Core DOM

A Formal Model of the Document Object Model

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November 9, 2026

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Abstract

In this AFP entry, we formalize the core of the Document Object Model (DOM). At its core, the DOM defines a tree-like data structure for representing documents in general and HTML documents in particular. It is the heart of any modern web browser.

Formalizing the key concepts of the DOM is a prerequisite for the formal reasoning over client-side JavaScript programs and for the analysis of security concepts in modern web browsers.

We present a formalization of the core DOM, with focus on the *node-tree* and the operations defined on node-trees, in Isabelle/HOL. We use the formalization to verify the functional correctness of the most important functions defined in the DOM standard. Moreover, our formalization is 1. *extensible*, i.e., can be extended without the need of re-proving already proven properties and 2. *executable*, i.e., we can generate executable code from our specification.

Keywords: Document Object Model, DOM, Formal Semantics, Isabelle/HOL

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1 Introduction

In a world in which more and more applications are offered as services on the internet, web browsers start to take on a similarly central role in our daily IT infrastructure as operating systems. Thus, web browsers should be developed as rigidly and formally as operating systems. While formal methods are a well-established technique in the development of operating systems (see, e. g., Klein [12] for an overview of formal verification of operating systems), there are few proposals for improving the development of web browsers using formal approaches [2, 9, 10, 13].

As a first step towards a verified client-side web application stack, we model and formally verify the Document Object Model (DOM) in Isabelle/HOL. The DOM [14, 15] is *the* central data structure of all modern web browsers. At its core, the Document Object Model (DOM), defines a tree-like data structure for representing documents in general and HTML documents in particular. Thus, the correctness of a DOM implementation is crucial for ensuring that a web browser displays web pages correctly. Moreover, the DOM is the core data structure underlying client-side JavaScript programs, i. e., client-side JavaScript programs are mostly programs that read, write, and update the DOM.

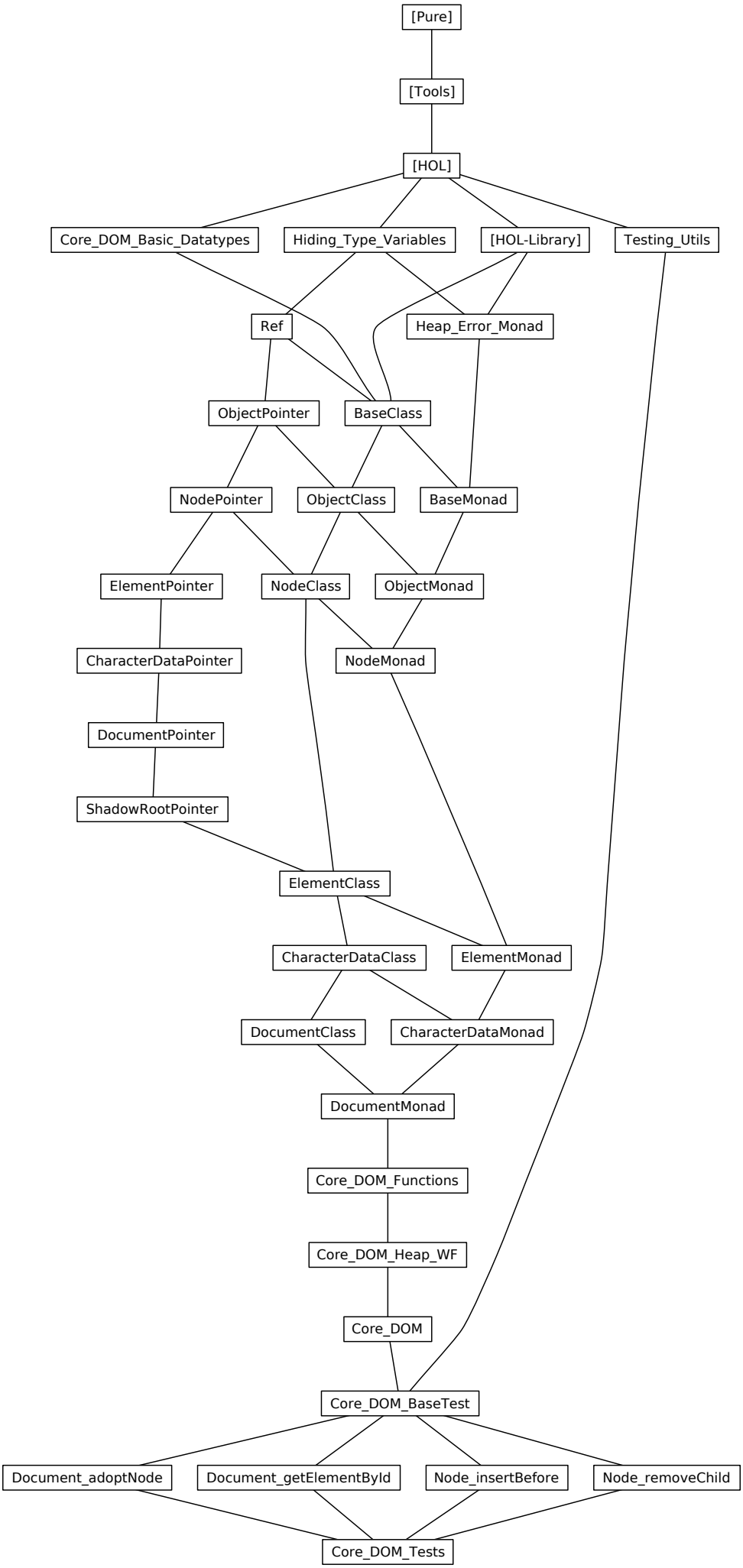
In more detail, we formalize the core DOM as a shallow embedding [11] in Isabelle/HOL. Our formalization is based on a typed data model for the *node-tree*, i. e., a data structure for representing XML-like documents in a tree structure. Furthermore, we formalize a typed heap for storing (partial) node-trees together with the necessary consistency constraints. Finally, we formalize the operations (as described in the DOM standard [15]) on this heap that allow manipulating node-trees.

Our machine-checked formalization of the DOM node tree [15] has the following desirable properties:

- It provides a *consistency guarantee.* Since all definitions in our formal semantics are conservative and all rules are derived, the logical consistency of the DOM node-tree is reduced to the consistency of HOL.
- It serves as a *technical basis for a proof system.* Based on the derived rules and specific setup of proof tactics over node-trees, our formalization provides a generic proof environment for the verification of programs manipulating node-trees.
- It is *executable*, which allows to validate its compliance to the standard by evaluating the compliance test suite on the formal model and
- It is *extensible* in the sense of [3, 7], i. e., properties proven over the core DOM do not need to be re-proven for object-oriented extensions such as the HTML document model.

The rest of this document is automatically generated from the formalization in Isabelle/HOL, i.e., all content is checked by Isabelle.[1] The structure follows the theory dependencies (see Figure 1.1): we start with introducing the technical preliminaries of our formalization (chapter 2). Next, we introduce the concepts of pointers (chapter 3) and classes (chapter 4), i.e., the core object-oriented datatypes of the DOM. On top of this data model, we define the functional behavior of the DOM classes, i.e., their methods (chapter 5). In chapter 6, we introduce the formalization of the functionality of the core DOM, i.e., the *main entry point for users* that want to use this AFP entry. Finally, we formalize the relevant compliance test cases in chapter 7.

[1] For a brief overview of the work, we refer the reader to [4].


[Pure]
[Tools]
[HOL]
Core_DOM_Basic_Datatypes
Hiding_Type_Variables
[HOL-Library]
Testing_Utils
Ref
Heap_Error_Monad
ObjectPointer
BaseClass
NodePointer
ObjectClass
BaseMonad
ElementPointer
NodeClass
ObjectMonad
CharacterDataPointer
NodeMonad
DocumentPointer
ShadowRootPointer
ElementClass
CharacterDataClass
ElementMonad
DocumentClass
CharacterDataMonad
DocumentMonad
Core_DOM_Functions
Core_DOM_Heap_WF
Core_DOM
Core_DOM_BaseTest
Document_adoptNode
Document_getElementById
Node_insertBefore
Node_removeChild
Core_DOM_Tests

2 Preliminaries

In this chapter, we introduce the technical preliminaries of our formalization of the core DOM, namely a mechanism for hiding type variables and the heap error monad.

2.1 Hiding Type Variables (Hiding_Type_Variables)

This theory[1] implements a mechanism for declaring default type variables for data types. This comes handy for complex data types with many type variables.

theory
  "Hiding_Type_Variables"
imports
  Main
keywords
    "register_default_tvars"
    "update_default_tvars_mode"::thy_decl
begin⟨*ML*⟩⟨*ML*⟩⟨*ML*⟩

2.1.1 Introduction

When modelling object-oriented data models in HOL with the goal of preserving *extensibility* (e.g., as described in [3, 7]) one needs to define type constructors with a large number of type variables. This can reduce the readability of the overall formalization. Thus, we use a short-hand notation in cases were the names of the type variables are known from the context. In more detail, this theory sets up both configurable print and parse translations that allows for replacing *all* type variables by *(_)*, e.g., a five-ary constructor *('a, 'b, 'c, 'd, 'e) hide_tvar_foo* can be shorted to *(_) hide_tvar_foo*. The use of this shorthand in output (printing) and input (parsing) is, on a per-type basis, user-configurable using the top-level commands *register_default_tvars* (for registering the names of the default type variables and the print/parse mode) and *update_default_tvars_mode* (for changing the print/parse mode dynamically).

