Faithful Logic Embeddings in HOL — Deep and Shallow (Isabelle/HOL dataset)

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Abstract

A recipe for the simultaneous deployment of different forms of deep and shallow embeddings of non-classical logics in classical higher-order logic is presented, which enables interactive or even automated faithfulness proofs between the logic embeddings. The approach, which is particularly fruitful for logic education, is explained in detail in an associated CADE conference paper. This paper presents the corresponding Isabelle/HOL dataset (which is only slightly modified to meet AFP requirements).

Contents

1 Introduction

The Isabelle/HOL dataset associated with [1] is presented. Sections 3, 4 and 5 present deep, maximally shallow, and minimally shallow embeddings of propositional modal logic (PML) in classical higher-order logic (HOL). These are connected, as a novel contribution, by automated faithfulness proofs given in Sect. 6. This connection ensures that these deep and shallow embeddings can now be used interchangeably in subsequent applications. Several experiments with the presented embeddings are presented in Sect. 7. The presented work is conceptual in nature and can be adapted to other non-classical logics. For more detailed explanations of the presented material, including a discussion of related works, see [1].

2 Preliminaries

The following preliminaries are are shared between all embeddings introduced in the remainder of this paper.

theory *PMLinHOL-preliminaries*
 imports *Main*
begin

— Type declarations common for both the deep and shallow embedding
typedecl w — Type for possible worlds
typedecl $\mathcal{S}$ — Type for propositional constant symbols
consts p::$\mathcal{S}$ q::$\mathcal{S}$ — Some propositional constant symbols
type-synonym $\mathcal{W}$ = w⇒*bool* — Type for sets of possible worlds
type-synonym $\mathcal{R}$ = w⇒w⇒*bool* — Type for accessibility relations
type-synonym $\mathcal{V}$ = $\mathcal{S}$⇒w⇒*bool* — Type for valuation functions

— Some useful predicates for accessibility relations
abbreviation(*input*) *reflexive* $\equiv \lambda R::\mathcal{R}.\ \forall x.\ R\ x\ x$
abbreviation(*input*) *symmetric* $\equiv \lambda R::\mathcal{R}.\ \forall x\ y.\ R\ x\ y \longrightarrow R\ y\ x$
abbreviation(*input*) *transitive* $\equiv \lambda R::\mathcal{R}.\ \forall x\ y\ z.\ (R\ x\ y \wedge R\ y\ z) \longrightarrow R\ x\ z$
abbreviation(*input*) *equivrel* $\equiv \lambda R::\mathcal{R}.$ *reflexive* $R \wedge$ *symmetric* $R \wedge$ *transitive* R
abbreviation(*input*) *irreflexive* $\equiv \lambda R::\mathcal{R}.\ \forall x.\ \neg R\ x\ x$
abbreviation(*input*) *euclidean* $\equiv \lambda R::\mathcal{R}.\ \forall x\ y\ z.\ R\ x\ y \wedge R\ x\ z \longrightarrow R\ y\ z$
abbreviation(*input*) *wellfounded* $\equiv \lambda R::\mathcal{R}.\ \forall P::\mathcal{W}.\ (\forall x.\ (\forall y.\ R\ y\ x \longrightarrow P\ y) \longrightarrow P\ x) \longrightarrow (\forall x.\ P\ x)$
abbreviation(*input*) *converserel* $\equiv \lambda R::\mathcal{R}.\ \lambda y::\mathrm{w}.\ \lambda x::\mathrm{w}.\ R\ x\ y$

abbreviation(*input*) *conversewf* ≡ λR::$\mathcal{R}$. *wellfounded* (*converserel R*)

— Bounded universal quantifier: $\forall x{:}W.\ \varphi$ stands for $\forall x.\ W\ x \longrightarrow \varphi\ x$
abbreviation(*input*) *BoundedAll*::$\mathcal{W}\Rightarrow\mathcal{W}\Rightarrow$*bool* **where** *BoundedAll W* $\varphi \equiv \forall x.\ W\ x \longrightarrow \varphi\ x$
syntax *-BoundedAll*:: *pttrn*$\Rightarrow\mathcal{W}\Rightarrow$*bool*$\Rightarrow$*bool* ((3∀ (-/:-)./ -) [0, 0, 10] 10)
translations $\forall x{:}W.\ \varphi \rightleftharpoons$ *CONST BoundedAll W* $(\lambda x.\ \varphi)$

— Backward implication; useful for aestethic reasons
abbreviation(*input*) *Bimp* (**infixr** ⟵ *50*) **where** $\varphi \longleftarrow \psi \equiv \psi \longrightarrow \varphi$

— Some further settings
declare[[*syntax-ambiguity-warning=false*]]
nitpick-params[*user-axioms*,*expect=genuine*,*timeout=60*]

end

3 Deep embedding of PML in HOL

theory *PMLinHOL-deep*
imports *PMLinHOL-preliminaries*
begin
— Deep embedding (of propositional modal logic in HOL)
datatype *PML* = *AtmD* $\mathcal{S}$ (-^{d}) | *NotD PML* ($\neg^d$) | *ImpD PML PML* (**infixr** $\supset^d$ *93*) | *BoxD PML* ($\Box^d$)

— Further logical connectives as definitions
definition *OrD* (**infixr** $\vee^d$ *92*) **where** $\varphi\vee^d\psi \equiv \neg^d\varphi \supset^d \psi$
definition *AndD* (**infixr** $\wedge^d$ *95*) **where** $\varphi\wedge^d\psi \equiv \neg^d(\varphi \supset^d \neg^d\psi)$
definition *DiaD* ($\Diamond^d$-) **where** $\Diamond^d\varphi \equiv \neg^d(\Box^d(\neg^d\varphi))$
definition *TopD* ($\top^d$) **where** $\top^d \equiv p^d \supset^d p^d$
definition *BotD* ($\bot^d$) **where** $\bot^d \equiv \neg^d \top^d$

— Definition of truth of a formula relative to a model $\langle W,R,V\rangle$ and possible world w
primrec *RelativeTruthD* :: $\mathcal{W}\Rightarrow\mathcal{R}\Rightarrow\mathcal{V}\Rightarrow$w$\Rightarrow$*PML*$\Rightarrow$*bool* (⟨-,-,-⟩,- $\models^d$ -) **where**
$\langle W,R,V\rangle,\ w \models^d a^d\ = (V\ a\ w)$
| $\langle W,R,V\rangle,\ w \models^d \neg^d\varphi = (\neg\ \langle W,R,V\rangle,\ w \models^d \varphi)$
| $\langle W,R,V\rangle,\ w \models^d \varphi \supset^d \psi = (\langle W,R,V\rangle,\ w \models^d \varphi \longrightarrow \langle W,R,V\rangle,\ w \models^d \psi)$
| $\langle W,R,V\rangle,\ w \models^d \Box^d\varphi = (\forall v{:}W.\ R\ w\ v \longrightarrow \langle W,R,V\rangle,\ v \models^d \varphi)$

— Definition of validity
definition *ValD* ($\models^d$ -) **where** $(\models^d \varphi) \equiv (\forall W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi)$

— Collection of definitions in a bag called DefD
named-theorems *DefD* **declare** *OrD-def*[*DefD*,*simp*] *AndD-def*[*DefD*,*simp*] *DiaD-def*[*DefD*,*simp*] *TopD-def*[*DefD*,*simp*] *BotD-def*[*DefD*,*simp*] *RelativeTruthD-def*[*DefD*,*simp*] *ValD-def*[*DefD*,*simp*]
end

4 Shallow embedding of PML in HOL (maximal)

theory *PMLinHOL-shallow*
imports *PMLinHOL-preliminaries*
begin

— Shallow embedding (of propositional modal logic in HOL)
type-synonym $\sigma = \mathcal{W}{\Rightarrow}\mathcal{R}{\Rightarrow}\mathcal{V}{\Rightarrow}\mathrm{w}{\Rightarrow}bool$
definition $AtmS{::}\mathcal{S}{\Rightarrow}\sigma$ (-^s) **where** $a^s \equiv \lambda W\ R\ V\ w.\ V\ a\ w$
definition $NegS{::}\sigma{\Rightarrow}\sigma$ ($\neg^s$) **where** $\neg^s\ \varphi \equiv \lambda W\ R\ V\ w.\ \neg(\varphi\ \ W\ R\ V\ w)$
definition $ImpS{::}\sigma{\Rightarrow}\sigma{\Rightarrow}\sigma$ (**infixr** $\supset^s$ *93*) **where** $\varphi \supset^s \psi \equiv \lambda W\ R\ V\ w.\ (\varphi\ \ W\ R\ V\ w) \longrightarrow (\psi\ \ W\ R\ V\ w)$
definition $BoxS{::}\sigma{\Rightarrow}\sigma$ ($\Box^s$) **where** $\Box^s\ \varphi \equiv \lambda W\ R\ V\ w.\ \forall v{:}W.\ R\ w\ v \longrightarrow (\varphi\ \ W\ R\ V\ v)$

