Faithful Logic Embeddings in HOL — Deep and Shallow (Isabelle/HOL dataset)

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Abstract

A recipe for the simultaneous deployment of different forms of deep and shallow embeddings of non-classical logics in classical higher-order logic is presented, which enables interactive or even automated faithfulness proofs between the logic embeddings. The approach, which is particularly fruitful for logic education, is explained in detail in an associated CADE conference paper. This paper presents the corresponding Isabelle/HOL dataset (which is only slightly modified to meet AFP requirements).

Contents

1 Introduction

The Isabelle/HOL dataset associated with [1] is presented. Sections 3, 4 and 5 present deep, maximally shallow, and minimally shallow embeddings of propositional modal logic (PML) in classical higher-order logic (HOL). These are connected, as a novel contribution, by automated faithfulness proofs given in Sect. 6. This connection ensures that these deep and shallow embeddings can now be used interchangeably in subsequent applications. Several experiments with the presented embeddings are presented in Sect. 7. The presented work is conceptual in nature and can be adapted to other non-classical logics. For more detailed explanations of the presented material, including a discussion of related works, see [1].

2 Preliminaries

The following preliminaries are are shared between all embeddings introduced in the remainder of this paper.

theory *PMLinHOL-preliminaries*
imports *Main*
begin

— Type declarations common for both the deep and shallow embedding
typedecl w — Type for possible worlds
typedecl $\mathcal{S}$ — Type for propositional constant symbols
consts p::$\mathcal{S}$ q::$\mathcal{S}$ — Some propositional constant symbols
type-synonym $\mathcal{W}$ = w⇒*bool* — Type for sets of possible worlds
type-synonym $\mathcal{R}$ = w⇒w⇒*bool* — Type for accessibility relations
type-synonym $\mathcal{V}$ = $\mathcal{S}$⇒w⇒*bool* — Type for valuation functions

— Some useful predicates for accessibility relations
abbreviation(*input*) $reflexive \equiv \lambda R::\mathcal{R}.\ \forall x.\ R\ x\ x$
abbreviation(*input*) $symmetric \equiv \lambda R::\mathcal{R}.\ \forall x\ y.\ R\ x\ y \longrightarrow R\ y\ x$
abbreviation(*input*) $transitive \equiv \lambda R::\mathcal{R}.\ \forall x\ y\ z.\ (R\ x\ y \wedge R\ y\ z) \longrightarrow R\ x\ z$
abbreviation(*input*) $equivrel \equiv \lambda R::\mathcal{R}.\ reflexive\ R \wedge symmetric\ R \wedge transitive\ R$
abbreviation(*input*) $irreflexive \equiv \lambda R::\mathcal{R}.\ \forall x.\ \neg R\ x\ x$
abbreviation(*input*) $euclidean \equiv \lambda R::\mathcal{R}.\ \forall x\ y\ z.\ R\ x\ y \wedge R\ x\ z \longrightarrow R\ y\ z$
abbreviation(*input*) $wellfounded \equiv \lambda R::\mathcal{R}.\ \forall P::\mathcal{W}.\ (\forall x.\ (\forall y.\ R\ y\ x \longrightarrow P\ y) \longrightarrow P\ x) \longrightarrow (\forall x.\ P\ x)$
abbreviation(*input*) $converserel \equiv \lambda R::\mathcal{R}.\ \lambda y::w.\ \lambda x::w.\ R\ x\ y$

abbreviation(*input*) *conversewf* $\equiv \lambda R{::}\mathcal{R}.\ wellfounded\ (converserel\ R)$

— Bounded universal quantifier: $\forall x{:}W.\ \varphi$ stands for $\forall x.\ W\ x \longrightarrow \varphi\ x$
abbreviation(*input*) *BoundedAll*::$\mathcal{W}\Rightarrow\mathcal{W}\Rightarrow bool$ **where** *BoundedAll* $W\ \varphi \equiv \forall x.$ $W\ x \longrightarrow \varphi\ x$
syntax *-BoundedAll*:: *pttrn*$\Rightarrow\mathcal{W}\Rightarrow bool\Rightarrow bool$ ((*3*∀(-/:-)./ -) [*0*, *0*, *10*] *10*)
translations $\forall x{:}W.\ \varphi \rightleftharpoons$ *CONST BoundedAll W* $(\lambda x.\ \varphi)$

— Backward implication; useful for aestethic reasons
abbreviation(*input*) *Bimp* (**infixr** $\longleftarrow$ *50*) **where** $\varphi \longleftarrow \psi \equiv \psi \longrightarrow \varphi$

— Some further settings
declare[[*syntax-ambiguity-warning=false*]]
nitpick-params[*user-axioms*,*expect=genuine*,*timeout=60*]

end

3 Deep embedding of PML in HOL

theory *PMLinHOL-deep*
 imports *PMLinHOL-preliminaries*
begin
— Deep embedding (of propositional modal logic in HOL)
datatype *PML* = *AtmD* $\mathcal{S}$ (-^d) | *NotD PML* $(\neg^d)$ | *ImpD PML PML* (**infixr** $\supset^d$ *93*) | *BoxD PML* $(\Box^d)$

— Further logical connectives as definitions
definition *OrD* (**infixr** $\vee^d$ *92*) **where** $\varphi\vee^d\psi \equiv \neg^d\varphi \supset^d \psi$
definition *AndD* (**infixr** $\wedge^d$ *95*) **where** $\varphi\wedge^d\psi \equiv \neg^d(\varphi \supset^d \neg^d\psi)$
definition *DiaD* $(\Diamond^d\text{-})$ **where** $\Diamond^d\varphi \equiv \neg^d(\Box^d(\neg^d\varphi))$
definition *TopD* $(\top^d)$ **where** $\top^d \equiv p^d \supset^d p^d$
definition *BotD* $(\bot^d)$ **where** $\bot^d \equiv \neg^d\ \top^d$