The input also supports short-hands for declaring default sorts (e.g., *(_::linorder)* specifies that all default variables need to be instances of the sort (type class) *linorder* and short-hands of overriding a suffice (or prefix) of the default type variables. For example, *('state) hide_tvar_foo _.* is a short-hand for *('a, 'b, 'c, 'd, 'state) hide_tvar_foo*. In this document, we omit the implementation details (we refer the interested reader to theory file) and continue directly with a few examples.

2.1.2 Example

Given the following type definition:

datatype *('a, 'b) hide_tvar_foobar = hide_tvar_foo 'a | hide_tvar_bar 'b*
type_synonym *('a, 'b, 'c, 'd) hide_tvar_baz = "('a+'b, 'a × 'b) hide_tvar_foobar"*

We can register default values for the type variables for the abstract data type as well as the type synonym:

register_default_tvars *"('alpha, 'beta) hide_tvar_foobar" (print_all,parse)*
register_default_tvars *"('alpha, 'beta, 'gamma, 'delta) hide_tvar_baz" (print_all,parse)*

This allows us to write

definition *hide_tvar_f::"(_) hide_tvar_foobar ⇒ (_) hide_tvar_foobar ⇒ (_) hide_tvar_foobar"*
    where *"hide_tvar_f a b = a"*
definition *hide_tvar_g::"(_) hide_tvar_baz ⇒ (_) hide_tvar_baz ⇒ (_) hide_tvar_baz"*
  where *"hide_tvar_g a b = a"*

[1] This theory can be used "stand-alone," i.e., this theory is not specific to the DOM formalization. The latest version is part of the "Isabelle Hacks" repository: `https://git.logicalhacking.com/adbrucker/isabelle-hacks/`.

Instead of specifying the type variables explicitely. This makes, in particular for type constructors with a large number of type variables, definitions much more concise. This syntax is also used in the output of antiquotations, e.g., *(x::(_) hide_tvar_baz ⇒ (_) hide_tvar_baz ⇒ (_) hide_tvar_baz) = hide_tvar_g*. Both the print translation and the parse translation can be disabled for each type individually:

update_default_tvars_mode *"_ hide_tvar_foobar" (noprint,noparse)*
update_default_tvars_mode *"_ hide_tvar_foobar" (noprint,noparse)*

Now, Isabelle's interactive output and the antiquotations will show all type variables, e.g., *(x::('a + 'b, 'a × 'b) hide_tvar_foobar ⇒ ('a + 'b, 'a × 'b) hide_tvar_foobar ⇒ ('a + 'b, 'a × 'b) hide_tvar_foobar) = hide_tvar_g*.

end

2.2 The Heap Error Monad (Heap_Error_Monad)

In this theory, we define a heap and error monad for modeling exceptions. This allows us to define composite methods similar to stateful programming in Haskell, but also to stay close to the official DOM specification.

theory
  Heap_Error_Monad
  imports
    Hiding_Type_Variables
    "HOL-Library.Monad_Syntax"
begin

2.2.1 The Program Data Type

datatype *('heap, 'e, 'result) prog = Prog (the_prog: "'heap ⇒ 'e + 'result × 'heap")*
register_default_tvars *"('heap, 'e, 'result) prog" (print, parse)*

2.2.2 Basic Functions

definition
  bind :: "(_, 'result) prog ⇒ ('result ⇒ (_, 'result2) prog) ⇒ (_, 'result2) prog"
  where
    "bind f g = Prog (λh. (case (the_prog f) h of Inr (x, h') ⇒ (the_prog (g x)) h'
                                                | Inl exception ⇒ Inl exception))"

adhoc_overloading *Monad_Syntax.bind ⇌ bind*

definition
  execute :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ ('e + 'result × 'heap)"
  (‹((_)/ ⊢ (_))› [51, 52] 55)
  where
    "execute h p = (the_prog p) h"

definition
  returns_result :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ 'result ⇒ bool"
  (‹((_)/ ⊢ (_)/ →r (_))› [60, 35, 61] 65)
  where
    "returns_result h p r ⟷ (case h ⊢ p of Inr (r', _) ⇒ r = r' | Inl _ ⇒ False)"

fun *select_result (‹|(_)|r›)*
  where
    "select_result (Inr (r, _)) = r"
  | "select_result (Inl _) = undefined"

lemma *returns_result_eq [elim]: "h ⊢ f →r y ⟹ h ⊢ f →r y' ⟹ y = y'"*
  ⟨*proof*⟩

definition
  returns_heap :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ 'heap ⇒ bool"
  (‹((_)/ ⊢ (_)/ →h (_))› [60, 35, 61] 65)

where
"returns_heap h p h' ⟷ (case h ⊢ p of Inr (_ , h'') ⇒ h' = h'' | Inl _ ⇒ False)"

fun *select_heap (‹|(_)|$_h$›)*
where
"select_heap (Inr (_, h)) = h"
| "select_heap (Inl _) = undefined"

lemma *returns_heap_eq [elim]: "h ⊢ f →$_h$ h' ⟹ h ⊢ f →$_h$ h'' ⟹ h' = h''"*
⟨*proof*⟩

definition
returns_result_heap :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ 'result ⇒ 'heap ⇒ bool"
(‹((_)/ ⊢ (_)/ →$_r$ (_) →$_h$ (_))› [60, 35, 61, 62] 65)
where
"returns_result_heap h p r h' ⟷ h ⊢ p →$_r$ r ∧ h ⊢ p →$_h$ h'"

lemma *return_result_heap_code [code]:*
"returns_result_heap h p r h' ⟷ (case h ⊢ p of Inr (r', h'') ⇒ r = r' ∧ h' = h'' | Inl _ ⇒ False)"
⟨*proof*⟩

fun *select_result_heap (‹|(_)|$_{rh}$›)*
where
"select_result_heap (Inr (r, h)) = (r, h)"
| "select_result_heap (Inl _) = undefined"

definition
returns_error :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ 'e ⇒ bool"
(‹((_)/ ⊢ (_)/ →$_e$ (_))› [60, 35, 61] 65)
where
"returns_error h p e = (case h ⊢ p of Inr _ ⇒ False | Inl e' ⇒ e = e')"

definition *is_OK :: "'heap ⇒ ('heap, 'e, 'result) prog ⇒ bool" (‹((_)/ ⊢ ok (_))› [75, 75])*
where
"is_OK h p = (case h ⊢ p of Inr _ ⇒ True | Inl _ ⇒ False)"

lemma *is_OK_returns_result_I [intro]: "h ⊢ f →$_r$ y ⟹ h ⊢ ok f"*
⟨*proof*⟩

lemma *is_OK_returns_result_E [elim]:*
assumes *"h ⊢ ok f"*
obtains *x* **where** *"h ⊢ f →$_r$ x"*
⟨*proof*⟩

lemma *is_OK_returns_heap_I [intro]: "h ⊢ f →$_h$ h' ⟹ h ⊢ ok f"*
⟨*proof*⟩

lemma *is_OK_returns_heap_E [elim]:*
assumes *"h ⊢ ok f"*
obtains *h'* **where** *"h ⊢ f →$_h$ h'"*
⟨*proof*⟩

lemma *select_result_I:*
assumes *"h ⊢ ok f"*
and *"⋀x. h ⊢ f →$_r$ x ⟹ P x"*
shows *"P |h ⊢ f|$_r$"*
⟨*proof*⟩

lemma *select_result_I2 [simp]:*
assumes *"h ⊢ f →$_r$ x"*
shows *"|h ⊢ f|$_r$ = x"*
⟨*proof*⟩

lemma *returns_result_select_result [simp]:*
assumes *"h ⊢ ok f"*
shows *"h ⊢ f →r |h ⊢ f|r"*
⟨*proof*⟩

lemma *select_result_E:*
assumes *"P |h ⊢ f|r"* **and** *"h ⊢ ok f"*
obtains *x* **where** *"h ⊢ f →r x"* **and** *"P x"*
⟨*proof*⟩

lemma *select_result_eq: "(⋀x .h ⊢ f →r x = h' ⊢ f →r x) ⟹ |h ⊢ f|r = |h' ⊢ f|r"*
⟨*proof*⟩

definition *error :: "'e ⇒ ('heap, 'e, 'result) prog"*
where
"error exception = Prog (λh. Inl exception)"

lemma *error_bind [iff]: "(error e ⤜ g) = error e"*
⟨*proof*⟩

lemma *error_returns_result [simp]: "¬ (h ⊢ error e →r y)"*
⟨*proof*⟩

lemma *error_returns_heap [simp]: "¬ (h ⊢ error e →h h')"*
⟨*proof*⟩

lemma *error_returns_error [simp]: "h ⊢ error e →e e"*
⟨*proof*⟩

definition *return :: "'result ⇒ ('heap, 'e, 'result) prog"*
where
"return result = Prog (λh. Inr (result, h))"

