A Machine-Checked Model for a Java-like Language, Virtual Machine and Compiler

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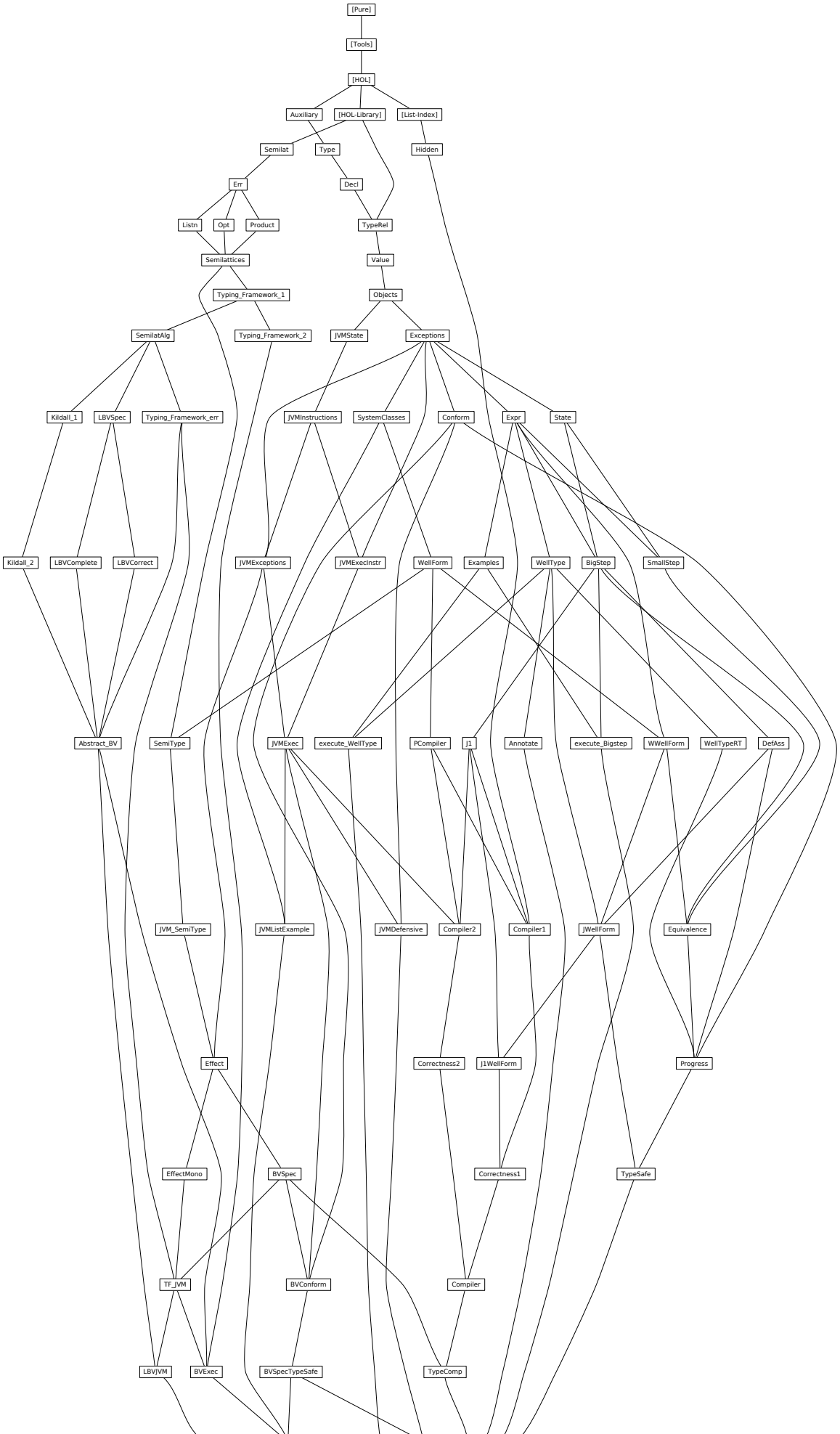
Chapter 1

Preface

This document contains the automatically generated listings of the Isabelle sources for the theories defining and analysing Jinja (a Java-like programming language), the Jinja Virtual Machine, and the compiler. To shorten the document, all proofs have been hidden. For a detailed exposition of these theories see the paper by Klein and Nipkow [1, 2].

1.1 Theory Dependencies

Figure 1.1 shows the dependencies between the Isabelle theories in the following sections.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Chapter 2

Jinja Source Language

2.1 Auxiliary Definitions

theory *Auxiliary* **imports** *Main* **begin**

lemma *nat-add-max-le*[*simp*]:
$((n::nat) + max\ i\ j \leq m) = (n + i \leq m \wedge n + j \leq m)$
⟨*proof*⟩
lemma *Suc-add-max-le*[*simp*]:
$(Suc(n + max\ i\ j) \leq m) = (Suc(n + i) \leq m \wedge Suc(n + j) \leq m)$⟨*proof*⟩

notation *Some* (⌊-⌋)

2.1.1 *distinct-fst*

definition *distinct-fst* :: $('a \times 'b)\ list \Rightarrow bool$
where
$distinct\text{-}fst \equiv distinct \circ map\ fst$

lemma *distinct-fst-Nil* [*simp*]:
distinct-fst []
⟨*proof*⟩
lemma *distinct-fst-Cons* [*simp*]:
$distinct\text{-}fst\ ((k,x)\#kxs) = (distinct\text{-}fst\ kxs \wedge (\forall\, y.\ (k,y) \notin set\ kxs))$⟨*proof*⟩

lemma *map-of-SomeI*:
$[\![\ distinct\text{-}fst\ kxs;\ (k,x) \in set\ kxs\ ]\!] \Longrightarrow map\text{-}of\ kxs\ k = Some\ x$⟨*proof*⟩

2.1.2 Using *list-all2* for relations

definition *fun-of* :: $('a \times 'b)\ set \Rightarrow 'a \Rightarrow 'b \Rightarrow bool$
where
$fun\text{-}of\ S \equiv \lambda x\ y.\ (x,y) \in S$

Convenience lemmas

lemma *rel-list-all2-Cons* [*iff*]:
$list\text{-}all2\ (fun\text{-}of\ S)\ (x\#xs)\ (y\#ys) =$
$((x,y) \in S \wedge list\text{-}all2\ (fun\text{-}of\ S)\ xs\ ys)$
⟨*proof*⟩

lemma *rel-list-all2-Cons1*:
list-all2 (*fun-of S*) (*x*#*xs*) *ys* =
(∃ *z zs*. *ys* = *z*#*zs* ∧ (*x*,*z*) ∈ *S* ∧ *list-all2* (*fun-of S*) *xs zs*)
⟨*proof*⟩
lemma *rel-list-all2-Cons2*:
list-all2 (*fun-of S*) *xs* (*y*#*ys*) =
(∃ *z zs*. *xs* = *z*#*zs* ∧ (*z*,*y*) ∈ *S* ∧ *list-all2* (*fun-of S*) *zs ys*)
⟨*proof*⟩
lemma *rel-list-all2-refl*:
(⋀*x*. (*x*,*x*) ∈ *S*) ⟹ *list-all2* (*fun-of S*) *xs xs*
⟨*proof*⟩
lemma *rel-list-all2-antisym*:
⟦ (⋀*x y*. ⟦(*x*,*y*) ∈ *S*; (*y*,*x*) ∈ *T*⟧ ⟹ *x* = *y*);
list-all2 (*fun-of S*) *xs ys*; *list-all2* (*fun-of T*) *ys xs* ⟧ ⟹ *xs* = *ys*
⟨*proof*⟩
lemma *rel-list-all2-trans*:
⟦ ⋀*a b c*. ⟦(*a*,*b*) ∈ *R*; (*b*,*c*) ∈ *S*⟧ ⟹ (*a*,*c*) ∈ *T*;
list-all2 (*fun-of R*) *as bs*; *list-all2* (*fun-of S*) *bs cs*⟧
⟹ *list-all2* (*fun-of T*) *as cs*
⟨*proof*⟩
lemma *rel-list-all2-update-cong*:
⟦ *i*<*size xs*; *list-all2* (*fun-of S*) *xs ys*; (*x*,*y*) ∈ *S* ⟧
⟹ *list-all2* (*fun-of S*) (*xs*[*i*:=*x*]) (*ys*[*i*:=*y*])
⟨*proof*⟩
lemma *rel-list-all2-nthD*:
⟦ *list-all2* (*fun-of S*) *xs ys*; *p* < *size xs* ⟧ ⟹ (*xs*!*p*,*ys*!*p*) ∈ *S*
⟨*proof*⟩
lemma *rel-list-all2I*:
⟦ *length a* = *length b*; ⋀*n*. *n* < *length a* ⟹ (*a*!*n*,*b*!*n*) ∈ *S* ⟧ ⟹ *list-all2* (*fun-of S*) *a b*
⟨*proof*⟩
end

2.2 Jinja types

theory *Type* **imports** *Auxiliary* **begin**

type-synonym *cname* = *string* — class names
type-synonym *mname* = *string* — method name
type-synonym *vname* = *string* — names for local/field variables

definition *Object* :: *cname*
where
Object ≡ ''*Object*''

definition *this* :: *vname*
where
this ≡ ''*this*''

— types
datatype *ty*
= *Void* — type of statements
| *Boolean*
| *Integer*

| *NT* — null type
| *Class cname* — class type

definition *is-refT* :: *ty* ⇒ *bool*
where
is-refT T ≡ *T* = *NT* ∨ (∃ *C*. *T* = *Class C*)

lemma [*iff*]: *is-refT NT*⟨*proof*⟩
lemma [*iff*]: *is-refT*(*Class C*)⟨*proof*⟩
lemma *refTE*:
⟦*is-refT T*; *T* = *NT* ⟹ *P*; ⋀*C*. *T* = *Class C* ⟹ *P* ⟧ ⟹ *P*⟨*proof*⟩
lemma *not-refTE*:
⟦ ¬*is-refT T*; *T* = *Void* ∨ *T* = *Boolean* ∨ *T* = *Integer* ⟹ *P* ⟧ ⟹ *P*⟨*proof*⟩
end

2.3 Class Declarations and Programs

theory *Decl* **imports** *Type* **begin**

type-synonym
fdecl = *vname* × *ty* — field declaration
type-synonym
'm mdecl = *mname* × *ty list* × *ty* × *'m* — method = name, arg. types, return type, body
type-synonym
'm class = *cname* × *fdecl list* × *'m mdecl list* — class = superclass, fields, methods
type-synonym
'm cdecl = *cname* × *'m class* — class declaration
type-synonym
'm prog = *'m cdecl list* — program

definition *class* :: *'m prog* ⇒ *cname* ⇀ *'m class*
where
class ≡ *map-of*

definition *is-class* :: *'m prog* ⇒ *cname* ⇒ *bool*
where
is-class P C ≡ *class P C* ≠ *None*

lemma *finite-is-class*: *finite* {*C*. *is-class P C*}
⟨*proof*⟩
definition *is-type* :: *'m prog* ⇒ *ty* ⇒ *bool*
where
is-type P T ≡
(*case T of Void* ⇒ *True* | *Boolean* ⇒ *True* | *Integer* ⇒ *True* | *NT* ⇒ *True*
| *Class C* ⇒ *is-class P C*)

lemma *is-type-simps* [*simp*]:
is-type P Void ∧ *is-type P Boolean* ∧ *is-type P Integer* ∧
is-type P NT ∧ *is-type P* (*Class C*) = *is-class P C*⟨*proof*⟩

abbreviation
types P == *Collect* (*is-type P*)

end

2.4 Relations between Jinja Types

theory *TypeRel* **imports**
HOL−Library.Transitive-Closure-Table
Decl
begin

2.4.1 The subclass relations

inductive-set
subcls1 :: *'m prog* ⇒ (*cname* × *cname*) *set*
and *subcls1'* :: *'m prog* ⇒ [*cname*, *cname*] ⇒ *bool* (- ⊢ - ≺¹ - [*71*,*71*,*71*] *70*)
for *P* :: *'m prog*
where
P ⊢ *C* ≺¹ *D* ≡ (*C*,*D*) ∈ *subcls1 P*
| *subcls1I*: ⟦*class P C* = *Some* (*D*,*rest*); *C* ≠ *Object*⟧ ⟹ *P* ⊢ *C* ≺¹ *D*

abbreviation
subcls :: *'m prog* ⇒ [*cname*, *cname*] ⇒ *bool* (- ⊢ - ≼* - [*71*,*71*,*71*] *70*)
where *P* ⊢ *C* ≼* *D* ≡ (*C*,*D*) ∈ (*subcls1 P*)*

lemma *subcls1D*: *P* ⊢ *C* ≺¹ *D* ⟹ *C* ≠ *Object* ∧ (∃*fs ms*. *class P C* = *Some* (*D*,*fs*,*ms*))⟨*proof*⟩
lemma [*iff*]: ¬ *P* ⊢ *Object* ≺¹ *C*⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *Object* ≼* *C*) = (*C* = *Object*)⟨*proof*⟩
lemma *subcls1-def2*:
subcls1 P =
(*SIGMA C*:{*C*. *is-class P C*}. {*D*. *C*≠*Object* ∧ *fst* (*the* (*class P C*))=*D*})⟨*proof*⟩
lemma *finite-subcls1*: *finite* (*subcls1 P*)⟨*proof*⟩

2.4.2 The subtype relations

inductive
widen :: *'m prog* ⇒ *ty* ⇒ *ty* ⇒ *bool* (- ⊢ - ≤ - [*71*,*71*,*71*] *70*)
for *P* :: *'m prog*
where
widen-refl[*iff*]: *P* ⊢ *T* ≤ *T*
| *widen-subcls*: *P* ⊢ *C* ≼* *D* ⟹ *P* ⊢ *Class C* ≤ *Class D*
| *widen-null*[*iff*]: *P* ⊢ *NT* ≤ *Class C*

abbreviation
widens :: *'m prog* ⇒ *ty list* ⇒ *ty list* ⇒ *bool*
(- ⊢ - [≤] - [*71*,*71*,*71*] *70*) **where**
widens P Ts Ts' ≡ *list-all2* (*widen P*) *Ts Ts'*

lemma [*iff*]: (*P* ⊢ *T* ≤ *Void*) = (*T* = *Void*)⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *T* ≤ *Boolean*) = (*T* = *Boolean*)⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *T* ≤ *Integer*) = (*T* = *Integer*)⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *Void* ≤ *T*) = (*T* = *Void*)⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *Boolean* ≤ *T*) = (*T* = *Boolean*)⟨*proof*⟩
lemma [*iff*]: (*P* ⊢ *Integer* ≤ *T*) = (*T* = *Integer*)⟨*proof*⟩

lemma *Class-widen*: *P* ⊢ *Class C* ≤ *T* ⟹ ∃*D*. *T* = *Class D*⟨*proof*⟩

lemma [*iff*]: $(P \vdash T \leq NT) = (T = NT)\langle proof\rangle$
lemma *Class-widen-Class* [*iff*]: $(P \vdash Class\ C \leq Class\ D) = (P \vdash C \preceq^* D)\langle proof\rangle$
lemma *widen-Class*: $(P \vdash T \leq Class\ C) = (T = NT \vee (\exists D.\ T = Class\ D \wedge P \vdash D \preceq^* C))\langle proof\rangle$

lemma *widen-trans*[*trans*]: $[\![P \vdash S \leq U;\ P \vdash U \leq T]\!] \Longrightarrow P \vdash S \leq T\langle proof\rangle$
lemma *widens-trans* [*trans*]: $[\![P \vdash Ss\ [\leq]\ Ts;\ P \vdash Ts\ [\leq]\ Us]\!] \Longrightarrow P \vdash Ss\ [\leq]\ Us\langle proof\rangle$

2.4.3 Method lookup

inductive
Methods :: $['m\ prog,\ cname,\ mname \rightharpoonup (ty\ list \times ty \times 'm) \times cname] \Rightarrow bool$
(- ⊢ - *sees'-methods* - [*51*,*51*,*51*] *50*)
for $P :: 'm\ prog$
where
sees-methods-Object:
$[\![\ class\ P\ Object = Some(D,fs,ms);\ Mm = map\text{-}option\ (\lambda m.\ (m,Object)) \circ map\text{-}of\ ms\ ]\!]$
$\Longrightarrow P \vdash Object\ sees\text{-}methods\ Mm$
| *sees-methods-rec*:
$[\![\ class\ P\ C = Some(D,fs,ms);\ C \neq Object;\ P \vdash D\ sees\text{-}methods\ Mm;$
$Mm' = Mm\ ++\ (map\text{-}option\ (\lambda m.\ (m,C)) \circ map\text{-}of\ ms)\ ]\!]$
$\Longrightarrow P \vdash C\ sees\text{-}methods\ Mm'$

lemma *sees-methods-fun*:
assumes *1*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge Mm'.\ P \vdash C\ sees\text{-}methods\ Mm' \Longrightarrow Mm' = Mm$
$\langle proof\rangle$
lemma *visible-methods-exist*:
$P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow Mm\ M = Some(m,D) \Longrightarrow$
$(\exists D'\ fs\ ms.\ class\ P\ D = Some(D',fs,ms) \wedge map\text{-}of\ ms\ M = Some\ m)$
$\langle proof\rangle$
lemma *sees-methods-decl-above*:
assumes *Csees*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $Mm\ M = Some(m,D) \Longrightarrow P \vdash C \preceq^* D$
$\langle proof\rangle$
lemma *sees-methods-idemp*:
assumes *Cmethods*: $P \vdash C\ sees\text{-}methods\ Mm$
shows $\bigwedge m\ D.\ Mm\ M = Some(m,D) \Longrightarrow$
$\exists Mm'.\ (P \vdash D\ sees\text{-}methods\ Mm') \wedge Mm'\ M = Some(m,D)\langle proof\rangle$

lemma *sees-methods-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C\ sees\text{-}methods\ Mm \Longrightarrow$
$\exists Mm'\ Mm_2.\ P \vdash C'\ sees\text{-}methods\ Mm' \wedge Mm' = Mm\ ++\ Mm_2 \wedge$
$(\forall M\ m\ D.\ Mm_2\ M = Some(m,D) \longrightarrow P \vdash D \preceq^* C)\langle proof\rangle$

definition *Method* :: $'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow ty\ list \Rightarrow ty \Rightarrow 'm \Rightarrow cname \Rightarrow bool$
(- ⊢ - *sees* -: -→- = - *in* - [*51*,*51*,*51*,*51*,*51*,*51*,*51*] *50*)
where
$P \vdash C\ sees\ M: Ts \to T = m\ in\ D \equiv$
$\exists Mm.\ P \vdash C\ sees\text{-}methods\ Mm \wedge Mm\ M = Some((Ts,T,m),D)$

definition *has-method* :: $'m\ prog \Rightarrow cname \Rightarrow mname \Rightarrow bool$ (- ⊢ - *has* - [*51*,*0*,*51*] *50*)
where

$P \vdash C$ *has* $M \equiv \exists\, Ts\ T\ m\ D.\ P \vdash C$ *sees* $M{:}Ts \to T = m$ *in* D

lemma *sees-method-fun*:
$[\![P \vdash C$ *sees* $M{:}TS \to T = m$ *in* $D;\ P \vdash C$ *sees* $M{:}TS' \to T' = m'$ *in* $D']\!]$
$\Longrightarrow TS' = TS \wedge T' = T \wedge m' = m \wedge D' = D$
⟨*proof*⟩
lemma *sees-method-decl-above*:
$P \vdash C$ *sees* $M{:}Ts \to T = m$ *in* $D \Longrightarrow P \vdash C \preceq^* D$
⟨*proof*⟩
lemma *visible-method-exists*:
$P \vdash C$ *sees* $M{:}Ts \to T = m$ *in* $D \Longrightarrow$
$\exists D'\ fs\ ms.$ *class* $P\ D =$ *Some*$(D',fs,ms) \wedge$ *map-of* $ms\ M =$ *Some*(Ts,T,m)⟨*proof*⟩

lemma *sees-method-idemp*:
$P \vdash C$ *sees* $M{:}Ts \to T{=}m$ *in* $D \Longrightarrow P \vdash D$ *sees* $M{:}Ts \to T{=}m$ *in* D
⟨*proof*⟩
lemma *sees-method-decl-mono*:
assumes *sub*: $P \vdash C' \preceq^* C$ **and**
C-sees: $P \vdash C$ *sees* $M{:}Ts \to T{=}m$ *in* D **and**
C'*-sees*: $P \vdash C'$ *sees* $M{:}Ts' \to T'{=}m'$ *in* D'
shows $P \vdash D' \preceq^* D$
⟨*proof*⟩
lemma *sees-method-is-class*:
$[\![\ P \vdash C$ *sees* $M{:}Ts \to T = m$ *in* $D\ ]\!] \Longrightarrow$ *is-class* $P\ C$⟨*proof*⟩

2.4.4 Field lookup

inductive
Fields :: $['m$ *prog*, *cname*, $((vname \times cname) \times ty)$ *list*$] \Rightarrow$ *bool*
(- ⊢ - *has'-fields* - $[51,51,51]$ 50)
for $P :: 'm$ *prog*
where
has-fields-rec:
$[\![$ *class* $P\ C =$ *Some*$(D,fs,ms);\ C \neq$ *Object*; $P \vdash D$ *has-fields* *FDTs*;
$FDTs' =$ *map* $(\lambda(F,T).\ ((F,C),T))$ *fs* @ *FDTs* $]\!]$
$\Longrightarrow P \vdash C$ *has-fields* $FDTs'$
| *has-fields-Object*:
$[\![$ *class* P *Object* = *Some*$(D,fs,ms);$ *FDTs* = *map* $(\lambda(F,T).\ ((F,$*Object*$),T))$ *fs* $]\!]$
$\Longrightarrow P \vdash$ *Object has-fields FDTs*

lemma *has-fields-fun*:
assumes *1*: $P \vdash C$ *has-fields* *FDTs*
shows $\bigwedge FDTs'.\ P \vdash C$ *has-fields* $FDTs' \Longrightarrow FDTs' = FDTs$
⟨*proof*⟩
lemma *all-fields-in-has-fields*:
assumes *sub*: $P \vdash C$ *has-fields* *FDTs*
shows $[\![\ P \vdash C \preceq^* D;$ *class* $P\ D =$ *Some*$(D',fs,ms);\ (F,T) \in$ *set* $fs\ ]\!]$
$\Longrightarrow ((F,D),T) \in$ *set* *FDTs*⟨*proof*⟩

lemma *has-fields-decl-above*:
assumes *fields*: $P \vdash C$ *has-fields* *FDTs*
shows $((F,D),T) \in$ *set* *FDTs* $\Longrightarrow P \vdash C \preceq^* D$⟨*proof*⟩

lemma *subcls-notin-has-fields*:

assumes *fields*: $P \vdash C$ *has-fields FDTs*
shows $((F,D),T) \in$ *set FDTs* $\Longrightarrow (D,C) \notin (subcls1\ P)^+ \langle proof \rangle$

lemma *has-fields-mono-lem*:
assumes *sub*: $P \vdash D \preceq^* C$
shows $P \vdash C$ *has-fields FDTs*
$\Longrightarrow \exists pre.\ P \vdash D$ *has-fields pre@FDTs* $\wedge\ dom(map\text{-}of\ pre) \cap dom(map\text{-}of\ FDTs) = \{\} \langle proof \rangle$

definition *has-field* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *has* -:- *in* - [*51*,*51*,*51*,*51*,*51*] *50*)
where
$P \vdash C$ *has* $F{:}T$ *in* $D \equiv$
$\exists FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$ *map-of FDTs* $(F,D) =$ *Some* T

lemma *has-field-mono*:
assumes *has*: $P \vdash C$ *has* $F{:}T$ *in* D **and** *sub*: $P \vdash C' \preceq^* C$
shows $P \vdash C'$ *has* $F{:}T$ *in* $D \langle proof \rangle$

definition *sees-field* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *ty* $\Rightarrow$ *cname* $\Rightarrow$ *bool*
(- $\vdash$ - *sees* -:- *in* - [*51*,*51*,*51*,*51*,*51*] *50*)
where
$P \vdash C$ *sees* $F{:}T$ *in* $D \equiv$
$\exists FDTs.\ P \vdash C$ *has-fields FDTs* $\wedge$
map-of (*map* $(\lambda((F,D),T).\ (F,(D,T)))$ *FDTs*) $F =$ *Some*(D,T)

lemma *map-of-remap-SomeD*:
map-of (*map* $(\lambda((k,k'),x).\ (k,(k',x)))\ t)\ k =$ *Some* $(k',x) \Longrightarrow$ *map-of* $t\ (k,\ k') =$ *Some* $x \langle proof \rangle$

lemma *has-visible-field*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C$ *has* $F{:}T$ *in* $D \langle proof \rangle$

lemma *sees-field-fun*:
$[\![P \vdash C$ *sees* $F{:}T$ *in* $D;\ P \vdash C$ *sees* $F{:}T'$ *in* $D]\!] \Longrightarrow T' = T \wedge D' = D \langle proof \rangle$

lemma *sees-field-decl-above*:
$P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow P \vdash C \preceq^* D \langle proof \rangle$
lemma *sees-field-idemp*:
assumes *sees*: $P \vdash C$ *sees* $F{:}T$ *in* D
shows $P \vdash D$ *sees* $F{:}T$ *in* $D \langle proof \rangle$

2.4.5 Functional lookup

definition *method* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ *mname* $\Rightarrow$ *cname* $\times$ *ty list* $\times$ *ty* $\times$ *'m*
where
method $P\ C\ M \equiv$ *THE* $(D,Ts,T,m).\ P \vdash C$ *sees* $M{:}Ts \to T = m$ *in* D

definition *field* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ *vname* $\Rightarrow$ *cname* $\times$ *ty*
where
field $P\ C\ F \equiv$ *THE* $(D,T).\ P \vdash C$ *sees* $F{:}T$ *in* D

definition *fields* :: *'m prog* $\Rightarrow$ *cname* $\Rightarrow$ $((vname \times cname) \times ty)$ *list*
where
fields $P\ C \equiv$ *THE FDTs*. $P \vdash C$ *has-fields FDTs*

lemma *fields-def2* [*simp*]: $P \vdash C$ *has-fields* $FDTs \Longrightarrow$ *fields* $P\ C = FDTs\langle proof\rangle$
lemma *field-def2* [*simp*]: $P \vdash C$ *sees* $F{:}T$ *in* $D \Longrightarrow$ *field* $P\ C\ F = (D,T)\langle proof\rangle$
lemma *method-def2* [*simp*]: $P \vdash C$ *sees* $M{:}\ Ts \to T = m$ *in* $D \Longrightarrow$ *method* $P\ C\ M = (D,Ts,T,m)\langle proof\rangle$

2.4.6 Code generator setup

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
subcls1p
$\langle proof\rangle$
declare *subcls1-def* [*code-pred-def*]

code-pred
(*modes*: $i \Rightarrow i \times o \Rightarrow bool$, $i \Rightarrow i \times i \Rightarrow bool$)
[*inductify*]
subcls1
$\langle proof\rangle$
definition *subcls'* **where** *subcls'* $G = (subcls1p\ G)^{\frown}{**}$
code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
[*inductify*]
subcls'
$\langle proof\rangle$

lemma *subcls-conv-subcls'* [*code-unfold*]:
$(subcls1\ G)^{\frown}{*} = \{(C,\ D).\ subcls'\ G\ C\ D\}$
$\langle proof\rangle$

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow bool$)
widen
$\langle proof\rangle$

code-pred
(*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
Fields
$\langle proof\rangle$

lemma *has-field-code* [*code-pred-intro*]:
$[\![\ P \vdash C$ *has-fields* $FDTs$; *map-of* $FDTs\ (F,\ D) = \lfloor T \rfloor\ ]\!]$
$\Longrightarrow P \vdash C$ *has* $F{:}T$ *in* D
$\langle proof\rangle$

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
has-field
$\langle proof\rangle$

lemma *sees-field-code* [*code-pred-intro*]:
$[\![\ P \vdash C$ *has-fields* $FDTs$; *map-of* $(map\ (\lambda((F,\ D),\ T).\ (F,\ D,\ T))\ FDTs)\ F = \lfloor (D,\ T) \rfloor\ ]\!]$
$\Longrightarrow P \vdash C$ *sees* $F{:}T$ *in* D
$\langle proof\rangle$

code-pred

(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow i \Rightarrow bool$,
$i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow bool$, $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
sees-field
⟨*proof*⟩

code-pred
(*modes*: $i \Rightarrow i \Rightarrow o \Rightarrow bool$)
Methods
⟨*proof*⟩

lemma *Method-code* [*code-pred-intro*]:
$[\![P \vdash C \text{ sees-methods } Mm;\ Mm\ M = \lfloor((Ts,\ T,\ m),\ D)\rfloor]\!]$
$\Longrightarrow P \vdash C \text{ sees } M\colon Ts \to T = m \text{ in } D$
⟨*proof*⟩

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow o \Rightarrow bool$,
$i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow bool$)
Method
⟨*proof*⟩

lemma *eval-Method-i-i-i-o-o-o-o-conv*:
Predicate.eval (*Method-i-i-i-o-o-o-o P C M*) = ($\lambda(Ts,\ T,\ m,\ D).\ P \vdash C \text{ sees } M\colon Ts \to T = m \text{ in } D$)
⟨*proof*⟩

lemma *method-code* [*code*]:
method P C M =
Predicate.the (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) ($\lambda(Ts,\ T,\ m,\ D).$ *Predicate.single* (D, Ts, T, m)))
⟨*proof*⟩

lemma *eval-Fields-conv*:
Predicate.eval (*Fields-i-i-o P C*) = ($\lambda FDTs.\ P \vdash C \text{ has-fields } FDTs$)
⟨*proof*⟩

lemma *fields-code* [*code*]:
fields P C = *Predicate.the* (*Fields-i-i-o P C*)
⟨*proof*⟩

lemma *eval-sees-field-i-i-i-o-o-conv*:
Predicate.eval (*sees-field-i-i-i-o-o P C F*) = ($\lambda(T,\ D).\ P \vdash C \text{ sees } F\colon T \text{ in } D$)
⟨*proof*⟩

lemma *eval-sees-field-i-i-i-o-i-conv*:
Predicate.eval (*sees-field-i-i-i-o-i P C F D*) = ($\lambda T.\ P \vdash C \text{ sees } F\colon T \text{ in } D$)
⟨*proof*⟩

lemma *field-code* [*code*]:
field P C F = *Predicate.the* (*Predicate.bind* (*sees-field-i-i-i-o-o P C F*) ($\lambda(T,\ D).$ *Predicate.single* (D, T)))
⟨*proof*⟩

2.5 Jinja Values

theory *Value* **imports** *TypeRel* **begin**

type-synonym *addr* = *nat*

datatype *val*
= *Unit* — dummy result value of void expressions
| *Null* — null reference
| *Bool bool* — Boolean value
| *Intg int* — integer value
| *Addr addr* — addresses of objects in the heap

primrec *the-Intg* :: *val* ⇒ *int* **where**
the-Intg (*Intg i*) = *i*

primrec *the-Addr* :: *val* ⇒ *addr* **where**
the-Addr (*Addr a*) = *a*

primrec *default-val* :: *ty* ⇒ *val* — default value for all types **where**
default-val Void = *Unit*
| *default-val Boolean* = *Bool False*
| *default-val Integer* = *Intg 0*
| *default-val NT* = *Null*
| *default-val* (*Class C*) = *Null*

end

2.6 Objects and the Heap

theory *Objects* **imports** *TypeRel Value* **begin**

2.6.1 Objects

type-synonym
fields = *vname* × *cname* ⇀ *val* — field name, defining class, value
type-synonym
obj = *cname* × *fields* — class instance with class name and fields

definition *obj-ty* :: *obj* ⇒ *ty*
where
obj-ty obj ≡ *Class* (*fst obj*)

definition *init-fields* :: ((*vname* × *cname*) × *ty*) *list* ⇒ *fields*
where
init-fields ≡ *map-of* ∘ *map* (λ(*F*,*T*). (*F*,*default-val T*))

— a new, blank object with default values in all fields:
definition *blank* :: 'm *prog* ⇒ *cname* ⇒ *obj*
where
blank P C ≡ (*C*,*init-fields* (*fields P C*))

lemma [*simp*]: *obj-ty* (*C*,*fs*) = *Class C*⟨*proof*⟩

2.6.2 Heap

type-synonym *heap* = *addr* ⇀ *obj*

abbreviation
cname-of :: *heap* ⇒ *addr* ⇒ *cname* **where**
cname-of hp a == *fst* (*the* (*hp a*))

definition *new-Addr* :: *heap* ⇒ *addr option*
where
new-Addr h ≡ *if* ∃ *a*. *h a* = *None then Some*(*LEAST a*. *h a* = *None*) *else None*

definition *cast-ok* :: *'m prog* ⇒ *cname* ⇒ *heap* ⇒ *val* ⇒ *bool*
where
cast-ok P C h v ≡ *v* = *Null* ∨ *P* ⊢ *cname-of h* (*the-Addr v*) ⪯* *C*

definition *hext* :: *heap* ⇒ *heap* ⇒ *bool* (- ⊴ - [*51*,*51*] *50*)
where
h ⊴ *h'* ≡ ∀ *a C fs*. *h a* = *Some*(*C*,*fs*) ⟶ (∃ *fs'*. *h' a* = *Some*(*C*,*fs'*))

primrec *typeof-h* :: *heap* ⇒ *val* ⇒ *ty option* (*typeof*_)
where
typeof$_h$ *Unit* = *Some Void*
| *typeof*$_h$ *Null* = *Some NT*
| *typeof*$_h$ (*Bool b*) = *Some Boolean*
| *typeof*$_h$ (*Intg i*) = *Some Integer*
| *typeof*$_h$ (*Addr a*) = (*case h a of None* ⇒ *None* | *Some*(*C*,*fs*) ⇒ *Some*(*Class C*))

lemma *new-Addr-SomeD*:
new-Addr h = *Some a* ⟹ *h a* = *None*
⟨*proof*⟩
lemma [*simp*]: (*typeof*$_h$ *v* = *Some Boolean*) = (∃ *b*. *v* = *Bool b*)
⟨*proof*⟩
lemma [*simp*]: (*typeof*$_h$ *v* = *Some Integer*) = (∃ *i*. *v* = *Intg i*)⟨*proof*⟩
lemma [*simp*]: (*typeof*$_h$ *v* = *Some NT*) = (*v* = *Null*)
⟨*proof*⟩
lemma [*simp*]: (*typeof*$_h$ *v* = *Some*(*Class C*)) = (∃ *a fs*. *v* = *Addr a* ∧ *h a* = *Some*(*C*,*fs*))
⟨*proof*⟩
lemma [*simp*]: *h a* = *Some*(*C*,*fs*) ⟹ *typeof*$_{(h(a\mapsto(C,fs')))}$ *v* = *typeof*$_h$ *v*
⟨*proof*⟩

For literal values the first parameter of *typeof* can be set to *Map.empty* because they do not contain addresses:

abbreviation
typeof :: *val* ⇒ *ty option* **where**
typeof v == *typeof-h Map.empty v*

lemma *typeof-lit-typeof*:
typeof v = *Some T* ⟹ *typeof*$_h$ *v* = *Some T*
⟨*proof*⟩
lemma *typeof-lit-is-type*:
typeof v = *Some T* ⟹ *is-type P T*
⟨*proof*⟩

2.6.3 Heap extension $\trianglelefteq$

lemma *hextI*: $\forall a\ C\ fs.\ h\ a = Some(C,fs) \longrightarrow (\exists fs'.\ h'\ a = Some(C,fs')) \Longrightarrow h \trianglelefteq h' \langle proof \rangle$
lemma *hext-objD*: $[\![\ h \trianglelefteq h';\ h\ a = Some(C,fs)\ ]\!] \Longrightarrow \exists fs'.\ h'\ a = Some(C,fs') \langle proof \rangle$
lemma *hext-refl* [*iff*]: $h \trianglelefteq h \langle proof \rangle$
lemma *hext-new* [*simp*]: $h\ a = None \Longrightarrow h \trianglelefteq h(a \mapsto x) \langle proof \rangle$
lemma *hext-trans*: $[\![\ h \trianglelefteq h';\ h' \trianglelefteq h''\ ]\!] \Longrightarrow h \trianglelefteq h'' \langle proof \rangle$
lemma *hext-upd-obj*: $h\ a = Some\ (C,fs) \Longrightarrow h \trianglelefteq h(a \mapsto (C,fs')) \langle proof \rangle$
lemma *hext-typeof-mono*: $[\![\ h \trianglelefteq h';\ typeof_h\ v = Some\ T\ ]\!] \Longrightarrow typeof_{h'}\ v = Some\ T \langle proof \rangle$

Code generator setup for *new-Addr*

definition *gen-new-Addr* :: *heap* ⇒ *addr* ⇒ *addr option*
where *gen-new-Addr h n* ≡ *if* ∃ *a. a* ≥ *n* ∧ *h a* = *None then Some*(*LEAST a. a* ≥ *n* ∧ *h a* = *None*) *else None*

lemma *new-Addr-code-code* [*code*]:
new-Addr h = *gen-new-Addr h 0*
⟨*proof*⟩

lemma *gen-new-Addr-code* [*code*]:
gen-new-Addr h n = (*if h n* = *None then Some n else gen-new-Addr h* (*Suc n*))
⟨*proof*⟩

end

2.7 Exceptions

theory *Exceptions* **imports** *Objects* **begin**

definition *NullPointer* :: *cname*
where
NullPointer ≡ *''NullPointer''*

definition *ClassCast* :: *cname*
where
ClassCast ≡ *''ClassCast''*

definition *OutOfMemory* :: *cname*
where
OutOfMemory ≡ *''OutOfMemory''*

definition *sys-xcpts* :: *cname set*
where
sys-xcpts ≡ {*NullPointer*, *ClassCast*, *OutOfMemory*}

definition *addr-of-sys-xcpt* :: *cname* ⇒ *addr*
where
addr-of-sys-xcpt s ≡ *if s* = *NullPointer then 0 else*
if s = *ClassCast then 1 else*
if s = *OutOfMemory then 2 else undefined*

definition *start-heap* :: *'c prog* ⇒ *heap*
where
start-heap G ≡ *Map.empty* (*addr-of-sys-xcpt NullPointer* ↦ *blank G NullPointer*)

$(addr\text{-}of\text{-}sys\text{-}xcpt\ ClassCast \mapsto blank\ G\ ClassCast)$
$(addr\text{-}of\text{-}sys\text{-}xcpt\ OutOfMemory \mapsto blank\ G\ OutOfMemory)$

definition *preallocated* :: *heap* ⇒ *bool*
where
preallocated h ≡ ∀ *C* ∈ *sys-xcpts*. ∃ *fs*. *h*(*addr-of-sys-xcpt C*) = *Some* (*C*,*fs*)

2.7.1 System exceptions

lemma [*simp*]: *NullPointer* ∈ *sys-xcpts* ∧ *OutOfMemory* ∈ *sys-xcpts* ∧ *ClassCast* ∈ *sys-xcpts*⟨*proof*⟩

lemma *sys-xcpts-cases* [*consumes 1*, *cases set*]:
⟦ *C* ∈ *sys-xcpts*; *P NullPointer*; *P OutOfMemory*; *P ClassCast*⟧ ⟹ *P C*⟨*proof*⟩

2.7.2 *preallocated*

lemma *preallocated-dom* [*simp*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ *addr-of-sys-xcpt C* ∈ *dom h*⟨*proof*⟩

lemma *preallocatedD*:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ ∃ *fs*. *h*(*addr-of-sys-xcpt C*) = *Some* (*C*, *fs*)⟨*proof*⟩

lemma *preallocatedE* [*elim?*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts*; ⋀*fs*. *h*(*addr-of-sys-xcpt C*) = *Some*(*C*,*fs*) ⟹ *P h C*⟧
⟹ *P h C*⟨*proof*⟩

lemma *cname-of-xcp* [*simp*]:
⟦ *preallocated h*; *C* ∈ *sys-xcpts* ⟧ ⟹ *cname-of h* (*addr-of-sys-xcpt C*) = *C*⟨*proof*⟩

lemma *typeof-ClassCast* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt ClassCast*)) = *Some*(*Class ClassCast*)⟨*proof*⟩

lemma *typeof-OutOfMemory* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt OutOfMemory*)) = *Some*(*Class OutOfMemory*)⟨*proof*⟩

lemma *typeof-NullPointer* [*simp*]:
preallocated h ⟹ $typeof_h$ (*Addr*(*addr-of-sys-xcpt NullPointer*)) = *Some*(*Class NullPointer*)⟨*proof*⟩

lemma *preallocated-hext*:
⟦ *preallocated h*; *h* ⊴ *h'* ⟧ ⟹ *preallocated h'*⟨*proof*⟩

lemma *preallocated-start*:
preallocated (*start-heap P*)⟨*proof*⟩

end

2.8 Expressions

theory *Expr*
imports *../Common/Exceptions*
begin

datatype *bop* = *Eq* | *Add* — names of binary operations

datatype *'a exp*
  = *new cname* — class instance creation
  | *Cast cname ('a exp)* — type cast
  | *Val val* — value
  | *BinOp ('a exp) bop ('a exp)* (- «-» - [*80,0,81*] *80*) — binary operation
  | *Var 'a* — local variable (incl. parameter)
  | *LAss 'a ('a exp)* (-:=- [*90,90*]*90*) — local assignment
  | *FAcc ('a exp) vname cname* (-•-{-} [*10,90,99*]*90*) — field access
  | *FAss ('a exp) vname cname ('a exp)* (-•-{-} := - [*10,90,99,90*]*90*) — field assignment
  | *Call ('a exp) mname ('a exp list)* (-•-'(-') [*90,99,0*] *90*) — method call
  | *Block 'a ty ('a exp)* ('{-:-; -})
  | *Seq ('a exp) ('a exp)* (-;;/ - [*61,60*]*60*)
  | *Cond ('a exp) ('a exp) ('a exp)* (*if '(-') -/ else* - [*80,79,79*]*70*)
  | *While ('a exp) ('a exp)* (*while '(-') -* [*80,79*]*70*)
  | *throw ('a exp)*
  | *TryCatch ('a exp) cname 'a ('a exp)* (*try -/ catch'(- -') -* [*0,99,80,79*] *70*)

type-synonym
  expr = *vname exp* — Jinja expression
type-synonym
  J-mb = *vname list* × *expr* — Jinja method body: parameter names and expression
type-synonym
  J-prog = *J-mb prog* — Jinja program

The semantics of binary operators:

fun *binop* :: *bop* × *val* × *val* ⇒ *val option* **where**
  binop(*Eq*,v_1,v_2) = *Some*(*Bool* (v_1 = v_2))
| *binop*(*Add*,*Intg* i_1,*Intg* i_2) = *Some*(*Intg*(i_1+i_2))
| *binop*(*bop*,v_1,v_2) = *None*

lemma [*simp*]:
  (*binop*(*Add*,v_1,v_2) = *Some v*) = (∃ i_1 i_2. v_1 = *Intg* i_1 ∧ v_2 = *Intg* i_2 ∧ *v* = *Intg*(i_1+i_2))⟨*proof*⟩

2.8.1 Syntactic sugar

abbreviation (*input*)
  InitBlock:: *'a* ⇒ *ty* ⇒ *'a exp* ⇒ *'a exp* ⇒ *'a exp* ((*1 '*{-:- := -;/ -})) **where**
  InitBlock V T e1 e2 == {*V*:*T*; *V* := *e1*;; *e2*}

abbreviation *unit* **where** *unit* == *Val Unit*
abbreviation *null* **where** *null* == *Val Null*
abbreviation *addr a* == *Val*(*Addr a*)
abbreviation *true* == *Val*(*Bool True*)
abbreviation *false* == *Val*(*Bool False*)

abbreviation
  Throw :: *addr* ⇒ *'a exp* **where**
  Throw a == *throw*(*Val*(*Addr a*))

abbreviation
  THROW :: *cname* ⇒ *'a exp* **where**
  THROW xc == *Throw*(*addr-of-sys-xcpt xc*)

2.8.2 Free Variables

primrec *fv* :: *expr* ⇒ *vname set* **and** *fvs* :: *expr list* ⇒ *vname set* **where**
  fv(*new C*) = {}
| *fv*(*Cast C e*) = *fv e*
| *fv*(*Val v*) = {}
| *fv*(*e*$_1$ «*bop*» *e*$_2$) = *fv e*$_1$ ∪ *fv e*$_2$
| *fv*(*Var V*) = {*V*}
| *fv*(*LAss V e*) = {*V*} ∪ *fv e*
| *fv*(*e*·*F*{*D*}) = *fv e*
| *fv*(*e*$_1$·*F*{*D*}:=*e*$_2$) = *fv e*$_1$ ∪ *fv e*$_2$
| *fv*(*e*·*M*(*es*)) = *fv e* ∪ *fvs es*
| *fv*({*V*:*T*; *e*}) = *fv e* − {*V*}
| *fv*(*e*$_1$;;*e*$_2$) = *fv e*$_1$ ∪ *fv e*$_2$
| *fv*(*if* (*b*) *e*$_1$ *else e*$_2$) = *fv b* ∪ *fv e*$_1$ ∪ *fv e*$_2$
| *fv*(*while* (*b*) *e*) = *fv b* ∪ *fv e*
| *fv*(*throw e*) = *fv e*
| *fv*(*try e*$_1$ *catch*(*C V*) *e*$_2$) = *fv e*$_1$ ∪ (*fv e*$_2$ − {*V*})
| *fvs*([]) = {}
| *fvs*(*e*#*es*) = *fv e* ∪ *fvs es*

lemma [*simp*]: *fvs*(*es*$_1$ @ *es*$_2$) = *fvs es*$_1$ ∪ *fvs es*$_2$⟨*proof*⟩
lemma [*simp*]: *fvs*(*map Val vs*) = {}⟨*proof*⟩
end

2.9 Program State

theory *State* **imports** *../Common/Exceptions* **begin**

type-synonym
  locals = *vname* ⇀ *val* — local vars, incl. params and "this"
type-synonym
  state = *heap* × *locals*

definition *hp* :: *state* ⇒ *heap*
where
  hp ≡ *fst*
definition *lcl* :: *state* ⇒ *locals*
where
  lcl ≡ *snd*
end

2.10 Big Step Semantics

theory *BigStep* **imports** *Expr State* **begin**

inductive
  eval :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
    (- ⊢ ((*1*⟨-,/-⟩) ⇒/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  and *evals* :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
    (- ⊢ ((*1*⟨-,/-⟩) [⇒]/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  for *P* :: *J-prog*
where

New:
$[\![$ *new-Addr* h = *Some* a; $P \vdash C$ *has-fields FDTs*; $h' = h(a \mapsto (C, \textit{init-fields FDTs}))$ $]\!]$
$\Longrightarrow P \vdash \langle \textit{new } C,(h,l)\rangle \Rightarrow \langle \textit{addr } a,(h',l)\rangle$

| *NewFail*:
new-Addr h = *None* $\Longrightarrow$
$P \vdash \langle \textit{new } C, (h,l)\rangle \Rightarrow \langle \textit{THROW OutOfMemory},(h,l)\rangle$

| *Cast*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{addr } a,(h,l)\rangle$; $h\ a = \textit{Some}(D,fs)$; $P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle \textit{Cast } C\ e,s_0\rangle \Rightarrow \langle \textit{addr } a,(h,l)\rangle$

| *CastNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{null},s_1\rangle \Longrightarrow$
$P \vdash \langle \textit{Cast } C\ e,s_0\rangle \Rightarrow \langle \textit{null},s_1\rangle$

| *CastFail*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{addr } a,(h,l)\rangle$; $h\ a = \textit{Some}(D,fs)$; $\neg\ P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash \langle \textit{Cast } C\ e,s_0\rangle \Rightarrow \langle \textit{THROW ClassCast},(h,l)\rangle$

| *CastThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{throw } e',s_1\rangle \Longrightarrow$
$P \vdash \langle \textit{Cast } C\ e,s_0\rangle \Rightarrow \langle \textit{throw } e',s_1\rangle$

| *Val*:
$P \vdash \langle \textit{Val } v,s\rangle \Rightarrow \langle \textit{Val } v,s\rangle$

| *BinOp*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \textit{Val } v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle \textit{Val } v_2,s_2\rangle$; $\textit{binop}(bop,v_1,v_2) = \textit{Some } v$ $]\!]$
$\Longrightarrow P \vdash \langle e_1\ \textit{«bop»}\ e_2,s_0\rangle \Rightarrow \langle \textit{Val } v,s_2\rangle$

| *BinOpThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \textit{throw } e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_1\ \textit{«bop»}\ e_2,\ s_0\rangle \Rightarrow \langle \textit{throw } e,s_1\rangle$

| *BinOpThrow2*:
$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \textit{Val } v_1,s_1\rangle$; $P \vdash \langle e_2,s_1\rangle \Rightarrow \langle \textit{throw } e,s_2\rangle$ $]\!]$
$\Longrightarrow P \vdash \langle e_1\ \textit{«bop»}\ e_2,s_0\rangle \Rightarrow \langle \textit{throw } e,s_2\rangle$

| *Var*:
$l\ V$ = *Some* $v \Longrightarrow$
$P \vdash \langle \textit{Var } V,(h,l)\rangle \Rightarrow \langle \textit{Val } v,(h,l)\rangle$

| *LAss*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{Val } v,(h,l)\rangle$; $l' = l(V \mapsto v)$ $]\!]$
$\Longrightarrow P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle \textit{unit},(h,l')\rangle$

| *LAssThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{throw } e',s_1\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s_0\rangle \Rightarrow \langle \textit{throw } e',s_1\rangle$

| *FAcc*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle \textit{addr } a,(h,l)\rangle$; $h\ a = \textit{Some}(C,fs)$; $fs(F,D) = \textit{Some } v$ $]\!]$

$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle \mathit{Val}\ v,(h,l)\rangle$

| *FAccNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{null},s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle \mathit{THROW\ NullPointer},s_1\rangle$

| *FAccThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *FAss*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{addr}\ a,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle \mathit{Val}\ v,(h_2,l_2)\rangle;$
$h_2\ a = \mathit{Some}(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle \mathit{unit},(h_2',l_2)\rangle$

| *FAssNull*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{null},s_1\rangle;\ \ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle \mathit{Val}\ v,s_2\rangle\ ]\!] \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle \mathit{THROW\ NullPointer},s_2\rangle$

| *FAssThrow1*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *FAssThrow2*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{Val}\ v,s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle \mathit{throw}\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_2\rangle$

| *CallObjThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *CallParamsThrow*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{Val}\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle \mathit{map}\ \mathit{Val}\ vs\ @\ \mathit{throw}\ ex\ \#\ es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle \mathit{throw}\ ex,s_2\rangle$

| *CallNull*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{null},s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle \mathit{map}\ \mathit{Val}\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle \mathit{THROW\ NullPointer},s_2\rangle$

| *Call*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{addr}\ a,s_1\rangle;\ \ P \vdash \langle ps,s_1\rangle\ [\Rightarrow]\ \langle \mathit{map}\ \mathit{Val}\ vs,(h_2,l_2)\rangle;$
$h_2\ a = \mathit{Some}(C,fs);\ \ P \vdash C\ \mathit{sees}\ M{:}Ts{\to}T = (pns,body)\ \mathit{in}\ D;$
$\mathit{length}\ vs = \mathit{length}\ pns;\ \ l_2' = [\mathit{this}{\mapsto}\mathit{Addr}\ a,\ pns[{\mapsto}]vs];$
$P \vdash \langle \mathit{body},(h_2,l_2')\rangle \Rightarrow \langle e',(h_3,l_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}M(ps),s_0\rangle \Rightarrow \langle e',(h_3,l_2)\rangle$

| *Block*:
$P \vdash \langle e_0,(h_0,l_0(V{:=}None))\rangle \Rightarrow \langle e_1,(h_1,l_1)\rangle \Longrightarrow$
$P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \Rightarrow \langle e_1,(h_1,l_1(V{:=}l_0\ V))\rangle$

| *Seq*:
$[\![\ P \vdash \langle e_0,s_0\rangle \Rightarrow \langle \mathit{Val}\ v,s_1\rangle;\ P \vdash \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$

| *SeqThrow*:
$P \vdash \langle e_0,s_0\rangle \Rightarrow \langle \mathit{throw}\ e,s_1\rangle \Longrightarrow$
$P \vdash \langle e_0;;e_1,s_0\rangle \Rightarrow \langle \mathit{throw}\ e,s_1\rangle$

| *CondT*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{true},s_1\rangle;\ P \vdash \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondF*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{false},s_1\rangle;\ P \vdash \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$

| *CondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{if}\ (e)\ e_1\ \mathit{else}\ e_2,\ s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *WhileF*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{false},s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{while}\ (e)\ c,s_0\rangle \Rightarrow \langle \mathit{unit},s_1\rangle$

| *WhileT*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{true},s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle \mathit{Val}\ v_1,s_2\rangle;\ P \vdash \langle \mathit{while}\ (e)\ c,s_2\rangle \Rightarrow \langle e_3,s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle \mathit{while}\ (e)\ c,s_0\rangle \Rightarrow \langle e_3,s_3\rangle$

| *WhileCondThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle\ \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{while}\ (e)\ c,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *WhileBodyThrow*:
$[\![\ P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{true},s_1\rangle;\ P \vdash \langle c,s_1\rangle \Rightarrow \langle \mathit{throw}\ e',s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle \mathit{while}\ (e)\ c,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_2\rangle$

| *Throw*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{addr}\ a,s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{throw}\ e,s_0\rangle \Rightarrow \langle \mathit{Throw}\ a,s_1\rangle$

| *ThrowNull*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{null},s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{throw}\ e,s_0\rangle \Rightarrow \langle \mathit{THROW\ NullPointer},s_1\rangle$

| *ThrowThrow*:
$P \vdash \langle e,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{throw}\ e,s_0\rangle \Rightarrow \langle \mathit{throw}\ e',s_1\rangle$

| *Try*:
$P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{Val}\ v_1,s_1\rangle \Longrightarrow$
$P \vdash \langle \mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2,s_0\rangle \Rightarrow \langle \mathit{Val}\ v_1,s_1\rangle$

| *TryCatch*:
$[\![\ P \vdash \langle e_1,s_0\rangle \Rightarrow \langle \mathit{Throw}\ a,(h_1,l_1)\rangle;\ \ h_1\ a = \mathit{Some}(D,fs);\ \ P \vdash D \preceq^*\ C;$
$\quad P \vdash \langle e_2,(h_1,l_1(V \mapsto \mathit{Addr}\ a))\rangle \Rightarrow \langle e_2',(h_2,l_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle \mathit{try}\ e_1\ \mathit{catch}(C\ V)\ e_2,s_0\rangle \Rightarrow \langle e_2',(h_2,l_2(V := l_1\ V))\rangle$

| *TryThrow*:

$[\![P \vdash \langle e_1,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle;\ h_1\ a = Some(D,fs);\ \neg P \vdash D \preceq^* C]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s_0\rangle \Rightarrow \langle Throw\ a,(h_1,l_1)\rangle$

| *Nil*:
$P \vdash \langle[],s\rangle\ [\Rightarrow]\ \langle[],s\rangle$

| *Cons*:
$[\![P \vdash \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle; P \vdash \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\Rightarrow]\ \langle Val\ v\ \#\ es',s_2\rangle$

| *ConsThrow*:
$P \vdash \langle e,\ s_0\rangle \Rightarrow \langle throw\ e',\ s_1\rangle \Longrightarrow$
$P \vdash \langle e\#es,\ s_0\rangle\ [\Rightarrow]\ \langle throw\ e'\ \#\ es,\ s_1\rangle$