— Definition of truth of a formula relative to a model $\langle W,R,V\rangle$ and possible world w
primrec *RelativeTruthD* :: $\mathcal{W}\Rightarrow\mathcal{R}\Rightarrow\mathcal{V}\Rightarrow$w$\Rightarrow PML\Rightarrow bool$ $(\langle\text{-},\text{-},\text{-}\rangle,\text{-} \models^d \text{-})$ **where**
 $\langle W,R,V\rangle,\ w \models^d a^d\ = (V\ a\ w)$
 | $\langle W,R,V\rangle,\ w \models^d \neg^d\varphi = (\neg\ \langle W,R,V\rangle,\ w \models^d \varphi)$
 | $\langle W,R,V\rangle,\ w \models^d \varphi \supset^d \psi = (\langle W,R,V\rangle,\ w \models^d \varphi \longrightarrow \langle W,R,V\rangle,\ w \models^d \psi)$
 | $\langle W,R,V\rangle,\ w \models^d \Box^d\varphi = (\forall v{:}W.\ R\ w\ v \longrightarrow \langle W,R,V\rangle,\ v \models^d \varphi)$

— Definition of validity
definition *ValD* $(\models^d \text{-})$ **where** $(\models^d \varphi) \equiv (\forall W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi)$

— Collection of definitions in a bag called DefD
named-theorems *DefD* **declare** *OrD-def*[*DefD*,*simp*] *AndD-def*[*DefD*,*simp*] *DiaD-def*[*DefD*,*simp*] *TopD-def*[*DefD*,*simp*] *BotD-def*[*DefD*,*simp*] *RelativeTruthD-def*[*DefD*,*simp*] *ValD-def*[*DefD*,*simp*]
end

4 Shallow embedding of PML in HOL (maximal)

theory *PMLinHOL-shallow*
imports *PMLinHOL-preliminaries*
begin

— Shallow embedding (of propositional modal logic in HOL)
type-synonym $\sigma = \mathcal{W}\Rightarrow\mathcal{R}\Rightarrow\mathcal{V}\Rightarrow \mathrm{w}\Rightarrow bool$
definition $AtmS::\mathcal{S}\Rightarrow\sigma$ (-^s) **where** $a^s \equiv \lambda W\ R\ V\ w.\ V\ a\ w$
definition $NegS::\sigma\Rightarrow\sigma$ $(\neg^s)$ **where** $\neg^s\ \varphi \equiv \lambda W\ R\ V\ w.\ \neg(\varphi\ \ W\ R\ V\ w)$
definition $ImpS::\sigma\Rightarrow\sigma\Rightarrow\sigma$ (**infixr** $\supset^s$ *93*) **where** $\varphi \supset^s \psi \equiv \lambda W\ R\ V\ w.\ (\varphi\ \ W\ R\ V\ w) \longrightarrow (\psi\ \ W\ R\ V\ w)$
definition $BoxS::\sigma\Rightarrow\sigma$ $(\Box^s)$ **where** $\Box^s\ \varphi \equiv \lambda W\ R\ V\ w.\ \forall v{:}W.\ R\ w\ v \longrightarrow (\varphi\ \ W\ R\ V\ v)$

— Further logical connectives as definitions
definition *OrS* (**infixr** $\vee^s$ *92*) **where** $\varphi \vee^s \psi \equiv \neg^s\varphi \supset^s \psi$
definition *AndS* (**infixr** $\wedge^s$ *95*) **where** $\varphi \wedge^s \psi \equiv \neg^s(\varphi \supset^s \neg^s\psi)$
definition *DiaS* $(\Diamond^s)$ **where** $\Diamond^s\varphi \equiv \neg^s\ (\Box^s\ (\neg^s\varphi))$
definition *TopS* $(\top^s)$ **where** $\top^s \equiv p^s \supset^s p^s$
definition *BotS* $(\bot^s)$ **where** $\bot^s \equiv \neg^s\ \top^s$

— Definition of truth of a formula relative to a model $\langle W,R,V\rangle$ and possible world w
definition $RelativeTruthS::\mathcal{W}\Rightarrow\mathcal{R}\Rightarrow\mathcal{V}\Rightarrow\mathrm{w}\Rightarrow\sigma\Rightarrow bool$ $(\langle\text{-},\text{-},\text{-}\rangle,\text{-}\models^s\ \text{-})$ **where** $\langle W,R,V\rangle,w \models^s \varphi \equiv \varphi\ \ W\ R\ V\ w$

— Definition of validity
definition *ValS* $(\models^s\ \text{-})$ **where** $\models^s\ \varphi \equiv \forall\ W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi$

— Collection of definitions in a bag called DefS
named-theorems *DefS* **declare** *AtmS-def*[*DefS*,*simp*] *NegS-def*[*DefS*,*simp*] *ImpS-def*[*DefS*,*simp*] *BoxS-def*[*DefS*,*simp*] *OrS-def*[*DefS*,*simp*] *AndS-def*[*DefS*,*simp*] *DiaS-def*[*DefS*,*simp*] *TopS-def*[*DefS*,*simp*] *BotS-def*[*DefS*,*simp*] *RelativeTruthS-def*[*DefS*,*simp*] *ValS-def*[*DefS*,*simp*]
end

5 Shallow embedding of PML in HOL (minimal)

theory *PMLinHOL-shallow-minimal*
imports *PMLinHOL-preliminaries*
begin

— The acessibility relation R and the valuation function V are introduced as constants at the meta-level HOL
consts $R::\mathcal{R}$ $V::\mathcal{V}$

— Shallow embedding (of propositional modal logic in HOL)
type-synonym $\sigma = \mathrm{w}\Rightarrow bool$
definition $AtmM::\mathcal{S}\Rightarrow\sigma$ (-^m) **where** $a^m \equiv \lambda w.\ V\ a\ w$

definition *NegM*::$\sigma \Rightarrow \sigma$ ($\neg^m$) **where** $\neg^m\ \varphi \equiv \lambda w. \neg \varphi\ w$
definition *ImpM*::$\sigma \Rightarrow \sigma \Rightarrow \sigma$ (**infixr** $\supset^m$ *93*) **where** $\varphi \supset^m \psi \equiv \lambda w.\ \varphi\ w \longrightarrow \psi\ w$
definition *BoxM*::$\sigma \Rightarrow \sigma$ ($\Box^m$) **where** $\Box^m\ \varphi \equiv \lambda w.\ \forall v.\ R\ w\ v \longrightarrow \varphi\ v$