lemma *return_ok [simp]: "h ⊢ ok (return x)"*
⟨*proof*⟩

lemma *return_bind [iff]: "(return x ⤜ g) = g x"*
⟨*proof*⟩

lemma *return_id [simp]: "f ⤜ return = f"*
⟨*proof*⟩

lemma *return_returns_result [iff]: "(h ⊢ return x →r y) = (x = y)"*
⟨*proof*⟩

lemma *return_returns_heap [iff]: "(h ⊢ return x →h h') = (h = h')"*
⟨*proof*⟩

lemma *return_returns_error [iff]: "¬ h ⊢ return x →e e"*
⟨*proof*⟩

definition *noop :: "('heap, 'e, unit) prog"*
where
"noop = return ()"

lemma *noop_returns_heap [simp]: "h ⊢ noop →h h' ⟷ h = h'"*
⟨*proof*⟩

definition *get_heap :: "('heap, 'e, 'heap) prog"*
where
"get_heap = Prog (λh. h ⊢ return h)"

lemma *get_heap_ok [simp]: "h ⊢ ok (get_heap)"*

⟨*proof*⟩

lemma *get_heap_returns_result [simp]: "(h ⊢ get_heap ⤜ (λh'. f h') →r x) = (h ⊢ f h →r x)"*
⟨*proof*⟩

lemma *get_heap_returns_heap [simp]: "(h ⊢ get_heap ⤜ (λh'. f h') →h h'') = (h ⊢ f h →h h'')"*
⟨*proof*⟩

lemma *get_heap_is_OK [simp]: "(h ⊢ ok (get_heap ⤜ (λh'. f h'))) = (h ⊢ ok (f h))"*
⟨*proof*⟩

lemma *get_heap_E [elim]: "(h ⊢ get_heap →r x) ⟹ x = h"*
⟨*proof*⟩

definition *return_heap :: "'heap ⇒ ('heap, 'e, unit) prog"*
where
"return_heap h = Prog (λ_. h ⊢ return ())"

lemma *return_heap_E [iff]: "(h ⊢ return_heap h' →h h'') = (h'' = h')"*
⟨*proof*⟩

lemma *return_heap_returns_result [simp]: "h ⊢ return_heap h' →r ()"*
⟨*proof*⟩

2.2.3 Pure Heaps

definition *pure :: "('heap, 'e, 'result) prog ⇒ 'heap ⇒ bool"*
where *"pure f h ⟷ h ⊢ ok f ⟶ h ⊢ f →h h"*

lemma *return_pure [simp]: "pure (return x) h"*
⟨*proof*⟩

lemma *error_pure [simp]: "pure (error e) h"*
⟨*proof*⟩

lemma *noop_pure [simp]: "pure (noop) h"*
⟨*proof*⟩

lemma *get_pure [simp]: "pure get_heap h"*
⟨*proof*⟩

lemma *pure_returns_heap_eq:*
"h ⊢ f →h h' ⟹ pure f h ⟹ h = h'"
⟨*proof*⟩

lemma *pure_eq_iff:*
"(∀h' x. h ⊢ f →r x ⟶ h ⊢ f →h h' ⟶ h = h') ⟷ pure f h"
⟨*proof*⟩

2.2.4 Bind

lemma *bind_assoc [simp]:*
"((bind f g) ⤜ h) = (f ⤜ (λx. (g x ⤜ h)))"
⟨*proof*⟩

lemma *bind_returns_result_E:*
assumes *"h ⊢ f ⤜ g →r y"*
obtains *x h'* **where** *"h ⊢ f →r x"* **and** *"h ⊢ f →h h'"* **and** *"h' ⊢ g x →r y"*
⟨*proof*⟩

lemma *bind_returns_result_E2:*
assumes *"h ⊢ f ⤜ g →r y"* **and** *"pure f h"*
obtains *x* **where** *"h ⊢ f →r x"* **and** *"h ⊢ g x →r y"*

⟨*proof*⟩

lemma *bind_returns_result_E3:*
assumes "h ⊢ f ⤜ g →r y" **and** "h ⊢ f →r x" **and** "pure f h"
shows "h ⊢ g x →r y"
⟨*proof*⟩

lemma *bind_returns_result_E4:*
assumes "h ⊢ f ⤜ g →r y" **and** "h ⊢ f →r x"
obtains h' **where** "h ⊢ f →h h'" **and** "h' ⊢ g x →r y"
⟨*proof*⟩

lemma *bind_returns_heap_E:*
assumes "h ⊢ f ⤜ g →h h''"
obtains x h' **where** "h ⊢ f →r x" **and** "h ⊢ f →h h'" **and** "h' ⊢ g x →h h''"
⟨*proof*⟩

lemma *bind_returns_heap_E2 [elim]:*
assumes "h ⊢ f ⤜ g →h h'" **and** "pure f h"
obtains x **where** "h ⊢ f →r x" **and** "h ⊢ g x →h h'"
⟨*proof*⟩

lemma *bind_returns_heap_E3 [elim]:*
assumes "h ⊢ f ⤜ g →h h'" **and** "h ⊢ f →r x" **and** "pure f h"
shows "h ⊢ g x →h h'"
⟨*proof*⟩

lemma *bind_returns_heap_E4:*
assumes "h ⊢ f ⤜ g →h h''" **and** "h ⊢ f →h h'"
obtains x **where** "h ⊢ f →r x" **and** "h' ⊢ g x →h h''"
⟨*proof*⟩

lemma *bind_returns_error_I [intro]:*
assumes "h ⊢ f →e e"
shows "h ⊢ f ⤜ g →e e"
⟨*proof*⟩

lemma *bind_returns_error_I3:*
assumes "h ⊢ f →r x" **and** "h ⊢ f →h h'" **and** "h' ⊢ g x →e e"
shows "h ⊢ f ⤜ g →e e"
⟨*proof*⟩

lemma *bind_returns_error_I2 [intro]:*
assumes "pure f h" **and** "h ⊢ f →r x" **and** "h ⊢ g x →e e"
shows "h ⊢ f ⤜ g →e e"
⟨*proof*⟩

lemma *bind_is_OK_E [elim]:*
assumes "h ⊢ ok (f ⤜ g)"
obtains x h' **where** "h ⊢ f →r x" **and** "h ⊢ f →h h'" **and** "h' ⊢ ok (g x)"
⟨*proof*⟩

lemma *bind_is_OK_E2:*
assumes "h ⊢ ok (f ⤜ g)" **and** "h ⊢ f →r x"
obtains h' **where** "h ⊢ f →h h'" **and** "h' ⊢ ok (g x)"
⟨*proof*⟩

lemma *bind_returns_result_I [intro]:*
assumes "h ⊢ f →r x" **and** "h ⊢ f →h h'" **and** "h' ⊢ g x →r y"
shows "h ⊢ f ⤜ g →r y"
⟨*proof*⟩

lemma *bind_pure_returns_result_I [intro]:*

assumes *"pure f h"* **and** *"h ⊢ f →r x"* **and** *"h ⊢ g x →r y"*
shows *"h ⊢ f ⤜ g →r y"*
⟨*proof*⟩

lemma *bind_pure_returns_result_I2 [intro]:*
assumes *"pure f h"* **and** *"h ⊢ ok f"* **and** *"⋀x. h ⊢ f →r x ⟹ h ⊢ g x →r y"*
shows *"h ⊢ f ⤜ g →r y"*
⟨*proof*⟩

lemma *bind_returns_heap_I [intro]:*
assumes *"h ⊢ f →r x"* **and** *"h ⊢ f →h h'"* **and** *"h' ⊢ g x →h h''"*
shows *"h ⊢ f ⤜ g →h h''"*
⟨*proof*⟩

lemma *bind_returns_heap_I2 [intro]:*
assumes *"h ⊢ f →h h'"* **and** *"⋀x. h ⊢ f →r x ⟹ h' ⊢ g x →h h''"*
shows *"h ⊢ f ⤜ g →h h''"*
⟨*proof*⟩

lemma *bind_is_OK_I [intro]:*
assumes *"h ⊢ f →r x"* **and** *"h ⊢ f →h h'"* **and** *"h' ⊢ ok (g x)"*
shows *"h ⊢ ok (f ⤜ g)"*
⟨*proof*⟩

lemma *bind_is_OK_I2 [intro]:*
assumes *"h ⊢ ok f"* **and** *"⋀x h'. h ⊢ f →r x ⟹ h ⊢ f →h h' ⟹ h' ⊢ ok (g x)"*
shows *"h ⊢ ok (f ⤜ g)"*
⟨*proof*⟩

lemma *bind_is_OK_pure_I [intro]:*
assumes *"pure f h"* **and** *"h ⊢ ok f"* **and** *"⋀x. h ⊢ f →r x ⟹ h ⊢ ok (g x)"*
shows *"h ⊢ ok (f ⤜ g)"*
⟨*proof*⟩

lemma *bind_pure_I:*
assumes *"pure f h"* **and** *"⋀x. h ⊢ f →r x ⟹ pure (g x) h"*
shows *"pure (f ⤜ g) h"*
⟨*proof*⟩

lemma *pure_pure:*
assumes *"h ⊢ ok f"* **and** *"pure f h"*
shows *"h ⊢ f →h h"*
⟨*proof*⟩

lemma *bind_returns_error_eq:*
assumes *"h ⊢ f →e e"*
and *"h ⊢ g →e e"*
shows *"h ⊢ f = h ⊢ g"*
⟨*proof*⟩