2.10.1 Final expressions

definition *final* :: *'a exp* ⇒ *bool*
where
$final\ e\ \equiv\ (\exists v.\ e = Val\ v) \vee (\exists a.\ e = Throw\ a)$

definition *finals*:: *'a exp list* ⇒ *bool*
where
$finals\ es\ \equiv\ (\exists vs.\ es = map\ Val\ vs) \vee (\exists vs\ a\ es'.\ es = map\ Val\ vs\ @\ Throw\ a\ \#\ es')$

lemma [*simp*]: $final(Val\ v)\langle proof\rangle$
lemma [*simp*]: $final(throw\ e) = (\exists a.\ e = addr\ a)\langle proof\rangle$
lemma *finalE*: $[\![final\ e;\ \bigwedge v.\ e = Val\ v \Longrightarrow R;\ \bigwedge a.\ e = Throw\ a \Longrightarrow R]\!] \Longrightarrow R\langle proof\rangle$
lemma [*iff*]: $finals\ []\langle proof\rangle$
lemma [*iff*]: $finals\ (Val\ v\ \#\ es) = finals\ es\langle proof\rangle$
lemma *finals-app-map*[*iff*]: $finals\ (map\ Val\ vs\ @\ es) = finals\ es\langle proof\rangle$
lemma [*iff*]: $finals\ (map\ Val\ vs)\langle proof\rangle$
lemma [*iff*]: $finals\ (throw\ e\ \#\ es) = (\exists a.\ e = addr\ a)\langle proof\rangle$
lemma *not-finals-ConsI*: $\neg final\ e \Longrightarrow \neg finals(e\#es)\langle proof\rangle$

lemma *eval-final*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow final\ e'$
and *evals-final*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow finals\ es'\langle proof\rangle$

lemma *eval-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \Rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and *evals-lcl-incr*: $P \vdash \langle es,(h_0,l_0)\rangle\ [\Rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1\langle proof\rangle$

Only used later, in the small to big translation, but is already a good sanity check:

lemma *eval-finalId*: $final\ e \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e,s\rangle\langle proof\rangle$

lemma *eval-finalsId*:
assumes *finals*: $finals\ es$ **shows** $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es,s\rangle\langle proof\rangle$

theorem *eval-hext*: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *evals-hext*: $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h'\langle proof\rangle$

end

2.11 Small Step Semantics

theory *SmallStep*
imports *Expr State*
begin

fun *blocks* :: *vname list* ∗ *ty list* ∗ *val list* ∗ *expr* ⇒ *expr*
where
blocks(*V*#*Vs*, *T*#*Ts*, *v*#*vs*, *e*) = {*V*:*T* := *Val v*; *blocks*(*Vs*,*Ts*,*vs*,*e*)}
|*blocks*([],[],[],*e*) = *e*

lemmas *blocks-induct* = *blocks.induct*[*split-format* (*complete*)]

lemma [*simp*]:
⟦ *size vs* = *size Vs*; *size Ts* = *size Vs* ⟧ ⟹ *fv*(*blocks*(*Vs*,*Ts*,*vs*,*e*)) = *fv e* − *set Vs*⟨*proof*⟩

definition *assigned* :: *vname* ⇒ *expr* ⇒ *bool*
where
assigned V e ≡ ∃ *v e*′. *e* = (*V* := *Val v*;; *e*′)

inductive-set
red :: *J-prog* ⇒ ((*expr* × *state*) × (*expr* × *state*)) *set*
and *reds* :: *J-prog* ⇒ ((*expr list* × *state*) × (*expr list* × *state*)) *set*
and *red*′ :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
(- ⊢ ((1⟨-,/-⟩) →/ (1⟨-,/-⟩)) [51,0,0,0,0] 81)
and *reds*′ :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
(- ⊢ ((1⟨-,/-⟩) [→]/ (1⟨-,/-⟩)) [51,0,0,0,0] 81)
for *P* :: *J-prog*
where

P ⊢ ⟨*e*,*s*⟩ → ⟨*e*′,*s*′⟩ ≡ ((*e*,*s*), *e*′,*s*′) ∈ *red P*
| *P* ⊢ ⟨*es*,*s*⟩ [→] ⟨*es*′,*s*′⟩ ≡ ((*es*,*s*), *es*′,*s*′) ∈ *reds P*

| *RedNew*:
⟦ *new-Addr h* = *Some a*; *P* ⊢ *C has-fields FDTs*; *h*′ = *h*(*a*↦(*C*,*init-fields FDTs*)) ⟧
⟹ *P* ⊢ ⟨*new C*, (*h*,*l*)⟩ → ⟨*addr a*, (*h*′,*l*)⟩

| *RedNewFail*:
new-Addr h = *None* ⟹
P ⊢ ⟨*new C*, (*h*,*l*)⟩ → ⟨*THROW OutOfMemory*, (*h*,*l*)⟩

| *CastRed*:
P ⊢ ⟨*e*,*s*⟩ → ⟨*e*′,*s*′⟩ ⟹
P ⊢ ⟨*Cast C e*, *s*⟩ → ⟨*Cast C e*′, *s*′⟩

| *RedCastNull*:
P ⊢ ⟨*Cast C null*, *s*⟩ → ⟨*null*,*s*⟩

| *RedCast*:
⟦ *hp s a* = *Some*(*D*,*fs*); *P* ⊢ *D* ⪯∗ *C* ⟧
⟹ *P* ⊢ ⟨*Cast C* (*addr a*), *s*⟩ → ⟨*addr a*, *s*⟩

| *RedCastFail*:
⟦ *hp s a* = *Some*(*D*,*fs*); ¬ *P* ⊢ *D* ⪯∗ *C* ⟧

$\Longrightarrow P \vdash \langle Cast\ C\ (addr\ a),\ s\rangle \to \langle THROW\ ClassCast,\ s\rangle$

| *BinOpRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e\ «bop»\ e_2,\ s\rangle \to \langle e'\ «bop»\ e_2,\ s'\rangle$

| *BinOpRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle(Val\ v_1)\ «bop»\ e,\ s\rangle \to \langle(Val\ v_1)\ «bop»\ e',\ s'\rangle$

| *RedBinOp*:
$binop(bop,v_1,v_2) = Some\ v \Longrightarrow$
$P \vdash \langle(Val\ v_1)\ «bop»\ (Val\ v_2),\ s\rangle \to \langle Val\ v,s\rangle$

| *RedVar*:
$lcl\ s\ V = Some\ v \Longrightarrow$
$P \vdash \langle Var\ V,s\rangle \to \langle Val\ v,s\rangle$

| *LAssRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle V{:=}e,s\rangle \to \langle V{:=}e',s'\rangle$

| *RedLAss*:
$P \vdash \langle V{:=}(Val\ v),\ (h,l)\rangle \to \langle unit,\ (h,l(V{\mapsto}v))\rangle$

| *FAccRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\},\ s\rangle \to \langle e'{\cdot}F\{D\},\ s'\rangle$

| *RedFAcc*:
$[\![\ hp\ s\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle(addr\ a){\cdot}F\{D\},\ s\rangle \to \langle Val\ v,s\rangle$

| *RedFAccNull*:
$P \vdash \langle null{\cdot}F\{D\},\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *FAssRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle$

| *FAssRed2*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \to \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle$

| *RedFAss*:
$h\ a = Some(C,fs) \Longrightarrow$
$P \vdash \langle(addr\ a){\cdot}F\{D\}{:=}(Val\ v),\ (h,l)\rangle \to \langle unit,\ (h(a \mapsto (C,fs((F,D) \mapsto v))),l)\rangle$

| *RedFAssNull*:
$P \vdash \langle null{\cdot}F\{D\}{:=}Val\ v,\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *CallObj*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e{\cdot}M(es),s\rangle \to \langle e'{\cdot}M(es),s'\rangle$

| *CallParams*:
$P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle (Val\ v)\cdot M(es),s\rangle \rightarrow \langle (Val\ v)\cdot M(es'),s'\rangle$

| *RedCall*:
$[\![\ hp\ s\ a = Some(C,fs);\ P \vdash C\ sees\ M:Ts\rightarrow T = (pns,body)\ in\ D;\ size\ vs = size\ pns;\ size\ Ts = size\ pns\ ]\!]$
$\Longrightarrow P \vdash \langle (addr\ a)\cdot M(map\ Val\ vs),\ s\rangle \rightarrow \langle blocks(this\#pns,\ Class\ D\#Ts,\ Addr\ a\#vs,\ body),\ s\rangle$

| *RedCallNull*:
$P \vdash \langle null\cdot M(map\ Val\ vs),s\rangle \rightarrow \langle THROW\ NullPointer,s\rangle$

| *BlockRedNone*:
$[\![\ P \vdash \langle e,\ (h,l(V:=None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = None;\ \neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V:T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V:T;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *BlockRedSome*:
$[\![\ P \vdash \langle e,\ (h,l(V:=None))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v;\neg\ assigned\ V\ e\ ]\!]$
$\Longrightarrow P \vdash \langle \{V:T;\ e\},\ (h,l)\rangle \rightarrow \langle \{V:T := Val\ v;\ e'\},\ (h',l'(V := l\ V))\rangle$

| *InitBlockRed*:
$[\![\ P \vdash \langle e,\ (h,l(V\mapsto v))\rangle \rightarrow \langle e',\ (h',l')\rangle;\ l'\ V = Some\ v'\ ]\!]$
$\Longrightarrow P \vdash \langle \{V:T := Val\ v;\ e\},\ (h,l)\rangle \rightarrow \langle \{V:T := Val\ v';\ e'\},\ (h',l'(V := l\ V))\rangle$

| *RedBlock*:
$P \vdash \langle \{V:T;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *RedInitBlock*:
$P \vdash \langle \{V:T := Val\ v;\ Val\ u\},\ s\rangle \rightarrow \langle Val\ u,\ s\rangle$

| *SeqRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e;;e_2,\ s\rangle \rightarrow \langle e';;e_2,\ s'\rangle$

| *RedSeq*:
$P \vdash \langle (Val\ v);;e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *CondRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle if\ (e')\ e_1\ else\ e_2,\ s'\rangle$

| *RedCondT*:
$P \vdash \langle if\ (true)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_1,\ s\rangle$

| *RedCondF*:
$P \vdash \langle if\ (false)\ e_1\ else\ e_2,\ s\rangle \rightarrow \langle e_2,\ s\rangle$

| *RedWhile*:
$P \vdash \langle while(b)\ c,\ s\rangle \rightarrow \langle if(b)\ (c;;while(b)\ c)\ else\ unit,\ s\rangle$

| *ThrowRed*:
$P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle throw\ e,\ s\rangle \rightarrow \langle throw\ e',\ s'\rangle$

| *RedThrowNull*:
$P \vdash \langle throw\ null,\ s\rangle \to \langle THROW\ NullPointer,\ s\rangle$

| *TryRed*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle try\ e\ catch(C\ V)\ e_2,\ s\rangle \to \langle try\ e'\ catch(C\ V)\ e_2,\ s'\rangle$

| *RedTry*:
$P \vdash \langle try\ (Val\ v)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Val\ v,\ s\rangle$

| *RedTryCatch*:
$[\![\ hp\ s\ a = Some(D,fs);\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle\{V{:}Class\ C := addr\ a;\ e_2\},\ s\rangle$

| *RedTryFail*:
$[\![\ hp\ s\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ (Throw\ a)\ catch(C\ V)\ e_2,\ s\rangle \to \langle Throw\ a,\ s\rangle$

| *ListRed1*:
$P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow$
$P \vdash \langle e\#es,s\rangle\ [\to]\ \langle e'\#es,s'\rangle$

| *ListRed2*:
$P \vdash \langle es,s\rangle\ [\to]\ \langle es',s'\rangle \Longrightarrow$
$P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\to]\ \langle Val\ v\ \#\ es',s'\rangle$

— Exception propagation

| *CastThrow*: $P \vdash \langle Cast\ C\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow1*: $P \vdash \langle(throw\ e)\ «bop»\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BinOpThrow2*: $P \vdash \langle(Val\ v_1)\ «bop»\ (throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *LAssThrow*: $P \vdash \langle V{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAccThrow*: $P \vdash \langle(throw\ e){\cdot}F\{D\},\ s\rangle \to \langle throw\ e,\ s\rangle$
| *FAssThrow1*: $P \vdash \langle(throw\ e){\cdot}F\{D\}{:=}e_2,\ s\rangle \to \langle throw\ e,s\rangle$
| *FAssThrow2*: $P \vdash \langle Val\ v{\cdot}F\{D\}{:=}(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowObj*: $P \vdash \langle(throw\ e){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CallThrowParams*: $[\![\ es = map\ Val\ vs\ @\ throw\ e\ \#\ es'\ ]\!] \Longrightarrow P \vdash \langle(Val\ v){\cdot}M(es),\ s\rangle \to \langle throw\ e,\ s\rangle$
| *BlockThrow*: $P \vdash \langle\{V{:}T;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *InitBlockThrow*: $P \vdash \langle\{V{:}T := Val\ v;\ Throw\ a\},\ s\rangle \to \langle Throw\ a,\ s\rangle$
| *SeqThrow*: $P \vdash \langle(throw\ e);;e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *CondThrow*: $P \vdash \langle if\ (throw\ e)\ e_1\ else\ e_2,\ s\rangle \to \langle throw\ e,\ s\rangle$
| *ThrowThrow*: $P \vdash \langle throw(throw\ e),\ s\rangle \to \langle throw\ e,\ s\rangle$

2.11.1 The reflexive transitive closure

abbreviation
Step :: *J-prog* ⇒ *expr* ⇒ *state* ⇒ *expr* ⇒ *state* ⇒ *bool*
(- ⊢ ((1⟨-,/-⟩) →*/ (1⟨-,/-⟩))) [51,0,0,0,0] 81)
where $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \equiv ((e,s),\ e',s') \in (red\ P)^*$

abbreviation
Steps :: *J-prog* ⇒ *expr list* ⇒ *state* ⇒ *expr list* ⇒ *state* ⇒ *bool*
(- ⊢ ((1⟨-,/-⟩) [→]*/ (1⟨-,/-⟩))) [51,0,0,0,0] 81)

where $P \vdash \langle es,s\rangle\ [\rightarrow]*\ \langle es',s'\rangle \equiv ((es,s),\ es',s') \in (reds\ P)^*$

lemma *converse-rtrancl-induct-red*[*consumes 1*]:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow* \langle e',(h',l')\rangle$
and $\bigwedge e\ h\ l.\ R\ e\ h\ l\ e\ h\ l$
and $\bigwedge e_0\ h_0\ l_0\ e_1\ h_1\ l_1\ e'\ h'\ l'.$
$[\![P \vdash \langle e_0,(h_0,l_0)\rangle \rightarrow \langle e_1,(h_1,l_1)\rangle;\ R\ e_1\ h_1\ l_1\ e'\ h'\ l']\!] \Longrightarrow R\ e_0\ h_0\ l_0\ e'\ h'\ l'$
shows $R\ e\ h\ l\ e'\ h'\ l' \langle proof\rangle$

2.11.2 Some easy lemmas

lemma [*iff*]: $\neg\ P \vdash \langle[],s\rangle\ [\rightarrow]\ \langle es',s'\rangle \langle proof\rangle$
lemma [*iff*]: $\neg\ P \vdash \langle Val\ v,s\rangle \rightarrow \langle e',s'\rangle \langle proof\rangle$
lemma [*iff*]: $\neg\ P \vdash \langle Throw\ a,s\rangle \rightarrow \langle e',s'\rangle \langle proof\rangle$

lemma *red-hext-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow h \trianglelefteq h'$
and *reds-hext-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow h \trianglelefteq h' \langle proof\rangle$

lemma *red-lcl-incr*: $P \vdash \langle e,(h_0,l_0)\rangle \rightarrow \langle e',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1$
and $P \vdash \langle es,(h_0,l_0)\rangle\ [\rightarrow]\ \langle es',(h_1,l_1)\rangle \Longrightarrow dom\ l_0 \subseteq dom\ l_1 \langle proof\rangle$

lemma *red-lcl-add*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow \langle e',(h',l_0{+}{+}l')\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge l_0.\ P \vdash \langle es,(h,l_0{+}{+}l)\rangle\ [\rightarrow]\ \langle es',(h',l_0{+}{+}l')\rangle) \langle proof\rangle$

lemma *Red-lcl-add*:
assumes $P \vdash \langle e,(h,l)\rangle \rightarrow* \langle e',(h',l')\rangle$ **shows** $P \vdash \langle e,(h,l_0{+}{+}l)\rangle \rightarrow* \langle e',(h',l_0{+}{+}l')\rangle \langle proof\rangle$

end

2.12 System Classes

theory *SystemClasses*
imports *Decl Exceptions*
begin

This theory provides definitions for the *Object* class, and the system exceptions.

definition *ObjectC* :: *'m cdecl*
where
ObjectC ≡ (*Object*, (*undefined*,[],[]))

definition *NullPointerC* :: *'m cdecl*
where
NullPointerC ≡ (*NullPointer*, (*Object*,[],[]))

definition *ClassCastC* :: *'m cdecl*
where
ClassCastC ≡ (*ClassCast*, (*Object*,[],[]))

definition *OutOfMemoryC* :: *'m cdecl*
where
OutOfMemoryC ≡ (*OutOfMemory*, (*Object*,[],[]))

definition *SystemClasses* :: *'m cdecl list*
where

SystemClasses ≡ [*ObjectC*, *NullPointerC*, *ClassCastC*, *OutOfMemoryC*]

end

2.13 Generic Well-formedness of programs

theory *WellForm* **imports** *TypeRel SystemClasses* **begin**

This theory defines global well-formedness conditions for programs but does not look inside method bodies. Hence it works for both Jinja and JVM programs. Well-typing of expressions is defined elsewhere (in theory *WellType*).

Because Jinja does not have method overloading, its policy for method overriding is the classical one: *covariant in the result type but contravariant in the argument types.* This means the result type of the overriding method becomes more specific, the argument types become more general.

type-synonym *'m wf-mdecl-test* = *'m prog* ⇒ *cname* ⇒ *'m mdecl* ⇒ *bool*

definition *wf-fdecl* :: *'m prog* ⇒ *fdecl* ⇒ *bool*
where
wf-fdecl P ≡ λ(*F*,*T*). *is-type P T*

definition *wf-mdecl* :: *'m wf-mdecl-test* ⇒ *'m wf-mdecl-test*
where
wf-mdecl wf-md P C ≡ λ(*M*,*Ts*,*T*,*mb*).
(∀ *T*∈*set Ts*. *is-type P T*) ∧ *is-type P T* ∧ *wf-md P C* (*M*,*Ts*,*T*,*mb*)

definition *wf-cdecl* :: *'m wf-mdecl-test* ⇒ *'m prog* ⇒ *'m cdecl* ⇒ *bool*
where
wf-cdecl wf-md P ≡ λ(*C*,(*D*,*fs*,*ms*)).
(∀ *f*∈*set fs*. *wf-fdecl P f*) ∧ *distinct-fst fs* ∧
(∀ *m*∈*set ms*. *wf-mdecl wf-md P C m*) ∧ *distinct-fst ms* ∧
(*C* ≠ *Object* ⟶
is-class P D ∧ ¬ *P* ⊢ *D* ≼* *C* ∧
(∀ (*M*,*Ts*,*T*,*m*)∈*set ms*.
∀ *D' Ts' T' m'*. *P* ⊢ *D sees M*:*Ts'* → *T'* = *m' in D'* ⟶
P ⊢ *Ts'* [≤] *Ts* ∧ *P* ⊢ *T* ≤ *T'*))

definition *wf-syscls* :: *'m prog* ⇒ *bool*
where
wf-syscls P ≡ {*Object*} ∪ *sys-xcpts* ⊆ *set*(*map fst P*)

definition *wf-prog* :: *'m wf-mdecl-test* ⇒ *'m prog* ⇒ *bool*
where
wf-prog wf-md P ≡ *wf-syscls P* ∧ (∀ *c* ∈ *set P*. *wf-cdecl wf-md P c*) ∧ *distinct-fst P*

2.13.1 Well-formedness lemmas

lemma *class-wf*:
⟦*class P C* = *Some c*; *wf-prog wf-md P*⟧ ⟹ *wf-cdecl wf-md P* (*C*,*c*)⟨*proof*⟩

lemma *class-Object* [*simp*]:
wf-prog wf-md P ⟹ ∃ *C fs ms*. *class P Object* = *Some* (*C*,*fs*,*ms*)⟨*proof*⟩

lemma *is-class-Object* [*simp*]:
$wf\text{-}prog\ wf\text{-}md\ P \Longrightarrow is\text{-}class\ P\ Object\langle proof\rangle$

lemma *is-class-xcpt*:
$[\![\ C \in sys\text{-}xcpts;\ wf\text{-}prog\ wf\text{-}md\ P\ ]\!] \Longrightarrow is\text{-}class\ P\ C\langle proof\rangle$

lemma *subcls1-wfD*:
assumes *sub1*: $P \vdash C \prec^1 D$ **and** *wf*: *wf-prog wf-md P*
shows $D \neq C \wedge (D,C) \notin (subcls1\ P)^+\langle proof\rangle$

lemma *wf-cdecl-supD*:
$[\![wf\text{-}cdecl\ wf\text{-}md\ P\ (C,D,r);\ C \neq Object]\!] \Longrightarrow is\text{-}class\ P\ D\langle proof\rangle$

lemma *subcls-asym*:
$[\![\ wf\text{-}prog\ wf\text{-}md\ P;\ (C,D) \in (subcls1\ P)^+\ ]\!] \Longrightarrow (D,C) \notin (subcls1\ P)^+\langle proof\rangle$

lemma *subcls-irrefl*:
$[\![\ wf\text{-}prog\ wf\text{-}md\ P;\ (C,D) \in (subcls1\ P)^+\ ]\!] \Longrightarrow C \neq D\langle proof\rangle$

lemma *acyclic-subcls1*:
$wf\text{-}prog\ wf\text{-}md\ P \Longrightarrow acyclic\ (subcls1\ P)\langle proof\rangle$

lemma *wf-subcls1*:
$wf\text{-}prog\ wf\text{-}md\ P \Longrightarrow wf\ ((subcls1\ P)^{-1})\langle proof\rangle$

lemma *single-valued-subcls1*:
$wf\text{-}prog\ wf\text{-}md\ G \Longrightarrow single\text{-}valued\ (subcls1\ G)\langle proof\rangle$

lemma *subcls-induct*:
$[\![\ wf\text{-}prog\ wf\text{-}md\ P;\ \bigwedge C.\ \forall D.\ (C,D) \in (subcls1\ P)^+ \longrightarrow Q\ D \Longrightarrow Q\ C\ ]\!] \Longrightarrow Q\ C\langle proof\rangle$

lemma *subcls1-induct-aux*:
assumes *is-class P C* **and** *wf*: *wf-prog wf-md P* **and** *QObj*: *Q Object*
shows
$[\![\ \bigwedge C\ D\ fs\ ms.$
$\quad [\![\ C \neq Object;\ is\text{-}class\ P\ C;\ class\ P\ C = Some\ (D,fs,ms) \wedge$
$\quad\ wf\text{-}cdecl\ wf\text{-}md\ P\ (C,D,fs,ms) \wedge P \vdash C \prec^1 D \wedge is\text{-}class\ P\ D \wedge Q\ D]\!] \Longrightarrow Q\ C\ ]\!]$
$\Longrightarrow Q\ C\langle proof\rangle$

lemma *subcls1-induct* [*consumes 2*, *case-names Object Subcls*]:
$[\![\ wf\text{-}prog\ wf\text{-}md\ P;\ is\text{-}class\ P\ C;\ Q\ Object;$
$\quad \bigwedge C\ D.\ [\![C \neq Object;\ P \vdash C \prec^1 D;\ is\text{-}class\ P\ D;\ Q\ D]\!] \Longrightarrow Q\ C\ ]\!]$
$\Longrightarrow Q\ C\langle proof\rangle$

lemma *subcls-C-Object*:
assumes *class*: *is-class P C* **and** *wf*: *wf-prog wf-md P*
shows $P \vdash C \preceq^* Object\langle proof\rangle$

lemma *is-type-pTs*:
assumes *wf-prog wf-md P* **and** $(C,S,fs,ms) \in set\ P$ **and** $(M,Ts,T,m) \in set\ ms$
shows $set\ Ts \subseteq types\ P\langle proof\rangle$

2.13.2 Well-formedness and method lookup

lemma *sees-wf-mdecl*:
assumes *wf*: *wf-prog wf-md P* **and** *sees*: $P \vdash C$ *sees* M:$Ts \to T = m$ *in* D
shows *wf-mdecl wf-md P D* $(M,Ts,T,m)\langle proof\rangle$

lemma *sees-method-mono* [*rule-format* (*no-asm*)]:
assumes *sub*: $P \vdash C' \preceq^* C$ **and** *wf*: *wf-prog wf-md P*
shows $\forall D\ Ts\ T\ m.\ P \vdash C$ *sees* M:$Ts \to T = m$ *in* $D \longrightarrow$
$(\exists D'\ Ts'\ T'\ m'.\ P \vdash C'$ *sees* M:$Ts' \to T' = m'$ *in* $D' \wedge P \vdash Ts\ [\leq]\ Ts' \wedge P \vdash T' \leq T)\langle proof\rangle$

lemma *sees-method-mono2*:
$[\![\ P \vdash C' \preceq^* C$; *wf-prog wf-md P*;
$P \vdash C$ *sees* M:$Ts \to T = m$ *in* D; $P \vdash C'$ *sees* M:$Ts' \to T' = m'$ *in* $D'\ ]\!]$
$\Longrightarrow P \vdash Ts\ [\leq]\ Ts' \wedge P \vdash T' \leq T\langle proof\rangle$

lemma *mdecls-visible*:
assumes *wf*: *wf-prog wf-md P* **and** *class*: *is-class P C*
shows $\bigwedge D\ fs\ ms.$ *class P C* $=$ *Some*(D,fs,ms)
$\Longrightarrow \exists Mm.\ P \vdash C$ *sees-methods* $Mm \wedge (\forall (M,Ts,T,m) \in set\ ms.\ Mm\ M = Some((Ts,T,m),C))\langle proof\rangle$

lemma *mdecl-visible*:
assumes *wf*: *wf-prog wf-md P* **and** C: $(C,S,fs,ms) \in set\ P$ **and** m: $(M,Ts,T,m) \in set\ ms$
shows $P \vdash C$ *sees* M:$Ts \to T = m$ *in* $C\langle proof\rangle$

lemma *Call-lemma*:
assumes *sees*: $P \vdash C$ *sees* M:$Ts \to T = m$ *in* D **and** *sub*: $P \vdash C' \preceq^* C$ **and** *wf*: *wf-prog wf-md P*
shows $\exists D'\ Ts'\ T'\ m'.$
$P \vdash C'$ *sees* M:$Ts' \to T' = m'$ *in* $D' \wedge P \vdash Ts\ [\leq]\ Ts' \wedge P \vdash T' \leq T \wedge P \vdash C' \preceq^* D'$
$\wedge$ *is-type P* $T' \wedge (\forall T \in set\ Ts'.$ *is-type P* $T) \wedge$ *wf-md P D'* $(M,Ts',T',m')\langle proof\rangle$

lemma *wf-prog-lift*:
assumes *wf*: *wf-prog* $(\lambda P\ C\ bd.\ A\ P\ C\ bd)\ P$
and *rule*:
$\bigwedge$*wf-md C M Ts C T m bd.*
wf-prog wf-md P $\Longrightarrow$
$P \vdash C$ *sees* M:$Ts \to T = m$ *in* $C \Longrightarrow$
set Ts $\subseteq$ *types P* $\Longrightarrow$
bd $= (M,Ts,T,m) \Longrightarrow$
A P C bd $\Longrightarrow$
B P C bd
shows *wf-prog* $(\lambda P\ C\ bd.\ B\ P\ C\ bd)\ P\langle proof\rangle$

2.13.3 Well-formedness and field lookup

lemma *wf-Fields-Ex*:
assumes *wf*: *wf-prog wf-md P* **and** *is-class P C*
shows $\exists FDTs.\ P \vdash C$ *has-fields* $FDTs\langle proof\rangle$

lemma *has-fields-types*:
$[\![\ P \vdash C$ *has-fields* $FDTs$; $(FD,T) \in set\ FDTs$; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type P* $T\langle proof\rangle$

lemma *sees-field-is-type*:
$[\![\ P \vdash C$ *sees* F:T *in* D; *wf-prog wf-md P* $]\!] \Longrightarrow$ *is-type P* $T\langle proof\rangle$
lemma *wf-syscls*:

set SystemClasses $\subseteq$ *set P* $\Longrightarrow$ *wf-syscls P*$\langle$*proof*$\rangle$
end

2.14 Weak well-formedness of Jinja programs

theory *WWellForm* **imports** *../Common/ WellForm Expr* **begin**

definition *wwf-J-mdecl* :: *J-prog* $\Rightarrow$ *cname* $\Rightarrow$ *J-mb mdecl* $\Rightarrow$ *bool*
where
wwf-J-mdecl P C $\equiv$ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
length Ts = *length pns* $\wedge$ *distinct pns* $\wedge$ *this* $\notin$ *set pns* $\wedge$ *fv body* $\subseteq$ {*this*} $\cup$ *set pns*

lemma *wwf-J-mdecl*[*simp*]:
wwf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) =
(*length Ts* = *length pns* $\wedge$ *distinct pns* $\wedge$ *this* $\notin$ *set pns* $\wedge$ *fv body* $\subseteq$ {*this*} $\cup$ *set pns*)$\langle$*proof*$\rangle$
abbreviation
wwf-J-prog :: *J-prog* $\Rightarrow$ *bool* **where**
wwf-J-prog == *wf-prog wwf-J-mdecl*

end

2.15 Equivalence of Big Step and Small Step Semantics

theory *Equivalence* **imports** *BigStep SmallStep WWellForm* **begin**

2.15.1 Small steps simulate big step

Cast

lemma *CastReds*:
P $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*e*′,*s*′$\rangle$ $\Longrightarrow$ *P* $\vdash$ $\langle$*Cast C e*,*s*$\rangle$ $\rightarrow$* $\langle$*Cast C e*′,*s*′$\rangle$$\langle$*proof*$\rangle$
lemma *CastRedsNull*:
P $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*null*,*s*′$\rangle$ $\Longrightarrow$ *P* $\vdash$ $\langle$*Cast C e*,*s*$\rangle$ $\rightarrow$* $\langle$*null*,*s*′$\rangle$$\langle$*proof*$\rangle$
lemma *CastRedsAddr*:
$[\![$ *P* $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*addr a*,*s*′$\rangle$; *hp s*′ *a* = *Some*(*D*,*fs*); *P* $\vdash$ *D* $\preceq^*$ *C* $]\!]$ $\Longrightarrow$
P $\vdash$ $\langle$*Cast C e*,*s*$\rangle$ $\rightarrow$* $\langle$*addr a*,*s*′$\rangle$$\langle$*proof*$\rangle$
lemma *CastRedsFail*:
$[\![$ *P* $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*addr a*,*s*′$\rangle$; *hp s*′ *a* = *Some*(*D*,*fs*); $\neg$ *P* $\vdash$ *D* $\preceq^*$ *C* $]\!]$ $\Longrightarrow$
P $\vdash$ $\langle$*Cast C e*,*s*$\rangle$ $\rightarrow$* $\langle$*THROW ClassCast*,*s*′$\rangle$$\langle$*proof*$\rangle$
lemma *CastRedsThrow*:
$[\![$ *P* $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*throw a*,*s*′$\rangle$ $]\!]$ $\Longrightarrow$ *P* $\vdash$ $\langle$*Cast C e*,*s*$\rangle$ $\rightarrow$* $\langle$*throw a*,*s*′$\rangle$$\langle$*proof*$\rangle$

LAss

lemma *LAssReds*:
P $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*e*′,*s*′$\rangle$ $\Longrightarrow$ *P* $\vdash$ $\langle$ *V*:=*e*,*s*$\rangle$ $\rightarrow$* $\langle$ *V*:=*e*′,*s*′$\rangle$$\langle$*proof*$\rangle$
lemma *LAssRedsVal*:
$[\![$ *P* $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*Val v*,(*h*′,*l*′)$\rangle$ $]\!]$ $\Longrightarrow$ *P* $\vdash$ $\langle$ *V*:=*e*,*s*$\rangle$ $\rightarrow$* $\langle$*unit*,(*h*′,*l*′(*V*$\mapsto$*v*))$\rangle$$\langle$*proof*$\rangle$
lemma *LAssRedsThrow*:
$[\![$ *P* $\vdash$ $\langle$*e*,*s*$\rangle$ $\rightarrow$* $\langle$*throw a*,*s*′$\rangle$ $]\!]$ $\Longrightarrow$ *P* $\vdash$ $\langle$ *V*:=*e*,*s*$\rangle$ $\rightarrow$* $\langle$*throw a*,*s*′$\rangle$$\langle$*proof*$\rangle$

BinOp

lemma *BinOp1Reds*:

$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle\ e\ «bop»\ e_2,\ s\rangle \to* \langle e'\ «bop»\ e_2,\ s'\rangle\langle proof\rangle$
lemma *BinOp2Reds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle(Val\ v)\ «bop»\ e,\ s\rangle \to* \langle(Val\ v)\ «bop»\ e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsVal*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v_2,s_2\rangle$
and *op*: $binop(bop,v_1,v_2) = Some\ v$
shows $P \vdash \langle e_1\ «bop»\ e_2,\ s_0\rangle \to* \langle Val\ v,s_2\rangle\langle proof\rangle$
lemma *BinOpRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e\ «bop»\ e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *BinOpRedsThrow2*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle e_1\ «bop»\ e_2,\ s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

FAcc

lemma *FAccReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},\ s\rangle \to* \langle e'{\cdot}F\{D\},\ s'\rangle\langle proof\rangle$
lemma *FAccRedsVal*:
$[\![P \vdash \langle e,s\rangle \to* \langle addr\ a,s'\rangle;\ hp\ s'\ a = Some(C,fs);\ fs(F,D) = Some\ v\ ]\!]$
$\Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *FAccRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *FAccRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\},s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$

FAss

lemma *FAssReds1*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle e'{\cdot}F\{D\}{:=}e_2,\ s'\rangle\langle proof\rangle$
lemma *FAssReds2*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle Val\ v{\cdot}F\{D\}{:=}e,\ s\rangle \to* \langle Val\ v{\cdot}F\{D\}{:=}e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsVal*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle addr\ a,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,(h_2,l_2)\rangle$
and *hC*: $Some(C,fs) = h_2\ a$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle unit,\ (h_2(a{\mapsto}(C,fs((F,D){\mapsto}v))),l_2)\rangle\langle proof\rangle$
lemma *FAssRedsNull*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle null,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle Val\ v,s_2\rangle$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,\ s_0\rangle \to* \langle THROW\ NullPointer,\ s_2\rangle\langle proof\rangle$
lemma *FAssRedsThrow1*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}F\{D\}{:=}e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$
lemma *FAssRedsThrow2*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle e_1{\cdot}F\{D\}{:=}e_2,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

;;

lemma *SeqReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle e';;e_2,\ s'\rangle\langle proof\rangle$
lemma *SeqRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ e',s'\rangle \Longrightarrow P \vdash \langle e;;e_2,\ s\rangle \to* \langle throw\ e',\ s'\rangle\langle proof\rangle$

lemma *SeqReds2*:
assumes e_1-*steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Val\ v_1,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,s_1\rangle \to* \langle e_2',s_2\rangle$
shows $P \vdash \langle e_1;;e_2,\ s_0\rangle \to* \langle e_2',s_2\rangle\langle proof\rangle$

If

lemma *CondReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,s\rangle \to* \langle if\ (e')\ e_1\ else\ e_2,s'\rangle\langle proof\rangle$
lemma *CondRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$
lemma *CondReds2T*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle true,s_1\rangle$
and e_1-*steps*: $P \vdash \langle e_1,\ s_1\rangle \to* \langle e',s_2\rangle$
shows $P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$
lemma *CondReds2F*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle false,s_1\rangle$
and e_2-*steps*: $P \vdash \langle e_2,\ s_1\rangle \to* \langle e',s_2\rangle$
shows $P \vdash \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \to* \langle e',s_2\rangle\langle proof\rangle$

While

lemma *WhileFReds*:
assumes *b-steps*: $P \vdash \langle b,s\rangle \to* \langle false,s'\rangle$
shows $P \vdash \langle while\ (b)\ c,s\rangle \to* \langle unit,s'\rangle\langle proof\rangle$
lemma *WhileRedsThrow*:
assumes *b-steps*: $P \vdash \langle b,s\rangle \to* \langle throw\ e,s'\rangle$
shows $P \vdash \langle while\ (b)\ c,s\rangle \to* \langle throw\ e,s'\rangle\langle proof\rangle$
lemma *WhileTReds*:
assumes *b-steps*: $P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle$
and *c-steps*: $P \vdash \langle c,s_1\rangle \to* \langle Val\ v_1,s_2\rangle$
and *while-steps*: $P \vdash \langle while\ (b)\ c,s_2\rangle \to* \langle e,s_3\rangle$
shows $P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle e,s_3\rangle\langle proof\rangle$
lemma *WhileTRedsThrow*:
assumes *b-steps*: $P \vdash \langle b,s_0\rangle \to* \langle true,s_1\rangle$
and *c-steps*: $P \vdash \langle c,s_1\rangle \to* \langle throw\ e,s_2\rangle$
shows $P \vdash \langle while\ (b)\ c,s_0\rangle \to* \langle throw\ e,s_2\rangle\langle proof\rangle$

Throw

lemma *ThrowReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ e',s'\rangle\langle proof\rangle$
lemma *ThrowRedsNull*:
$P \vdash \langle e,s\rangle \to* \langle null,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle THROW\ NullPointer,s'\rangle\langle proof\rangle$
lemma *ThrowRedsThrow*:
$P \vdash \langle e,s\rangle \to* \langle throw\ a,s'\rangle \Longrightarrow P \vdash \langle throw\ e,s\rangle \to* \langle throw\ a,s'\rangle\langle proof\rangle$

InitBlock

lemma *InitBlockReds-aux*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow$
$\forall h\ l\ h'\ l'\ v.\ s = (h,l(V\mapsto v)) \longrightarrow s' = (h',l') \longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle\{V{:}T := Val(the(l'\ V));\ e'\},(h',l'(V:=(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockReds*:
$P \vdash \langle e,\ (h,l(V\mapsto v))\rangle \to* \langle e',\ (h',l')\rangle \Longrightarrow$

$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle\{V{:}T := Val(the(l'\ V));\ e'\},(h',l'(V{:=}(l\ V)))\rangle\langle proof\rangle$
lemma *InitBlockRedsFinal*:
$[\![\ P \vdash \langle e,(h,l(V{\mapsto}v))\rangle \to* \langle e',(h',l')\rangle;\ final\ e'\ ]\!] \Longrightarrow$
$P \vdash \langle\{V{:}T := Val\ v;\ e\},(h,l)\rangle \to* \langle e',(h',\ l'(V := l\ V))\rangle\langle proof\rangle$

Block

lemma *BlockRedsFinal*:
assumes *reds*: $P \vdash \langle e_0,s_0\rangle \to* \langle e_2,(h_2,l_2)\rangle$ **and** *fin*: *final* e_2
shows $\bigwedge h_0\ l_0.\ s_0 = (h_0,l_0(V{:=}None)) \Longrightarrow P \vdash \langle\{V{:}T;\ e_0\},(h_0,l_0)\rangle \to* \langle e_2,(h_2,l_2(V{:=}l_0\ V))\rangle\langle proof\rangle$

try-catch

lemma *TryReds*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \to* \langle try\ e'\ catch(C\ V)\ e_2,s'\rangle\langle proof\rangle$
lemma *TryRedsVal*:
$P \vdash \langle e,s\rangle \to* \langle Val\ v,s'\rangle \Longrightarrow P \vdash \langle try\ e\ catch(C\ V)\ e_2,s\rangle \to* \langle Val\ v,s'\rangle\langle proof\rangle$
lemma *TryCatchRedsFinal*:
assumes e_1*-steps*: $P \vdash \langle e_1,s_0\rangle \to* \langle Throw\ a,(h_1,l_1)\rangle$
and $h_1 a$: $h_1\ a = Some(D,fs)$ **and** *sub*: $P \vdash D \preceq^* C$
and e_2*-steps*: $P \vdash \langle e_2,\ (h_1,\ l_1(V \mapsto Addr\ a))\rangle \to* \langle e_2{}',\ (h_2,l_2)\rangle$
and *final*: *final* $e_2{}'$
shows $P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,\ s_0\rangle \to* \langle e_2{}',\ (h_2,\ (l_2{::}locals)(V := l_1\ V))\rangle\langle proof\rangle$
lemma *TryRedsFail*:
$[\![\ P \vdash \langle e_1,s\rangle \to* \langle Throw\ a,(h,l)\rangle;\ h\ a = Some(D,fs);\ \neg\ P \vdash D \preceq^*\ C\ ]\!]$
$\Longrightarrow P \vdash \langle try\ e_1\ catch(C\ V)\ e_2,s\rangle \to* \langle Throw\ a,(h,l)\rangle\langle proof\rangle$

List

lemma *ListReds1*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e\#es,s\rangle\ [\to]*\ \langle e'\ \#\ es,s'\rangle\langle proof\rangle$
lemma *ListReds2*:
$P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle Val\ v\ \#\ es,s\rangle\ [\to]*\ \langle Val\ v\ \#\ es',s'\rangle\langle proof\rangle$
lemma *ListRedsVal*:
$[\![\ P \vdash \langle e,s_0\rangle \to* \langle Val\ v,s_1\rangle;\ P \vdash \langle es,s_1\rangle\ [\to]*\ \langle es',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle e\#es,s_0\rangle\ [\to]*\ \langle Val\ v\ \#\ es',s_2\rangle\langle proof\rangle$

Call

First a few lemmas on what happens to free variables during redction.

lemma assumes *wf*: *wwf-J-prog P*
shows *Red-fv*: $P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow fv\ e' \subseteq fv\ e$
and $P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow fvs\ es' \subseteq fvs\ es\langle proof\rangle$

lemma *Red-dom-lcl*:
$P \vdash \langle e,(h,l)\rangle \to \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e$ **and**
$P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fvs\ es\langle proof\rangle$
lemma *Reds-dom-lcl*:
assumes *wf*: *wwf-J-prog P*
shows $P \vdash \langle e,(h,l)\rangle \to* \langle e',(h',l')\rangle \Longrightarrow dom\ l' \subseteq dom\ l \cup fv\ e\langle proof\rangle$

Now a few lemmas on the behaviour of blocks during reduction.

lemma *override-on-upd-lemma*:
$(override\text{-}on\ f\ (g(a{\mapsto}b))\ A)(a := g\ a) = override\text{-}on\ f\ g\ (insert\ a\ A)\langle proof\rangle$

lemma *blocksReds*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ distinct\ Vs;$
$P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to* \langle e',\ (h',l')\rangle\ ]\!]$
$\Longrightarrow P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to* \langle blocks(Vs,Ts,map\ (the \circ l')\ Vs,e'),\ (h',override\text{-}on\ l'\ l\ (set\ Vs))\rangle\langle proof\rangle$

lemma *blocksFinal*:
$\bigwedge l.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts;\ final\ e\ ]\!] \Longrightarrow$
$P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to* \langle e,\ (h,l)\rangle\langle proof\rangle$

lemma *blocksRedsFinal*:
assumes *wf*: *length Vs = length Ts length vs = length Ts distinct Vs*
and *reds*: $P \vdash \langle e,\ (h,l(Vs\ [\mapsto]\ vs))\rangle \to* \langle e',\ (h',l')\rangle$
and *fin*: *final e'* **and** *l''*: $l'' = override\text{-}on\ l'\ l\ (set\ Vs)$
shows $P \vdash \langle blocks(Vs,Ts,vs,e),\ (h,l)\rangle \to* \langle e',\ (h',l'')\rangle\langle proof\rangle$

An now the actual method call reduction lemmas.

lemma *CallRedsObj*:
$P \vdash \langle e,s\rangle \to* \langle e',s'\rangle \Longrightarrow P \vdash \langle e{\cdot}M(es),s\rangle \to* \langle e'{\cdot}M(es),s'\rangle\langle proof\rangle$

lemma *CallRedsParams*:
$P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle \Longrightarrow P \vdash \langle(Val\ v){\cdot}M(es),s\rangle \to* \langle(Val\ v){\cdot}M(es'),s'\rangle\langle proof\rangle$

lemma *CallRedsFinal*:
assumes *wwf*: *wwf-J-prog P*
and $P \vdash \langle e,s_0\rangle \to* \langle addr\ a,s_1\rangle$
$P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,(h_2,l_2)\rangle$
$h_2\ a = Some(C,fs)\ P \vdash C\ sees\ M{:}Ts\to T = (pns,body)\ in\ D$
$size\ vs = size\ pns$
and l_2': $l_2' = [this \mapsto Addr\ a,\ pns[\mapsto]vs]$
and *body*: $P \vdash \langle body,(h_2,l_2')\rangle \to* \langle ef,(h_3,l_3)\rangle$
and *final ef*
shows $P \vdash \langle e{\cdot}M(es),\ s_0\rangle \to* \langle ef,(h_3,l_2)\rangle\langle proof\rangle$

lemma *CallRedsThrowParams*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle Val\ v,s_1\rangle$
and *es-steps*: $P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs_1\ @\ throw\ a\ \#\ es_2,s_2\rangle$
shows $P \vdash \langle e{\cdot}M(es),s_0\rangle \to* \langle throw\ a,s_2\rangle\langle proof\rangle$

lemma *CallRedsThrowObj*:
$P \vdash \langle e,s0\rangle \to* \langle throw\ a,s_1\rangle \Longrightarrow P \vdash \langle e{\cdot}M(es),s0\rangle \to* \langle throw\ a,s_1\rangle\langle proof\rangle$

lemma *CallRedsNull*:
assumes *e-steps*: $P \vdash \langle e,s_0\rangle \to* \langle null,s_1\rangle$
and *es-steps*: $P \vdash \langle es,s_1\rangle\ [\to]*\ \langle map\ Val\ vs,s_2\rangle$
shows $P \vdash \langle e{\cdot}M(es),s_0\rangle \to* \langle THROW\ NullPointer,s_2\rangle\langle proof\rangle$

The main Theorem

lemma assumes *wwf*: *wwf-J-prog P*
shows *big-by-small*: $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
and *bigs-by-smalls*: $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \Longrightarrow P \vdash \langle es,s\rangle\ [\to]*\ \langle es',s'\rangle\langle proof\rangle$

2.15.2 Big steps simulates small step

This direction was carried out by Norbert Schirmer and Daniel Wasserrab.

The big step equivalent of *RedWhile*:

lemma *unfold-while*:
$P \vdash \langle while(b)\ c,s\rangle \Rightarrow \langle e',s'\rangle \ = \ P \vdash \langle if(b)\ (c;;while(b)\ c)\ else\ (unit),s\rangle \Rightarrow \langle e',s'\rangle \langle proof\rangle$

lemma *blocksEval*:
$\bigwedge Ts\ vs\ l\ l'.\ [\![size\ ps = size\ Ts;\ size\ ps = size\ vs;\ P \vdash \langle blocks(ps,Ts,vs,e),(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle\ ]\!]$
$\Longrightarrow \exists\ l''.\ P \vdash \langle e,(h,l(ps[\mapsto]vs))\rangle \Rightarrow \langle e',(h',l'')\rangle \langle proof\rangle$

lemma
assumes *wf*: *wwf-J-prog P*
shows *eval-restrict-lcl*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fv\ e \subseteq W \Longrightarrow P \vdash \langle e,(h,l|`W)\rangle \Rightarrow \langle e',(h',l'|`W)\rangle)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge W.\ fvs\ es \subseteq W \Longrightarrow P \vdash \langle es,(h,l|`W)\rangle\ [\Rightarrow]\ \langle es',(h',l'|`W)\rangle)\langle proof\rangle$

lemma *eval-notfree-unchanged*:
$P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fv\ e \Longrightarrow l'\ V = l\ V)$
and $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow (\bigwedge V.\ V \notin fvs\ es \Longrightarrow l'\ V = l\ V)\langle proof\rangle$

lemma *eval-closed-lcl-unchanged*:
$[\![\ P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle;\ fv\ e = \{\}\ ]\!] \Longrightarrow l' = l\langle proof\rangle$

lemma *list-eval-Throw*:
assumes *eval-e*: $P \vdash \langle throw\ x,s\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle map\ Val\ vs\ @\ throw\ x\ \#\ es',s\rangle\ [\Rightarrow]\ \langle map\ Val\ vs\ @\ e'\ \#\ es',s'\rangle \langle proof\rangle$

The key lemma:

lemma
assumes *wf*: *wwf-J-prog P*
shows *extend-1-eval*:
$P \vdash \langle e,s\rangle \rightarrow \langle e'',s''\rangle \Longrightarrow\ (\bigwedge s'\ e'.\ P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle)$
and *extend-1-evals*:
$P \vdash \langle es,t\rangle\ [\rightarrow]\ \langle es'',t''\rangle \Longrightarrow (\bigwedge t'\ es'.\ P \vdash \langle es'',t''\rangle\ [\Rightarrow]\ \langle es',t'\rangle \Longrightarrow P \vdash \langle es,t\rangle\ [\Rightarrow]\ \langle es',t'\rangle)\langle proof\rangle$

Its extension to $\rightarrow*$:

lemma *extend-eval*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle e,s\rangle \rightarrow* \langle e'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle e'',s''\rangle \Rightarrow \langle e',s'\rangle$
shows $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \langle proof\rangle$

lemma *extend-evals*:
assumes *wf*: *wwf-J-prog P*
and *reds*: $P \vdash \langle es,s\rangle\ [\rightarrow]* \langle es'',s''\rangle$ **and** *eval-rest*: $P \vdash \langle es'',s''\rangle\ [\Rightarrow]\ \langle es',s'\rangle$
shows $P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \langle proof\rangle$

Finally, small step semantics can be simulated by big step semantics:

theorem
assumes *wf*: *wwf-J-prog P*
shows *small-by-big*: $[\![P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle;\ final\ e']\!] \Longrightarrow P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$
and $[\![P \vdash \langle es,s\rangle\ [\rightarrow]* \langle es',s'\rangle;\ finals\ es']\!] \Longrightarrow P \vdash \langle es,s\rangle\ [\Rightarrow]\ \langle es',s'\rangle \langle proof\rangle$

2.15.3 Equivalence

And now, the crowning achievement:

corollary *big-iff-small*:
$wwf\text{-}J\text{-}prog\ P \Longrightarrow$
$P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle \ = \ (P \vdash \langle e,s\rangle \rightarrow* \langle e',s'\rangle \wedge final\ e')\langle proof\rangle$

end

2.16 Well-typedness of Jinja expressions

theory *WellType*
imports *../Common/Objects Expr*
begin

type-synonym
$env = vname \rightharpoonup ty$

inductive
$WT :: [J\text{-}prog, env, expr, ty] \Rightarrow bool$
$(-,- \vdash - :: - \ [51,51,51]50)$
and $WTs :: [J\text{-}prog, env, expr\ list, ty\ list] \Rightarrow bool$
$(-,- \vdash - [::] - \ [51,51,51]50)$
for $P :: J\text{-}prog$
where

$WTNew$:
$is\text{-}class\ P\ C \Longrightarrow$
$P,E \vdash new\ C :: Class\ C$

| $WTCast$:
$[\![\ P,E \vdash e :: Class\ D;\ \ is\text{-}class\ P\ C;\ \ P \vdash C \preceq^* D \vee P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P,E \vdash Cast\ C\ e :: Class\ C$

| $WTVal$:
$typeof\ v = Some\ T \Longrightarrow$
$P,E \vdash Val\ v :: T$

| $WTVar$:
$E\ V = Some\ T \Longrightarrow$
$P,E \vdash Var\ V :: T$

| $WTBinOpEq$:
$[\![\ P,E \vdash e_1 :: T_1;\ \ P,E \vdash e_2 :: T_2;\ P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1\ ]\!]$
$\Longrightarrow P,E \vdash e_1\ «Eq»\ e_2 :: Boolean$

| $WTBinOpAdd$:
$[\![\ P,E \vdash e_1 :: Integer;\ \ P,E \vdash e_2 :: Integer\ ]\!]$
$\Longrightarrow P,E \vdash e_1\ «Add»\ e_2 :: Integer$

| $WTLAss$:
$[\![\ E\ V = Some\ T;\ \ P,E \vdash e :: T';\ \ P \vdash T' \leq T;\ \ V \neq this\ ]\!]$
$\Longrightarrow P,E \vdash V{:=}e :: Void$

| *WTFAcc*:
⟦ $P,E \vdash e :: Class\ C$; $P \vdash C\ sees\ F{:}T\ in\ D$ ⟧
⟹ $P,E \vdash e{\cdot}F\{D\} :: T$

| *WTFAss*:
⟦ $P,E \vdash e_1 :: Class\ C$; $P \vdash C\ sees\ F{:}T\ in\ D$; $P,E \vdash e_2 :: T'$; $P \vdash T' \leq T$ ⟧
⟹ $P,E \vdash e_1{\cdot}F\{D\}{:=}e_2 :: Void$

| *WTCall*:
⟦ $P,E \vdash e :: Class\ C$; $P \vdash C\ sees\ M{:}Ts \to T = (pns,body)\ in\ D$;
$P,E \vdash es\ [::]\ Ts'$; $P \vdash Ts'\ [\leq]\ Ts$ ⟧
⟹ $P,E \vdash e{\cdot}M(es) :: T$

| *WTBlock*:
⟦ *is-type* $P\ T$; $P,E(V \mapsto T) \vdash e :: T'$ ⟧
⟹ $P,E \vdash \{V{:}T;\ e\} :: T'$

| *WTSeq*:
⟦ $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$ ⟧
⟹ $P,E \vdash e_1;;e_2 :: T_2$
| *WTCond*:
⟦ $P,E \vdash e :: Boolean$; $P,E \vdash e_1{::}T_1$; $P,E \vdash e_2{::}T_2$;
$P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1$; $P \vdash T_1 \leq T_2 \longrightarrow T = T_2$; $P \vdash T_2 \leq T_1 \longrightarrow T = T_1$ ⟧
⟹ $P,E \vdash if\ (e)\ e_1\ else\ e_2 :: T$

| *WTWhile*:
⟦ $P,E \vdash e :: Boolean$; $P,E \vdash c{::}T$ ⟧
⟹ $P,E \vdash while\ (e)\ c :: Void$

| *WTThrow*:
$P,E \vdash e :: Class\ C$ ⟹
$P,E \vdash throw\ e :: Void$

| *WTTry*:
⟦ $P,E \vdash e_1 :: T$; $P,E(V \mapsto Class\ C) \vdash e_2 :: T$; *is-class* $P\ C$ ⟧
⟹ $P,E \vdash try\ e_1\ catch(C\ V)\ e_2 :: T$

— well-typed expression lists

| *WTNil*:
$P,E \vdash []\ [::]\ []$

| *WTCons*:
⟦ $P,E \vdash e :: T$; $P,E \vdash es\ [::]\ Ts$ ⟧
⟹ $P,E \vdash e\#es\ [::]\ T\#Ts$

lemma [*iff*]: $(P,E \vdash []\ [::]\ Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash e\#es\ [::]\ T\#Ts) = (P,E \vdash e :: T \wedge P,E \vdash es\ [::]\ Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash (e\#es)\ [::]\ Ts) =$
$(\exists\, U\ Us.\ Ts = U\#Us \wedge P,E \vdash e :: U \wedge P,E \vdash es\ [::]\ Us)\langle proof\rangle$
lemma [*iff*]: $\bigwedge Ts.\ (P,E \vdash es_1\ @\ es_2\ [::]\ Ts) =$
$(\exists\, Ts_1\ Ts_2.\ Ts = Ts_1\ @\ Ts_2 \wedge P,E \vdash es_1\ [::]\ Ts_1 \wedge P,E \vdash es_2[::]Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash Val\ v :: T = (typeof\ v = Some\ T)\langle proof\rangle$

lemma [*iff*]: $P,E \vdash Var\ V :: T = (E\ V = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E \vdash e_1;;e_2 :: T_2 = (\exists\, T_1.\ P,E \vdash e_1::T_1 \wedge P,E \vdash e_2::T_2)\langle proof\rangle$
lemma [*iff*]: $(P,E \vdash \{V:T;\ e\} :: T') = (is\text{-}type\ P\ T \wedge P,E(V\mapsto T) \vdash e :: T')\langle proof\rangle$

lemma *wt-env-mono*:
$P,E \vdash e :: T \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash e :: T)$ **and**
$P,E \vdash es\ [::]\ Ts \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E' \vdash es\ [::]\ Ts)\langle proof\rangle$

lemma *WT-fv*: $P,E \vdash e :: T \Longrightarrow fv\ e \subseteq dom\ E$
and $P,E \vdash es\ [::]\ Ts \Longrightarrow fvs\ es \subseteq dom\ E\langle proof\rangle$

