failed)''*
                    *name (λ-. (name, Var VTChan 0)))*

**definition** *emptyProc* :: *pState*
  — The empty process.

```
where
  emptyProc = (|pid = 0, vars = lm.empty (), pc = 0, channels = [], idx = 0 |)

lemma vardict-inv-empty:
  vardict-inv ss proc (lm.empty())
⟨proof⟩

lemma emptyProc-cl-inv[simp]:
  cl-inv (g, emptyProc)
⟨proof⟩

lemma emptyProc-pState-inv:
  assumes program-inv prog
  shows pState-inv prog emptyProc
⟨proof⟩

fun mkProc
  :: 'a gState-scheme ⇒ pState
    ⇒ String.literal ⇒ expr list ⇒ process ⇒ nat
    ⇒ 'a gState-scheme * pState
where
  mkProc g p name args (sidx, start, argDecls, decls) pidN = (
    let start = case start of
                  Index x ⇒ x
                | - ⇒ abortv STR ''Process start is not index: '' name (λ-. 0)
    in
     — sanity check
     if length args ≠ length argDecls
     then abortv STR ''Signature mismatch: '' name (λ-. (g,emptyProc))
     else
       let
         — evaluate args (in the context of the calling process)
         eArgs = map (exprArith g p) args;

         — replace the init part of argDecls
         argVars = map modProcArg (zip argDecls eArgs);

         — add -pid to vars
         pidI = integer-of-nat pidN;
         argVars = (STR ''-pid'', Var (VTBounded 0 pidI) pidI)#argVars;
         argVars = lm.to-map argVars;

         — our new process
        p = (| pid = pidN, vars = argVars, pc = start, channels = [], idx = sidx |)
       in
         — apply the declarations
         foldl (λ(g,p) d. mkVarChannel d (apsnd ∘ pState.vars-update) g p)
               (g,p)
               decls)
```

**lemma** *mkProc-gState-progress-rel*:
**assumes** *gState-inv prog g*
**shows** (*g*, *fst* (*mkProc g p name args* (*processes prog* !! *sidx*) *pidN*)) ∈
*gState-progress-rel prog*
⟨*proof*⟩

**lemmas** *mkProc-gState-inv* =
*mkProc-gState-progress-rel*[*THEN gState-progress-rel-gState-invI2*]

**lemma** *mkProc-pState-inv*:
**assumes** *program-inv prog*
**and** *gState-inv prog g*
**and** *pidN* ≤ *max-procs* **and** *pidN* > *0*
**and** *sidx* < *IArray.length* (*processes prog*)
**and** *fst* (*processes prog* !! *sidx*) = *sidx*
**shows** *pState-inv prog* (*snd* (*mkProc g p name args* (*processes prog* !! *sidx*) *pidN*))
⟨*proof*⟩
**including** *integer.lifting*
⟨*proof*⟩

**lemma** *mkProc-cl-inv*:
**assumes** *cl-inv* (*g*,*p*)
**shows** *cl-inv* (*mkProc g p name args* (*processes prog* !! *sidx*) *pidN*)
⟨*proof*⟩

**declare** *mkProc.simps*[*simp del*]

**definition** *runProc*
:: *String.literal* ⇒ *expr list* ⇒ *program*
⇒ *'a gState-scheme* ⇒ *pState*
⇒ *'a gState-scheme* ∗ *pState*
**where**
*runProc name args prog g p* = (
*if length* (*procs g*) ≥ *max-procs*
*then abort STR ''Too many processes''* (λ-. (*g*,*p*))
*else let pid* = *length* (*procs g*) + *1 in*
*case lm.lookup name* (*proc-data prog*) *of*
*None* ⇒ *abortv STR ''No such process: '' name*
(λ-. (*g*,*p*))
| *Some proc-idx* ⇒
*let* (*g'*, *proc*) = *mkProc g p name args* (*processes prog* !! *proc-idx*) *pid*
*in* (*g'*(|*procs* := *procs g* @ [*proc*]|), *p*))

**lemma** *runProc-gState-progress-rel*:
**assumes** *program-inv prog*
**and** *gState-inv prog g*
**and** *pState-inv prog p*
**and** *cl-inv* (*g*,*p*)

**shows** (*g*, *fst* (*runProc name args prog g p*)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemmas** *runProc-gState-inv* =
*runProc-gState-progress-rel*[*THEN gState-progress-rel-gState-invI2*]

**lemma** *runProc-pState-id*:
*snd* (*runProc name args prog g p*) = *p*
⟨*proof*⟩

**lemma** *runProc-pState-inv*:
**assumes** *pState-inv prog p*
**shows** *pState-inv prog* (*snd* (*runProc name args prog g p*))
⟨*proof*⟩

**lemma** *runProc-cl-inv*:
**assumes** *program-inv prog*
**and** *gState-inv prog g*
**and** *pState-inv prog p*
**and** *cl-inv* (*g*,*p*)
**shows** *cl-inv* (*runProc name args prog g p*)
⟨*proof*⟩

## 5.7 AST to edges

**type-synonym** *ast* = *AST.module list*

In this section, the AST is translated into the transition system.

Handling atomic blocks is non-trivial. Therefore, we do this in an extra pass: *lp* and *hp* are the positions of the start and the end of the atomic block. Every edge pointing into this range is therefore marked as *Atomic*. If they are pointing somewhere else, they are set to *InAtomic*, meaning: they start *in* an atomic block, but leave it afterwards.

**definition** *atomize* :: *nat* ⇒ *nat* ⇒ *edge list* ⇒ *edge list* **where**
*atomize lp hp es* = *fold* (λ*e es*.
*let e′* = *case target e of*
*LabelJump - None* ⇒
— Labels are checked again later on, when they
— are going to be resolved. Hence it is safe to say
— *atomic* here, especially as the later algorithm
— relies on targets in atomic blocks to be marked as such.
*e*(| *atomic* := *InAtomic* |)
| *LabelJump - (Some via)* ⇒
*if lp* ≤ *via* ∧ *hp* ≥ *via then e*(| *atomic* := *Atomic* |)
*else e*(| *atomic* := *InAtomic* |)
| *Index p′* ⇒
*if lp* ≤ *p′* ∧ *hp* ≥ *p′ then e*(| *atomic* := *Atomic* |)
*else e*(| *atomic* := *InAtomic* |)

```
    in e'#es) es []

fun skip — No-(edge)
where
  skip (lbls, pri, pos, nxt, -) =
  ([[(|cond = ECExpr (ExprConst 1),
      effect = EEId, target = nxt, prio = pri,
      atomic = NonAtomic|)]], Index pos, lbls)
```

The AST is walked backwards. This allows to know the next state directly. Parameters used:

**lbls** Map of Labels

**pri** Current priority

**pos** Current position in the array

**nxt** Next state

**onxt** Previous 'next state' (where to jump after a 'do')

**inBlock** Needed for certain constructs to calculate the layout of the array

```
fun stepToState
  :: step
      ⇒ (labels * integer * nat * edgeIndex * edgeIndex option * bool)
      ⇒ edge list list * edgeIndex * labels
and stmntToState
  :: stmnt
      ⇒ (labels * integer * nat * edgeIndex * edgeIndex option * bool)
      ⇒ edge list list * edgeIndex * labels
where
  stepToState (StepStmnt s None) data = stmntToState s data
| stepToState (StepStmnt s (Some u)) (lbls, pri, pos, nxt, onxt, -) = (
    let
      — the unless part
      (ues,-,lbls') = stmntToState u (lbls, pri, pos, nxt, onxt, True);
      u = last ues; ues = butlast ues;
      pos' = pos + length ues;

      — find minimal current priority
      pri = min-prio u pri;

      — the guarded part –
      — priority is decreased, because there is now a new unless part with
      — higher prio
      (ses,spos,lbls'') = stmntToState s (lbls', pri − 1, pos', nxt, onxt, False);

      — add an edge to the unless part for each generated state
```

```
      ses = map (List.append u) ses
    in
      (ues@ses,spos,lbls'))

| stepToState (StepDecl decls) (lbls, pri, pos, nxt, onxt, -) = (
    let edgeF = λd (lbls,pri,pos,nxt,-).
      ([[(|cond = ECTrue, effect = EEDecl d, target = nxt,
           prio = pri, atomic = NonAtomic|)]], Index pos, lbls)
    in
      step-fold edgeF decls lbls pri pos nxt onxt False)

| stepToState StepSkip (lbls,-,-,nxt,-) = ([],nxt,lbls)

| stmntToState (StmntAtomic steps) (lbls, pri, pos, nxt, onxt, inBlock) = (
    let (es,pos',lbls') = step-fold stepToState steps lbls pri pos nxt onxt inBlock in
    let es' = map (atomize pos (pos + length es)) es in
    (es', pos', lbls'))

| stmntToState (StmntLabeled l s) (lbls, pri, pos, d) = (
    let
        (es, pos', lbls) = stmntToState s (lbls, pri, pos, d);

        — We don't resolve goto-chains. If the labeled stmnt returns only a jump,
        — use this goto state.
        lpos = case pos' of Index p ⇒ p | - ⇒ pos;
        lbls' = add-label l lbls lpos
    in
      (es, pos', lbls'))

| stmntToState (StmntDo stepss) (lbls, pri, pos, nxt, onxt, inBlock) = (
    let
      — construct the different branches
      — nxt in those branches points current pos (it is a loop after all)
      — onxt then is the current nxt (needed for break, f.ex.)
      (-,-,lbls,es,is) = step-foldL stepToState stepss lbls pri
                                    (pos+1) (Index pos) (Some nxt);

      — put the branch starting points (is) into the array
      es' = concat is # es
    in
      if inBlock then
          — inside another DO or IF or UNLESS
          — ⟶ append branches again, so they can be consumed
          (es' @ [concat is], Index pos, lbls)
      else
          (es', Index pos, lbls)
  )

| stmntToState (StmntIf stepss) (lbls, pri, pos, nxt, onxt, -) = (
```

```
    let (pos,-,lbls,es,is) = step-foldL stepToState stepss lbls pri pos nxt onxt
    in (es @ [concat is], Index pos, lbls))

| stmntToState (StmntSeq steps) (lbls, pri, pos, nxt, onxt, inBlock) =
    step-fold stepToState steps lbls pri pos nxt onxt inBlock


| stmntToState (StmntAssign v e) (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECTrue, effect = EEAssign v e, target = nxt, prio = pri,
      atomic = NonAtomic⦈]], Index pos, lbls)

| stmntToState (StmntAssert e) (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECTrue, effect = EEAssert e, target = nxt, prio = pri,
      atomic = NonAtomic⦈]], Index pos, lbls)

| stmntToState (StmntCond e) (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECExpr e, effect = EEId, target = nxt, prio = pri,
      atomic = NonAtomic⦈]], Index pos, lbls)

| stmntToState StmntElse (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECElse, effect = EEId, target = nxt, prio = pri,
      atomic = NonAtomic ⦈]], Index pos, lbls)

| stmntToState StmntBreak (lbls,pri,-,-,Some onxt,-) =
  ([[⦇cond = ECTrue, effect = EEGoto, target = onxt, prio = pri,
      atomic = NonAtomic ⦈]], onxt, lbls)
| stmntToState StmntBreak (-,-,-,-,None,-) =
  abort STR ''Misplaced break'' (λ-. ([],Index 0,lm.empty()))

| stmntToState (StmntRun n args) (lbls, pri, pos, nxt, onxt, -) =
  ([[⦇cond = ECRun n, effect = EERun n args, target = nxt, prio = pri,
      atomic = NonAtomic ⦈]], Index pos,lbls)

| stmntToState (StmntGoTo l) (lbls, pri, pos, -) =
  ([[⦇cond = ECTrue, effect = EEGoto, target = LabelJump l None, prio = pri,
      atomic = NonAtomic ⦈]], LabelJump l (Some pos), lbls)

| stmntToState (StmntSend v e srt) (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECSend v, effect = EESend v e srt, target = nxt, prio = pri,
      atomic = NonAtomic ⦈]], Index pos, lbls)

| stmntToState (StmntRecv v r srt rem) (lbls, pri, pos, nxt, -) =
  ([[⦇cond = ECRecv v r srt, effect = EERecv v r srt rem, target = nxt, prio =
pri,
      atomic = NonAtomic ⦈]], Index pos, lbls)

| stmntToState StmntSkip d = skip d
```

### 5.7.1 Setup

```
definition endState :: edge list where
  — An extra state added to each process marking its end.
  endState = [(| cond = ECFalse, effect = EEEnd, target = Index 0, prio = 0,
                 atomic = NonAtomic |)]

definition resolveLabel :: String.literal ⇒ labels ⇒ nat where
  resolveLabel l lbls = (
    case lm.lookup l lbls of
      None ⇒ abortv STR ''Unresolved label: '' l (λ-. 0)
    | Some pos ⇒ pos)

primrec resolveLabels :: edge list list ⇒ labels ⇒ edge list ⇒ edge list where
  resolveLabels - - [] = []
| resolveLabels edges lbls (e#es) = (
    let check-atomic = λpos. fold (λe a. a ∧ inAtomic e) (edges ! pos) True in
    case target e of
      Index - ⇒ e
    | LabelJump l None ⇒
        let pos = resolveLabel l lbls in
          e(|target := Index pos,
             atomic := if inAtomic e then
                         if check-atomic pos then Atomic
                         else InAtomic
                       else NonAtomic |)
    | LabelJump l (Some via) ⇒
        let pos = resolveLabel l lbls in
          e(|target := Index pos,
             — NB: isAtomic instead of inAtomic, cf atomize()
             atomic := if isAtomic e then
                         if check-atomic pos ∧ check-atomic via then Atomic
                         else InAtomic
                       else atomic e |)
  ) # (resolveLabels edges lbls es)

definition calculatePrios :: edge list list ⇒ (integer * edge list) list where
  calculatePrios ess = map (λes. (min-prio es 0, es)) ess

definition toStates :: step list ⇒ states * edgeIndex * labels where
  toStates steps = (
    let
      (states,pos,lbls) = step-fold stepToState steps (lm.empty())
                                    0 1 (Index 0) None False;
      pos = (case pos of
               Index - ⇒ pos
             | LabelJump l - ⇒ Index (resolveLabel l lbls));
      states = endState # states;
      states = map (resolveLabels states lbls) states;
      states = calculatePrios states
```

```
    in
    case pos of Index s ⇒
          if s < length states then (IArray states, pos, lbls)
           else abort STR ''Start index out of bounds'' (λ-. (IArray states, Index 0,
lbls)))

lemma toStates-inv:
  assumes toStates steps = (ss,start,lbls)
  shows ∃ s. start = Index s ∧ s < IArray.length ss
    and IArray.length ss > 0
  ⟨proof⟩


primrec toProcess
  :: nat ⇒ proc ⇒ states * nat * String.literal * (labels * process)
 where
  toProcess sidx (ProcType act name args decls steps) = (
     let
       (states, start, lbls) = toStates steps;
       act = (case act of
                None ⇒ 0
              | Some None ⇒ 1
              | Some (Some x) ⇒ nat-of-integer x)
     in
        (states, act, name, lbls, sidx, start, args, decls))
| toProcess sidx (Init decls steps) = (
      let (states, start, lbls) = toStates steps in
      (states, 1, STR '':init:'', lbls, sidx, start, [], decls))

lemma toProcess-sidx:
  toProcess sidx p = (ss,a,n,l,idx,r) ⟹ idx = sidx
⟨proof⟩

lemma toProcess-states-nonempty:
  toProcess sidx p = (ss,a,n,l,idx,r) ⟹ IArray.length ss > 0
⟨proof⟩

lemma toProcess-start:
  toProcess sidx p = (ss,a,n,l,idx,start,r)
   ⟹ ∃ s. start = Index s ∧ s < IArray.length ss
⟨proof⟩


lemma toProcess-startE:
  assumes toProcess sidx p = (ss,a,n,l,idx,start,r)
  obtains s where start = Index s s < IArray.length ss
  ⟨proof⟩
```