— Further logical connectives as definitions
definition *OrS* (**infixr** $\vee^s$ *92*) **where** $\varphi \vee^s \psi \equiv \neg^s\varphi \supset^s \psi$
definition *AndS* (**infixr** $\wedge^s$ *95*) **where** $\varphi \wedge^s \psi \equiv \neg^s(\varphi \supset^s \neg^s\psi)$
definition *DiaS* ($\Diamond^s$) **where** $\Diamond^s\varphi \equiv \neg^s\ (\Box^s\ (\neg^s\varphi))$
definition *TopS* ($\top^s$) **where** $\top^s \equiv p^s \supset^s p^s$
definition *BotS* ($\bot^s$) **where** $\bot^s \equiv \neg^s\ \top^s$

— Definition of truth of a formula relative to a model $\langle W,R,V\rangle$ and possible world w
definition $RelativeTruthS{::}\mathcal{W}{\Rightarrow}\mathcal{R}{\Rightarrow}\mathcal{V}{\Rightarrow}\mathrm{w}{\Rightarrow}\sigma{\Rightarrow}bool$ ($\langle\text{-},\text{-},\text{-}\rangle,\text{-}{\models^s}\ \text{-}$) **where** $\langle W,R,V\rangle,w \models^s \varphi \equiv \varphi\ \ W\ R\ V\ w$

— Definition of validity
definition *ValS* ($\models^s$ -) **where** $\models^s \varphi \equiv \forall\ W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi$

— Collection of definitions in a bag called DefS
named-theorems *DefS* **declare** *AtmS-def*[*DefS*,*simp*] *NegS-def*[*DefS*,*simp*] *ImpS-def*[*DefS*,*simp*] *BoxS-def*[*DefS*,*simp*] *OrS-def*[*DefS*,*simp*] *AndS-def*[*DefS*,*simp*] *DiaS-def*[*DefS*,*simp*] *TopS-def*[*DefS*,*simp*] *BotS-def*[*DefS*,*simp*] *RelativeTruthS-def*[*DefS*,*simp*] *ValS-def*[*DefS*,*simp*]
end

5 Shallow embedding of PML in HOL (minimal)

theory *PMLinHOL-shallow-minimal*
imports *PMLinHOL-preliminaries*
begin

— The acessibility relation R and the valuation function V are introduced as constants at the meta-level HOL
consts $R{::}\mathcal{R}\ \ V{::}\mathcal{V}$

— Shallow embedding (of propositional modal logic in HOL)
type-synonym $\sigma = \mathrm{w}{\Rightarrow}bool$
definition $AtmM{::}\mathcal{S}{\Rightarrow}\sigma$ (-^m) **where** $a^m \equiv \lambda w.\ V\ a\ w$

definition *NegM*::$\sigma \Rightarrow \sigma$ ($\neg^m$) **where** $\neg^m\ \varphi \equiv \lambda w.\neg\varphi\ w$
definition *ImpM*::$\sigma \Rightarrow \sigma \Rightarrow \sigma$ (**infixr** $\supset^m$ *93*) **where** $\varphi \supset^m \psi \equiv \lambda w.\ \varphi\ w \longrightarrow \psi\ w$
definition *BoxM*::$\sigma \Rightarrow \sigma$ ($\Box^m$) **where** $\Box^m\ \varphi \equiv \lambda w.\ \forall v.\ R\ w\ v \longrightarrow \varphi\ v$

— Further logical connectives as definitions
definition *OrM* (**infixr** $\vee^m$ *92*) **where** $\varphi \vee^m \psi \equiv \neg^m\varphi \supset^m \psi$
definition *AndM* (**infixr** $\wedge^m$ *95*) **where** $\varphi \wedge^m \psi \equiv \neg^m(\varphi \supset^m \neg^m\psi)$
definition *DiaM* ($\Diamond^m$-) **where** $\Diamond^m\varphi \equiv \neg^m\ (\Box^m\ (\neg^m\varphi))$
definition *TopM* ($\top^m$) **where** $\top^m \equiv p^m \supset^m p^m$
definition *BotM* ($\bot^m$) **where** $\bot^m \equiv \neg^m\ \top^m$

— Definition of truth of a formula relative to a model $\langle W,R,V\rangle$ and a possible world w
definition *RelativeTruthM*::w$\Rightarrow\sigma\Rightarrow$*bool* (-$\models^m$ -) **where** $w \models^m \varphi \equiv \varphi\ w$

— Definition of validity
definition *ValM* ($\models^m$ -) **where** $\models^m \varphi \equiv \forall w$::w. $w \models^m \varphi$

— Collection of definitions in a bag called DefM
named-theorems *DefM* **declare** *AtmM-def*[*DefM*,*simp*] *NegM-def*[*DefM*,*simp*] *ImpM-def*[*DefM*,*simp*] *BoxM-def*[*DefM*,*simp*] *OrM-def*[*DefM*,*simp*] *AndM-def*[*DefM*,*simp*] *DiaM-def*[*DefM*,*simp*] *TopM-def*[*DefM*,*simp*] *BotM-def*[*DefM*,*simp*] *RelativeTruthM-def*[*DefM*,*simp*] *ValM-def*[*DefM*,*simp*]
end

6 Automated faithfulness proofs

theory *PMLinHOL-faithfulness*
imports *PMLinHOL-deep PMLinHOL-shallow PMLinHOL-shallow-minimal*
begin

— Mappings: deep to maximal shallow and deep to minimal shallow
primrec *DpToShMax* ($(\!|$-$|\!)$) **where** $(\!|\varphi^d|\!) = \varphi^s \mid (\!|\neg^d\ \varphi|\!) = \neg^s\ (\!|\varphi|\!) \mid (\!|\varphi \supset^d \psi|\!) = (\!|\varphi|\!) \supset^s (\!|\psi|\!) \mid (\!|\Box^d\ \varphi|\!) = \Box^s\ (\!|\varphi|\!)$
primrec *DpToShMin* ($[\![$-$]\!]$) **where** $[\![\varphi^d]\!] = \varphi^m \mid [\![\neg^d\ \varphi]\!] = \neg^m\ [\![\varphi]\!] \mid [\![\varphi \supset^d \psi]\!] = [\![\varphi]\!] \supset^m [\![\psi]\!] \mid [\![\Box^d\ \varphi]\!] = \Box^m\ [\![\varphi]\!]$

— Proving faithfulness between deep and maximal shallow
theorem *Faithful1a*: $\forall\ W\ R\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \longleftrightarrow \langle W,R,V\rangle,w \models^s (\!|\varphi|\!)$ **apply** *induct* **by** *auto*
theorem *Faithful1b*: $\models^d \varphi \longleftrightarrow\ \models^s (\!|\varphi|\!)$ **using** *Faithful1a* **by** *auto*

— Proving faithfulness between deep and minimal shallow
theorem *Faithful2*: $\forall w.\ \langle(\lambda x$::w. *True*$),R,V\rangle,w \models^d \varphi \longleftrightarrow w \models^m [\![\varphi]\!]$ **apply** *induct* **by** *auto*

— Proving faithfulness maximal shallow and minimal shallow
theorem *Faithful3*: $\forall w.\ \langle(\lambda x$::w. *True*$),R,V\rangle,w \models^s (\!|\varphi|\!) \longleftrightarrow w \models^m [\![\varphi]\!]$ **apply** *induct* **by** *auto*

— Additional check for soundness for the minimal shallow embedding
lemma *Sound1*: $\models^m \psi \longleftrightarrow (\exists \varphi.\ \psi = [\![\varphi]\!] \wedge \models^d \varphi)$ **by** (*smt Faithful2 DefM DefD RelativeTruthD.simps ext*[*of* ψ $[\![x \supset^d x]\!]$])
lemma *Sound2*: $\models^m \psi \longleftrightarrow (\exists \varphi.\ \psi = [\![\varphi]\!] \wedge \models^m [\![\varphi]\!])$ **using** *Sound1* **by** *blast*
end