2.17 Runtime Well-typedness

theory *WellTypeRT*
imports *WellType*
begin

inductive
WTrt :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr* ⇒ *ty* ⇒ *bool*
and *WTrts* :: *J-prog* ⇒ *heap* ⇒ *env* ⇒ *expr list* ⇒ *ty list* ⇒ *bool*
and *WTrt2* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
(-,-,- ⊢ - : - [*51*,*51*,*51*]*50*)
and *WTrts2* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
(-,-,- ⊢ - [:] - [*51*,*51*,*51*]*50*)
for P :: *J-prog* **and** h :: *heap*
where

$P,E,h \vdash e : T \equiv WTrt\ P\ h\ E\ e\ T$
| $P,E,h \vdash es[:]Ts \equiv WTrts\ P\ h\ E\ es\ Ts$

| *WTrtNew*:
is-class $P\ C \Longrightarrow$
$P,E,h \vdash new\ C : Class\ C$

| *WTrtCast*:
⟦ $P,E,h \vdash e : T$; *is-refT* T; *is-class* $P\ C$ ⟧
⟹ $P,E,h \vdash Cast\ C\ e : Class\ C$

| *WTrtVal*:
$typeof_h\ v = Some\ T \Longrightarrow$
$P,E,h \vdash Val\ v : T$

| *WTrtVar*:
$E\ V = Some\ T \Longrightarrow$
$P,E,h \vdash Var\ V : T$

| *WTrtBinOpEq*:
⟦ $P,E,h \vdash e_1 : T_1$; $P,E,h \vdash e_2 : T_2$ ⟧
⟹ $P,E,h \vdash e_1\ «Eq»\ e_2 : Boolean$

| *WTrtBinOpAdd*:
⟦ $P,E,h \vdash e_1 : Integer$; $P,E,h \vdash e_2 : Integer$ ⟧
⟹ $P,E,h \vdash e_1\ «Add»\ e_2 : Integer$

| *WTrtLAss*:
$[\![E\ V = Some\ T;\ \ P,E,h \vdash e : T';\ \ P \vdash T' \leq T]\!]$
$\Longrightarrow P,E,h \vdash V{:=}e : Void$

| *WTrtFAcc*:
$[\![P,E,h \vdash e : Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D]\!] \Longrightarrow$
$P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAccNT*:
$P,E,h \vdash e : NT \Longrightarrow$
$P,E,h \vdash e{\cdot}F\{D\} : T$

| *WTrtFAss*:
$[\![P,E,h \vdash e_1 : Class\ C;\ \ P \vdash C\ has\ F{:}T\ in\ D;\ P,E,h \vdash e_2 : T_2;\ \ P \vdash T_2 \leq T]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtFAssNT*:
$[\![P,E,h \vdash e_1{:}NT;\ P,E,h \vdash e_2 : T_2]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 : Void$

| *WTrtCall*:
$[\![P,E,h \vdash e : Class\ C;\ P \vdash C\ sees\ M{:}Ts \to T = (pns,body)\ in\ D;$
$\quad P,E,h \vdash es\ [:]\ Ts';\ P \vdash Ts'\ [\leq]\ Ts]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtCallNT*:
$[\![P,E,h \vdash e : NT;\ P,E,h \vdash es\ [:]\ Ts]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) : T$

| *WTrtBlock*:
$P,E(V{\mapsto}T),h \vdash e : T' \Longrightarrow$
$P,E,h \vdash \{V{:}T;\ e\} : T'$

| *WTrtSeq*:
$[\![P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2]\!]$
$\Longrightarrow P,E,h \vdash e_1;;e_2 : T_2$

| *WTrtCond*:
$[\![P,E,h \vdash e : Boolean;\ \ P,E,h \vdash e_1{:}T_1;\ \ P,E,h \vdash e_2{:}T_2;$
$\quad P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1;\ P \vdash T_1 \leq T_2 \longrightarrow T = T_2;\ P \vdash T_2 \leq T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 : T$

| *WTrtWhile*:
$[\![P,E,h \vdash e : Boolean;\ \ P,E,h \vdash c{:}T]\!]$
$\Longrightarrow\ \ P,E,h \vdash while(e)\ c : Void$

| *WTrtThrow*:
$[\![P,E,h \vdash e : T_r;\ is\text{-}refT\ T_r]\!] \Longrightarrow$
$P,E,h \vdash throw\ e : T$

| *WTrtTry*:
$[\![P,E,h \vdash e_1 : T_1;\ \ P,E(V \mapsto Class\ C),h \vdash e_2 : T_2;\ P \vdash T_1 \leq T_2]\!]$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 : T_2$

— well-typed expression lists

| *WTrtNil*:
  $P,E,h \vdash [] [:] []$

| *WTrtCons*:
  $[\![P,E,h \vdash e : T;\ \ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
  $\Longrightarrow\ \ P,E,h \vdash e\#es\ [:]\ T\#Ts$

2.17.1 Easy consequences

lemma [*iff*]: $(P,E,h \vdash [] [:] Ts) = (Ts = [])\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash e\#es\ [:]\ T\#Ts) = (P,E,h \vdash e : T \wedge P,E,h \vdash es\ [:]\ Ts)\langle proof\rangle$
lemma [*iff*]: $(P,E,h \vdash (e\#es)\ [:]\ Ts) =$
  $(\exists U\ Us.\ Ts = U\#Us \wedge P,E,h \vdash e : U \wedge P,E,h \vdash es\ [:]\ Us)\langle proof\rangle$
lemma [*simp*]: $\forall Ts.\ (P,E,h \vdash es_1\ @\ es_2\ [:]\ Ts) =$
  $(\exists Ts_1\ Ts_2.\ Ts = Ts_1\ @\ Ts_2 \wedge P,E,h \vdash es_1\ [:]\ Ts_1\ \&\ P,E,h \vdash es_2[:]Ts_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Val\ v : T = (typeof_h\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Var\ v : T = (E\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash e_1;;e_2 : T_2 = (\exists T_1.\ P,E,h \vdash e_1:T_1 \wedge P,E,h \vdash e_2:T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash \{V:T;\ e\} : T' = (P,E(V\mapsto T),h \vdash e : T')\langle proof\rangle$

2.17.2 Some interesting lemmas

lemma *WTrts-Val*[*simp*]:
$\bigwedge Ts.\ (P,E,h \vdash map\ Val\ vs\ [:]\ Ts) = (map\ (typeof_h)\ vs = map\ Some\ Ts)\langle proof\rangle$

lemma *WTrts-same-length*: $\bigwedge Ts.\ P,E,h \vdash es\ [:]\ Ts \Longrightarrow length\ es = length\ Ts\langle proof\rangle$

lemma *WTrt-env-mono*:
  $P,E,h \vdash e : T \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E',h \vdash e : T)$ **and**
  $P,E,h \vdash es\ [:]\ Ts \Longrightarrow (\bigwedge E'.\ E \subseteq_m E' \Longrightarrow P,E',h \vdash es\ [:]\ Ts)\langle proof\rangle$

lemma *WTrt-hext-mono*: $P,E,h \vdash e : T \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash e : T$
and *WTrts-hext-mono*: $P,E,h \vdash es\ [:]\ Ts \Longrightarrow h \trianglelefteq h' \Longrightarrow P,E,h' \vdash es\ [:]\ Ts\langle proof\rangle$

lemma *WT-implies-WTrt*: $P,E \vdash e :: T \Longrightarrow P,E,h \vdash e : T$
and *WTs-implies-WTrts*: $P,E \vdash es\ [::]\ Ts \Longrightarrow P,E,h \vdash es\ [:]\ Ts\langle proof\rangle$

end

2.18 Definite assignment

theory *DefAss* **imports** *BigStep* **begin**

2.18.1 Hypersets

type-synonym *'a hyperset* = *'a set option*

definition *hyperUn* :: *'a hyperset* ⇒ *'a hyperset* ⇒ *'a hyperset* (**infixl** ⊔ *65*)
where
  $A \sqcup B \equiv$ *case A of None* ⇒ *None*

$| \lfloor A \rfloor \Rightarrow (case\ B\ of\ None \Rightarrow None \mid \lfloor B \rfloor \Rightarrow \lfloor A \cup B \rfloor)$

definition *hyperInt* :: *'a hyperset* ⇒ *'a hyperset* ⇒ *'a hyperset* (**infixl** ⊓ *70*)
where
$A \sqcap B \equiv case\ A\ of\ None \Rightarrow B$
$| \lfloor A \rfloor \Rightarrow (case\ B\ of\ None \Rightarrow \lfloor A \rfloor \mid \lfloor B \rfloor \Rightarrow \lfloor A \cap B \rfloor)$

definition *hyperDiff1* :: *'a hyperset* ⇒ *'a* ⇒ *'a hyperset* (**infixl** ⊖ *65*)
where
$A \ominus a \equiv case\ A\ of\ None \Rightarrow None \mid \lfloor A \rfloor \Rightarrow \lfloor A - \{a\} \rfloor$

definition *hyper-isin* :: *'a* ⇒ *'a hyperset* ⇒ *bool* (**infix** ∈∈ *50*)
where
$a \in\in A \equiv case\ A\ of\ None \Rightarrow True \mid \lfloor A \rfloor \Rightarrow a \in A$

definition *hyper-subset* :: *'a hyperset* ⇒ *'a hyperset* ⇒ *bool* (**infix** ⊑ *50*)
where
$A \sqsubseteq B \equiv case\ B\ of\ None \Rightarrow True$
$| \lfloor B \rfloor \Rightarrow (case\ A\ of\ None \Rightarrow False \mid \lfloor A \rfloor \Rightarrow A \subseteq B)$

lemmas *hyperset-defs* =
hyperUn-def hyperInt-def hyperDiff1-def hyper-isin-def hyper-subset-def

lemma [*simp*]: $\lfloor\{\}\rfloor \sqcup A = A \wedge A \sqcup \lfloor\{\}\rfloor = A\langle proof \rangle$
lemma [*simp*]: $\lfloor A \rfloor \sqcup \lfloor B \rfloor = \lfloor A \cup B \rfloor \wedge \lfloor A \rfloor \ominus a = \lfloor A - \{a\} \rfloor\langle proof \rangle$
lemma [*simp*]: $None \sqcup A = None \wedge A \sqcup None = None\langle proof \rangle$
lemma [*simp*]: $a \in\in None \wedge None \ominus a = None\langle proof \rangle$
lemma *hyperUn-assoc*: $(A \sqcup B) \sqcup C = A \sqcup (B \sqcup C)\langle proof \rangle$
lemma *hyper-insert-comm*: $A \sqcup \lfloor\{a\}\rfloor = \lfloor\{a\}\rfloor \sqcup A \wedge A \sqcup (\lfloor\{a\}\rfloor \sqcup B) = \lfloor\{a\}\rfloor \sqcup (A \sqcup B)\langle proof \rangle$

2.18.2 Definite assignment

primrec
$\mathcal{A}$ :: *'a exp* ⇒ *'a hyperset*
and $\mathcal{A}s$ :: *'a exp list* ⇒ *'a hyperset*
where
$\mathcal{A}\ (new\ C) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (Cast\ C\ e) = \mathcal{A}\ e$
$| \mathcal{A}\ (Val\ v) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (e_1 \ll bop \gg e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$| \mathcal{A}\ (Var\ V) = \lfloor\{\}\rfloor$
$| \mathcal{A}\ (LAss\ V\ e) = \lfloor\{V\}\rfloor \sqcup \mathcal{A}\ e$
$| \mathcal{A}\ (e{\cdot}F\{D\}) = \mathcal{A}\ e$
$| \mathcal{A}\ (e_1{\cdot}F\{D\}{:=}e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$| \mathcal{A}\ (e{\cdot}M(es)) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$
$| \mathcal{A}\ (\{V{:}T;\ e\}) = \mathcal{A}\ e \ominus V$
$| \mathcal{A}\ (e_1;;e_2) = \mathcal{A}\ e_1 \sqcup \mathcal{A}\ e_2$
$| \mathcal{A}\ (if\ (e)\ e_1\ else\ e_2) = \mathcal{A}\ e \sqcup (\mathcal{A}\ e_1 \sqcap \mathcal{A}\ e_2)$
$| \mathcal{A}\ (while\ (b)\ e) = \mathcal{A}\ b$
$| \mathcal{A}\ (throw\ e) = None$
$| \mathcal{A}\ (try\ e_1\ catch(C\ V)\ e_2) = \mathcal{A}\ e_1 \sqcap (\mathcal{A}\ e_2 \ominus V)$

$| \mathcal{A}s\ ([]) = \lfloor\{\}\rfloor$
$| \mathcal{A}s\ (e\#es) = \mathcal{A}\ e \sqcup \mathcal{A}s\ es$

primrec
  $\mathcal{D}$ :: *'a exp* ⇒ *'a hyperset* ⇒ *bool*
  and $\mathcal{D}s$ :: *'a exp list* ⇒ *'a hyperset* ⇒ *bool*
where
  $\mathcal{D}$ (*new C*) *A* = *True*
| $\mathcal{D}$ (*Cast C e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*Val v*) *A* = *True*
| $\mathcal{D}$ (e_1 «*bop*» e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*Var V*) *A* = (*V* ∈∈ *A*)
| $\mathcal{D}$ (*LAss V e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*e*·*F*{*D*}) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (e_1·*F*{*D*}:=e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*e*·*M*(*es*)) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}s$ *es* (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ ({*V*:*T*; *e*}) *A* = $\mathcal{D}$ *e* (*A* ⊖ *V*)
| $\mathcal{D}$ (e_1;;e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ e_1))
| $\mathcal{D}$ (*if* (*e*) e_1 *else* e_2) *A* =
  ($\mathcal{D}$ *e A* ∧ $\mathcal{D}$ e_1 (*A* ⊔ $\mathcal{A}$ *e*) ∧ $\mathcal{D}$ e_2 (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ (*while* (*e*) *c*) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}$ *c* (*A* ⊔ $\mathcal{A}$ *e*))
| $\mathcal{D}$ (*throw e*) *A* = $\mathcal{D}$ *e A*
| $\mathcal{D}$ (*try* e_1 *catch*(*C V*) e_2) *A* = ($\mathcal{D}$ e_1 *A* ∧ $\mathcal{D}$ e_2 (*A* ⊔ ⌊{*V*}⌋))

| $\mathcal{D}s$ ([]) *A* = *True*
| $\mathcal{D}s$ (*e*#*es*) *A* = ($\mathcal{D}$ *e A* ∧ $\mathcal{D}s$ *es* (*A* ⊔ $\mathcal{A}$ *e*))

lemma *As-map-Val*[*simp*]: $\mathcal{A}s$ (*map Val vs*) = ⌊{}⌋⟨*proof*⟩
lemma *D-append*[*iff*]: ⋀*A*. $\mathcal{D}s$ (*es* @ *es'*) *A* = ($\mathcal{D}s$ *es A* ∧ $\mathcal{D}s$ *es'* (*A* ⊔ $\mathcal{A}s$ *es*))⟨*proof*⟩

lemma *A-fv*: ⋀*A*. $\mathcal{A}$ *e* = ⌊*A*⌋ ⟹ *A* ⊆ *fv e*
and ⋀*A*. $\mathcal{A}s$ *es* = ⌊*A*⌋ ⟹ *A* ⊆ *fvs es*⟨*proof*⟩

lemma *sqUn-lem*: *A* ⊑ *A'* ⟹ *A* ⊔ *B* ⊑ *A'* ⊔ *B*⟨*proof*⟩
lemma *diff-lem*: *A* ⊑ *A'* ⟹ *A* ⊖ *b* ⊑ *A'* ⊖ *b*⟨*proof*⟩

lemma *D-mono*: ⋀*A A'*. *A* ⊑ *A'* ⟹ $\mathcal{D}$ *e A* ⟹ $\mathcal{D}$ (*e*::*'a exp*) *A'*
and *Ds-mono*: ⋀*A A'*. *A* ⊑ *A'* ⟹ $\mathcal{D}s$ *es A* ⟹ $\mathcal{D}s$ (*es*::*'a exp list*) *A'*⟨*proof*⟩

lemma *D-mono'*: $\mathcal{D}$ *e A* ⟹ *A* ⊑ *A'* ⟹ $\mathcal{D}$ *e A'*
and *Ds-mono'*: $\mathcal{D}s$ *es A* ⟹ *A* ⊑ *A'* ⟹ $\mathcal{D}s$ *es A'*⟨*proof*⟩

end

2.19 Conformance Relations for Type Soundness Proofs

theory *Conform*
imports *Exceptions*
begin

definition *conf* :: *'m prog* ⇒ *heap* ⇒ *val* ⇒ *ty* ⇒ *bool* (-,- ⊢ - :≤ - [*51*,*51*,*51*,*51*] *50*)
where
  P,*h* ⊢ *v* :≤ *T* ≡
  ∃ *T'*. *typeof*$_h$ *v* = *Some T'* ∧ *P* ⊢ *T'* ≤ *T*

definition *oconf* :: *'m prog* ⇒ *heap* ⇒ *obj* ⇒ *bool* (-,- ⊢ - √ [*51*,*51*,*51*] *50*)
where
P,*h* ⊢ *obj* √ ≡
let (*C*,*fs*) = *obj in* ∀ *F D T*. *P* ⊢ *C has F*:*T in D* ⟶
(∃ *v*. *fs*(*F*,*D*) = *Some v* ∧ *P*,*h* ⊢ *v* :≤ *T*)

definition *hconf* :: *'m prog* ⇒ *heap* ⇒ *bool* (- ⊢ - √ [*51*,*51*] *50*)
where
P ⊢ *h* √ ≡
(∀ *a obj*. *h a* = *Some obj* ⟶ *P*,*h* ⊢ *obj* √) ∧ *preallocated h*

definition *lconf* :: *'m prog* ⇒ *heap* ⇒ (*vname* ⇀ *val*) ⇒ (*vname* ⇀ *ty*) ⇒ *bool* (-,- ⊢ - *'*(:≤*'*) - [*51*,*51*,*51*,*51*] *50*)
where
P,*h* ⊢ *l* (:≤) *E* ≡
∀ *V v*. *l V* = *Some v* ⟶ (∃ *T*. *E V* = *Some T* ∧ *P*,*h* ⊢ *v* :≤ *T*)

abbreviation
confs :: *'m prog* ⇒ *heap* ⇒ *val list* ⇒ *ty list* ⇒ *bool*
(-,- ⊢ - [:≤] - [*51*,*51*,*51*,*51*] *50*) **where**
P,*h* ⊢ *vs* [:≤] *Ts* ≡ *list-all2* (*conf P h*) *vs Ts*

2.19.1 Value conformance :≤

lemma *conf-Null* [*simp*]: *P*,*h* ⊢ *Null* :≤ *T* = *P* ⊢ *NT* ≤ *T*⟨*proof*⟩
lemma *typeof-conf*[*simp*]: *typeof*$_h$ *v* = *Some T* ⟹ *P*,*h* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *typeof-lit-conf*[*simp*]: *typeof v* = *Some T* ⟹ *P*,*h* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *defval-conf*[*simp*]: *P*,*h* ⊢ *default-val T* :≤ *T*⟨*proof*⟩
lemma *conf-upd-obj*: *h a* = *Some*(*C*,*fs*) ⟹ (*P*,*h*(*a*↦(*C*,*fs'*)) ⊢ *x* :≤ *T*) = (*P*,*h* ⊢ *x* :≤ *T*)⟨*proof*⟩
lemma *conf-widen*: *P*,*h* ⊢ *v* :≤ *T* ⟹ *P* ⊢ *T* ≤ *T'* ⟹ *P*,*h* ⊢ *v* :≤ *T'*⟨*proof*⟩
lemma *conf-hext*: *h* ⊴ *h'* ⟹ *P*,*h* ⊢ *v* :≤ *T* ⟹ *P*,*h'* ⊢ *v* :≤ *T*⟨*proof*⟩
lemma *conf-ClassD*: *P*,*h* ⊢ *v* :≤ *Class C* ⟹
v = *Null* ∨ (∃ *a obj T*. *v* = *Addr a* ∧ *h a* = *Some obj* ∧ *obj-ty obj* = *T* ∧ *P* ⊢ *T* ≤ *Class C*)⟨*proof*⟩
lemma *conf-NT* [*iff*]: *P*,*h* ⊢ *v* :≤ *NT* = (*v* = *Null*)⟨*proof*⟩
lemma *non-npD*: ⟦ *v* ≠ *Null*; *P*,*h* ⊢ *v* :≤ *Class C* ⟧
⟹ ∃ *a C' fs*. *v* = *Addr a* ∧ *h a* = *Some*(*C'*,*fs*) ∧ *P* ⊢ *C'* ≼* *C*⟨*proof*⟩

2.19.2 Value list conformance [:≤]

lemma *confs-widens* [*trans*]: ⟦*P*,*h* ⊢ *vs* [:≤] *Ts*; *P* ⊢ *Ts* [≤] *Ts'*⟧ ⟹ *P*,*h* ⊢ *vs* [:≤] *Ts'*⟨*proof*⟩
lemma *confs-rev*: *P*,*h* ⊢ *rev s* [:≤] *t* = (*P*,*h* ⊢ *s* [:≤] *rev t*)⟨*proof*⟩
lemma *confs-conv-map*:
⋀*Ts'*. *P*,*h* ⊢ *vs* [:≤] *Ts'* = (∃ *Ts*. *map typeof*$_h$ *vs* = *map Some Ts* ∧ *P* ⊢ *Ts* [≤] *Ts'*)⟨*proof*⟩
lemma *confs-hext*: *P*,*h* ⊢ *vs* [:≤] *Ts* ⟹ *h* ⊴ *h'* ⟹ *P*,*h'* ⊢ *vs* [:≤] *Ts*⟨*proof*⟩
lemma *confs-Cons2*: *P*,*h* ⊢ *xs* [:≤] *y*#*ys* = (∃ *z zs*. *xs* = *z*#*zs* ∧ *P*,*h* ⊢ *z* :≤ *y* ∧ *P*,*h* ⊢ *zs* [:≤] *ys*)⟨*proof*⟩

2.19.3 Object conformance

lemma *oconf-hext*: *P*,*h* ⊢ *obj* √ ⟹ *h* ⊴ *h'* ⟹ *P*,*h'* ⊢ *obj* √⟨*proof*⟩
lemma *oconf-init-fields*:
P ⊢ *C has-fields FDTs* ⟹ *P*,*h* ⊢ (*C*, *init-fields FDTs*) √
⟨*proof*⟩

lemma *oconf-fupd* [*intro?*]:

⟦ $P \vdash C$ *has* $F{:}T$ *in* D; $P,h \vdash v :\leq T$; $P,h \vdash (C,fs)\ \surd$ ⟧
$\Longrightarrow P,h \vdash (C, fs((F,D)\mapsto v))\ \surd\langle proof\rangle$

2.19.4 Heap conformance

lemma *hconfD*: ⟦ $P \vdash h\ \surd$; $h\ a = Some\ obj$ ⟧ $\Longrightarrow P,h \vdash obj\ \surd\langle proof\rangle$
lemma *hconf-new*: ⟦ $P \vdash h\ \surd$; $h\ a = None$; $P,h \vdash obj\ \surd$ ⟧ $\Longrightarrow P \vdash h(a\mapsto obj)\ \surd\langle proof\rangle$
lemma *hconf-upd-obj*: ⟦ $P \vdash h\surd$; $h\ a = Some(C,fs)$; $P,h \vdash (C,fs')\surd$ ⟧ $\Longrightarrow P \vdash h(a\mapsto(C,fs'))\surd\langle proof\rangle$

2.19.5 Local variable conformance

lemma *lconf-hext*: ⟦ $P,h \vdash l\ (:\leq)\ E$; $h \trianglelefteq h'$ ⟧ $\Longrightarrow P,h' \vdash l\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd*:
⟦ $P,h \vdash l\ (:\leq)\ E$; $P,h \vdash v :\leq T$; $E\ V = Some\ T$ ⟧ $\Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-empty*[*iff*]: $P,h \vdash Map.empty\ (:\leq)\ E\langle proof\rangle$
lemma *lconf-upd2*: ⟦$P,h \vdash l\ (:\leq)\ E$; $P,h \vdash v :\leq T$⟧ $\Longrightarrow P,h \vdash l(V\mapsto v)\ (:\leq)\ E(V\mapsto T)\langle proof\rangle$

end

2.20 Progress of Small Step Semantics

theory *Progress*
imports *Equivalence WellTypeRT DefAss ../Common/Conform*
begin

lemma *final-addrE*:
⟦ $P,E,h \vdash e : Class\ C$; *final* e;
$\bigwedge a.\ e = addr\ a \Longrightarrow R$;
$\bigwedge a.\ e = Throw\ a \Longrightarrow R$ ⟧ $\Longrightarrow R\langle proof\rangle$

lemma *finalRefE*:
⟦ $P,E,h \vdash e : T$; *is-refT* T; *final* e;
$e = null \Longrightarrow R$;
$\bigwedge a\ C.$ ⟦ $e = addr\ a$; $T = Class\ C$ ⟧ $\Longrightarrow R$;
$\bigwedge a.\ e = Throw\ a \Longrightarrow R$ ⟧ $\Longrightarrow R\langle proof\rangle$

Derivation of new induction scheme for well typing:

inductive
WTrt' :: [*J-prog*,*heap*,*env*,*expr*,*ty*] ⇒ *bool*
and *WTrts'* :: [*J-prog*,*heap*,*env*,*expr list*, *ty list*] ⇒ *bool*
and *WTrt2'* :: [*J-prog*,*env*,*heap*,*expr*,*ty*] ⇒ *bool*
(-,-,- ⊢ - :′′ - [51,51,51]50)
and *WTrts2'* :: [*J-prog*,*env*,*heap*,*expr list*, *ty list*] ⇒ *bool*
(-,-,- ⊢ - [:′] - [51,51,51]50)
for P :: *J-prog* **and** h :: *heap*
where
$P,E,h \vdash e :' T \equiv WTrt'\ P\ h\ E\ e\ T$
| $P,E,h \vdash es\ [:']\ Ts \equiv WTrts'\ P\ h\ E\ es\ Ts$

| *is-class* $P\ C \Longrightarrow P,E,h \vdash new\ C :' Class\ C$
| ⟦ $P,E,h \vdash e :' T$; *is-refT* T; *is-class* $P\ C$ ⟧
$\Longrightarrow P,E,h \vdash Cast\ C\ e :' Class\ C$
| $typeof_h\ v = Some\ T \Longrightarrow P,E,h \vdash Val\ v :' T$
| $E\ v = Some\ T \Longrightarrow P,E,h \vdash Var\ v :' T$

$| [\![P,E,h \vdash e_1 :' T_1;\ P,E,h \vdash e_2 :' T_2]\!]$
$\Longrightarrow P,E,h \vdash e_1\ «Eq»\ e_2 :' Boolean$
$| [\![P,E,h \vdash e_1 :' Integer;\ P,E,h \vdash e_2 :' Integer]\!]$
$\Longrightarrow P,E,h \vdash e_1\ «Add»\ e_2 :' Integer$
$| [\![P,E,h \vdash Var\ V :' T;\ P,E,h \vdash e :' T';\ P \vdash T' \leq T]\!]$
$\Longrightarrow P,E,h \vdash V{:=}e :' Void$
$| [\![P,E,h \vdash e :' Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D]\!] \Longrightarrow P,E,h \vdash e{\cdot}F\{D\} :' T$
$| P,E,h \vdash e :' NT \Longrightarrow P,E,h \vdash e{\cdot}F\{D\} :' T$
$| [\![P,E,h \vdash e_1 :' Class\ C;\ P \vdash C\ has\ F{:}T\ in\ D;$
$P,E,h \vdash e_2 :' T_2;\ P \vdash T_2 \leq T]\!]$
$\Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 :' Void$
$| [\![P,E,h \vdash e_1{:}'NT;\ P,E,h \vdash e_2 :' T_2]\!] \Longrightarrow P,E,h \vdash e_1{\cdot}F\{D\}{:=}e_2 :' Void$
$| [\![P,E,h \vdash e :' Class\ C;\ P \vdash C\ sees\ M{:}Ts \to T = (pns,body)\ in\ D;$
$P,E,h \vdash es\ [:']\ Ts';\ P \vdash Ts'\ [\leq]\ Ts]\!]$
$\Longrightarrow P,E,h \vdash e{\cdot}M(es) :' T$
$| [\![P,E,h \vdash e :' NT;\ P,E,h \vdash es\ [:']\ Ts]\!] \Longrightarrow P,E,h \vdash e{\cdot}M(es) :' T$
$| P,E,h \vdash [\,]\ [:']\ [\,]$
$| [\![P,E,h \vdash e :' T;\ P,E,h \vdash es\ [:']\ Ts]\!] \Longrightarrow P,E,h \vdash e\#es\ [:']\ T\#Ts$
$| [\![typeof_h\ v = Some\ T_1;\ P \vdash T_1 \leq T;\ P,E(V{\mapsto}T),h \vdash e_2 :' T_2]\!]$
$\Longrightarrow P,E,h \vdash \{V{:}T := Val\ v;\ e_2\} :' T_2$
$| [\![P,E(V{\mapsto}T),h \vdash e :' T';\ \neg\ assigned\ V\ e]\!] \Longrightarrow P,E,h \vdash \{V{:}T;\ e\} :' T'$
$| [\![P,E,h \vdash e_1{:}'\ T_1;\ P,E,h \vdash e_2{:}'T_2]\!] \Longrightarrow P,E,h \vdash e_1;;e_2 :' T_2$
$| [\![P,E,h \vdash e :' Boolean;\ P,E,h \vdash e_1{:}'\ T_1;\ P,E,h \vdash e_2{:}'\ T_2;$
$P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1;$
$P \vdash T_1 \leq T_2 \longrightarrow T = T_2;\ P \vdash T_2 \leq T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E,h \vdash if\ (e)\ e_1\ else\ e_2 :' T$

$| [\![P,E,h \vdash e :' Boolean;\ P,E,h \vdash c{:}'\ T]\!]$
$\Longrightarrow P,E,h \vdash while(e)\ c :' Void$
$| [\![P,E,h \vdash e :' T_r;\ is\text{-}refT\ T_r]\!] \Longrightarrow P,E,h \vdash throw\ e :' T$
$| [\![P,E,h \vdash e_1 :' T_1;\ P,E(V \mapsto Class\ C),h \vdash e_2 :' T_2;\ P \vdash T_1 \leq T_2]\!]$
$\Longrightarrow P,E,h \vdash try\ e_1\ catch(C\ V)\ e_2 :' T_2$

lemma [*iff*]: $P,E,h \vdash e_1;;e_2 :' T_2 = (\exists\, T_1.\ P,E,h \vdash e_1{:}'\ T_1 \wedge P,E,h \vdash e_2{:}'\ T_2)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Val\ v :' T = (typeof_h\ v = Some\ T)\langle proof\rangle$
lemma [*iff*]: $P,E,h \vdash Var\ v :' T = (E\ v = Some\ T)\langle proof\rangle$

lemma *wt-wt'*: $P,E,h \vdash e : T \Longrightarrow P,E,h \vdash e :' T$
and *wts-wts'*: $P,E,h \vdash es\ [:]\ Ts \Longrightarrow P,E,h \vdash es\ [:']\ Ts\langle proof\rangle$

lemma *wt'-wt*: $P,E,h \vdash e :' T \Longrightarrow P,E,h \vdash e : T$
and *wts'-wts*: $P,E,h \vdash es\ [:']\ Ts \Longrightarrow P,E,h \vdash es\ [:]\ Ts\langle proof\rangle$

corollary *wt'-iff-wt*: $(P,E,h \vdash e :' T) = (P,E,h \vdash e : T)\langle proof\rangle$

corollary *wts'-iff-wts*: $(P,E,h \vdash es\ [:']\ Ts) = (P,E,h \vdash es\ [:]\ Ts)\langle proof\rangle$
theorem assumes *wf*: *wwf-J-prog P* **and** *hconf*: $P \vdash h\ \surd$
shows *progress*: $P,E,h \vdash e : T \Longrightarrow$
$(\bigwedge l.\ [\![\mathcal{D}\ e\ \lfloor dom\ l \rfloor;\ \neg\ final\ e]\!] \Longrightarrow \exists\, e'\ s'.\ P \vdash \langle e,(h,l)\rangle \to \langle e',s'\rangle)$
and $P,E,h \vdash es\ [:]\ Ts \Longrightarrow$
$(\bigwedge l.\ [\![\mathcal{D}s\ es\ \lfloor dom\ l \rfloor;\ \neg\ finals\ es]\!] \Longrightarrow \exists\, es'\ s'.\ P \vdash \langle es,(h,l)\rangle\ [\to]\ \langle es',s'\rangle)\langle proof\rangle$

end

2.21 Well-formedness Constraints

theory *JWellForm*
imports *../Common/ WellForm WWellForm WellType DefAss*
begin

definition *wf-J-mdecl* :: *J-prog* ⇒ *cname* ⇒ *J-mb mdecl* ⇒ *bool*
where
wf-J-mdecl P C ≡ λ(*M*,*Ts*,*T*,(*pns*,*body*)).
length Ts = *length pns* ∧
distinct pns ∧
this ∉ *set pns* ∧
(∃ *T'*. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T'* ∧ *P* ⊢ *T'* ≤ *T*) ∧
𝒟 *body* ⌊{*this*} ∪ *set pns*⌋

lemma *wf-J-mdecl*[*simp*]:
wf-J-mdecl P C (*M*,*Ts*,*T*,*pns*,*body*) ≡
(*length Ts* = *length pns* ∧
distinct pns ∧
this ∉ *set pns* ∧
(∃ *T'*. *P*,[*this*↦*Class C*,*pns*[↦]*Ts*] ⊢ *body* :: *T'* ∧ *P* ⊢ *T'* ≤ *T*) ∧
𝒟 *body* ⌊{*this*} ∪ *set pns*⌋)⟨*proof*⟩

abbreviation
wf-J-prog :: *J-prog* ⇒ *bool* **where**
wf-J-prog == *wf-prog wf-J-mdecl*

lemma *wf-J-prog-wf-J-mdecl*:
⟦ *wf-J-prog P*; (*C*, *D*, *fds*, *mths*) ∈ *set P*; *jmdcl* ∈ *set mths* ⟧
⟹ *wf-J-mdecl P C jmdcl*⟨*proof*⟩

lemma *wf-mdecl-wwf-mdecl*: *wf-J-mdecl P C Md* ⟹ *wwf-J-mdecl P C Md*⟨*proof*⟩

lemma *wf-prog-wwf-prog*: *wf-J-prog P* ⟹ *wwf-J-prog P*⟨*proof*⟩

end

2.22 Type Safety Proof

theory *TypeSafe*
imports *Progress JWellForm*
begin

2.22.1 Basic preservation lemmas

First two easy preservation lemmas.

theorem *red-preserves-hconf*:
P ⊢ ⟨*e*,(*h*,*l*)⟩ → ⟨*e'*,(*h'*,*l'*)⟩ ⟹ (⋀*T E*. ⟦ *P*,*E*,*h* ⊢ *e* : *T*; *P* ⊢ *h* √ ⟧ ⟹ *P* ⊢ *h'* √)
and *reds-preserves-hconf*:
P ⊢ ⟨*es*,(*h*,*l*)⟩ [→] ⟨*es'*,(*h'*,*l'*)⟩ ⟹ (⋀*Ts E*. ⟦ *P*,*E*,*h* ⊢ *es* [:] *Ts*; *P* ⊢ *h* √ ⟧ ⟹ *P* ⊢ *h'* √)⟨*proof*⟩

theorem *red-preserves-lconf*:
P ⊢ ⟨*e*,(*h*,*l*)⟩ → ⟨*e'*,(*h'*,*l'*)⟩ ⟹

$(\bigwedge T\ E.\ [\![\ P,E,h \vdash e{:}T;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)$
and *reds-preserves-lconf*:
$P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge Ts\ E.\ [\![\ P,E,h \vdash es[:]Ts;\ P,h \vdash l\ (:\leq)\ E\ ]\!] \Longrightarrow P,h' \vdash l'\ (:\leq)\ E)\langle proof\rangle$

Preservation of definite assignment more complex and requires a few lemmas first.

lemma [*iff*]: $\bigwedge A.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts]\!] \Longrightarrow$
$\mathcal{D}\ (blocks\ (Vs,Ts,vs,e))\ A = \mathcal{D}\ e\ (A \sqcup \lfloor set\ Vs\rfloor)\langle proof\rangle$

lemma *red-lA-incr*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}\ e \sqsubseteq \lfloor dom\ l'\rfloor \sqcup \mathcal{A}\ e'$
and *reds-lA-incr*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \lfloor dom\ l\rfloor \sqcup \mathcal{A}s\ es \sqsubseteq \lfloor dom\ l'\rfloor \sqcup \mathcal{A}s\ es'\langle proof\rangle$

Now preservation of definite assignment.

lemma assumes *wf*: *wf-J-prog P*
shows *red-preserves-defass*:
$P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow \mathcal{D}\ e\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom\ l'\rfloor$
and $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow \mathcal{D}s\ es\ \lfloor dom\ l\rfloor \Longrightarrow \mathcal{D}s\ es'\ \lfloor dom\ l'\rfloor\langle proof\rangle$

Combining conformance of heap and local variables:

definition *sconf* :: *J-prog* ⇒ *env* ⇒ *state* ⇒ *bool* (-,- ⊢ - √ [51,51,51]50)
where
$P,E \vdash s\ \surd \equiv$ *let* $(h,l) = s$ *in* $P \vdash h\ \surd \wedge P,h \vdash l\ (:\leq)\ E$

lemma *red-preserves-sconf*:
$[\![\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd\langle proof\rangle$
lemma *reds-preserves-sconf*:
$[\![\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E,hp\ s \vdash es\ [:]\ Ts;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd\langle proof\rangle$

2.22.2 Subject reduction

lemma *wt-blocks*:
$\bigwedge E.\ [\![\ length\ Vs = length\ Ts;\ length\ vs = length\ Ts\ ]\!] \Longrightarrow$
$(P,E,h \vdash blocks(Vs,Ts,vs,e) : T) =$
$(P,E(Vs[\mapsto]Ts),h \vdash e{:}T \wedge (\exists Ts'.\ map\ (typeof_h)\ vs = map\ Some\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts))\langle proof\rangle$

theorem assumes *wf*: *wf-J-prog P*
shows *subject-reduction2*: $P \vdash \langle e,(h,l)\rangle \rightarrow \langle e',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ T.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists T'.\ P,E,h' \vdash e'{:}T' \wedge P \vdash T' \leq T)$
and *subjects-reduction2*: $P \vdash \langle es,(h,l)\rangle\ [\rightarrow]\ \langle es',(h',l')\rangle \Longrightarrow$
$(\bigwedge E\ Ts.\ [\![\ P,E \vdash (h,l)\ \surd;\ P,E,h \vdash es\ [:]\ Ts\ ]\!]$
$\Longrightarrow \exists Ts'.\ P,E,h' \vdash es'\ [:]\ Ts' \wedge P \vdash Ts'\ [\leq]\ Ts)\langle proof\rangle$

corollary *subject-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle e,s\rangle \rightarrow \langle e',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists T'.\ P,E,hp\ s' \vdash e'{:}T' \wedge P \vdash T' \leq T\langle proof\rangle$
corollary *subjects-reduction*:
$[\![\ wf\text{-}J\text{-}prog\ P;\ P \vdash \langle es,s\rangle\ [\rightarrow]\ \langle es',s'\rangle;\ P,E \vdash s\ \surd;\ P,E,hp\ s \vdash es[:]Ts\ ]\!]$
$\Longrightarrow \exists Ts'.\ P,E,hp\ s' \vdash es'[:]Ts' \wedge P \vdash Ts'\ [\leq]\ Ts\langle proof\rangle$

2.22.3 Lifting to $\rightarrow*$

Now all these preservation lemmas are first lifted to the transitive closure ...

lemma *Red-preserves-sconf*:

assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $\bigwedge T.\ [\![\ P,E,hp\ s \vdash e : T;\ P,E \vdash s\ \surd\ ]\!] \Longrightarrow P,E \vdash s'\ \surd \langle proof\rangle$

lemma *Red-preserves-defass*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor \Longrightarrow \mathcal{D}\ e'\ \lfloor dom(lcl\ s')\rfloor$
$\langle proof\rangle$

lemma *Red-preserves-type*:
assumes *wf*: *wf-J-prog P* **and** *Red*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $!!T.\ [\![\ P,E \vdash s\surd;\ P,E,hp\ s \vdash e{:}T\ ]\!]$
$\Longrightarrow \exists T'.\ P \vdash T' \le T \wedge P,E,hp\ s' \vdash e'{:}T' \langle proof\rangle$

2.22.4 Lifting to ⇒

. . . and now to the big step semantics, just for fun.

lemma *eval-preserves-sconf*:
$[\![$ *wf-J-prog P*; $P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle$; $P,E \vdash e::T$; $P,E \vdash s\surd\ ]\!] \Longrightarrow P,E \vdash s'\surd\langle proof\rangle$

lemma *eval-preserves-type*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P \vdash \langle e,s\rangle \Rightarrow \langle e',s'\rangle;\ P,E \vdash s\surd;\ P,E \vdash e::T\ ]\!]$
$\Longrightarrow \exists T'.\ P \vdash T' \le T \wedge P,E,hp\ s' \vdash e'{:}T' \langle proof\rangle$

2.22.5 The final polish

The above preservation lemmas are now combined and packed nicely.

definition *wf-config* :: *J-prog* ⇒ *env* ⇒ *state* ⇒ *expr* ⇒ *ty* ⇒ *bool* (-,-,- ⊢ - : - √ [51,0,0,0,0]50)
where
$P,E,s \vdash e{:}T\ \surd \ \equiv\ P,E \vdash s\ \surd \wedge P,E,hp\ s \vdash e{:}T$

theorem *Subject-reduction*: **assumes** *wf*: *wf-J-prog P*
shows $P \vdash \langle e,s\rangle \to \langle e',s'\rangle \Longrightarrow P,E,s \vdash e : T\ \surd$
$\Longrightarrow \exists T'.\ P,E,s' \vdash e' : T'\ \surd \wedge P \vdash T' \le T \langle proof\rangle$

theorem *Subject-reductions*:
assumes *wf*: *wf-J-prog P* **and** *reds*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
shows $\bigwedge T.\ P,E,s \vdash e{:}T\ \surd \Longrightarrow \exists T'.\ P,E,s' \vdash e'{:}T'\ \surd \wedge P \vdash T' \le T \langle proof\rangle$

corollary *Progress*: **assumes** *wf*: *wf-J-prog P*
shows $[\![\ P,E,s\ \vdash e : T\ \surd;\ \mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor;\ \neg\ final\ e\ ]\!] \Longrightarrow \exists e'\ s'.\ P \vdash \langle e,s\rangle \to \langle e',s'\rangle \langle proof\rangle$

corollary *TypeSafety*:
fixes *s*::*state*
assumes *wf*: *wf-J-prog P* **and** *sconf*: $P,E \vdash s\ \surd$ **and** *wt*: $P,E \vdash e::T$
and $\mathcal{D}$: $\mathcal{D}\ e\ \lfloor dom(lcl\ s)\rfloor$
and *steps*: $P \vdash \langle e,s\rangle \to* \langle e',s'\rangle$
and *nstep*: $\neg(\exists e''\ s''.\ P \vdash \langle e',s'\rangle \to \langle e'',s''\rangle)$
shows $(\exists v.\ e' = Val\ v \wedge P,hp\ s' \vdash v :\le T) \vee$
$(\exists a.\ e' = Throw\ a \wedge a \in dom(hp\ s'))\langle proof\rangle$

end

2.23 Program annotation

theory *Annotate* **imports** *WellType* **begin**

inductive
  Anno :: [*J-prog*,*env*, *expr* , *expr*] ⇒ *bool*
    (-,- ⊢ - ⇝ - [51,0,0,51]50)
  and *Annos* :: [*J-prog*,*env*, *expr list*, *expr list*] ⇒ *bool*
    (-,- ⊢ - [⇝] - [51,0,0,51]50)
  for *P* :: *J-prog*
where

  AnnoNew: *P*,*E* ⊢ *new C* ⇝ *new C*
| *AnnoCast*: *P*,*E* ⊢ *e* ⇝ *e'* ⟹ *P*,*E* ⊢ *Cast C e* ⇝ *Cast C e'*
| *AnnoVal*: *P*,*E* ⊢ *Val v* ⇝ *Val v*
| *AnnoVarVar*: *E V* = ⌊*T*⌋ ⟹ *P*,*E* ⊢ *Var V* ⇝ *Var V*
| *AnnoVarField*: ⟦ *E V* = *None*; *E this* = ⌊*Class C*⌋; *P* ⊢ *C sees V*:*T in D* ⟧
    ⟹ *P*,*E* ⊢ *Var V* ⇝ *Var this*·*V*{*D*}
| *AnnoBinOp*:
  ⟦ *P*,*E* ⊢ *e1* ⇝ *e1'*; *P*,*E* ⊢ *e2* ⇝ *e2'* ⟧
  ⟹ *P*,*E* ⊢ *e1* «*bop*» *e2* ⇝ *e1'* «*bop*» *e2'*
| *AnnoLAssVar*:
  ⟦ *E V* = ⌊*T*⌋; *P*,*E* ⊢ *e* ⇝ *e'* ⟧ ⟹ *P*,*E* ⊢ *V*:=*e* ⇝ *V*:=*e'*
| *AnnoLAssField*:
  ⟦ *E V* = *None*; *E this* = ⌊*Class C*⌋; *P* ⊢ *C sees V*:*T in D*; *P*,*E* ⊢ *e* ⇝ *e'* ⟧
  ⟹ *P*,*E* ⊢ *V*:=*e* ⇝ *Var this*·*V*{*D*} := *e'*
| *AnnoFAcc*:
  ⟦ *P*,*E* ⊢ *e* ⇝ *e'*; *P*,*E* ⊢ *e'* :: *Class C*; *P* ⊢ *C sees F*:*T in D* ⟧
  ⟹ *P*,*E* ⊢ *e*·*F*{[]} ⇝ *e'*·*F*{*D*}
| *AnnoFAss*: ⟦ *P*,*E* ⊢ *e1* ⇝ *e1'*; *P*,*E* ⊢ *e2* ⇝ *e2'*;
    P,*E* ⊢ *e1'* :: *Class C*; *P* ⊢ *C sees F*:*T in D* ⟧
    ⟹ *P*,*E* ⊢ *e1*·*F*{[]} := *e2* ⇝ *e1'*·*F*{*D*} := *e2'*
| *AnnoCall*:
  ⟦ *P*,*E* ⊢ *e* ⇝ *e'*; *P*,*E* ⊢ *es* [⇝] *es'* ⟧
  ⟹ *P*,*E* ⊢ *Call e M es* ⇝ *Call e' M es'*
| *AnnoBlock*:
  P,*E*(*V* ↦ *T*) ⊢ *e* ⇝ *e'* ⟹ *P*,*E* ⊢ {*V*:*T*; *e*} ⇝ {*V*:*T*; *e'*}
| *AnnoComp*: ⟦ *P*,*E* ⊢ *e1* ⇝ *e1'*; *P*,*E* ⊢ *e2* ⇝ *e2'* ⟧
    ⟹ *P*,*E* ⊢ *e1*;;*e2* ⇝ *e1'*;;*e2'*
| *AnnoCond*: ⟦ *P*,*E* ⊢ *e* ⇝ *e'*; *P*,*E* ⊢ *e1* ⇝ *e1'*; *P*,*E* ⊢ *e2* ⇝ *e2'* ⟧
    ⟹ *P*,*E* ⊢ *if* (*e*) *e1 else e2* ⇝ *if* (*e'*) *e1' else e2'*
| *AnnoLoop*: ⟦ *P*,*E* ⊢ *e* ⇝ *e'*; *P*,*E* ⊢ *c* ⇝ *c'* ⟧
    ⟹ *P*,*E* ⊢ *while* (*e*) *c* ⇝ *while* (*e'*) *c'*
| *AnnoThrow*: *P*,*E* ⊢ *e* ⇝ *e'* ⟹ *P*,*E* ⊢ *throw e* ⇝ *throw e'*
| *AnnoTry*: ⟦ *P*,*E* ⊢ *e1* ⇝ *e1'*; *P*,*E*(*V* ↦ *Class C*) ⊢ *e2* ⇝ *e2'* ⟧
    ⟹ *P*,*E* ⊢ *try e1 catch*(*C V*) *e2* ⇝ *try e1' catch*(*C V*) *e2'*

| *AnnoNil*: *P*,*E* ⊢ [] [⇝] []
| *AnnoCons*: ⟦ *P*,*E* ⊢ *e* ⇝ *e'*; *P*,*E* ⊢ *es* [⇝] *es'* ⟧
    ⟹ *P*,*E* ⊢ *e*#*es* [⇝] *e'*#*es'*

end

2.24 Example Expressions

theory *Examples* **imports** *Expr* **begin**

definition *classObject*::*J-mb cdecl*
where
classObject == (''*Object*'','''',[],[])

definition *classI* :: *J-mb cdecl*
where
classI ==
(''*I*'', *Object*,
[],
[(''*mult*'',[*Integer*,*Integer*],*Integer*,[''*i*'',''*j*''],
if (*Var* ''*i*'' «*Eq*» *Val*(*Intg* 0)) (*Val*(*Intg* 0))
else Var ''*j*'' «*Add*»
Var this · ''*mult*''([*Var* ''*i*'' «*Add*» *Val*(*Intg* (− 1)),*Var* ''*j*''])
])

definition *classL* :: *J-mb cdecl*
where
classL ==
(''*L*'', *Object*,
[(''*F*'',*Integer*), (''*N*'',*Class* ''*L*'')],
[(''*app*'',[*Class* ''*L*''],*Void*,[''*l*''],
if (*Var this* · ''*N*''{''*L*''} «*Eq*» *null*)
(*Var this* · ''*N*''{''*L*''} := *Var* ''*l*'')
else (*Var this* · ''*N*''{''*L*''}) · ''*app*''([*Var* ''*l*''])
])

definition *testExpr-BuildList* :: *expr*
where
testExpr-BuildList ==
{''*l1*'':*Class* ''*L*'' := *new* ''*L*'';
Var ''*l1*''·''*F*''{''*L*''} := *Val*(*Intg* 1);;
{''*l2*'':*Class* ''*L*'' := *new* ''*L*'';
Var ''*l2*''· ''*F*''{''*L*''} := *Val*(*Intg* 2);;
{''*l3*'':*Class* ''*L*'' := *new* ''*L*'';
Var ''*l3*''· ''*F*''{''*L*''} := *Val*(*Intg* 3);;
{''*l4*'':*Class* ''*L*'' := *new* ''*L*'';
Var ''*l4*''· ''*F*''{''*L*''} := *Val*(*Intg* 4);;
Var ''*l1*''·''*app*''([*Var* ''*l2*'']);;
Var ''*l1*''·''*app*''([*Var* ''*l3*'']);;
Var ''*l1*''·''*app*''([*Var* ''*l4*''])}}}}

definition *testExpr1* ::*expr*
where
testExpr1 == *Val*(*Intg* 5)
definition *testExpr2* ::*expr*
where
testExpr2 == *BinOp* (*Val*(*Intg* 5)) *Add* (*Val*(*Intg* 6))

definition *testExpr3* ::*expr*
where
testExpr3 == *BinOp* (*Var* ''*V*'') *Add* (*Val*(*Intg* 6))
definition *testExpr4* ::*expr*
where
testExpr4 == ''*V*'' := *Val*(*Intg* 6)
definition *testExpr5* ::*expr*
where
testExpr5 == *new* ''*Object*'';; {''*V*'':(*Class* ''*C*'') := *new* ''*C*''; *Var* ''*V*''•''*F*''{''*C*''} := *Val*(*Intg* 42)}
definition *testExpr6* ::*expr*
where
testExpr6 == {''*V*'':(*Class* ''*I*'') := *new* ''*I*''; *Var* ''*V*''•''*mult*''([*Val*(*Intg* 40),*Val*(*Intg* 4)])}

definition *mb-isNull*:: *expr*
where
mb-isNull == *Var this* • ''*test*''{''*A*''} «*Eq*» *null*

definition *mb-add*:: *expr*
where
mb-add == (*Var this* • ''*int*''{''*A*''} :=(*Var this* • ''*int*''{''*A*''} «*Add*» *Var* ''*i*''));; (*Var this* • ''*int*''{''*A*''})

definition *mb-mult-cond*:: *expr*
where
mb-mult-cond == (*Var* ''*j*'' «*Eq*» *Val* (*Intg* 0)) «*Eq*» *Val* (*Bool False*)

definition *mb-mult-block*:: *expr*
where
mb-mult-block == ''*temp*'':=(*Var* ''*temp*'' «*Add*» *Var* ''*i*'');;''*j*'':=(*Var* ''*j*'' «*Add*» *Val* (*Intg* (− 1)))

definition *mb-mult*:: *expr*
where
mb-mult == {''*temp*'':*Integer*:=*Val* (*Intg* 0); *While* (*mb-mult-cond*) *mb-mult-block*;; (*Var this* • ''*int*''{''*A*''} := *Var* ''*temp*'';; *Var* ''*temp*'')}

definition *classA*:: *J-mb cdecl*
where
classA ==
(''*A*'', *Object*,
[(''*int*'',*Integer*),
(''*test*'',*Class* ''*A*'')],
[(''*isNull*'',[],*Boolean*,[], *mb-isNull*),
(''*add*'',[*Integer*],*Integer*,[''*i*''], *mb-add*),
(''*mult*'',[*Integer*,*Integer*],*Integer*,[''*i*'',''*j*''], *mb-mult*)])

definition *testExpr-ClassA*:: *expr*
where
testExpr-ClassA ==
{''*A1*'':*Class* ''*A*'':= *new* ''*A*'';
{''*A2*'':*Class* ''*A*'':= *new* ''*A*'';
{''*testint*'':*Integer*:= *Val* (*Intg* 5);
(*Var* ''*A2*''• ''*int*''{''*A*''} := (*Var* ''*A1*''• ''*add*''([*Var* ''*testint*''])));;

(*Var ''A2''· ''int''{''A''} := (Var ''A1''· ''add''([Var ''testint'']));;*
Var ''A2''· ''mult''([Var ''A2''· ''int''{''A''}, Var ''testint'']) }}}

end

2.25 Code Generation For BigStep

theory *execute-Bigstep*
imports
BigStep Examples
HOL−Library.Code-Target-Numeral
begin

inductive *map-val* :: *expr list* ⇒ *val list* ⇒ *bool*
where
Nil: *map-val* [] []
| *Cons*: *map-val xs ys* ⟹ *map-val* (*Val y* # *xs*) (*y* # *ys*)

inductive *map-val2* :: *expr list* ⇒ *val list* ⇒ *expr list* ⇒ *bool*
where
Nil: *map-val2* [] [] []
| *Cons*: *map-val2 xs ys zs* ⟹ *map-val2* (*Val y* # *xs*) (*y* # *ys*) *zs*
| *Throw*: *map-val2* (*throw e* # *xs*) [] (*throw e* # *xs*)

theorem *map-val-conv*: (*xs* = *map Val ys*) = *map-val xs ys*⟨*proof*⟩
theorem *map-val2-conv*:
(*xs* = *map Val ys* @ *throw e* # *zs*) = *map-val2 xs ys* (*throw e* # *zs*)⟨*proof*⟩
lemma *CallNull2*:
⟦ *P* ⊢ ⟨*e*,*s*$_0$⟩ ⇒ ⟨*null*,*s*$_1$⟩; *P* ⊢ ⟨*ps*,*s*$_1$⟩ [⇒] ⟨*evs*,*s*$_2$⟩; *map-val evs vs* ⟧
⟹ *P* ⊢ ⟨*e*·*M*(*ps*),*s*$_0$⟩ ⇒ ⟨*THROW NullPointer*,*s*$_2$⟩
⟨*proof*⟩

lemma *CallParamsThrow2*:
⟦ *P* ⊢ ⟨*e*,*s*$_0$⟩ ⇒ ⟨*Val v*,*s*$_1$⟩; *P* ⊢ ⟨*es*,*s*$_1$⟩ [⇒] ⟨*evs*,*s*$_2$⟩;
map-val2 evs vs (*throw ex* # *es''*) ⟧
⟹ *P* ⊢ ⟨*e*·*M*(*es*),*s*$_0$⟩ ⇒ ⟨*throw ex*,*s*$_2$⟩
⟨*proof*⟩

lemma *Call2*:
⟦ *P* ⊢ ⟨*e*,*s*$_0$⟩ ⇒ ⟨*addr a*,*s*$_1$⟩; *P* ⊢ ⟨*ps*,*s*$_1$⟩ [⇒] ⟨*evs*,(*h*$_2$,*l*$_2$)⟩;
map-val evs vs;
h$_2$ *a* = *Some*(*C*,*fs*); *P* ⊢ *C sees M*:*Ts*→*T* = (*pns*,*body*) *in D*;
length vs = *length pns*; *l*$_2$′ = [*this*↦*Addr a*, *pns*[↦]*vs*];
P ⊢ ⟨*body*,(*h*$_2$,*l*$_2$′)⟩ ⇒ ⟨*e*′,(*h*$_3$,*l*$_3$)⟩ ⟧
⟹ *P* ⊢ ⟨*e*·*M*(*ps*),*s*$_0$⟩ ⇒ ⟨*e*′,(*h*$_3$,*l*$_2$)⟩
⟨*proof*⟩

code-pred
(*modes*: *i* ⇒ *o* ⇒ *bool*)
map-val
⟨*proof*⟩

code-pred
(*modes*: $i \Rightarrow o \Rightarrow o \Rightarrow bool$)
map-val2
⟨*proof*⟩

lemmas [*code-pred-intro*] =
eval-evals.New eval-evals.NewFail
eval-evals.Cast eval-evals.CastNull eval-evals.CastFail eval-evals.CastThrow
eval-evals.Val eval-evals.Var
eval-evals.BinOp eval-evals.BinOpThrow1 eval-evals.BinOpThrow2
eval-evals.LAss eval-evals.LAssThrow
eval-evals.FAcc eval-evals.FAccNull eval-evals.FAccThrow
eval-evals.FAss eval-evals.FAssNull
eval-evals.FAssThrow1 eval-evals.FAssThrow2
eval-evals.CallObjThrow

declare *CallNull2* [*code-pred-intro CallNull2*]
declare *CallParamsThrow2* [*code-pred-intro CallParamsThrow2*]
declare *Call2* [*code-pred-intro Call2*]

lemmas [*code-pred-intro*] =
eval-evals.Block
eval-evals.Seq eval-evals.SeqThrow
eval-evals.CondT eval-evals.CondF eval-evals.CondThrow
eval-evals.WhileF eval-evals.WhileT
eval-evals.WhileCondThrow

declare *eval-evals.WhileBodyThrow* [*code-pred-intro WhileBodyThrow2*]

lemmas [*code-pred-intro*] =
eval-evals.Throw eval-evals.ThrowNull
eval-evals.ThrowThrow
eval-evals.Try eval-evals.TryCatch eval-evals.TryThrow
eval-evals.Nil eval-evals.Cons eval-evals.ConsThrow