The main construction function. Takes an AST and returns an initial state,

and the program (= transition system).

```
definition setUp :: ast ⇒ program × gState where
  setUp ast = (
      let
       (decls, procs, -) = preprocess ast;
       assertVar = Var (VTBounded 0 1) 0;

       pre-procs = map (case-prod toProcess) (List.enumerate 1 procs);

       procs = IArray ((0, Index 0, [], []) # map (λ(-,-,-,-,p). p) pre-procs);
       labels = IArray (lm.empty() # map (λ(-,-,-,l,-). l) pre-procs);
       states = IArray (IArray [(0,[])] # map (λ(s,-). s) pre-procs);
       names = IArray (STR ''invalid'' # map (λ(-,-,n,-). n) pre-procs);

       proc-data = lm.to-map (map (λ(-,-,n,-,idx,-). (n,idx)) pre-procs);

       prog = (| processes = procs, labels = labels, states = states,
                 proc-names = names, proc-data = proc-data |);

       g = (| vars = lm.sng (STR ''--assert--'') assertVar,
              channels = [InvChannel], timeout = False, procs = [] |);
       g' = foldl (λg d.
                    fst (mkVarChannel d (apfst ∘ gState.vars-update) g emptyProc)
                  ) g decls;
       g'' = foldl (λg (-,a,name,-).
                    foldl (λg name.
                             fst (runProc name [] prog g emptyProc)
                           ) g (replicate a name)
                  ) g' pre-procs
      in
       (prog, g''))

lemma setUp-program-inv':
  program-inv (fst (setUp ast))
⟨proof⟩

lemma setUp-program-inv:
  assumes setUp ast = (prog,g)
  shows program-inv prog
⟨proof⟩

lemma setUp-gState-inv:
  assumes setUp ast = (prog, g)
  shows gState-inv prog g
⟨proof⟩
```

## 5.8 Semantic Engine

After constructing the transition system, we are missing the final part: The successor function on this system. We use SPIN-nomenclature and call it *semantic engine*.

**definition** *assertVar* ≡ *VarRef True* (*STR ''--assert--''*) *None*

### 5.8.1 Evaluation of Edges

```
fun evalRecvArgs
  :: recvArg list ⇒ integer list ⇒ gState_I ⇒ pState ⇒ gState_I * pState
where
  evalRecvArgs [] [] g l = (g,l)
| evalRecvArgs - [] g l =
    abort STR ''Length mismatch on receiving.'' (λ-. (g,l))
| evalRecvArgs [] - g l =
    abort STR ''Length mismatch on receiving.'' (λ-. (g,l))
| evalRecvArgs (r#rs) (v#vs) g l = (
    let (g,l) =
      case r of
        RecvArgVar var ⇒ setVar var v g l
      | - ⇒ (g,l)
    in evalRecvArgs rs vs g l)

primrec evalCond
  :: edgeCond ⇒ gState_I ⇒ pState ⇒ bool
where
  evalCond ECTrue - - ⟷ True
| evalCond ECFalse - - ⟷ False
| evalCond (ECExpr e) g l ⟷ exprArith g l e ≠ 0
| evalCond (ECRun -) g l ⟷ length (procs g) < 255
| evalCond ECElse g l ⟷ gState_I.else g
| evalCond (ECSend v) g l ⟷
    withChannel v (λ- c.
      case c of
        Channel cap - q ⇒ integer-of-nat (length q) < cap
      | HSChannel - ⇒ True) g l
| evalCond (ECRecv v rs srt) g l ⟷
    withChannel v (λi c.
      case c of
        HSChannel - ⇒ handshake g ≠ 0 ∧ recvArgsCheck g l rs (hsdata g)
      | - ⇒ pollCheck g l c rs srt) g l

fun evalHandshake
  :: edgeCond ⇒ nat ⇒  gState_I ⇒ pState ⇒ bool
where
  evalHandshake (ECRecv v - -) h g l
   ⟷ h = 0
    ∨ withChannel v (λi c. case c of
```

```
                                HSChannel - ⇒ i = h
                              | Channel - - - ⇒ False) g l
| evalHandshake - h - - ⟷ h = 0

primrec evalEffect
  :: edgeEffect ⇒ program ⇒ gState_I ⇒ pState ⇒ gState_I * pState
where
  evalEffect EEEnd - g l = (g,l)
| evalEffect EEId - g l = (g,l)
| evalEffect EEGoto - g l = (g,l)
| evalEffect (EEAssign v e) - g l = setVar v (exprArith g l e) g l
| evalEffect (EEDecl d) - g l = mkVarChannelProc d g l
| evalEffect (EERun name args) prog g l = runProc name args prog g l
| evalEffect (EEAssert e) - g l = (
    if exprArith g l e = 0
    then setVar assertVar 1 g l
    else (g,l))
| evalEffect (EESend v es srt) - g l = withChannel v (λi c.
    let
      ab = λ-. abort STR ''Length mismatch on sending.'' (λ-. (g,l));
      es = map (exprArith g l) es
    in
      if ¬ for-all (λx. x ≥ min-var-value ∧ x ≤ max-var-value) es
      then abort STR ''Invalid Channel'' (λ-. (g,l))
      else
        case c of
          Channel cap ts q ⇒
            if length ts ≠ length es ∨ ¬ (length q < max-array-size)
            then ab()
            else let
                   q' = if ¬ srt then q@[es]
                        else let
                          q = map lexlist q;
                          q' = insort (lexlist es) q
                        in map unlex q';
                   g = gState.channels-update (λcs.
                         cs[ i := Channel cap ts q' ]) g
                 in (g,l)
        | HSChannel ts ⇒
            if length ts ≠ length es then ab()
            else (g(|hsdata := es, handshake := i|), l)
        | InvChannel ⇒ abort STR ''Trying to send on invalid channel'' (λ-. (g,l))
  ) g l
| evalEffect (EERecv v rs srt rem) - g l = withChannel v (λi c.
    case c of
      Channel cap ts qs ⇒
        if qs = [] then abort STR ''Recv from empty channel'' (λ-. (g,l))
        else
          let
```

```
            (q', qs') = if ¬ srt then (hd qs, tl qs)
                        else apfst the (find-remove (recvArgsCheck g l rs) qs);
            (g,l) = evalRecvArgs rs q' g l;
            g = if rem
                then gState.channels-update (λcs. cs[ i := Channel cap ts qs']) g
                else g
                  — messages are not removed – so no need to update anything
          in (g,l)
      | HSChannel - ⇒
          let (g,l) = evalRecvArgs rs (hsdata g) g l in
          let g = g(| handshake := 0, hsdata := [] |)
          in (g,l)
      | InvChannel ⇒ abort STR ''Receiving on invalid channel'' (λ-. (g,l))
    ) g l

lemma statesDecls-effect:
  assumes ef ∈ effect ‘ edgeSet ss
  and ef = EEDecl d
  shows d ∈ statesDecls ss
⟨proof⟩

lemma evalRecvArgs-pState-inv:
  assumes pState-inv prog p
  shows pState-inv prog (snd (evalRecvArgs rargs xs g p))
  ⟨proof⟩

lemma evalRecvArgs-pState-inv':
  assumes evalRecvArgs rargs xs g p = (g', p')
  and pState-inv prog p
  shows pState-inv prog p'
⟨proof⟩

lemma evalRecvArgs-gState-progress-rel:
  assumes gState-inv prog g
  shows (g, fst (evalRecvArgs rargs xs g p)) ∈ gState-progress-rel prog
  ⟨proof⟩

lemmas evalRecvArgs-gState-inv =
  evalRecvArgs-gState-progress-rel[THEN gState-progress-rel-gState-invI2]

lemma evalRecvArgs-cl-inv:
  assumes cl-inv (g,p)
  shows cl-inv (evalRecvArgs rargs xs g p)
  ⟨proof⟩

lemma evalEffect-pState-inv:
  assumes pState-inv prog p
  and gState-inv prog g
  and cl-inv (g,p)
```