— Further logical connectives as definitions
definition *OrM* (**infixr** $\vee^m$ *92*) **where** $\varphi \vee^m \psi \equiv \neg^m \varphi \supset^m \psi$
definition *AndM* (**infixr** $\wedge^m$ *95*) **where** $\varphi \wedge^m \psi \equiv \neg^m(\varphi \supset^m \neg^m \psi)$
definition *DiaM* ($\Diamond^m$-) **where** $\Diamond^m \varphi \equiv \neg^m\ (\Box^m\ (\neg^m \varphi))$
definition *TopM* ($\top^m$) **where** $\top^m \equiv p^m \supset^m p^m$
definition *BotM* ($\bot^m$) **where** $\bot^m \equiv \neg^m\ \top^m$

— Definition of truth of a formula relative to a model $\langle W,R,V \rangle$ and a possible world w
definition *RelativeTruthM*::w$\Rightarrow \sigma \Rightarrow$*bool* (-$\models^m$ -) **where** $w \models^m \varphi \equiv \varphi\ w$

— Definition of validity
definition *ValM* ($\models^m$ -) **where** $\models^m \varphi \equiv \forall w$::w. $w \models^m \varphi$

— Collection of definitions in a bag called DefM
named-theorems *DefM* **declare** *AtmM-def*[*DefM*,*simp*] *NegM-def*[*DefM*,*simp*] *ImpM-def*[*DefM*,*simp*] *BoxM-def*[*DefM*,*simp*] *OrM-def*[*DefM*,*simp*] *AndM-def*[*DefM*,*simp*] *DiaM-def*[*DefM*,*simp*] *TopM-def*[*DefM*,*simp*] *BotM-def*[*DefM*,*simp*] *RelativeTruthM-def*[*DefM*,*simp*] *ValM-def*[*DefM*,*simp*]
end

6 Automated faithfulness proofs

theory *PMLinHOL-faithfulness*
imports *PMLinHOL-deep PMLinHOL-shallow PMLinHOL-shallow-minimal*
begin

— Mappings: deep to maximal shallow and deep to minimal shallow
primrec *DpToShMax* ($(\!|$-$|\!)$) **where** $(\!|\varphi^d|\!) = \varphi^s \mid (\!|\neg^d\ \varphi|\!) = \neg^s\ (\!|\varphi|\!) \mid (\!|\varphi \supset^d \psi|\!) = (\!|\varphi|\!) \supset^s (\!|\psi|\!) \mid (\!|\Box^d\ \varphi|\!) = \Box^s\ (\!|\varphi|\!)$
primrec *DpToShMin* ($[\![$-$]\!]$) **where** $[\![\varphi^d]\!] = \varphi^m \mid [\![\neg^d\ \varphi]\!] = \neg^m\ [\![\varphi]\!] \mid [\![\varphi \supset^d \psi]\!] = [\![\varphi]\!] \supset^m [\![\psi]\!] \mid [\![\Box^d\ \varphi]\!] = \Box^m\ [\![\varphi]\!]$

— Proving faithfulness between deep and maximal shallow
theorem *Faithful1a*: $\forall\ W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \longleftrightarrow \langle W,R,V\rangle,w \models^s (\!|\varphi|\!)$ $\langle proof \rangle$
theorem *Faithful1b*: $\models^d \varphi \longleftrightarrow\ \models^s (\!|\varphi|\!)$ $\langle proof \rangle$
theorem *Faithful2*: $\forall w.\ \langle(\lambda x$::w. *True*$),R,V\rangle,w \models^d \varphi \longleftrightarrow w \models^m [\![\varphi]\!]$ $\langle proof \rangle$
theorem *Faithful3*: $\forall w.\ \langle(\lambda x$::w. *True*$),R,V\rangle,w \models^s (\!|\varphi|\!) \longleftrightarrow w \models^m [\![\varphi]\!]$ $\langle proof \rangle$
lemma *Sound1*: $\models^m \psi \longleftrightarrow (\exists \varphi.\ \psi = [\![\varphi]\!] \wedge\ \models^d \varphi)$ $\langle proof \rangle$
lemma *Sound2*: $\models^m \psi \longleftrightarrow (\exists \varphi.\ \psi = [\![\varphi]\!] \wedge\ \models^m [\![\varphi]\!])$ $\langle proof \rangle$
end

7 Appendix: proof automation tests

7.1 Tests with the deep embedding

theory *PMLinHOL-deep-tests*
imports *PMLinHOL-deep*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^d \varphi \supset^d (\psi \supset^d \varphi)$ ⟨*proof*⟩
lemma *H2*: $\models^d (\varphi \supset^d (\psi \supset^d \gamma)) \supset^d ((\varphi \supset^d \psi) \supset^d (\varphi \supset^d \gamma))$ ⟨*proof*⟩
lemma *H3*: $\models^d (\neg^d\varphi \supset^d \neg^d\psi) \supset^d (\psi \supset^d \varphi)$ ⟨*proof*⟩
lemma *MP*: $\models^d \varphi \Longrightarrow \models^d (\varphi \supset^d \psi) \Longrightarrow \models^d \psi$ ⟨*proof*⟩
lemma *HCderived1*: $\models^d (\varphi \supset^d \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
⟨*proof*⟩

lemma *HCderived2*: $\models^d \varphi \supset^d (\neg^d\varphi \supset^d \psi)$ ⟨*proof*⟩
lemma *HCderived3*: $\models^d (\neg^d\varphi \supset^d \varphi) \supset^d \varphi$ ⟨*proof*⟩
lemma *HCderived4*: $\models^d (\varphi \supset^d \psi) \supset^d (\neg^d\psi \supset^d \neg^d\varphi)$ ⟨*proof*⟩
lemma *Nec*: $\models^d \varphi \Longrightarrow \models^d \Box^d\varphi$ ⟨*proof*⟩
lemma *Dist*: $\models^d \Box^d(\varphi \supset^d \psi) \supset^d (\Box^d\varphi \supset^d \Box^d\psi)$ ⟨*proof*⟩
lemma *cM*: *reflexive R* $\longleftrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \varphi)$ — sledgehammer: Proof found ⟨*proof*⟩
lemma *cBa*: *symmetric R* $\longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi))$ ⟨*proof*⟩
lemma *cBb*: *symmetric R* $\longleftarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi))$ — sledgehammer: No proof ⟨*proof*⟩
lemma *c4a*: *transitive R* $\longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi))$ ⟨*proof*⟩
lemma *c4b*: *transitive R* $\longleftarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi))$ — sledgehammer: No proof ⟨*proof*⟩
lemma *reflexive R* $\longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi))$ **nitpick**[*card* w=*3*] ⟨*proof*⟩
lemma $\models^d \varphi \supset^d \Box^d\varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] ⟨*proof*⟩
lemma $\models^d \Box^d(\Box^d\varphi \supset^d \Box^d\psi) \vee^d \Box^d(\Box^d\psi \supset^d \Box^d\varphi)$ **nitpick**[*card* w=*3*] ⟨*proof*⟩
lemma $\models^d (\Diamond^d(\Box^d \varphi)) \supset^d \Box^d(\Diamond^d \varphi)$ **nitpick**[*card* w=*2*] ⟨*proof*⟩
lemma *KB*: *symmetric R* $\longrightarrow (\forall \varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi))$ ⟨*proof*⟩
lemma *K4B*: *symmetric R* $\wedge$ *transitive R* $\longrightarrow (\forall \varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d \Box^d\psi) \vee^d \Box^d(\Box^d\psi \supset^d \Box^d\varphi))$ ⟨*proof*⟩
end