2.2.5 Map

fun *map_M :: "('x ⇒ ('heap, 'e, 'result) prog) ⇒ 'x list ⇒ ('heap, 'e, 'result list) prog"*
where

```
  "map_M f [] = return []"
| "map_M f (x#xs) = do {
    y ← f x;
    ys ← map_M f xs;
    return (y # ys)
  }"
```

lemma *map_M_ok_I [intro]:*
"(⋀x. x ∈ set xs ⟹ h ⊢ ok (f x)) ⟹ (⋀x. x ∈ set xs ⟹ pure (f x) h) ⟹ h ⊢ ok (map_M f xs)"

⟨*proof*⟩

lemma *map_M_pure_I : "⋀h. (⋀x. x ∈ set xs ⟹ pure (f x) h) ⟹ pure (map_M f xs) h"*
⟨*proof*⟩

lemma *map_M_pure_E :*
assumes *"h ⊢ map_M g xs →r ys"* **and** *"x ∈ set xs"* **and** *"⋀x h. x ∈ set xs ⟹ pure (g x) h"*
obtains *y* **where** *"h ⊢ g x →r y"* **and** *"y ∈ set ys"*
⟨*proof*⟩

lemma *map_M_pure_E2:*
assumes *"h ⊢ map_M g xs →r ys"* **and** *"y ∈ set ys"* **and** *"⋀x h. x ∈ set xs ⟹ pure (g x) h"*
obtains *x* **where** *"h ⊢ g x →r y"* **and** *"x ∈ set xs"*
⟨*proof*⟩

2.2.6 Forall

fun *forall_M :: "('y ⇒ ('heap, 'e, 'result) prog) ⇒ 'y list ⇒ ('heap, 'e, unit) prog"*
where
"forall_M P [] = return ()"
| "forall_M P (x # xs) = do {
P x;
forall_M P xs
}"

lemma *pure_forall_M_I: "(⋀x. x ∈ set xs ⟹ pure (P x) h) ⟹ pure (forall_M P xs) h"*
⟨*proof*⟩

2.2.7 Fold

fun *fold_M :: "('result ⇒ 'y ⇒ ('heap, 'e, 'result) prog) ⇒ 'result ⇒ 'y list*
⇒ ('heap, 'e, 'result) prog"
where
"fold_M f d [] = return d" |
"fold_M f d (x # xs) = do { y ← f d x; fold_M f y xs }"

lemma *fold_M_pure_I : "(⋀d x. pure (f d x) h) ⟹ (⋀d. pure (fold_M f d xs) h)"*
⟨*proof*⟩

2.2.8 Filter

fun *filter_M :: "('x ⇒ ('heap, 'e, bool) prog) ⇒ 'x list ⇒ ('heap, 'e, 'x list) prog"*
where
"filter_M P [] = return []"
| "filter_M P (x#xs) = do {
p ← P x;
ys ← filter_M P xs;
return (if p then x # ys else ys)
}"

lemma *filter_M_pure_I [intro]: "(⋀x. x ∈ set xs ⟹ pure (P x) h) ⟹ pure (filter_M P xs)h"*
⟨*proof*⟩

lemma *filter_M_is_OK_I [intro]:*
"(⋀x. x ∈ set xs ⟹ h ⊢ ok (P x)) ⟹ (⋀x. x ∈ set xs ⟹ pure (P x) h) ⟹ h ⊢ ok (filter_M P xs)"
⟨*proof*⟩

lemma *filter_M_not_more_elements:*
assumes *"h ⊢ filter_M P xs →r ys"* **and** *"⋀x. x ∈ set xs ⟹ pure (P x) h"* **and** *"x ∈ set ys"*
shows *"x ∈ set xs"*
⟨*proof*⟩

lemma *filter_M_in_result_if_ok:*
assumes "h ⊢ filter_M P xs →r ys" **and** "⋀h x. x ∈ set xs ⟹ pure (P x) h" **and** "x ∈ set xs" **and**
"h ⊢ P x →r True"
shows "x ∈ set ys"
⟨*proof*⟩

lemma *filter_M_holds_for_result:*
assumes "h ⊢ filter_M P xs →r ys" **and** "x ∈ set ys" **and** "⋀x h. x ∈ set xs ⟹ pure (P x) h"
shows "h ⊢ P x →r True"
⟨*proof*⟩

lemma *filter_M_empty_I:*
assumes "⋀x. pure (P x) h"
and "∀x ∈ set xs. h ⊢ P x →r False"
shows "h ⊢ filter_M P xs →r []"
⟨*proof*⟩

lemma *filter_M_subset_2:* "h ⊢ filter_M P xs →r ys ⟹ h' ⊢ filter_M P xs →r ys'
⟹ (⋀x. pure (P x) h) ⟹ (⋀x. pure (P x) h')
⟹ (∀b. ∀x ∈ set xs. h ⊢ P x →r True ⟶ h' ⊢ P x →r b ⟶ b)
⟹ set ys ⊆ set ys'"
⟨*proof*⟩

lemma *filter_M_subset:* "h ⊢ filter_M P xs →r ys ⟹ set ys ⊆ set xs"
⟨*proof*⟩

lemma *filter_M_distinct:* "h ⊢ filter_M P xs →r ys ⟹ distinct xs ⟹ distinct ys"
⟨*proof*⟩

lemma *filter_M_filter:* "h ⊢ filter_M P xs →r ys ⟹ (⋀x. x ∈ set xs ⟹ pure (P x) h)
⟹ (∀x ∈ set xs. h ⊢ ok P x) ∧ ys = filter (λx. |h ⊢ P x|r) xs"
⟨*proof*⟩

lemma *filter_M_filter2:* "(⋀x. x ∈ set xs ⟹ pure (P x) h ∧ h ⊢ ok P x)
⟹ filter (λx. |h ⊢ P x|r) xs = ys ⟹ h ⊢ filter_M P xs →r ys"
⟨*proof*⟩

lemma *filter_ex1:* "∃!x ∈ set xs. P x ⟹ P x ⟹ x ∈ set xs ⟹ distinct xs
⟹ filter P xs = [x]"
⟨*proof*⟩

lemma *filter_M_ex1:*
assumes "h ⊢ filter_M P xs →r ys"
and "x ∈ set xs"
and "∃!x ∈ set xs. h ⊢ P x →r True"
and "⋀x. x ∈ set xs ⟹ pure (P x) h"
and "distinct xs"
and "h ⊢ P x →r True"
shows "ys = [x]"
⟨*proof*⟩

lemma *filter_M_eq:*
assumes "⋀x. pure (P x) h" **and** "⋀x. pure (P x) h'"
and "⋀b x. x ∈ set xs ⟹ h ⊢ P x →r b = h' ⊢ P x →r b"
shows "h ⊢ filter_M P xs →r ys ⟷ h' ⊢ filter_M P xs →r ys"
⟨*proof*⟩

2.2.9 Map Filter

definition *map_filter_M :: "('x ⇒ ('heap, 'e, 'y option) prog) ⇒ 'x list*
⇒ ('heap, 'e, 'y list) prog"
where
"map_filter_M f xs = do {

```
    ys_opts ← map_M f xs;
    ys_no_opts ← filter_M (λx. return (x ≠ None)) ys_opts;
    map_M (λx. return (the x)) ys_no_opts
  }"
```

lemma *map_filter_M_pure:* `"(⋀x h. x ∈ set xs ⟹ pure (f x) h) ⟹ pure (map_filter_M f xs) h"`
⟨*proof*⟩

lemma *map_filter_M_pure_E:*
assumes `"h ⊢ (map_filter_M::('x ⇒ ('heap, 'e, 'y option) prog) ⇒ 'x list ⇒ ('heap, 'e, 'y list) prog) f xs →r ys"` **and** `"y ∈ set ys"` **and** `"⋀x h. x ∈ set xs ⟹ pure (f x) h"`
obtains *x* **where** `"h ⊢ f x →r Some y"` **and** `"x ∈ set xs"`
⟨*proof*⟩

2.2.10 Iterate

fun *iterate_M* `:: "('heap, 'e, 'result) prog list ⇒ ('heap, 'e, 'result) prog"`
where
`"iterate_M [] = return undefined"`
`| "iterate_M (x # xs) = x ⤜ (λ_. iterate_M xs)"`

lemma *iterate_M_concat:*
assumes `"h ⊢ iterate_M xs →h h'"`
and `"h' ⊢ iterate_M ys →h h''"`
shows `"h ⊢ iterate_M (xs @ ys) →h h''"`
⟨*proof*⟩