**and** $e \in effect\ `\ edgeSet\ (states\ prog\ !!\ pState.idx\ p)$
**shows** $pState\text{-}inv\ prog\ (snd\ (evalEffect\ e\ prog\ g\ p))$
⟨*proof*⟩

**lemma** *evalEffect-gState-progress-rel*:
**assumes** *program-inv prog*
**and** *gState-inv prog g*
**and** *pState-inv prog p*
**and** *cl-inv* $(g,p)$
**shows** $(g,\ fst\ (evalEffect\ e\ prog\ g\ p)) \in gState\text{-}progress\text{-}rel\ prog$
⟨*proof*⟩

**lemma** *evalEffect-cl-inv*:
**assumes** *cl-inv* $(g,p)$
**and** *program-inv prog*
**and** *gState-inv prog g*
**and** *pState-inv prog p*
**shows** *cl-inv* $(evalEffect\ e\ prog\ g\ p)$
⟨*proof*⟩

### 5.8.2 Executable edges

To find a successor global state, we first need to find all those edges which are executable (i. e. the condition evaluates to true).

**type-synonym** $choices = (edge * pState)\ list$
— A choice is an executable edge and the process it belongs to.

**definition** $getChoices :: gState_I \Rightarrow pState \Rightarrow edge\ list \Rightarrow choices$ **where**
$getChoices\ g\ p = foldl\ (\lambda E\ e.$
$\quad if\ evalHandshake\ (cond\ e)\ (handshake\ g)\ g\ p \wedge evalCond\ (cond\ e)\ g\ p$
$\quad then\ (e,p)\#E$
$\quad else\ E)\ []$

**lemma** *getChoices-sub-edges-fst*:
$fst\ `\ set\ (getChoices\ g\ p\ es) \subseteq set\ es$
⟨*proof*⟩

**lemma** *getChoices-sub-edges*:
$(a,b) \in set\ (getChoices\ g\ p\ es) \Longrightarrow a \in set\ es$
⟨*proof*⟩

**lemma** *getChoices-p-snd*:
$snd\ `\ set\ (getChoices\ g\ p\ es) \subseteq \{p\}$
⟨*proof*⟩

**lemma** *getChoices-p*:
$(a,b) \in set\ (getChoices\ g\ p\ es) \Longrightarrow b = p$
⟨*proof*⟩

```
definition sort-by-pri where
  sort-by-pri min-pri edges = foldl (λes e.
      let idx = nat-of-integer (abs (prio e))
      in if idx > min-pri
         then abort STR ''Invalid priority'' (λ-. es)
         else let ep = e # (es ! idx) in es[idx := ep]
      ) (replicate (min-pri + 1) []) edges

lemma sort-by-pri-edges':
  assumes set edges ⊆ A
  shows set (sort-by-pri min-pri edges) ⊆ {xs. set xs ⊆ A}
⟨proof⟩

lemma sort-by-pri-edges:
  assumes set edges ⊆ A
  and es ∈ set (sort-by-pri min-pri edges)
  shows set es ⊆ A
⟨proof⟩

lemma sort-by-pri-length:
  length (sort-by-pri min-pri edges) = min-pri + 1
⟨proof⟩

definition executable
  :: states iarray ⇒ gState_I ⇒ choices nres
  — Find all executable edges
 where
  executable ss g = (
      let procs = procs g in
      nfoldli procs (λ-. True) (λp E.
        if (exclusive g = 0 ∨ exclusive g = pid p) then do {
          let (min-pri, edges) = (ss !! pState.idx p) !! pc p;
          ASSERT(set edges ⊆ edgeSet (ss !! pState.idx p));

          (E',-,-) ←
            if min-pri = 0 then do {
              WHILE_T (λ(E,brk,-). E = [] ∧ brk = 0) (λ (-, -, ELSE). do {
                let g = g(|gState_I.else := ELSE|);
                    E = getChoices g p edges
                in
                    if E = [] then (
                      if ¬ ELSE then RETURN (E, 0::nat, True)
                      else RETURN (E, 1, False))
                    else RETURN (E, 1, ELSE) }) ([], 0::nat, False)
            } else do {
              let min-pri = nat-of-integer (abs min-pri);
              let pri-edges = sort-by-pri min-pri edges;
              ASSERT (∀ es ∈ set pri-edges.
                        set es ⊆ edgeSet (ss !! pState.idx p));
```

```
                let pri-edges = IArray pri-edges;

                WHILE_T (λ(E,pri,-). E = [] ∧ pri ≤ min-pri) (λ(-, pri, ELSE). do
{
                    let es = pri-edges !! pri;
                    let g = g(|gState_I.else := ELSE|);
                    let E = getChoices g p es;
                    if E = [] then (
                        if ¬ ELSE then RETURN (E,pri,True)
                        else RETURN (E, pri + 1, False))
                    else RETURN (E, pri, ELSE) }) ([], 0, False)
                };
            RETURN (E'@E)
        } else RETURN E
      ) []
 )
```

**definition**
*while-rel1* =
  *measure* (λ*x*. *if x* = [] *then 1 else 0*)
<∗*lex*∗> *measure* (λ*x*. *if x* = *0 then 1 else 0*)
<∗*lex*∗> *measure* (λ*x*. *if* ¬ *x then 1 else 0*)

**lemma** *wf-while-rel1*:
*wf while-rel1*
⟨*proof*⟩

**definition**
*while-rel2 mp* =
  *measure* (λ*x*. *if x* = [] *then 1 else 0*)
<∗*lex*∗> *measure* (λ*x*. (*mp* + *1*) − *x*)
<∗*lex*∗> *measure* (λ*x*. *if* ¬ *x then 1 else 0*)

**lemma** *wf-while-rel2*:
*wf* (*while-rel2 mp*)
⟨*proof*⟩

**lemma** *executable-edgeSet*:
**assumes** *gState-inv prog g*
**and** *program-inv prog*
**and** *ss* = *states prog*
**shows** *executable ss g*
  ≤ *SPEC* (λ*cs*. ∀(*e*,*p*) ∈ *set cs*.
                *e* ∈ *edgeSet* (*ss* !! *pState.idx p*)
              ∧ *pState-inv prog p*
              ∧ *cl-inv* (*g*,*p*))
⟨*proof*⟩