7 Appendix: proof automation tests

7.1 Tests with the deep embedding

theory *PMLinHOL-deep-tests*
imports *PMLinHOL-deep*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^d \varphi \supset^d (\psi \supset^d \varphi)$ **by** *auto*
lemma *H2*: $\models^d (\varphi \supset^d (\psi \supset^d \gamma)) \supset^d ((\varphi \supset^d \psi) \supset^d (\varphi \supset^d \gamma))$ **by** *auto*
lemma *H3*: $\models^d (\neg^d\varphi \supset^d \neg^d\psi) \supset^d (\psi \supset^d \varphi)$ **by** *auto*
lemma *MP*: $\models^d \varphi \Longrightarrow \models^d (\varphi \supset^d \psi) \Longrightarrow \models^d \psi$ **by** *auto*

— Reasoning with the Hilbert calculus: interactive and fully automated
lemma *HCderived1*: $\models^d (\varphi \supset^d \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
proof –
have *1*: $\models^d \varphi \supset^d ((\psi \supset^d \varphi) \supset^d \varphi)$ **using** *H1* **by** *auto*
have *2*: $\models^d (\varphi \supset^d ((\psi \supset^d \varphi) \supset^d \varphi)) \supset^d ((\varphi \supset^d (\psi \supset^d \varphi)) \supset^d (\varphi \supset^d \varphi))$ **using** *H2* **by** *auto*
have *3*: $\models^d (\varphi \supset^d (\psi \supset^d \varphi)) \supset^d (\varphi \supset^d \varphi)$ **using** *1 2 MP* **by** *meson*
have *4*: $\models^d \varphi \supset^d (\psi \supset^d \varphi)$ **using** *H1* **by** *auto*
thus *?thesis* **using** *3 4 MP* **by** *meson*
qed

lemma *HCderived2*: $\models^d \varphi \supset^d (\neg^d\varphi \supset^d \psi)$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived3*: $\models^d (\neg^d\varphi \supset^d \varphi) \supset^d \varphi$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived4*: $\models^d (\varphi \supset^d \psi) \supset^d (\neg^d\psi \supset^d \neg^d\varphi)$ **by** *auto*

— Modal logic: the schematic necessitation rule and distribution axiom are implied
lemma *Nec*: $\models^d \varphi \Longrightarrow \models^d \Box^d\varphi$ **by** *auto*
lemma *Dist*: $\models^d \Box^d(\varphi \supset^d \psi) \supset^d (\Box^d\varphi \supset^d \Box^d\psi)$ **by** *auto*

— Correspondence theory: correct statements
lemma *cM*: *reflexive* $R \longleftrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \varphi)$ — sledgehammer: Proof found **oops**
lemma *cBa*: *symmetric* $R \longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi))$ **by** *auto*
lemma *cBb*: *symmetric* $R \longleftarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi))$ — sledgehammer: No proof **oops**

lemma *c4a*: *transitive R* $\longrightarrow$ ($\forall \varphi$ *W V*. $\forall$ *w*: *W*. $\langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi)$)
by (*metis RelativeTruthD.simps*)
lemma *c4b*: *transitive R* $\longleftarrow$ ($\forall \varphi$ *W V*. $\forall$ *w*: *W*. $\langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi)$)
— sledgehammer: No proof **oops**

— Correspondence theory: incorrect statements
lemma *reflexive R* $\longrightarrow$ ($\forall \varphi$ *W V*. $\forall$ *w*: *W*. $\langle W,R,V\rangle,w \models^d \Box^d\varphi \supset^d \Box^d(\Box^d\varphi)$)
nitpick[*card* w=*3*] **oops** — nitpick: Cex.

— Simple, incorrect validity statements
lemma $\models^d \varphi \supset^d \Box^d\varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] **oops** — nitpick: Counterexample: modal collapse not implied
lemma $\models^d \Box^d(\Box^d\varphi \supset^d \Box^d\psi) \vee^d \Box^d(\Box^d\psi \supset^d \Box^d\varphi)$ **nitpick**[*card* w=*3*] **oops**
— nitpick: Counterexample
lemma $\models^d (\Diamond^d(\Box^d \varphi)) \supset^d \Box^d(\Diamond^d \varphi)$ **nitpick**[*card* w=*2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5
lemma *KB*: *symmetric R* $\longrightarrow$ ($\forall \varphi\ \psi$ *W V*. $\forall$ *w*: *W*. $\langle W,R,V\rangle,w \models^d (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi)$) **by** *auto*
lemma *K4B*: *symmetric R* $\wedge$ *transitive R* $\longrightarrow$ ($\forall \varphi\ \psi$ *W V*. $\forall$ *w*: *W*. $\langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d \Box^d\psi) \vee^d \Box^d(\Box^d\psi \supset^d \Box^d\varphi)$) **by** (*smt OrD-def RelativeTruthD.simps*)
end

theory *PMLinHOL-deep-further-tests*
imports *PMLinHOL-deep-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^d (\Box^d(\varphi \supset^d \psi)) \supset^d ((\Diamond^d\varphi) \supset^d \Diamond^d\psi)$ **by** *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy
lemma *T1a*: $\models^d \Box^d p^d \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ **by** *auto* — fast automation in meta-logic HOL
lemma *T1b*: $\models^d \Box^d p^d \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ — alternative interactive proof in modal object logic K
proof –
have *1*: $\models^d p^d \supset^d (q^d \supset^d (p^d \wedge^d q^d))$ **unfolding** *AndD-def* **using** *H1 H2 H3 MP* **by** *metis*
have *2*: $\models^d \Box^d(p^d \supset^d (q^d \supset^d (p^d \wedge^d q^d)))$ **using** *1 Nec* **by** *metis*
have *3*: $\models^d \Box^d p^d \supset^d \Box^d(q^d \supset^d (p^d \wedge^d q^d))$ **using** *2 Dist MP* **by** *metis*
have *4*: $\models^d (\Box^d(q^d \supset^d (p^d \wedge^d q^d))) \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ **using** *K-Dia* **by** *metis*
have *5*: $\models^d \Box^d p^d \supset^d ((\Diamond^d q^d) \supset^d \Diamond^d(p^d \wedge^d q^d))$ **using** *3 4 H1 H2 MP* **by** *metis*
thus *?thesis* **.**
qed
end

theory *PMLinHOL-deep-Loeb-tests*
imports *PMLinHOL-deep*
begin

— Löb axiom: with the deep embedding automated reasoning tools are not very responsive
lemma *Loeb1*: $\forall\varphi.\ \models^d \Box^d(\ \Box^d\varphi \supset^d \varphi) \supset^d \Box^d\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*] **oops**
— nitpick: Counterexample
lemma *Loeb2*: $(\mathit{conversewf}\ R \wedge \mathit{transitive}\ R) \longrightarrow (\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3*: $(\mathit{conversewf}\ R \wedge \mathit{transitive}\ R) \longleftarrow (\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3a*: $\mathit{conversewf}\ R \longleftarrow (\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3b*: $\mathit{transitive}\ R \longleftarrow (\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3c*: $\mathit{irreflexive}\ R \longleftarrow (\forall\varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^d \Box^d(\Box^d\varphi \supset^d\varphi) \supset^d \Box^d\varphi)$ — sledgehammer: No Proof **oops**
end

7.2 Tests with the maximal shallow embedding

theory *PMLinHOL-shallow-tests*
imports *PMLinHOL-shallow*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^s \varphi \supset^s (\psi \supset^s \varphi)$ **by** *auto*
lemma *H2*: $\models^s (\varphi \supset^s (\psi \supset^s \gamma)) \supset^s ((\varphi \supset^s \psi) \supset^s (\varphi \supset^s \gamma))$ **by** *auto*
lemma *H3*: $\models^s (\neg^s\varphi \supset^s \neg^s\psi) \supset^s (\psi \supset^s \varphi)$ **by** *auto*
lemma *MP*: $\models^s \varphi \Longrightarrow \models^s (\varphi \supset^s \psi) \Longrightarrow \models^s \psi$ **by** *auto*