2.2.11 Miscellaneous Rules

lemma *execute_bind_simp:*
assumes `"h ⊢ f →r x"` **and** `"h ⊢ f →h h'"`
shows `"h ⊢ f ⤜ g = h' ⊢ g x"`
⟨*proof*⟩

lemma *bind_cong [fundef_cong]:*
fixes *f1 f2* `:: "('heap, 'e, 'result) prog"`
and *g1 g2* `:: "'result ⇒ ('heap, 'e, 'result2) prog"`
assumes `"h ⊢ f1 = h ⊢ f2"`
and `"⋀y h'. h ⊢ f1 →r y ⟹ h ⊢ f1 →h h' ⟹ h' ⊢ g1 y = h' ⊢ g2 y"`
shows `"h ⊢ (f1 ⤜ g1) = h ⊢ (f2 ⤜ g2)"`
⟨*proof*⟩

lemma *bind_cong_2:*
assumes `"pure f h"` **and** `"pure f h'"`
and `"⋀x. h ⊢ f →r x = h' ⊢ f →r x"`
and `"⋀x. h ⊢ f →r x ⟹ h ⊢ g x →r y = h' ⊢ g x →r y'"`
shows `"h ⊢ f ⤜ g →r y = h' ⊢ f ⤜ g →r y'"`
⟨*proof*⟩

lemma *bind_case_cong [fundef_cong]:*
assumes `"x = x'"` **and** `"⋀a. x = Some a ⟹ f a h = f' a h"`
shows `"(case x of Some a ⇒ f a | None ⇒ g) h = (case x' of Some a ⇒ f' a | None ⇒ g) h"`
⟨*proof*⟩

2.2.12 Reasoning About Reads and Writes

definition *preserved* `:: "('heap, 'e, 'result) prog ⇒ 'heap ⇒ 'heap ⇒ bool"`
where
`"preserved f h h' ⟷ (∀x. h ⊢ f →r x ⟷ h' ⊢ f →r x)"`

lemma *preserved_code [code]:*
`"preserved f h h' = (((h ⊢ ok f) ∧ (h' ⊢ ok f) ∧ |h ⊢ f|r = |h' ⊢ f|r) ∨ ((¬h ⊢ ok f) ∧ (¬h' ⊢ ok f)))"`

⟨*proof*⟩

lemma *reflp_preserved_f [simp]: "reflp (preserved f)"*
⟨*proof*⟩
lemma *transp_preserved_f [simp]: "transp (preserved f)"*
⟨*proof*⟩

definition
all_args :: "('a ⇒ ('heap, 'e, 'result) prog) ⇒ ('heap, 'e, 'result) prog set"
where
"all_args f = (⋃arg. {f arg})"

definition
reads :: "('heap ⇒ 'heap ⇒ bool) set ⇒ ('heap, 'e, 'result) prog ⇒ 'heap
⇒ 'heap ⇒ bool"
where
"reads S getter h h' ⟷ (∀P ∈ S. reflp P ∧ transp P) ∧ ((∀P ∈ S. P h h')
⟶ preserved getter h h')"

lemma *reads_singleton [simp]: "reads {preserved f} f h h'"*
⟨*proof*⟩

lemma *reads_bind_pure:*
assumes *"pure f h"* **and** *"pure f h'"*
and *"reads S f h h'"*
and *"⋀x. h ⊢ f →ᵣ x ⟹ reads S (g x) h h'"*
shows *"reads S (f ⤜ g) h h'"*
⟨*proof*⟩

lemma *reads_insert_writes_set_left:*
"∀P ∈ S. reflp P ∧ transp P ⟹ reads {getter} f h h' ⟹ reads (insert getter S) f h h'"
⟨*proof*⟩

lemma *reads_insert_writes_set_right:*
"reflp getter ⟹ transp getter ⟹ reads S f h h' ⟹ reads (insert getter S) f h h'"
⟨*proof*⟩

lemma *reads_subset:*
"reads S f h h' ⟹ ∀P ∈ S' - S. reflp P ∧ transp P ⟹ S ⊆ S' ⟹ reads S' f h h'"
⟨*proof*⟩

lemma *return_reads [simp]: "reads {} (return x) h h'"*
⟨*proof*⟩

lemma *error_reads [simp]: "reads {} (error e) h h'"*
⟨*proof*⟩

lemma *noop_reads [simp]: "reads {} noop h h'"*
⟨*proof*⟩

lemma *filter_M_reads:*
assumes *"⋀x. x ∈ set xs ⟹ pure (P x) h"* **and** *"⋀x. x ∈ set xs ⟹ pure (P x) h'"*
and *"⋀x. x ∈ set xs ⟹ reads S (P x) h h'"*
and *"∀P ∈ S. reflp P ∧ transp P"*
shows *"reads S (filter_M P xs) h h'"*
⟨*proof*⟩

definition *writes ::*
"('heap, 'e, 'result) prog set ⇒ ('heap, 'e, 'result2) prog ⇒ 'heap ⇒ 'heap ⇒ bool"
where
"writes S setter h h'

```
    ⟷ (h ⊢ setter →h h' ⟶ (∃progs. set progs ⊆ S ∧ h ⊢ iterate_M progs →h h'))"
```

lemma *writes_singleton [simp]: "writes (all_args f) (f a) h h'"*
⟨*proof*⟩

lemma *writes_singleton2 [simp]: "writes {f} f h h'"*
⟨*proof*⟩

lemma *writes_union_left_I:*
assumes *"writes S f h h'"*
shows *"writes (S ∪ S') f h h'"*
⟨*proof*⟩

lemma *writes_union_right_I:*
assumes *"writes S' f h h'"*
shows *"writes (S ∪ S') f h h'"*
⟨*proof*⟩

lemma *writes_union_minus_split:*
assumes *"writes (S - S2) f h h'"*
and *"writes (S' - S2) f h h'"*
shows *"writes ((S ∪ S') - S2) f h h'"*
⟨*proof*⟩

lemma *writes_subset: "writes S f h h' ⟹ S ⊆ S' ⟹ writes S' f h h'"*
⟨*proof*⟩

lemma *writes_error [simp]: "writes S (error e) h h'"*
⟨*proof*⟩

lemma *writes_not_ok [simp]: "¬h ⊢ ok f ⟹ writes S f h h'"*
⟨*proof*⟩

lemma *writes_pure [simp]:*
assumes *"pure f h"*
shows *"writes S f h h'"*
⟨*proof*⟩

lemma *writes_bind:*
assumes *"⋀h2. writes S f h h2"*
assumes *"⋀x h2. h ⊢ f →r x ⟹ h ⊢ f →h h2 ⟹ writes S (g x) h2 h'"*
shows *"writes S (f ⤜ g) h h'"*
⟨*proof*⟩

lemma *writes_bind_pure:*
assumes *"pure f h"*
assumes *"⋀x. h ⊢ f →r x ⟹ writes S (g x) h h'"*
shows *"writes S (f ⤜ g) h h'"*
⟨*proof*⟩

lemma *writes_small_big:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →h h'"*
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →h h' ⟹ P h h'"*
assumes *"reflp P"*
assumes *"transp P"*
shows *"P h h'"*
⟨*proof*⟩

lemma *reads_writes_preserved:*
assumes *"reads SR getter h h'"*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →h h'"*

assumes "⋀h h'. ∀w ∈ SW. h ⊢ w →$_h$ h' ⟶ (∀r ∈ SR. r h h')"
shows "h ⊢ getter →$_r$ x ⟷ h' ⊢ getter →$_r$ x"
⟨*proof*⟩

lemma *reads_writes_separate_forwards:*
assumes "reads SR getter h h'"
assumes "writes SW setter h h'"
assumes "h ⊢ setter →$_h$ h'"
assumes "h ⊢ getter →$_r$ x"
assumes "⋀h h'. ∀w ∈ SW. h ⊢ w →$_h$ h' ⟶ (∀r ∈ SR. r h h')"
shows "h' ⊢ getter →$_r$ x"
⟨*proof*⟩

lemma *reads_writes_separate_backwards:*
assumes "reads SR getter h h'"
assumes "writes SW setter h h'"
assumes "h ⊢ setter →$_h$ h'"
assumes "h' ⊢ getter →$_r$ x"
assumes "⋀h h'. ∀w ∈ SW. h ⊢ w →$_h$ h' ⟶ (∀r ∈ SR. r h h')"
shows "h ⊢ getter →$_r$ x"
⟨*proof*⟩

end

3 References and Pointers

In this chapter, we introduce a generic type for object-oriented references and typed pointers for each class type defined in the DOM standard.