**lemma** *executable-edgeSet'*:

**assumes** *gState-inv prog g*
**and** *program-inv prog*
**shows** *executable* (*states prog*) *g*
≤ *SPEC* (λ*cs*. ∀ (*e*,*p*) ∈ *set cs*.
*e* ∈ *edgeSet* ((*states prog*) !! *pState.idx p*)
∧ *pState-inv prog p*
∧ *cl-inv*(*g*,*p*))
⟨*proof*⟩

**schematic-goal** *executable-refine*:
*RETURN* (*?ex s g*) ≤ *executable s g*
⟨*proof*⟩

**concrete-definition** *executable-impl* **for** *s g* **uses** *executable-refine*

### 5.8.3 Successor calculation

**function** $to_I$ **where**
$to_I$ (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *t*, *procs* = *p* |)
= (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *False*, *procs* = *p*,
*handshake* = *0*, *hsdata* = [], *exclusive* = *0*, $gState_I$.*else* = *False* |)
⟨*proof*⟩
**termination** ⟨*proof*⟩

**function** $from_I$ **where**
$from_I$ (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *t*, *procs* = *p*, ... = *m* |)
= (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *t*, *procs* = *p* |)
⟨*proof*⟩
**termination** ⟨*proof*⟩

**function** $reset_I$ **where**
$reset_I$ (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *t*, *procs* = *p*,
*handshake* = *hs*, *hsdata* = *hsd*, *exclusive* = -, $gState_I$.*else* = - |)
= (| *gState.vars* = *v*, *channels* = *ch*, *timeout* = *False*, *procs* = *p*,
*handshake* = *0*, *hsdata* = *if hs* ≠ *0 then hsd else* [], *exclusive* = *0*,
$gState_I$.*else* = *False* |)
⟨*proof*⟩
**termination** ⟨*proof*⟩

**lemma** *gState-inv-*$to_I$:
*gState-inv prog g* = *gState-inv prog* ($to_I$ *g*)
⟨*proof*⟩

**lemma** *gState-inv-*$from_I$:
*gState-inv prog g* = *gState-inv prog* ($from_I$ *g*)
⟨*proof*⟩

**lemma** *gState-inv-*$reset_I$:
*gState-inv prog g* = *gState-inv prog* ($reset_I$ *g*)

⟨*proof*⟩

**lemmas** *gState-inv-I-simps* =
*gState-inv-to*$_I$ *gState-inv-from*$_I$ *gState-inv-reset*$_I$

**definition** *removeProcs*
— Remove ended processes, if there is no running one with a higher pid.
**where**
*removeProcs ps* = *foldr* (λ*p* (*dead*,*sd*,*ps*,*dcs*).
*if dead* ∧ *pc p* = *0 then* (*True*, *True*, *ps*, *pState.channels p* @ *dcs*)
*else* (*False*, *sd*, *p*#*ps*, *dcs*)) *ps* (*True*, *False*, [], [])

**lemma** *removeProcs-subset′*:
*set* (*fst* (*snd* (*snd* (*removeProcs ps*)))) ⊆ *set ps*
⟨*proof*⟩

**lemma** *removeProcs-length′*:
*length* (*fst* (*snd* (*snd* (*removeProcs ps*)))) ≤ *length ps*
⟨*proof*⟩

**lemma** *removeProcs-subset*:
*removeProcs ps* = (*dead*,*sd*,*ps′*,*dcs*) ⟹ *set ps′* ⊆ *set ps*
⟨*proof*⟩

**lemma** *removeProcs-length*:
*removeProcs ps* = (*dead*,*sd*,*ps′*,*dcs*) ⟹ *length ps′* ≤ *length ps*
⟨*proof*⟩

**definition** *cleanChans* :: *integer list* ⇒ *channels* ⇒ *channels*
— Mark channels of closed processes as invalid.
**where**
*cleanChans dchans cs* = *snd* (*foldl* (λ(*i*,*cs*) *c*.
*if List.member dchans i then* (*i* + *1*, *cs*@[*InvChannel*])
*else* (*i* + *1*, *cs*@[*c*])) (*0*, []) *cs*)

**lemma** *cleanChans-channel-inv*:
**assumes** *set cs* ⊆ *Collect channel-inv*
**shows** *set* (*cleanChans dchans cs*) ⊆ *Collect channel-inv*
⟨*proof*⟩

**lemma** *cleanChans-length*:
*length* (*cleanChans dchans cs*) = *length cs*
⟨*proof*⟩

**definition** *checkDeadProcs* :: *′a gState-scheme* ⇒ *′a gState-scheme* **where**
*checkDeadProcs g* = (
*let* (-, *soDied*, *procs*, *dchans*) = *removeProcs* (*procs g*) *in*
*if soDied then*

$g(\!|\ procs := procs,\ channels := cleanChans\ dchans\ (channels\ g)\ |\!)$
*else g*)

**lemma** *checkDeadProcs-gState-progress-rel*:
**assumes** *gState-inv prog g*
**shows** (*g*, *checkDeadProcs g*) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *gState-progress-rel-exclusive*:
(*g*, *g'*) ∈ *gState-progress-rel prog*
⟹ (*g*, *g'*(|*exclusive* := *p*|)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**definition** *applyEdge*
:: *program* ⇒ *edge* ⇒ *pState* ⇒ $gState_I$ ⇒ $gState_I$ *nres*
**where**
*applyEdge prog e p g* = *do* {

*let* (*g'*,*p'*) = *evalEffect* (*effect e*) *prog g p*;
*ASSERT* ((*g*,*g'*) ∈ *gState-progress-rel prog*);
*ASSERT* (*pState-inv prog p'*);
*ASSERT* (*cl-inv* (*g'*,*p'*));

*let p''* = (*case target e of Index t* ⇒
*if t* < *IArray.length* (*states prog* !! *pState.idx p'*) *then p'*(|*pc* := *t*|)
*else abort STR ''Edge target out of bounds''* (λ-. *p'*)
| - ⇒ *abort STR ''Edge target not Index''* (λ-. *p'*));
*ASSERT* (*pState-inv prog p''*);

*let g''* = *g'*(|*procs* := *list-update* (*procs g'*) (*pid p''* − 1) *p''*|);
*ASSERT* ((*g'*,*g''*) ∈ *gState-progress-rel prog*);

*let g'''* = (*if isAtomic e* ∧ *handshake g''* = 0
*then g''*(|*exclusive* := *pid p''* |)
*else g''*);
*ASSERT* ((*g'*,*g'''*) ∈ *gState-progress-rel prog*);

*let* $g_f$ = (*if pc p''* = 0 *then checkDeadProcs g''' else g'''*);
*ASSERT* ((*g'''*,$g_f$) ∈ *gState-progress-rel prog*);
*RETURN* $g_f$ }