theory *PMLinHOL-deep-further-tests*
imports *PMLinHOL-deep-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^d (\Box^d(\varphi \supset^d \psi)) \supset^d ((\Diamond^d\varphi) \supset^d \Diamond^d\psi)$ ⟨*proof*⟩

lemma *T1a*: $\models^d \Box^d p^d \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ ⟨*proof*⟩
lemma *T1b*: $\models^d \Box^d p^d \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ — alternative interactive proof in modal object logic K
⟨*proof*⟩
end

theory *PMLinHOL-deep-Loeb-tests*
imports *PMLinHOL-deep*
begin

— Löb axiom: with the deep embedding automated reasoning tools are not very responsive
lemma *Loeb1*: $\forall \varphi.\ \models^d \Box^d(\ \Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*] ⟨*proof*⟩
lemma *Loeb2*: (*conversewf R* ∧ *transitive R*) ⟶ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3*: (*conversewf R* ∧ *transitive R*) ⟵ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3a*: *conversewf R* ⟵ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3b*: *transitive R* ⟵ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3c*: *irreflexive R* ⟵ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
end

7.2 Tests with the maximal shallow embedding

theory *PMLinHOL-shallow-tests*
imports *PMLinHOL-shallow*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^s \varphi \supset^s (\psi \supset^s \varphi)$ ⟨*proof*⟩
lemma *H2*: $\models^s (\varphi \supset^s (\psi \supset^s \gamma)) \supset^s ((\varphi \supset^s \psi) \supset^s (\varphi \supset^s \gamma))$ ⟨*proof*⟩
lemma *H3*: $\models^s (\neg^s\varphi \supset^s \neg^s\psi) \supset^s (\psi \supset^s \varphi)$ ⟨*proof*⟩
lemma *MP*: $\models^s \varphi \Longrightarrow \models^s (\varphi \supset^s \psi) \Longrightarrow \models^s \psi$ ⟨*proof*⟩
lemma *HCderived1*: $\models^s (\varphi \supset^s \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
⟨*proof*⟩

lemma *HCderived2*: $\models^s \varphi \supset^s (\neg^s\varphi \supset^s \psi)$ ⟨*proof*⟩
lemma *HCderived3*: $\models^s (\neg^s\varphi \supset^s \varphi) \supset^s \varphi$ ⟨*proof*⟩
lemma *HCderived4*: $\models^s (\varphi \supset^s \psi) \supset^s (\neg^s\psi \supset^s \neg^s\varphi)$ ⟨*proof*⟩
lemma *Nec*: $\models^s \varphi \Longrightarrow \models^s \Box^s\varphi$ ⟨*proof*⟩
lemma *Dist*: $\models^s \Box^s(\varphi \supset^s \psi) \supset^s (\Box^s\varphi \supset^s \Box^s\psi)$ ⟨*proof*⟩
lemma *cM*:*reflexive R* ⟷ $(\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \varphi)$ — sledgehammer: Proof found ⟨*proof*⟩

lemma *cBa*: *symmetric R* $\longrightarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi))$ ⟨*proof*⟩
lemma *cBb*: *symmetric R* $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi))$ — sledgehammer: No proof ⟨*proof*⟩
lemma *c4a*: *transitive R* $\longrightarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ ⟨*proof*⟩
lemma *c4b*: *transitive R* $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ — sledgehammer: No proof ⟨*proof*⟩
lemma *reflexive R* $\longrightarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ **nitpick**[*card* w=*3*] ⟨*proof*⟩
lemma $\models^s \varphi \supset^s \Box^s\varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] ⟨*proof*⟩
lemma $\models^s \Box^s(\ \Box^s\varphi \supset^s\Box^s\psi) \vee^s \Box^s(\ \Box^s\psi \supset^s\Box^s\varphi)$ ⟨*proof*⟩
lemma $\models^s (\Diamond^s(\Box^s\ \varphi)) \supset^s\Box^s(\Diamond^s\ \varphi)$ **nitpick**[*card* w=*2*] ⟨*proof*⟩
lemma *KB*: *symmetric R* $\longrightarrow$ $(\forall\varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s (\Diamond^s(\Box^s\varphi)) \supset^s\Box^s(\Diamond^s\varphi))$ ⟨*proof*⟩
lemma *K4B*: *symmetric R* $\wedge$ *transitive R* $\longrightarrow$ $(\forall\varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\ \Box^s\varphi \supset^s\Box^s\psi) \vee^s \Box^s(\ \Box^s\psi \supset^s\Box^s\varphi))$ ⟨*proof*⟩
end

theory *PMLinHOL-shallow-further-tests*
  imports *PMLinHOL-shallow-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^s (\Box^s(\varphi \supset^s \psi)) \supset^s ((\Diamond^s\varphi) \supset^s \Diamond^s\psi)$ ⟨*proof*⟩
lemma *T1a*:$\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s\ (p^s \wedge^s\ q^s))$ ⟨*proof*⟩
lemma *T1b*: $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s\ (p^s \wedge^s\ q^s))$ — alternative interactive proof in modal object logic K
  ⟨*proof*⟩
end

theory *PMLinHOL-shallow-Loeb-tests*
  imports *PMLinHOL-shallow*
begin
— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive
lemma *Loeb1*: $\forall\varphi.\ \models^s \Box^s(\ \Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*] ⟨*proof*⟩
lemma *Loeb2*: (*conversewf R* $\wedge$ *transitive R*) $\longrightarrow$$(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found ⟨*proof*⟩
lemma *Loeb3*: (*conversewf R* $\wedge$ *transitive R*) $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3a*: *conversewf R* $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found ⟨*proof*⟩
lemma *Loeb3b*: *transitive R* $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3c*: *irreflexive R* $\longleftarrow$ $(\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found ⟨*proof*⟩
end