3.1 References (Ref)

This theory, we introduce a generic reference. All our typed pointers include such a reference, which allows us to distinguish pointers of the same type, but also to iterate over all pointers in a set.

```
theory
  Ref
  imports
    "../preliminaries/Hiding_Type_Variables"
begin

instantiation sum :: (linorder, linorder) linorder
begin
definition less_eq_sum :: "'a + 'b ⇒ 'a + 'b ⇒ bool"
  where
    "less_eq_sum t t' = (case t of
      Inl l ⇒ (case t' of
        Inl l' ⇒ l ≤ l'
      | Inr r' ⇒ True)
    | Inr r ⇒ (case t' of
        Inl l' ⇒ False
      | Inr r' ⇒ r ≤ r'))"
definition less_sum :: "'a + 'b ⇒ 'a + 'b ⇒ bool"
  where
    "less_sum t t' ≡ t ≤ t' ∧ ¬ t' ≤ t"
instance ⟨proof⟩
end

type_synonym ref = nat
consts cast :: 'a

end
```

3.2 Object (ObjectPointer)

In this theory, we introduce the typed pointer for the class Object. This class is the common superclass of our class model.

```
theory ObjectPointer
  imports
    Ref
begin

datatype 'object_ptr object_ptr = Ext 'object_ptr
register_default_tvars "'object_ptr object_ptr"

instantiation object_ptr :: (linorder) linorder
begin
definition less_eq_object_ptr :: "'object_ptr::linorder object_ptr ⇒ 'object_ptr object_ptr ⇒ bool"
  where "less_eq_object_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j))"
definition less_object_ptr :: "'object_ptr::linorder object_ptr ⇒ 'object_ptr object_ptr ⇒ bool"
```

where *"less_object_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"*
instance ⟨*proof*⟩
end

end

3.3 Node (NodePointer)

In this theory, we introduce the typed pointers for the class Node.

theory *NodePointer*
imports
ObjectPointer
begin

datatype *'node_ptr node_ptr = Ext 'node_ptr*
register_default_tvars *"'node_ptr node_ptr"*

type_synonym *('object_ptr, 'node_ptr) object_ptr = "('node_ptr node_ptr + 'object_ptr) object_ptr"*
register_default_tvars *"('object_ptr, 'node_ptr) object_ptr"*

definition $cast_{node_ptr2object_ptr}$ *:: "(_) node_ptr ⇒ (_) object_ptr"*
where
"$cast_{node_ptr2object_ptr}$ *ptr = object_ptr.Ext (Inl ptr)"*

definition $cast_{object_ptr2node_ptr}$ *:: "(_) object_ptr ⇒ (_) node_ptr option"*
where
"$cast_{object_ptr2node_ptr}$ *object_ptr = (case object_ptr of object_ptr.Ext (Inl node_ptr)*
⇒ Some node_ptr | _ ⇒ None)"

adhoc_overloading *cast* ⇌ $cast_{node_ptr2object_ptr}$ $cast_{object_ptr2node_ptr}$

definition *is_node_ptr_kind :: "(_) object_ptr ⇒ bool"*
where
"is_node_ptr_kind ptr = ($cast_{object_ptr2node_ptr}$ *ptr ≠ None)"*

instantiation *node_ptr :: (linorder) linorder*
begin
definition *less_eq_node_ptr :: "(_::linorder) node_ptr ⇒ (_) node_ptr ⇒ bool"*
where *"less_eq_node_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j))"*
definition *less_node_ptr :: "(_::linorder) node_ptr ⇒ (_) node_ptr ⇒ bool"*
where *"less_node_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"*
instance
⟨*proof*⟩
end

lemma *node_ptr_casts_commute [simp]:*
"$cast_{object_ptr2node_ptr}$ *ptr = Some node_ptr ⟷* $cast_{node_ptr2object_ptr}$ *node_ptr = ptr"*
⟨*proof*⟩

lemma *node_ptr_casts_commute2 [simp]:*
"$cast_{object_ptr2node_ptr}$ *(*$cast_{node_ptr2object_ptr}$ *node_ptr) = Some node_ptr"*
⟨*proof*⟩

lemma *node_ptr_casts_commute3 [simp]:*
assumes *"is_node_ptr_kind ptr"*
shows *"*$cast_{node_ptr2object_ptr}$ *(the (*$cast_{object_ptr2node_ptr}$ *ptr)) = ptr"*
⟨*proof*⟩

lemma *is_node_ptr_kind_obtains:*
assumes *"is_node_ptr_kind ptr"*
obtains *node_ptr* **where** *"*$cast_{object_ptr2node_ptr}$ *ptr = Some node_ptr"*
⟨*proof*⟩

lemma *is_node_ptr_kind_none:*
assumes *"¬is_node_ptr_kind ptr"*
shows *"cast$_{object_ptr2node_ptr}$ ptr = None"*
⟨*proof*⟩

lemma *is_node_ptr_kind_cast [simp]: "is_node_ptr_kind (cast$_{node_ptr2object_ptr}$ node_ptr)"*
⟨*proof*⟩

lemma *cast$_{node_ptr2object_ptr}$_inject [simp]:*
"cast$_{node_ptr2object_ptr}$ x = cast$_{node_ptr2object_ptr}$ y ⟷ x = y"
⟨*proof*⟩

lemma *cast$_{object_ptr2node_ptr}$_ext_none [simp]:*
"cast$_{object_ptr2node_ptr}$ (object_ptr.Ext (Inr (Inr (Inr object_ext_ptr)))) = None"
⟨*proof*⟩

lemma *node_ptr_inclusion [simp]:*
"cast$_{node_ptr2object_ptr}$ node_ptr ∈ cast$_{node_ptr2object_ptr}$ ‘ node_ptrs ⟷ node_ptr ∈ node_ptrs"
⟨*proof*⟩
end

3.4 Element (ElementPointer)

In this theory, we introduce the typed pointers for the class Element.

theory *ElementPointer*
imports
NodePointer
begin

datatype *'element_ptr element_ptr = Ref (the_ref: ref) | Ext 'element_ptr*
register_default_tvars *"'element_ptr element_ptr"*

type_synonym *('node_ptr, 'element_ptr) node_ptr*
= "('element_ptr element_ptr + 'node_ptr) node_ptr"
register_default_tvars *"('node_ptr, 'element_ptr) node_ptr"*
type_synonym *('object_ptr, 'node_ptr, 'element_ptr) object_ptr*
= "('object_ptr, 'element_ptr element_ptr + 'node_ptr) object_ptr"
register_default_tvars *"('object_ptr, 'node_ptr, 'element_ptr) object_ptr"*

definition *cast$_{element_ptr2element_ptr}$ :: "(_) element_ptr ⇒ (_) element_ptr"*
where
"cast$_{element_ptr2element_ptr}$ = id"

definition *cast$_{element_ptr2node_ptr}$ :: "(_) element_ptr ⇒ (_) node_ptr"*
where
"cast$_{element_ptr2node_ptr}$ ptr = node_ptr.Ext (Inl ptr)"

abbreviation *cast$_{element_ptr2object_ptr}$ :: "(_) element_ptr ⇒ (_) object_ptr"*
where
"cast$_{element_ptr2object_ptr}$ ptr ≡ cast$_{node_ptr2object_ptr}$ (cast$_{element_ptr2node_ptr}$ ptr)"

definition *cast$_{node_ptr2element_ptr}$ :: "(_) node_ptr ⇒ (_) element_ptr option"*
where
"cast$_{node_ptr2element_ptr}$ node_ptr = (case node_ptr of node_ptr.Ext (Inl element_ptr)
⇒ Some element_ptr | _ ⇒ None)"