**lemma** *applyEdge-gState-progress-rel*:
**assumes** *program-inv prog*
**and** *gState-inv prog g*
**and** *pState-inv prog p*
**and** *cl-inv* (*g*,*p*)
**and** *e* ∈ *edgeSet* (*states prog* !! *pState.idx p*)
**shows** *applyEdge prog e p g* ≤ *SPEC* (λ*g'*. (*g*,*g'*) ∈ *gState-progress-rel prog*)
⟨*proof*⟩

```
schematic-goal applyEdge-refine:
  RETURN (?ae prog e p g) ≤ applyEdge prog e p g
⟨proof⟩

concrete-definition applyEdge-impl for e p g uses applyEdge-refine

definition nexts
  :: program ⇒ gState ⇒ gState ls nres
  — The successor function
where
  nexts prog g = (
    let
      f = from_I;
      g = to_I g
    in

      REC (λD g. do {
        E ← executable (states prog) g;
        if E = [] then
          if handshake g ≠ 0 then
            — HS not possible – remove current step
            RETURN (ls.empty())
          else if exclusive g ≠ 0 then
            — Atomic blocks – just return current state
            RETURN (ls.sng (f g))
          else if ¬ timeout g then
            — Set timeout
            D (g(|timeout := True|))
          else
            — If all else fails: stutter
            RETURN (ls.sng (f (reset_I g)))
        else
          — Setting the internal variables (exclusive, handshake, ...) to 0
          — is safe – they are either set by the edges, or not thought
          — to be used outside executable.
          let g = reset_I g in
          nfoldli E (λ-. True) (λ(e,p) G.
              applyEdge prog e p g ≫= (λ g'.
              if handshake g' ≠ 0 ∨ isAtomic e then do {
                G_R ← D g';
                if ls.isEmpty G_R ∧ handshake g' = 0 then
                  — this only happens if the next step is a handshake, which fails
                  — hence we stay at the current state
                  RETURN (ls.ins (f g') G)
                else
                  RETURN (ls.union G_R G)
              } else RETURN (ls.ins (f g') G))) (ls.empty())
      }) g
```

$\gg\!=$ ($\lambda G$. *if ls.isEmpty G then RETURN* (*ls.sng* (*f g*)) *else RETURN G*)
)

**lemma** *gState-progress-rel-intros*:
(*to*$_I$ *g*, *gI'*) ∈ *gState-progress-rel prog*
⟹ (*g*, *from*$_I$ *gI'*) ∈ *gState-progress-rel prog*
(*gI*, *gI'*) ∈ *gState-progress-rel prog*
⟹ (*gI*, *reset*$_I$ *gI'*) ∈ *gState-progress-rel prog*
(*to*$_I$ *g*, *gI'*) ∈ *gState-progress-rel prog*
⟹ (*to*$_I$ *g*, *gI'*(|*timeout* := *t*|)) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *gState-progress-rel-step-intros*:
(*to*$_I$ *g*, *g'*) ∈ *gState-progress-rel prog*
⟹ (*reset*$_I$ *g'*, *g''*) ∈ *gState-progress-rel prog*
⟹ (*g*, *from*$_I$ *g''*) ∈ *gState-progress-rel prog*
(*to*$_I$ *g*, *g'*) ∈ *gState-progress-rel prog*
⟹ (*reset*$_I$ *g'*, *g''*) ∈ *gState-progress-rel prog*
⟹ (*to*$_I$ *g*, *g''*) ∈ *gState-progress-rel prog*
⟨*proof*⟩

**lemma** *cl-inv-reset*$_I$:
*cl-inv*(*g*,*p*) ⟹ *cl-inv*(*reset*$_I$ *g*, *p*)
⟨*proof*⟩

**lemmas** *refine-helpers* =
*gState-progress-rel-intros gState-progress-rel-step-intros cl-inv-reset*$_I$

**lemma** *nexts-SPEC*:
**assumes** *gState-inv prog g*
**and** *program-inv prog*
**shows** *nexts prog g* ≤ *SPEC* (λ*gs*. ∀ *g'* ∈ *ls.α gs*. (*g*,*g'*) ∈ *gState-progress-rel prog*)
⟨*proof*⟩

**lemma** *RETURN-dRETURN*:
*RETURN f* ≤ *f'* ⟹ *nres-of* (*dRETURN f*) ≤ *f'*
⟨*proof*⟩

**lemma** *executable-dRETURN*:
*nres-of* (*dRETURN* (*executable-impl prog g*)) ≤ *executable prog g*
⟨*proof*⟩

**lemma** *applyEdge-dRETURN*:
*nres-of* (*dRETURN* (*applyEdge-impl prog e p g*)) ≤ *applyEdge prog e p g*
⟨*proof*⟩

**schematic-goal** *nexts-code-aux*:
*nres-of* (*?nexts prog g*) ≤ *nexts prog g*

⟨*proof*⟩

**concrete-definition** *nexts-code-aux* **for** *prog g* **uses** *nexts-code-aux*
**prepare-code-thms** *nexts-code-aux-def*

### 5.8.4 Handle non-termination

A Promela model may include non-terminating parts. Therefore we cannot guarantee, that *nexts* will actually terminate. To avoid having to deal with this in the model checker, we fail in case of non-termination.

**definition** *SUCCEED-abort* **where**
*SUCCEED-abort msg dm m* = (
*case m of*
*RES X* ⇒ *if X*={} *then Code.abort msg* (λ-. *dm*) *else RES X*
| - ⇒ *m*)

**definition** *dSUCCEED-abort* **where**
*dSUCCEED-abort msg dm m* = (
*case m of*
*dSUCCEEDi* ⇒ *Code.abort msg* (λ-. *dm*)
| - ⇒ *m*)

**definition** *ref-succeed* **where**
*ref-succeed m m′* ⟷ *m* ≤ *m′* ∧ (*m*=*SUCCEED* ⟶ *m′*=*SUCCEED*)

**lemma** *dSUCCEED-abort-SUCCEED-abort*:
⟦ *RETURN dm′* ≤ *dm*; *ref-succeed* (*nres-of m′*) *m* ⟧
⟹ *nres-of* (*dSUCCEED-abort msg* (*dRETURN dm′*) (*m′*))
≤ *SUCCEED-abort msg dm m*
⟨*proof*⟩

The final successor function now incorporates:

1. *nexts*
2. handling of non-termination

**definition** *nexts-code* **where**
*nexts-code prog g* =
*the-res* (*dSUCCEED-abort* (*STR ''The Universe is broken!''*)
(*dRETURN* (*ls.sng g*))
(*nexts-code-aux prog g*))

**lemma** *nexts-code-SPEC*:
**assumes** *gState-inv prog g*
**and** *program-inv prog*
**shows** *g′* ∈ *ls.α* (*nexts-code prog g*)
⟹ (*g*,*g′*) ∈ *gState-progress-rel prog*
⟨*proof*⟩

## 5.9 Finiteness of the state space

**inductive-set** *reachable-states*
  **for** $P$ :: *program*
  **and** $g_s$ :: *gState* — start state
**where**
$g_s \in$ *reachable-states* $P$ $g_s$ |
$g \in$ *reachable-states* $P$ $g_s \Longrightarrow x \in ls.\alpha$ (*nexts-code* $P$ $g$)
$\Longrightarrow x \in$ *reachable-states* $P$ $g_s$

**lemmas** *reachable-states-induct*[*case-names init step*] =
  *reachable-states.induct*[*split-format* (*complete*)]

**lemma** *reachable-states-finite*:
  **assumes** *program-inv prog*
  **and** *gState-inv prog g*
  **shows** *finite* (*reachable-states prog g*)
⟨*proof*⟩

## 5.10 Traces

When trying to generate a lasso, we have a problem: We only have a list of global states. But what are the transitions to come from one to the other?