7.3 Tests with the minimal shallow embedding

theory *PMLinHOL-shallow-minimal-tests*
imports *PMLinHOL-shallow-minimal*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^m \varphi \supset^m (\psi \supset^m \varphi)$ ⟨*proof*⟩
lemma *H2*: $\models^m (\varphi \supset^m (\psi \supset^m \gamma)) \supset^m ((\varphi \supset^m \psi) \supset^m (\varphi \supset^m \gamma))$ ⟨*proof*⟩
lemma *H3*: $\models^m (\neg^m\varphi \supset^m \neg^m\psi) \supset^m (\psi \supset^m \varphi)$ ⟨*proof*⟩
lemma *MP*: $\models^m \varphi \Longrightarrow \models^m (\varphi \supset^m \psi) \Longrightarrow \models^m \psi$ ⟨*proof*⟩
lemma *HCderived1*: $\models^m (\varphi \supset^m \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
⟨*proof*⟩

lemma *HCderived2*: $\models^m \varphi \supset^m (\neg^m\varphi \supset^m \psi)$ ⟨*proof*⟩
lemma *HCderived3*: $\models^m (\neg^m\varphi \supset^m \varphi) \supset^m \varphi$ ⟨*proof*⟩
lemma *HCderived4*: $\models^m (\varphi \supset^m \psi) \supset^m (\neg^m\psi \supset^m \neg^m\varphi)$ ⟨*proof*⟩
lemma *Nec*: $\models^m \varphi \Longrightarrow \models^m \Box^m\varphi$ ⟨*proof*⟩
lemma *Dist*: $\models^m \Box^m(\varphi \supset^m \psi) \supset^m (\Box^m\varphi \supset^m \Box^m\psi)$ ⟨*proof*⟩
lemma *cM*: *reflexive R* $\longleftrightarrow (\forall \varphi.\ \models^m \Box^m\varphi \supset^m \varphi)$ ⟨*proof*⟩
lemma *cBa*: *symmetric R* $\longrightarrow (\forall \varphi.\ \models^m \varphi \supset^m \Box^m\Diamond^m\varphi)$ ⟨*proof*⟩
lemma *cBb*: *symmetric R* $\longleftarrow (\forall \varphi.\ \models^m \varphi \supset^m \Box^m\Diamond^m\varphi)$ ⟨*proof*⟩
lemma *c4a*: *transitive R* $\longrightarrow (\forall \varphi.\ \models^m \Box^m\varphi \supset^m \Box^m(\Box^m\varphi))$ ⟨*proof*⟩
lemma *c4b*: *transitive R* $\longleftarrow (\forall \varphi.\ \models^m \Box^m\varphi \supset^m \Box^m(\Box^m\varphi))$ ⟨*proof*⟩
lemma *reflexive R* $\longrightarrow (\forall \varphi.\ \models^m \Box^m\varphi \supset^m \Box^m(\Box^m\varphi))$ **nitpick**[*card* w=*3*,*show-all*]
⟨*proof*⟩
lemma $\models^m \varphi \supset^m \Box^m\varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] ⟨*proof*⟩
lemma $\models^m \Box^m(\ \Box^m\varphi \supset^m \Box^m\psi) \vee^m \Box^m(\ \Box^m\psi \supset^m \Box^m\varphi)$ **nitpick**[*card* w=*3*]
⟨*proof*⟩
lemma $\models^m (\Diamond^m(\Box^m\ \varphi)) \supset^m \Box^m(\Diamond^m\ \varphi)$ **nitpick**[*card* w=*2*] ⟨*proof*⟩
lemma *KB*: *symmetric R* $\longrightarrow (\forall \varphi\ \psi. \models^m (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m(\Diamond^m\varphi))$ ⟨*proof*⟩
lemma *K4B*: *symmetric R* $\wedge$ *transitive R* $\longrightarrow (\forall \varphi\ \psi.\ \models^m \Box^m(\ \Box^m\varphi \supset^m \Box^m\psi) \vee^m \Box^m(\ \Box^m\psi \supset^m \Box^m\varphi))$ ⟨*proof*⟩
end

theory *PMLinHOL-shallow-minimal-further-tests*
imports *PMLinHOL-shallow-minimal-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^m (\Box^m(\varphi \supset^m \psi)) \supset^m ((\Diamond^m\varphi) \supset^m \Diamond^m\psi)$ ⟨*proof*⟩
lemma *T1a*: $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ ⟨*proof*⟩
lemma *T1b*: $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ — alternative interactive proof in modal object logic K
⟨*proof*⟩
end

theory *PMLinHOL-shallow-minimal-Loeb-tests*

imports *PMLinHOL-shallow-minimal*
begin
— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive
lemma *Loeb1*: $\forall \varphi.\ \models^m \Box^m(\ \Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*] ⟨*proof*⟩
lemma *Loeb2*: (*conversewf* R ∧ *transitive* R) $\longrightarrow$($\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$) — sh: Proof found ⟨*proof*⟩
lemma *Loeb3*: (*conversewf* R ∧ *transitive* R) $\longleftarrow$ ($\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$) — sh: No Proof ⟨*proof*⟩
lemma *Loeb3a*: *conversewf* R $\longleftarrow$ ($\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$) ⟨*proof*⟩
lemma *Loeb3b*: *transitive* R $\longleftarrow$ ($\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$) — sledgehammer: No Proof ⟨*proof*⟩
lemma *Loeb3c*: *irreflexive* R $\longleftarrow$ ($\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$) — sledgehammer: Proof found ⟨*proof*⟩
end