abbreviation *cast$_{object_ptr2element_ptr}$ :: "(_) object_ptr ⇒ (_) element_ptr option"*
where
"cast$_{object_ptr2element_ptr}$ ptr ≡ (case cast$_{object_ptr2node_ptr}$ ptr of
Some node_ptr ⇒ cast$_{node_ptr2element_ptr}$ node_ptr

```
                                  | None ⇒ None)"

adhoc_overloading cast ⇌ cast_element_ptr2node_ptr cast_element_ptr2object_ptr
  cast_node_ptr2element_ptr cast_object_ptr2element_ptr cast_element_ptr2element_ptr

consts is_element_ptr_kind :: 'a
definition is_element_ptr_kind_node_ptr :: "(_) node_ptr ⇒ bool"
  where
    "is_element_ptr_kind_node_ptr ptr = (case cast_node_ptr2element_ptr ptr of Some _ ⇒ True | _ ⇒ False)"

abbreviation is_element_ptr_kind_object_ptr :: "(_) object_ptr ⇒ bool"
  where
    "is_element_ptr_kind_object_ptr ptr ≡ (case cast ptr of
                                             Some node_ptr ⇒ is_element_ptr_kind_node_ptr node_ptr
                                           | None ⇒ False)"

adhoc_overloading is_element_ptr_kind ⇌ is_element_ptr_kind_object_ptr is_element_ptr_kind_node_ptr
lemmas is_element_ptr_kind_def = is_element_ptr_kind_node_ptr_def

consts is_element_ptr :: 'a
definition is_element_ptr_element_ptr :: "(_) element_ptr ⇒ bool"
  where
    "is_element_ptr_element_ptr ptr = (case ptr of element_ptr.Ref _ ⇒ True | _ ⇒ False)"

abbreviation is_element_ptr_node_ptr :: "(_) node_ptr ⇒ bool"
  where
    "is_element_ptr_node_ptr ptr ≡ (case cast ptr of
                                      Some element_ptr ⇒ is_element_ptr_element_ptr element_ptr
                                    | _ ⇒ False)"

abbreviation is_element_ptr_object_ptr :: "(_) object_ptr ⇒ bool"
  where
    "is_element_ptr_object_ptr ptr ≡ (case cast ptr of
                                        Some node_ptr ⇒ is_element_ptr_node_ptr node_ptr
                                      | None ⇒ False)"

adhoc_overloading is_element_ptr ⇌ is_element_ptr_object_ptr is_element_ptr_node_ptr is_element_ptr_element_ptr
lemmas is_element_ptr_def = is_element_ptr_element_ptr_def

consts is_element_ptr_ext :: 'a
abbreviation "is_element_ptr_ext_element_ptr ptr ≡ ¬ is_element_ptr_element_ptr ptr"

abbreviation "is_element_ptr_ext_node_ptr ptr ≡ is_element_ptr_kind ptr ∧ (¬ is_element_ptr_node_ptr ptr)"

abbreviation "is_element_ptr_ext_object_ptr ptr ≡ is_element_ptr_kind ptr ∧ (¬ is_element_ptr_object_ptr ptr)"
adhoc_overloading is_element_ptr_ext ⇌ is_element_ptr_ext_object_ptr is_element_ptr_ext_node_ptr


instantiation element_ptr :: (linorder) linorder
begin
definition
  less_eq_element_ptr :: "(_::linorder) element_ptr ⇒ (_)element_ptr ⇒ bool"
  where
    "less_eq_element_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j | Ref _ ⇒ False)
                                        | Ref i ⇒ (case y of Ext _ ⇒ True | Ref j ⇒ i ≤ j))"
definition
  less_element_ptr :: "(_::linorder) element_ptr ⇒ (_) element_ptr ⇒ bool"
  where "less_element_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"
instance
  ⟨proof⟩
end

lemma is_element_ptr_ref [simp]: "is_element_ptr (element_ptr.Ref n)"
```

⟨*proof*⟩

lemma *element_ptr_casts_commute [simp]:*
"cast$_{node_ptr2element_ptr}$ node_ptr = Some element_ptr ⟷ cast$_{element_ptr2node_ptr}$ element_ptr = node_ptr"
⟨*proof*⟩

lemma *element_ptr_casts_commute2 [simp]:*
"(cast$_{node_ptr2element_ptr}$ (cast$_{element_ptr2node_ptr}$ element_ptr) = Some element_ptr)"
⟨*proof*⟩

lemma *element_ptr_casts_commute3 [simp]:*
assumes *"is_element_ptr_kind$_{node_ptr}$ node_ptr"*
shows *"cast$_{element_ptr2node_ptr}$ (the (cast$_{node_ptr2element_ptr}$ node_ptr)) = node_ptr"*
⟨*proof*⟩

lemma *is_element_ptr_kind_obtains:*
assumes *"is_element_ptr_kind node_ptr"*
obtains *element_ptr* **where** *"node_ptr = cast$_{element_ptr2node_ptr}$ element_ptr"*
⟨*proof*⟩

lemma *is_element_ptr_kind_none:*
assumes *"¬is_element_ptr_kind node_ptr"*
shows *"cast$_{node_ptr2element_ptr}$ node_ptr = None"*
⟨*proof*⟩

lemma *is_element_ptr_kind_cast [simp]:*
"is_element_ptr_kind (cast$_{element_ptr2node_ptr}$ element_ptr)"
⟨*proof*⟩

lemma *cast$_{element_ptr2node_ptr}$_inject [simp]:*
"cast$_{element_ptr2node_ptr}$ x = cast$_{element_ptr2node_ptr}$ y ⟷ x = y"
⟨*proof*⟩

lemma *cast$_{node_ptr2element_ptr}$_ext_none [simp]:*
"cast$_{node_ptr2element_ptr}$ (node_ptr.Ext (Inr (Inr node_ext_ptr))) = None"
⟨*proof*⟩

lemma *is_element_ptr_implies_kind [dest]: "is_element_ptr$_{node_ptr}$ ptr ⟹ is_element_ptr_kind$_{node_ptr}$ ptr"*
⟨*proof*⟩

end

3.5 CharacterData (CharacterDataPointer)

In this theory, we introduce the typed pointers for the class CharacterData.

theory *CharacterDataPointer*
imports
ElementPointer
begin

datatype *'character_data_ptr character_data_ptr = Ref (the_ref: ref) | Ext 'character_data_ptr*
register_default_tvars *"'character_data_ptr character_data_ptr"*
type_synonym *('node_ptr, 'element_ptr, 'character_data_ptr) node_ptr*
= "('character_data_ptr character_data_ptr + 'node_ptr, 'element_ptr) node_ptr"
register_default_tvars *"('node_ptr, 'element_ptr, 'character_data_ptr) node_ptr"*
type_synonym *('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr) object_ptr*
= "('object_ptr, 'character_data_ptr character_data_ptr + 'node_ptr, 'element_ptr) object_ptr"
register_default_tvars *"('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr) object_ptr"*

definition *cast$_{character_data_ptr2node_ptr}$ :: "(_) character_data_ptr ⇒ (_) node_ptr"*
where
"cast$_{character_data_ptr2node_ptr}$ ptr = node_ptr.Ext (Inr (Inl ptr))"

```
abbreviation cast_character_data_ptr2object_ptr :: "(_) character_data_ptr ⇒ (_) object_ptr"
  where
    "cast_character_data_ptr2object_ptr ptr ≡ cast_node_ptr2object_ptr (cast_character_data_ptr2node_ptr ptr)"

definition cast_node_ptr2character_data_ptr :: "(_) node_ptr ⇒ (_) character_data_ptr option"
  where
    "cast_node_ptr2character_data_ptr node_ptr = (case node_ptr of
                      node_ptr.Ext (Inr (Inl character_data_ptr)) ⇒ Some character_data_ptr
                    | _ ⇒ None)"

abbreviation cast_object_ptr2character_data_ptr :: "(_) object_ptr ⇒ (_) character_data_ptr option"
  where
    "cast_object_ptr2character_data_ptr ptr ≡ (case cast_object_ptr2node_ptr ptr of
                              Some node_ptr ⇒ cast_node_ptr2character_data_ptr node_ptr
                            | None ⇒ None)"

adhoc_overloading cast ⇌ cast_character_data_ptr2node_ptr cast_character_data_ptr2object_ptr
  cast_node_ptr2character_data_ptr cast_object_ptr2character_data_ptr

consts is_character_data_ptr_kind :: 'a
definition is_character_data_ptr_kind_node_ptr :: "(_) node_ptr ⇒ bool"
  where
    "is_character_data_ptr_kind_node_ptr ptr = (case cast_node_ptr2character_data_ptr ptr
                                          of Some _ ⇒ True | _ ⇒ False)"

abbreviation is_character_data_ptr_kind_object_ptr :: "(_) object_ptr ⇒ bool"
  where
    "is_character_data_ptr_kind_object_ptr ptr ≡ (case cast ptr of
                      Some node_ptr ⇒ is_character_data_ptr_kind_node_ptr node_ptr
                    | None ⇒ False)"

adhoc_overloading is_character_data_ptr_kind ⇌ is_character_data_ptr_kind_object_ptr
  is_character_data_ptr_kind_node_ptr
lemmas is_character_data_ptr_kind_def = is_character_data_ptr_kind_node_ptr_def

consts is_character_data_ptr :: 'a
definition is_character_data_ptr_character_data_ptr :: "(_) character_data_ptr ⇒ bool"
  where
    "is_character_data_ptr_character_data_ptr ptr = (case ptr
                                of character_data_ptr.Ref _ ⇒ True | _ ⇒ False)"

abbreviation is_character_data_ptr_node_ptr :: "(_) node_ptr ⇒ bool"
  where
    "is_character_data_ptr_node_ptr ptr ≡ (case cast ptr of
      Some character_data_ptr ⇒ is_character_data_ptr_character_data_ptr character_data_ptr
    | _ ⇒ False)"

abbreviation is_character_data_ptr_object_ptr :: "(_) object_ptr ⇒ bool"
  where
    "is_character_data_ptr_object_ptr ptr ≡ (case cast_object_ptr2node_ptr ptr of
      Some node_ptr ⇒ is_character_data_ptr_node_ptr node_ptr
    | None ⇒ False)"

adhoc_overloading is_character_data_ptr ⇌
  is_character_data_ptr_object_ptr is_character_data_ptr_node_ptr is_character_data_ptr_character_data_ptr
lemmas is_character_data_ptr_def = is_character_data_ptr_character_data_ptr_def

consts is_character_data_ptr_ext :: 'a
abbreviation
  "is_character_data_ptr_ext_character_data_ptr ptr ≡ ¬ is_character_data_ptr_character_data_ptr ptr"

abbreviation "is_character_data_ptr_ext_node_ptr ptr ≡ (case cast_node_ptr2character_data_ptr ptr of
```

```
  Some character_data_ptr ⇒ is_character_data_ptr_ext_character_data_ptr character_data_ptr
| None ⇒ False)"

abbreviation "is_character_data_ptr_ext_object_ptr ptr ≡ (case cast_object_ptr2node_ptr ptr of
  Some node_ptr ⇒ is_character_data_ptr_ext_node_ptr node_ptr
| None ⇒ False)"

adhoc_overloading is_character_data_ptr_ext ⇌
  is_character_data_ptr_ext_object_ptr is_character_data_ptr_ext_node_ptr is_character_data_ptr_ext_character_data_ptr

instantiation character_data_ptr :: (linorder) linorder
begin
definition
  less_eq_character_data_ptr :: "(_::linorder) character_data_ptr ⇒ (_) character_data_ptr ⇒ bool"
  where
    "less_eq_character_data_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j | Ref _ ⇒ False)
                                               | Ref i ⇒ (case y of Ext _ ⇒ True | Ref j ⇒ i ≤ j))"
definition
  less_character_data_ptr :: "(_::linorder) character_data_ptr ⇒ (_) character_data_ptr ⇒ bool"
  where "less_character_data_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"
instance
  ⟨proof⟩
end

lemma is_character_data_ptr_ref [simp]: "is_character_data_ptr (character_data_ptr.Ref n)"
  ⟨proof⟩

lemma cast_element_ptr_not_character_data_ptr [simp]:
  "(cast_element_ptr2node_ptr element_ptr ≠ cast_character_data_ptr2node_ptr character_data_ptr)"
  "(cast_character_data_ptr2node_ptr character_data_ptr ≠ cast_element_ptr2node_ptr element_ptr)"
  ⟨proof⟩

lemma is_character_data_ptr_kind_not_element_ptr [simp]:
  "¬ is_character_data_ptr_kind (cast_element_ptr2node_ptr element_ptr)"
  ⟨proof⟩
lemma is_element_ptr_kind_not_character_data_ptr [simp]:
  "¬ is_element_ptr_kind (cast_character_data_ptr2node_ptr character_data_ptr)"
  ⟨proof⟩

lemma is_character_data_ptr_kind_cast [simp]:
  "is_character_data_ptr_kind (cast_character_data_ptr2node_ptr character_data_ptr)"
  ⟨proof⟩

lemma character_data_ptr_casts_commute [simp]:
  "cast_node_ptr2character_data_ptr node_ptr = Some character_data_ptr
   ⟷ cast_character_data_ptr2node_ptr character_data_ptr = node_ptr"
  ⟨proof⟩

lemma character_data_ptr_casts_commute2 [simp]:
  "(cast_node_ptr2character_data_ptr (cast_character_data_ptr2node_ptr character_data_ptr) = Some character_data_ptr)"
  ⟨proof⟩

lemma character_data_ptr_casts_commute3 [simp]:
  assumes "is_character_data_ptr_kind_node_ptr node_ptr"
  shows "cast_character_data_ptr2node_ptr (the (cast_node_ptr2character_data_ptr node_ptr)) = node_ptr"
  ⟨proof⟩

lemma is_character_data_ptr_kind_obtains:
  assumes "is_character_data_ptr_kind_node_ptr node_ptr"
  obtains character_data_ptr where "cast_character_data_ptr2node_ptr character_data_ptr = node_ptr"
  ⟨proof⟩

lemma is_character_data_ptr_kind_none:
```