This problem shall be tackled by *replay*: Given two states, it generates a list of transitions that was taken.

```
definition replay :: program ⇒ gState ⇒ gState ⇒ choices nres where
  replay prog g₁ g₂ = (
    let
      g₁ = to_I g₁;
      check = λg. from_I g = g₂
    in
      REC_T (λD g. do {
      E ← executable (states prog) g;
      if E = [] then
        if check g then RETURN []
        else if ¬ timeout g then D (g(|timeout := True|))
        else abort STR ''Stuttering should not occur on replay''
                 (λ-. RETURN [])
      else
         let g = reset_I g in
         nfoldli E (λE. E = []) (λ(e,p) -.
           applyEdge prog e p g ≫= (λg'.
             if handshake g' ≠ 0 ∨ isAtomic e then do {
               E_R ← D g';
               if E_R = [] then
                  if check g' then RETURN [(e,p)] else RETURN []
               else
                  RETURN ((e,p) # E_R)
             } else if check g' then RETURN [(e,p)] else RETURN [])
```

```
            ) []
      }) g₁
)

lemma abort-refine[refine-transfer]:
  nres-of (f ()) ≤ F () ⟹ nres-of (abort s f) ≤ abort s F
  f() ≠ dSUCCEED ⟹ abort s f ≠ dSUCCEED
⟨proof⟩

schematic-goal replay-code-aux:
  RETURN (?replay prog g₁ g₂) ≤ replay prog g₁ g₂
⟨proof⟩

concrete-definition replay-code for prog g₁ g₂ uses replay-code-aux
prepare-code-thms replay-code-def
```

### 5.10.1 Printing of traces

```
definition procDescr
  :: (integer ⇒ string) ⇒ program ⇒ pState ⇒ string
where
  procDescr f prog p = (
    let
      name = String.explode (proc-names prog !! pState.idx p);
      id = f (integer-of-nat (pid p))
    in
      name @ '' ('' @ id @ '')'')

definition printInitial
  :: (integer ⇒ string) ⇒ program ⇒ gState ⇒ string
where
  printInitial f prog g₀ = (
      let psS = printList (procDescr f prog) (procs g₀) [] [] ''  '' in
      ''Initially running: '' @ psS)

abbreviation lf ≡ CHR 0x0A

fun printConfig
  :: (integer ⇒ string) ⇒ program ⇒ gState option ⇒ gState ⇒ string
where
  printConfig f prog None g₀ = printInitial f prog g₀
| printConfig f prog (Some g₀) g₁ = (
          let eps = replay-code prog g₀ g₁ in
          let print = (λ(e,p). procDescr f prog p @ '': '' @ printEdge f (pc p) e)
          in if eps = [] ∧ g₁ = g₀ then ''    -- stutter --''
             else printList print eps [] [] (lf#''    ''))

definition printConfigFromAST f ≡ printConfig f o fst o setUp
```

### 5.11 Code export

**code-identifier**
  **code-module** *PromelaInvariants* ⇀ (*SML*) *Promela*
| **code-module** *PromelaDatastructures* ⇀ (*SML*) *Promela*

**definition** *executable-triv prog g* = *executable-impl* (*snd prog*) *g*
**definition** *apply-triv prog g ep* = *applyEdge-impl prog* (*fst ep*) (*snd ep*) (*reset*$_I$ *g*)

**export-code**
  *setUp printProcesses printConfigFromAST nexts-code executable-triv apply-triv extractLTLs lookupLTL*
  **checking** *SML*

**export-code**
  *setUp printProcesses printConfigFromAST nexts-code executable-triv apply-triv extractLTLs lookupLTL*
  **in** *SML*
  **file** ‹*Promela.sml*›

**end**

## 6 LTL integration

**theory** *PromelaLTL*
**imports**
  *Promela*
  *LTL.LTL*
**begin**

We have a semantic engine for Promela. But we need to have an integration with LTL – more specificly, we must know when a proposition is true in a global state. This is achieved in this theory.

### 6.1 LTL optimization

For efficiency reasons, we do not store the whole *expr* on the labels of a system automaton, but *nat* instead. This index then is used to look up the corresponding *expr*.

**type-synonym** *APs* = *expr iarray*

**primrec** *ltlc-aps-list′* :: *'a ltlc* ⇒ *'a list* ⇒ *'a list*
**where**
  *ltlc-aps-list′ True-ltlc l* = *l*
| *ltlc-aps-list′ False-ltlc l* = *l*
| *ltlc-aps-list′* (*Prop-ltlc p*) *l* = (*if List.member l p then l else p*#*l*)
| *ltlc-aps-list′* (*Not-ltlc x*) *l* = *ltlc-aps-list′ x l*
| *ltlc-aps-list′* (*Next-ltlc x*) *l* = *ltlc-aps-list′ x l*

| *ltlc-aps-list'* (*Final-ltlc x*) *l* = *ltlc-aps-list' x l*
| *ltlc-aps-list'* (*Global-ltlc x*) *l* = *ltlc-aps-list' x l*
| *ltlc-aps-list'* (*And-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*Or-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*Implies-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*Until-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*Release-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*WeakUntil-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)
| *ltlc-aps-list'* (*StrongRelease-ltlc x y*) *l* = *ltlc-aps-list' y* (*ltlc-aps-list' x l*)

**lemma** *ltlc-aps-list'-correct*:
*set* (*ltlc-aps-list'* $\varphi$ *l*) = *atoms-ltlc* $\varphi$ ∪ *set l*
⟨*proof*⟩

**lemma** *ltlc-aps-list'-distinct*:
*distinct l* ⟹ *distinct* (*ltlc-aps-list'* $\varphi$ *l*)
⟨*proof*⟩

**definition** *ltlc-aps-list* :: *'a ltlc* ⇒ *'a list*
**where**
*ltlc-aps-list* $\varphi$ = *ltlc-aps-list'* $\varphi$ []

**lemma** *ltlc-aps-list-correct*:
*set* (*ltlc-aps-list* $\varphi$) = *atoms-ltlc* $\varphi$
⟨*proof*⟩

**lemma** *ltlc-aps-list-distinct*:
*distinct* (*ltlc-aps-list* $\varphi$)
⟨*proof*⟩

**primrec** *idx'* :: *nat* ⇒ *'a list* ⇒ *'a* ⇒ *nat option* **where**
*idx'* - [] - = *None*
| *idx' ctr* (*x*#*xs*) *y* = (*if x* = *y then Some ctr else idx'* (*ctr*+*1*) *xs y*)

**definition** *idx* = *idx' 0*

**lemma** *idx'-correct*:
**assumes** *distinct xs*
**shows** *idx' ctr xs y* = *Some n* ⟷ *n* ≥ *ctr* ∧ *n* < *length xs* + *ctr* ∧ *xs* ! (*n*−*ctr*)
= *y*
⟨*proof*⟩

**lemma** *idx-correct*:
**assumes** *distinct xs*
**shows** *idx xs y* = *Some n* ⟷ *n* < *length xs* ∧ *xs* ! *n* = *y*
⟨*proof*⟩

**lemma** *idx-dom*:
**assumes** *distinct xs*

**shows** *dom* (*idx xs*) = *set xs*
⟨*proof*⟩

**lemma** *idx-image-self*:
**assumes** *distinct xs*
**shows** (*the* ∘ *idx xs*) ‘ *set xs* = {..<*length xs*}
⟨*proof*⟩

**lemma** *idx-ran*:
**assumes** *distinct xs*
**shows** *ran* (*idx xs*) = {..<*length xs*}
⟨*proof*⟩

**lemma** *idx-inj-on-dom*:
**assumes** *distinct xs*
**shows** *inj-on* (*idx xs*) (*dom* (*idx xs*))
⟨*proof*⟩

**definition** *ltl-convert* :: *expr ltlc* ⇒ *APs* × *nat ltlc* **where**
*ltl-convert* φ = (
*let APs* = *ltlc-aps-list* φ;
φ$_i$ = *map-ltlc* (*the* ∘ *idx APs*) φ
*in* (*IArray APs*, φ$_i$))

