

# Deriving class instances for datatypes.\*

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## Abstract

We provide a framework for registering automatic methods to derive class instances of datatypes, as it is possible using Haskell’s “deriving Ord, Show, . . .” feature.

We further implemented such automatic methods to derive (linear) orders or hash-functions which are required in the Isabelle Collection Framework [1] and the Container Framework [2]. Moreover, for the tactic of Huffman and Krauss to show that a datatype is countable, we implemented a wrapper so that this tactic becomes accessible in our framework.

Our formalization was performed as part of the IsaFoR/CeTA project<sup>1</sup> [3]. With our new tactic we could completely remove tedious proofs for linear orders of two datatypes.

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<sup>1</sup><http://cl-informatik.uibk.ac.at/software/ceta>

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## 1 Important Information

The described generators are outdated as they are based on the old datatype package. Generators for the new datatypes are available in the AFP entry “Deriving”.

```
theory Derive-Aux
imports
  Deriving.Derive-Manager
begin
```

ML-file  $\langle$ derive-aux.ML $\rangle$

end

## 2 Generating linear orders for datatypes

```
theory Order-Generator
imports
  Derive-Aux
begin
```

### 2.1 Introduction

The order generator registers itself at the derive-manager for the classes *ord*, *order*, and *linorder*. To be more precise, it automatically generates the two functions  $(\leq)$  and  $(<)$  for some datatype *dtype* and proves the following instantiations.

- instantiation *dtype* :: (ord,...,ord) ord
- instantiation *dtype* :: (order,...,order) order
- instantiation *dtype* :: (linorder,...,linorder) linorder

All the non-recursive types that are used in the datatype must have similar instantiations. For recursive type-dependencies this is automatically generated.

For example, for the `datatype tree = Leaf nat | Node "tree list"` we require that `nat` is already in `linorder`, whereas for `list` nothing is required, since for the `tree` datatype the `list` is only used recursively.

However, if we define `datatype tree = Leaf "nat list" | Node tree tree` then `list` must provide the above instantiations.

Note that when calling the generator for `linorder`, it will automatically also derive the instantiations for `order`, which in turn invokes the generator for `ord`. A later invocation of `linorder` after `order` or `ord` is not possible.

## 2.2 Implementation Notes

The generator uses the recursors from the datatype package to define a lexicographic order. E.g., for a declaration `datatype 'a tree = Empty | Node "'a tree" 'a "'a tree"` this will semantically result in

```
(Empty < Node _ _ _) = True
(Node l1 l2 l3 < Node r1 r2 r3) =
  (l1 < r1 || l1 = r1 && (l2 < r2 || l2 = r2 && l3 < r3))
(_ < _) = False
(l <= r) = (l < r || l = r)
```

The desired properties (like  $\llbracket x < y; y < z \rrbracket \implies x < z$ ) of the orders are all proven using induction (with the induction theorem from the datatype on  $x$ ), and afterwards there is a case distinction on the remaining variables, i.e., here  $y$  and  $z$ . If the constructors of  $x$ ,  $y$ , and  $z$  are different always some basic tactic is invoked. In the other case (identical constructors) for each property a dedicated tactic was designed.

## 2.3 Features and Limitations

The order generator has been developed mainly for datatypes without explicit mutual recursion. For mutual recursive datatypes—like `datatype a = C b and b = D a a`—only for the first mentioned datatype—here `a`—the instantiations of the order-classes are derived.

Indirect recursion like in `datatype tree = Leaf nat | Node "tree list"` should work without problems.

## 2.4 Installing the generator

**lemma** *linear-cases*:  $(x :: 'a :: linorder) = y \vee x < y \vee y < x$  **by** *auto*

**ML-file**  $\langle$ *order-generator.ML* $\rangle$

end

### 3 Hash functions

```
theory Hash-Generator
imports
  Collections.HashCode
  Derive-Aux
begin
```

#### 3.1 Introduction

The interface for hash-functions is defined in the class *hashable* which has been developed as part of the Isabelle Collection Framework [1]. It requires a hash-function (*hashcode*), a bounded hash-function (*bounded-hashcode*), and a default hash-table size (*def-hashmap-size*).