8 Test Examples: Formula classification

8.1 Tests with the deep embedding: axiom schemata

theory *PMLinHOL-deep-further-tests-1*
imports *PMLinHOL-deep-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms

consts φ::*PML* ψ::*PML*
abbreviation(*input*) *F1* $\equiv (\Diamond^d(\Diamond^d\varphi)) \supset^d \Diamond^d\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d\varphi$ — holds in KB4
abbreviation(*input*) *F4* $\equiv (\Box^d(\Diamond^d(\Box^d(\Diamond^d\varphi)))) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB/K4
abbreviation(*input*) *F5* $\equiv (\Diamond^d(\varphi \wedge^d (\Diamond^d\psi))) \supset^d ((\Diamond^d\varphi) \wedge^d (\Diamond^d\psi))$ — holds in K4
abbreviation(*input*) *F6* $\equiv ((\Box^d(\varphi \supset^d \psi)) \wedge^d (\Diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\Diamond^d\psi)$ — holds in KB4
abbreviation(*input*) *F7* $\equiv (\Diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \Diamond^d\varphi))$ — holds in KB4
abbreviation(*input*) *F8* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \Diamond^d\varphi)$ — holds in KT/KB
abbreviation(*input*) *F9* $\equiv ((\Box^d(\Diamond^d\varphi)) \wedge^d (\Box^d(\Diamond^d(\neg^d \varphi)))) \supset^d \Diamond^d(\Diamond^d\varphi)$ — holds in KT
abbreviation(*input*) *F10* $\equiv ((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩

lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$

— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F4$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$

— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩

lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \land (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩

lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩

lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$

— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩

lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F10$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=16 (with simp 10, without simp 6), none=74 (with simp 0, without simp 74), unknwn=70 (with simp 70, without simp 0) Sledgehammer: proof=33 (with simp 16, without simp 17), no prf=127 (with simp 64, without simp 63)

— No conflict

end

8.2 Tests with the deep embedding: semantic frame conditions

theory *PMLinHOL-deep-further-tests-2*
imports *PMLinHOL-deep-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with semantic conditions

abbreviation(*input*) *refl* (r) **where** r $\equiv \lambda R.$ *reflexive R*
abbreviation(*input*) *sym* (s) **where** s $\equiv \lambda R.$ *symmetric R*
abbreviation(*input*) *tra* (t) **where** t $\equiv \lambda R.$ *transitive R*

consts φ::*PML* ψ::*PML*
abbreviation(*input*) *F1* $\equiv (\Diamond^d(\Diamond^d\varphi)) \supset^d \Diamond^d\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d\varphi$ — holds in KB4
abbreviation(*input*) *F4* $\equiv (\Box^d(\Diamond^d(\Box^d(\Diamond^d\varphi)))) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB/K4
abbreviation(*input*) *F5* $\equiv (\Diamond^d(\varphi \wedge^d (\Diamond^d\psi))) \supset^d ((\Diamond^d\varphi) \wedge^d (\Diamond^d\psi))$ — holds in K4
abbreviation(*input*) *F6* $\equiv ((\Box^d(\varphi \supset^d \psi)) \wedge^d (\Diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\Diamond^d\psi)$ — holds in KB4
abbreviation(*input*) *F7* $\equiv (\Diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \Diamond^d\varphi))$ — holds in KB4
abbreviation(*input*) *F8* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \Diamond^d\varphi)$ — holds in KT/KB
abbreviation(*input*) *F9* $\equiv ((\Box^d(\Diamond^d\varphi)) \wedge^d (\Box^d(\Diamond^d(\neg^d \varphi)))) \supset^d \Diamond^d(\Diamond^d\varphi)$ — holds in KT
abbreviation(*input*) *F10* $\equiv ((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *KTB*: $\forall\, w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall\, w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall\, w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall\, w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall\, w{:}W.$ $(\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall\, w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall\, w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $\forall\, w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall\, w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $\forall\, w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall\, w{:}W.$ $(\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma K: $\forall\, w{:}W.\ (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma $S5$: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma $S4$: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma $KB4$: $\forall\, w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma KTB: $\forall\, w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma KT: $\forall\, w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma KB: $\forall\, w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma $K4$: $\forall\, w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma K: $\forall\, w{:}W.\ (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

experiment begin
lemma $S5$: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma $S4$: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma $KB4$: $\forall\, w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma KTB: $\forall\, w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma KT: $\forall\, w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma KB: $\forall\, w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$

— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K4*: $\forall\, w{:}W.\ \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *K*: $\forall\, w{:}W.\ (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=32 (with simp 8, without simp 24), none=56 (with simp 0, without simp 56), unknwn=72 (with simp 72, without simp 0) Sledgehammer: proof=70 (with simp 38, without simp 32), no prf=90 (with simp 42, without simp 48)

— No conflicts
end

8.3 Tests with the (maximal) shallow embedding: axiom schemata

theory *PMLinHOL-shallow-further-tests-1* **imports** *PMLinHOL-shallow-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms

consts $\varphi{::}\sigma\ \psi{::}\sigma$
abbreviation(*input*) $F1 \equiv (\Diamond^s(\Diamond^s\varphi)) \supset^s \Diamond^s\varphi$ — holds in K4
abbreviation(*input*) $F2 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) $F3 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$ — holds in KB4
abbreviation(*input*) $F4 \equiv (\Box^s(\Diamond^s(\Box^s(\Diamond^s\varphi)))) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB/K4
abbreviation(*input*) $F5 \equiv (\Diamond^s(\varphi \wedge^s (\Diamond^s\psi))) \supset^s ((\Diamond^s\varphi) \wedge^s (\Diamond^s\psi))$ — holds in K4
abbreviation(*input*) $F6 \equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Diamond^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\Diamond^s\psi)$ — holds in KB4
abbreviation(*input*) $F7 \equiv (\Diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \Diamond^s\varphi))$ — holds in KB4
abbreviation(*input*) $F8 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \Diamond^s\varphi)$ — holds in KT/KB
abbreviation(*input*) $F9 \equiv ((\Box^s(\Diamond^s\varphi)) \wedge^s (\Box^s(\Diamond^s(\neg^s \varphi)))) \supset^s \Diamond^s(\Diamond^s\varphi)$ — holds in KT
abbreviation(*input*) $F10 \equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩

lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf

⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩

lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf

$\langle proof \rangle$
lemma *KB*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
$\langle proof \rangle$
lemma *K4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
$\langle proof \rangle$
lemma *K*: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
$\langle proof \rangle$
end

experiment begin
lemma *S5*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
$\langle proof \rangle$
lemma *S4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
$\langle proof \rangle$
lemma *KB4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
$\langle proof \rangle$
lemma *KTB*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
$\langle proof \rangle$
lemma *KT*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
$\langle proof \rangle$
lemma *KB*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
$\langle proof \rangle$
lemma *K4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
$\langle proof \rangle$
lemma *K*: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
$\langle proof \rangle$
end

experiment begin
lemma *S5*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi$