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow o \Rightarrow bool$ *as execute*)
eval
⟨*proof*⟩

notation *execute* (- ⊢ ((1⟨-,/-⟩) ⇒/ ⟨'-, '-⟩) [51,0,0] 81)

definition *test1* = [] ⊢ ⟨*testExpr1*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test2* = [] ⊢ ⟨*testExpr2*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test3* = [] ⊢ ⟨*testExpr3*,(*Map.empty*,*Map.empty*(''*V*''↦*Intg* 77))⟩ ⇒ ⟨-,-⟩
definition *test4* = [] ⊢ ⟨*testExpr4*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *test5* = [(''*Object*'',(''''',[],[])),(''*C*'',(''*Object*'',[(''*F*'',*Integer*)],[]))] ⊢ ⟨*testExpr5*,(*Map.empty*,*Map.empty*)⟩
⇒ ⟨-,-⟩
definition *test6* = [(''*Object*'',(''''',[],[])), *classI*] ⊢ ⟨*testExpr6*,(*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩

definition *V* = ''*V*''
definition *C* = ''*C*''
definition *F* = ''*F*''

⟨*ML*⟩

definition *test7* = [*classObject*, *classL*] ⊢ ⟨*testExpr-BuildList*, (*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩

definition *L* = ″*L*″
definition *N* = ″*N*″

⟨*ML*⟩

definition *test8* = [*classObject*, *classA*] ⊢ ⟨*testExpr-ClassA*, (*Map.empty*,*Map.empty*)⟩ ⇒ ⟨-,-⟩
definition *i* = ″*int*″
definition *t* = ″*test*″
definition *A* = ″*A*″

⟨*ML*⟩

end

2.26 Code Generation For WellType

theory *execute-WellType*
imports
WellType Examples
begin

lemma *WTCond1*:
⟦*P*,*E* ⊢ *e* :: *Boolean*; *P*,*E* ⊢ e_1::T_1; *P*,*E* ⊢ e_2::T_2; *P* ⊢ $T_1 \leq T_2$;
P ⊢ $T_2 \leq T_1 \longrightarrow T_2 = T_1$ ⟧ ⟹ *P*,*E* ⊢ *if* (*e*) e_1 *else* e_2 :: T_2
⟨*proof*⟩

lemma *WTCond2*:
⟦*P*,*E* ⊢ *e* :: *Boolean*; *P*,*E* ⊢ e_1::T_1; *P*,*E* ⊢ e_2::T_2; *P* ⊢ $T_2 \leq T_1$;
P ⊢ $T_1 \leq T_2 \longrightarrow T_1 = T_2$ ⟧ ⟹ *P*,*E* ⊢ *if* (*e*) e_1 *else* e_2 :: T_1
⟨*proof*⟩

lemmas [*code-pred-intro*] =
WT-WTs.WTNew
WT-WTs.WTCast
WT-WTs.WTVal
WT-WTs.WTVar
WT-WTs.WTBinOpEq
WT-WTs.WTBinOpAdd
WT-WTs.WTLAss
WT-WTs.WTFAcc
WT-WTs.WTFAss
WT-WTs.WTCall
WT-WTs.WTBlock
WT-WTs.WTSeq

declare
WTCond1 [*code-pred-intro WTCond1*]

WTCond2 [*code-pred-intro WTCond2*]

lemmas [*code-pred-intro*] =
WT-WTs.WTWhile
WT-WTs.WTThrow
WT-WTs.WTTry
WT-WTs.WTNil
WT-WTs.WTCons

code-pred
(*modes*: $i \Rightarrow i \Rightarrow i \Rightarrow i \Rightarrow$ *bool as type-check*, $i \Rightarrow i \Rightarrow i \Rightarrow o \Rightarrow$ *bool as infer-type*)
WT
⟨*proof*⟩

notation *infer-type* (-,- ⊢ - :: '- [*51*,*51*,*51*]*100*)

definition *test1* **where** *test1* = [],*Map.empty* ⊢ *testExpr1* :: -
definition *test2* **where** *test2* = [], *Map.empty* ⊢ *testExpr2* :: -
definition *test3* **where** *test3* = [], *Map.empty*(''*V*'' ↦ *Integer*) ⊢ *testExpr3* :: -
definition *test4* **where** *test4* = [], *Map.empty*(''*V*'' ↦ *Integer*) ⊢ *testExpr4* :: -
definition *test5* **where** *test5* = [*classObject*, (''*C*'',(''*Object*'',[(''*F*'',*Integer*)],[]))], *Map.empty* ⊢ *testExpr5* :: -
definition *test6* **where** *test6* = [*classObject*, *classI*], *Map.empty* ⊢ *testExpr6* :: -

⟨*ML*⟩

definition *testmb-isNull* **where** *testmb-isNull* = [*classObject*, *classA*], *Map.empty*([*this*] [↦] [*Class* ''*A*'']) ⊢ *mb-isNull* :: -
definition *testmb-add* **where** *testmb-add* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*''] [↦] [*Class* ''*A*'',*Integer*]) ⊢ *mb-add* :: -
definition *testmb-mult-cond* **where** *testmb-mult-cond* = [*classObject*, *classA*], *Map.empty*([*this*,''*j*''] [↦] [*Class* ''*A*'',*Integer*]) ⊢ *mb-mult-cond* :: -
definition *testmb-mult-block* **where** *testmb-mult-block* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*'',''*j*'',''*temp*''] [↦] [*Class* ''*A*'',*Integer*,*Integer*,*Integer*]) ⊢ *mb-mult-block* :: -
definition *testmb-mult* **where** *testmb-mult* = [*classObject*, *classA*], *Map.empty*([*this*,''*i*'',''*j*''] [↦] [*Class* ''*A*'',*Integer*,*Integer*]) ⊢ *mb-mult* :: -

⟨*ML*⟩

definition *test* **where** *test* = [*classObject*, *classA*], *Map.empty* ⊢ *testExpr-ClassA* :: -

⟨*ML*⟩

end

Chapter 3

Jinja Virtual Machine

3.1 State of the JVM

theory *JVMState* **imports** *../Common/Objects* **begin**

3.1.1 Frame Stack

type-synonym
pc = *nat*

type-synonym
frame = *val list* × *val list* × *cname* × *mname* × *pc*
— operand stack
— registers (including this pointer, method parameters, and local variables)
— name of class where current method is defined
— parameter types
— program counter within frame

3.1.2 Runtime State

type-synonym
jvm-state = *addr option* × *heap* × *frame list*
— exception flag, heap, frames

end

3.2 Instructions of the JVM

theory *JVMInstructions* **imports** *JVMState* **begin**

datatype
instr = *Load nat* — load from local variable
| *Store nat* — store into local variable
| *Push val* — push a value (constant)
| *New cname* — create object
| *Getfield vname cname* — Fetch field from object
| *Putfield vname cname* — Set field in object
| *Checkcast cname* — Check whether object is of given type
| *Invoke mname nat* — inv. instance meth of an object

| *Return* — return from method
| *Pop* — pop top element from opstack
| *IAdd* — integer addition
| *Goto int* — goto relative address
| *CmpEq* — equality comparison
| *IfFalse int* — branch if top of stack false
| *Throw* — throw top of stack as exception

type-synonym
bytecode = *instr list*

type-synonym
ex-entry = *pc* × *pc* × *cname* × *pc* × *nat*
— start-pc, end-pc, exception type, handler-pc, remaining stack depth

type-synonym
ex-table = *ex-entry list*

type-synonym
jvm-method = *nat* × *nat* × *bytecode* × *ex-table*
— max stacksize
— number of local variables. Add 1 + no. of parameters to get no. of registers
— instruction sequence
— exception handler table

type-synonym
jvm-prog = *jvm-method prog*

end

3.3 JVM Instruction Semantics

theory *JVMExecInstr*
imports *JVMInstructions JVMState ../Common/Exceptions*
begin

primrec
exec-instr :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
cname, *mname*, *pc*, *frame list*] => *jvm-state*
where
exec-instr-Load:
exec-instr (*Load n*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, ((*loc* ! *n*) # *stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*)

| *exec-instr* (*Store n*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*tl stk*, *loc*[*n*:=*hd stk*], C_0, M_0, *pc*+*1*)#*frs*)

| *exec-instr-Push*:
exec-instr (*Push v*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*v* # *stk*, *loc*, C_0, M_0, *pc*+*1*)#*frs*)

| *exec-instr-New*:
exec-instr (*New C*) *P h stk loc* C_0 M_0 *pc frs* =

```
  (case new-Addr h of
    None ⇒   (Some (addr-of-sys-xcpt OutOfMemory), h, (stk, loc, C₀, M₀, pc)#frs)
  | Some a ⇒ (None, h(a↦blank P C), (Addr a#stk, loc, C₀, M₀, pc+1)#frs))

| exec-instr (Getfield F C) P h stk loc C₀ M₀ pc frs =
  (let v      = hd stk;
       xp'    = if v=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       (D,fs) = the(h(the-Addr v))
   in (xp', h, (the(fs(F,C))#(tl stk), loc, C₀, M₀, pc+1)#frs))

| exec-instr (Putfield F C) P h stk loc C₀ M₀ pc frs =
  (let v    = hd stk;
       r    = hd (tl stk);
       xp'  = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       a    = the-Addr r;
       (D,fs) = the (h a);
       h'   = h(a ↦ (D, fs((F,C) ↦ v)))
   in (xp', h', (tl (tl stk), loc, C₀, M₀, pc+1)#frs))

| exec-instr (Checkcast C) P h stk loc C₀ M₀ pc frs =
  (let v   = hd stk;
       xp' = if ¬cast-ok P C h v then ⌊addr-of-sys-xcpt ClassCast⌋ else None
   in (xp', h, (stk, loc, C₀, M₀, pc+1)#frs))

| exec-instr-Invoke:
 exec-instr (Invoke M n) P h stk loc C₀ M₀ pc frs =
  (let ps  = take n stk;
       r   = stk!n;
       xp' = if r=Null then ⌊addr-of-sys-xcpt NullPointer⌋ else None;
       C   = fst(the(h(the-Addr r)));
       (D,M',Ts,mxs,mxl₀,ins,xt)= method P C M;
       f'  = ([],[r]@(rev ps)@(replicate mxl₀ undefined),D,M,0)
   in (xp', h, f'#(stk, loc, C₀, M₀, pc)#frs))

| exec-instr Return P h stk₀ loc₀ C₀ M₀ pc frs =
  (if frs=[] then (None, h, []) else
   let v = hd stk₀;
       (stk,loc,C,m,pc) = hd frs;
       n = length (fst (snd (method P C₀ M₀)))
   in (None, h, (v#(drop (n+1) stk),loc,C,m,pc+1)#tl frs))

| exec-instr Pop P h stk loc C₀ M₀ pc frs =
     (None, h, (tl stk, loc, C₀, M₀, pc+1)#frs)

| exec-instr IAdd P h stk loc C₀ M₀ pc frs =
  (let i₂ = the-Intg (hd stk);
       i₁ = the-Intg (hd (tl stk))
   in (None, h, (Intg (i₁+i₂)#(tl (tl stk)), loc, C₀, M₀, pc+1)#frs))

| exec-instr (IfFalse i) P h stk loc C₀ M₀ pc frs =
  (let pc' = if hd stk = Bool False then nat(int pc+i) else pc+1
   in (None, h, (tl stk, loc, C₀, M₀, pc')#frs))

| exec-instr CmpEq P h stk loc C₀ M₀ pc frs =
```

(*let* v_2 = *hd stk*;
v_1 = *hd* (*tl stk*)
in (*None*, *h*, (*Bool* (v_1=v_2) # *tl* (*tl stk*), *loc*, C_0, M_0, *pc+1*)#*frs*))

| *exec-instr-Goto*:
exec-instr (*Goto i*) *P h stk loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*, C_0, M_0, *nat*(*int pc+i*))#*frs*)

| *exec-instr Throw P h stk loc* C_0 M_0 *pc frs* =
(*let xp'* = *if hd stk* = *Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else* ⌊*the-Addr*(*hd stk*)⌋
in (*xp'*, *h*, (*stk*, *loc*, C_0, M_0, *pc*)#*frs*))

lemma *exec-instr-Store*:
exec-instr (*Store n*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*[*n*:=*v*], C_0, M_0, *pc+1*)#*frs*)
⟨*proof*⟩

lemma *exec-instr-Getfield*:
exec-instr (*Getfield F C*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp'* = *if v=Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
(*D*,*fs*) = *the*(*h*(*the-Addr v*))
in (*xp'*, *h*, (*the*(*fs*(*F*,*C*))#*stk*, *loc*, C_0, M_0, *pc+1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Putfield*:
exec-instr (*Putfield F C*) *P h* (*v*#*r*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp'* = *if r=Null then* ⌊*addr-of-sys-xcpt NullPointer*⌋ *else None*;
a = *the-Addr r*;
(*D*,*fs*) = *the* (*h a*);
h' = *h*(*a* ↦ (*D*, *fs*((*F*,*C*) ↦ *v*)))
in (*xp'*, *h'*, (*stk*, *loc*, C_0, M_0, *pc+1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Checkcast*:
exec-instr (*Checkcast C*) *P h* (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*let xp'* = *if* ¬*cast-ok P C h v then* ⌊*addr-of-sys-xcpt ClassCast*⌋ *else None*
in (*xp'*, *h*, (*v*#*stk*, *loc*, C_0, M_0, *pc+1*)#*frs*))
⟨*proof*⟩

lemma *exec-instr-Return*:
exec-instr Return P h (*v*#*stk*$_0$) *loc*$_0$ C_0 M_0 *pc frs* =
(*if frs*=[] *then* (*None*, *h*, []) *else*
let (*stk*,*loc*,*C*,*m*,*pc*) = *hd frs*;
n = *length* (*fst* (*snd* (*method P* C_0 M_0)))
in (*None*, *h*, (*v*#(*drop* (*n+1*) *stk*),*loc*,*C*,*m*,*pc+1*)#*tl frs*))
⟨*proof*⟩

lemma *exec-instr-IPop*:
exec-instr Pop P h (*v*#*stk*) *loc* C_0 M_0 *pc frs* =
(*None*, *h*, (*stk*, *loc*, C_0, M_0, *pc+1*)#*frs*)
⟨*proof*⟩

lemma *exec-instr-IAdd*:

exec-instr IAdd P h (Intg i_2 # Intg i_1 # stk) loc C_0 M_0 pc frs =
(None, h, (Intg (i_1+i_2)#stk, loc, C_0, M_0, pc+1)#frs)
⟨*proof*⟩

lemma *exec-instr-IfFalse*:
exec-instr (IfFalse i) P h (v#stk) loc C_0 M_0 pc frs =
(let pc′ = if v = Bool False then nat(int pc+i) else pc+1
in (None, h, (stk, loc, C_0, M_0, pc′)#frs))
⟨*proof*⟩

lemma *exec-instr-CmpEq*:
exec-instr CmpEq P h (v_2#v_1#stk) loc C_0 M_0 pc frs =
(None, h, (Bool (v_1=v_2) # stk, loc, C_0, M_0, pc+1)#frs)
⟨*proof*⟩

lemma *exec-instr-Throw*:
exec-instr Throw P h (v#stk) loc C_0 M_0 pc frs =
(let xp′ = if v = Null then ⌊addr-of-sys-xcpt NullPointer⌋ else ⌊the-Addr v⌋
in (xp′, h, (v#stk, loc, C_0, M_0, pc)#frs))
⟨*proof*⟩

end

3.4 Exception handling in the JVM

theory *JVMExceptions* **imports** *JVMInstructions ../Common/Exceptions* **begin**

definition *matches-ex-entry* :: *'m prog ⇒ cname ⇒ pc ⇒ ex-entry ⇒ bool*
where
matches-ex-entry P C pc xcp ≡
let (s, e, C′, h, d) = xcp in
s ≤ pc ∧ pc < e ∧ P ⊢ C ≼ C′*

primrec *match-ex-table* :: *'m prog ⇒ cname ⇒ pc ⇒ ex-table ⇒ (pc × nat) option*
where
match-ex-table P C pc [] = None
| *match-ex-table P C pc (e#es) = (if matches-ex-entry P C pc e*
then Some (snd(snd(snd e)))
else match-ex-table P C pc es)

abbreviation
ex-table-of :: *jvm-prog ⇒ cname ⇒ mname ⇒ ex-table* **where**
ex-table-of P C M == snd (snd (snd (snd (snd (snd(method P C M))))))

primrec *find-handler* :: *jvm-prog ⇒ addr ⇒ heap ⇒ frame list ⇒ jvm-state*
where
find-handler P a h [] = (Some a, h, [])
| *find-handler P a h (fr#frs) =*
(let (stk,loc,C,M,pc) = fr in
case match-ex-table P (cname-of h a) pc (ex-table-of P C M) of
None ⇒ find-handler P a h frs

| *Some pc-d* ⇒ (*None*, *h*, (*Addr a* # *drop* (*size stk* − *snd pc-d*) *stk*, *loc*, *C*, *M*, *fst pc-d*)#*frs*))

end

3.5 Program Execution in the JVM

theory *JVMExec*
imports *JVMExecInstr JVMExceptions*
begin

abbreviation
instrs-of :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *instr list* **where**
instrs-of P C M == *fst*(*snd*(*snd*(*snd*(*snd*(*snd*(*method P C M*))))))

fun *exec* :: *jvm-prog* × *jvm-state* => *jvm-state option* **where** — single step execution
exec (*P*, *xp*, *h*, []) = *None*

| *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) =
(*let*
i = *instrs-of P C M* ! *pc*;
(*xcpt'*, *h'*, *frs'*) = *exec-instr i P h stk loc C M pc frs*
in Some(*case xcpt' of*
None ⇒ (*None*,*h'*,*frs'*)
| *Some a* ⇒ *find-handler P a h* ((*stk*,*loc*,*C*,*M*,*pc*)#*frs*)))

| *exec* (*P*, *Some xa*, *h*, *frs*) = *None*

— relational view
inductive-set
exec-1 :: *jvm-prog* ⇒ (*jvm-state* × *jvm-state*) *set*
and *exec-1'* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state* ⇒ *bool*
(- ⊢/ - −*jvm*→$_1$/ - [*61*,*61*,*61*] *60*)
for *P* :: *jvm-prog*
where
P ⊢ *σ* −*jvm*→$_1$ *σ'* ≡ (*σ*,*σ'*) ∈ *exec-1 P*
| *exec-1I*: *exec* (*P*,*σ*) = *Some σ'* ⟹ *P* ⊢ *σ* −*jvm*→$_1$ *σ'*

— reflexive transitive closure:
definition *exec-all* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state* ⇒ *bool*
((- ⊢/ - −*jvm*→/ -) [*61*,*61*,*61*]*60*) **where**

exec-all-def1: *P* ⊢ *σ* −*jvm*→ *σ'* ⟷ (*σ*,*σ'*) ∈ (*exec-1 P*)*

notation (*ASCII*)
exec-all (- |−/ - −*jvm*−>/ - [*61*,*61*,*61*]*60*)

lemma *exec-1-eq*:
exec-1 P = {(*σ*,*σ'*). *exec* (*P*,*σ*) = *Some σ'*}⟨*proof*⟩
lemma *exec-1-iff*:
P ⊢ *σ* −*jvm*→$_1$ *σ'* = (*exec* (*P*,*σ*) = *Some σ'*)⟨*proof*⟩
lemma *exec-all-def*:
P ⊢ *σ* −*jvm*→ *σ'* = ((*σ*,*σ'*) ∈ {(*σ*,*σ'*). *exec* (*P*,*σ*) = *Some σ'*}*)⟨*proof*⟩

lemma *jvm-refl*[*iff*]: $P \vdash \sigma\ -jvm\rightarrow\ \sigma\langle proof\rangle$
lemma *jvm-trans*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''\langle proof\rangle$
lemma *jvm-one-step1*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\rightarrow_1\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''\langle proof\rangle$
lemma *jvm-one-step2*[*trans*]:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma'\ -jvm\rightarrow_1\ \sigma''\ ]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow\ \sigma''\langle proof\rangle$
lemma *exec-all-conf*:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ \sigma';\ P \vdash \sigma\ -jvm\rightarrow\ \sigma''\ ]\!]$
$\Longrightarrow P \vdash \sigma'\ -jvm\rightarrow\ \sigma'' \vee P \vdash \sigma''\ -jvm\rightarrow\ \sigma'\langle proof\rangle$

lemma *exec-all-finalD*: $P \vdash (x,\ h,\ []) -jvm\rightarrow \sigma \Longrightarrow \sigma = (x,\ h,\ [])\langle proof\rangle$
lemma *exec-all-deterministic*:
$[\![\ P \vdash \sigma\ -jvm\rightarrow\ (x,h,[]);\ P \vdash \sigma\ -jvm\rightarrow\ \sigma'\ ]\!] \Longrightarrow P \vdash \sigma'\ -jvm\rightarrow\ (x,h,[])\langle proof\rangle$

The start configuration of the JVM: in the start heap, we call a method m of class C in program P. The *this* pointer of the frame is set to *Null* to simulate a static method invokation.

definition *start-state* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *jvm-state* **where**
start-state P C M =
(*let* (*D*,*Ts*,*T*,*mxs*,mxl_0,*b*) = *method P C M in*
(*None*, *start-heap P*, [([], *Null* # *replicate* mxl_0 *undefined*, *C*, *M*, *0*)]))

end

3.6 A Defensive JVM

theory *JVMDefensive*
imports *JVMExec ../Common/Conform*
begin

Extend the state space by one element indicating a type error (or other abnormal termination)

datatype *'a type-error* = *TypeError* | *Normal 'a*

fun *is-Addr* :: *val* ⇒ *bool* **where**
is-Addr (*Addr a*) ⟷ *True*
| *is-Addr v* ⟷ *False*

fun *is-Intg* :: *val* ⇒ *bool* **where**
is-Intg (*Intg i*) ⟷ *True*
| *is-Intg v* ⟷ *False*

fun *is-Bool* :: *val* ⇒ *bool* **where**
is-Bool (*Bool b*) ⟷ *True*
| *is-Bool v* ⟷ *False*

definition *is-Ref* :: *val* ⇒ *bool* **where**
is-Ref v ⟷ *v* = *Null* ∨ *is-Addr v*

primrec *check-instr* :: [*instr*, *jvm-prog*, *heap*, *val list*, *val list*,
cname, *mname*, *pc*, *frame list*] ⇒ *bool* **where**
check-instr-Load:
check-instr (*Load n*) *P h stk loc C* M_0 *pc frs* =

$(n < \mathit{length}\ \mathit{loc})$

| *check-instr-Store*:
$\mathit{check\text{-}instr}\ (\mathit{Store}\ n)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(0 < \mathit{length}\ \mathit{stk} \wedge n < \mathit{length}\ \mathit{loc})$

| *check-instr-Push*:
$\mathit{check\text{-}instr}\ (\mathit{Push}\ v)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(\neg \mathit{is\text{-}Addr}\ v)$

| *check-instr-New*:
$\mathit{check\text{-}instr}\ (\mathit{New}\ C)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$\mathit{is\text{-}class}\ P\ C$

| *check-instr-Getfield*:
$\mathit{check\text{-}instr}\ (\mathit{Getfield}\ F\ C)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(0 < \mathit{length}\ \mathit{stk} \wedge (\exists\, C'\ T.\ P \vdash C\ \mathit{sees}\ F{:}T\ \mathit{in}\ C') \wedge$
$(\mathit{let}\ (C',\ T) = \mathit{field}\ P\ C\ F;\ \mathit{ref} = \mathit{hd}\ \mathit{stk}\ \mathit{in}$
$C' = C \wedge \mathit{is\text{-}Ref}\ \mathit{ref} \wedge (\mathit{ref} \neq \mathit{Null} \longrightarrow$
$h\ (\mathit{the\text{-}Addr}\ \mathit{ref}) \neq \mathit{None} \wedge$
$(\mathit{let}\ (D,vs) = \mathit{the}\ (h\ (\mathit{the\text{-}Addr}\ \mathit{ref}))\ \mathit{in}$
$P \vdash D \preceq^* C \wedge \mathit{vs}\ (F,C) \neq \mathit{None} \wedge P,h \vdash \mathit{the}\ (\mathit{vs}\ (F,C)) :\leq T))))$

| *check-instr-Putfield*:
$\mathit{check\text{-}instr}\ (\mathit{Putfield}\ F\ C)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(1 < \mathit{length}\ \mathit{stk} \wedge (\exists\, C'\ T.\ P \vdash C\ \mathit{sees}\ F{:}T\ \mathit{in}\ C') \wedge$
$(\mathit{let}\ (C',\ T) = \mathit{field}\ P\ C\ F;\ v = \mathit{hd}\ \mathit{stk};\ \mathit{ref} = \mathit{hd}\ (\mathit{tl}\ \mathit{stk})\ \mathit{in}$
$C' = C \wedge \mathit{is\text{-}Ref}\ \mathit{ref} \wedge (\mathit{ref} \neq \mathit{Null} \longrightarrow$
$h\ (\mathit{the\text{-}Addr}\ \mathit{ref}) \neq \mathit{None} \wedge$
$(\mathit{let}\ D = \mathit{fst}\ (\mathit{the}\ (h\ (\mathit{the\text{-}Addr}\ \mathit{ref})))\ \mathit{in}$
$P \vdash D \preceq^* C \wedge P,h \vdash v :\leq T))))$

| *check-instr-Checkcast*:
$\mathit{check\text{-}instr}\ (\mathit{Checkcast}\ C)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(0 < \mathit{length}\ \mathit{stk} \wedge \mathit{is\text{-}class}\ P\ C \wedge \mathit{is\text{-}Ref}\ (\mathit{hd}\ \mathit{stk}))$

| *check-instr-Invoke*:
$\mathit{check\text{-}instr}\ (\mathit{Invoke}\ M\ n)\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(n < \mathit{length}\ \mathit{stk} \wedge \mathit{is\text{-}Ref}\ (\mathit{stk}!n) \wedge$
$(\mathit{stk}!n \neq \mathit{Null} \longrightarrow$
$(\mathit{let}\ a = \mathit{the\text{-}Addr}\ (\mathit{stk}!n);$
$C = \mathit{cname\text{-}of}\ h\ a;$
$Ts = \mathit{fst}\ (\mathit{snd}\ (\mathit{method}\ P\ C\ M))$
$\mathit{in}\ h\ a \neq \mathit{None} \wedge P \vdash C\ \mathit{has}\ M \wedge$
$P,h \vdash \mathit{rev}\ (\mathit{take}\ n\ \mathit{stk})\ [:\leq]\ Ts)))$

| *check-instr-Return*:
$\mathit{check\text{-}instr}\ \mathit{Return}\ P\ h\ \mathit{stk}\ \mathit{loc}\ C_0\ M_0\ \mathit{pc}\ \mathit{frs} =$
$(0 < \mathit{length}\ \mathit{stk} \wedge ((0 < \mathit{length}\ \mathit{frs}) \longrightarrow$
$(P \vdash C_0\ \mathit{has}\ M_0) \wedge$
$(\mathit{let}\ v = \mathit{hd}\ \mathit{stk};$
$T = \mathit{fst}\ (\mathit{snd}\ (\mathit{snd}\ (\mathit{method}\ P\ C_0\ M_0)))$
$\mathit{in}\ P,h \vdash v :\leq T)))$

| *check-instr-Pop*:
  check-instr Pop P h stk loc C_0 M_0 *pc frs* =
  (*0* < *length stk*)

| *check-instr-IAdd*:
  check-instr IAdd P h stk loc C_0 M_0 *pc frs* =
  (*1* < *length stk* ∧ *is-Intg* (*hd stk*) ∧ *is-Intg* (*hd* (*tl stk*)))

| *check-instr-IfFalse*:
  check-instr (*IfFalse b*) *P h stk loc* C_0 M_0 *pc frs* =
  (*0* < *length stk* ∧ *is-Bool* (*hd stk*) ∧ *0* ≤ *int pc*+*b*)

| *check-instr-CmpEq*:
  check-instr CmpEq P h stk loc C_0 M_0 *pc frs* =
  (*1* < *length stk*)

| *check-instr-Goto*:
  check-instr (*Goto b*) *P h stk loc* C_0 M_0 *pc frs* =
  (*0* ≤ *int pc*+*b*)

| *check-instr-Throw*:
  check-instr Throw P h stk loc C_0 M_0 *pc frs* =
  (*0* < *length stk* ∧ *is-Ref* (*hd stk*))

definition *check* :: *jvm-prog* ⇒ *jvm-state* ⇒ *bool* **where**
  check P σ = (*let* (*xcpt*, *h*, *frs*) = σ *in*
    (*case frs of* [] ⇒ *True* | (*stk*,*loc*,*C*,*M*,*pc*)#*frs*′ ⇒
     P ⊢ *C has M* ∧
     (*let* (*C*′,*Ts*,*T*,*mxs*,*mxl*$_0$,*ins*,*xt*) = *method P C M*; *i* = *ins*!*pc in*
      pc < *size ins* ∧ *size stk* ≤ *mxs* ∧
      check-instr i P h stk loc C M pc frs′)))

definition *exec-d* :: *jvm-prog* ⇒ *jvm-state* ⇒ *jvm-state option type-error* **where**
  exec-d P σ = (*if check P* σ *then Normal* (*exec* (*P*, σ)) *else TypeError*)

inductive-set
  exec-1-d :: *jvm-prog* ⇒ (*jvm-state type-error* × *jvm-state type-error*) *set*
  and *exec-1-d*′ :: *jvm-prog* ⇒ *jvm-state type-error* ⇒ *jvm-state type-error* ⇒ *bool*
        (- ⊢ - −*jvmd*→$_1$ - [*61*,*61*,*61*]*60*)
  for *P* :: *jvm-prog*
where
  P ⊢ σ −*jvmd*→$_1$ σ′ ≡ (σ,σ′) ∈ *exec-1-d P*
| *exec-1-d-ErrorI*: *exec-d P* σ = *TypeError* ⟹ *P* ⊢ *Normal* σ −*jvmd*→$_1$ *TypeError*
| *exec-1-d-NormalI*: *exec-d P* σ = *Normal* (*Some* σ′) ⟹ *P* ⊢ *Normal* σ −*jvmd*→$_1$ *Normal* σ′

— reflexive transitive closure:
definition *exec-all-d* :: *jvm-prog* ⇒ *jvm-state type-error* ⇒ *jvm-state type-error* ⇒ *bool*
  (- ⊢ - −*jvmd*→ - [*61*,*61*,*61*]*60*) **where**
  exec-all-d-def1: *P* ⊢ σ −*jvmd*→ σ′ ⟷ (σ,σ′) ∈ (*exec-1-d P*)*

notation (*ASCII*)
  exec-all-d (- |− - −*jvmd*−> - [*61*,*61*,*61*]*60*)

lemma *exec-1-d-eq*:
exec-1-d P = {(*s*,*t*). ∃σ. *s* = *Normal* σ ∧ *t* = *TypeError* ∧ *exec-d P* σ = *TypeError*} ∪
{(*s*,*t*). ∃σ σ′. *s* = *Normal* σ ∧ *t* = *Normal* σ′ ∧ *exec-d P* σ = *Normal* (*Some* σ′)}
⟨*proof*⟩

declare *split-paired-All* [*simp del*]
declare *split-paired-Ex* [*simp del*]

lemma *if-neq* [*dest*!]:
(*if P then A else B*) ≠ *B* ⟹ *P*
⟨*proof*⟩

lemma *exec-d-no-errorI* [*intro*]:
check P σ ⟹ *exec-d P* σ ≠ *TypeError*
⟨*proof*⟩

theorem *no-type-error-commutes*:
exec-d P σ ≠ *TypeError* ⟹ *exec-d P* σ = *Normal* (*exec* (*P*, σ))
⟨*proof*⟩

lemma *defensive-imp-aggressive*:
P ⊢ (*Normal* σ) −*jvmd*→ (*Normal* σ′) ⟹ *P* ⊢ σ −*jvm*→ σ′⟨*proof*⟩
end

3.7 Example for generating executable code from JVM semantics

theory *JVMListExample*
imports
../Common/SystemClasses
JVMExec
HOL−Library.Code-Target-Numeral
begin

definition *list-name* :: *string*
where
list-name == *''list''*

definition *test-name* :: *string*
where
test-name == *''test''*

definition *val-name* :: *string*
where
val-name == *''val''*

definition *next-name* :: *string*
where
next-name == *''next''*

definition *append-name* :: *string*
where
append-name == *''append''*

definition *makelist-name* :: *string*
where
makelist-name == *''makelist''*

definition *append-ins* :: *bytecode*
where
append-ins ==
[*Load 0*,
Getfield next-name list-name,
Load 0,
Getfield next-name list-name,
Push Null,
CmpEq,
IfFalse 7,
Pop,
Load 0,
Load 1,
Putfield next-name list-name,
Push Unit,
Return,
Load 1,
Invoke append-name 1,
Return]

definition *list-class* :: *jvm-method class*
where
list-class ==
(*Object*,
[(*val-name*, *Integer*), (*next-name*, *Class list-name*)],
[(*append-name*, [*Class list-name*], *Void*,
(*3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)]))])

definition *make-list-ins* :: *bytecode*
where
make-list-ins ==
[*New list-name*,
Store 0,
Load 0,
Push (*Intg 1*),
Putfield val-name list-name,
New list-name,
Store 1,
Load 1,
Push (*Intg 2*),
Putfield val-name list-name,
New list-name,
Store 2,
Load 2,
Push (*Intg 3*),
Putfield val-name list-name,

Load 0,
Load 1,
Invoke append-name 1,
Pop,
Load 0,
Load 2,
Invoke append-name 1,
Return]

definition *test-class* :: *jvm-method class*
where
test-class ==
(*Object*, [],
[(*makelist-name*, [], *Void*, (*3*, *2*, *make-list-ins*, []))])

definition *E* :: *jvm-prog*
where
E == *SystemClasses* @ [(*list-name*, *list-class*), (*test-name*, *test-class*)]

definition *undefined-cname* :: *cname*
where [*code del*]: *undefined-cname* = *undefined*
declare *undefined-cname-def*[*symmetric*, *code-unfold*]
code-printing constant *undefined-cname* ⇀ (*SML*) *object*

definition *undefined-val* :: *val*
where [*code del*]: *undefined-val* = *undefined*
declare *undefined-val-def*[*symmetric*, *code-unfold*]
code-printing constant *undefined-val* ⇀ (*SML*) *Unit*

lemmas [*code-unfold*] = *SystemClasses-def* [*unfolded ObjectC-def NullPointerC-def ClassCastC-def OutOfMemoryC-def*]

definition *test* = *exec* (*E*, *start-state E test-name makelist-name*)

⟨*ML*⟩

end

Chapter 4

Bytecode Verifier

4.1 Semilattices

theory *Semilat*
imports *Main HOL−Library.While-Combinator*
begin

type-synonym *'a ord* = *'a* ⇒ *'a* ⇒ *bool*
type-synonym *'a binop* = *'a* ⇒ *'a* ⇒ *'a*
type-synonym *'a sl* = *'a set* × *'a ord* × *'a binop*

definition *lesub* :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool*
where *lesub x r y* ⟷ *r x y*

definition *lesssub* :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool*
where *lesssub x r y* ⟷ *lesub x r y* ∧ *x* ≠ *y*

definition *plussub* :: *'a* ⇒ (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'b* ⇒ *'c*
where *plussub x f y* = *f x y*

notation (*ASCII*)
lesub ((- /<='-- -) [*50*, *1000*, *51*] *50*) **and**
lesssub ((- /<'-- -) [*50*, *1000*, *51*] *50*) **and**
plussub ((- /+'-- -) [*65*, *1000*, *66*] *65*)

notation
lesub ((- /⊑_ -) [*50*, *0*, *51*] *50*) **and**
lesssub ((- /⊏_ -) [*50*, *0*, *51*] *50*) **and**
plussub ((- /⊔_ -) [*65*, *0*, *66*] *65*)

abbreviation (*input*)
lesub1 :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool* ((- /⊑_ -) [*50*, *1000*, *51*] *50*)
where *x* ⊑*r* *y* == *x* ⊑*r* *y*

abbreviation (*input*)
lesssub1 :: *'a* ⇒ *'a ord* ⇒ *'a* ⇒ *bool* ((- /⊏_ -) [*50*, *1000*, *51*] *50*)
where *x* ⊏*r* *y* == *x* ⊏*r* *y*

abbreviation (*input*)

plussub1 :: $'a \Rightarrow ('a \Rightarrow 'b \Rightarrow 'c) \Rightarrow 'b \Rightarrow 'c$ ((- /$\sqcup$_ -) [65, 1000, 66] 65)
where $x \sqcup_f y == x \sqcup_f y$

definition *ord* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a$ *ord*
where
ord r = $(\lambda x\ y.\ (x,y) \in r)$

definition *order* :: $'a$ *ord* $\Rightarrow$ $'a$ *set* $\Rightarrow$ *bool*
where
order r A $\longleftrightarrow$ $(\forall x{\in}A.\ x \sqsubseteq_r x) \wedge (\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r x \longrightarrow x{=}y) \wedge (\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r y \wedge y \sqsubseteq_r z \longrightarrow x \sqsubseteq_r z)$

definition *top* :: $'a$ *ord* $\Rightarrow$ $'a$ $\Rightarrow$ *bool*
where
top r T $\longleftrightarrow$ $(\forall x.\ x \sqsubseteq_r T)$

definition *acc* :: $'a$ *ord* $\Rightarrow$ *bool*
where
acc r $\longleftrightarrow$ *wf* $\{(y,x).\ x \sqsubset_r y\}$

definition *closed* :: $'a$ *set* $\Rightarrow$ $'a$ *binop* $\Rightarrow$ *bool*
where
closed A f $\longleftrightarrow$ $(\forall x{\in}A.\ \forall y{\in}A.\ x \sqcup_f y \in A)$

definition *semilat* :: $'a$ *sl* $\Rightarrow$ *bool*
where
semilat = $(\lambda(A,r,f).$ *order r A* $\wedge$ *closed A f* $\wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z))$

definition *is-ub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a$ $\Rightarrow$ $'a$ $\Rightarrow$ $'a$ $\Rightarrow$ *bool*
where
is-ub r x y u $\longleftrightarrow$ $(x,u){\in}r \wedge (y,u){\in}r$

definition *is-lub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a$ $\Rightarrow$ $'a$ $\Rightarrow$ $'a$ $\Rightarrow$ *bool*
where
is-lub r x y u $\longleftrightarrow$ *is-ub r x y u* $\wedge$ $(\forall z.$ *is-ub r x y z* $\longrightarrow$ $(u,z){\in}r)$

definition *some-lub* :: $('a \times 'a)$ *set* $\Rightarrow$ $'a$ $\Rightarrow$ $'a$ $\Rightarrow$ $'a$
where
some-lub r x y = $(SOME\ z.$ *is-lub r x y z*$)$

locale *Semilat* =
fixes *A* :: $'a$ *set*
fixes *r* :: $'a$ *ord*
fixes *f* :: $'a$ *binop*
assumes *semilat*: *semilat* (*A*, *r*, *f*)

lemma *order-refl* [*simp*, *intro*]: *order r A* $\Longrightarrow$ $x \in A \Longrightarrow x \sqsubseteq_r x$
⟨*proof*⟩
lemma *order-antisym*: $[\![$ *order r A*; $x \sqsubseteq_r y$; $y \sqsubseteq_r x$; $x \in A$; $y \in A$ $]\!] \Longrightarrow x = y$
⟨*proof*⟩
lemma *order-trans*: $[\![$ *order r A*; $x \sqsubseteq_r y$; $y \sqsubseteq_r z$; $x \in A$; $y \in A$; $z \in A$ $]\!] \Longrightarrow x \sqsubseteq_r z$

⟨*proof*⟩
lemma *order-less-irrefl* [*intro*, *simp*]: *order r A* ⟹ $x \in A \Longrightarrow \neg\ x \sqsubset_r x$
⟨*proof*⟩
lemma *order-less-trans*: ⟦ *order r A*; $x \sqsubset_r y$; $y \sqsubset_r z$; $x \in A$; $y \in A$; $z \in A$ ⟧ ⟹ $x \sqsubset_r z$
⟨*proof*⟩
lemma *topD* [*simp*, *intro*]: *top r T* ⟹ $x \sqsubseteq_r T$
⟨*proof*⟩
lemma *top-le-conv* [*simp*]: ⟦ *order r A*; *top r T*; $x \in A$; $T \in A$⟧ ⟹ $(T \sqsubseteq_r x) = (x = T)$
⟨*proof*⟩
lemma *semilat-Def*:
semilat(*A*,*r*,*f*) ⟷ *order r A* ∧ *closed A f* ∧
$(\forall x{\in}A.\ \forall y{\in}A.\ x \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ y \sqsubseteq_r x \sqcup_f y) \wedge$
$(\forall x{\in}A.\ \forall y{\in}A.\ \forall z{\in}A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z \longrightarrow x \sqcup_f y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *orderI* [*simp*, *intro*]: *order r A*
⟨*proof*⟩
lemma (**in** *Semilat*) *closedI* [*simp*, *intro*]: *closed A f*
⟨*proof*⟩
lemma *closedD*: ⟦ *closed A f*; $x{\in}A$; $y{\in}A$ ⟧ ⟹ $x \sqcup_f y \in A$
⟨*proof*⟩
lemma *closed-UNIV* [*simp*]: *closed UNIV f*
⟨*proof*⟩
lemma (**in** *Semilat*) *closed-f* [*simp*, *intro*]: ⟦$x \in A$; $y \in A$⟧ ⟹ $x \sqcup_f y \in A$
⟨*proof*⟩
lemma (**in** *Semilat*) *refl-r* [*intro*, *simp*]: $x \in A \Longrightarrow x \sqsubseteq_r x$ ⟨*proof*⟩

lemma (**in** *Semilat*) *antisym-r* [*intro?*]: ⟦ $x \sqsubseteq_r y$; $y \sqsubseteq_r x$; $x \in A$; $y \in A$ ⟧ ⟹ $x = y$
⟨*proof*⟩
lemma (**in** *Semilat*) *trans-r* [*trans*, *intro?*]: ⟦$x \sqsubseteq_r y$; $y \sqsubseteq_r z$; $x \in A$; $y \in A$; $z \in A$ ⟧ ⟹ $x \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub1* [*simp*, *intro?*]: ⟦ $x \in A$; $y \in A$ ⟧ ⟹ $x \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩
lemma (**in** *Semilat*) *ub2* [*simp*, *intro?*]: ⟦ $x \in A$; $y \in A$ ⟧ ⟹ $y \sqsubseteq_r x \sqcup_f y$
⟨*proof*⟩
lemma (**in** *Semilat*) *lub* [*simp*, *intro?*]:
⟦ $x \sqsubseteq_r z$; $y \sqsubseteq_r z$; $x \in A$; $y \in A$; $z \in A$ ⟧ ⟹ $x \sqcup_f y \sqsubseteq_r z$
⟨*proof*⟩
lemma (**in** *Semilat*) *plus-le-conv* [*simp*]:
⟦ $x \in A$; $y \in A$; $z \in A$ ⟧ ⟹ $(x \sqcup_f y \sqsubseteq_r z) = (x \sqsubseteq_r z \wedge y \sqsubseteq_r z)$
⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow x \sqcup_f y = y$ (**is** *?P* ⟷ *?Q*)⟨*proof*⟩
lemma (**in** *Semilat*) *le-iff-plus-unchanged2*:
assumes $x \in A$ **and** $y \in A$
shows $x \sqsubseteq_r y \longleftrightarrow y \sqcup_f x = y$ (**is** *?P* ⟷ *?Q*)⟨*proof*⟩
lemma (**in** *Semilat*) *plus-assoc* [*simp*]:
assumes *a*: $a \in A$ **and** *b*: $b \in A$ **and** *c*: $c \in A$
shows $a \sqcup_f (b \sqcup_f c) = a \sqcup_f b \sqcup_f c$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-com-lemma*:
⟦$a \in A$; $b \in A$⟧ ⟹ $a \sqcup_f b \sqsubseteq_r b \sqcup_f a$⟨*proof*⟩
lemma (**in** *Semilat*) *plus-commutative*:
⟦$a \in A$; $b \in A$⟧ ⟹ $a \sqcup_f b = b \sqcup_f a$

⟨*proof*⟩
lemma *is-lubD*:
is-lub r x y u ⟹ *is-ub r x y u* ∧ (∀ *z*. *is-ub r x y z* ⟶ (*u*,*z*) ∈ *r*)
⟨*proof*⟩
lemma *is-ubI*:
⟦ (*x*,*u*) ∈ *r*; (*y*,*u*) ∈ *r* ⟧ ⟹ *is-ub r x y u*
⟨*proof*⟩
lemma *is-ubD*:
is-ub r x y u ⟹ (*x*,*u*) ∈ *r* ∧ (*y*,*u*) ∈ *r*
⟨*proof*⟩

lemma *is-lub-bigger1* [*iff*]:
is-lub (*r*^*) *x y y* = ((*x*,*y*)∈*r*^*)⟨*proof*⟩
lemma *is-lub-bigger2* [*iff*]:
is-lub (*r*^*) *x y x* = ((*y*,*x*)∈*r*^*)⟨*proof*⟩
lemma *extend-lub*:
⟦ *single-valued r*; *is-lub* (*r*^*) *x y u*; (*x'*,*x*) ∈ *r* ⟧
⟹ ∃ *v*. *is-lub* (*r*^*) *x'* *y v*⟨*proof*⟩
lemma *single-valued-has-lubs* [*rule-format*]:
⟦ *single-valued r*; (*x*,*u*) ∈ *r*^* ⟧ ⟹ (∀ *y*. (*y*,*u*) ∈ *r*^* ⟶
(∃ *z*. *is-lub* (*r*^*) *x y z*))⟨*proof*⟩
lemma *some-lub-conv*:
⟦ *acyclic r*; *is-lub* (*r*^*) *x y u* ⟧ ⟹ *some-lub* (*r*^*) *x y* = *u*⟨*proof*⟩
lemma *is-lub-some-lub*:
⟦ *single-valued r*; *acyclic r*; (*x*,*u*)∈*r*^*; (*y*,*u*)∈*r*^* ⟧
⟹ *is-lub* (*r*^*) *x y* (*some-lub* (*r*^*) *x y*)
⟨*proof*⟩

4.1.1 An executable lub-finder

definition *exec-lub* :: ('*a* ∗ '*a*) *set* ⇒ ('*a* ⇒ '*a*) ⇒ '*a binop*
where
exec-lub r f x y = *while* (λ*z*. (*x*,*z*) ∉ *r**) *f y*

lemma *exec-lub-refl*: *exec-lub r f T T* = *T*
⟨*proof*⟩

lemma *acyclic-single-valued-finite*:
⟦*acyclic r*; *single-valued r*; (*x*,*y*) ∈ *r**⟧
⟹ *finite* (*r* ∩ {*a*. (*x*, *a*) ∈ *r**} × {*b*. (*b*, *y*) ∈ *r**})⟨*proof*⟩

lemma *exec-lub-conv*:
⟦ *acyclic r*; ∀ *x y*. (*x*,*y*) ∈ *r* ⟶ *f x* = *y*; *is-lub* (*r**) *x y u* ⟧ ⟹
exec-lub r f x y = *u*⟨*proof*⟩
lemma *is-lub-exec-lub*:
⟦ *single-valued r*; *acyclic r*; (*x*,*u*):*r*^*; (*y*,*u*):*r*^*; ∀ *x y*. (*x*,*y*) ∈ *r* ⟶ *f x* = *y* ⟧
⟹ *is-lub* (*r*^*) *x y* (*exec-lub r f x y*)
⟨*proof*⟩
end

4.2 The Error Type

theory *Err*
imports *Semilat*

begin

datatype *'a err* = *Err* | *OK 'a*

type-synonym *'a ebinop* = *'a* ⇒ *'a* ⇒ *'a err*
type-synonym *'a esl* = *'a set* × *'a ord* × *'a ebinop*

primrec *ok-val* :: *'a err* ⇒ *'a*
where
ok-val (*OK x*) = *x*

definition *lift* :: (*'a* ⇒ *'b err*) ⇒ (*'a err* ⇒ *'b err*)
where
lift f e = (*case e of Err* ⇒ *Err* | *OK x* ⇒ *f x*)

definition *lift2* :: (*'a* ⇒ *'b* ⇒ *'c err*) ⇒ *'a err* ⇒ *'b err* ⇒ *'c err*
where
lift2 f e$_1$ *e*$_2$ =
(*case e*$_1$ *of Err* ⇒ *Err* | *OK x* ⇒ (*case e*$_2$ *of Err* ⇒ *Err* | *OK y* ⇒ *f x y*))

definition *le* :: *'a ord* ⇒ *'a err ord*
where
le r e$_1$ *e*$_2$ =
(*case e*$_2$ *of Err* ⇒ *True* | *OK y* ⇒ (*case e*$_1$ *of Err* ⇒ *False* | *OK x* ⇒ *x* ⊑$_r$ *y*))

definition *sup* :: (*'a* ⇒ *'b* ⇒ *'c*) ⇒ (*'a err* ⇒ *'b err* ⇒ *'c err*)
where
sup f = *lift2* (λ*x y*. *OK* (*x* ⊔$_f$ *y*))

definition *err* :: *'a set* ⇒ *'a err set*
where
err A = *insert Err* {*OK x*|*x*. *x*∈*A*}

definition *esl* :: *'a sl* ⇒ *'a esl*
where
esl = (λ(*A*,*r*,*f*). (*A*, *r*, λ*x y*. *OK*(*f x y*)))

definition *sl* :: *'a esl* ⇒ *'a err sl*
where
sl = (λ(*A*,*r*,*f*). (*err A*, *le r*, *lift2 f*))

abbreviation
err-semilat :: *'a esl* ⇒ *bool* **where**
err-semilat L == *semilat*(*sl L*)

primrec *strict* :: (*'a* ⇒ *'b err*) ⇒ (*'a err* ⇒ *'b err*)
where
strict f Err = *Err*
| *strict f* (*OK x*) = *f x*

lemma *err-def'*:
err A = *insert Err* {*x*. ∃ *y*∈*A*. *x* = *OK y*}⟨*proof*⟩
lemma *strict-Some* [*simp*]:
(*strict f x* = *OK y*) = (∃ *z*. *x* = *OK z* ∧ *f z* = *OK y*)⟨*proof*⟩

lemma *not-Err-eq*: $(x \neq Err) = (\exists\, a.\ x = OK\ a)\langle proof\rangle$
lemma *not-OK-eq*: $(\forall\, y.\ x \neq OK\ y) = (x = Err)\langle proof\rangle$
lemma *unfold-lesub-err*: $e1 \sqsubseteq_{le\ r} e2 = le\ r\ e1\ e2\langle proof\rangle$
lemma *le-err-refl*: $\forall\, x.\ x \sqsubseteq_r x \Longrightarrow e \sqsubseteq_{le\ r} e\langle proof\rangle$
lemma *le-err-refl'*: $(\forall\, x{\in}A.\ x \sqsubseteq_r x) \Longrightarrow e \in err\ A \Longrightarrow e \sqsubseteq_{le\ r} e\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle$

4.3 More about Options

theory *Opt* **imports** *Err* **begin**

definition *le* :: *'a ord* ⇒ *'a option ord*
where
le r o$_1$ *o*$_2$ =
(*case o*$_2$ *of None* ⇒ *o*$_1$=*None* | *Some y* ⇒ (*case o*$_1$ *of None* ⇒ *True* | *Some x* ⇒ *x* ⊑$_r$ *y*))

definition *opt* :: *'a set* ⇒ *'a option set*
where
opt A = *insert None* {*Some y* |*y*. *y* ∈ *A*}

definition *sup* :: *'a ebinop* ⇒ *'a option ebinop*
where
sup f o$_1$ *o*$_2$ =
(*case o*$_1$ *of None* ⇒ *OK o*$_2$
| *Some x* ⇒ (*case o*$_2$ *of None* ⇒ *OK o*$_1$
| *Some y* ⇒ (*case f x y of Err* ⇒ *Err* | *OK z* ⇒ *OK* (*Some z*))))

definition *esl* :: *'a esl* ⇒ *'a option esl*
where
esl = (λ(*A*,*r*,*f*). (*opt A*, *le r*, *sup f*))

lemma *unfold-le-opt*:
o$_1$ ⊑$_{le\ r}$ *o*$_2$ =
(*case o*$_2$ *of None* ⇒ *o*$_1$=*None* |
Some y ⇒ (*case o*$_1$ *of None* ⇒ *True* | *Some x* ⇒ *x* ⊑$_r$ *y*))⟨*proof*⟩
lemma *le-opt-refl*: *order r A* ⟹ *x* ∈ *opt A* ⟹ *x* ⊑$_{le\ r}$ *x*⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩

4.4 Products as Semilattices

theory *Product*
imports *Err*
begin

definition *le* :: *'a ord* ⇒ *'b ord* ⇒ (*'a* × *'b*) *ord*
where
le r$_A$ *r*$_B$ = (λ(*a*$_1$,*b*$_1$) (*a*$_2$,*b*$_2$). *a*$_1$ ⊑$_{r_A}$ *a*$_2$ ∧ *b*$_1$ ⊑$_{r_B}$ *b*$_2$)

definition *sup* :: *'a ebinop* ⇒ *'b ebinop* ⇒ (*'a* × *'b*) *ebinop*
where
sup f g = (λ(*a*$_1$,*b*$_1$)(*a*$_2$,*b*$_2$). *Err.sup Pair* (*a*$_1$ ⊔$_f$ *a*$_2$) (*b*$_1$ ⊔$_g$ *b*$_2$))

definition *esl* :: *'a esl* ⇒ *'b esl* ⇒ (*'a* × *'b*) *esl*
where

esl = $(\lambda(A,r_A,f_A)\ (B,r_B,f_B).\ (A \times B,\ le\ r_A\ r_B,\ sup\ f_A\ f_B))$

abbreviation
lesubprod :: $'a \times 'b \Rightarrow ('a \Rightarrow 'a \Rightarrow bool) \Rightarrow ('b \Rightarrow 'b \Rightarrow bool) \Rightarrow 'a \times 'b \Rightarrow bool$
((- /$\sqsubseteq'$(-,-') -) [50, 0, 0, 51] 50) **where**
$p \sqsubseteq(rA,rB)\ q == p \sqsubseteq_{Product.le\ rA\ rB} q$

lemma *unfold-lesub-prod*: $x \sqsubseteq(r_A,r_B)\ y = le\ r_A\ r_B\ x\ y\langle proof\rangle$
lemma *le-prod-Pair-conv* [*iff*]: $((a_1,b_1) \sqsubseteq(r_A,r_B)\ (a_2,b_2)) = (a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2)\langle proof\rangle$
lemma *less-prod-Pair-conv*:
$((a_1,b_1) \sqsubset_{Product.le\ r_A\ r_B} (a_2,b_2)) =$
$(a_1 \sqsubset_{r_A} a_2\ \&\ b_1 \sqsubseteq_{r_B} b_2 \mid a_1 \sqsubseteq_{r_A} a_2\ \&\ b_1 \sqsubset_{r_B} b_2)\langle proof\rangle$
lemma *order-le-prodI* [*iff*]: $(order\ r_A\ \ A\ \&\ order\ r_B\ B) \Longrightarrow order\ (Product.le\ r_A\ r_B)\ (A \times B)$
$\langle proof\rangle$

lemma *order-le-prodE*: $A \neq \{\} \Longrightarrow B \neq \{\} \Longrightarrow order\ (Product.le\ r_A\ r_B)\ (A \times B) \Longrightarrow (order\ r_A\ \ A\ \&\ order\ r_B\ B)$
$\langle proof\rangle$

lemma *order-le-prod* [*iff*]: $A \neq \{\} \Longrightarrow B \neq \{\} \Longrightarrow order(Product.le\ r_A\ r_B)\ (A \times B) = (order\ r_A\ A\ \&\ order\ r_B\ B)\langle proof\rangle$
lemma *acc-le-prodI* [*intro*!]:
$[\![\ acc\ r_A;\ acc\ r_B\ ]\!] \Longrightarrow acc(Product.le\ r_A\ r_B)\langle proof\rangle$

lemma *closed-lift2-sup*:
$[\![\ closed\ (err\ A)\ (lift2\ f);\ closed\ (err\ B)\ (lift2\ g)\ ]\!] \Longrightarrow$
$closed\ (err(A\times B))\ (lift2(sup\ f\ g))\langle proof\rangle$
lemma *unfold-plussub-lift2*: $e_1 \sqcup_{lift2\ f} e_2 = lift2\ f\ e_1\ e_2\langle proof\rangle$

lemma *plus-eq-Err-conv* [*simp*]:
assumes $x\in A\ \ y\in A\ \ semilat(err\ A,\ Err.le\ r,\ lift2\ f)$
shows $(x \sqcup_f y = Err) = (\neg(\exists z\in A.\ x \sqsubseteq_r z \wedge y \sqsubseteq_r z))\langle proof\rangle$
lemma *err-semilat-Product-esl*:
$\bigwedge L_1\ L_2.\ [\![\ err\text{-}semilat\ L_1;\ err\text{-}semilat\ L_2\ ]\!] \Longrightarrow err\text{-}semilat(Product.esl\ L_1\ L_2)\langle proof\rangle$
end

4.5 Fixed Length Lists

theory *Listn*
imports *Err HOL−Library.NList*
begin

definition *le* :: $'a\ ord \Rightarrow ('a\ list)ord$
where
$le\ r = list\text{-}all2\ (\lambda x\ y.\ x \sqsubseteq_r y)$

abbreviation
lesublist :: $'a\ list \Rightarrow 'a\ ord \Rightarrow 'a\ list \Rightarrow bool$ ((- /[$\sqsubseteq$-] -) [50, 0, 51] 50) **where**
$x\ [\sqsubseteq_r]\ y == x <=-(Listn.le\ r)\ y$

abbreviation
lesssublist :: $'a\ list \Rightarrow 'a\ ord \Rightarrow 'a\ list \Rightarrow bool$ ((- /[$\sqsubset$-] -) [50, 0, 51] 50) **where**
$x\ [\sqsubset_r]\ y == x <-(Listn.le\ r)\ y$

abbreviation
plussublist :: *'a list* ⇒ (*'a* ⇒ *'b* ⇒ *'c*) ⇒ *'b list* ⇒ *'c list*
((- /[⊔-] -) [*65*, *0*, *66*] *65*) **where**
x [$\sqcup_f$] y == $x \sqcup_{map2\ f}\ y$

primrec *coalesce* :: *'a err list* ⇒ *'a list err*
where
coalesce [] = *OK*[]
| *coalesce* (*ex#exs*) = *Err.sup* (#) *ex* (*coalesce exs*)

definition *sl* :: *nat* ⇒ *'a sl* ⇒ *'a list sl*
where
sl n = (λ(*A*,*r*,*f*). (*nlists n A*, *le r*, *map2 f*))

definition *sup* :: (*'a* ⇒ *'b* ⇒ *'c err*) ⇒ *'a list* ⇒ *'b list* ⇒ *'c list err*
where
sup f = (λ*xs ys*. *if size xs* = *size ys then coalesce*(*xs* [$\sqcup_f$] *ys*) *else Err*)

definition *upto-esl* :: *nat* ⇒ *'a esl* ⇒ *'a list esl*
where
upto-esl m = (λ(*A*,*r*,*f*). (*Union*{*nlists n A* |*n*. $n \le m$}, *le r*, *sup f*))

lemmas [*simp*] = *set-update-subsetI*

lemma *unfold-lesub-list*: *xs* [$\sqsubseteq_r$] *ys* = *Listn.le r xs ys*⟨*proof*⟩
lemma *Nil-le-conv* [*iff*]: ([] [$\sqsubseteq_r$] *ys*) = (*ys* = [])⟨*proof*⟩
lemma *Cons-notle-Nil* [*iff*]: ¬ *x#xs* [$\sqsubseteq_r$] []⟨*proof*⟩
lemma *Cons-le-Cons* [*iff*]: *x#xs* [$\sqsubseteq_r$] *y#ys* = ($x \sqsubseteq_r y$ ∧ *xs* [$\sqsubseteq_r$] *ys*)⟨*proof*⟩
lemma *list-update-le-cong*:
⟦ *i*<*size xs*; *xs* [$\sqsubseteq_r$] *ys*; $x \sqsubseteq_r y$ ⟧ ⟹ *xs*[*i*:=*x*] [$\sqsubseteq_r$] *ys*[*i*:=*y*]⟨*proof*⟩

lemma *le-listD*: ⟦ *xs* [$\sqsubseteq_r$] *ys*; *p* < *size xs* ⟧ ⟹ *xs*!*p* $\sqsubseteq_r$ *ys*!*p*⟨*proof*⟩
lemma *le-list-refl*: ∀ *x*. $x \sqsubseteq_r x$ ⟹ *xs* [$\sqsubseteq_r$] *xs*⟨*proof*⟩

lemma *le-list-trans*:
assumes *ord*: *order r A*
and *xs*: *xs* ∈ *nlists n A* **and** *ys*: *ys* ∈ *nlists n A* **and** *zs*: *zs* ∈ *nlists n A*
and *xs* [$\sqsubseteq_r$] *ys* **and** *ys* [$\sqsubseteq_r$] *zs*
shows *xs* [$\sqsubseteq_r$] *zs*⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*

4.6 Typing and Dataflow Analysis Framework

theory *Typing-Framework-1* **imports** *Semilattices* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

type-synonym
's step-type = *nat* ⇒ *'s* ⇒ (*nat* × *'s*) *list*

definition *stable* :: *'s ord* ⇒ *'s step-type* ⇒ *'s list* ⇒ *nat* ⇒ *bool*
where
stable r step τs p ⟷ (∀ (*q*,τ) ∈ *set* (*step p* (*τs*!*p*)). $\tau \sqsubseteq_r$ *τs*!*q*)

definition *stables* :: *'s ord* ⇒ *'s step-type* ⇒ *'s list* ⇒ *bool*
where
stables r step τs ⟷ (∀ *p* < *size τs*. *stable r step τs p*)

definition *wt-step* :: *'s ord* ⇒ *'s* ⇒ *'s step-type* ⇒ *'s list* ⇒ *bool*
where
wt-step r T step τs ⟷ (∀ *p*<*size τs*. *τs*!*p* ≠ *T* ∧ *stable r step τs p*)

end

4.7 More on Semilattices

theory *SemilatAlg*
imports *Typing-Framework-1*
begin

definition *lesubstep-type* :: (*nat* × *'s*) *set* ⇒ *'s ord* ⇒ (*nat* × *'s*) *set* ⇒ *bool*
((- /{⊑*r*-} -) [*50*, *0*, *51*] *50*)
where *A* {⊑*r*} *B* ≡ ∀ (*p*,*τ*) ∈ *A*. ∃ *τ'*. (*p*,*τ'*) ∈ *B* ∧ *τ* ⊑*r* *τ'*

notation (*ASCII*)
lesubstep-type ((- /{<=*r*--} -) [*50*, *0*, *51*] *50*)

primrec *pluslussub* :: *'a list* ⇒ (*'a* ⇒ *'a* ⇒ *'a*) ⇒ *'a* ⇒ *'a* ((- /⨆*r*- -) [*65*, *0*, *66*] *65*)
where
pluslussub [] *f y* = *y*
| *pluslussub* (*x*#*xs*) *f y* = *pluslussub xs f* (*x* ⊔*f* *y*)
definition *bounded* :: *'s step-type* ⇒ *nat* ⇒ *bool*
where
bounded step n ⟷ (∀ *p*<*n*. ∀ *τ*. ∀ (*q*,*τ'*) ∈ *set* (*step p τ*). *q*<*n*)

definition *pres-type* :: *'s step-type* ⇒ *nat* ⇒ *'s set* ⇒ *bool*
where
pres-type step n A ⟷ (∀ *τ*∈*A*. ∀ *p*<*n*. ∀ (*q*,*τ'*) ∈ *set* (*step p τ*). *τ'* ∈ *A*)

definition *mono* :: *'s ord* ⇒ *'s step-type* ⇒ *nat* ⇒ *'s set* ⇒ *bool*
where
mono r step n A ⟷
(∀ *τ p τ'*. *τ* ∈ *A* ∧ *p*<*n* ∧ *τ* ⊑*r* *τ'* ⟶ *set* (*step p τ*) {⊑*r*} *set* (*step p τ'*))

lemma [*iff*]: {} {⊑*r*} *B*
⟨*proof*⟩
lemma [*iff*]: (*A* {⊑*r*} {}) = (*A* = {})
⟨*proof*⟩
lemma *lesubstep-union*:
⟦ *A*1 {⊑*r*} *B*1; *A*2 {⊑*r*} *B*2 ⟧ ⟹ *A*1 ∪ *A*2 {⊑*r*} *B*1 ∪ *B*2
⟨*proof*⟩
lemma *pres-typeD*:
⟦ *pres-type step n A*; *s*∈*A*; *p*<*n*; (*q*,*s'*)∈*set* (*step p s*) ⟧ ⟹ *s'* ∈ *A*⟨*proof*⟩
lemma *monoD*:
⟦ *mono r step n A*; *p* < *n*; *s*∈*A*; *s* ⊑*r* *t* ⟧ ⟹ *set* (*step p s*) {⊑*r*} *set* (*step p t*)⟨*proof*⟩
lemma *boundedD*:

⟦ *bounded step n*; *p* < *n*; (*q*,*t*) ∈ *set* (*step p xs*) ⟧ ⟹ *q* < *n*⟨*proof*⟩
lemma *lesubstep-type-refl* [*simp*, *intro*]:
(⋀*x*. *x* ⊑$_r$ *x*) ⟹ *A* {⊑$_r$} *A*⟨*proof*⟩
lemma *lesub-step-typeD*:
A {⊑$_r$} *B* ⟹ (*x*,*y*) ∈ *A* ⟹ ∃ *y'*. (*x*, *y'*) ∈ *B* ∧ *y* ⊑$_r$ *y'*⟨*proof*⟩

lemma *list-update-le-listI* [*rule-format*]:
set xs ⊆ *A* ⟶ *set ys* ⊆ *A* ⟶ *xs* [⊑$_r$] *ys* ⟶ *p* < *size xs* ⟶
x ⊑$_r$ *ys*!*p* ⟶ *semilat*(*A*,*r*,*f*) ⟶ *x*∈*A* ⟶
xs[*p* := *x* ⊔$_f$ *xs*!*p*] [⊑$_r$] *ys*⟨*proof*⟩
lemma *plusplus-closed*: **assumes** *Semilat A r f* **shows**
⋀*y*. ⟦ *set x* ⊆ *A*; *y* ∈ *A*⟧ ⟹ *x* ⨆$_f$ *y* ∈ *A*⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub2*:
⋀*y*. ⟦ *set x* ⊆ *A*; *y* ∈ *A*⟧ ⟹ *y* ⊑$_r$ *x* ⨆$_f$ *y*⟨*proof*⟩

lemma (**in** *Semilat*) *pp-ub1*:
shows ⋀*y*. ⟦*set ls* ⊆ *A*; *y* ∈ *A*; *x* ∈ *set ls*⟧ ⟹ *x* ⊑$_r$ *ls* ⨆$_f$ *y*⟨*proof*⟩

lemma (**in** *Semilat*) *pp-lub*:
assumes *z*: *z* ∈ *A*
shows
⋀*y*. *y* ∈ *A* ⟹ *set xs* ⊆ *A* ⟹ ∀ *x* ∈ *set xs*. *x* ⊑$_r$ *z* ⟹ *y* ⊑$_r$ *z* ⟹ *xs* ⨆$_f$ *y* ⊑$_r$ *z*⟨*proof*⟩

lemma *ub1'*: **assumes** *Semilat A r f*
shows ⟦∀ (*p*,*s*) ∈ *set S*. *s* ∈ *A*; *y* ∈ *A*; (*a*,*b*) ∈ *set S*⟧
⟹ *b* ⊑$_r$ *map snd* [(*p'*, *t'*) ← *S*. *p'* = *a*] ⨆$_f$ *y*⟨*proof*⟩

lemma *plusplus-empty*:
∀ *s'*. (*q*, *s'*) ∈ *set S* ⟶ *s'* ⊔$_f$ *ss* ! *q* = *ss* ! *q* ⟹
(*map snd* [(*p'*, *t'*) ← *S*. *p'* = *q*] ⨆$_f$ *ss* ! *q*) = *ss* ! *q*⟨*proof*⟩

end

4.8 Lifting the Typing Framework to err, app, and eff

theory *Typing-Framework-err* **imports** *SemilatAlg* **begin**

definition *wt-err-step* :: *'s ord* ⇒ *'s err step-type* ⇒ *'s err list* ⇒ *bool*
where
wt-err-step r step τs ⟷ *wt-step* (*Err.le r*) *Err step τs*

definition *wt-app-eff* :: *'s ord* ⇒ (*nat* ⇒ *'s* ⇒ *bool*) ⇒ *'s step-type* ⇒ *'s list* ⇒ *bool*
where
wt-app-eff r app step τs ⟷
(∀ *p* < *size τs*. *app p* (*τs*!*p*) ∧ (∀ (*q*,*τ*) ∈ *set* (*step p* (*τs*!*p*)). *τ* <=*-r* *τs*!*q*))

definition *map-snd* :: (*'b* ⇒ *'c*) ⇒ (*'a* × *'b*) *list* ⇒ (*'a* × *'c*) *list*
where
map-snd f = *map* (λ(*x*,*y*). (*x*, *f y*))

definition *error* :: *nat* ⇒ (*nat* × *'a err*) *list*
where

error n = map (λx. (x,Err)) [0..<n]

definition *err-step :: nat ⇒ (nat ⇒ 's ⇒ bool) ⇒ 's step-type ⇒ 's err step-type*
where
err-step n app step p t =
(case t of
Err ⇒ error n
| OK τ ⇒ if app p τ then map-snd OK (step p τ) else error n)

definition *app-mono :: 's ord ⇒ (nat ⇒ 's ⇒ bool) ⇒ nat ⇒ 's set ⇒ bool*
where
app-mono r app n A ⟷
(∀ s p t. s ∈ A ∧ p < n ∧ s ⊑_r t ⟶ app p t ⟶ app p s)

lemmas *err-step-defs = err-step-def map-snd-def error-def*

lemma *bounded-err-stepD*:
⟦ bounded (err-step n app step) n;
p < n; app p a; (q,b) ∈ set (step p a) ⟧ ⟹ q < n⟨proof⟩

lemma *in-map-sndD*: *(a,b) ∈ set (map-snd f xs) ⟹ ∃ b'. (a,b') ∈ set xs⟨proof⟩*

lemma *bounded-err-stepI*:
∀ p. p < n ⟶ (∀ s. ap p s ⟶ (∀ (q,s') ∈ set (step p s). q < n))
⟹ bounded (err-step n ap step) n⟨proof⟩

lemma *bounded-lift*:
bounded step n ⟹ bounded (err-step n app step) n⟨proof⟩

lemma *le-list-map-OK* [*simp*]:
⋀b. (map OK a [⊑_Err.le r] map OK b) = (a [⊑_r] b)⟨proof⟩

lemma *map-snd-lessI*:
set xs {⊑_r} set ys ⟹ set (map-snd OK xs) {⊑_Err.le r} set (map-snd OK ys)⟨proof⟩

lemma *mono-lift*:
⟦ order r A; app-mono r app n A; bounded (err-step n app step) n;
∀ s p t. s ∈ A ∧ p < n ∧ s ⊑_r t ⟶ app p t ⟶ set (step p s) {⊑_r} set (step p t) ⟧
⟹ mono (Err.le r) (err-step n app step) n (err A)⟨proof⟩
lemma *in-errorD*: *(x,y) ∈ set (error n) ⟹ y = Err⟨proof⟩*
lemma *pres-type-lift*:
∀ s∈A. ∀ p. p < n ⟶ app p s ⟶ (∀ (q, s')∈set (step p s). s' ∈ A)
⟹ pres-type (err-step n app step) n (err A)⟨proof⟩

lemma *wt-err-imp-wt-app-eff*:
assumes *wt*: *wt-err-step r (err-step (size ts) app step) ts*
assumes *b*: *bounded (err-step (size ts) app step) (size ts)*
shows *wt-app-eff r app step (map ok-val ts)⟨proof⟩*

lemma *wt-app-eff-imp-wt-err*:
assumes *app-eff*: *wt-app-eff r app step ts*
assumes *bounded*: *bounded (err-step (size ts) app step) (size ts)*

shows *wt-err-step r* (*err-step* (*size ts*) *app step*) (*map OK ts*)⟨*proof*⟩
end

4.9 Kildall's Algorithm

theory *Kildall-1*
imports *SemilatAlg*
begin

primrec *merges* :: *'s binop* ⇒ (*nat* × *'s*) *list* ⇒ *'s list* ⇒ *'s list*
where
merges f [] *τs* = *τs*
| *merges f* (*p'*#*ps*) *τs* = (*let* (*p*,*τ*) = *p' in merges f ps* (*τs*[*p* := *τ* ⊔$_f$ *τs*!*p*]))

lemmas [*simp*] = *Let-def Semilat.le-iff-plus-unchanged* [*OF Semilat.intro*, *symmetric*]

lemma (**in** *Semilat*) *nth-merges*:
⋀*ss*. ⟦*p* < *length ss*; *ss* ∈ *nlists n A*; ∀(*p*,*t*)∈*set ps*. *p*<*n* ∧ *t*∈*A* ⟧ ⟹
(*merges f ps ss*)!*p* = *map snd* [(*p'*,*t'*) ← *ps*. *p'*=*p*] ⨆$_f$ *ss*!*p*
(**is** ⋀*ss*. ⟦-; -; *?steptype ps*⟧ ⟹ *?P ss ps*)⟨*proof*⟩

lemma *length-merges* [*simp*]:
⋀*ss*. *size*(*merges f ps ss*) = *size ss*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-preserves-type-lemma*:
shows ∀*xs*. *xs* ∈ *nlists n A* ⟶ (∀(*p*,*x*) ∈ *set ps*. *p*<*n* ∧ *x*∈*A*)
⟶ *merges f ps xs* ∈ *nlists n A*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-preserves-type* [*simp*]:
⟦ *xs* ∈ *nlists n A*; ∀(*p*,*x*) ∈ *set ps*. *p*<*n* ∧ *x*∈*A* ⟧
⟹ *merges f ps xs* ∈ *nlists n A*
⟨*proof*⟩

lemma (**in** *Semilat*) *list-update-le-listI* [*rule-format*]:
set xs ⊆ *A* ⟶ *set ys* ⊆ *A* ⟶ *xs* [⊑$_r$] *ys* ⟶ *p* < *size xs* ⟶
x ⊑$_r$ *ys*!*p* ⟶ *x*∈*A* ⟶ *xs*[*p* := *x* ⊔$_f$ *xs*!*p*] [⊑$_r$] *ys*⟨*proof*⟩
lemma (**in** *Semilat*) *merges-pres-le-ub*:
assumes *set ts* ⊆ *A* *set ss* ⊆ *A*
∀(*p*,*t*)∈*set ps*. *t* ⊑$_r$ *ts*!*p* ∧ *t* ∈ *A* ∧ *p* < *size ts* *ss* [⊑$_r$] *ts*
shows *merges f ps ss* [⊑$_r$] *ts*⟨*proof*⟩

end

4.10 Kildall's Algorithm

theory *Kildall-2*
imports *SemilatAlg Kildall-1*
begin

primrec *propa* :: *'s binop* ⇒ (*nat* × *'s*) *list* ⇒ *'s list* ⇒ *nat set* ⇒ *'s list* ∗ *nat set*
where
  propa f [] *τs w* = (*τs*,*w*)
| *propa f* (*q'*#*qs*) *τs w* = (*let* (*q*,*τ*) = *q'*;
  u = *τ* ⊔*f* *τs*!*q*;
  w' = (*if u* = *τs*!*q then w else insert q w*)
  in propa f qs (*τs*[*q* := *u*]) *w'*)

definition *iter* :: *'s binop* ⇒ *'s step-type* ⇒
  's list ⇒ *nat set* ⇒ *'s list* × *nat set*
where
  iter f step τs w =
  while (λ(*τs*,*w*). *w* ≠ {})
  (λ(*τs*,*w*). *let p* = *SOME p*. *p* ∈ *w*
  in propa f (*step p* (*τs*!*p*)) *τs* (*w*−{*p*}))
  (*τs*,*w*)

definition *unstables* :: *'s ord* ⇒ *'s step-type* ⇒ *'s list* ⇒ *nat set*
where
  unstables r step τs = {*p*. *p* < *size τs* ∧ ¬*stable r step τs p*}

definition *kildall* :: *'s ord* ⇒ *'s binop* ⇒ *'s step-type* ⇒ *'s list* ⇒ *'s list*
where
  kildall r f step τs = *fst*(*iter f step τs* (*unstables r step τs*))

lemma (**in** *Semilat*) *merges-incr-lemma*:
 ∀ *xs*. *xs* ∈ *nlists n A* ⟶ (∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x* ∈ *A*) ⟶ *xs* [⊑*r*] *merges f ps xs*
  ⟨*proof*⟩

lemma (**in** *Semilat*) *merges-incr*:
 ⟦ *xs* ∈ *nlists n A*; ∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x* ∈ *A* ⟧
  ⟹ *xs* [⊑*r*] *merges f ps xs*
  ⟨*proof*⟩

lemma (**in** *Semilat*) *merges-same-conv* [*rule-format*]:
 (∀ *xs*. *xs* ∈ *nlists n A* ⟶ (∀ (*p*,*x*)∈*set ps*. *p*<*size xs* ∧ *x*∈*A*) ⟶
  (*merges f ps xs* = *xs*) = (∀ (*p*,*x*)∈*set ps*. *x* ⊑*r* *xs*!*p*))⟨*proof*⟩
lemma *decomp-propa*:
  ⋀*ss w*. (∀ (*q*,*t*)∈*set qs*. *q* < *size ss*) ⟹
  propa f qs ss w =
  (*merges f qs ss*, {*q*. ∃ *t*.(*q*,*t*)∈*set qs* ∧ *t* ⊔*f* *ss*!*q* ≠ *ss*!*q*} ∪ *w*)⟨*proof*⟩
lemma (**in** *Semilat*) *stable-pres-lemma*:
shows ⟦*pres-type step n A*; *bounded step n*;
  ss ∈ *nlists n A*; *p* ∈ *w*; ∀ *q*∈*w*. *q* < *n*;
  ∀ *q*. *q* < *n* ⟶ *q* ∉ *w* ⟶ *stable r step ss q*; *q* < *n*;
  ∀ *s'*. (*q*,*s'*) ∈ *set* (*step p* (*ss*!*p*)) ⟶ *s'* ⊔*f* *ss*!*q* = *ss*!*q*;
  q ∉ *w* ∨ *q* = *p* ⟧

$\Longrightarrow$ *stable r step (merges f (step p (ss!p)) ss) q⟨proof⟩*

lemma (**in** *Semilat*) *merges-bounded-lemma*:
⟦ *mono r step n A*; *bounded step n*; *pres-type step n A*;
∀(*p'*,*s'*) ∈ *set* (*step p* (*ss*!*p*)). *s'* ∈ *A*; *ss* ∈ *nlists n A*; *ts* ∈ *nlists n A*; *p* < *n*;
ss [⊑$_r$] *ts*; ∀ *p*. *p* < *n* ⟶ *stable r step ts p* ⟧
$\Longrightarrow$ *merges f* (*step p* (*ss*!*p*)) *ss* [⊑$_r$] *ts⟨proof⟩*

lemma *termination-lemma*: **assumes** *Semilat A r f*
shows ⟦ *ss* ∈ *nlists n A*; ∀(*q*,*t*)∈*set qs*. *q*<*n* ∧ *t*∈*A*; *p*∈*w* ⟧ $\Longrightarrow$
ss [⊑$_r$] *merges f qs ss* ∨
merges f qs ss = *ss* ∧ {*q*. ∃ *t*. (*q*,*t*)∈*set qs* ∧ *t* ⊔$_f$ *ss*!*q* ≠ *ss*!*q*} ∪ (*w*−{*p*}) ⊂ *w⟨proof⟩*
end

4.11 The Lightweight Bytecode Verifier

theory *LBVSpec*
imports *SemilatAlg Opt*
begin

type-synonym
's certificate = *'s list*

primrec *merge* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *nat* ⇒ (*nat* × *'s*) *list* ⇒ *'s* ⇒ *'s*
where
merge cert f r T pc [] *x* = *x*
| *merge cert f r T pc* (*s*#*ss*) *x* = *merge cert f r T pc ss* (*let* (*pc'*,*s'*) = *s in*
if pc'=*pc*+*1 then s'* ⊔$_f$ *x*
else if s' ⊑$_r$ *cert*!*pc' then x*
else T)

definition *wtl-inst* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-inst cert f r T step pc s = *merge cert f r T pc* (*step pc s*) (*cert*!(*pc*+*1*))

definition *wtl-cert* :: *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-cert cert f r T B step pc s =
(*if cert*!*pc* = *B then*
wtl-inst cert f r T step pc s
else
if s ⊑$_r$ *cert*!*pc then wtl-inst cert f r T step pc* (*cert*!*pc*) *else T*)

primrec *wtl-inst-list* :: *'a list* ⇒ *'s certificate* ⇒ *'s binop* ⇒ *'s ord* ⇒ *'s* ⇒ *'s* ⇒
's step-type ⇒ *nat* ⇒ *'s* ⇒ *'s*
where
wtl-inst-list [] *cert f r T B step pc s* = *s*
| *wtl-inst-list* (*i*#*is*) *cert f r T B step pc s* =
(*let s'* = *wtl-cert cert f r T B step pc s in*

if s′ = T ∨ s = T then T else wtl-inst-list is cert f r T B step (pc+1) s′)

definition *cert-ok* :: *′s certificate ⇒ nat ⇒ ′s ⇒ ′s ⇒ ′s set ⇒ bool*
where
cert-ok cert n T B A ⟷ (∀ i < n. cert!i ∈ A ∧ cert!i ≠ T) ∧ (cert!n = B)

definition *bottom* :: *′a ord ⇒ ′a ⇒ bool*
where
bottom r B ⟷ (∀ x. B ⊑r x)

locale *lbv* = *Semilat* +
fixes *T* :: *′a* (⊤)
fixes *B* :: *′a* (⊥)
fixes *step* :: *′a step-type*
assumes *top*: *top r* ⊤
assumes *T-A*: ⊤ ∈ *A*
assumes *bot*: *bottom r* ⊥
assumes *B-A*: ⊥ ∈ *A*

fixes *merge* :: *′a certificate ⇒ nat ⇒ (nat × ′a) list ⇒ ′a ⇒ ′a*
defines *mrg-def*: *merge cert ≡ LBVSpec.merge cert f r* ⊤

fixes *wti* :: *′a certificate ⇒ nat ⇒ ′a ⇒ ′a*
defines *wti-def*: *wti cert ≡ wtl-inst cert f r* ⊤ *step*

fixes *wtc* :: *′a certificate ⇒ nat ⇒ ′a ⇒ ′a*
defines *wtc-def*: *wtc cert ≡ wtl-cert cert f r* ⊤ ⊥ *step*

fixes *wtl* :: *′b list ⇒ ′a certificate ⇒ nat ⇒ ′a ⇒ ′a*
defines *wtl-def*: *wtl ins cert ≡ wtl-inst-list ins cert f r* ⊤ ⊥ *step*

lemma (**in** *lbv*) *wti*:
wti c pc s = merge c pc (step pc s) (c!(pc+1))
⟨*proof*⟩
lemma (**in** *lbv*) *wtc*:
wtc c pc s = (if c!pc = ⊥ then wti c pc s else if s ⊑r c!pc then wti c pc (c!pc) else ⊤)
⟨*proof*⟩
lemma *cert-okD1* [*intro?*]:
cert-ok c n T B A ⟹ pc < n ⟹ c!pc ∈ A
⟨*proof*⟩
lemma *cert-okD2* [*intro?*]:
cert-ok c n T B A ⟹ c!n = B
⟨*proof*⟩
lemma *cert-okD3* [*intro?*]:
cert-ok c n T B A ⟹ B ∈ A ⟹ pc < n ⟹ c!Suc pc ∈ A
⟨*proof*⟩
lemma *cert-okD4* [*intro?*]:
cert-ok c n T B A ⟹ pc < n ⟹ c!pc ≠ T
⟨*proof*⟩
declare *Let-def* [*simp*]

4.11.1 more semilattice lemmas

lemma (**in** *lbv*) *sup-top* [*simp*, *elim*]:
assumes *x*: $x \in A$
shows $x \sqcup_f \top = \top \langle proof \rangle$
lemma (**in** *lbv*) *plusplussup-top* [*simp*, *elim*]:
$set\ xs \subseteq A \Longrightarrow xs \bigsqcup_f \top = \top$
$\langle proof \rangle$

lemma (**in** *Semilat*) *pp-ub1'*:
assumes *S*: $snd`set\ S \subseteq A$
assumes *y*: $y \in A$ **and** *ab*: $(a, b) \in set\ S$
shows $b \sqsubseteq_r map\ snd\ [(p', t') \leftarrow S\ .\ p' = a] \bigsqcup_f y \langle proof \rangle$
lemma (**in** *lbv*) *bottom-le* [*simp*, *intro*!]: $\bot \sqsubseteq_r x$
$\langle proof \rangle$

lemma (**in** *lbv*) *le-bottom* [*simp*]: $x \in A \Longrightarrow x \sqsubseteq_r \bot = (x = \bot)$
$\langle proof \rangle$

4.11.2 merge

lemma (**in** *lbv*) *merge-Nil* [*simp*]:
$merge\ c\ pc\ []\ x = x\ \langle proof \rangle$

lemma (**in** *lbv*) *merge-Cons* [*simp*]:
$merge\ c\ pc\ (l\#ls)\ x = merge\ c\ pc\ ls\ (if\ fst\ l{=}pc{+}1\ then\ snd\ l\ +{-}f\ x$
$else\ if\ snd\ l \sqsubseteq_r c!fst\ l\ then\ x$
$else\ \top)$
$\langle proof \rangle$

lemma (**in** *lbv*) *merge-Err* [*simp*]:
$snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ \top = \top$
$\langle proof \rangle$

lemma (**in** *lbv*) *merge-not-top*:
$\bigwedge x.\ snd`set\ ss \subseteq A \Longrightarrow merge\ c\ pc\ ss\ x \neq \top \Longrightarrow$
$\forall (pc',s') \in set\ ss.\ (pc' \neq pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc')$
(**is** $\bigwedge x.\ ?set\ ss \Longrightarrow ?merge\ ss\ x \Longrightarrow ?P\ ss) \langle proof \rangle$

lemma (**in** *lbv*) *merge-def*:
shows
$\bigwedge x.\ x \in A \Longrightarrow snd`set\ ss \subseteq A \Longrightarrow$
$merge\ c\ pc\ ss\ x =$
$(if\ \forall (pc',s') \in set\ ss.\ pc'{\neq}pc{+}1 \longrightarrow s' \sqsubseteq_r c!pc'\ then$
$map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1] \bigsqcup_f x$
$else\ \top)$
(**is** $\bigwedge x.\ - \Longrightarrow - \Longrightarrow ?merge\ ss\ x = ?if\ ss\ x$ **is** $\bigwedge x.\ - \Longrightarrow - \Longrightarrow ?P\ ss\ x) \langle proof \rangle$
lemma (**in** *lbv*) *merge-not-top-s*:
assumes *x*: $x \in A$ **and** *ss*: $snd`set\ ss \subseteq A$
assumes *m*: $merge\ c\ pc\ ss\ x \neq \top$
shows $merge\ c\ pc\ ss\ x = (map\ snd\ [(p',t') \leftarrow ss.\ p'{=}pc{+}1] \bigsqcup_f x) \langle proof \rangle$

4.11.3 wtl-inst-list

lemmas [*iff*] = *not-Err-eq*

lemma (**in** *lbv*) *wtl-Nil* [*simp*]: *wtl* [] *c pc s* = *s*
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-Cons* [*simp*]:
wtl (*i*#*is*) *c pc s* =
(*let s′* = *wtc c pc s in if s′* = ⊤ ∨ *s* = ⊤ *then* ⊤ *else wtl is c* (*pc*+1) *s′*)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-Cons-not-top*:
wtl (*i*#*is*) *c pc s* ≠ ⊤ =
(*wtc c pc s* ≠ ⊤ ∧ *s* ≠ *T* ∧ *wtl is c* (*pc*+1) (*wtc c pc s*) ≠ ⊤)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-top* [*simp*]: *wtl ls c pc* ⊤ = ⊤
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-not-top*:
wtl ls c pc s ≠ ⊤ ⟹ *s* ≠ ⊤
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-append* [*simp*]:
⋀*pc s*. *wtl* (*a*@*b*) *c pc s* = *wtl b c* (*pc*+*length a*) (*wtl a c pc s*)
⟨*proof*⟩

lemma (**in** *lbv*) *wtl-take*:
wtl is c pc s ≠ ⊤ ⟹ *wtl* (*take pc′ is*) *c pc s* ≠ ⊤
(**is** *?wtl is* ≠ - ⟹ -)⟨*proof*⟩
lemma *take-Suc*:
∀ *n*. *n* < *length l* ⟶ *take* (*Suc n*) *l* = (*take n l*)@[*l*!*n*] (**is** *?P l*)⟨*proof*⟩
lemma (**in** *lbv*) *wtl-Suc*:
assumes *suc*: *pc*+1 < *length is*
assumes *wtl*: *wtl* (*take pc is*) *c* 0 *s* ≠ ⊤
shows *wtl* (*take* (*pc*+1) *is*) *c* 0 *s* = *wtc c pc* (*wtl* (*take pc is*) *c* 0 *s*)⟨*proof*⟩
lemma (**in** *lbv*) *wtl-all*:
assumes *all*: *wtl is c* 0 *s* ≠ ⊤ (**is** *?wtl is* ≠ -)
assumes *pc*: *pc* < *length is*
shows *wtc c pc* (*wtl* (*take pc is*) *c* 0 *s*) ≠ ⊤⟨*proof*⟩

4.11.4 preserves-type

lemma (**in** *lbv*) *merge-pres*:
assumes *s0*: *snd*‘*set ss* ⊆ *A* **and** *x*: *x* ∈ *A*
shows *merge c pc ss x* ∈ *A*⟨*proof*⟩
lemma *pres-typeD2*:
pres-type step n A ⟹ *s* ∈ *A* ⟹ *p* < *n* ⟹ *snd*‘*set* (*step p s*) ⊆ *A*
⟨*proof*⟩

lemma (**in** *lbv*) *wti-pres* [*intro?*]:
assumes *pres*: *pres-type step n A*
assumes *cert*: *c*!(*pc*+1) ∈ *A*
assumes *s-pc*: *s* ∈ *A pc* < *n*

shows *wti c pc s* ∈ *A*⟨*proof*⟩
lemma (**in** *lbv*) *wtc-pres*:
assumes *pres-type step n A*
assumes *c*!*pc* ∈ *A* **and** *c*!(*pc*+*1*) ∈ *A*
assumes *s* ∈ *A* **and** *pc* < *n*
shows *wtc c pc s* ∈ *A*⟨*proof*⟩
lemma (**in** *lbv*) *wtl-pres*:
assumes *pres*: *pres-type step* (*length is*) *A*
assumes *cert*: *cert-ok c* (*length is*) ⊤ ⊥ *A*
assumes *s*: *s* ∈ *A*
assumes *all*: *wtl is c 0 s* ≠ ⊤
shows *pc* < *length is* ⟹ *wtl* (*take pc is*) *c 0 s* ∈ *A*
(**is** *?len pc* ⟹ *?wtl pc* ∈ *A*)⟨*proof*⟩
end

4.12 Correctness of the LBV

theory *LBVCorrect*
imports *LBVSpec Typing-Framework-1*
begin

locale *lbvs* = *lbv* +
fixes s_0 :: *'a*
fixes *c* :: *'a list*
fixes *ins* :: *'b list*
fixes *τs* :: *'a list*
defines *phi-def*:
τs ≡ *map* (λ*pc*. *if c*!*pc* = ⊥ *then wtl* (*take pc ins*) *c 0* s_0 *else c*!*pc*)
[*0*..<*size ins*]

assumes *bounded*: *bounded step* (*size ins*)
assumes *cert*: *cert-ok c* (*size ins*) ⊤ ⊥ *A*
assumes *pres*: *pres-type step* (*size ins*) *A*

lemma (**in** *lbvs*) *phi-None* [*intro?*]:
⟦ *pc* < *size ins*; *c*!*pc* = ⊥ ⟧ ⟹ *τs*!*pc* = *wtl* (*take pc ins*) *c 0* s_0⟨*proof*⟩
lemma (**in** *lbvs*) *phi-Some* [*intro?*]:
⟦ *pc* < *size ins*; *c*!*pc* ≠ ⊥ ⟧ ⟹ *τs*!*pc* = *c*!*pc*⟨*proof*⟩
lemma (**in** *lbvs*) *phi-len* [*simp*]: *size τs* = *size ins*⟨*proof*⟩
lemma (**in** *lbvs*) *wtl-suc-pc*:
assumes *all*: *wtl ins c 0* s_0 ≠ ⊤
assumes *pc*: *pc*+*1* < *size ins*
assumes *sA*: s_0 ∈ *A*
shows *wtl* (*take* (*pc*+*1*) *ins*) *c 0* s_0 ⊑$_r$ *τs*!(*pc*+*1*)⟨*proof*⟩
lemma (**in** *lbvs*) *wtl-stable*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤
assumes s_0: s_0 ∈ *A* **and** *pc*: *pc* < *size ins*
shows *stable r step τs pc*⟨*proof*⟩
lemma (**in** *lbvs*) *phi-not-top*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤ **and** *pc*: *pc* < *size ins*
shows *τs*!*pc* ≠ ⊤⟨*proof*⟩
lemma (**in** *lbvs*) *phi-in-A*:
assumes *wtl*: *wtl ins c 0* s_0 ≠ ⊤ **and** s_0: s_0 ∈ *A*

shows $\tau s \in nlists\ (size\ ins)\ A \langle proof \rangle$
lemma (**in** *lbvs*) *phi0*:
assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** *0*: $0 < size\ ins$ **and** s_0: $s_0 \in A$
shows $s_0 \sqsubseteq_r \tau s!0 \langle proof \rangle$

theorem (**in** *lbvs*) *wtl-sound*:
assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$ **and** s_0: $s_0 \in A$
shows $\exists \tau s.\ wt\text{-}step\ r\ \top\ step\ \tau s \langle proof \rangle$

theorem (**in** *lbvs*) *wtl-sound-strong*:
assumes *wtl*: $wtl\ ins\ c\ 0\ s_0 \neq \top$
assumes s_0: $s_0 \in A$ **and** *ins*: $0 < size\ ins$
shows $\exists \tau s \in nlists\ (size\ ins)\ A.\ wt\text{-}step\ r\ \top\ step\ \tau s \wedge s_0 \sqsubseteq_r \tau s!0 \langle proof \rangle$
end

4.13 Completeness of the LBV

theory *LBVComplete*
imports *LBVSpec Typing-Framework-1*
begin

definition *is-target* :: *'s step-type* ⇒ *'s list* ⇒ *nat* ⇒ *bool* **where**
$is\text{-}target\ step\ \tau s\ pc' \longleftrightarrow (\exists pc\ s'.\ pc' \neq pc+1 \wedge pc < size\ \tau s \wedge (pc',s') \in set\ (step\ pc\ (\tau s!pc)))$

definition *make-cert* :: *'s step-type* ⇒ *'s list* ⇒ *'s* ⇒ *'s certificate* **where**
$make\text{-}cert\ step\ \tau s\ B = map\ (\lambda pc.\ if\ is\text{-}target\ step\ \tau s\ pc\ then\ \tau s!pc\ else\ B)\ [0..<size\ \tau s] @ [B]$

lemma [*code*]:
$is\text{-}target\ step\ \tau s\ pc' =$
$list\text{-}ex\ (\lambda pc.\ pc' \neq pc+1 \wedge List.member\ (map\ fst\ (step\ pc\ (\tau s!pc)))\ pc')\ [0..<size\ \tau s] \langle proof \rangle$
locale *lbvc* = *lbv* +
fixes τs :: *'a list*
fixes *c* :: *'a list*
defines *cert-def*: $c \equiv make\text{-}cert\ step\ \tau s\ \bot$

assumes *mono*: $mono\ r\ step\ (size\ \tau s)\ A$
assumes *pres*: $pres\text{-}type\ step\ (size\ \tau s)\ A$
assumes τs: $\forall pc < size\ \tau s.\ \tau s!pc \in A \wedge \tau s!pc \neq \top$
assumes *bounded*: $bounded\ step\ (size\ \tau s)$

assumes *B-neq-T*: $\bot \neq \top$

lemma (**in** *lbvc*) *cert*: $cert\text{-}ok\ c\ (size\ \tau s)\ \top\ \bot\ A \langle proof \rangle$
lemmas [*simp del*] = *split-paired-Ex*

lemma (**in** *lbvc*) *cert-target* [*intro?*]:
$[\![(pc',s') \in set\ (step\ pc\ (\tau s!pc));$
$pc' \neq pc+1;\ pc < size\ \tau s;\ pc' < size\ \tau s\ ]\!]$
$\Longrightarrow c!pc' = \tau s!pc' \langle proof \rangle$
lemma (**in** *lbvc*) *cert-approx* [*intro?*]:
$[\![pc < size\ \tau s;\ c!pc \neq \bot\ ]\!] \Longrightarrow c!pc = \tau s!pc \langle proof \rangle$
lemma (**in** *lbv*) *le-top* [*simp, intro*]: $x <=\text{-}r\ \top \langle proof \rangle$

lemma (**in** *lbv*) *merge-mono*:
assumes *less*: *set* ss_2 $\{\sqsubseteq_r\}$ *set* ss_1
assumes x: $x \in A$
assumes ss_1: *snd*'*set* $ss_1 \subseteq A$
assumes ss_2: *snd*'*set* $ss_2 \subseteq A$
assumes *boun*: $\forall x{\in}(fst`set\ ss_1).\ x < size\ \tau s$
assumes *cert*: *cert-ok c* (*size* τs) *T B A*
shows *merge c pc* ss_2 *x* $\sqsubseteq_r$ *merge c pc* ss_1 *x* (**is** *?*s_2 $\sqsubseteq_r$ *?*s_1)⟨*proof*⟩
lemma (**in** *lbvc*) *wti-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows *wti c pc* $s_2 \sqsubseteq_r$ *wti c pc* s_1 (**is** *?*$s_2' \sqsubseteq_r$ *?*s_1')⟨*proof*⟩
lemma (**in** *lbvc*) *wtc-mono*:
assumes *less*: $s_2 \sqsubseteq_r s_1$
assumes *pc*: $pc < size\ \tau s$ **and** s_1: $s_1 \in A$ **and** s_2: $s_2 \in A$
shows *wtc c pc* $s_2 \sqsubseteq_r$ *wtc c pc* s_1 (**is** *?*$s_2' \sqsubseteq_r$ *?*s_1')⟨*proof*⟩
lemma (**in** *lbv*) *top-le-conv* [*simp*]: $x \in A \Longrightarrow \top \sqsubseteq_r x = (x = \top)$⟨*proof*⟩
lemma (**in** *lbv*) *neq-top* [*simp*, *elim*]: $[\![\ x \sqsubseteq_r y;\ y \neq \top;\ y \in A\ ]\!] \Longrightarrow x \neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *stable-wti*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows *wti c pc* (τs!*pc*) $\neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wti-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: *Suc pc* $< size\ \tau s$
shows *wti c pc* (τs!*pc*) $\sqsubseteq_r$ τs!*Suc pc* (**is** *?wti* $\sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *stable-wtc*:
assumes *stable*: *stable r step* τs *pc* **and** *pc*: $pc < size\ \tau s$
shows *wtc c pc* (τs!*pc*) $\neq \top$⟨*proof*⟩
lemma (**in** *lbvc*) *wtc-less*:
assumes *stable*: *stable r step* τs *pc* **and** *suc-pc*: *Suc pc* $< size\ \tau s$
shows *wtc c pc* (τs!*pc*) $\sqsubseteq_r$ τs!*Suc pc* (**is** *?wtc* $\sqsubseteq_r$ -)⟨*proof*⟩
lemma (**in** *lbvc*) *wt-step-wtl-lemma*:
assumes *wt-step*: *wt-step r* $\top$ *step* τs
shows $\bigwedge$*pc s. pc*+*size ls* = *size* $\tau s \Longrightarrow s \sqsubseteq_r \tau s!pc \Longrightarrow s \in A \Longrightarrow s{\neq}\top \Longrightarrow$
wtl ls c pc s $\neq \top$
(**is** $\bigwedge$*pc s.* - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ - $\Longrightarrow$ *?wtl ls pc s* $\neq$ -)⟨*proof*⟩
theorem (**in** *lbvc*) *wtl-complete*:
assumes *wt*: *wt-step r* $\top$ *step* τs
assumes *s*: $s \sqsubseteq_r \tau s!0$ $s \in A$ $s \neq \top$ **and** *eq*: *size ins* = *size* τs
shows *wtl ins c 0 s* $\neq \top$⟨*proof*⟩
end

4.14 The Jinja Type System as a Semilattice

theory *SemiType*
imports *../Common/ WellForm ../DFA/Semilattices*
begin

definition *super* :: *'a prog* ⇒ *cname* ⇒ *cname*
where *super P C* ≡ *fst* (*the* (*class P C*))

lemma *superI*:
(*C*,*D*) ∈ *subcls1 P* ⟹ *super P C* = *D*
⟨*proof*⟩

primrec *the-Class* :: *ty* ⇒ *cname*
where
the-Class (*Class C*) = *C*

definition *sup* :: *'c prog* ⇒ *ty* ⇒ *ty* ⇒ *ty err*
where
sup P T$_1$ *T*$_2$ ≡
if is-refT T$_1$ ∧ *is-refT T*$_2$ *then*
OK (*if T*$_1$ = *NT then T*$_2$ *else*
if T$_2$ = *NT then T*$_1$ *else*
(*Class* (*exec-lub* (*subcls1 P*) (*super P*) (*the-Class T*$_1$) (*the-Class T*$_2$))))
else
(*if T*$_1$ = *T*$_2$ *then OK T*$_1$ *else Err*)

lemma *sup-def'*:
sup P = (λ*T*$_1$ *T*$_2$.
if is-refT T$_1$ ∧ *is-refT T*$_2$ *then*
OK (*if T*$_1$ = *NT then T*$_2$ *else*
if T$_2$ = *NT then T*$_1$ *else*
(*Class* (*exec-lub* (*subcls1 P*) (*super P*) (*the-Class T*$_1$) (*the-Class T*$_2$))))
else
(*if T*$_1$ = *T*$_2$ *then OK T*$_1$ *else Err*))
⟨*proof*⟩

abbreviation
subtype :: *'c prog* ⇒ *ty* ⇒ *ty* ⇒ *bool*
where *subtype P* ≡ *widen P*

definition *esl* :: *'c prog* ⇒ *ty esl*
where
esl P ≡ (*types P*, *subtype P*, *sup P*)

lemma *is-class-is-subcls*:
wf-prog m P ⟹ *is-class P C* = *P* ⊢ *C* ≼* *Object*⟨*proof*⟩

lemma *subcls-antisym*:
⟦*wf-prog m P*; *P* ⊢ *C* ≼* *D*; *P* ⊢ *D* ≼* *C*⟧ ⟹ *C* = *D*
⟨*proof*⟩

lemma *widen-antisym*:
⟦ *wf-prog m P*; *P* ⊢ *T* ≤ *U*; *P* ⊢ *U* ≤ *T* ⟧ ⟹ *T* = *U*⟨*proof*⟩
lemma *order-widen* [*intro*,*simp*]:
wf-prog m P ⟹ *order* (*subtype P*) (*types P*)⟨*proof*⟩

lemma *NT-widen*:
P ⊢ *NT* ≤ *T* = (*T* = *NT* ∨ (∃ *C*. *T* = *Class C*))⟨*proof*⟩

lemma *Class-widen2*: *P* ⊢ *Class C* ≤ *T* = (∃ *D*. *T* = *Class D* ∧ *P* ⊢ *C* ≼* *D*)⟨*proof*⟩
lemma *wf-converse-subcls1-impl-acc-subtype*:

wf ((*subcls1 P*)^−1) ⟹ *acc* (*subtype P*)⟨*proof*⟩
lemma *wf-subtype-acc* [*intro*, *simp*]:
wf-prog wf-mb P ⟹ *acc* (*subtype P*)⟨*proof*⟩
lemma *exec-lub-refl* [*simp*]: *exec-lub r f T T* = *T*⟨*proof*⟩
lemma *closed-err-types*:
wf-prog wf-mb P ⟹ *closed* (*err* (*types P*)) (*lift2* (*sup P*))⟨*proof*⟩

lemma *sup-subtype-greater*:
⟦ *wf-prog wf-mb P*; *is-type P t1*; *is-type P t2*; *sup P t1 t2* = *OK s* ⟧
⟹ *subtype P t1 s* ∧ *subtype P t2 s*⟨*proof*⟩
lemma *sup-subtype-smallest*:
⟦ *wf-prog wf-mb P*; *is-type P a*; *is-type P b*; *is-type P c*;
subtype P a c; *subtype P b c*; *sup P a b* = *OK d* ⟧
⟹ *subtype P d c*⟨*proof*⟩
lemma *sup-exists*:
⟦ *subtype P a c*; *subtype P b c* ⟧ ⟹ ∃ *T*. *sup P a b* = *OK T*⟨*proof*⟩
lemma *err-semilat-JType-esl*:
wf-prog wf-mb P ⟹ *err-semilat* (*esl P*)⟨*proof*⟩

end

4.15 The JVM Type System as Semilattice

theory *JVM-SemiType* **imports** *SemiType* **begin**

type-synonym ty_l = *ty err list*
type-synonym ty_s = *ty list*
type-synonym ty_i = ty_s × ty_l
type-synonym ty_i' = ty_i *option*
type-synonym ty_m = ty_i' *list*
type-synonym ty_P = *mname* ⇒ *cname* ⇒ ty_m

definition *stk-esl* :: *'c prog* ⇒ *nat* ⇒ ty_s *esl*
where
stk-esl P mxs ≡ *upto-esl mxs* (*SemiType.esl P*)

definition *loc-sl* :: *'c prog* ⇒ *nat* ⇒ ty_l *sl*
where
loc-sl P mxl ≡ *Listn.sl mxl* (*Err.sl* (*SemiType.esl P*))

definition *sl* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ ty_i' *err sl*
where
sl P mxs mxl ≡
Err.sl(*Opt.esl*(*Product.esl* (*stk-esl P mxs*) (*Err.esl*(*loc-sl P mxl*))))

definition *states* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ ty_i' *err set*
where *states P mxs mxl* ≡ *fst*(*sl P mxs mxl*)

definition *le* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ ty_i' *err ord*
where
le P mxs mxl ≡ *fst*(*snd*(*sl P mxs mxl*))

definition *sup* :: *'c prog* ⇒ *nat* ⇒ *nat* ⇒ *ty$_i$' err binop*
where
sup P mxs mxl ≡ *snd*(*snd*(*sl P mxs mxl*))

definition *sup-ty-opt* :: [*'c prog*,*ty err*,*ty err*] ⇒ *bool*
(- ⊢ - ≤$_\top$ - [*71*,*71*,*71*] *70*)
where
sup-ty-opt P ≡ *Err.le* (*subtype P*)

definition *sup-state* :: [*'c prog*,*ty$_i$*,*ty$_i$*] ⇒ *bool*
(- ⊢ - ≤$_i$ - [*71*,*71*,*71*] *70*)
where
sup-state P ≡ *Product.le* (*Listn.le* (*subtype P*)) (*Listn.le* (*sup-ty-opt P*))

definition *sup-state-opt* :: [*'c prog*,*ty$_i$'*,*ty$_i$'*] ⇒ *bool*
(- ⊢ - ≤'' - [*71*,*71*,*71*] *70*)
where
sup-state-opt P ≡ *Opt.le* (*sup-state P*)

abbreviation
sup-loc :: [*'c prog*,*ty$_l$*,*ty$_l$*] ⇒ *bool* (- ⊢ - [≤$_\top$] - [*71*,*71*,*71*] *70*)
where *P* ⊢ *LT* [≤$_\top$] *LT'* ≡ *list-all2* (*sup-ty-opt P*) *LT LT'*

notation (*ASCII*)
sup-ty-opt (- |- - <=*T* - [*71*,*71*,*71*] *70*) **and**
sup-state (- |- - <=*i* - [*71*,*71*,*71*] *70*) **and**
sup-state-opt (- |- - <=' - [*71*,*71*,*71*] *70*) **and**
sup-loc (- |- - [<=*T*] - [*71*,*71*,*71*] *70*)

4.15.1 Unfolding

lemma *JVM-states-unfold*:
states P mxs mxl ≡ *err*(*opt*((*Union* {*nlists n* (*types P*) |*n*. *n* <= *mxs*}) ×
nlists mxl (*err*(*types P*))))⟨*proof*⟩
lemma *JVM-le-unfold*:
le P m n ≡
Err.le(*Opt.le*(*Product.le*(*Listn.le*(*subtype P*))(*Listn.le*(*Err.le*(*subtype P*)))))⟨*proof*⟩
lemma *sl-def2*:
JVM-SemiType.sl P mxs mxl ≡
(*states P mxs mxl*, *JVM-SemiType.le P mxs mxl*, *JVM-SemiType.sup P mxs mxl*)⟨*proof*⟩

lemma *JVM-le-conv*:
le P m n (*OK t1*) (*OK t2*) = *P* ⊢ *t1* ≤' *t2*⟨*proof*⟩
lemma *JVM-le-Err-conv*:
le P m n = *Err.le* (*sup-state-opt P*)⟨*proof*⟩
lemma *err-le-unfold* [*iff*]:
Err.le r (*OK a*) (*OK b*) = *r a b*⟨*proof*⟩

4.15.2 Semilattice

lemma *order-sup-state-opt'* [*intro*, *simp*]:
wf-prog wf-mb P ⟹

order (*sup-state-opt P*) (*opt* ((⋃ {*nlists n* (*types P*) |*n*. *n* ≤ *mxs*}) × *nlists* (*Suc* (*length Ts* + mxl_0)) (*err* (*types P*))))⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨*proof*⟩⟨pro