— Reasoning with the Hilbert calculus: interactive and fully automated
lemma *HCderived1*: $\models^s (\varphi \supset^s \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
proof −
have *1*: $\models^s \varphi \supset^s ((\psi \supset^s \varphi) \supset^s \varphi)$ **using** *H1* **by** *auto*
have *2*: $\models^s (\varphi \supset^s ((\psi \supset^s \varphi) \supset^s \varphi)) \supset^s ((\varphi \supset^s (\psi \supset^s \varphi)) \supset^s (\varphi \supset^s \varphi))$ **using** *H2* **by** *auto*
have *3*: $\models^s (\varphi \supset^s (\psi \supset^s \varphi)) \supset^s (\varphi \supset^s \varphi)$ **using** *1 2 MP* **by** *meson*
have *4*: $\models^s \varphi \supset^s (\psi \supset^s \varphi)$ **using** *H1* **by** *auto*
thus *?thesis* **using** *3 4 MP* **by** *meson*
qed

lemma *HCderived2*: $\models^s \varphi \supset^s (\neg^s\varphi \supset^s \psi)$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived3*: $\models^s (\neg^s\varphi \supset^s \varphi) \supset^s \varphi$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived4*: $\models^s (\varphi \supset^s \psi) \supset^s (\neg^s\psi \supset^s \neg^s\varphi)$ **by** *auto*

— Modal logic: the schematic necessitation rule and distribution axiom are implied
lemma *Nec*: $\models^s \varphi \Longrightarrow \models^s \Box^s\varphi$ **by** *auto*
lemma *Dist*:$\models^s \Box^s(\varphi \supset^s \psi) \supset^s (\Box^s\varphi \supset^s \Box^s\psi)$ **by** *auto*

— Correspondence theory: correct statements
lemma *cM*:*reflexive* $R \longleftrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \varphi)$ — sledgehammer: Proof found **oops**
lemma *cBa*: *symmetric* $R \longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi))$ **by** *auto*
lemma *cBb*: *symmetric* $R \longleftarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi))$ — sledgehammer: No proof **oops**
lemma *c4a*: *transitive* $R \longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ **by** (*smt DefS*)
lemma *c4b*: *transitive* $R \longleftarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ — sledgehammer: No proof **oops**

— Correspondence theory: incorrect statements
lemma *reflexive* $R \longrightarrow (\forall \varphi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s\varphi \supset^s \Box^s(\Box^s\varphi))$ **nitpick**[*card* w=*3*] **oops** — nitpick: Counterexample

— Simple, incorrect validity statements
lemma $\models^s \varphi \supset^s \Box^s\varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] **oops** — nitpick: Counterexample: modal collapse not implied
lemma $\models^s \Box^s(\ \Box^s\varphi \supset^s\Box^s\psi) \vee^s \Box^s(\ \Box^s\psi \supset^s\Box^s\varphi)$ **oops** — *nitpick*[*card* w=*3*] returns: unknown
lemma $\models^s (\Diamond^s(\Box^s\ \varphi)) \supset^s\Box^s(\Diamond^s\ \varphi)$ **nitpick**[*card* w=*2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5
lemma *KB*: *symmetric* $R \longrightarrow (\forall \varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s (\Diamond^s(\Box^s\varphi)) \supset^s\Box^s(\Diamond^s\varphi))$ **by** *auto*
lemma *K4B*: *symmetric* $R \wedge$ *transitive* $R \longrightarrow (\forall \varphi\ \psi\ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\ \Box^s\varphi \supset^s\Box^s\psi) \vee^s \Box^s(\ \Box^s\psi \supset^s\Box^s\varphi))$ **by** (*smt DefS*)
end

theory *PMLinHOL-shallow-further-tests*
  imports *PMLinHOL-shallow-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^s (\Box^s(\varphi \supset^s \psi)) \supset^s ((\Diamond^s\varphi) \supset^s \Diamond^s\psi)$ **by** *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy
lemma *T1a*:$\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s\ (p^s \wedge^s q^s))$ **by** *auto*— fast automation in meta-logic HOL
lemma *T1b*: $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s\ (p^s \wedge^s\ q^s))$ — alternative interactive proof in modal object logic K
  proof –
    have *1*: $\models^s\ p^s \supset^s (q^s \supset^s (p^s \wedge^s\ q^s))$ **unfolding** *AndS-def* **using** *H1 H2 H3*

MP **by** *metis*
have *2*: $\models^s \Box^s(p^s \supset^s (q^s \supset^s (p^s \wedge^s q^s)))$ **using** *1 Nec* **by** *metis*
have *3*: $\models^s \Box^s p^s \supset^s \Box^s(q^s \supset^s (p^s \wedge^s q^s))$ **using** *2 Dist MP* **by** *metis*
have *4*: $\models^s (\Box^s(q^s \supset^s (p^s \wedge^s q^s))) \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s(p^s \wedge^s q^s))$ **using** *K-Dia* **by** *metis*
have *5*: $\models^s \Box^s p^s \supset^s ((\Diamond^s q^s) \supset^s \Diamond^s (p^s \wedge^s q^s))$ **using** *3 4 H1 H2 MP* **by** *metis*

thus *?thesis* **.**
qed
end

theory *PMLinHOL-shallow-Loeb-tests*
imports *PMLinHOL-shallow*
begin
— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive
lemma *Loeb1*: $\forall \varphi.\ \models^s \Box^s(\ \Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*] **oops**
— nitpick: Counterexample
lemma *Loeb2*: $(conversewf\ R \wedge transitive\ R) \longrightarrow(\forall \varphi\ \ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found **oops**
lemma *Loeb3*: $(conversewf\ R \wedge transitive\ R) \longleftarrow (\forall \varphi\ \ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3a*: $conversewf\ R \longleftarrow (\forall \varphi\ \ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found **oops**
lemma *Loeb3b*: $transitive\ R \longleftarrow (\forall \varphi\ \ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3c*: $irreflexive\ R \longleftarrow (\forall \varphi\ \ W\ V.\ \forall w{:}W.\ \langle W,R,V\rangle,w \models^s \Box^s(\Box^s\varphi \supset^s\varphi) \supset^s \Box^s\varphi)$ — sledgehammer: Proof found **oops**
end

7.3 Tests with the minimal shallow embedding

theory *PMLinHOL-shallow-minimal-tests*
imports *PMLinHOL-shallow-minimal*
begin

— Hilbert calculus: proving that the schematic axioms and rules implied by the embedding
lemma *H1*: $\models^m \varphi \supset^m (\psi \supset^m \varphi)$ **by** *auto*
lemma *H2*: $\models^m (\varphi \supset^m (\psi \supset^m \gamma)) \supset^m ((\varphi \supset^m \psi) \supset^m (\varphi \supset^m \gamma))$ **by** *auto*
lemma *H3*: $\models^m (\neg^m\varphi\supset^m \neg^m\psi) \supset^m (\psi \supset^m \varphi)$ **by** *auto*
lemma *MP*: $\models^m \varphi \Longrightarrow \models^m (\varphi\supset^m \psi) \Longrightarrow \models^m \psi$ **by** *auto*

— Reasoning with the Hilbert calculus: interactive and fully automated
lemma *HCderived1*: $\models^m (\varphi \supset^m \varphi)$ — sledgehammer(HC1 HC2 HC3 MP) returns: by (metis HC1 HC2 MP)
proof –
have *1*: $\models^m \varphi \supset^m ((\psi \supset^m \varphi) \supset^m \varphi)$ **using** *H1* **by** *auto*
have *2*: $\models^m (\varphi \supset^m ((\psi \supset^m \varphi) \supset^m \varphi)) \supset^m ((\varphi \supset^m (\psi \supset^m \varphi)) \supset^m (\varphi \supset^m \varphi))$

using *H2* **by** *auto*
  have *3*: $\models^m (\varphi \supset^m (\psi \supset^m \varphi)) \supset^m (\varphi \supset^m \varphi)$ **using** *1 2 MP* **by** *meson*
  have *4*: $\models^m \varphi \supset^m (\psi \supset^m \varphi)$ **using** *H1* **by** *auto*
  thus *?thesis* **using** *3 4 MP* **by** *meson*
 qed

lemma *HCderived2*: $\models^m \varphi \supset^m (\neg^m \varphi \supset^m \psi)$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived3*: $\models^m (\neg^m \varphi \supset^m \varphi) \supset^m \varphi$ **by** (*metis H1 H2 H3 MP*)
lemma *HCderived4*: $\models^m (\varphi \supset^m \psi) \supset^m (\neg^m \psi \supset^m \neg^m \varphi)$ **by** *auto*