$\supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=32 (with simp 16, without simp 16), none=0, unknwn=128 (with simp 64, without simp 64) Sledgehammer: proof=32 (with simp 24, without simp 8), no prf=128 (with simp 56, without simp 72)

— No conflict
end

8.4 Tests with the (maximal) shallow embedding: semantic frame conditions

theory *PMLinHOL-shallow-further-tests-2*
imports *PMLinHOL-shallow-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with semantic conditions
abbreviation(*input*) *refl* (r) **where** r $\equiv \lambda R.\ reflexive\ R$

abbreviation(*input*) *sym* (s) **where** s ≡ λR. *symmetric R*
abbreviation(*input*) *tra* (t) **where** t ≡ λR. *transitive R*

consts φ::σ ψ::σ
abbreviation(*input*) *F1* $\equiv (\Diamond^s(\Diamond^s\varphi)) \supset^s \Diamond^s\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$ — holds in KB4
abbreviation(*input*) *F4* $\equiv (\Box^s(\Diamond^s(\Box^s(\Diamond^s\varphi)))) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) *F5* $\equiv (\Diamond^s(\varphi \wedge^s (\Diamond^s\psi))) \supset^s ((\Diamond^s\varphi) \wedge^s (\Diamond^s\psi))$ — holds in K4
abbreviation(*input*) *F6* $\equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Diamond^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\Diamond^s\psi)$ — holds in KB4
abbreviation(*input*) *F7* $\equiv (\Diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \Diamond^s\varphi))$ — holds in KB4
abbreviation(*input*) *F8* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \Diamond^s\varphi)$ — holds in KT and in KB
abbreviation(*input*) *F9* $\equiv ((\Box^s(\Diamond^s\varphi)) \wedge^s (\Box^s(\Diamond^s(\neg^s \varphi)))) \supset^s \Diamond^s(\Diamond^s\varphi)$ — holds in KT
abbreviation(*input*) *F10* $\equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W$. r R ∧ s R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W$. r R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W$. s R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W$. r R ∧ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W$. r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W$. s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W$. t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *K*: $\forall w{:}W$. $(\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin

lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F3)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf

⟨*proof*⟩

lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F5)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

end

experiment begin

lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

⟨*proof*⟩

lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

⟨*proof*⟩

lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F6)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

end

experiment begin

lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

⟨*proof*⟩

lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$

— nitpick[expect=unknown] — sledgehammer — unkn — proof

⟨*proof*⟩

lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf

⟨*proof*⟩

lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
⟨*proof*⟩
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=0, unknwn=76 (with simp 38, without simp 38) Sledgehammer: proof=66 (with simp 38, without simp 28), no prf=94 (with simp 42, without simp 52)

— No conflicts
end

8.5 Tests with the (minimal) shallow embedding: axiom schemata

theory *PMLinHOL-shallow-minimal-further-tests-1*
imports *PMLinHOL-shallow-minimal* — C.B., 2025
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms
abbreviation(*input*) $AxT \equiv \forall \varphi.\ \models^m (\Box^m\varphi) \supset^m \varphi$
abbreviation(*input*) $AxB \equiv \forall \varphi.\ \models^m \varphi \supset^m \Box^m(\Diamond^m\varphi)$
abbreviation(*input*) $Ax4 \equiv \forall \varphi.\ \models^m (\Box^m\varphi) \supset^m \Box^m(\Box^m\varphi)$

consts $\varphi{::}\sigma$ $\psi{::}\sigma$
abbreviation(*input*) $F1 \equiv (\Diamond^m(\Diamond^m\varphi)) \supset^m \Diamond^m\varphi$ — holds in K4
abbreviation(*input*) $F2 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) $F3 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m\varphi$ — holds in KB4
abbreviation(*input*) $F4 \equiv (\Box^m(\Diamond^m(\Box^m(\Diamond^m\varphi)))) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB/K4
abbreviation(*input*) $F5 \equiv (\Diamond^m(\varphi \wedge^m (\Diamond^m\psi))) \supset^m ((\Diamond^m\varphi) \wedge^m (\Diamond^m\psi))$ — holds in K4
abbreviation(*input*) $F6 \equiv ((\Box^m(\varphi \supset^m \psi)) \wedge^m (\Diamond^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\Diamond^m\psi)$ — holds in KB4
abbreviation(*input*) $F7 \equiv (\Diamond^m\varphi) \supset^m (\Box^m(\varphi \vee^m \Diamond^m\varphi))$ — holds in KB4
abbreviation(*input*) $F8 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m (\varphi \vee^m \Diamond^m\varphi)$ — holds in KT/KB
abbreviation(*input*) $F9 \equiv ((\Box^m(\Diamond^m\varphi)) \wedge^m (\Box^m(\Diamond^m(\neg^m \varphi)))) \supset^m \Diamond^m(\Diamond^m\varphi)$ — holds in KT
abbreviation(*input*) $F10 \equiv ((\Box^m(\varphi \supset^m \Box^m\psi)) \wedge^m (\Box^m(\Diamond^m(\neg^m\psi)))) \supset^m \neg^m(\Box^m\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

nitpick-params
experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KT*: $AxT \longrightarrow \models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: $\models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F2$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F2$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F2$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $AxT \longrightarrow \models^m F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F2$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F3$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F3$

— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $AxT \longrightarrow \models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F4$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F4$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F4$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F4$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $AxT \longrightarrow \models^m F4$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F4$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F4$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
⟨*proof*⟩
lemma *K*: $\models^m F4$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F5$
— nitpick[expect=none] — sledgehammer — none — proof