4.16 Effect of Instructions on the State Type

theory *Effect*
imports *JVM-SemiType ../JVM/JVMExceptions*
begin

— FIXME
locale *prog* =
  fixes *P* :: *'a prog*

locale *jvm-method* = *prog* +
  fixes *mxs* :: *nat*
  fixes mxl_0 :: *nat*
  fixes *Ts* :: *ty list*
  fixes T_r :: *ty*
  fixes *is* :: *instr list*
  fixes *xt* :: *ex-table*

  fixes *mxl* :: *nat*
  defines *mxl-def*: *mxl* ≡ *1+size Ts+*mxl_0

Program counter of successor instructions:

primrec *succs* :: *instr* ⇒ ty_i ⇒ *pc* ⇒ *pc list* **where**
  succs (*Load idx*) τ *pc* = [*pc+1*]
| *succs* (*Store idx*) τ *pc* = [*pc+1*]
| *succs* (*Push v*) τ *pc* = [*pc+1*]
| *succs* (*Getfield F C*) τ *pc* = [*pc+1*]
| *succs* (*Putfield F C*) τ *pc* = [*pc+1*]
| *succs* (*New C*) τ *pc* = [*pc+1*]
| *succs* (*Checkcast C*) τ *pc* = [*pc+1*]
| *succs Pop* τ *pc* = [*pc+1*]
| *succs IAdd* τ *pc* = [*pc+1*]
| *succs CmpEq* τ *pc* = [*pc+1*]
| *succs-IfFalse*:
    succs (*IfFalse b*) τ *pc* = [*pc+1*, *nat* (*int pc* + *b*)]
| *succs-Goto*:
    succs (*Goto b*) τ *pc* = [*nat* (*int pc* + *b*)]
| *succs-Return*:
    succs Return τ *pc* = []
| *succs-Invoke*:
    succs (*Invoke M n*) τ *pc* = (*if* (*fst* τ)!*n* = *NT then* [] *else* [*pc+1*])
| *succs-Throw*:
    succs Throw τ *pc* = []

Effect of instruction on the state type:

fun *the-class*:: *ty* ⇒ *cname* **where**
  the-class (*Class C*) = *C*

fun eff_i :: *instr* × *'m prog* × ty_i ⇒ ty_i **where**
  eff_i*-Load*:
    eff_i (*Load n*, *P*, (*ST*, *LT*)) = (*ok-val* (*LT* ! *n*) # *ST*, *LT*)

| *eff$_i$-Store*:
  eff$_i$ (*Store n*, *P*, (*T*#*ST*, *LT*)) = (*ST*, *LT*[*n*:= *OK T*])
| *eff$_i$-Push*:
  eff$_i$ (*Push v*, *P*, (*ST*, *LT*)) = (*the* (*typeof v*) # *ST*, *LT*)
| *eff$_i$-Getfield*:
  eff$_i$ (*Getfield F C*, *P*, (*T*#*ST*, *LT*)) = (*snd* (*field P C F*) # *ST*, *LT*)
| *eff$_i$-Putfield*:
  eff$_i$ (*Putfield F C*, *P*, (T_1#T_2#*ST*, *LT*)) = (*ST*,*LT*)
| *eff$_i$-New*:
  eff$_i$ (*New C*, *P*, (*ST*,*LT*)) = (*Class C* # *ST*, *LT*)
| *eff$_i$-Checkcast*:
  eff$_i$ (*Checkcast C*, *P*, (*T*#*ST*,*LT*)) = (*Class C* # *ST*,*LT*)
| *eff$_i$-Pop*:
  eff$_i$ (*Pop*, *P*, (*T*#*ST*,*LT*)) = (*ST*,*LT*)
| *eff$_i$-IAdd*:
  eff$_i$ (*IAdd*, *P*,(T_1#T_2#*ST*,*LT*)) = (*Integer*#*ST*,*LT*)
| *eff$_i$-CmpEq*:
  eff$_i$ (*CmpEq*, *P*, (T_1#T_2#*ST*,*LT*)) = (*Boolean*#*ST*,*LT*)
| *eff$_i$-IfFalse*:
  eff$_i$ (*IfFalse b*, *P*, (T_1#*ST*,*LT*)) = (*ST*,*LT*)
| *eff$_i$-Invoke*:
  eff$_i$ (*Invoke M n*, *P*, (*ST*,*LT*)) =
  (*let C* = *the-class* (*ST*!*n*); (*D*,*Ts*,T_r,*b*) = *method P C M*
  in (T_r # *drop* (*n*+*1*) *ST*, *LT*))
| *eff$_i$-Goto*:
  eff$_i$ (*Goto n*, *P*, *s*) = *s*

fun *is-relevant-class* :: *instr* ⇒ '*m prog* ⇒ *cname* ⇒ *bool* **where**
  rel-Getfield:
    is-relevant-class (*Getfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* ⪯* *C*)
| *rel-Putfield*:
    is-relevant-class (*Putfield F D*) = (λ*P C*. *P* ⊢ *NullPointer* ⪯* *C*)
| *rel-Checcast*:
    is-relevant-class (*Checkcast D*) = (λ*P C*. *P* ⊢ *ClassCast* ⪯* *C*)
| *rel-New*:
    is-relevant-class (*New D*) = (λ*P C*. *P* ⊢ *OutOfMemory* ⪯* *C*)
| *rel-Throw*:
    is-relevant-class Throw = (λ*P C*. *True*)
| *rel-Invoke*:
    is-relevant-class (*Invoke M n*) = (λ*P C*. *True*)
| *rel-default*:
    is-relevant-class i = (λ*P C*. *False*)

definition *is-relevant-entry* :: '*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-entry* ⇒ *bool* **where**
  is-relevant-entry P i pc e ⟷ (*let* (*f*,*t*,*C*,*h*,*d*) = *e in is-relevant-class i P C* ∧ *pc* ∈ {*f*..<*t*})

definition *relevant-entries* :: '*m prog* ⇒ *instr* ⇒ *pc* ⇒ *ex-table* ⇒ *ex-table* **where**
  relevant-entries P i pc = *filter* (*is-relevant-entry P i pc*)

definition *xcpt-eff* :: *instr* ⇒ '*m prog* ⇒ *pc* ⇒ ty_i
  ⇒ *ex-table* ⇒ (*pc* × ty_i') *list* **where**
  xcpt-eff i P pc τ *et* = (*let* (*ST*,*LT*) = τ *in*
  map (λ(*f*,*t*,*C*,*h*,*d*). (*h*, *Some* (*Class C*#*drop* (*size ST* − *d*) *ST*, *LT*))) (*relevant-entries P i pc et*))

definition *norm-eff* :: *instr* ⇒ *'m prog* ⇒ *nat* ⇒ *ty*$_i$ ⇒ (*pc* × *ty*$_i$*'*) *list* **where**
norm-eff i P pc τ = *map* (λ*pc'*. (*pc'*,*Some* (*eff*$_i$ (*i*,*P*,*τ*)))) (*succs i τ pc*)

definition *eff* :: *instr* ⇒ *'m prog* ⇒ *pc* ⇒ *ex-table* ⇒ *ty*$_i$*'* ⇒ (*pc* × *ty*$_i$*'*) *list* **where**
eff i P pc et t = (*case t of*
None ⇒ []
| *Some τ* ⇒ (*norm-eff i P pc τ*) @ (*xcpt-eff i P pc τ et*))

lemma *eff-None*:
eff i P pc xt None = []
⟨*proof*⟩

lemma *eff-Some*:
eff i P pc xt (*Some τ*) = *norm-eff i P pc τ* @ *xcpt-eff i P pc τ xt*
⟨*proof*⟩

Conditions under which eff is applicable:

fun *app*$_i$ :: *instr* × *'m prog* × *pc* × *nat* × *ty* × *ty*$_i$ ⇒ *bool* **where**
app$_i$*-Load*:
app$_i$ (*Load n*, *P*, *pc*, *mxs*, *T*$_r$, (*ST*,*LT*)) =
(*n* < *length LT* ∧ *LT* ! *n* ≠ *Err* ∧ *length ST* < *mxs*)
| *app*$_i$*-Store*:
app$_i$ (*Store n*, *P*, *pc*, *mxs*, *T*$_r$, (*T*#*ST*, *LT*)) =
(*n* < *length LT*)
| *app*$_i$*-Push*:
app$_i$ (*Push v*, *P*, *pc*, *mxs*, *T*$_r$, (*ST*,*LT*)) =
(*length ST* < *mxs* ∧ *typeof v* ≠ *None*)
| *app*$_i$*-Getfield*:
app$_i$ (*Getfield F C*, *P*, *pc*, *mxs*, *T*$_r$, (*T*#*ST*, *LT*)) =
(∃ *T*$_f$. *P* ⊢ *C sees F*:*T*$_f$ *in C* ∧ *P* ⊢ *T* ≤ *Class C*)
| *app*$_i$*-Putfield*:
app$_i$ (*Putfield F C*, *P*, *pc*, *mxs*, *T*$_r$, (*T*$_1$#*T*$_2$#*ST*, *LT*)) =
(∃ *T*$_f$. *P* ⊢ *C sees F*:*T*$_f$ *in C* ∧ *P* ⊢ *T*$_2$ ≤ (*Class C*) ∧ *P* ⊢ *T*$_1$ ≤ *T*$_f$)
| *app*$_i$*-New*:
app$_i$ (*New C*, *P*, *pc*, *mxs*, *T*$_r$, (*ST*,*LT*)) =
(*is-class P C* ∧ *length ST* < *mxs*)
| *app*$_i$*-Checkcast*:
app$_i$ (*Checkcast C*, *P*, *pc*, *mxs*, *T*$_r$, (*T*#*ST*,*LT*)) =
(*is-class P C* ∧ *is-refT T*)
| *app*$_i$*-Pop*:
app$_i$ (*Pop*, *P*, *pc*, *mxs*, *T*$_r$, (*T*#*ST*,*LT*)) =
True
| *app*$_i$*-IAdd*:
app$_i$ (*IAdd*, *P*, *pc*, *mxs*, *T*$_r$, (*T*$_1$#*T*$_2$#*ST*,*LT*)) = (*T*$_1$ = *T*$_2$ ∧ *T*$_1$ = *Integer*)
| *app*$_i$*-CmpEq*:
app$_i$ (*CmpEq*, *P*, *pc*, *mxs*, *T*$_r$, (*T*$_1$#*T*$_2$#*ST*,*LT*)) =
(*T*$_1$ = *T*$_2$ ∨ *is-refT T*$_1$ ∧ *is-refT T*$_2$)
| *app*$_i$*-IfFalse*:
app$_i$ (*IfFalse b*, *P*, *pc*, *mxs*, *T*$_r$, (*Boolean*#*ST*,*LT*)) =
(0 ≤ *int pc* + *b*)
| *app*$_i$*-Goto*:
app$_i$ (*Goto b*, *P*, *pc*, *mxs*, *T*$_r$, *s*) =
(0 ≤ *int pc* + *b*)

| app_i-*Return*:
  app_i (*Return*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*,*LT*)) =
  (*P* ⊢ *T* ≤ T_r)
| app_i-*Throw*:
  app_i (*Throw*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*,*LT*)) =
  is-refT T
| app_i-*Invoke*:
  app_i (*Invoke M n*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
  (*n* < *length ST* ∧
  (*ST*!*n* ≠ *NT* ⟶
    (∃ *C D Ts T m*. *ST*!*n* = *Class C* ∧ *P* ⊢ *C sees M*:*Ts* → *T* = *m in D* ∧
      P ⊢ *rev* (*take n ST*) [≤] *Ts*)))

| app_i-*default*:
  app_i (*i*,*P*, *pc*,*mxs*,T_r,*s*) = *False*

definition *xcpt-app* :: *instr* ⇒ '*m prog* ⇒ *pc* ⇒ *nat* ⇒ *ex-table* ⇒ ty_i ⇒ *bool* **where**
xcpt-app i P pc mxs xt τ ⟷ (∀(*f*,*t*,*C*,*h*,*d*) ∈ *set* (*relevant-entries P i pc xt*). *is-class P C* ∧ *d* ≤ *size* (*fst τ*) ∧ *d* < *mxs*)

definition *app* :: *instr* ⇒ '*m prog* ⇒ *nat* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒ *ex-table* ⇒ ty_i' ⇒ *bool* **where**
app i P mxs T_r *pc mpc xt t* = (*case t of None* ⇒ *True* | *Some τ* ⇒
app_i (*i*,*P*,*pc*,*mxs*,T_r,*τ*) ∧ *xcpt-app i P pc mxs xt τ* ∧
(∀(*pc*',*τ*') ∈ *set* (*eff i P pc xt t*). *pc*' < *mpc*))

lemma *app-Some*:
app i P mxs T_r *pc mpc xt* (*Some τ*) =
(app_i (*i*,*P*,*pc*,*mxs*,T_r,*τ*) ∧ *xcpt-app i P pc mxs xt τ* ∧
(∀(*pc*',*s*') ∈ *set* (*eff i P pc xt* (*Some τ*)). *pc*' < *mpc*))
⟨*proof*⟩

locale *eff* = *jvm-method* +
fixes eff_i **and** app_i **and** *eff* **and** *app*
fixes *norm-eff* **and** *xcpt-app* **and** *xcpt-eff*

fixes *mpc*
defines *mpc* ≡ *size is*

defines eff_i *i τ* ≡ *Effect*.eff_i (*i*,*P*,*τ*)
notes eff_i-*simps* [*simp*] = *Effect*.eff_i.*simps* [**where** *P* = *P*, *folded* eff_i-*def*]

defines app_i *i pc τ* ≡ *Effect*.app_i (*i*, *P*, *pc*, *mxs*, T_r, *τ*)
notes app_i-*simps* [*simp*] = *Effect*.app_i.*simps* [**where** *P*=*P* **and** *mxs*=*mxs* **and** T_r=T_r, *folded* app_i-*def*]

defines *xcpt-eff i pc τ* ≡ *Effect.xcpt-eff i P pc τ xt*
notes *xcpt-eff* = *Effect.xcpt-eff-def* [*of* - *P* - - *xt*, *folded xcpt-eff-def*]

defines *norm-eff i pc τ* ≡ *Effect.norm-eff i P pc τ*
notes *norm-eff* = *Effect.norm-eff-def* [*of* - *P*, *folded norm-eff-def* eff_i-*def*]

defines *eff i pc* ≡ *Effect.eff i P pc xt*

notes *eff* = *Effect.eff-def* [*of* - *P* - *xt*, *folded eff-def norm-eff-def xcpt-eff-def*]

defines *xcpt-app i pc* $\tau \equiv$ *Effect.xcpt-app i P pc mxs xt* τ
notes *xcpt-app* = *Effect.xcpt-app-def* [*of* - *P* - *mxs xt*, *folded xcpt-app-def*]

defines *app i pc* $\equiv$ *Effect.app i P mxs* T_r *pc mpc xt*
notes *app* = *Effect.app-def* [*of* - *P mxs* T_r - *mpc xt*, *folded app-def xcpt-app-def* app_i*-def eff-def*]

lemma *length-cases2*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows *P s*
⟨*proof*⟩

lemma *length-cases3*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ ST\ LT.\ P\ (l\#ST,LT)$
shows *P s*⟨*proof*⟩
lemma *length-cases4*:
assumes $\bigwedge LT.\ P\ ([],LT)$
assumes $\bigwedge l\ LT.\ P\ ([l],LT)$
assumes $\bigwedge l\ l'\ LT.\ P\ ([l,l'],LT)$
assumes $\bigwedge l\ l'\ ST\ LT.\ P\ (l\#l'\#ST,LT)$
shows *P s*⟨*proof*⟩

simp rules for *app*

lemma *appNone*[*simp*]: *app i P mxs* T_r *pc mpc et None* = *True*
⟨*proof*⟩

lemma *appLoad*[*simp*]:
app_i (*Load idx*, *P*, T_r, *mxs*, *pc*, *s*) = ($\exists\, ST\ LT.\ s = (ST,LT) \wedge idx < length\ LT \wedge LT!idx \neq Err \wedge length\ ST < mxs$)
⟨*proof*⟩

lemma *appStore*[*simp*]:
app_i (*Store idx*,*P*,*pc*,*mxs*,T_r,*s*) = ($\exists\, ts\ ST\ LT.\ s = (ts\#ST,LT) \wedge idx < length\ LT$)
⟨*proof*⟩

lemma *appPush*[*simp*]:
app_i (*Push v*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists\, ST\ LT.\ s = (ST,LT) \wedge length\ ST < mxs \wedge typeof\ v \neq None$)
⟨*proof*⟩

lemma *appGetField*[*simp*]:
app_i (*Getfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
($\exists$ *oT vT ST LT. s* = (*oT*#*ST*, *LT*) $\wedge$
P $\vdash$ *C sees F*:*vT in C* $\wedge$ *P* $\vdash$ *oT* $\leq$ (*Class C*))
⟨*proof*⟩

lemma *appPutField*[*simp*]:

app_i (*Putfield F C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *vT vT′ oT ST LT*. *s* = (*vT*#*oT*#*ST*, *LT*) ∧
P ⊢ *C sees F*:*vT′ in C* ∧ *P* ⊢ *oT* ≤ (*Class C*) ∧ *P* ⊢ *vT* ≤ *vT′*)
⟨*proof*⟩

lemma *appNew*[*simp*]:
app_i (*New C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s*=(*ST*,*LT*) ∧ *is-class P C* ∧ *length ST* < *mxs*)
⟨*proof*⟩

lemma *appCheckcast*[*simp*]:
app_i (*Checkcast C*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *is-class P C* ∧ *is-refT T*)
⟨*proof*⟩

lemma app_i*Pop*[*simp*]:
app_i (*Pop*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ts ST LT*. *s* = (*ts*#*ST*,*LT*))
⟨*proof*⟩

lemma *appIAdd*[*simp*]:
app_i (*IAdd*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *ST LT*. *s* = (*Integer*#*Integer*#*ST*,*LT*))⟨*proof*⟩

lemma *appIfFalse* [*simp*]:
app_i (*IfFalse b*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ *ST LT*. *s* = (*Boolean*#*ST*,*LT*) ∧ 0 ≤ *int pc* + *b*)⟨*proof*⟩
lemma *appCmpEq*[*simp*]:
app_i (*CmpEq*,*P*,*pc*,*mxs*,T_r,*s*) =
(∃ T_1 T_2 *ST LT*. *s* = (T_1#T_2#*ST*,*LT*) ∧ (¬*is-refT* T_1 ∧ T_2 = T_1 ∨ *is-refT* T_1 ∧ *is-refT* T_2))
⟨*proof*⟩

lemma *appReturn*[*simp*]:
app_i (*Return*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s* = (*T*#*ST*,*LT*) ∧ *P* ⊢ *T* ≤ T_r)
⟨*proof*⟩

lemma *appThrow*[*simp*]:
app_i (*Throw*,*P*,*pc*,*mxs*,T_r,*s*) = (∃ *T ST LT*. *s*=(*T*#*ST*,*LT*) ∧ *is-refT T*)
⟨*proof*⟩

lemma *effNone*:
(*pc′*, *s′*) ∈ *set* (*eff i P pc et None*) ⟹ *s′* = *None*
⟨*proof*⟩

some helpers to make the specification directly executable:

lemma *relevant-entries-append* [*simp*]:
relevant-entries P i pc (*xt* @ *xt′*) = *relevant-entries P i pc xt* @ *relevant-entries P i pc xt′*
⟨*proof*⟩

lemma *xcpt-app-append* [*iff*]:
xcpt-app i P pc mxs (*xt*@*xt′*) *τ* = (*xcpt-app i P pc mxs xt τ* ∧ *xcpt-app i P pc mxs xt′ τ*)
⟨*proof*⟩

lemma *xcpt-eff-append* [*simp*]:
xcpt-eff i P pc τ (*xt*@*xt′*) = *xcpt-eff i P pc τ xt* @ *xcpt-eff i P pc τ xt′*
⟨*proof*⟩

lemma *app-append* [*simp*]:
app i P pc T mxs mpc (*xt*@*xt'*) τ = (*app i P pc T mxs mpc xt* τ $\wedge$ *app i P pc T mxs mpc xt'* τ)
⟨*proof*⟩

end

4.17 Monotonicity of eff and app

theory *EffectMono* **imports** *Effect* **begin**

declare *not-Err-eq* [*iff*]

lemma *app$_i$-mono*:
assumes *wf*: *wf-prog p P*
assumes *less*: $P \vdash \tau \leq_i \tau'$
shows *app$_i$* (*i*,*P*,*mxs*,*mpc*,*rT*,τ') $\Longrightarrow$ *app$_i$* (*i*,*P*,*mxs*,*mpc*,*rT*,τ)⟨*proof*⟩
lemma *succs-mono*:
assumes *wf*: *wf-prog p P* **and** *app$_i$*: *app$_i$* (*i*,*P*,*mxs*,*mpc*,*rT*,τ')
shows $P \vdash \tau \leq_i \tau' \Longrightarrow$ *set* (*succs i* τ *pc*) $\subseteq$ *set* (*succs i* τ' *pc*)⟨*proof*⟩

lemma *app-mono*:
assumes *wf*: *wf-prog p P*
assumes *less'*: $P \vdash \tau \leq' \tau'$
shows *app i P m rT pc mpc xt* τ' $\Longrightarrow$ *app i P m rT pc mpc xt* τ⟨*proof*⟩

lemma *eff$_i$-mono*:
assumes *wf*: *wf-prog p P*
assumes *less*: $P \vdash \tau \leq_i \tau'$
assumes *app$_i$*: *app i P m rT pc mpc xt* (*Some* τ')
assumes *succs*: *succs i* τ *pc* $\neq$ [] *succs i* τ' *pc* $\neq$ []
shows $P \vdash$ *eff$_i$* (*i*,*P*,τ) $\leq_i$ *eff$_i$* (*i*,*P*,τ')⟨*proof*⟩
end

4.18 The Bytecode Verifier

theory *BVSpec*
imports *Effect*
begin

This theory contains a specification of the BV. The specification describes correct typings of method bodies; it corresponds to type *checking*.

definition
— The method type only contains declared classes:
check-types :: *'m prog* ⇒ *nat* ⇒ *nat* ⇒ *ty$_i$' err list* ⇒ *bool*
where
check-types P mxs mxl $\tau s \equiv$ *set* $\tau s \subseteq$ *states P mxs mxl*

— An instruction is welltyped if it is applicable and its effect
— is compatible with the type at all successor instructions:
definition
wt-instr :: [*'m prog*,*ty*,*nat*,*pc*,*ex-table*,*instr*,*pc*,*ty$_m$*] ⇒ *bool*
(-,-,-,-,- ⊢ -,- :: - [*60*,*0*,*0*,*0*,*0*,*0*,*0*,*61*] *60*)

where
P,*T*,*mxs*,*mpc*,*xt* ⊢ *i*,*pc* :: *τs* ≡
app i P mxs T pc mpc xt (*τs*!*pc*) ∧
(∀ (*pc*′,*τ*′) ∈ *set* (*eff i P pc xt* (*τs*!*pc*)). *P* ⊢ *τ*′ ≤′ *τs*!*pc*′)

— The type at *pc=0* conforms to the method calling convention:
definition *wt-start* :: [′*m prog*,*cname*,*ty list*,*nat*,*ty*$_m$] ⇒ *bool*
where
wt-start P C Ts mxl$_0$ *τs* ≡
P ⊢ *Some* ([],*OK* (*Class C*)#*map OK Ts*@*replicate mxl*$_0$ *Err*) ≤′ *τs*!*0*

— A method is welltyped if the body is not empty,
— if the method type covers all instructions and mentions
— declared classes only, if the method calling convention is respected, and
— if all instructions are welltyped.
definition *wt-method* :: [′*m prog*,*cname*,*ty list*,*ty*,*nat*,*nat*,*instr list*,
ex-table,*ty*$_m$] ⇒ *bool*
where
wt-method P C Ts T$_r$ *mxs mxl*$_0$ *is xt τs* ≡
0 < *size is* ∧ *size τs* = *size is* ∧
check-types P mxs (*1*+*size Ts*+*mxl*$_0$) (*map OK τs*) ∧
wt-start P C Ts mxl$_0$ *τs* ∧
(∀ *pc* < *size is*. *P*,*T*$_r$,*mxs*,*size is*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*)

— A program is welltyped if it is wellformed and all methods are welltyped
definition *wf-jvm-prog-phi* :: *ty*$_P$ ⇒ *jvm-prog* ⇒ *bool* (*wf*′*-jvm*′*-prog*$_-$)
where
wf-jvm-prog$_\Phi$ ≡
wf-prog (λ*P C* (*M*,*Ts*,*T*$_r$,(*mxs*,*mxl*$_0$,*is*,*xt*)).
wt-method P C Ts T$_r$ *mxs mxl*$_0$ *is xt* (Φ *C M*))

definition *wf-jvm-prog* :: *jvm-prog* ⇒ *bool*
where
wf-jvm-prog P ≡ ∃ Φ. *wf-jvm-prog*$_\Phi$ *P*

lemma *wt-jvm-progD*:
wf-jvm-prog$_\Phi$ *P* ⟹ ∃ *wt*. *wf-prog wt P*⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-instr*:
assumes *wf*: *wf-jvm-prog*$_\Phi$ *P* **and**
sees: *P* ⊢ *C sees M*:*Ts* → *T* = (*mxs*,*mxl*$_0$,*ins*,*xt*) *in C* **and**
pc: *pc* < *size ins*
shows *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*⟨*proof*⟩
lemma *wt-jvm-prog-impl-wt-start*:
assumes *wf*: *wf-jvm-prog*$_\Phi$ *P* **and**
sees: *P* ⊢ *C sees M*:*Ts* → *T* = (*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*
shows *0* < *size ins* ∧ *wt-start P C Ts mxl*$_0$ (Φ *C M*)⟨*proof*⟩

4.19 The Typing Framework for the JVM

theory *TF-JVM*
imports *../DFA/Typing-Framework-err EffectMono BVSpec*
begin

definition *exec* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list* ⇒ ty_i' *err step-type*
where
exec G maxs rT et bs ≡
err-step (*size bs*) (λ*pc*. *app* (*bs*!*pc*) *G maxs rT pc* (*size bs*) *et*)
(λ*pc*. *eff* (*bs*!*pc*) *G pc et*)

locale *JVM-sl* =
fixes *P* :: *jvm-prog* **and** *mxs* **and** mxl_0 **and** *n*
fixes *Ts* :: *ty list* **and** *is* **and** *xt* **and** T_r

fixes *mxl* **and** *A* **and** *r* **and** *f* **and** *app* **and** *eff* **and** *step*
defines [*simp*]: *mxl* ≡ 1+*size Ts*+mxl_0
defines [*simp*]: *A* ≡ *states P mxs mxl*
defines [*simp*]: *r* ≡ *JVM-SemiType.le P mxs mxl*
defines [*simp*]: *f* ≡ *JVM-SemiType.sup P mxs mxl*

defines [*simp*]: *app* ≡ λ*pc*. *Effect.app* (*is*!*pc*) *P mxs* T_r *pc* (*size is*) *xt*
defines [*simp*]: *eff* ≡ λ*pc*. *Effect.eff* (*is*!*pc*) *P pc xt*
defines [*simp*]: *step* ≡ *err-step* (*size is*) *app eff*

defines [*simp*]: *n* ≡ *size is*

locale *start-context* = *JVM-sl* +
fixes *p* **and** *C*
assumes *wf*: *wf-prog p P*
assumes *C*: *is-class P C*
assumes *Ts*: *set Ts* ⊆ *types P*

fixes *first* :: ty_i' **and** *start*
defines [*simp*]:
first ≡ *Some* ([],*OK* (*Class C*) # *map OK Ts* @ *replicate* mxl_0 *Err*)
defines [*simp*]:
start ≡ *OK first* # *replicate* (*size is* − 1) (*OK None*)

4.19.1 Connecting JVM and Framework

lemma (**in** *start-context*) *semi*: *semilat* (*A*, *r*, *f*)
⟨*proof*⟩

lemma (**in** *JVM-sl*) *step-def-exec*: *step* ≡ *exec P mxs* T_r *xt is*
⟨*proof*⟩

lemma *special-ex-swap-lemma* [*iff*]:
(? *X*. (? *n*. *X* = *A n* & *P n*) & *Q X*) = (? *n*. *Q*(*A n*) & *P n*)
⟨*proof*⟩

lemma *ex-in-nlists* [*iff*]:
(∃ *n*. *ST* ∈ *nlists n A* ∧ *n* ≤ *mxs*) = (*set ST* ⊆ *A* ∧ *size ST* ≤ *mxs*)
⟨*proof*⟩

lemma *singleton-nlists*:
(∃ *n*. [*Class C*] ∈ *nlists n* (*types P*) ∧ *n* ≤ *mxs*) = (*is-class P C* ∧ 0 < *mxs*)
⟨*proof*⟩

lemma *set-drop-subset*:
set xs $\subseteq$ *A* $\Longrightarrow$ *set* (*drop n xs*) $\subseteq$ *A*
⟨*proof*⟩

lemma *Suc-minus-minus-le*:
n < *mxs* $\Longrightarrow$ *Suc* (*n* − (*n* − *b*)) $\leq$ *mxs*
⟨*proof*⟩

lemma *in-nlistsE*:
⟦ *xs* $\in$ *nlists n A*; ⟦*size xs* = *n*; *set xs* $\subseteq$ *A*⟧ $\Longrightarrow$ *P* ⟧ $\Longrightarrow$ *P*
⟨*proof*⟩

declare *is-relevant-entry-def* [*simp*]
declare *set-drop-subset* [*simp*]

theorem (**in** *start-context*) *exec-pres-type*:
pres-type step (*size is*) *A*⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]
declare *set-drop-subset* [*simp del*]

lemma *lesubstep-type-simple*:
xs [$\sqsubseteq_{Product.le\ (=)\ r}$] *ys* $\Longrightarrow$ *set xs* {$\sqsubseteq_r$} *set ys*⟨*proof*⟩
declare *is-relevant-entry-def* [*simp del*]

lemma *conjI2*: ⟦ *A*; *A* $\Longrightarrow$ *B* ⟧ $\Longrightarrow$ *A* ∧ *B* ⟨*proof*⟩

lemma (**in** *JVM-sl*) *eff-mono*:
⟦*wf-prog p P*; *pc* < *length is*; *s* $\sqsubseteq_{sup\text{-}state\text{-}opt\ P}$ *t*; *app pc t*⟧
$\Longrightarrow$ *set* (*eff pc s*) {$\sqsubseteq_{sup\text{-}state\text{-}opt\ P}$} *set* (*eff pc t*)⟨*proof*⟩
lemma (**in** *JVM-sl*) *bounded-step*: *bounded step* (*size is*)⟨*proof*⟩
theorem (**in** *JVM-sl*) *step-mono*:
wf-prog wf-mb P $\Longrightarrow$ *mono r step* (*size is*) *A*⟨*proof*⟩

lemma (**in** *start-context*) *first-in-A* [*iff*]: *OK first* $\in$ *A*
⟨*proof*⟩

lemma (**in** *JVM-sl*) *wt-method-def2*:
wt-method P C′ Ts T_r *mxs* mxl_0 *is xt* τs =
(*is* ≠ [] ∧
size τs = *size is* ∧
OK ‘ *set* τs $\subseteq$ *states P mxs mxl* ∧
wt-start P C′ Ts mxl_0 τs ∧
wt-app-eff (*sup-state-opt P*) *app eff* τs)⟨*proof*⟩

end

4.20 Typing and Dataflow Analysis Framework

theory *Typing-Framework-2* **imports** *Typing-Framework-1* **begin**

The relationship between dataflow analysis and a welltyped-instruction predicate.

definition *is-bcv* :: *'s ord* ⇒ *'s* ⇒ *'s step-type* ⇒ *nat* ⇒ *'s set* ⇒ (*'s list* ⇒ *'s list*) ⇒ *bool*
where
is-bcv r T step n A bcv ⟷ (∀ *τs*$_0$ ∈ *nlists n A*.
(∀ *p*<*n*. (*bcv τs*$_0$)!*p* ≠ *T*) = (∃ *τs* ∈ *nlists n A*. *τs*$_0$ [⊑$_r$] *τs* ∧ *wt-step r T step τs*))

end

4.21 Kildall for the JVM

theory *BVExec*
imports *../DFA/Abstract-BV TF-JVM ../DFA/Typing-Framework-2*
begin

definition *kiljvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒
instr list ⇒ *ex-table* ⇒ *ty*$_i$' *err list* ⇒ *ty*$_i$' *err list*
where
kiljvm P mxs mxl T$_r$ *is xt* ≡
kildall (*JVM-SemiType.le P mxs mxl*) (*JVM-SemiType.sup P mxs mxl*)
(*exec P mxs T*$_r$ *xt is*)

definition *wt-kildall* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
instr list ⇒ *ex-table* ⇒ *bool*
where
wt-kildall P C' Ts T$_r$ *mxs mxl*$_0$ *is xt* ≡
0 < *size is* ∧
(*let first* = *Some* ([],[*OK* (*Class C'*)]@(*map OK Ts*)@(*replicate mxl*$_0$ *Err*));
start = *OK first*#(*replicate* (*size is* − *1*) (*OK None*));
result = *kiljvm P mxs* (*1*+*size Ts*+*mxl*$_0$) *T*$_r$ *is xt start*
in ∀ *n* < *size is*. *result*!*n* ≠ *Err*)

definition *wf-jvm-prog*$_k$ :: *jvm-prog* ⇒ *bool*
where
wf-jvm-prog$_k$ *P* ≡
wf-prog (λ*P C'* (*M*,*Ts*,*T*$_r$,(*mxs*,*mxl*$_0$,*is*,*xt*)). *wt-kildall P C' Ts T*$_r$ *mxs mxl*$_0$ *is xt*) *P*

context *start-context*
begin

lemma *Cons-less-Conss3* [*simp*]:
x#*xs* [⊏$_r$] *y*#*ys* = (*x* ⊏$_r$ *y* ∧ *xs* [⊑$_r$] *ys* ∨ *x* = *y* ∧ *xs* [⊏$_r$] *ys*)
⟨*proof*⟩

lemma *acc-le-listI3* [*intro*!]:
acc r ⟹ *acc* (*Listn.le r*)
⟨*proof*⟩

lemma *wf-jvm*: *wf* {(*ss'*, *ss*). *ss* [⊏$_r$] *ss'*}
⟨*proof*⟩

lemma *iter-properties-bv*[*rule-format*]:
shows ⟦ ∀ *p*∈*w0*. *p* < *n*; *ss0* ∈ *nlists n A*; ∀ *p*<*n*. *p* ∉ *w0* ⟶ *stable r step ss0 p* ⟧ ⟹
iter f step ss0 w0 = (*ss'*,*w'*) ⟶

$ss' \in nlists\ n\ A \wedge stables\ r\ step\ ss' \wedge ss0\ [\sqsubseteq_r]\ ss' \wedge$
$(\forall\, ts{\in}nlists\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow ss'\ [\sqsubseteq_r]\ ts)\langle proof\rangle$

lemma *kildall-properties-bv*:
shows $[\![\ ss0 \in nlists\ n\ A\ ]\!] \Longrightarrow$
$kildall\ r\ f\ step\ ss0 \in nlists\ n\ A \wedge$
$stables\ r\ step\ (kildall\ r\ f\ step\ ss0) \wedge$
$ss0\ [\sqsubseteq_r]\ kildall\ r\ f\ step\ ss0 \wedge$
$(\forall\, ts{\in}nlists\ n\ A.\ ss0\ [\sqsubseteq_r]\ ts \wedge stables\ r\ step\ ts \longrightarrow$
$kildall\ r\ f\ step\ ss0\ [\sqsubseteq_r]\ ts)\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle\langle proof\rangle$

4.22 LBV for the JVM

theory *LBVJVM*
imports *../DFA/Abstract-BV TF-JVM*
begin

type-synonym *prog-cert* = *cname* ⇒ *mname* ⇒ ty_i' *err list*

definition *check-cert* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *nat* ⇒ ty_i' *err list* ⇒ *bool*
where
check-cert P mxs mxl n cert ≡ *check-types P mxs mxl cert* ∧ *size cert* = *n*+*1* ∧
(∀ *i*<*n*. *cert*!*i* ≠ *Err*) ∧ *cert*!*n* = *OK None*

definition *lbvjvm* :: *jvm-prog* ⇒ *nat* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒
ty_i' *err list* ⇒ *instr list* ⇒ ty_i' *err* ⇒ ty_i' *err*
where
lbvjvm P mxs maxr T_r *et cert bs* ≡
wtl-inst-list bs cert (*JVM-SemiType.sup P mxs maxr*) (*JVM-SemiType.le P mxs maxr*) *Err* (*OK None*) (*exec P mxs* T_r *et bs*) *0*

definition *wt-lbv* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ ty_i' *err list* ⇒ *instr list* ⇒ *bool*
where
wt-lbv P C Ts T_r *mxs* mxl_0 *et cert ins* ≡
check-cert P mxs (*1*+*size Ts*+mxl_0) (*size ins*) *cert* ∧
0 < *size ins* ∧
(*let start* = *Some* ([],(*OK* (*Class C*))#((*map OK Ts*))@(*replicate* mxl_0 *Err*));
result = *lbvjvm P mxs* (*1*+*size Ts*+mxl_0) T_r *et cert ins* (*OK start*)
in result ≠ *Err*)

definition *wt-jvm-prog-lbv* :: *jvm-prog* ⇒ *prog-cert* ⇒ *bool*
where
wt-jvm-prog-lbv P cert ≡
wf-prog (λ*P C* (*mn*,*Ts*,T_r,(*mxs*,mxl_0,*b*,*et*)). *wt-lbv P C Ts* T_r *mxs* mxl_0 *et* (*cert C mn*) *b*) *P*

definition *mk-cert* :: *jvm-prog* ⇒ *nat* ⇒ *ty* ⇒ *ex-table* ⇒ *instr list*
⇒ ty_m ⇒ ty_i' *err list*
where
mk-cert P mxs T_r *et bs phi* ≡ *make-cert* (*exec P mxs* T_r *et bs*) (*map OK phi*) (*OK None*)

definition *prg-cert* :: *jvm-prog* ⇒ ty_P ⇒ *prog-cert*
where

prg-cert P phi C mn ≡ *let* $(C,Ts,T_r,(mxs,mxl_0,ins,et))$ = *method P C mn*
in mk-cert P mxs T_r *et ins* (*phi C mn*)

lemma *check-certD* [*intro?*]:
check-cert P mxs mxl n cert ⟹ *cert-ok cert n Err* (*OK None*) (*states P mxs mxl*)
⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-step*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ $\tau s \in$ *nlists* (*size is*) *A. wt-step r Err step* $\tau s \wedge$ *OK first* $\sqsubseteq_r$ $\tau s!0$⟨*proof*⟩

lemma (**in** *start-context*) *wt-lbv-wt-method*:
assumes *lbv*: *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*
shows ∃ τs. *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs⟨*proof*⟩

lemma (**in** *start-context*) *wt-method-wt-lbv*:
assumes *wt*: *wt-method P C Ts* T_r *mxs* mxl_0 *is xt* τs
defines [*simp*]: *cert* ≡ *mk-cert P mxs* T_r *xt is* τs

shows *wt-lbv P C Ts* T_r *mxs* mxl_0 *xt cert is*⟨*proof*⟩

theorem *jvm-lbv-correct*:
wt-jvm-prog-lbv P Cert ⟹ *wf-jvm-prog P*⟨*proof*⟩
theorem *jvm-lbv-complete*:
assumes *wt*: *wf-jvm-prog*$_\Phi$ *P*
shows *wt-jvm-prog-lbv P* (*prg-cert P* Φ)⟨*proof*⟩
end

4.23 BV Type Safety Invariant

theory *BVConform*
imports *BVSpec ../JVM/JVMExec ../Common/Conform*
begin

definition *confT* :: *'c prog* ⇒ *heap* ⇒ *val* ⇒ *ty err* ⇒ *bool*
(-,- ⊢ - :≤$_\top$ - [*51*,*51*,*51*,*51*] *50*)
where
P,*h* ⊢ *v* :≤$_\top$ *E* ≡ *case E of Err* ⇒ *True* | *OK T* ⇒ *P*,*h* ⊢ *v* :≤ *T*

notation (*ASCII*)
confT (-,- |− - :<=*T* - [*51*,*51*,*51*,*51*] *50*)

abbreviation
confTs :: *'c prog* ⇒ *heap* ⇒ *val list* ⇒ *ty*$_l$ ⇒ *bool*
(-,- ⊢ - [:≤$_\top$] - [*51*,*51*,*51*,*51*] *50*) **where**
P,*h* ⊢ *vs* [:≤$_\top$] *Ts* ≡ *list-all2* (*confT P h*) *vs Ts*

notation (*ASCII*)
confTs (-,- |− - [:<=*T*] - [*51*,*51*,*51*,*51*] *50*)

definition *conf-f* :: *jvm-prog* ⇒ *heap* ⇒ *ty*$_i$ ⇒ *bytecode* ⇒ *frame* ⇒ *bool*

where
conf-f P h ≡ λ(*ST*,*LT*) *is* (*stk*,*loc*,*C*,*M*,*pc*).
P,*h* ⊢ *stk* [:≤] *ST* ∧ *P*,*h* ⊢ *loc* [:≤$_\top$] *LT* ∧ *pc* < *size is*

lemma *conf-f-def2*:
conf-f P h (*ST*,*LT*) *is* (*stk*,*loc*,*C*,*M*,*pc*) ≡
P,*h* ⊢ *stk* [:≤] *ST* ∧ *P*,*h* ⊢ *loc* [:≤$_\top$] *LT* ∧ *pc* < *size is*
⟨*proof*⟩

primrec *conf-fs* :: [*jvm-prog*,*heap*,*ty*$_P$,*mname*,*nat*,*ty*,*frame list*] ⇒ *bool*
where
conf-fs P h Φ *M*$_0$ *n*$_0$ *T*$_0$ [] = *True*
| *conf-fs P h* Φ *M*$_0$ *n*$_0$ *T*$_0$ (*f*#*frs*) =
(*let* (*stk*,*loc*,*C*,*M*,*pc*) = *f in*
(∃ *ST LT Ts T mxs mxl*$_0$ *is xt*.
Φ *C M* ! *pc* = *Some* (*ST*,*LT*) ∧
(*P* ⊢ *C sees M*:*Ts* → *T* = (*mxs*,*mxl*$_0$,*is*,*xt*) *in C*) ∧
(∃ *D Ts*′ *T*′ *m D*′.
is!*pc* = (*Invoke M*$_0$ *n*$_0$) ∧ *ST*!*n*$_0$ = *Class D* ∧
P ⊢ *D sees M*$_0$:*Ts*′ → *T*′ = *m in D*′ ∧ *P* ⊢ *T*$_0$ ≤ *T*′) ∧
conf-f P h (*ST*, *LT*) *is f* ∧ *conf-fs P h* Φ *M* (*size Ts*) *T frs*))

definition *correct-state* :: [*jvm-prog*,*ty*$_P$,*jvm-state*] ⇒ *bool* (-,- ⊢ - √ [*61*,*0*,*0*] *61*)
where
correct-state P Φ ≡ λ(*xp*,*h*,*frs*).
case xp of
None ⇒ (*case frs of*
[] ⇒ *True*
| (*f*#*fs*) ⇒ *P*⊢ *h*√ ∧
(*let* (*stk*,*loc*,*C*,*M*,*pc*) = *f*
in ∃ *Ts T mxs mxl*$_0$ *is xt* τ.
(*P* ⊢ *C sees M*:*Ts*→*T* = (*mxs*,*mxl*$_0$,*is*,*xt*) *in C*) ∧
Φ *C M* ! *pc* = *Some* τ ∧
conf-f P h τ *is f* ∧ *conf-fs P h* Φ *M* (*size Ts*) *T fs*))
| *Some x* ⇒ *frs* = []

notation
correct-state (-,- |− - [*ok*] [*61*,*0*,*0*] *61*)

4.23.1 Values and ⊤

lemma *confT-Err* [*iff*]: *P*,*h* ⊢ *x* :≤$_\top$ *Err*
⟨*proof*⟩

lemma *confT-OK* [*iff*]: *P*,*h* ⊢ *x* :≤$_\top$ *OK T* = (*P*,*h* ⊢ *x* :≤ *T*)
⟨*proof*⟩

lemma *confT-cases*:
P,*h* ⊢ *x* :≤$_\top$ *X* = (*X* = *Err* ∨ (∃ *T*. *X* = *OK T* ∧ *P*,*h* ⊢ *x* :≤ *T*))
⟨*proof*⟩

lemma *confT-hext* [*intro?*, *trans*]:

⟦ P,$h \vdash x :\leq_\top T$; $h \trianglelefteq h'$ ⟧ $\Longrightarrow$ P,$h' \vdash x :\leq_\top T$
⟨*proof*⟩

lemma *confT-widen* [*intro?*, *trans*]:
⟦ P,$h \vdash x :\leq_\top T$; $P \vdash T \leq_\top T'$ ⟧ $\Longrightarrow$ P,$h \vdash x :\leq_\top T'$
⟨*proof*⟩

4.23.2 Stack and Registers

lemmas *confTs-Cons1* [*iff*] = *list-all2-Cons1* [*of confT P h*] **for** P h

lemma *confTs-confT-sup*:
⟦ P,$h \vdash loc$ [$:\leq_\top$] LT; $n < size\ LT$; $LT!n = OK\ T$; $P \vdash T \leq T'$ ⟧
$\Longrightarrow$ P,$h \vdash (loc!n) :\leq T'$⟨*proof*⟩
lemma *confTs-hext* [*intro?*]:
P,$h \vdash loc$ [$:\leq_\top$] $LT \Longrightarrow h \trianglelefteq h' \Longrightarrow P$,$h' \vdash loc$ [$:\leq_\top$] LT
⟨*proof*⟩

lemma *confTs-widen* [*intro?*, *trans*]:
P,$h \vdash loc$ [$:\leq_\top$] $LT \Longrightarrow P \vdash LT$ [$\leq_\top$] $LT' \Longrightarrow P$,$h \vdash loc$ [$:\leq_\top$] LT'
⟨*proof*⟩

lemma *confTs-map* [*iff*]:
$\bigwedge vs.\ (P$,$h \vdash vs$ [$:\leq_\top$] $map\ OK\ Ts) = (P$,$h \vdash vs$ [$:\leq$] $Ts)$
⟨*proof*⟩

lemma *reg-widen-Err* [*iff*]:
$\bigwedge LT.\ (P \vdash replicate\ n\ Err$ [$\leq_\top$] $LT) = (LT = replicate\ n\ Err)$
⟨*proof*⟩

lemma *confTs-Err* [*iff*]:
P,$h \vdash replicate\ n\ v$ [$:\leq_\top$] $replicate\ n\ Err$
⟨*proof*⟩

4.23.3 correct-frames

lemmas [*simp del*] = *fun-upd-apply*

lemma *conf-fs-hext*:
$\bigwedge M\ n\ T_r$.
⟦ *conf-fs* $P\ h\ \Phi\ M\ n\ T_r$ *frs*; $h \trianglelefteq h'$ ⟧ $\Longrightarrow$ *conf-fs* $P\ h'\ \Phi\ M\ n\ T_r$ *frs*⟨*proof*⟩
end

4.24 BV Type Safety Proof

theory *BVSpecTypeSafe*
imports *BVConform*
begin

This theory contains proof that the specification of the bytecode verifier only admits type safe programs.

4.24.1 Preliminaries

Simp and intro setup for the type safety proof:

lemmas *defs1 = correct-state-def conf-f-def wt-instr-def eff-def norm-eff-def app-def xcpt-app-def*

lemmas *widen-rules* [*intro*] = *conf-widen confT-widen confs-widens confTs-widen*

4.24.2 Exception Handling

For the *Invoke* instruction the BV has checked all handlers that guard the current *pc*.

lemma *Invoke-handlers*:
match-ex-table P C pc xt = Some (pc′,d′) $\Longrightarrow$
$\exists$ *(f,t,D,h,d)* $\in$ *set (relevant-entries P (Invoke n M) pc xt).*
P $\vdash$ *C* $\preceq^*$ *D* $\wedge$ *pc* $\in$ {*f..<t*} $\wedge$ *pc′ = h* $\wedge$ *d′ = d*
⟨*proof*⟩

We can prove separately that the recursive search for exception handlers (*find-handler*) in the frame stack results in a conforming state (if there was no matching exception handler in the current frame). We require that the exception is a valid heap address, and that the state before the exception occurred conforms.

term *find-handler*
lemma *uncaught-xcpt-correct*:
assumes *wt*: *wf-jvm-prog*$_\Phi$ *P*
assumes *h*: *h xcp = Some obj*
shows $\bigwedge f$. *P*,Φ $\vdash$ (*None*, *h*, *f#frs*)$\surd$ $\Longrightarrow$ *P*,Φ $\vdash$ (*find-handler P xcp h frs*) $\surd$
(**is** $\bigwedge f$. *?correct* (*None*, *h*, *f#frs*) $\Longrightarrow$ *?correct* (*?find frs*))⟨*proof*⟩

The requirement of lemma *uncaught-xcpt-correct* (that the exception is a valid reference on the heap) is always met for welltyped instructions and conformant states:

lemma *exec-instr-xcpt-h*:
⟦ *fst* (*exec-instr* (*ins*!*pc*) *P h stk vars Cl M pc frs*) = *Some xcp*;
P,*T*,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ *C M*;
P,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$ ⟧
$\Longrightarrow$ $\exists$ *obj*. *h xcp = Some obj*
(**is** ⟦ *?xcpt*; *?wt*; *?correct* ⟧ $\Longrightarrow$ *?thesis*)⟨*proof*⟩
lemma *conf-sys-xcpt*:
⟦*preallocated h*; *C* $\in$ *sys-xcpts*⟧ $\Longrightarrow$ *P*,*h* $\vdash$ *Addr* (*addr-of-sys-xcpt C*) :$\leq$ *Class C*
⟨*proof*⟩

lemma *match-ex-table-SomeD*:
match-ex-table P C pc xt = Some (pc′,d′) $\Longrightarrow$
$\exists$ *(f,t,D,h,d)* $\in$ *set xt*. *matches-ex-entry P C pc* (*f*,*t*,*D*,*h*,*d*) $\wedge$ *h = pc′* $\wedge$ *d=d′*
⟨*proof*⟩

Finally we can state that, whenever an exception occurs, the next state always conforms:

lemma *xcpt-correct*:
fixes *σ′* :: *jvm-state*
assumes *wtp*: *wf-jvm-prog*$_\Phi$ *P*
assumes *meth*: *P* $\vdash$ *C sees M*:*Ts*$\rightarrow$*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*
assumes *wt*: *P*,*T*,*mxs*,*size ins*,*xt* $\vdash$ *ins*!*pc*,*pc* :: Φ *C M*
assumes *xp*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *Some xcp*
assumes *s′*: *Some σ′ = exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *correct*: *P*,Φ $\vdash$ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)$\surd$
shows *P*,Φ $\vdash$ *σ′*$\surd$⟨*proof*⟩

4.24.3 Single Instructions

In this section we prove for each single (welltyped) instruction that the state after execution of the instruction still conforms. Since we have already handled exceptions above, we can now assume that no exception occurs in this step.

declare *defs1* [*simp*]

lemma *Invoke-correct*:
fixes σ' :: *jvm-state*
assumes *wtprog*: *wf-jvm-prog*$_\Phi$ *P*
assumes *meth-C*: *P* ⊢ *C sees M*:*Ts*→*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*
assumes *ins*: *ins* ! *pc* = *Invoke M' n*
assumes *wti*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
assumes σ': *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *approx*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√
assumes *no-xcp*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*
shows *P*,Φ ⊢ σ'√⟨*proof*⟩
declare *list-all2-Cons2* [*iff*]

lemma *Return-correct*:
fixes σ' :: *jvm-state*
assumes *wt-prog*: *wf-jvm-prog*$_\Phi$ *P*
assumes *meth*: *P* ⊢ *C sees M*:*Ts*→*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*
assumes *ins*: *ins* ! *pc* = *Return*
assumes *wt*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *correct*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√

shows *P*,Φ ⊢ σ'√⟨*proof*⟩
declare *sup-state-opt-any-Some* [*iff*]
declare *not-Err-eq* [*iff*]

lemma *Load-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*;
ins!*pc* = *Load idx*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√
⟨*proof*⟩

declare [[*simproc del*: *list-to-set-comprehension*]]

lemma *Store-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*;
ins!*pc* = *Store idx*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩

lemma *Push-correct*:

⟦ *wf-prog wt P*;
  $P \vdash C$ *sees* M:$Ts \to T = (mxs, mxl_0, ins, xt)$ *in* C;
  ins!*pc* = *Push v*;
  $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$;
  Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
  $P, \Phi \vdash (None, h, (stk, loc, C, M, pc)\#frs)\surd$ ⟧
⟹ $P, \Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Cast-conf2*:
  ⟦ *wf-prog ok P*; $P, h \vdash v :\leq T$; *is-refT T*; *cast-ok P C h v*;
    $P \vdash Class\ C \leq T'$; *is-class P C*⟧
  ⟹ $P, h \vdash v :\leq T' \langle proof\rangle$

lemma *Checkcast-correct*:
⟦ *wf-jvm-prog*$_\Phi$ *P*;
  $P \vdash C$ *sees* M:$Ts \to T = (mxs, mxl_0, ins, xt)$ *in* C;
  ins!*pc* = *Checkcast D*;
  $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$;
  Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
  $P, \Phi \vdash (None, h, (stk, loc, C, M, pc)\#frs)\surd$;
  fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* ⟧
⟹ $P, \Phi \vdash \sigma'\surd\langle proof\rangle$
declare *split-paired-All* [*simp del*]

lemmas *widens-Cons* [*iff*] = *list-all2-Cons1* [*of widen P*] **for** *P*

lemma *Getfield-correct*:
  fixes σ' :: *jvm-state*
  assumes *wf*: *wf-prog wt P*
  assumes *mC*: $P \vdash C$ *sees* M:$Ts \to T = (mxs, mxl_0, ins, xt)$ *in* C
  assumes *i*: *ins*!*pc* = *Getfield F D*
  assumes *wt*: $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$
  assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
  assumes *cf*: $P, \Phi \vdash (None, h, (stk, loc, C, M, pc)\#frs)\surd$
  assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

  shows $P, \Phi \vdash \sigma'\surd\langle proof\rangle$
lemma *Putfield-correct*:
  fixes σ' :: *jvm-state*
  assumes *wf*: *wf-prog wt P*
  assumes *mC*: $P \vdash C$ *sees* M:$Ts \to T = (mxs, mxl_0, ins, xt)$ *in* C
  assumes *i*: *ins*!*pc* = *Putfield F D*
  assumes *wt*: $P, T, mxs, size\ ins, xt \vdash ins!pc, pc :: \Phi\ C\ M$
  assumes *s'*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
  assumes *cf*: $P, \Phi \vdash (None, h, (stk, loc, C, M, pc)\#frs)\surd$
  assumes *xc*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*

  shows $P, \Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *has-fields-b-fields*:
  $P \vdash C$ *has-fields FDTs* ⟹ *fields P C* = *FDTs*⟨*proof*⟩

lemma *oconf-blank* [*intro*, *simp*]:
  ⟦*is-class P C*; *wf-prog wt P*⟧ ⟹ $P, h \vdash blank\ P\ C\ \surd\langle proof\rangle$

lemma *obj-ty-blank* [*iff*]: *obj-ty* (*blank P C*) = *Class C*
⟨*proof*⟩

lemma *New-correct*:
fixes σ' :: *jvm-state*
assumes *wf*: *wf-prog wt P*
assumes *meth*: *P* ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*
assumes *ins*: *ins*!*pc* = *New X*
assumes *wt*: *P*,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*
assumes *exec*: *Some* σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)
assumes *conf*: *P*,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√
assumes *no-x*: *fst* (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None*
shows *P*,Φ ⊢ σ'√⟨*proof*⟩

lemma *Goto-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*;
ins ! *pc* = *Goto branch*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩

lemma *IfFalse-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*;
ins ! *pc* = *IfFalse branch*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩
lemma *CmpEq-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*;
ins ! *pc* = *CmpEq*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩
lemma *Pop-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*;
ins ! *pc* = *Pop*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
P,Φ ⊢ (*None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*)√ ⟧
⟹ *P*,Φ ⊢ σ'√⟨*proof*⟩

lemma *IAdd-correct*:
⟦ *wf-prog wt P*;
P ⊢ *C sees M*:*Ts*→*T*=(*mxs*,mxl_0,*ins*,*xt*) *in C*;
ins ! *pc* = *IAdd*;
P,*T*,*mxs*,*size ins*,*xt* ⊢ *ins*!*pc*,*pc* :: Φ *C M*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;