— Modal logic: the schematic necessitation rule and distribution axiom are implied
lemma *Nec*: $\models^m \varphi \Longrightarrow \models^m \Box^m \varphi$ **by** (*smt DefM*)
lemma *Dist*: $\models^m \Box^m(\varphi \supset^m \psi) \supset^m (\Box^m \varphi \supset^m \Box^m \psi)$ **by** *auto*

— Correspondence theory: correct statements
lemma *cM*:*reflexive R* $\longleftrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \varphi)$ **by** *auto*
lemma *cBa*: *symmetric R* $\longrightarrow (\forall \varphi. \models^m \varphi \supset^m \Box^m \Diamond^m \varphi)$ **by** *auto*
lemma *cBb*: *symmetric R* $\longleftarrow (\forall \varphi. \models^m \varphi \supset^m \Box^m \Diamond^m \varphi)$ **by** (*metis DefM*)
lemma *c4a*: *transitive R* $\longrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m(\Box^m \varphi))$ **by** (*smt DefM*)
lemma *c4b*: *transitive R* $\longleftarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m(\Box^m \varphi))$ **by** *auto*

— Correspondence theory: incorrect statements
lemma *reflexive R* $\longrightarrow (\forall \varphi. \models^m \Box^m \varphi \supset^m \Box^m(\Box^m \varphi))$ **nitpick**[*card* w=*3*,*show-all*]
oops — nitpick: Counterexample

— Simple, incorrect validity statements
lemma $\models^m \varphi \supset^m \Box^m \varphi$ **nitpick**[*card* w=*2*, *card* $\mathcal{S}$= *1*] **oops** — nitpick: Counterexample: modal collapse not implied
lemma $\models^m \Box^m(\Box^m \varphi \supset^m \Box^m \psi) \vee^m \Box^m(\Box^m \psi \supset^m \Box^m \varphi)$ **nitpick**[*card* w=*3*] **oops** — nitpick: Counterexample
lemma $\models^m (\Diamond^m(\Box^m \varphi)) \supset^m \Box^m(\Diamond^m \varphi)$ **nitpick**[*card* w=*2*] **oops** — nitpick: Counterexample

— Implied axiom schemata in S5
lemma *KB*: *symmetric R* $\longrightarrow (\forall \varphi\ \psi. \models^m (\Diamond^m(\Box^m \varphi)) \supset^m \Box^m(\Diamond^m \varphi))$ **by** *auto*
lemma *K4B*: *symmetric R* $\wedge$ *transitive R* $\longrightarrow (\forall \varphi\ \psi. \models^m \Box^m(\Box^m \varphi \supset^m \Box^m \psi) \vee^m \Box^m(\Box^m \psi \supset^m \Box^m \varphi))$ **by** (*smt DefM*)
end

theory *PMLinHOL-shallow-minimal-further-tests*
 imports *PMLinHOL-shallow-minimal-tests*
begin

— Implied modal principle
lemma *K-Dia*: $\models^m (\Box^m(\varphi \supset^m \psi)) \supset^m ((\Diamond^m \varphi) \supset^m \Diamond^m \psi)$ **by** *auto*

— Example 6.10 of Sider (2009) Logic for Philosophy
lemma *T1a*: $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ **by** *auto* — fast automation in meta-logic HOL

lemma *T1b*: $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ — alternative interactive proof in modal object logic K
proof −
have *1*: $\models^m p^m \supset^m (q^m \supset^m (p^m \wedge^m q^m))$ **unfolding** *AndM-def* **using** *H1 H2 H3 MP* **by** *metis*
have *2*: $\models^m \Box^m(p^m \supset^m (q^m \supset^m (p^m \wedge^m q^m)))$ **using** *1 Nec* **by** *metis*
have *3*: $\models^m \Box^m p^m \supset^m \Box^m(q^m \supset^m (p^m \wedge^m q^m))$ **using** *2 Dist MP* **by** *metis*
have *4*: $\models^m (\Box^m(q^m \supset^m (p^m \wedge^m q^m))) \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ **using** *K-Dia* **by** *metis*
have *5*: $\models^m \Box^m p^m \supset^m ((\Diamond^m q^m) \supset^m \Diamond^m(p^m \wedge^m q^m))$ **using** *3 4 H1 H2 MP* **by** *metis*
thus *?thesis* **.**
qed
end

theory *PMLinHOL-shallow-minimal-Loeb-tests*
imports *PMLinHOL-shallow-minimal*
begin
— Löb axiom: with the minimal shallow embedding automated reasoning tools are still partly responsive
lemma *Loeb1*: $\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi$ **nitpick**[*card* w=*1*,*card* $\mathcal{S}$=*1*]
oops — nitpick: Counterexample.
lemma *Loeb2*: (*conversewf R* ∧ *transitive R*) $\longrightarrow(\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi)$ — sh: Proof found **oops**
lemma *Loeb3*: (*conversewf R* ∧ *transitive R*) $\longleftarrow (\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi)$ — sh: No Proof **oops**
lemma *Loeb3a*: *conversewf R* $\longleftarrow (\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi)$ **unfolding** *DefM* **by** *blast*
lemma *Loeb3b*: *transitive R* $\longleftarrow (\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi)$ — sledgehammer: No Proof **oops**
lemma *Loeb3c*: *irreflexive R* $\longleftarrow (\forall \varphi.\ \models^m \Box^m(\Box^m\varphi \supset^m\varphi) \supset^m \Box^m\varphi)$ — sledgehammer: Proof found **oops**
end

8 Test Examples: Formula classification

8.1 Tests with the deep embedding: axiom schemata

theory *PMLinHOL-deep-further-tests-1*
imports *PMLinHOL-deep-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms

consts φ::*PML* ψ::*PML*
abbreviation(*input*) *F1* $\equiv (\Diamond^d(\Diamond^d\varphi)) \supset^d \Diamond^d\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^d(\Box^d\varphi)) \supset^d \Box^d\varphi$ — holds in KB4

abbreviation(*input*) $F4 \equiv (\Box^d(\Diamond^d(\Box^d(\Diamond^d\varphi)))) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB/K4
abbreviation(*input*) $F5 \equiv (\Diamond^d(\varphi \wedge^d (\Diamond^d\psi))) \supset^d ((\Diamond^d\varphi) \wedge^d (\Diamond^d\psi))$ — holds in K4
abbreviation(*input*) $F6 \equiv ((\Box^d(\varphi \supset^d \psi)) \wedge^d (\Diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\Diamond^d\psi)$ — holds in KB4
abbreviation(*input*) $F7 \equiv (\Diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \Diamond^d\varphi))$ — holds in KB4
abbreviation(*input*) $F8 \equiv (\Diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \Diamond^d\varphi)$ — holds in KT/KB
abbreviation(*input*) $F9 \equiv ((\Box^d(\Diamond^d\varphi)) \wedge^d (\Box^d(\Diamond^d(\neg^d \varphi)))) \supset^d \Diamond^d(\Diamond^d\varphi)$ — holds in KT
abbreviation(*input*) $F10 \equiv ((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

by (smt ($z3$) $PML.simps$(22))
lemma K: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^d F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma $S5$: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma $S4$: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma $KB4$: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (smt ($z3$) $PML.simps$(22))
lemma KTB: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (smt ($z3$) $PML.simps$(22))
lemma KT: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma KB: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (smt ($z3$) $PML.simps$(22))
lemma $K4$: $\forall\, w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma K: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^d F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F3$
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apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops

lemma *S4*: $\forall\, w{:}W.\ (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F4$
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=genuine] — sledgehammer — ctex — no prf
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experiment begin
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— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *S4*: $\forall\, w{:}W.\ (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F5$
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— nitpick[expect=none] — sledgehammer — none — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — proof
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by (*smt* (*z3*) *PML.simps*(*22*))
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F5$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F6$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

oops
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F6*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F6*

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F6*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d$ *F6*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F7*

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F7*
— nitpick[expect=none] — sledgehammer — none — no prf
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lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F7*
— nitpick[expect=none] — sledgehammer — none — no prf
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lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F7*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
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experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$

— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
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apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*z3*) *PML.simps*(*22*))
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

oops
lemma *K*: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^d F8$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall\, w{:}W.\ (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis* (*mono-tags*, *lifting*) *PML.simps*(*22*))
lemma *S4*: $\forall\, w{:}W.\ (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — proof
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
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lemma *KTB*: $\forall\, w{:}W.\ (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\,\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d F9$
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
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— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^d F9$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis* (*mono-tags*, *lifting*) *PML.simps*(*22*))
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis* (*mono-tags*, *lifting*) *PML.simps*(*22*))
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
by (*smt* (*verit*) *PML.simps*(*22*,*24*))
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \varphi) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
by (*smt* (*verit*) *PML.simps*(*22*,*24*))
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d \varphi \supset^d \Box^d(\Diamond^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^d (\Box^d\varphi) \supset^d \Box^d(\Box^d\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^d$ *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=16 (with simp 10, without simp 6), none=74 (with simp 0, without simp 74), unknwn=70 (with simp 70, without simp 0) Sledgehammer: proof=33 (with simp 16, without simp 17), no prf=127 (with simp 64, without simp 63)

— No conflict
end

8.2 Tests with the deep embedding: semantic frame conditions

theory *PMLinHOL-deep-further-tests-2*
  imports *PMLinHOL-deep-tests*
begin
 — What is the weakest modal logic in which the following test formulas F1-F10 are provable?
 — Test with semantic conditions

abbreviation(*input*) *refl* (r) **where** r ≡ λ*R*. *reflexive R*
abbreviation(*input*) *sym* (s) **where** s ≡ λ*R*. *symmetric R*
abbreviation(*input*) *tra* (t) **where** t ≡ λ*R*. *transitive R*

consts φ::*PML* ψ::*PML*
abbreviation(*input*) *F1* ≡ $(\Diamond^d(\Diamond^d\varphi)) \supset^d \Diamond^d\varphi$ — holds in K4
abbreviation(*input*) *F2* ≡ $(\Diamond^d(\Box^d\varphi)) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB
abbreviation(*input*) *F3* ≡ $(\Diamond^d(\Box^d\varphi)) \supset^d \Box^d\varphi$ — holds in KB4
abbreviation(*input*) *F4* ≡ $(\Box^d(\Diamond^d(\Box^d(\Diamond^d\varphi)))) \supset^d \Box^d(\Diamond^d\varphi)$ — holds in KB/K4
abbreviation(*input*) *F5* ≡ $(\Diamond^d(\varphi \wedge^d (\Diamond^d\psi))) \supset^d ((\Diamond^d\varphi) \wedge^d (\Diamond^d\psi))$ — holds in K4
abbreviation(*input*) *F6* ≡ $((\Box^d(\varphi \supset^d \psi)) \wedge^d (\Diamond^d(\Box^d(\neg^d\psi)))) \supset^d \neg^d(\Diamond^d\psi)$ — holds in KB4
abbreviation(*input*) *F7* ≡ $(\Diamond^d\varphi) \supset^d (\Box^d(\varphi \vee^d \Diamond^d\varphi))$ — holds in KB4
abbreviation(*input*) *F8* ≡ $(\Diamond^d(\Box^d\varphi)) \supset^d (\varphi \vee^d \Diamond^d\varphi)$ — holds in KT/KB
abbreviation(*input*) *F9* ≡ $((\Box^d(\Diamond^d\varphi)) \wedge^d (\Box^d(\Diamond^d(\neg^d \varphi)))) \supset^d \Diamond^d(\Diamond^d\varphi)$ — holds in KT
abbreviation(*input*) *F10* ≡ $((\Box^d(\varphi \supset^d \Box^d\psi)) \wedge^d (\Box^d(\Diamond^d(\neg^d\psi)))) \supset^d \neg^d(\Box^d\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: ∀ *w*:*W*. r *R* ∧ s *R* ∧ t *R* ⟶ (⟨*W*,*R*,*V*⟩,*w* $\models^d$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by *blast*
lemma *S4*: ∀ *w*:*W*. r *R* ∧ t *R* ⟶ (⟨*W*,*R*,*V*⟩,*w* $\models^d$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by *blast*
lemma *KB4*: ∀ *w*:*W*. s *R* ∧ t *R* ⟶ (⟨*W*,*R*,*V*⟩,*w* $\models^d$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
  by *blast*
lemma *KTB*: ∀ *w*:*W*. r *R* ∧ s *R* ⟶ (⟨*W*,*R*,*V*⟩,*w* $\models^d$ *F1*)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
  oops
lemma *KT*: ∀ *w*:*W*. r *R* ⟶ (⟨*W*,*R*,*V*⟩,*w* $\models^d$ *F1*)

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ \mathrm{r}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ \mathrm{s}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^d F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *S4*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *KB4*: $\forall w{:}W.\ \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof

apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops

lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F5)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F6)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

by *blast*
lemma *S4*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ \mathrm{r}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ \mathrm{s}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^d F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.\ \mathrm{s}\ R \wedge \mathrm{t}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.\ \mathrm{r}\ R \wedge \mathrm{s}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.\ \mathrm{r}\ R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F9)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r R ∧ s R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s R ∧ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.$ r R ∧ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^d F10)$
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=32 (with simp 8, without simp 24), none=56 (with simp 0, without simp 56), unknwn=72 (with simp 72, without simp 0) Sledgehammer: proof=70 (with simp 38, without simp 32), no prf=90 (with simp 42, without simp 48)

— No conflicts
end

8.3 Tests with the (maximal) shallow embedding: axiom schemata

theory *PMLinHOL-shallow-further-tests-1* **imports** *PMLinHOL-shallow-tests*

begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms

consts φ::σ ψ::σ
abbreviation(*input*) *F1* $\equiv (\Diamond^s(\Diamond^s\varphi)) \supset^s \Diamond^s\varphi$ — holds in K4
abbreviation(*input*) *F2* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) *F3* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$ — holds in KB4
abbreviation(*input*) *F4* $\equiv (\Box^s(\Diamond^s(\Box^s(\Diamond^s\varphi)))) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB/K4
abbreviation(*input*) *F5* $\equiv (\Diamond^s(\varphi \wedge^s (\Diamond^s\psi))) \supset^s ((\Diamond^s\varphi) \wedge^s (\Diamond^s\psi))$ — holds in K4
abbreviation(*input*) *F6* $\equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Diamond^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\Diamond^s\psi)$ — holds in KB4
abbreviation(*input*) *F7* $\equiv (\Diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \Diamond^s\varphi))$ — holds in KB4
abbreviation(*input*) *F8* $\equiv (\Diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \Diamond^s\varphi)$ — holds in KT/KB
abbreviation(*input*) *F9* $\equiv ((\Box^s(\Diamond^s\varphi)) \wedge^s (\Box^s(\Diamond^s(\neg^s \varphi)))) \supset^s \Diamond^s(\Diamond^s\varphi)$ — holds in KT
abbreviation(*input*) *F10* $\equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F1$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F1$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F1*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s$ *F1*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s$ *F2*
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf

oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F2$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F3$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin

lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
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— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
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lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F4$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf

apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*) *NegS-def*)
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*smt* (*verit*, *del-insts*) *NegS-def*)
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F5$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s$

$\varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F6$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops

lemma *KB*: $\forall\, w{:}W.\ (\forall\, \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K4*: $\forall\, w{:}W.\ (\forall\, \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall\, w{:}W.\ \langle W,R,V\rangle,w \models^s F7$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall\, w{:}W.\ (\forall\, \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *S4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB4*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KTB*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KT*: $\forall\, w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *KB*: $\forall\, w{:}W.\ (\forall\, \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *K4*: $\forall\, w{:}W.\ (\forall\, \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf

apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F8$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *S4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KB4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KT*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KB*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall \varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F9$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$

— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *S4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis NegS-def*)
lemma *KB4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \wedge (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis* (*lifting*) *BoxS-def NegS-def*)
lemma *KT*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \varphi) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by (*metis* (*lifting*) *BoxS-def NegS-def*)
lemma *KB*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s \varphi \supset^s \Box^s(\Diamond^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.\ (\forall\varphi.\ \langle W,R,V\rangle,w \models^s (\Box^s\varphi) \supset^s \Box^s(\Box^s\varphi)) \longrightarrow \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.\ \langle W,R,V\rangle,w \models^s F10$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=32 (with simp 16, without simp 16), none=0, unknwn=128 (with simp 64, without simp 64) Sledgehammer: proof=32 (with simp 24, without simp 8), no prf=128 (with simp 56, without simp 72)