The *hashcode* function for each datatype are created by instantiating the recursors of that datatype appropriately. E.g., for datatype `'a test = C1 'a 'a | C2 "'a test list"` we get a hash-function which is equivalent to

```
hashcode (C1 a b) = c1 * hashcode a + c2 * hashcode b
hashcode (C2 Nil) = c3
hashcode (C2 (a # as)) = c4 * hashcode a + c5 * hashcode as
```

where each  $c_i$  is a non-negative 32-bit number which is dependent on the datatype name, the constructor name, and the occurrence of the argument (i.e., in the example  $c_1$  and  $c_2$  will usually be different numbers.) These parameters are used in linear combination with prime numbers to hopefully get some useful hash-function.

The *bounded-hashcode* functions are constructed in the same way, except that after each arithmetic operation a modulo operation is performed.

Finally, the default hash-table size is just set to 10, following Java's default hash-table constructor.

#### 3.2 Features and Limitations

We get same limitation as for the order generator. For mutual recursive datatypes, only for the first mentioned datatype the instantiations of the *hashable*-class are derived.

#### 3.3 Installing the generator

**lemma** *hash-mod-lemma*:  $1 < (n :: nat) \implies x \bmod n < n$  **by** *auto*

**ML-file** `<hash-generator.ML>`

**end**

## 4 Loading derive-commands

```
theory Derive  
imports  
  Order-Generator  
  Hash-Generator  
  Deriving.Countable-Generator  
begin
```

We just load the commands to derive (linear) orders, hash-functions, and the command to show that a datatype is countable, so that now all of them are available. There are further generators available in the AFP entries of lightweight containers and Show.

```
print-derives
```

**end**

## 5 Examples

```
theory Derive-Examples  
imports  
  Derive  
  HOL.Rat  
begin
```

### 5.1 Register standard existing types

```
derive linorder list sum prod
```

### 5.2 Without nested recursion

```
datatype 'a bintree = BEmpty | BNode 'a bintree 'a 'a bintree
```

```
derive linorder bintree  
derive hashable bintree  
derive countable bintree
```

### 5.3 Using other datatypes

```
datatype nat-list-list = NNil | CCons nat list nat-list-list
```

```
derive linorder nat-list-list  
derive hashable nat-list-list  
derive countable nat-list-list
```

## 5.4 Explicit mutual recursion

**datatype**

```
'a mtree = MEmpty | MNode 'a 'a mtree-list and  
'a mtree-list = MNil | MCons 'a mtree 'a mtree-list
```

**derive** *linorder mtree*

**derive** *hashable mtree*

**derive** *countable mtree*

## 5.5 Implicit mutual recursion

**datatype** 'a tree = Empty | Node 'a 'a tree list

**datatype-compatible** tree

**derive** *linorder tree*

**derive** *hashable tree*

**derive** *countable tree*

**datatype** 'a ttree = TEmpty | TNode 'a 'a ttree list tree

**datatype-compatible** ttree

**derive** *linorder ttree*

**derive** *hashable ttree*

**derive** *countable ttree*

## 5.6 Examples from IsaFoR

**datatype** ('f,'v) term = Var 'v | Fun 'f ('f,'v) term list

**datatype-compatible** term

**datatype** ('f, 'l) lab =

```
Lab ('f, 'l) lab 'l  
| FunLab ('f, 'l) lab ('f, 'l) lab list  
| UnLab 'f  
| Sharp ('f, 'l) lab
```

**datatype-compatible** lab

**derive** *linorder term lab*

**derive** *countable term lab*

**derive** *hashable term lab*

## 5.7 A complex datatype

The following datatype has nested indirect recursion, mutual recursion and uses other datatypes.

```

datatype ('a, 'b) complex =
  C1 nat 'a ttree |
  C2 ('a, 'b) complex list tree tree 'b ('a, 'b) complex ('a, 'b) complex2 ttree list
and ('a, 'b) complex2 = D1 ('a, 'b) complex ttree

datatype-compat complex complex2

derive linorder complex
derive hashable complex
derive countable complex

end

```

## 6 Acknowledgements

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## References

- [1] P. Lammich and A. Lochbihler. The Isabelle collections framework. In *Proc. ITP'10*, volume 6172 of *LNCS*, pages 339–354, 2010.
- [2] A. Lochbihler. Light-weight containers for isabelle: Efficient, extensible, nestable. In *Proc. ITP'13*, volume 7998 of *LNCS*, pages 116–132, 2013.
- [3] R. Thiemann and C. Sternagel. Certification of termination proofs using CeTA. In *Proc. TPHOLs'09*, volume 5674 of *LNCS*, pages 452–468, 2009.