$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

lemma *Throw-correct*:
$[\![$ *wf-prog wt P*;
P $\vdash$ *C sees M*:*Ts*$\rightarrow$*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*;
ins ! *pc* = *Throw*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*) ;
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$;
fst (*exec-instr* (*ins*!*pc*) *P h stk loc C M pc frs*) = *None* $]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd$
$\langle proof\rangle$

The next theorem collects the results of the sections above, i.e. exception handling and the execution step for each instruction. It states type safety for single step execution: in welltyped programs, a conforming state is transformed into another conforming state when one instruction is executed.

theorem *instr-correct*:
$[\![$ *wf-jvm-prog*$_\Phi$ *P*;
P $\vdash$ *C sees M*:*Ts*$\rightarrow$*T*=(*mxs*,*mxl*$_0$,*ins*,*xt*) *in C*;
Some σ' = *exec* (*P*, *None*, *h*, (*stk*,*loc*,*C*,*M*,*pc*)#*frs*);
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd\ ]\!]$
$\Longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

4.24.4 Main

lemma *correct-state-impl-Some-method*:
$P,\Phi \vdash (None,\ h,\ (stk,loc,C,M,pc)\#frs)\surd$
$\Longrightarrow \exists m\ Ts\ T.\ P \vdash C\ sees\ M{:}Ts\rightarrow T = m\ in\ C$
$\langle proof\rangle$

lemma *BV-correct-1* [*rule-format*]:
$\bigwedge\sigma.\ [\![$ *wf-jvm-prog*$_\Phi$ *P*; $P,\Phi \vdash \sigma\surd]\!] \Longrightarrow P \vdash \sigma\ -jvm\rightarrow_1\ \sigma' \longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$

theorem *progress*:
$[\![$ *xp*=*None*; *frs*$\neq$[] $]\!] \Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ -jvm\rightarrow_1\ \sigma'$
$\langle proof\rangle$

lemma *progress-conform*:
$[\![$*wf-jvm-prog*$_\Phi$ *P*; $P,\Phi \vdash (xp,h,frs)\surd$; *xp*=*None*; *frs*$\neq$[]$]\!]$
$\Longrightarrow \exists\sigma'.\ P \vdash (xp,h,frs)\ -jvm\rightarrow_1\ \sigma' \wedge P,\Phi \vdash \sigma'\surd\langle proof\rangle$
theorem *BV-correct* [*rule-format*]:
$[\![$ *wf-jvm-prog*$_\Phi$ *P*; $P \vdash \sigma\ -jvm\rightarrow\ \sigma'\ ]\!] \Longrightarrow P,\Phi \vdash \sigma\surd \longrightarrow P,\Phi \vdash \sigma'\surd\langle proof\rangle$
lemma *hconf-start*:
assumes *wf*: *wf-prog wf-mb P*
shows *P* $\vdash$ (*start-heap P*) $\surd\langle proof\rangle$
lemma *BV-correct-initial*:
shows $[\![$ *wf-jvm-prog*$_\Phi$ *P*; *P* $\vdash$ *C sees M*:[]$\rightarrow$*T* = *m in C* $]\!]$
$\Longrightarrow P,\Phi \vdash$ *start-state P C M* $\surd\langle proof\rangle$
theorem *typesafe*:
assumes *welltyped*: *wf-jvm-prog*$_\Phi$ *P*
assumes *main-method*: *P* $\vdash$ *C sees M*:[]$\rightarrow$*T* = *m in C*
shows *P* $\vdash$ *start-state P C M* $-jvm\rightarrow\ \sigma\ \Longrightarrow\ P,\Phi \vdash \sigma\ \surd\langle proof\rangle$
end

4.25 Welltyped Programs produce no Type Errors

theory *BVNoTypeError*
imports *../JVM/JVMDefensive BVSpecTypeSafe*
begin

lemma *has-methodI*:
P ⊢ *C sees M*:*Ts*→*T* = *m in D* ⟹ *P* ⊢ *C has M*
⟨*proof*⟩

Some simple lemmas about the type testing functions of the defensive JVM:

lemma *typeof-NoneD* [*simp*,*dest*]: *typeof v* = *Some x* ⟹ ¬*is-Addr v*
⟨*proof*⟩

lemma *is-Ref-def2*:
is-Ref v = (*v* = *Null* ∨ (∃ *a*. *v* = *Addr a*))
⟨*proof*⟩

lemma [*iff*]: *is-Ref Null* ⟨*proof*⟩

lemma *is-RefI* [*intro*, *simp*]: *P*,*h* ⊢ *v* :≤ *T* ⟹ *is-refT T* ⟹ *is-Ref v*⟨*proof*⟩
lemma *is-IntgI* [*intro*, *simp*]: *P*,*h* ⊢ *v* :≤ *Integer* ⟹ *is-Intg v*⟨*proof*⟩
lemma *is-BoolI* [*intro*, *simp*]: *P*,*h* ⊢ *v* :≤ *Boolean* ⟹ *is-Bool v*⟨*proof*⟩
declare *defs1* [*simp del*]

lemma *wt-jvm-prog-states*:
⟦ *wf-jvm-prog*$_\Phi$ *P*; *P* ⊢ *C sees M*: *Ts*→*T* = (*mxs*, *mxl*, *ins*, *et*) *in C*;
Φ *C M* ! *pc* = τ; *pc* < *size ins* ⟧
⟹ *OK* τ ∈ *states P mxs* (1+*size Ts*+*mxl*)⟨*proof*⟩

The main theorem: welltyped programs do not produce type errors if they are started in a conformant state.

theorem *no-type-error*:
fixes σ :: *jvm-state*
assumes *welltyped*: *wf-jvm-prog*$_\Phi$ *P* **and** *conforms*: *P*,Φ ⊢ σ √
shows *exec-d P* σ ≠ *TypeError*⟨*proof*⟩

The theorem above tells us that, in welltyped programs, the defensive machine reaches the same result as the aggressive one (after arbitrarily many steps).

theorem *welltyped-aggressive-imp-defensive*:
wf-jvm-prog$_\Phi$ *P* ⟹ *P*,Φ ⊢ σ √ ⟹ *P* ⊢ σ −*jvm*→ σ′
⟹ *P* ⊢ (*Normal* σ) −*jvmd*→ (*Normal* σ′)⟨*proof*⟩

As corollary we get that the aggressive and the defensive machine are equivalent for welltyped programs (if started in a conformant state or in the canonical start state)

corollary *welltyped-commutes*:
fixes σ :: *jvm-state*
assumes *wf*: *wf-jvm-prog*$_\Phi$ *P* **and** *conforms*: *P*,Φ ⊢ σ √
shows *P* ⊢ (*Normal* σ) −*jvmd*→ (*Normal* σ′) = *P* ⊢ σ −*jvm*→ σ′
⟨*proof*⟩

corollary *welltyped-initial-commutes*:
assumes *wf*: *wf-jvm-prog P*
assumes *meth*: *P* ⊢ *C sees M*:[]→*T* = *b in C*

defines *start*: $\sigma \equiv$ *start-state P C M*
shows $P \vdash (Normal\ \sigma)\ -jvmd\rightarrow\ (Normal\ \sigma') = P \vdash \sigma\ -jvm\rightarrow\ \sigma'$
⟨*proof*⟩

lemma *not-TypeError-eq* [*iff*]:
$x \neq TypeError = (\exists\, t.\ x = Normal\ t)$
⟨*proof*⟩

locale *cnf* =
fixes *P* **and** Φ **and** σ
assumes *wf*: *wf-jvm-prog*$_\Phi$ *P*
assumes *cnf*: *correct-state P* Φ σ

theorem (**in** *cnf*) *no-type-errors*:
$P \vdash (Normal\ \sigma)\ -jvmd\rightarrow\ \sigma' \Longrightarrow \sigma' \neq TypeError$
⟨*proof*⟩

locale *start* =
fixes *P* **and** *C* **and** *M* **and** σ **and** *T* **and** *b*
assumes *wf*: *wf-jvm-prog P*
assumes *sees*: $P \vdash C\ sees\ M{:}[]\rightarrow T = b\ in\ C$
defines $\sigma \equiv Normal\ (start\text{-}state\ P\ C\ M)$

corollary (**in** *start*) *bv-no-type-error*:
shows $P \vdash \sigma\ -jvmd\rightarrow\ \sigma' \Longrightarrow \sigma' \neq TypeError$
⟨*proof*⟩

end

4.26 Example Welltypings

theory *BVExample*
imports *../JVM/JVMListExample BVSpecTypeSafe BVExec*
HOL−Library.Code-Target-Numeral
begin

This theory shows type correctness of the example program in section 3.7 (p. 70) by explicitly providing a welltyping. It also shows that the start state of the program conforms to the welltyping; hence type safe execution is guaranteed.

4.26.1 Setup

lemma *distinct-classes'*:
list-name ≠ *test-name*
list-name ≠ *Object*
list-name ≠ *ClassCast*
list-name ≠ *OutOfMemory*
list-name ≠ *NullPointer*
test-name ≠ *Object*
test-name ≠ *OutOfMemory*
test-name ≠ *ClassCast*

test-name ≠ *NullPointer*
ClassCast ≠ *NullPointer*
ClassCast ≠ *Object*
NullPointer ≠ *Object*
OutOfMemory ≠ *ClassCast*
OutOfMemory ≠ *NullPointer*
OutOfMemory ≠ *Object*
⟨*proof*⟩

lemmas *distinct-classes* = *distinct-classes'* *distinct-classes'* [*symmetric*]

lemma *distinct-fields*:
val-name ≠ *next-name*
next-name ≠ *val-name*
⟨*proof*⟩

Abbreviations for definitions we will have to use often in the proofs below:

lemmas *system-defs* = *SystemClasses-def ObjectC-def NullPointerC-def*
OutOfMemoryC-def ClassCastC-def
lemmas *class-defs* = *list-class-def test-class-def*

These auxiliary proofs are for efficiency: class lookup, subclass relation, method and field lookup are computed only once:

lemma *class-Object* [*simp*]:
class E Object = *Some* (*undefined*, [],[])
⟨*proof*⟩

lemma *class-NullPointer* [*simp*]:
class E NullPointer = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-OutOfMemory* [*simp*]:
class E OutOfMemory = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-ClassCast* [*simp*]:
class E ClassCast = *Some* (*Object*, [], [])
⟨*proof*⟩

lemma *class-list* [*simp*]:
class E list-name = *Some list-class*
⟨*proof*⟩

lemma *class-test* [*simp*]:
class E test-name = *Some test-class*
⟨*proof*⟩

lemma *E-classes* [*simp*]:
{*C*. *is-class E C*} = {*list-name*, *test-name*, *NullPointer*,
ClassCast, *OutOfMemory*, *Object*}
⟨*proof*⟩

The subclass releation spelled out:

lemma *subcls1*:

subcls1 E = {(*list-name*,*Object*), (*test-name*,*Object*), (*NullPointer*, *Object*), (*ClassCast*, *Object*), (*OutOfMemory*, *Object*)}⟨*proof*⟩

The subclass relation is acyclic; hence its converse is well founded:

lemma *notin-rtrancl*:
(*a*,*b*) ∈ *r** ⟹ *a* ≠ *b* ⟹ (⋀*y*. (*a*,*y*) ∉ *r*) ⟹ *False*
⟨*proof*⟩

lemma *acyclic-subcls1-E*: *acyclic* (*subcls1 E*)⟨*proof*⟩
lemma *wf-subcls1-E*: *wf* ((*subcls1 E*)$^{-1}$)⟨*proof*⟩

Method and field lookup:

lemma *method-append* [*simp*]:
method E list-name append-name =
(*list-name*, [*Class list-name*], *Void*, *3*, *0*, *append-ins*, [(*1*, *2*, *NullPointer*, *7*, *0*)])⟨*proof*⟩
lemma *method-makelist* [*simp*]:
method E test-name makelist-name =
(*test-name*, [], *Void*, *3*, *2*, *make-list-ins*, [])⟨*proof*⟩
lemma *field-val* [*simp*]:
field E list-name val-name = (*list-name*, *Integer*)⟨*proof*⟩
lemma *field-next* [*simp*]:
field E list-name next-name = (*list-name*, *Class list-name*)⟨*proof*⟩
lemma [*simp*]: *fields E Object* = []
⟨*proof*⟩

lemma [*simp*]: *fields E NullPointer* = []
⟨*proof*⟩

lemma [*simp*]: *fields E ClassCast* = []
⟨*proof*⟩

lemma [*simp*]: *fields E OutOfMemory* = []
⟨*proof*⟩

lemma [*simp*]: *fields E test-name* = []⟨*proof*⟩
lemmas [*simp*] = *is-class-def*

4.26.2 Program structure

The program is structurally wellformed:

lemma *wf-struct*:
wf-prog (λ*G C mb*. *True*) *E* (**is** *wf-prog ?mb E*)⟨*proof*⟩

4.26.3 Welltypings

We show welltypings of the methods *append-name* in class *list-name*, and *makelist-name* in class *test-name*:

lemmas *eff-simps* [*simp*] = *eff-def norm-eff-def xcpt-eff-def*

definition *phi-append* :: ty_m (φ_a)
where
φ_a ≡ *map* (λ(*x*,*y*). *Some* (*x*, *map OK y*)) [

```
(                                  [], [Class list-name, Class list-name]),
(                        [Class list-name], [Class list-name, Class list-name]),
(                        [Class list-name], [Class list-name, Class list-name]),
(     [Class list-name, Class list-name], [Class list-name, Class list-name]),
(     [Class list-name, Class list-name], [Class list-name, Class list-name]),
([NT, Class list-name, Class list-name], [Class list-name, Class list-name]),
(              [Boolean, Class list-name], [Class list-name, Class list-name]),

(                      [Class Object], [Class list-name, Class list-name]),
(                                  [], [Class list-name, Class list-name]),
(                        [Class list-name], [Class list-name, Class list-name]),
(     [Class list-name, Class list-name], [Class list-name, Class list-name]),
(                                  [], [Class list-name, Class list-name]),
(                               [Void], [Class list-name, Class list-name]),

(                        [Class list-name], [Class list-name, Class list-name]),
(     [Class list-name, Class list-name], [Class list-name, Class list-name]),
(                               [Void], [Class list-name, Class list-name])]
```

The next definition and three proof rules implement an algorithm to enumarate natural numbers. The command *apply* (*elim pc-end pc-next pc-0* transforms a goal of the form

$pc < n \Longrightarrow P\ pc$

into a series of goals

$P\ (0::'a)$

$P\ (Suc\ 0)$

...

$P\ n$

```
definition intervall :: nat ⇒ nat ⇒ nat ⇒ bool (- ∈ [-, -'))
where
  x ∈ [a, b) ≡ a ≤ x ∧ x < b

lemma pc-0: x < n ⟹ (x ∈ [0, n) ⟹ P x) ⟹ P x
  ⟨proof⟩

lemma pc-next: x ∈ [n0, n) ⟹ P n0 ⟹ (x ∈ [Suc n0, n) ⟹ P x) ⟹ P x⟨proof⟩
lemma pc-end: x ∈ [n,n) ⟹ P x
  ⟨proof⟩


lemma types-append [simp]: check-types E 3 (Suc (Suc 0)) (map OK φa)⟨proof⟩
lemma wt-append [simp]:
  wt-method E list-name [Class list-name] Void 3 0 append-ins
          [(Suc 0, 2, NullPointer, 7, 0)] φa⟨proof⟩
```

Some abbreviations for readability

```
abbreviation Clist == Class list-name
abbreviation Ctest == Class test-name
```

definition *phi-makelist* :: ty_m (φ_m)
where
$\varphi_m \equiv$ *map* (λ(*x*,*y*). *Some* (*x*, *y*)) [
([], [*OK Ctest*, *Err* , *Err*]),
([*Clist*], [*OK Ctest*, *Err* , *Err*]),
([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *Err* , *Err*]),

([], [*OK Clist*, *Err* , *Err*]),
([*Clist*], [*OK Clist*, *Err* , *Err*]),
([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *Err*]),

([], [*OK Clist*, *OK Clist*, *Err*]),
([*Clist*], [*OK Clist*, *OK Clist*, *Err*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Integer*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),

([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Clist*, *Clist*], [*OK Clist*, *OK Clist*, *OK Clist*]),
([*Void*], [*OK Clist*, *OK Clist*, *OK Clist*])]

lemma *types-makelist* [*simp*]: *check-types E 3* (*Suc* (*Suc* (*Suc 0*))) (*map OK* φ_m)⟨*proof*⟩
lemma *wt-makelist* [*simp*]:
wt-method E test-name [] *Void 3 2 make-list-ins* [] φ_m⟨*proof*⟩
lemma *wf-md'E*:
⟦ *wf-prog wf-md P*;
⋀*C S fs ms m*.⟦(*C*,*S*,*fs*,*ms*) ∈ *set P*; *m* ∈ *set ms*⟧ ⟹ *wf-md' P C m* ⟧
⟹ *wf-prog wf-md' P*⟨*proof*⟩

The whole program is welltyped:

definition *Phi* :: ty_P (Φ)
where
Φ *C mn* ≡ *if C* = *test-name* ∧ *mn* = *makelist-name then* φ_m *else*
if C = *list-name* ∧ *mn* = *append-name then* φ_a *else* []

lemma *wf-prog*:
wf-jvm-prog$_\Phi$ *E*⟨*proof*⟩

4.26.4 Conformance

Execution of the program will be typesafe, because its start state conforms to the welltyping:

lemma *E*,Φ ⊢ *start-state E test-name makelist-name* √⟨*proof*⟩

4.26.5 Example for code generation: inferring method types

definition *test-kil* :: *jvm-prog* ⇒ *cname* ⇒ *ty list* ⇒ *ty* ⇒ *nat* ⇒ *nat* ⇒
ex-table ⇒ *instr list* ⇒ ty_i' *err list*
where
test-kil G C pTs rT mxs mxl et instr ≡
(*let first* = *Some* ([],(*OK* (*Class C*))#(*map OK pTs*)@(*replicate mxl Err*));
start = *OK first*#(*replicate* (*size instr* − 1) (*OK None*))
in kiljvm G mxs (1+*size pTs*+*mxl*) *rT instr et start*)

lemma [*code*]:
unstables r step ss =
fold (λ*p A*. *if* ¬*stable r step ss p then insert p A else A*) [*0*..<*size ss*] {}
⟨*proof*⟩

definition *some-elem* :: *'a set* ⇒ *'a* **where** [*code del*]:
some-elem = (%*S*. *SOME x*. *x* : *S*)
code-printing
constant *some-elem* ⇀ (*SML*) (*case*/ - *of*/ *Set*/ *xs*/ =>/ *hd*/ *xs*)

This code setup is just a demonstration and *not* sound!

notepad begin
⟨*proof*⟩
end

lemma [*code*]:
iter f step ss w = *while* (λ(*ss*, *w*). ¬ *Set.is-empty w*)
(λ(*ss*, *w*).
let p = *some-elem w in propa f* (*step p* (*ss* ! *p*)) *ss* (*w* − {*p*}))
(*ss*, *w*)
⟨*proof*⟩

lemma *JVM-sup-unfold* [*code*]:
JVM-SemiType.sup S m n = *lift2* (*Opt.sup*
(*Product.sup* (*Listn.sup* (*SemiType.sup S*))
(λ*x y*. *OK* (*map2* (*lift2* (*SemiType.sup S*)) *x y*))))
⟨*proof*⟩

lemmas [*code*] = *SemiType.sup-def* [*unfolded exec-lub-def*] *JVM-le-unfold*

lemmas [*code*] = *lesub-def plussub-def*

lemma [*code*]:
is-refT T = (*case T of NT* ⇒ *True* | *Class C* ⇒ *True* | - ⇒ *False*)
⟨*proof*⟩

declare app_i.*simps* [*code*]

lemma [*code*]:
app_i (*Getfield F C*, *P*, *pc*, *mxs*, T_r, (*T*#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if P* ⊢ *T* ≤ *Class C then Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:
app$_i$ (*Putfield F C*, *P*, *pc*, *mxs*, T_r, (T_1#T_2#*ST*, *LT*)) =
Predicate.holds (*Predicate.bind* (*sees-field-i-i-i-o-i P C F C*) (λT_f. *if P* ⊢ T_2 ≤ (*Class C*) ∧ *P* ⊢ T_1 ≤ T_f *then Predicate.single* () *else bot*))
⟨*proof*⟩

lemma [*code*]:
app$_i$ (*Invoke M n*, *P*, *pc*, *mxs*, T_r, (*ST*,*LT*)) =
(*n* < *length ST* ∧
(*ST*!*n* ≠ *NT* ⟶
(*case ST*!*n of*
Class C ⇒ *Predicate.holds* (*Predicate.bind* (*Method-i-i-i-o-o-o-o P C M*) (λ(*Ts*, *T*, *m*, *D*). *if P* ⊢ *rev* (*take n ST*) [≤] *Ts then Predicate.single* () *else bot*))
| - ⇒ *False*)))
⟨*proof*⟩

lemmas [*code*] =
SemiType.sup-def [*unfolded exec-lub-def*]
widen.equation
is-relevant-class.simps

definition *test1* **where**
test1 = *test-kil E list-name* [*Class list-name*] *Void 3 0*
[(*Suc 0*, *2*, *NullPointer*, *7*, *0*)] *append-ins*
definition *test2* **where**
test2 = *test-kil E test-name* [] *Void 3 2* [] *make-list-ins*
definition *test3* **where** *test3* = φ_a
definition *test4* **where** *test4* = φ_m

⟨*ML*⟩

end

Chapter 5

Compilation

5.1 An Intermediate Language

theory *J1* **imports** *../J/BigStep* **begin**

type-synonym $expr_1 = nat\ exp$
type-synonym $J_1\text{-}prog = expr_1\ prog$
type-synonym $state_1 = heap \times (val\ list)$

primrec
  max-vars :: *'a exp* ⇒ *nat*
  and *max-varss* :: *'a exp list* ⇒ *nat*
where
  max-vars(*new C*) = *0*
| *max-vars*(*Cast C e*) = *max-vars e*
| *max-vars*(*Val v*) = *0*
| *max-vars*(e_1 «*bop*» e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*Var V*) = *0*
| *max-vars*(*V*:=*e*) = *max-vars e*
| *max-vars*(*e*·*F*{*D*}) = *max-vars e*
| *max-vars*(*FAss* e_1 *F D* e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*e*·*M*(*es*)) = *max* (*max-vars e*) (*max-varss es*)
| *max-vars*({*V*:*T*; *e*}) = *max-vars e* + *1*
| *max-vars*(e_1;;e_2) = *max* (*max-vars* e_1) (*max-vars* e_2)
| *max-vars*(*if* (*e*) e_1 *else* e_2) =
   max (*max-vars e*) (*max* (*max-vars* e_1) (*max-vars* e_2))
| *max-vars*(*while* (*b*) *e*) = *max* (*max-vars b*) (*max-vars e*)
| *max-vars*(*throw e*) = *max-vars e*
| *max-vars*(*try* e_1 *catch*(*C V*) e_2) = *max* (*max-vars* e_1) (*max-vars* e_2 + *1*)

| *max-varss* [] = *0*
| *max-varss* (*e*#*es*) = *max* (*max-vars e*) (*max-varss es*)

inductive
  $eval_1$ :: $J_1\text{-}prog \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow expr_1 \Rightarrow state_1 \Rightarrow bool$
        (- ⊢$_1$ ((*1*⟨-,/-⟩) ⇒/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  and $evals_1$ :: $J_1\text{-}prog \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow expr_1\ list \Rightarrow state_1 \Rightarrow bool$
         (- ⊢$_1$ ((*1*⟨-,/-⟩) [⇒]/ (*1*⟨-,/-⟩)) [*51*,*0*,*0*,*0*,*0*] *81*)
  for P :: $J_1\text{-}prog$
where

New_1:
$[\![$ *new-Addr* h = *Some* a; $P \vdash C$ *has-fields FDTs*; $h' = h(a \mapsto (C, \textit{init-fields FDTs}))$ $]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{new } C, (h,l)\rangle \Rightarrow \langle \textit{addr } a, (h',l)\rangle$
| $NewFail_1$:
new-Addr h = *None* $\Longrightarrow$
$P \vdash_1 \langle \textit{new } C, (h,l)\rangle \Rightarrow \langle \textit{THROW OutOfMemory}, (h,l)\rangle$

| $Cast_1$:
$[\![P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{addr } a, (h,l)\rangle$; $h\ a$ = *Some*(D,fs); $P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{Cast } C\ e, s_0\rangle \Rightarrow \langle \textit{addr } a, (h,l)\rangle$
| $CastNull_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{null}, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \textit{Cast } C\ e, s_0\rangle \Rightarrow \langle \textit{null}, s_1\rangle$
| $CastFail_1$:
$[\![P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{addr } a, (h,l)\rangle$; $h\ a$ = *Some*(D,fs); $\neg\ P \vdash D \preceq^* C$ $]\!]$
$\Longrightarrow P \vdash_1 \langle \textit{Cast } C\ e, s_0\rangle \Rightarrow \langle \textit{THROW ClassCast}, (h,l)\rangle$
| $CastThrow_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \textit{Cast } C\ e, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle$

| Val_1:
$P \vdash_1 \langle \textit{Val } v, s\rangle \Rightarrow \langle \textit{Val } v, s\rangle$

| $BinOp_1$:
$[\![P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \textit{Val } v_1, s_1\rangle$; $P \vdash_1 \langle e_2, s_1\rangle \Rightarrow \langle \textit{Val } v_2, s_2\rangle$; $\textit{binop}(bop, v_1, v_2) = \textit{Some } v$ $]\!]$
$\Longrightarrow P \vdash_1 \langle e_1\ \textit{«bop»}\ e_2, s_0\rangle \Rightarrow \langle \textit{Val } v, s_2\rangle$
| $BinOpThrow_{11}$:
$P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \textit{throw } e, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1\ \textit{«bop»}\ e_2, s_0\rangle \Rightarrow \langle \textit{throw } e, s_1\rangle$
| $BinOpThrow_{21}$:
$[\![P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \textit{Val } v_1, s_1\rangle$; $P \vdash_1 \langle e_2, s_1\rangle \Rightarrow \langle \textit{throw } e, s_2\rangle$ $]\!]$
$\Longrightarrow P \vdash_1 \langle e_1\ \textit{«bop»}\ e_2, s_0\rangle \Rightarrow \langle \textit{throw } e, s_2\rangle$

| Var_1:
$[\![ls!i = v$; $i < \textit{size } ls$ $]\!] \Longrightarrow$
$P \vdash_1 \langle \textit{Var } i, (h,ls)\rangle \Rightarrow \langle \textit{Val } v, (h,ls)\rangle$

| $LAss_1$:
$[\![P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{Val } v, (h,ls)\rangle$; $i < \textit{size } ls$; $ls' = ls[i := v]$ $]\!]$
$\Longrightarrow P \vdash_1 \langle i := e, s_0\rangle \Rightarrow \langle \textit{unit}, (h,ls')\rangle$
| $LAssThrow_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle i := e, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle$

| $FAcc_1$:
$[\![P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{addr } a, (h,ls)\rangle$; $h\ a$ = *Some*(C,fs); $fs(F,D)$ = *Some* v $]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}F\{D\}, s_0\rangle \Rightarrow \langle \textit{Val } v, (h,ls)\rangle$
| $FAccNull_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{null}, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\}, s_0\rangle \Rightarrow \langle \textit{THROW NullPointer}, s_1\rangle$
| $FAccThrow_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}F\{D\}, s_0\rangle \Rightarrow \langle \textit{throw } e', s_1\rangle$

| $FAss_1$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,(h_2,l_2)\rangle;$
$h_2\ a = Some(C,fs);\ fs' = fs((F,D)\mapsto v);\ h_2' = h_2(a\mapsto(C,fs'))\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}:= e_2,s_0\rangle \Rightarrow \langle unit,(h_2',l_2)\rangle$
| $FAssNull_1$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ \ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle Val\ v,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}:= e_2,s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $FAssThrow_{11}$:
$P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_1{\cdot}F\{D\}:= e_2,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $FAssThrow_{21}$:
$[\![P \vdash_1 \langle e_1,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle throw\ e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_1{\cdot}F\{D\}:= e_2,s_0\rangle \Rightarrow \langle throw\ e',s_2\rangle$

| $CallObjThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$
| $CallNull_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle null,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle THROW\ NullPointer,s_2\rangle$
| $Call_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle addr\ a,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle map\ Val\ vs,(h_2,ls_2)\rangle;$
$h_2\ a = Some(C,fs);\ P \vdash C\ sees\ M{:}Ts\to T = body\ in\ D;$
$size\ vs = size\ Ts;\ ls_2' = (Addr\ a)\ \#\ vs\ @\ replicate\ (max\text{-}vars\ body)\ undefined;$
$P \vdash_1 \langle body,(h_2,ls_2')\rangle \Rightarrow \langle e',(h_3,ls_3)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle e',(h_3,ls_2)\rangle$
| $CallParamsThrow_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle es,s_1\rangle\ [\Rightarrow]\ \langle es',s_2\rangle;$
$es' = map\ Val\ vs\ @\ throw\ ex\ \#\ es_2\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e{\cdot}M(es),s_0\rangle \Rightarrow \langle throw\ ex,s_2\rangle$

| $Block_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle e',s_1\rangle \Longrightarrow P \vdash_1 \langle Block\ i\ T\ e,s_0\rangle \Rightarrow \langle e',s_1\rangle$

| Seq_1:
$[\![P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle Val\ v,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e_2,s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle e_2,s_2\rangle$
| $SeqThrow_1$:
$P \vdash_1 \langle e_0,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e_0;;e_1,s_0\rangle \Rightarrow \langle throw\ e,s_1\rangle$

| $CondT_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle true,s_1\rangle;\ P \vdash_1 \langle e_1,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondF_1$:
$[\![P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle;\ P \vdash_1 \langle e_2,s_1\rangle \Rightarrow \langle e',s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,s_0\rangle \Rightarrow \langle e',s_2\rangle$
| $CondThrow_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle \Longrightarrow$
$P \vdash_1 \langle if\ (e)\ e_1\ else\ e_2,\ s_0\rangle \Rightarrow \langle throw\ e',s_1\rangle$

| $WhileF_1$:
$P \vdash_1 \langle e,s_0\rangle \Rightarrow \langle false,s_1\rangle \Longrightarrow$

$P \vdash_1 \langle \mathit{while}\ (e)\ c, s_0\rangle \Rightarrow \langle \mathit{unit}, s_1\rangle$
| *WhileT*$_1$:
$[\![\ P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{true}, s_1\rangle;\ P \vdash_1 \langle c, s_1\rangle \Rightarrow \langle \mathit{Val}\ v_1, s_2\rangle;$
$P \vdash_1 \langle \mathit{while}\ (e)\ c, s_2\rangle \Rightarrow \langle e_3, s_3\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \mathit{while}\ (e)\ c, s_0\rangle \Rightarrow \langle e_3, s_3\rangle$
| *WhileCondThrow*$_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \mathit{while}\ (e)\ c, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_1\rangle$
| *WhileBodyThrow*$_1$:
$[\![\ P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{true}, s_1\rangle;\ P \vdash_1 \langle c, s_1\rangle \Rightarrow \langle \mathit{throw}\ e', s_2\rangle]\!]$
$\Longrightarrow P \vdash_1 \langle \mathit{while}\ (e)\ c, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_2\rangle$

| *Throw*$_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{addr}\ a, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \mathit{throw}\ e, s_0\rangle \Rightarrow \langle \mathit{Throw}\ a, s_1\rangle$
| *ThrowNull*$_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{null}, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \mathit{throw}\ e, s_0\rangle \Rightarrow \langle \mathit{THROW}\ \mathit{NullPointer}, s_1\rangle$
| *ThrowThrow*$_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \mathit{throw}\ e, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_1\rangle$

| *Try*$_1$:
$P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \mathit{Val}\ v_1, s_1\rangle \Longrightarrow$
$P \vdash_1 \langle \mathit{try}\ e_1\ \mathit{catch}(C\ i)\ e_2, s_0\rangle \Rightarrow \langle \mathit{Val}\ v_1, s_1\rangle$
| *TryCatch*$_1$:
$[\![\ P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \mathit{Throw}\ a, (h_1, ls_1)\rangle;$
$h_1\ a = \mathit{Some}(D, fs);\ P \vdash D \preceq^* C;\ i < \mathit{length}\ ls_1;$
$P \vdash_1 \langle e_2, (h_1, ls_1[i := \mathit{Addr}\ a])\rangle \Rightarrow \langle e_2', (h_2, ls_2)\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \mathit{try}\ e_1\ \mathit{catch}(C\ i)\ e_2, s_0\rangle \Rightarrow \langle e_2', (h_2, ls_2)\rangle$
| *TryThrow*$_1$:
$[\![\ P \vdash_1 \langle e_1, s_0\rangle \Rightarrow \langle \mathit{Throw}\ a, (h_1, ls_1)\rangle;\ h_1\ a = \mathit{Some}(D, fs);\ \neg\ P \vdash D \preceq^* C\ ]\!]$
$\Longrightarrow P \vdash_1 \langle \mathit{try}\ e_1\ \mathit{catch}(C\ i)\ e_2, s_0\rangle \Rightarrow \langle \mathit{Throw}\ a, (h_1, ls_1)\rangle$

| *Nil*$_1$:
$P \vdash_1 \langle [], s\rangle\ [\Rightarrow]\ \langle [], s\rangle$

| *Cons*$_1$:
$[\![\ P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{Val}\ v, s_1\rangle;\ P \vdash_1 \langle es, s_1\rangle\ [\Rightarrow]\ \langle es', s_2\rangle\ ]\!]$
$\Longrightarrow P \vdash_1 \langle e \# es, s_0\rangle\ [\Rightarrow]\ \langle \mathit{Val}\ v\ \#\ es', s_2\rangle$
| *ConsThrow*$_1$:
$P \vdash_1 \langle e, s_0\rangle \Rightarrow \langle \mathit{throw}\ e', s_1\rangle \Longrightarrow$
$P \vdash_1 \langle e \# es, s_0\rangle\ [\Rightarrow]\ \langle \mathit{throw}\ e'\ \#\ es,\ s_1\rangle$

lemma *eval*$_1$*-preserves-len*:
$P \vdash_1 \langle e_0, (h_0, ls_0)\rangle \Rightarrow \langle e_1, (h_1, ls_1)\rangle \Longrightarrow \mathit{length}\ ls_0 = \mathit{length}\ ls_1$
and *evals*$_1$*-preserves-len*:
$P \vdash_1 \langle es_0, (h_0, ls_0)\rangle\ [\Rightarrow]\ \langle es_1, (h_1, ls_1)\rangle \Longrightarrow \mathit{length}\ ls_0 = \mathit{length}\ ls_1 \langle \mathit{proof}\rangle$

lemma *evals*$_1$*-preserves-elen*:
$\bigwedge es'\ s\ s'.\ P \vdash_1 \langle es, s\rangle\ [\Rightarrow]\ \langle es', s'\rangle \Longrightarrow \mathit{length}\ es = \mathit{length}\ es' \langle \mathit{proof}\rangle$

lemma *eval*$_1$*-final*: $P \vdash_1 \langle e, s\rangle \Rightarrow \langle e', s'\rangle \Longrightarrow \mathit{final}\ e'$
and *evals*$_1$*-final*: $P \vdash_1 \langle es, s\rangle\ [\Rightarrow]\ \langle es', s'\rangle \Longrightarrow \mathit{finals}\ es' \langle \mathit{proof}\rangle$

end

5.2 Well-Formedness of Intermediate Language

theory *J1WellForm*
imports *../J/JWellForm J1*
begin

5.2.1 Well-Typedness

type-synonym
env$_1$ = *ty list* — type environment indexed by variable number

inductive
WT$_1$ :: [*J*$_1$-*prog*,*env*$_1$, *expr*$_1$, *ty*] ⇒ *bool*
((-,- ⊢$_1$/ - :: -) [*51*,*51*,*51*]*50*)
and *WTs*$_1$ :: [*J*$_1$-*prog*,*env*$_1$, *expr*$_1$ *list*, *ty list*] ⇒ *bool*
((-,- ⊢$_1$/ - [::] -) [*51*,*51*,*51*]*50*)
for *P* :: *J*$_1$-*prog*
where

WTNew$_1$:
is-class P C ⟹
P,*E* ⊢$_1$ *new C* :: *Class C*

| *WTCast*$_1$:
⟦ *P*,*E* ⊢$_1$ *e* :: *Class D*; *is-class P C*; *P* ⊢ *C* ≼* *D* ∨ *P* ⊢ *D* ≼* *C* ⟧
⟹ *P*,*E* ⊢$_1$ *Cast C e* :: *Class C*

| *WTVal*$_1$:
typeof v = *Some T* ⟹
P,*E* ⊢$_1$ *Val v* :: *T*

| *WTVar*$_1$:
⟦ *E*!*i* = *T*; *i* < *size E* ⟧
⟹ *P*,*E* ⊢$_1$ *Var i* :: *T*

| *WTBinOp*$_1$:
⟦ *P*,*E* ⊢$_1$ *e*$_1$ :: *T*$_1$; *P*,*E* ⊢$_1$ *e*$_2$ :: *T*$_2$;
case bop of Eq ⇒ (*P* ⊢ *T*$_1$ ≤ *T*$_2$ ∨ *P* ⊢ *T*$_2$ ≤ *T*$_1$) ∧ *T* = *Boolean*
| *Add* ⇒ *T*$_1$ = *Integer* ∧ *T*$_2$ = *Integer* ∧ *T* = *Integer* ⟧
⟹ *P*,*E* ⊢$_1$ *e*$_1$ «*bop*» *e*$_2$ :: *T*

| *WTLAss*$_1$:
⟦ *E*!*i* = *T*; *i* < *size E*; *P*,*E* ⊢$_1$ *e* :: *T*′; *P* ⊢ *T*′ ≤ *T* ⟧
⟹ *P*,*E* ⊢$_1$ *i*:=*e* :: *Void*

| *WTFAcc*$_1$:
⟦ *P*,*E* ⊢$_1$ *e* :: *Class C*; *P* ⊢ *C sees F*:*T in D* ⟧
⟹ *P*,*E* ⊢$_1$ *e*·*F*{*D*} :: *T*

| *WTFAss*$_1$:
⟦ *P*,*E* ⊢$_1$ *e*$_1$ :: *Class C*; *P* ⊢ *C sees F*:*T in D*; *P*,*E* ⊢$_1$ *e*$_2$ :: *T*′; *P* ⊢ *T*′ ≤ *T* ⟧

$\Longrightarrow P,E \vdash_1 e_1{\cdot}F\{D\} := e_2 :: Void$

| $WTCall_1$:
$[\![P,E \vdash_1 e :: Class\ C;\ P \vdash C\ sees\ M{:}Ts' \to T = m\ in\ D;$
$P,E \vdash_1 es\ [::]\ Ts;\ P \vdash Ts\ [\leq]\ Ts']\!]$
$\Longrightarrow P,E \vdash_1 e{\cdot}M(es) :: T$

| $WTBlock_1$:
$[\![is\text{-}type\ P\ T;\ P,E@[T] \vdash_1 e{::}T']\!]$
$\Longrightarrow P,E \vdash_1 \{i{:}T;\ e\} :: T'$

| $WTSeq_1$:
$[\![P,E \vdash_1 e_1{::}T_1;\ P,E \vdash_1 e_2{::}T_2]\!]$
$\Longrightarrow P,E \vdash_1 e_1;;e_2 :: T_2$

| $WTCond_1$:
$[\![P,E \vdash_1 e :: Boolean;\ P,E \vdash_1 e_1{::}T_1;\ P,E \vdash_1 e_2{::}T_2;$
$P \vdash T_1 \leq T_2 \vee P \vdash T_2 \leq T_1;\ P \vdash T_1 \leq T_2 \longrightarrow T = T_2;\ P \vdash T_2 \leq T_1 \longrightarrow T = T_1]\!]$
$\Longrightarrow P,E \vdash_1 if\ (e)\ e_1\ else\ e_2 :: T$

| $WTWhile_1$:
$[\![P,E \vdash_1 e :: Boolean;\ P,E \vdash_1 c{::}T]\!]$
$\Longrightarrow P,E \vdash_1 while\ (e)\ c :: Void$

| $WTThrow_1$:
$P,E \vdash_1 e :: Class\ C \Longrightarrow$
$P,E \vdash_1 throw\ e :: Void$

| $WTTry_1$:
$[\![P,E \vdash_1 e_1 :: T;\ P,E@[Class\ C] \vdash_1 e_2 :: T;\ is\text{-}class\ P\ C]\!]$
$\Longrightarrow P,E \vdash_1 try\ e_1\ catch(C\ i)\ e_2 :: T$

| $WTNil_1$:
$P,E \vdash_1 [] [::] []$

| $WTCons_1$:
$[\![P,E \vdash_1 e :: T;\ P,E \vdash_1 es\ [::]\ Ts]\!]$
$\Longrightarrow P,E \vdash_1 e\#es\ [::]\ T\#Ts$

lemma $WTs_1\text{-}same\text{-}size$: $\bigwedge Ts.\ P,E \vdash_1 es\ [::]\ Ts \Longrightarrow size\ es = size\ Ts\langle proof\rangle$

lemma $WT_1\text{-}unique$:
$P,E \vdash_1 e :: T_1 \Longrightarrow (\bigwedge T_2.\ P,E \vdash_1 e :: T_2 \Longrightarrow T_1 = T_2)$ **and**
$P,E \vdash_1 es\ [::]\ Ts_1 \Longrightarrow (\bigwedge Ts_2.\ P,E \vdash_1 es\ [::]\ Ts_2 \Longrightarrow Ts_1 = Ts_2)\langle proof\rangle$

lemma assumes wf: $wf\text{-}prog\ p\ P$
shows $WT_1\text{-}is\text{-}type$: $P,E \vdash_1 e :: T \Longrightarrow set\ E \subseteq types\ P \Longrightarrow is\text{-}type\ P\ T$
and $P,E \vdash_1 es\ [::]\ Ts \Longrightarrow True\langle proof\rangle$

5.2.2 Well-formedness

primrec $\mathcal{B} :: expr_1 \Rightarrow nat \Rightarrow bool$
and $\mathcal{B}s :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$\mathcal{B}\ (new\ C)\ i = True$ |

$\mathcal{B}\ (Cast\ C\ e)\ i = \mathcal{B}\ e\ i\ |$
$\mathcal{B}\ (Val\ v)\ i = True\ |$
$\mathcal{B}\ (e_1\ «bop»\ e_2)\ i = (\mathcal{B}\ e_1\ i \wedge \mathcal{B}\ e_2\ i)\ |$
$\mathcal{B}\ (Var\ j)\ i = True\ |$
$\mathcal{B}\ (e{\cdot}F\{D\})\ i = \mathcal{B}\ e\ i\ |$
$\mathcal{B}\ (j{:=}e)\ i = \mathcal{B}\ e\ i\ |$
$\mathcal{B}\ (e_1{\cdot}F\{D\} := e_2)\ i = (\mathcal{B}\ e_1\ i \wedge \mathcal{B}\ e_2\ i)\ |$
$\mathcal{B}\ (e{\cdot}M(es))\ i = (\mathcal{B}\ e\ i \wedge \mathcal{B}s\ es\ i)\ |$
$\mathcal{B}\ (\{j{:}T\ ;\ e\})\ i = (i = j \wedge \mathcal{B}\ e\ (i{+}1))\ |$
$\mathcal{B}\ (e_1;;e_2)\ i = (\mathcal{B}\ e_1\ i \wedge \mathcal{B}\ e_2\ i)\ |$
$\mathcal{B}\ (if\ (e)\ e_1\ else\ e_2)\ i = (\mathcal{B}\ e\ i \wedge \mathcal{B}\ e_1\ i \wedge \mathcal{B}\ e_2\ i)\ |$
$\mathcal{B}\ (throw\ e)\ i = \mathcal{B}\ e\ i\ |$
$\mathcal{B}\ (while\ (e)\ c)\ i = (\mathcal{B}\ e\ i \wedge \mathcal{B}\ c\ i)\ |$
$\mathcal{B}\ (try\ e_1\ catch(C\ j)\ e_2)\ i = (\mathcal{B}\ e_1\ i \wedge i{=}j \wedge \mathcal{B}\ e_2\ (i{+}1))\ |$

$\mathcal{B}s\ []\ i = True\ |$
$\mathcal{B}s\ (e\#es)\ i = (\mathcal{B}\ e\ i \wedge \mathcal{B}s\ es\ i)$

definition *wf-J$_1$-mdecl* :: *J$_1$-prog* ⇒ *cname* ⇒ *expr$_1$ mdecl* ⇒ *bool*
where
wf-J$_1$-mdecl P C ≡ λ(*M*,*Ts*,*T*,*body*).
(∃ *T'*. *P*,*Class C*#*Ts* ⊢$_1$ *body* :: *T'* ∧ *P* ⊢ *T'* ≤ *T*) ∧
$\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*)

lemma *wf-J$_1$-mdecl*[*simp*]:
wf-J$_1$-mdecl P C (*M*,*Ts*,*T*,*body*) ≡
((∃ *T'*. *P*,*Class C*#*Ts* ⊢$_1$ *body* :: *T'* ∧ *P* ⊢ *T'* ≤ *T*) ∧
$\mathcal{D}$ *body* ⌊{..*size Ts*}⌋ ∧ $\mathcal{B}$ *body* (*size Ts* + *1*))⟨*proof*⟩
abbreviation *wf-J$_1$-prog* == *wf-prog wf-J$_1$-mdecl*

end

5.3 Program Compilation

theory *PCompiler*
imports *../Common/WellForm*
begin

definition *compM* :: ('*a* ⇒ '*b*) ⇒ '*a mdecl* ⇒ '*b mdecl*
where
compM f ≡ λ(*M*, *Ts*, *T*, *m*). (*M*, *Ts*, *T*, *f m*)

definition *compC* :: ('*a* ⇒ '*b*) ⇒ '*a cdecl* ⇒ '*b cdecl*
where
compC f ≡ λ(*C*,*D*,*Fdecls*,*Mdecls*). (*C*,*D*,*Fdecls*, *map* (*compM f*) *Mdecls*)

definition *compP* :: ('*a* ⇒ '*b*) ⇒ '*a prog* ⇒ '*b prog*
where
compP f ≡ *map* (*compC f*)

Compilation preserves the program structure. Therfore lookup functions either commute with compilation (like method lookup) or are preserved by it (like the subclass relation).

lemma *map-of-map4*:

map-of (*map* (λ(*x*,*a*,*b*,*c*).(*x*,*a*,*b*,*f c*)) *ts*) =
map-option (λ(*a*,*b*,*c*).(*a*,*b*,*f c*)) ∘ (*map-of ts*)⟨*proof*⟩

lemma *class-compP*:
class P C = *Some* (*D*, *fs*, *ms*)
⟹ *class* (*compP f P*) *C* = *Some* (*D*, *fs*, *map* (*compM f*) *ms*)⟨*proof*⟩

lemma *class-compPD*:
class (*compP f P*) *C* = *Some* (*D*, *fs*, *cms*)
⟹ ∃ *ms*. *class P C* = *Some*(*D*,*fs*,*ms*) ∧ *cms* = *map* (*compM f*) *ms*⟨*proof*⟩

lemma [*simp*]: *is-class* (*compP f P*) *C* = *is-class P C*⟨*proof*⟩

lemma [*simp*]: *class* (*compP f P*) *C* = *map-option* (λ*c*. *snd*(*compC f* (*C*,*c*))) (*class P C*)⟨*proof*⟩

lemma *sees-methods-compP*:
P ⊢ *C sees-methods Mm* ⟹
compP f P ⊢ *C sees-methods* (*map-option* (λ((*Ts*,*T*,*m*),*D*). ((*Ts*,*T*,*f m*),*D*)) ∘ *Mm*)⟨*proof*⟩

lemma *sees-method-compP*:
P ⊢ *C sees M*: *Ts*→*T* = *m in D* ⟹
compP f P ⊢ *C sees M*: *Ts*→*T* = (*f m*) *in D*⟨*proof*⟩

lemma [*simp*]:
P ⊢ *C sees M*: *Ts*→*T* = *m in D* ⟹
method (*compP f P*) *C M* = (*D*,*Ts*,*T*,*f m*)⟨*proof*⟩

lemma *sees-methods-compPD*:
⟦ *cP* ⊢ *C sees-methods Mm*′; *cP* = *compP f P* ⟧ ⟹
∃ *Mm*. *P* ⊢ *C sees-methods Mm* ∧
Mm′ = (*map-option* (λ((*Ts*,*T*,*m*),*D*). ((*Ts*,*T*,*f m*),*D*)) ∘ *Mm*)⟨*proof*⟩

lemma *sees-method-compPD*:
compP f P ⊢ *C sees M*: *Ts*→*T* = *fm in D* ⟹
∃ *m*. *P* ⊢ *C sees M*: *Ts*→*T* = *m in D* ∧ *f m* = *fm*⟨*proof*⟩

lemma [*simp*]: *subcls1*(*compP f P*) = *subcls1 P*⟨*proof*⟩

lemma *compP-widen*[*simp*]: (*compP f P* ⊢ *T* ≤ *T*′) = (*P* ⊢ *T* ≤ *T*′)⟨*proof*⟩

lemma [*simp*]: (*compP f P* ⊢ *Ts* [≤] *Ts*′) = (*P* ⊢ *Ts* [≤] *Ts*′)⟨*proof*⟩

lemma [*simp*]: *is-type* (*compP f P*) *T* = *is-type P T*⟨*proof*⟩

lemma [*simp*]: (*compP* (*f*::′*a*⇒′*b*) *P* ⊢ *C has-fields FDTs*) = (*P* ⊢ *C has-fields FDTs*)⟨*proof*⟩

lemma [*simp*]: *fields* (*compP f P*) *C* = *fields P C*⟨*proof*⟩

lemma [*simp*]: (*compP f P* ⊢ *C sees F*:*T in D*) = (*P* ⊢ *C sees F*:*T in D*)⟨*proof*⟩

lemma [*simp*]: *field* (*compP f P*) *F D* = *field P F D*⟨*proof*⟩

5.3.1 Invariance of *wf-prog* under compilation

lemma [*iff*]: *distinct-fst* (*compP f P*) = *distinct-fst P*⟨*proof*⟩

lemma [*iff*]: *distinct-fst* (*map* (*compM f*) *ms*) = *distinct-fst ms*⟨*proof*⟩

lemma [*iff*]: *wf-syscls* (*compP f P*) = *wf-syscls P*⟨*proof*⟩

lemma [*iff*]: *wf-fdecl* (*compP f P*) = *wf-fdecl P*⟨*proof*⟩

lemma *set-compP*:
((C,D,*fs*,*ms'*) ∈ *set*(*compP f P*)) =
(∃ *ms*. (C,D,*fs*,*ms*) ∈ *set P* ∧ *ms'* = *map* (*compM f*) *ms*)⟨*proof*⟩

lemma *wf-cdecl-compPI*:
⟦ ⋀C M *Ts* T *m*.
⟦ *wf-mdecl* wf_1 *P* C (M,*Ts*,T,*m*); *P* ⊢ C *sees* M:*Ts*→T = *m* *in* C ⟧
⟹ *wf-mdecl* wf_2 (*compP f P*) C (M,*Ts*,T, *f m*);
∀ *x*∈*set P*. *wf-cdecl* wf_1 *P x*; *x* ∈ *set* (*compP f P*); *wf-prog p P* ⟧
⟹ *wf-cdecl* wf_2 (*compP f P*) *x*⟨*proof*⟩

lemma *wf-prog-compPI*:
assumes *lift*:
⋀C M *Ts* T *m*.
⟦ *P* ⊢ C *sees* M:*Ts*→T = *m* *in* C; *wf-mdecl* wf_1 *P* C (M,*Ts*,T,*m*) ⟧
⟹ *wf-mdecl* wf_2 (*compP f P*) C (M,*Ts*,T, *f m*)
and *wf*: *wf-prog* wf_1 *P*
shows *wf-prog* wf_2 (*compP f P*)⟨*proof*⟩

end
theory *Hidden*
imports *List−Index.List-Index*
begin

definition *hidden* :: *'a list* ⇒ *nat* ⇒ *bool* **where**
hidden xs i ≡ *i* < *size xs* ∧ *xs*!*i* ∈ *set*(*drop* (*i*+*1*) *xs*)

lemma *hidden-last-index*: *x* ∈ *set xs* ⟹ *hidden* (*xs* @ [*x*]) (*last-index xs x*)
⟨*proof*⟩

lemma *hidden-inacc*: *hidden xs i* ⟹ *last-index xs x* ≠ *i*
⟨*proof*⟩

lemma [*simp*]: *hidden xs i* ⟹ *hidden* (*xs*@[*x*]) *i*
⟨*proof*⟩

lemma *fun-upds-apply*:
(*m*(*xs*[↦]*ys*)) *x* =
(*let xs'* = *take* (*size ys*) *xs*
in if x ∈ *set xs' then Some*(*ys* ! *last-index xs' x*) *else m x*)
⟨*proof*⟩

lemma *map-upds-apply-eq-Some*:
$((m(xs[\mapsto]ys))\ x = Some\ y) =$
$(let\ xs' = take\ (size\ ys)\ xs$
$in\ if\ x \in set\ xs'\ then\ ys\ !\ last\text{-}index\ xs'\ x = y\ else\ m\ x = Some\ y)$
⟨*proof*⟩

lemma *map-upds-upd-conv-last-index*:
$[\![x \in set\ xs;\ size\ xs \le size\ ys\ ]\!]$
$\Longrightarrow m(xs[\mapsto]ys)(x \mapsto y) = m(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])$
⟨*proof*⟩

end

5.4 Compilation Stage 1

theory *Compiler1* **imports** *PCompiler J1 Hidden* **begin**

Replacing variable names by indices.

primrec $compE_1$ :: *vname list* ⇒ *expr* ⇒ $expr_1$
and $compEs_1$ :: *vname list* ⇒ *expr list* ⇒ $expr_1$ *list* **where**
$compE_1\ Vs\ (new\ C) = new\ C$
| $compE_1\ Vs\ (Cast\ C\ e) = Cast\ C\ (compE_1\ Vs\ e)$
| $compE_1\ Vs\ (Val\ v) = Val\ v$
| $compE_1\ Vs\ (e_1\ «bop»\ e_2) = (compE_1\ Vs\ e_1)\ «bop»\ (compE_1\ Vs\ e_2)$
| $compE_1\ Vs\ (Var\ V) = Var(last\text{-}index\ Vs\ V)$
| $compE_1\ Vs\ (V{:=}e) = (last\text{-}index\ Vs\ V){:=}\ (compE_1\ Vs\ e)$
| $compE_1\ Vs\ (e{\cdot}F\{D\}) = (compE_1\ Vs\ e){\cdot}F\{D\}$
| $compE_1\ Vs\ (e_1{\cdot}F\{D\}{:=}e_2) = (compE_1\ Vs\ e_1){\cdot}F\{D\} := (compE_1\ Vs\ e_2)$
| $compE_1\ Vs\ (e{\cdot}M(es)) = (compE_1\ Vs\ e){\cdot}M(compEs_1\ Vs\ es)$
| $compE_1\ Vs\ \{V{:}T;\ e\} = \{(size\ Vs){:}T;\ compE_1\ (Vs@[V])\ e\}$
| $compE_1\ Vs\ (e_1;;e_2) = (compE_1\ Vs\ e_1);;(compE_1\ Vs\ e_2)$
| $compE_1\ Vs\ (if\ (e)\ e_1\ else\ e_2) = if\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ e_1)\ else\ (compE_1\ Vs\ e_2)$
| $compE_1\ Vs\ (while\ (e)\ c) = while\ (compE_1\ Vs\ e)\ (compE_1\ Vs\ c)$
| $compE_1\ Vs\ (throw\ e) = throw\ (compE_1\ Vs\ e)$
| $compE_1\ Vs\ (try\ e_1\ catch(C\ V)\ e_2) =$
$try(compE_1\ Vs\ e_1)\ catch(C\ (size\ Vs))\ (compE_1\ (Vs@[V])\ e_2)$

| $compEs_1\ Vs\ []\ \ = []$
| $compEs_1\ Vs\ (e\#es) = compE_1\ Vs\ e\ \#\ compEs_1\ Vs\ es$

lemma [*simp*]: $compEs_1\ Vs\ es = map\ (compE_1\ Vs)\ es$⟨*proof*⟩

primrec fin_1:: *expr* ⇒ $expr_1$ **where**
$fin_1(Val\ v) = Val\ v$
| $fin_1(throw\ e) = throw(fin_1\ e)$

lemma *comp-final*: *final e* $\Longrightarrow compE_1\ Vs\ e = fin_1\ e$⟨*proof*⟩

lemma [*simp*]:
$\bigwedge Vs.\ max\text{-}vars\ (compE_1\ Vs\ e) = max\text{-}vars\ e$
and $\bigwedge Vs.\ max\text{-}varss\ (compEs_1\ Vs\ es) = max\text{-}varss\ es$⟨*proof*⟩