— No conflict
end

8.4 Tests with the (maximal) shallow embedding: semantic frame conditions

theory *PMLinHOL-shallow-further-tests-2*
imports *PMLinHOL-shallow-tests*
begin
— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with semantic conditions
abbreviation(*input*) *refl* (r) **where** r ≡ $\lambda R.$ *reflexive R*
abbreviation(*input*) *sym* (s) **where** s ≡ $\lambda R.$ *symmetric R*
abbreviation(*input*) *tra* (t) **where** t ≡ $\lambda R.$ *transitive R*

consts $\varphi{::}\sigma$ $\psi{::}\sigma$
abbreviation(*input*) $F1 \equiv (\Diamond^s(\Diamond^s\varphi)) \supset^s \Diamond^s\varphi$ — holds in K4
abbreviation(*input*) $F2 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) $F3 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s \Box^s\varphi$ — holds in KB4
abbreviation(*input*) $F4 \equiv (\Box^s(\Diamond^s(\Box^s(\Diamond^s\varphi)))) \supset^s \Box^s(\Diamond^s\varphi)$ — holds in KB
abbreviation(*input*) $F5 \equiv (\Diamond^s(\varphi \wedge^s (\Diamond^s\psi))) \supset^s ((\Diamond^s\varphi) \wedge^s (\Diamond^s\psi))$ — holds in K4
abbreviation(*input*) $F6 \equiv ((\Box^s(\varphi \supset^s \psi)) \wedge^s (\Diamond^s(\Box^s(\neg^s\psi)))) \supset^s \neg^s(\Diamond^s\psi)$ — holds in KB4
abbreviation(*input*) $F7 \equiv (\Diamond^s\varphi) \supset^s (\Box^s(\varphi \vee^s \Diamond^s\varphi))$ — holds in KB4
abbreviation(*input*) $F8 \equiv (\Diamond^s(\Box^s\varphi)) \supset^s (\varphi \vee^s \Diamond^s\varphi)$ — holds in KT and in KB
abbreviation(*input*) $F9 \equiv ((\Box^s(\Diamond^s\varphi)) \wedge^s (\Box^s(\Diamond^s(\neg^s \varphi)))) \supset^s \Diamond^s(\Diamond^s\varphi)$ — holds in KT
abbreviation(*input*) $F10 \equiv ((\Box^s(\varphi \supset^s \Box^s\psi)) \wedge^s (\Box^s(\Diamond^s(\neg^s\psi)))) \supset^s \neg^s(\Box^s\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf

apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K*: $\forall w{:}W. (\langle W,R,V\rangle,w \models^s F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W. (\langle W,R,V\rangle,w \models^s F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops

end

experiment begin
lemma *S5*: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: $\forall\, w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall\, w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $\forall\, w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall\, w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall\, w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall\, w{:}W.$ $(\langle W,R,V\rangle, w \models^{s} F3)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall\, w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *metis*
lemma *S4*: $\forall\, w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *metis*
lemma *KB4*: $\forall\, w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle, w \models^{s} F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof

by *metis*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *meson*
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F4)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F5)$

— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^s F5)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.\ (\langle W,R,V\rangle,w \models^s F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*

lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof

apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops

end

experiment begin
lemma *S5*: $\forall w{:}W.$ r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *S4*: $\forall w{:}W.$ r $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB4*: $\forall w{:}W.$ s $R \wedge$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: $\forall w{:}W.$ r $R \wedge$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KT*: $\forall w{:}W.$ r $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=unknown] — sledgehammer — unkn — no prf
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *blast*
lemma *KB*: $\forall w{:}W.$ s $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: $\forall w{:}W.$ t $R \longrightarrow (\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\forall w{:}W.$ $(\langle W,R,V\rangle,w \models^s F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=0, unknwn=76 (with simp 38, without simp 38) Sledgehammer: proof=66 (with simp 38, without simp 28), no prf=94 (with simp 42, without simp 52)

— No conflicts
end

8.5 Tests with the (minimal) shallow embedding: axiom schemata

theory *PMLinHOL-shallow-minimal-further-tests-1*
imports *PMLinHOL-shallow-minimal* — C.B., 2025
begin

— What is the weakest modal logic in which the following test formulas F1-F10 are provable?
— Test with schematic axioms
abbreviation(*input*) $AxT \equiv \forall \varphi.\ \models^m (\Box^m\varphi) \supset^m \varphi$
abbreviation(*input*) $AxB \equiv \forall \varphi.\ \models^m \varphi \supset^m \Box^m(\Diamond^m\varphi)$
abbreviation(*input*) $Ax4 \equiv \forall \varphi.\ \models^m (\Box^m\varphi) \supset^m \Box^m(\Box^m\varphi)$

consts $\varphi{::}\sigma\ \psi{::}\sigma$
abbreviation(*input*) $F1 \equiv (\Diamond^m(\Diamond^m\varphi)) \supset^m \Diamond^m\varphi$ — holds in K4
abbreviation(*input*) $F2 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) $F3 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m \Box^m\varphi$ — holds in KB4
abbreviation(*input*) $F4 \equiv (\Box^m(\Diamond^m(\Box^m(\Diamond^m\varphi)))) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB/K4
abbreviation(*input*) $F5 \equiv (\Diamond^m(\varphi \wedge^m (\Diamond^m\psi))) \supset^m ((\Diamond^m\varphi) \wedge^m (\Diamond^m\psi))$ — holds in K4
abbreviation(*input*) $F6 \equiv ((\Box^m(\varphi \supset^m \psi)) \wedge^m (\Diamond^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\Diamond^m\psi)$ — holds in KB4
abbreviation(*input*) $F7 \equiv (\Diamond^m\varphi) \supset^m (\Box^m(\varphi \vee^m \Diamond^m\varphi))$ — holds in KB4
abbreviation(*input*) $F8 \equiv (\Diamond^m(\Box^m\varphi)) \supset^m (\varphi \vee^m \Diamond^m\varphi)$ — holds in KT/KB
abbreviation(*input*) $F9 \equiv ((\Box^m(\Diamond^m\varphi)) \wedge^m (\Box^m(\Diamond^m(\neg^m \varphi)))) \supset^m \Diamond^m(\Diamond^m\varphi)$ — holds in KT
abbreviation(*input*) $F10 \equiv ((\Box^m(\varphi \supset^m \Box^m\psi)) \wedge^m (\Box^m(\Diamond^m(\neg^m\psi)))) \supset^m \neg^m(\Box^m\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

nitpick-params
experiment begin
lemma *S5*: $AxT \wedge AxB \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: $AxT \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: $AxB \wedge Ax4 \longrightarrow \models^m F1$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: $AxT \wedge AxB \longrightarrow \models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: $AxT \longrightarrow \models^m F1$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: $AxB \longrightarrow \models^m F1$

— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F1*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K*: $\models^m$ *F1*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* $\wedge$ *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F2*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *S4*: *AxT* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F2*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F2*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* $\wedge$ *AxB* $\longrightarrow \models^m$ *F2*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KT*: *AxT* $\longrightarrow \models^m$ *F2*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* $\longrightarrow \models^m$ *F2*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F2*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F2*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin

lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ $\models^m$ *F3*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *S4*: *AxT* ∧ *Ax4* ⟶ $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F3*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: *AxT* ⟶ $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* ⟶ $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* ⟶ $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F3*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ $\models^m$ *F4*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *S4*: *AxT* ∧ *Ax4* ⟶ $\models^m$ *F4*
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *smt*
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F4*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F4*
— nitpick[expect=none] — sledgehammer — none — proof

apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KT*: *AxT* $\longrightarrow \models^m$ *F4*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* $\longrightarrow \models^m$ *F4*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F4*
— nitpick[expect=unknown] — sledgehammer — unkn — proof
apply *simp* — nitpick[expect=unknown] — sledgehammer — unkn — proof
by *smt*
lemma *K*: $\models^m$ *F4*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* $\wedge$ *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F5*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: *AxT* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F5*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F5*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: *AxT* $\wedge$ *AxB* $\longrightarrow \models^m$ *F5*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: *AxT* $\longrightarrow \models^m$ *F5*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* $\longrightarrow \models^m$ *F5*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F5*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*

lemma *K*: $\models^m$ *F5*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* $\wedge$ *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F6*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *S4*: *AxT* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F6*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* $\wedge$ *AxB* $\longrightarrow \models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: *AxT* $\longrightarrow \models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* $\longrightarrow \models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F6*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* $\wedge$ *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F7*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *S4*: *AxT* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

oops
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F7*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: *AxT* ⟶ $\models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: *AxB* ⟶ $\models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* ⟶ $\models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F7*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ $\models^m$ *F8*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: *AxT* ∧ *Ax4* ⟶ $\models^m$ *F8*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ $\models^m$ *F8*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *KTB*: *AxT* ∧ *AxB* ⟶ $\models^m$ *F8*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: *AxT* ⟶ $\models^m$ *F8*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: *AxB* ⟶ $\models^m$ *F8*