⟨*proof*⟩

lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F5$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F5$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KT*: $AxT \longrightarrow \models^m F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KB*: $AxB \longrightarrow \models^m F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *K4*: $Ax4 \longrightarrow \models^m F5$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *K*: $\models^m F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

end

experiment begin

lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F6$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F6$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KT*: $AxT \longrightarrow \models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *KB*: $AxB \longrightarrow \models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *K4*: $Ax4 \longrightarrow \models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

lemma *K*: $\models^m F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩

end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F7$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F7$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: $AxT \longrightarrow \models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: $AxT \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: $AxB \longrightarrow \models^m F8$
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: $Ax4 \longrightarrow \models^m F8$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m$ *F8*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ $\models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: *AxT* ∧ *Ax4* ⟶ $\models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: *AxT* ⟶ $\models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: *AxB* ⟶ $\models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: *Ax4* ⟶ $\models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ $\models^m$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *S4*: *AxT* ∧ *Ax4* ⟶ $\models^m$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
⟨*proof*⟩
lemma *KT*: *AxT* ⟶ $\models^m$ *F10*
— nitpick[expect=none] — sledgehammer — none — proof

⟨*proof*⟩
lemma *KB*: *AxB* $\longrightarrow$ $\models^m$ *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: *Ax4* $\longrightarrow$ $\models^m$ *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: $\models^m$ *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=72 (with simp 36, without simp 36), unknwn=4 (with simp 2, without simp 2) Sledgehammer: proof=73 (with simp 38, without simp 35), no prf=87(with simp 42, without simp 45)

— No conflicts
end

8.6 Tests with the (minimal) shallow embedding: semantic frame conditions

theory *PMLinHOL-shallow-minimal-further-tests-2*
imports *PMLinHOL-shallow-minimal*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with semantic conditions
abbreviation(*input*) *refl* (r) **where** r $\equiv$ $\lambda R.$ *reflexive R*
abbreviation(*input*) *sym* (s) **where** s $\equiv$ $\lambda R.$ *symmetric R*
abbreviation(*input*) *tra* (t) **where** t $\equiv$ $\lambda R.$ *transitive R*

consts φ::σ ψ::σ
abbreviation(*input*) *F1* $\equiv (\Diamond^m(\Diamond^m\varphi)) \supset^m \Diamond^m\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m\varphi$ — holds in KB4
abbreviation(*input*) *F4* $\equiv (\Box^m(\Diamond^m(\Box^m(\Diamond^m\varphi)))) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) *F5* $\equiv (\Diamond^m(\varphi \wedge^m (\Diamond^m\psi))) \supset^m ((\Diamond^m\varphi) \wedge^m (\Diamond^m\psi))$ — holds in K4
abbreviation(*input*) *F6* $\equiv ((\Box^m(\varphi \supset^m \psi)) \wedge^m (\Diamond^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\Diamond^m\psi)$ — holds in KB4
abbreviation(*input*) *F7* $\equiv (\Diamond^m\varphi) \supset^m (\Box^m(\varphi \vee^m \Diamond^m\varphi))$ — holds in KB4
abbreviation(*input*) *F8* $\equiv (\Diamond^m(\Box^m\varphi)) \supset^m (\varphi \vee^m \Diamond^m\varphi)$ — holds in KT and in KB
abbreviation(*input*) *F9* $\equiv ((\Box^m(\Diamond^m\varphi)) \wedge^m (\Box^m(\Diamond^m(\neg^m \varphi)))) \supset^m \Diamond^m(\Diamond^m\varphi)$ — holds in KT
abbreviation(*input*) *F10* $\equiv ((\Box^m(\varphi \supset^m \Box^m\psi)) \wedge^m (\Box^m(\Diamond^m(\neg^m\psi)))) \supset^m \neg^m(\Box^m\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: r *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t *R* ⟶ (⊨^{m} *F1*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: (⊨^{m} *F1*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: r *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *K4*: t *R* ⟶ (⊨^{m} *F2*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F2*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: r *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t *R* ⟶ (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: r *R* ⟶ (⊨^{m} *F4*)

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: t *R* ⟶ (⊨^{m} *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: (⊨^{m} *F4*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: r *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t *R* ⟶ (⊨^{m} *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K*: (⊨^{m} *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F6*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F6*)
— nitpick[expect=none] — sledgehammer — none — proof

⟨*proof*⟩
lemma *KTB*: r R ∧ s R ⟶ (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: r R ⟶ (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s R ⟶ (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t R ⟶ (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F6*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r R ∧ s R ∧ t R ⟶ (⊨^{m} *F7*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r R ∧ t R ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB4*: s R ∧ t R ⟶ (⊨^{m} *F7*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r R ∧ s R ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KT*: r R ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KB*: s R ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t R ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r R ∧ s R ∧ t R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩

lemma *S4*: r R ∧ t R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s R ∧ t R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KTB*: r R ∧ s R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: r R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: s R ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *K4*: t R ⟶ (⊨^{m} *F8*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F8*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r R ∧ s R ∧ t R ⟶ (⊨^{m} *F9*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r R ∧ t R ⟶ (⊨^{m} *F9*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s R ∧ t R ⟶ (⊨^{m} *F9*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: r R ∧ s R ⟶ (⊨^{m} *F9*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: r R ⟶ (⊨^{m} *F9*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: s R ⟶ (⊨^{m} *F9*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t R ⟶ (⊨^{m} *F9*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: (⊨^{m} *F9*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

experiment begin
lemma *S5*: r R ∧ s R ∧ t R ⟶ ($\models^m$ *F10*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *S4*: r R ∧ t R ⟶ ($\models^m$ *F10*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB4*: s R ∧ t R ⟶ ($\models^m$ *F10*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *KTB*: r R ∧ s R ⟶ ($\models^m$ *F10*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KT*: r R ⟶ ($\models^m$ *F10*)
— nitpick[expect=none] — sledgehammer — none — proof
⟨*proof*⟩
lemma *KB*: s R ⟶ ($\models^m$ *F10*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K4*: t R ⟶ ($\models^m$ *F10*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
lemma *K*: ($\models^m$ *F10*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
⟨*proof*⟩
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=76 (with simp 38, without simp 38), unknwn=0 Sledgehammer: proof=76 (with simp 38, without simp 38), no prf=84 (with simp 42, without simp 42)

— No conflicts
end

References

[1] C. Benzmüller. Faithful logic embeddings in HOL — deep and shallow. In C. Barrett and U. Waldmann, editors, *Automated Deduction – CADE-30 – 30th International Conference on Automated Deduction, Stuttgart, Germany, July 28-31, 2025, Proceedings*, volume 15943 of *Lecture Notes in Computer Science*, pages 280–302. Springer, 2025. (preprint: arXiv:2502.19311).