Compiling programs:

definition $compP_1 :: J\text{-}prog \Rightarrow J_1\text{-}prog$
where
$compP_1 \equiv compP\ (\lambda(pns,body).\ compE_1\ (this\#pns)\ body)$

end

5.5 Correctness of Stage 1

theory *Correctness1*
imports *J1WellForm Compiler1*
begin

5.5.1 Correctness of program compilation

primrec $unmod :: expr_1 \Rightarrow nat \Rightarrow bool$
and $unmods :: expr_1\ list \Rightarrow nat \Rightarrow bool$ **where**
$unmod\ (new\ C)\ i = True\ |$
$unmod\ (Cast\ C\ e)\ i = unmod\ e\ i\ |$
$unmod\ (Val\ v)\ i = True\ |$
$unmod\ (e_1\ «bop»\ e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (Var\ i)\ j = True\ |$
$unmod\ (i{:=}e)\ j = (i \neq j \wedge unmod\ e\ j)\ |$
$unmod\ (e{\cdot}F\{D\})\ i = unmod\ e\ i\ |$
$unmod\ (e_1{\cdot}F\{D\}{:=}e_2)\ i = (unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (e{\cdot}M(es))\ i = (unmod\ e\ i \wedge unmods\ es\ i)\ |$
$unmod\ \{j{:}T;\ e\}\ i = unmod\ e\ i\ |$
$unmod\ (e_1;;e_2)\ i = (unmod\ e_1\ i \wedge\ unmod\ e_2\ i)\ |$
$unmod\ (if\ (e)\ e_1\ else\ e_2)\ i = (unmod\ e\ i \wedge unmod\ e_1\ i \wedge unmod\ e_2\ i)\ |$
$unmod\ (while\ (e)\ c)\ i = (unmod\ e\ i \wedge unmod\ c\ i)\ |$
$unmod\ (throw\ e)\ i = unmod\ e\ i\ |$
$unmod\ (try\ e_1\ catch(C\ i)\ e_2)\ j = (unmod\ e_1\ j \wedge (if\ i{=}j\ then\ False\ else\ unmod\ e_2\ j))\ |$

$unmods\ ([])\ i = True\ |$
$unmods\ (e\#es)\ i = (unmod\ e\ i \wedge unmods\ es\ i)$

lemma *hidden-unmod*: $\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmod\ (compE_1\ Vs\ e)\ i$ **and**
$\bigwedge Vs.\ hidden\ Vs\ i \Longrightarrow unmods\ (compEs_1\ Vs\ es)\ i\langle proof\rangle$

lemma *eval₁-preserves-unmod*:
$[\![\ P \vdash_1 \langle e,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle;\ unmod\ e\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i$
and $[\![\ P \vdash_1 \langle es,(h,ls)\rangle\ [\Rightarrow]\ \langle es',(h',ls')\rangle;\ unmods\ es\ i;\ i < size\ ls\ ]\!]$
$\Longrightarrow ls\ !\ i = ls'\ !\ i\langle proof\rangle$

lemma *LAss-lem*:
$[\![x \in set\ xs;\ size\ xs \leq size\ ys\ ]\!]$
$\Longrightarrow m_1 \subseteq_m m_2(xs[\mapsto]ys) \Longrightarrow m_1(x{\mapsto}y) \subseteq_m m_2(xs[\mapsto]ys[last\text{-}index\ xs\ x := y])\langle proof\rangle$**lemma** *Block-lem*:
fixes $l :: 'a \rightharpoonup 'b$
assumes *0*: $l \subseteq_m [Vs\ [\mapsto]\ ls]$
and *1*: $l' \subseteq_m [Vs\ [\mapsto]\ ls',\ V{\mapsto}v]$
and *hidden*: $V \in set\ Vs \Longrightarrow ls\ !\ last\text{-}index\ Vs\ V = ls'\ !\ last\text{-}index\ Vs\ V$
and *size*: $size\ ls = size\ ls' \quad size\ Vs < size\ ls'$

shows $l'(V := l\ V) \subseteq_m [Vs\ [\mapsto]\ ls']\langle proof\rangle$

The main theorem:

theorem assumes *wf*: *wwf-J-prog P*
shows $eval_1$-*eval*: $P \vdash \langle e,(h,l)\rangle \Rightarrow \langle e',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fv\ e \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}vars\ e \leq size\ ls\ ]\!]$
$\Longrightarrow \exists ls'.\ compP_1\ P \vdash_1 \langle compE_1\ Vs\ e,(h,ls)\rangle \Rightarrow \langle fin_1\ e',(h',ls')\rangle \wedge l' \subseteq_m [Vs[\mapsto]ls'])$
and $evals_1$-*evals*: $P \vdash \langle es,(h,l)\rangle\ [\Rightarrow]\ \langle es',(h',l')\rangle$
$\Longrightarrow (\bigwedge Vs\ ls.\ [\![\ fvs\ es \subseteq set\ Vs;\ \ l \subseteq_m [Vs[\mapsto]ls];\ size\ Vs + max\text{-}varss\ es \leq size\ ls\ ]\!]$
$\Longrightarrow \exists ls'.\ compP_1\ P \vdash_1 \langle compEs_1\ Vs\ es,(h,ls)\rangle\ [\Rightarrow]\ \langle compEs_1\ Vs\ es',(h',ls')\rangle \wedge$
$l' \subseteq_m [Vs[\mapsto]ls'])\langle proof\rangle$

5.5.2 Preservation of well-formedness

The compiler preserves well-formedness. Is less trivial than it may appear. We start with two simple properties: preservation of well-typedness

lemma $compE_1$-*pres-wt*: $\bigwedge Vs\ Ts\ U.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash e :: U;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P, Ts \vdash_1 compE_1\ Vs\ e :: U$
and $\bigwedge Vs\ Ts\ Us.$
$[\![\ P,[Vs[\mapsto]Ts] \vdash es\ [::]\ Us;\ size\ Ts = size\ Vs\ ]\!]$
$\Longrightarrow compP\ f\ P, Ts \vdash_1 compEs_1\ Vs\ es\ [::]\ Us\langle proof\rangle$

and the correct block numbering:

lemma $\mathcal{B}$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}\ (compE_1\ Vs\ e)\ n$
and $\mathcal{B}s$: $\bigwedge Vs\ n.\ size\ Vs = n \Longrightarrow \mathcal{B}s\ (compEs_1\ Vs\ es)\ n\langle proof\rangle$

The main complication is preservation of definite assignment $\mathcal{D}$.

lemma *image-last-index*: $A \subseteq set(xs@[x]) \Longrightarrow last\text{-}index\ (xs\ @\ [x])\ `\ A =$
$(if\ x \in A\ then\ insert\ (size\ xs)\ (last\text{-}index\ xs\ `\ (A-\{x\}))\ else\ last\text{-}index\ xs\ `\ A)\langle proof\rangle$

lemma A-$compE_1$-*None*[*simp*]:
$\bigwedge Vs.\ \mathcal{A}\ e = None \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = None$
and $\bigwedge Vs.\ \mathcal{A}s\ es = None \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = None\langle proof\rangle$

lemma A-$compE_1$:
$\bigwedge A\ Vs.\ [\![\ \mathcal{A}\ e = \lfloor A\rfloor;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}\ (compE_1\ Vs\ e) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ \mathcal{A}s\ es = \lfloor A\rfloor;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow \mathcal{A}s\ (compEs_1\ Vs\ es) = \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma D-*None*[*iff*]: $\mathcal{D}\ (e::'a\ exp)\ None$ **and** [*iff*]: $\mathcal{D}s\ (es::'a\ exp\ list)\ None\langle proof\rangle$

lemma D-*last-index*-$compE_1$:
$\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fv\ e \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}\ e\ \lfloor A\rfloor \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor$
and $\bigwedge A\ Vs.\ [\![\ A \subseteq set\ Vs;\ fvs\ es \subseteq set\ Vs\ ]\!] \Longrightarrow$
$\mathcal{D}s\ es\ \lfloor A\rfloor \Longrightarrow \mathcal{D}s\ (compEs_1\ Vs\ es)\ \lfloor last\text{-}index\ Vs\ `\ A\rfloor\langle proof\rangle$

lemma *last-index-image-set*: $distinct\ xs \Longrightarrow last\text{-}index\ xs\ `\ set\ xs = \{..<size\ xs\}\langle proof\rangle$

lemma D-$compE_1$:
$[\![\ \mathcal{D}\ e\ \lfloor set\ Vs\rfloor;\ fv\ e \subseteq set\ Vs;\ distinct\ Vs\ ]\!] \Longrightarrow \mathcal{D}\ (compE_1\ Vs\ e)\ \lfloor\{..<length\ Vs\}\rfloor\langle proof\rangle$

lemma D-$compE_1{}'$:

assumes $\mathcal{D}\ e\ \lfloor set(V\#Vs)\rfloor$ **and** $fv\ e \subseteq set(V\#Vs)$ **and** $distinct(V\#Vs)$
shows $\mathcal{D}\ (compE_1\ (V\#Vs)\ e)\ \lfloor\{..length\ Vs\}\rfloor\langle proof\rangle$

lemma $compP_1\text{-}pres\text{-}wf$: $wf\text{-}J\text{-}prog\ P \Longrightarrow wf\text{-}J_1\text{-}prog\ (compP_1\ P)\langle proof\rangle$

end

5.6 Compilation Stage 2

theory *Compiler2*
imports *PCompiler J1 ../JVM/JVMExec*
begin

primrec $compE_2 :: expr_1 \Rightarrow instr\ list$
and $compEs_2 :: expr_1\ list \Rightarrow instr\ list$ **where**
$compE_2\ (new\ C) = [New\ C]$
| $compE_2\ (Cast\ C\ e) = compE_2\ e\ @\ [Checkcast\ C]$
| $compE_2\ (Val\ v) = [Push\ v]$
| $compE_2\ (e_1\ «bop»\ e_2) = compE_2\ e_1\ @\ compE_2\ e_2\ @$
$(case\ bop\ of\ Eq \Rightarrow [CmpEq]$
$\quad | \ Add \Rightarrow [IAdd])$
| $compE_2\ (Var\ i) = [Load\ i]$
| $compE_2\ (i{:=}e) = compE_2\ e\ @\ [Store\ i, Push\ Unit]$
| $compE_2\ (e{\cdot}F\{D\}) = compE_2\ e\ @\ [Getfield\ F\ D]$
| $compE_2\ (e_1{\cdot}F\{D\} := e_2) =$
$\quad compE_2\ e_1\ @\ compE_2\ e_2\ @\ [Putfield\ F\ D, Push\ Unit]$
| $compE_2\ (e{\cdot}M(es)) = compE_2\ e\ @\ compEs_2\ es\ @\ [Invoke\ M\ (size\ es)]$
| $compE_2\ (\{i{:}T;\ e\}) = compE_2\ e$
| $compE_2\ (e_1;;e_2) = compE_2\ e_1\ @\ [Pop]\ @\ compE_2\ e_2$
| $compE_2\ (if\ (e)\ e_1\ else\ e_2) =$
$\quad (let\ cnd\ \ = compE_2\ e;$
$\quad\ \ thn\ \ = compE_2\ e_1;$
$\quad\ \ els\ \ = compE_2\ e_2;$
$\quad\ \ test\ = IfFalse\ (int(size\ thn + 2));$
$\quad\ \ thnex = Goto\ (int(size\ els + 1))$
$\quad in\ cnd\ @\ [test]\ @\ thn\ @\ [thnex]\ @\ els)$
| $compE_2\ (while\ (e)\ c) =$
$\quad (let\ cnd\ \ = compE_2\ e;$
$\quad\ \ bdy\ \ = compE_2\ c;$
$\quad\ \ test\ = IfFalse\ (int(size\ bdy + 3));$
$\quad\ \ loop\ = Goto\ (-int(size\ bdy + size\ cnd + 2))$
$\quad in\ cnd\ @\ [test]\ @\ bdy\ @\ [Pop]\ @\ [loop]\ @\ [Push\ Unit])$
| $compE_2\ (throw\ e) = compE_2\ e\ @\ [instr.Throw]$
| $compE_2\ (try\ e_1\ catch(C\ i)\ e_2) =$
$\quad (let\ catch = compE_2\ e_2$
$\quad in\ compE_2\ e_1\ @\ [Goto\ (int(size\ catch){+}2), Store\ i]\ @\ catch)$

| $compEs_2\ []\ \ \ = []$
| $compEs_2\ (e\#es) = compE_2\ e\ @\ compEs_2\ es$

Compilation of exception table. Is given start address of code to compute absolute addresses necessary in exception table.

primrec $compxE_2\ :: expr_1\ \ \ \ \Rightarrow pc \Rightarrow nat \Rightarrow ex\text{-}table$

```
  and compxEs2 :: expr1 list ⇒ pc ⇒ nat ⇒ ex-table where
  compxE2 (new C) pc d = []
| compxE2 (Cast C e) pc d = compxE2 e pc d
| compxE2 (Val v) pc d = []
| compxE2 (e1 «bop» e2) pc d =
   compxE2 e1 pc d @ compxE2 e2 (pc + size(compE2 e1)) (d+1)
| compxE2 (Var i) pc d = []
| compxE2 (i:=e) pc d = compxE2 e pc d
| compxE2 (e·F{D}) pc d = compxE2 e pc d
| compxE2 (e1·F{D} := e2) pc d =
   compxE2 e1 pc d @ compxE2 e2 (pc + size(compE2 e1)) (d+1)
| compxE2 (e·M(es)) pc d =
   compxE2 e pc d @ compxEs2 es (pc + size(compE2 e)) (d+1)
| compxE2 ({i:T; e}) pc d = compxE2 e pc d
| compxE2 (e1;;e2) pc d =
   compxE2 e1 pc d @ compxE2 e2 (pc+size(compE2 e1)+1) d
| compxE2 (if (e) e1 else e2) pc d =
       (let pc1   = pc + size(compE2 e) + 1;
            pc2   = pc1 + size(compE2 e1) + 1
        in compxE2 e pc d @ compxE2 e1 pc1 d @ compxE2 e2 pc2 d)
| compxE2 (while (b) e) pc d =
   compxE2 b pc d @ compxE2 e (pc+size(compE2 b)+1) d
| compxE2 (throw e) pc d = compxE2 e pc d
| compxE2 (try e1 catch(C i) e2) pc d =
   (let pc1 = pc + size(compE2 e1)
    in compxE2 e1 pc d @ compxE2 e2 (pc1+2) d @ [(pc,pc1,C,pc1+1,d)])

| compxEs2 [] pc d    = []
| compxEs2 (e#es) pc d = compxE2 e pc d @ compxEs2 es (pc+size(compE2 e)) (d+1)

primrec max-stack :: expr1 ⇒ nat
  and max-stacks :: expr1 list ⇒ nat where
  max-stack (new C) = 1
| max-stack (Cast C e) = max-stack e
| max-stack (Val v) = 1
| max-stack (e1 «bop» e2) = max (max-stack e1) (max-stack e2) + 1
| max-stack (Var i) = 1
| max-stack (i:=e) = max-stack e
| max-stack (e·F{D}) = max-stack e
| max-stack (e1·F{D} := e2) = max (max-stack e1) (max-stack e2) + 1
| max-stack (e·M(es)) = max (max-stack e) (max-stacks es) + 1
| max-stack ({i:T; e}) = max-stack e
| max-stack (e1;;e2) = max (max-stack e1) (max-stack e2)
| max-stack (if (e) e1 else e2) =
  max (max-stack e) (max (max-stack e1) (max-stack e2))
| max-stack (while (e) c) = max (max-stack e) (max-stack c)
| max-stack (throw e) = max-stack e
| max-stack (try e1 catch(C i) e2) = max (max-stack e1) (max-stack e2)

| max-stacks [] = 0
| max-stacks (e#es) = max (max-stack e) (1 + max-stacks es)

lemma max-stack1: 1 ≤ max-stack e⟨proof⟩
```

definition *compMb*$_2$ :: *expr*$_1$ ⇒ *jvm-method*
where
compMb$_2$ ≡ λ*body*.
let ins = *compE*$_2$ *body* @ [*Return*];
xt = *compxE*$_2$ *body 0 0*
in (*max-stack body*, *max-vars body*, *ins*, *xt*)

definition *compP*$_2$ :: *J*$_1$*-prog* ⇒ *jvm-prog*
where
compP$_2$ ≡ *compP compMb*$_2$

lemma *compMb*$_2$ [*simp*]:
compMb$_2$ *e* = (*max-stack e*, *max-vars e*, *compE*$_2$ *e* @ [*Return*], *compxE*$_2$ *e 0 0*)⟨*proof*⟩

end

5.7 Correctness of Stage 2

theory *Correctness2*
imports *HOL−Library.Sublist Compiler2*
begin

5.7.1 Instruction sequences

How to select individual instructions and subsequences of instructions from a program given the class, method and program counter.

definition *before* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr list* ⇒ *bool*
((-,-,-,-/ ▷ -) [*51*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M*,*pc* ▷ *is* ⟷ *prefix is* (*drop pc* (*instrs-of P C M*))

definition *at* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *nat* ⇒ *instr* ⇒ *bool*
((-,-,-,-/ ▷ -) [*51*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M*,*pc* ▷ *i* ⟷ (∃ *is*. *drop pc* (*instrs-of P C M*) = *i*#*is*)

lemma [*simp*]: *P*,*C*,*M*,*pc* ▷ []⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ▷ (*i*#*is*) = (*P*,*C*,*M*,*pc* ▷ *i* ∧ *P*,*C*,*M*,*pc* + *1* ▷ *is*)⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ▷ (*is*$_1$ @ *is*$_2$) = (*P*,*C*,*M*,*pc* ▷ *is*$_1$ ∧ *P*,*C*,*M*,*pc* + *size is*$_1$ ▷ *is*$_2$)⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M*,*pc* ▷ *i* ⟹ *instrs-of P C M* ! *pc* = *i*⟨*proof*⟩

lemma *beforeM*:
P ⊢ *C sees M*: *Ts*→*T* = *body in D* ⟹
compP$_2$ *P*,*D*,*M*,*0* ▷ *compE*$_2$ *body* @ [*Return*]⟨*proof*⟩

This lemma executes a single instruction by rewriting:

lemma [*simp*]:
P,*C*,*M*,*pc* ▷ *instr* ⟹
(*P* ⊢ (*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*) −*jvm*→ *σ*′) =
((*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*) = *σ*′ ∨
(∃ *σ*. *exec*(*P*,(*None*, *h*, (*vs*,*ls*,*C*,*M*,*pc*) # *frs*)) = *Some σ* ∧ *P* ⊢ *σ* −*jvm*→ *σ*′))⟨*proof*⟩

5.7.2 Exception tables

definition *pcs* :: *ex-table* ⇒ *nat set*
where
pcs xt ≡ ⋃(*f*,*t*,*C*,*h*,*d*) ∈ *set xt*. {*f* ..< *t*}

lemma *pcs-subset*:
shows ⋀*pc d*. *pcs*(*compxE*$_2$ *e pc d*) ⊆ {*pc*..<*pc*+*size*(*compE*$_2$ *e*)}
and ⋀*pc d*. *pcs*(*compxEs*$_2$ *es pc d*) ⊆ {*pc*..<*pc*+*size*(*compEs*$_2$ *es*)}⟨*proof*⟩

lemma [*simp*]: *pcs* [] = {}⟨*proof*⟩

lemma [*simp*]: *pcs* (*x*#*xt*) = {*fst x* ..< *fst*(*snd x*)} ∪ *pcs xt*⟨*proof*⟩

lemma [*simp*]: *pcs*(*xt*$_1$ @ *xt*$_2$) = *pcs xt*$_1$ ∪ *pcs xt*$_2$⟨*proof*⟩

lemma [*simp*]: *pc* < *pc*$_0$ ∨ *pc*$_0$+*size*(*compE*$_2$ *e*) ≤ *pc* ⟹ *pc* ∉ *pcs*(*compxE*$_2$ *e pc*$_0$ *d*)⟨*proof*⟩

lemma [*simp*]: *pc* < *pc*$_0$ ∨ *pc*$_0$+*size*(*compEs*$_2$ *es*) ≤ *pc* ⟹ *pc* ∉ *pcs*(*compxEs*$_2$ *es pc*$_0$ *d*)⟨*proof*⟩

lemma [*simp*]: *pc*$_1$ + *size*(*compE*$_2$ *e*$_1$) ≤ *pc*$_2$ ⟹ *pcs*(*compxE*$_2$ *e*$_1$ *pc*$_1$ *d*$_1$) ∩ *pcs*(*compxE*$_2$ *e*$_2$ *pc*$_2$ *d*$_2$) = {}⟨*proof*⟩

lemma [*simp*]: *pc*$_1$ + *size*(*compE*$_2$ *e*) ≤ *pc*$_2$ ⟹ *pcs*(*compxE*$_2$ *e pc*$_1$ *d*$_1$) ∩ *pcs*(*compxEs*$_2$ *es pc*$_2$ *d*$_2$) = {}⟨*proof*⟩

lemma [*simp*]:
pc ∉ *pcs xt*$_0$ ⟹ *match-ex-table P C pc* (*xt*$_0$ @ *xt*$_1$) = *match-ex-table P C pc xt*$_1$⟨*proof*⟩

lemma [*simp*]: ⟦ *x* ∈ *set xt*; *pc* ∉ *pcs xt* ⟧ ⟹ ¬ *matches-ex-entry P D pc x*⟨*proof*⟩

lemma [*simp*]:
assumes *xe*: *xe* ∈ *set*(*compxE*$_2$ *e pc d*) **and** *outside*: *pc*′ < *pc* ∨ *pc*+*size*(*compE*$_2$ *e*) ≤ *pc*′
shows ¬ *matches-ex-entry P C pc*′ *xe*⟨*proof*⟩

lemma [*simp*]:
assumes *xe*: *xe* ∈ *set*(*compxEs*$_2$ *es pc d*) **and** *outside*: *pc*′ < *pc* ∨ *pc*+*size*(*compEs*$_2$ *es*) ≤ *pc*′
shows ¬ *matches-ex-entry P C pc*′ *xe*⟨*proof*⟩

lemma *match-ex-table-app*[*simp*]:
∀ *xte* ∈ *set xt*$_1$. ¬ *matches-ex-entry P D pc xte* ⟹
match-ex-table P D pc (*xt*$_1$ @ *xt*) = *match-ex-table P D pc xt*⟨*proof*⟩

lemma [*simp*]:
∀ *x* ∈ *set xtab*. ¬ *matches-ex-entry P C pc x* ⟹
match-ex-table P C pc xtab = *None*⟨*proof*⟩

lemma *match-ex-entry*:
matches-ex-entry P C pc (*start*, *end*, *catch-type*, *handler*) =
(*start* ≤ *pc* ∧ *pc* < *end* ∧ *P* ⊢ *C* ⪯* *catch-type*)⟨*proof*⟩

definition *caught* :: *jvm-prog* ⇒ *pc* ⇒ *heap* ⇒ *addr* ⇒ *ex-table* ⇒ *bool* **where**
caught P pc h a xt ⟷
(∃ *entry* ∈ *set xt*. *matches-ex-entry P* (*cname-of h a*) *pc entry*)

definition *beforex* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *ex-table* ⇒ *nat set* ⇒ *nat* ⇒ *bool*
((*2*-,/-,/- ▷/ - /'/ -,/-) [*51*,*0*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M* ▷ *xt* / *I*,*d* ⟷
(∃ *xt*$_0$ *xt*$_1$. *ex-table-of P C M* = *xt*$_0$ @ *xt* @ *xt*$_1$ ∧ *pcs xt*$_0$ ∩ *I* = {} ∧ *pcs xt* ⊆ *I* ∧
(∀ *pc* ∈ *I*. ∀ *C pc*′ *d*′. *match-ex-table P C pc xt*$_1$ = ⌊(*pc*′,*d*′)⌋ ⟶ *d*′ ≤ *d*))

definition *dummyx* :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *ex-table* ⇒ *nat set* ⇒ *nat* ⇒ *bool* ((*2*-,-,- ▷/ - /'/-,-) [*51*,*0*,*0*,*0*,*0*,*51*] *50*) **where**
P,*C*,*M* ▷ *xt*/*I*,*d* ⟷ *P*,*C*,*M* ▷ *xt*/*I*,*d*

abbreviation
beforex$_0$ *P C M d I xt xt*$_0$ *xt*$_1$
≡ *ex-table-of P C M* = *xt*$_0$ @ *xt* @ *xt*$_1$ ∧ *pcs xt*$_0$ ∩ *I* = {}
∧ *pcs xt* ⊆ *I* ∧ (∀ *pc* ∈ *I*. ∀ *C pc*′ *d*′. *match-ex-table P C pc xt*$_1$ = ⌊(*pc*′,*d*′)⌋ ⟶ *d*′ ≤ *d*)

lemma *beforex-beforex*$_0$*-eq*:
P,*C*,*M* ▷ *xt* / *I*,*d* ≡ ∃ *xt*$_0$ *xt*$_1$. *beforex*$_0$ *P C M d I xt xt*$_0$ *xt*$_1$
⟨*proof*⟩

lemma *beforexD1*: *P*,*C*,*M* ▷ *xt* / *I*,*d* ⟹ *pcs xt* ⊆ *I*⟨*proof*⟩

lemma *beforex-mono*: ⟦ *P*,*C*,*M* ▷ *xt*/*I*,*d*′; *d*′ ≤ *d* ⟧ ⟹ *P*,*C*,*M* ▷ *xt*/*I*,*d*⟨*proof*⟩

lemma [*simp*]: *P*,*C*,*M* ▷ *xt*/*I*,*d* ⟹ *P*,*C*,*M* ▷ *xt*/*I*,*Suc d*⟨*proof*⟩

lemma *beforex-append*[*simp*]:
pcs xt$_1$ ∩ *pcs xt*$_2$ = {} ⟹
P,*C*,*M* ▷ *xt*$_1$ @ *xt*$_2$/*I*,*d* =
(*P*,*C*,*M* ▷ *xt*$_1$/*I*−*pcs xt*$_2$,*d* ∧ *P*,*C*,*M* ▷ *xt*$_2$/*I*−*pcs xt*$_1$,*d* ∧ *P*,*C*,*M* ▷ *xt*$_1$@*xt*$_2$/*I*,*d*)⟨*proof*⟩

lemma *beforex-appendD1*:
assumes *bx*: *P*,*C*,*M* ▷ *xt*$_1$ @ *xt*$_2$ @ [(*f*,*t*,*D*,*h*,*d*)] / *I*,*d*
and *pcs*: *pcs xt*$_1$ ⊆ *J* **and** *JI*: *J* ⊆ *I* **and** *Jpcs*: *J* ∩ *pcs xt*$_2$ = {}
shows *P*,*C*,*M* ▷ *xt*$_1$ / *J*,*d*⟨*proof*⟩

lemma *beforex-appendD2*:
assumes *bx*: *P*,*C*,*M* ▷ *xt*$_1$ @ *xt*$_2$ @ [(*f*,*t*,*D*,*h*,*d*)] / *I*,*d*
and *pcs*: *pcs xt*$_2$ ⊆ *J* **and** *JI*: *J* ⊆ *I* **and** *Jpcs*: *J* ∩ *pcs xt*$_1$ = {}
shows *P*,*C*,*M* ▷ *xt*$_2$ / *J*,*d*⟨*proof*⟩

lemma *beforexM*:
P ⊢ *C sees M*: *Ts*→*T* = *body in D* ⟹
compP$_2$ *P*,*D*,*M* ▷ *compxE*$_2$ *body 0 0*/{..<*size*(*compE*$_2$ *body*)},*0*⟨*proof*⟩

lemma *match-ex-table-SomeD2*:
assumes *met*: *match-ex-table P D pc* (*ex-table-of P C M*) = ⌊(*pc*′,*d*′)⌋
and *bx*: *P*,*C*,*M* ▷ *xt*/*I*,*d*
and *nmet*: ∀ *x* ∈ *set xt*. ¬ *matches-ex-entry P D pc x* **and** *pcI*: *pc* ∈ *I*
shows *d*′ ≤ *d*⟨*proof*⟩

lemma *match-ex-table-SomeD1*:
⟦ *match-ex-table P D pc* (*ex-table-of P C M*) = ⌊(*pc*′,*d*′)⌋;

$P,C,M \rhd xt \;/\; I,d;\; pc \in I;\; pc \notin pcs\ xt \;\rrbracket \Longrightarrow d' \leq d\langle proof\rangle$

5.7.3 The correctness proof

definition
handle :: *jvm-prog* ⇒ *cname* ⇒ *mname* ⇒ *addr* ⇒ *heap* ⇒ *val list* ⇒ *val list* ⇒ *nat* ⇒ *frame list* ⇒ *jvm-state* **where**
handle P C M a h vs ls pc frs = *find-handler P a h* ((*vs*,*ls*,*C*,*M*,*pc*) # *frs*)

lemma *handle-Cons*:
⟦ $P,C,M \rhd xt/I,d$; $d \leq size\ vs$; $pc \in I$;
∀ $x \in set\ xt.\ \neg$ *matches-ex-entry P* (*cname-of h xa*) *pc x* ⟧ ⟹
handle P C M xa h (*v* # *vs*) *ls pc frs* = *handle P C M xa h vs ls pc frs*⟨*proof*⟩
lemma *handle-append*:
assumes *bx*: $P,C,M \rhd xt/I,d$ **and** *d*: $d \leq size\ vs$
and *pcI*: $pc \in I$ **and** *pc-not*: $pc \notin pcs\ xt$
shows *handle P C M xa h* (*ws* @ *vs*) *ls pc frs* = *handle P C M xa h vs ls pc frs*⟨*proof*⟩

lemma *aux-isin*[*simp*]: ⟦ $B \subseteq A$; $a \in B$ ⟧ ⟹ $a \in A\langle proof\rangle$

lemma fixes P_1 **defines** [*simp*]: $P \equiv compP_2\ P_1$
shows *Jcc*:
$P_1 \vdash_1 \langle e,(h_0,ls_0)\rangle \Rightarrow \langle ef,(h_1,ls_1)\rangle \Longrightarrow$
(⋀*C M pc v xa vs frs I*.
⟦ $P,C,M,pc \rhd compE_2\ e$; $P,C,M \rhd compxE_2\ e\ pc\ (size\ vs)/I,size\ vs$;
$\{pc..<pc+size(compE_2\ e)\} \subseteq I$ ⟧ ⟹
(*ef* = *Val v* ⟶
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(v\#vs,ls_1,C,M,pc+size(compE_2\ e))\#frs)$)
∧
(*ef* = *Throw xa* ⟶
(∃ pc_1. $pc \leq pc_1 \wedge pc_1 < pc + size(compE_2\ e) \wedge$
¬ *caught P* pc_1 h_1 *xa* ($compxE_2\ e\ pc\ (size\ vs)$) ∧
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$ *handle P C M xa* h_1 *vs* ls_1 pc_1 *frs*)))
and $P_1 \vdash_1 \langle es,(h_0,ls_0)\rangle\ [\Rightarrow]\ \langle fs,(h_1,ls_1)\rangle \Longrightarrow$
(⋀*C M pc ws xa es' vs frs I*.
⟦ $P,C,M,pc \rhd compEs_2\ es$; $P,C,M \rhd compxEs_2\ es\ pc\ (size\ vs)/I,size\ vs$;
$\{pc..<pc+size(compEs_2\ es)\} \subseteq I$ ⟧ ⟹
(*fs* = *map Val ws* ⟶
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$
$(None,h_1,(rev\ ws\ @\ vs,ls_1,C,M,pc+size(compEs_2\ es))\#frs)$)
∧
(*fs* = *map Val ws* @ *Throw xa* # *es'* ⟶
(∃ pc_1. $pc \leq pc_1 \wedge pc_1 < pc + size(compEs_2\ es) \wedge$
¬ *caught P* pc_1 h_1 *xa* ($compxEs_2\ es\ pc\ (size\ vs)$) ∧
$P \vdash (None,h_0,(vs,ls_0,C,M,pc)\#frs)\ -jvm\rightarrow$ *handle P C M xa* h_1 *vs* ls_1 pc_1 *frs*)))⟨*proof*⟩

lemma *atLeast0AtMost*[*simp*]: $\{0::nat..n\} = \{..n\}$
⟨*proof*⟩

lemma *atLeast0LessThan*[*simp*]: $\{0::nat..<n\} = \{..<n\}$

⟨*proof*⟩

fun *exception* :: *'a exp* ⇒ *addr option* **where**
  exception (*Throw a*) = *Some a*
| *exception e* = *None*

lemma *comp₂-correct*:
assumes *method*: $P_1 \vdash C$ *sees* M:*Ts*→*T* = *body in C*
  and *eval*: $P_1 \vdash_1 \langle body,(h,ls)\rangle \Rightarrow \langle e',(h',ls')\rangle$
shows *compP₂* $P_1 \vdash (None,h,[([],ls,C,M,0)]) -jvm\rightarrow (exception\ e',h',[])\langle proof\rangle$
end

5.8 Combining Stages 1 and 2

theory *Compiler*
imports *Correctness1 Correctness2*
begin

definition *J2JVM* :: *J-prog* ⇒ *jvm-prog*
where
  J2JVM ≡ *compP₂* ∘ *compP₁*

theorem *comp-correct*:
assumes *wwf*: *wwf-J-prog P*
and *method*: $P \vdash C$ *sees* M:*Ts*→*T* = (*pns*,*body*) *in C*
and *eval*: $P \vdash \langle body,(h,[this\#pns\ [\mapsto]\ vs])\rangle \Rightarrow \langle e',(h',l')\rangle$
and *sizes*: *size vs* = *size pns* + *1* *size rest* = *max-vars body*
shows *J2JVM* $P \vdash (None,h,[([],vs@rest,C,M,0)]) -jvm\rightarrow (exception\ e',h',[])\langle proof\rangle$

end

5.9 Preservation of Well-Typedness

theory *TypeComp*
imports *Compiler* ../*BV*/*BVSpec*
begin

locale *TC0* =
  fixes *P* :: *J₁-prog* **and** *mxl* :: *nat*
begin

definition *ty E e* = (*THE T*. $P,E \vdash_1 e :: T$)

definition *tyₗ E A'* = *map* (λ*i*. *if i* ∈ *A'* ∧ *i* < *size E then OK*(*E*!*i*) *else Err*) [*0*..<*mxl*]

definition *tyᵢ' ST E A* = (*case A of None* ⇒ *None* | ⌊*A'*⌋ ⇒ *Some*(*ST*, *tyₗ E A'*))

definition *after E A ST e* = *tyᵢ'* (*ty E e* # *ST*) *E* (*A* ⊔ 𝒜 *e*)

end

lemma (**in** *TC0*) *ty-def2* [*simp*]: $P,E \vdash_1 e :: T \Longrightarrow ty\ E\ e = T\langle proof\rangle$

lemma (**in** *TC0*) [*simp*]: $ty_i'\ ST\ E\ None = None\langle proof\rangle$
lemma (**in** *TC0*) *ty$_l$-app-diff*[*simp*]:
$ty_l\ (E@[T])\ (A - \{size\ E\}) = ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_i$'-app-diff*[*simp*]:
$ty_i'\ ST\ (E\ @\ [T])\ (A \ominus size\ E) = ty_i'\ ST\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_l$-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_l\ E\ A'\ [\leq_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_i$'-antimono*:
$A \subseteq A' \Longrightarrow P \vdash ty_i'\ ST\ E\ \lfloor A'\rfloor \leq' ty_i'\ ST\ E\ \lfloor A\rfloor\langle proof\rangle$

lemma (**in** *TC0*) *ty$_l$-env-antimono*:
$P \vdash ty_l\ (E@[T])\ A\ [\leq_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_i$'-env-antimono*:
$P \vdash ty_i'\ ST\ (E@[T])\ A \leq' ty_i'\ ST\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_i$'-incr*:
$P \vdash ty_i'\ ST\ (E\ @\ [T])\ \lfloor insert\ (size\ E)\ A\rfloor \leq' ty_i'\ ST\ E\ \lfloor A\rfloor\langle proof\rangle$

lemma (**in** *TC0*) *ty$_l$-incr*:
$P \vdash ty_l\ (E\ @\ [T])\ (insert\ (size\ E)\ A)\ [\leq_\top]\ ty_l\ E\ A\langle proof\rangle$

lemma (**in** *TC0*) *ty$_l$-in-types*:
$set\ E \subseteq types\ P \Longrightarrow ty_l\ E\ A \in nlists\ mxl\ (err\ (types\ P))\langle proof\rangle$
locale *TC1* = *TC0*
begin

primrec *compT* :: *ty list* ⇒ *nat hyperset* ⇒ *ty list* ⇒ $expr_1$ ⇒ ty_i' *list* **and**
compTs :: *ty list* ⇒ *nat hyperset* ⇒ *ty list* ⇒ $expr_1$ *list* ⇒ ty_i' *list* **where**
compT E A ST (*new C*) = []
| *compT E A ST* (*Cast C e*) =
compT E A ST e @ [*after E A ST e*]
| *compT E A ST* (*Val v*) = []
| *compT E A ST* (e_1 «*bop*» e_2) =
(*let* ST_1 = *ty E* e_1#*ST*; A_1 = $A \sqcup \mathcal{A}\ e_1$ *in*
compT E A ST e_1 @ [*after E A ST* e_1] @
compT E A_1 ST_1 e_2 @ [*after E* A_1 ST_1 e_2])
| *compT E A ST* (*Var i*) = []
| *compT E A ST* (*i* := *e*) = *compT E A ST e* @
[*after E A ST e*, $ty_i'\ ST\ E\ (A \sqcup \mathcal{A}\ e \sqcup \lfloor\{i\}\rfloor)$]
| *compT E A ST* (*e*·*F*{*D*}) =
compT E A ST e @ [*after E A ST e*]
| *compT E A ST* (e_1·*F*{*D*} := e_2) =
(*let* ST_1 = *ty E* e_1#*ST*; A_1 = $A \sqcup \mathcal{A}\ e_1$; A_2 = $A_1 \sqcup \mathcal{A}\ e_2$ *in*
compT E A ST e_1 @ [*after E A ST* e_1] @
compT E A_1 ST_1 e_2 @ [*after E* A_1 ST_1 e_2] @
[$ty_i'\ ST\ E\ A_2$])
| *compT E A ST* {*i*:*T*; *e*} = *compT* (*E*@[*T*]) (*A*⊖*i*) *ST e*
| *compT E A ST* (e_1;;e_2) =
(*let* A_1 = $A \sqcup \mathcal{A}\ e_1$ *in*
compT E A ST e_1 @ [*after E A ST* e_1, $ty_i'\ ST\ E\ A_1$] @

$compT\ E\ A_1\ ST\ e_2)$
| $compT\ E\ A\ ST\ (if\ (e)\ e_1\ else\ e_2) =$
$(let\ A_0 = A \sqcup \mathcal{A}\ e;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$compT\ E\ A_0\ ST\ e_1\ @\ [after\ E\ A_0\ ST\ e_1,\ \tau]\ @$
$compT\ E\ A_0\ ST\ e_2)$
| $compT\ E\ A\ ST\ (while\ (e)\ c) =$
$(let\ A_0 = A \sqcup \mathcal{A}\ e;\ \ A_1 = A_0 \sqcup \mathcal{A}\ c;\ \tau = ty_i'\ ST\ E\ A_0\ in$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e,\ \tau]\ @$
$compT\ E\ A_0\ ST\ c\ @\ [after\ E\ A_0\ ST\ c,\ ty_i'\ ST\ E\ A_1,\ ty_i'\ ST\ E\ A_0])$
| $compT\ E\ A\ ST\ (throw\ e) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$
| $compT\ E\ A\ ST\ (e{\cdot}M(es)) =$
$compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$compTs\ E\ (A \sqcup \mathcal{A}\ e)\ (ty\quad E\ e\ \#\ ST)\ es$
| $compT\ E\ A\ ST\ (try\ e_1\ catch(C\ i)\ e_2) =$
$compT\ E\ A\ ST\ e_1\ @\ [after\ E\ A\ ST\ e_1]\ @$
$[ty_i'\ (Class\ C\#ST)\ E\ A,\ ty_i'\ ST\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)]\ @$
$compT\ (E@[Class\ C])\ (A \sqcup \lfloor\{i\}\rfloor)\ ST\ e_2$
| $compTs\ E\ A\ ST\ [] = []$
| $compTs\ \ E\ A\ ST\ (e\#es) = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]\ @$
$compTs\ E\ (A \sqcup (\mathcal{A}\ e))\ (ty\ E\ e\ \#\ ST)\ es$

definition $compT_a :: ty\ list \Rightarrow nat\ hyperset \Rightarrow ty\ list \Rightarrow expr_1 \Rightarrow ty_i'\ list$ **where**
$compT_a\ E\ A\ ST\ e = compT\ E\ A\ ST\ e\ @\ [after\ E\ A\ ST\ e]$

end

lemma *compE$_2$-not-Nil*[*simp*]: $compE_2\ e \neq []\langle proof\rangle$
lemma (**in** *TC1*) *compT-sizes*[*simp*]:
shows $\bigwedge E\ A\ ST.\ size(compT\ E\ A\ ST\ e) = size(compE_2\ e) - 1$
and $\bigwedge E\ A\ ST.\ size(compTs\ E\ A\ ST\ es) = size(compEs_2\ es)\langle proof\rangle$

lemma (**in** *TC1*) [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compT\ E\ None\ ST\ e)$
and [*simp*]: $\bigwedge ST\ E.\ \lfloor\tau\rfloor \notin set\ (compTs\ E\ None\ ST\ es)\langle proof\rangle$

lemma (**in** *TC0*) *pair-eq-ty$_i$'-conv*:
$(\lfloor(ST,\ LT)\rfloor = ty_i'\ ST_0\ E\ A) =$
$(case\ A\ of\ None \Rightarrow False \mid Some\ A \Rightarrow (ST = ST_0 \wedge LT = ty_l\ E\ A))\langle proof\rangle$

lemma (**in** *TC0*) *pair-conv-ty$_i$'*:
$\lfloor(ST,\ ty_l\ E\ A)\rfloor = ty_i'\ ST\ E\ \lfloor A\rfloor\langle proof\rangle$

lemma (**in** *TC1*) *compT-LT-prefix*:
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compT\ E\ A\ ST_0\ e);\ \mathcal{B}\ e\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \leq' ty_i'\ ST\ E\ A$
and
$\bigwedge E\ A\ ST_0.\ [\![\ \lfloor(ST,LT)\rfloor \in set(compTs\ E\ A\ ST_0\ es);\ \mathcal{B}s\ es\ (size\ E)\ ]\!]$
$\Longrightarrow P \vdash \lfloor(ST,LT)\rfloor \leq' ty_i'\ ST\ E\ A\langle proof\rangle$

lemma [*iff*]: $OK\ None \in states\ P\ mxs\ mxl\langle proof\rangle$
lemma (**in** *TC0*) *after-in-states*:
assumes *wf*: *wf-prog p P* **and** *wt*: $P,E \vdash_1 e :: T$
and *Etypes*: *set E* $\subseteq$ *types P* **and** *STtypes*: *set ST* $\subseteq$ *types P*
and *stack*: *size ST* + *max-stack e* $\leq$ *mxs*

shows *OK* (*after E A ST e*) ∈ *states P mxs mxl*⟨*proof*⟩

lemma (**in** *TC0*) *OK-ty$_i$'-in-statesI*[*simp*]:
⟦ *set E* ⊆ *types P*; *set ST* ⊆ *types P*; *size ST* ≤ *mxs* ⟧
⟹ *OK* (*ty$_i$' ST E A*) ∈ *states P mxs mxl*⟨*proof*⟩

lemma *is-class-type-aux*: *is-class P C* ⟹ *is-type P* (*Class C*)⟨*proof*⟩
theorem (**in** *TC1*) *compT-states*:
assumes *wf*: *wf-prog p P*
shows ⋀*E T A ST*.
⟦ *P*,*E* ⊢$_1$ *e* :: *T*; *set E* ⊆ *types P*; *set ST* ⊆ *types P*;
size ST + *max-stack e* ≤ *mxs*; *size E* + *max-vars e* ≤ *mxl* ⟧
⟹ *OK* ‘ *set*(*compT E A ST e*) ⊆ *states P mxs mxl*
and ⋀*E Ts A ST*.
⟦ *P*,*E* ⊢$_1$ *es*[::]*Ts*; *set E* ⊆ *types P*; *set ST* ⊆ *types P*;
size ST + *max-stacks es* ≤ *mxs*; *size E* + *max-varss es* ≤ *mxl* ⟧
⟹ *OK* ‘ *set*(*compTs E A ST es*) ⊆ *states P mxs mxl*⟨*proof*⟩

definition *shift* :: *nat* ⇒ *ex-table* ⇒ *ex-table*
where
shift n xt ≡ *map* (λ(*from*,*to*,*C*,*handler*,*depth*). (*from*+*n*,*to*+*n*,*C*,*handler*+*n*,*depth*)) *xt*

lemma [*simp*]: *shift 0 xt* = *xt*⟨*proof*⟩
lemma [*simp*]: *shift n* [] = []⟨*proof*⟩
lemma [*simp*]: *shift n* (*xt$_1$* @ *xt$_2$*) = *shift n xt$_1$* @ *shift n xt$_2$*⟨*proof*⟩
lemma [*simp*]: *shift m* (*shift n xt*) = *shift* (*m*+*n*) *xt*⟨*proof*⟩
lemma [*simp*]: *pcs* (*shift n xt*) = {*pc*+*n*|*pc*. *pc* ∈ *pcs xt*}⟨*proof*⟩

lemma *shift-compxE$_2$*:
shows ⋀*pc pc' d*. *shift pc* (*compxE$_2$ e pc' d*) = *compxE$_2$ e* (*pc'* + *pc*) *d*
and ⋀*pc pc' d*. *shift pc* (*compxEs$_2$ es pc' d*) = *compxEs$_2$ es* (*pc'* + *pc*) *d*⟨*proof*⟩

lemma *compxE$_2$-size-convs*[*simp*]:
shows *n* ≠ *0* ⟹ *compxE$_2$ e n d* = *shift n* (*compxE$_2$ e 0 d*)
and *n* ≠ *0* ⟹ *compxEs$_2$ es n d* = *shift n* (*compxEs$_2$ es 0 d*)⟨*proof*⟩
locale *TC2* = *TC1* +
fixes *T$_r$* :: *ty* **and** *mxs* :: *pc*
begin

definition
wt-instrs :: *instr list* ⇒ *ex-table* ⇒ *ty$_i$' list* ⇒ *bool*
((⊢ -, - /[::]/ -) [*0*,*0*,*51*] *50*) **where**
⊢ *is*,*xt* [::] *τs* ⟷ *size is* < *size τs* ∧ *pcs xt* ⊆ {*0*..<*size is*} ∧
(∀ *pc*< *size is*. *P*,*T$_r$*,*mxs*,*size τs*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*)

end

notation *TC2.wt-instrs* ((-,-,- ⊢/ -, - /[::]/ -) [*50*,*50*,*50*,*50*,*50*,*51*] *50*)

lemma (**in** *TC2*) [*simp*]: *τs* ≠ [] ⟹ ⊢ [],[] [::] *τs*⟨*proof*⟩
lemma [*simp*]: *eff i P pc et None* = []⟨*proof*⟩
lemma *wt-instr-appR*:
⟦ *P*,*T*,*m*,*mpc*,*xt* ⊢ *is*!*pc*,*pc* :: *τs*;

$pc < size\ is;\ size\ is < size\ \tau s;\ mpc \le size\ \tau s;\ mpc \le mpc'\ ]\!]$
$\Longrightarrow P,T,m,mpc',xt \vdash is!pc,pc :: \tau s@\tau s'\langle proof\rangle$

lemma *relevant-entries-shift* [*simp*]:
$relevant\text{-}entries\ P\ i\ (pc+n)\ (shift\ n\ xt) = shift\ n\ (relevant\text{-}entries\ P\ i\ pc\ xt)\langle proof\rangle$

lemma [*simp*]:
$xcpt\text{-}eff\ i\ P\ (pc+n)\ \tau\ (shift\ n\ xt) =$
$map\ (\lambda(pc,\tau).\ (pc + n, \tau))\ (xcpt\text{-}eff\ i\ P\ pc\ \tau\ xt)\langle proof\rangle$

lemma [*simp*]:
$app_i\ (i,\ P,\ pc,\ m,\ T,\ \tau) \Longrightarrow$
$eff\ i\ P\ (pc+n)\ (shift\ n\ xt)\ (Some\ \tau) =$
$map\ (\lambda(pc,\tau).\ (pc+n,\tau))\ (eff\ i\ P\ pc\ xt\ (Some\ \tau))\langle proof\rangle$

lemma [*simp*]:
$xcpt\text{-}app\ i\ P\ (pc+n)\ mxs\ (shift\ n\ xt)\ \tau = xcpt\text{-}app\ i\ P\ pc\ mxs\ xt\ \tau\langle proof\rangle$

lemma *wt-instr-appL*:
assumes $P,T,m,mpc,xt \vdash i,pc :: \tau s$ **and** $pc < size\ \tau s$ **and** $mpc \le size\ \tau s$
shows $P,T,m,mpc + size\ \tau s',shift\ (size\ \tau s')\ xt \vdash i,pc+size\ \tau s' :: \tau s'@\tau s\langle proof\rangle$

lemma *wt-instr-Cons*:
assumes *wti*: $P,T,m,mpc - 1,[] \vdash i,pc - 1 :: \tau s$
and *pcl*: $0 < pc$ **and** *mpcl*: $0 < mpc$
and *pcu*: $pc < size\ \tau s + 1$ **and** *mpcu*: $mpc \le size\ \tau s + 1$
shows $P,T,m,mpc,[] \vdash i,pc :: \tau\#\tau s\langle proof\rangle$

lemma *wt-instr-append*:
assumes *wti*: $P,T,m,mpc - size\ \tau s',[] \vdash i,pc - size\ \tau s' :: \tau s$
and *pcl*: $size\ \tau s' \le pc$ **and** *mpcl*: $size\ \tau s' \le mpc$
and *pcu*: $pc < size\ \tau s + size\ \tau s'$ **and** *mpcu*: $mpc \le size\ \tau s + size\ \tau s'$
shows $P,T,m,mpc,[] \vdash i,pc :: \tau s'@\tau s\langle proof\rangle$

lemma *xcpt-app-pcs*:
$pc \notin pcs\ xt \Longrightarrow xcpt\text{-}app\ i\ P\ pc\ mxs\ xt\ \tau\langle proof\rangle$

lemma *xcpt-eff-pcs*:
$pc \notin pcs\ xt \Longrightarrow xcpt\text{-}eff\ i\ P\ pc\ \tau\ xt = []\langle proof\rangle$

lemma *pcs-shift*:
$pc < n \Longrightarrow pc \notin pcs\ (shift\ n\ xt)\langle proof\rangle$

lemma *wt-instr-appRx*:
$[\![\ P,T,m,mpc,xt \vdash is!pc,pc :: \tau s;\ pc < size\ is;\ size\ is < size\ \tau s;\ mpc \le size\ \tau s\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt\ @\ shift\ (size\ is)\ xt' \vdash is!pc,pc :: \tau s\langle proof\rangle$

lemma *wt-instr-appLx*:
$[\![\ P,T,m,mpc,xt \vdash i,pc :: \tau s;\ pc \notin pcs\ xt'\ ]\!]$
$\Longrightarrow P,T,m,mpc,xt'@xt \vdash i,pc :: \tau s\langle proof\rangle$

lemma (**in** *TC2*) *wt-instrs-extR*:
$\vdash is,xt\ [::]\ \tau s \Longrightarrow \vdash is,xt\ [::]\ \tau s\ @\ \tau s'\langle proof\rangle$

lemma (**in** *TC2*) *wt-instrs-ext*:
assumes wt_1: $\vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2$ **and** wt_2: $\vdash is_2,xt_2$ [::] τs_2
and *τs-size*: $size\ \tau s_1 = size\ is_1$
shows $\vdash is_1 @ is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2$ [::] $\tau s_1 @ \tau s_2 \langle proof \rangle$
corollary (**in** *TC2*) *wt-instrs-ext2*:
$[\![\vdash is_2,xt_2$ [::] $\tau s_2; \vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2;\ size\ \tau s_1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1 @ is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2$ [::] $\tau s_1 @ \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-ext-prefix* [*trans*]:
$[\![\vdash is_1,xt_1$ [::] $\tau s_1 @ \tau s_2; \vdash is_2,xt_2$ [::] $\tau s_3;$
$size\ \tau s_1 = size\ is_1;\ prefix\ \tau s_3\ \tau s_2\ ]\!]$
$\Longrightarrow \vdash is_1 @ is_2,\ xt_1\ @\ shift\ (size\ is_1)\ xt_2$ [::] $\tau s_1 @ \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-app*:
assumes is_1: $\vdash is_1,xt_1$ [::] $\tau s_1 @ [\tau]$
assumes is_2: $\vdash is_2,xt_2$ [::] $\tau \# \tau s_2$
assumes *s*: $size\ \tau s_1 = size\ is_1$
shows $\vdash is_1 @ is_2,\ xt_1 @ shift\ (size\ is_1)\ xt_2$ [::] $\tau s_1 @ \tau \# \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-app-last*[*trans*]:
assumes $\vdash is_2,xt_2$ [::] $\tau \# \tau s_2$ $\vdash is_1,xt_1$ [::] τs_1
$last\ \tau s_1 = \tau$ $size\ \tau s_1 = size\ is_1 + 1$
shows $\vdash is_1 @ is_2,\ xt_1 @ shift\ (size\ is_1)\ xt_2$ [::] $\tau s_1 @ \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-append-last*[*trans*]:
assumes *wtis*: $\vdash is,xt$ [::] τs **and** *wti*: $P,T_r,mxs,mpc,[] \vdash i,pc :: \tau s$
and *pc*: $pc = size\ is$ **and** *mpc*: $mpc = size\ \tau s$ **and** *is-τs*: $size\ is + 1 < size\ \tau s$
shows $\vdash is @ [i],xt$ [::] $\tau s \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-app2*:
$[\![\vdash is_2,xt_2$ [::] $\tau' \# \tau s_2;\ \vdash is_1,xt_1$ [::] $\tau \# \tau s_1 @ [\tau'];$
$xt' = xt_1\ @\ shift\ (size\ is_1)\ xt_2;\ \ size\ \tau s_1 + 1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1 @ is_2,xt'$ [::] $\tau \# \tau s_1 @ \tau' \# \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-app2-simp*[*trans*,*simp*]:
$[\![\vdash is_2,xt_2$ [::] $\tau' \# \tau s_2;\ \vdash is_1,xt_1$ [::] $\tau \# \tau s_1 @ [\tau'];\ size\ \tau s_1 + 1 = size\ is_1\ ]\!]$
$\Longrightarrow \vdash is_1 @ is_2,\ xt_1 @ shift\ (size\ is_1)\ xt_2$ [::] $\tau \# \tau s_1 @ \tau' \# \tau s_2 \langle proof \rangle$

corollary (**in** *TC2*) *wt-instrs-Cons*[*simp*]:
$[\![\ \tau s \neq [];\ \vdash [i],[]$ [::] $[\tau,\tau'];\ \vdash is,xt$ [::] $\tau' \# \tau s\ ]\!]$
$\Longrightarrow \vdash i \# is,shift\ 1\ xt$ [::] $\tau \# \tau' \# \tau s \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle \langle proof \rangle$
theory *Jinja*
imports
J/TypeSafe
J/Annotate

J/execute-Bigstep
J/execute-WellType
JVM/JVMDefensive
JVM/JVMListExample
BV/BVExec
BV/LBVJVM
BV/BVNoTypeError
BV/BVExample

Compiler/TypeComp
begin

end

Bibliography

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