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by (*metis NegM-def*)
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F8*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F8*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: *AxT* $\wedge$ *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: *AxT* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: *AxB* $\wedge$ *Ax4* $\longrightarrow \models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: *AxT* $\wedge$ *AxB* $\longrightarrow \models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: *AxT* $\longrightarrow \models^m$ *F9*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: *AxB* $\longrightarrow \models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* $\longrightarrow \models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $\models^m$ *F9*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin

lemma *S5*: *AxT* ∧ *AxB* ∧ *Ax4* ⟶ ⊨^{m} *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: *AxT* ∧ *Ax4* ⟶ ⊨^{m} *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: *AxB* ∧ *Ax4* ⟶ ⊨^{m} *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: *AxT* ∧ *AxB* ⟶ ⊨^{m} *F10*
— nitpick[expect=none] — sledgehammer — none — no prf
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: *AxT* ⟶ ⊨^{m} *F10*
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: *AxB* ⟶ ⊨^{m} *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: *Ax4* ⟶ ⊨^{m} *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: ⊨^{m} *F10*
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=72 (with simp 36, without simp 36), unknwn=4 (with simp 2, without simp 2) Sledgehammer: proof=73 (with simp 38, without simp 35), no prf=87(with simp 42, without simp 45)

— No conflicts
end

8.6 Tests with the (minimal) shallow embedding: semantic frame conditions

theory *PMLinHOL-shallow-minimal-further-tests-2*
imports *PMLinHOL-shallow-minimal*
begin
— What is the weakest modal logic in which the following test formulas F1-F10

are provable?
— Test with semantic conditions
abbreviation(*input*) *refl* (r) **where** r ≡ $\lambda R.$ *reflexive* R
abbreviation(*input*) *sym* (s) **where** s ≡ $\lambda R.$ *symmetric* R
abbreviation(*input*) *tra* (t) **where** t ≡ $\lambda R.$ *transitive* R

consts $\varphi::\sigma$ $\psi::\sigma$
abbreviation(*input*) *F1* ≡ $(\Diamond^m(\Diamond^m\varphi)) \supset^m \Diamond^m\varphi$ — holds in K4
abbreviation(*input*) *F2* ≡ $(\Diamond^m(\Box^m\varphi)) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) *F3* ≡ $(\Diamond^m(\Box^m\varphi)) \supset^m \Box^m\varphi$ — holds in KB4
abbreviation(*input*) *F4* ≡ $(\Box^m(\Diamond^m(\Box^m(\Diamond^m\varphi)))) \supset^m \Box^m(\Diamond^m\varphi)$ — holds in KB
abbreviation(*input*) *F5* ≡ $(\Diamond^m(\varphi \wedge^m (\Diamond^m\psi))) \supset^m ((\Diamond^m\varphi) \wedge^m (\Diamond^m\psi))$ — holds in K4
abbreviation(*input*) *F6* ≡ $((\Box^m(\varphi \supset^m \psi)) \wedge^m (\Diamond^m(\Box^m(\neg^m\psi)))) \supset^m \neg^m(\Diamond^m\psi)$ — holds in KB4
abbreviation(*input*) *F7* ≡ $(\Diamond^m\varphi) \supset^m (\Box^m(\varphi \vee^m \Diamond^m\varphi))$ — holds in KB4
abbreviation(*input*) *F8* ≡ $(\Diamond^m(\Box^m\varphi)) \supset^m (\varphi \vee^m \Diamond^m\varphi)$ — holds in KT and in KB
abbreviation(*input*) *F9* ≡ $((\Box^m(\Diamond^m\varphi)) \wedge^m (\Box^m(\Diamond^m(\neg^m \varphi)))) \supset^m \Diamond^m(\Diamond^m\varphi)$ — holds in KT
abbreviation(*input*) *F10* ≡ $((\Box^m(\varphi \supset^m \Box^m\psi)) \wedge^m (\Box^m(\Diamond^m(\neg^m\psi)))) \supset^m \neg^m(\Box^m\psi)$ — holds in KT

declare *imp-cong*[*cong del*]

experiment begin
lemma *S5*: r R ∧ s R ∧ t R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
  by *metis*
lemma *S4*: r R ∧ t R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
  by *metis*
lemma *KB4*: s R ∧ t R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=none] — sledgehammer — none — proof
  apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
  by *metis*
lemma *KTB*: r R ∧ s R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma *KT*: r R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  oops
lemma *KB*: s R ⟶ ($\models^m$ *F1*)
  — nitpick[expect=genuine] — sledgehammer — ctex — no prf
  apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf

oops
lemma *K4*: t $R \longrightarrow (\models^m F1)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K*: $(\models^m F1)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r $R \wedge$ t $R \longrightarrow (\models^m F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: s $R \wedge$ t $R \longrightarrow (\models^m F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r $R \wedge$ s $R \longrightarrow (\models^m F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: r $R \longrightarrow (\models^m F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s $R \longrightarrow (\models^m F2)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K4*: t $R \longrightarrow (\models^m F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $(\models^m F2)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m F3)$
— nitpick[expect=none] — sledgehammer — none — proof

apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r *R* ∧ t *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: s *R* ∧ t *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r *R* ∧ s *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: r *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t *R* ⟶ ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: ($\models^m$ *F3*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r *R* ∧ t *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: s *R* ∧ t *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r *R* ∧ s *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*

lemma *KT*: r *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K4*: t *R* ⟶ ($\models^m$ *F4*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K*: ($\models^m$ *F4*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r *R* ∧ t *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: s *R* ∧ t *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r *R* ∧ s *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: r *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t *R* ⟶ ($\models^m$ *F5*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *K*: ($\models^m$ *F5*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf

apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r $R \wedge$ t $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: s $R \wedge$ t $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r $R \wedge$ s $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: r $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t $R \longrightarrow (\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $(\models^m\ F6)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m\ F7)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r $R \wedge$ t $R \longrightarrow (\models^m\ F7)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB4*: s $R \wedge$ t $R \longrightarrow (\models^m\ F7)$

— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KT*: r *R* ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KB*: s *R* ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t *R* ⟶ (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: (⊨^{m} *F7*)
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r *R* ∧ s *R* ∧ t *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r *R* ∧ t *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: s *R* ∧ t *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KTB*: r *R* ∧ s *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: r *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: s *R* ⟶ (⊨^{m} *F8*)
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof

by *metis*
lemma *K4*: t $R \longrightarrow (\models^m F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $(\models^m F8)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r $R \wedge$ t $R \longrightarrow (\models^m F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: s $R \wedge$ t $R \longrightarrow (\models^m F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: r $R \wedge$ s $R \longrightarrow (\models^m F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: r $R \longrightarrow (\models^m F9)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: s $R \longrightarrow (\models^m F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t $R \longrightarrow (\models^m F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $(\models^m F9)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

experiment begin
lemma *S5*: r $R \wedge$ s $R \wedge$ t $R \longrightarrow (\models^m F10)$
— nitpick[expect=none] — sledgehammer — none — proof

apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *S4*: r $R \wedge$ t $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB4*: s $R \wedge$ t $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *KTB*: r $R \wedge$ s $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KT*: r $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=none] — sledgehammer — none — proof
apply *simp* — nitpick[expect=none] — sledgehammer — none — proof
by *metis*
lemma *KB*: s $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K4*: t $R \longrightarrow (\models^m\ F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
lemma *K*: $(\models^m\ F10)$
— nitpick[expect=genuine] — sledgehammer — ctex — no prf
apply *simp* — nitpick[expect=genuine] — sledgehammer — ctex — no prf
oops
end

— Summary of experiments: Nitpick: ctex=84 (with simp 42, without simp 42), none=76 (with simp 38, without simp 38), unknwn=0 Sledgehammer: proof=76 (with simp 38, without simp 38), no prf=84 (with simp 42, without simp 42)

— No conflicts
end

References

[1] C. Benzmüller. Faithful logic embeddings in HOL — deep and shallow. In C. Barrett and U. Waldmann, editors, *Automated Deduction – CADE-30 – 30th International Conference on Automated Deduction, Stuttgart, Germany, July 28-31, 2025, Proceedings*, volume 15943 of *Lecture Notes in Computer Science*, pages 280–302. Springer, 2025. (preprint: arXiv:2502.19311).