**lemma** *ltl-convert-correct*:
**assumes** *ltl-convert* φ = (*APs*, φ$_i$)
**shows** *atoms-ltlc* φ = *set* (*IArray.list-of APs*) (**is** *?P1*)
**and** *atoms-ltlc* φ$_i$ = {..<*IArray.length APs*} (**is** *?P2*)
**and** φ$_i$ = *map-ltlc* (*the* ∘ *idx* (*IArray.list-of APs*)) φ (**is** *?P3*)
**and** *distinct* (*IArray.list-of APs*)
⟨*proof*⟩

**definition** *prepare*
:: - × (*program* ⇒ *unit*) ⇒ *ast* ⇒ *expr ltlc* ⇒ (*program* × *APs* × *gState*) × *nat ltlc*
**where**
*prepare cfg ast* φ ≡
*let*
(*prog*,*g*$_0$) = *Promela.setUp ast*;
(*APs*,φ$_i$) = *PromelaLTL.ltl-convert* φ
*in*
((*prog*, *APs*, *g*$_0$), φ$_i$)

**lemma** *prepare-instrument*[*code*]:
*prepare cfg ast* φ ≡
*let*
(-,*printF*) = *cfg*;
- = *PromelaStatistics.start* ();
(*prog*,*g*$_0$) = *Promela.setUp ast*;

```
      - = printF prog;
      (APs,φ_i) = PromelaLTL.ltl-convert φ;
      - = PromelaStatistics.stop-timer ()
    in
      ((prog, APs, g_0), φ_i)
⟨proof⟩
```

**export-code** *prepare* **checking** *SML*

## 6.2 Language of a Promela program

**definition** *propValid* :: *APs* ⇒ *gState* ⇒ *nat* ⇒ *bool* **where**
*propValid APs g i* ⟷ *i* < *IArray.length APs* ∧ *exprArith g emptyProc* (*APs*!!*i*) ≠ *0*

**definition** *promela-E* :: *program* ⇒ (*gState* × *gState*) *set*
— Transition relation of a promela program
**where**
*promela-E prog* ≡ {(*g*,*g'*). *g'* ∈ *ls.α* (*nexts-code prog g*)}

**definition** *promela-E-ltl* :: *program* × *APs* ⇒ (*gState* × *gState*) *set* **where**
*promela-E-ltl* = *promela-E* ∘ *fst*

**definition** *promela-is-run'* :: *program* × *gState* ⇒ *gState word* ⇒ *bool*
— Predicate defining runs of promela programs
**where**
*promela-is-run' progg r* ≡
  *let* (*prog*,*g*$_0$)=*progg in*
    *r 0* = *g*$_0$
  ∧ (∀ *i*. *r* (*Suc i*) ∈ *ls.α* (*nexts-code prog* (*r i*)))

**abbreviation** *promela-is-run* ≡ *promela-is-run'* ∘ *setUp*

**definition** *promela-is-run-ltl* :: *program* × *APs* × *gState* ⇒ *gState word* ⇒ *bool*
**where**
*promela-is-run-ltl promg r* ≡ *let* (*prog*,*APs*,*g*) = *promg in promela-is-run'* (*prog*,*g*) *r*

**definition** *promela-props* :: *gState* ⇒ *expr set*
**where**
*promela-props g* = {*e*. *exprArith g emptyProc e* ≠ *0*}

**definition** *promela-props-ltl* :: *APs* ⇒ *gState* ⇒ *nat set*
**where**
*promela-props-ltl APs g* ≡ *Collect* (*propValid APs g*)

**definition** *promela-language* :: *ast* ⇒ *expr set word set* **where**
*promela-language ast* ≡ {*promela-props* ∘ *r* | *r*. *promela-is-run ast r*}

**definition** *promela-language-ltl* :: *program* × *APs* × *gState* ⇒ *nat set word set*
**where**
*promela-language-ltl promg* ≡ *let* (*prog*,*APs*,*g*) = *promg in*
{*promela-props-ltl APs* ∘ *r* | *r*. *promela-is-run-ltl promg r*}

**lemma** *promela-props-ltl-map-aprops*:
**assumes** *ltl-convert* φ = (*APs*,φ*i*)
**shows** *promela-props-ltl APs* =
*map-props* (*idx* (*IArray.list-of APs*)) ∘ *promela-props*
⟨*proof*⟩

**lemma** *promela-run-in-language-iff*:
**assumes** *conv*: *ltl-convert* φ = (*APs*,φ*i*)
**shows** *promela-props* ∘ ξ ∈ *language-ltlc* φ
⟷ *promela-props-ltl APs* ∘ ξ ∈ *language-ltlc* φ*i* (**is** *?L* ⟷ *?R*)
⟨*proof*⟩

**lemma** *promela-language-sub-iff*:
**assumes** *conv*: *ltl-convert* φ = (*APs*,φ*i*)
**and** *setUp*: *setUp ast* = (*prog*,*g*)
**shows** *promela-language-ltl* (*prog*,*APs*,*g*) ⊆ *language-ltlc* φ*i* ⟷ *promela-language ast* ⊆ *language-ltlc* φ
⟨*proof*⟩

**hide-const** (**open**) *abort abortv*
*err errv*
*usc*
*warn the-warn with-warn*

**hide-const** (**open**) *idx idx′*
**end**
**theory** *PromelaLTLConv*
**imports**
*Promela*
*LTL.LTL*
**begin**

## 6.3 Proposition types and conversion

LTL formulae and propositions are also generated by an SML parser. Hence we have the same setup as for Promela itself: Mirror the data structures and (sometimes) map them to new ones.

This theory is intended purely to be used by frontend code to convert from *propc* to *expr*. The other theories work on *expr* directly.

While we could of course convert directly, that would introduce yet a semantic level.

```
datatype binOp = Eq | Le | LEq | Gr | GEq

datatype ident = Ident String.literal integer option

datatype propc = CProp ident
               | BProp binOp ident ident
               | BExpProp binOp ident integer

fun identConv :: ident ⇒ varRef where
  identConv (Ident name None) = VarRef True name None
| identConv (Ident name (Some i)) = VarRef True name (Some (ExprConst i))

definition ident2expr :: ident ⇒ expr where
  ident2expr = ExprVarRef ∘ identConv

primrec binOpConv :: binOp ⇒ PromelaDatastructures.binOp where
  binOpConv Eq = BinOpEq
| binOpConv Le = BinOpLe
| binOpConv LEq = BinOpLEq
| binOpConv Gr = BinOpGr
| binOpConv GEq = BinOpGEq

primrec propc2expr :: propc ⇒ expr where
  propc2expr (CProp ident) =
     ExprBinOp BinOpEq (ident2expr ident) (ExprConst 1)
| propc2expr (BProp bop il ir) =
     ExprBinOp (binOpConv bop) (ident2expr il) (ident2expr ir)
| propc2expr (BExpProp bop il ir) =
     ExprBinOp (binOpConv bop) (ident2expr il) (ExprConst ir)

definition ltl-conv :: propc ltlc ⇒ expr ltlc where
  ltl-conv = map-ltlc propc2expr

definition printPropc
  :: (integer ⇒ char list) ⇒ propc ⇒ char list
where
  printPropc f p = printExpr f (propc2expr p)
```

The semantics of a *propc* is given just for reference.

```
definition evalPropc :: gState ⇒ propc ⇒ bool where
  evalPropc g p ⟷ exprArith g emptyProc (propc2expr p) ≠ 0

end
```

# References


[1] Promela manual pages. http://spinroot.com/spin/Man/promela.html. Accessed: 2013-02-07.

[2] G. J. Holzmann. *The Spin Model Checker — Primer and Reference Manual*. Addison-Wesley, 2003.