assumes "¬*is_character_data_ptr_kind*$_{node_ptr}$ *node_ptr*"
shows "*cast*$_{node_ptr2character_data_ptr}$ *node_ptr* = *None*"
⟨*proof*⟩

lemma *cast*$_{character_data_ptr2node_ptr}$*_inject [simp]:*
"*cast*$_{character_data_ptr2node_ptr}$ *x* = *cast*$_{character_data_ptr2node_ptr}$ *y* ⟷ *x* = *y*"
⟨*proof*⟩

lemma *cast*$_{node_ptr2character_data_ptr}$*_ext_none [simp]:*
"*cast*$_{node_ptr2character_data_ptr}$ *(node_ptr.Ext (Inr (Inr node_ext_ptr))) = None*"
⟨*proof*⟩

end

3.6 Document (DocumentPointer)

In this theory, we introduce the typed pointers for the class Document.

theory *DocumentPointer*
imports
CharacterDataPointer
begin

datatype *'document_ptr document_ptr = Ref (the_ref: ref) | Ext 'document_ptr*
register_default_tvars *"'document_ptr document_ptr"*
type_synonym *('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr) object_ptr*
= "('document_ptr document_ptr + 'object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr) object_ptr"
register_default_tvars *"('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr) object_ptr"*

definition *cast*$_{document_ptr2object_ptr}$ *:: "(_)document_ptr ⇒ (_) object_ptr"*
where
"cast$_{document_ptr2object_ptr}$ *ptr = object_ptr.Ext (Inr (Inl ptr))"*

definition *cast*$_{object_ptr2document_ptr}$ *:: "(_) object_ptr ⇒ (_) document_ptr option"*
where
"cast$_{object_ptr2document_ptr}$ *ptr = (case ptr of*
object_ptr.Ext (Inr (Inl document_ptr)) ⇒ Some document_ptr
| _ ⇒ None)"

adhoc_overloading *cast* ⇌ *cast*$_{document_ptr2object_ptr}$ *cast*$_{object_ptr2document_ptr}$

definition *is_document_ptr_kind :: "(_) object_ptr ⇒ bool"*
where
"is_document_ptr_kind ptr = (case cast$_{object_ptr2document_ptr}$ *ptr of*
Some _ ⇒ True | None ⇒ False)"

consts *is_document_ptr :: 'a*
definition *is_document_ptr*$_{document_ptr}$ *:: "(_) document_ptr ⇒ bool"*
where
"is_document_ptr$_{document_ptr}$ *ptr = (case ptr of document_ptr.Ref _ ⇒ True | _ ⇒ False)"*

abbreviation *is_document_ptr*$_{object_ptr}$ *:: "(_) object_ptr ⇒ bool"*
where
"is_document_ptr$_{object_ptr}$ *ptr ≡ (case cast*$_{object_ptr2document_ptr}$ *ptr of*
Some document_ptr ⇒ is_document_ptr$_{document_ptr}$ *document_ptr*
| None ⇒ False)"
adhoc_overloading *is_document_ptr* ⇌ *is_document_ptr*$_{object_ptr}$ *is_document_ptr*$_{document_ptr}$
lemmas *is_document_ptr_def = is_document_ptr*$_{document_ptr}$*_def*

consts *is_document_ptr_ext :: 'a*
abbreviation *"is_document_ptr_ext*$_{document_ptr}$ *ptr ≡ ¬ is_document_ptr*$_{document_ptr}$ *ptr"*

abbreviation "is_document_ptr_ext$_{object_ptr}$ ptr ≡ (case cast$_{object_ptr2document_ptr}$ ptr of
  Some document_ptr ⇒ is_document_ptr_ext$_{document_ptr}$ document_ptr
| None ⇒ False)"
adhoc_overloading is_document_ptr_ext ⇌ is_document_ptr_ext$_{object_ptr}$ is_document_ptr_ext$_{document_ptr}$

instantiation document_ptr :: (linorder) linorder
begin
definition less_eq_document_ptr :: "(_::linorder) document_ptr ⇒ (_) document_ptr ⇒ bool"
  where "less_eq_document_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j | Ref _ ⇒ False)
                                        | Ref i ⇒ (case y of Ext _ ⇒ True | Ref j ⇒ i ≤ j))"
definition less_document_ptr :: "(_::linorder) document_ptr ⇒ (_) document_ptr ⇒ bool"
  where "less_document_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"
instance
  ⟨*proof*⟩
end

lemma is_document_ptr_ref [simp]: "is_document_ptr (document_ptr.Ref n)"
  ⟨*proof*⟩

lemma cast_document_ptr_not_node_ptr [simp]:
  "cast$_{document_ptr2object_ptr}$ document_ptr ≠ cast$_{node_ptr2object_ptr}$ node_ptr"
  "cast$_{node_ptr2object_ptr}$ node_ptr ≠ cast$_{document_ptr2object_ptr}$ document_ptr"
  ⟨*proof*⟩

lemma document_ptr_no_node_ptr_cast [simp]:
  "¬ is_document_ptr_kind (cast$_{node_ptr2object_ptr}$ node_ptr)"
  ⟨*proof*⟩
lemma node_ptr_no_document_ptr_cast [simp]:
  "¬ is_node_ptr_kind (cast$_{document_ptr2object_ptr}$ document_ptr)"
  ⟨*proof*⟩

lemma document_ptr_document_ptr_cast [simp]:
  "is_document_ptr_kind (cast$_{document_ptr2object_ptr}$ document_ptr)"
  ⟨*proof*⟩

lemma document_ptr_casts_commute [simp]:
  "cast$_{object_ptr2document_ptr}$ ptr = Some document_ptr ⟷ cast$_{document_ptr2object_ptr}$ document_ptr = ptr"
  ⟨*proof*⟩

lemma document_ptr_casts_commute2 [simp]:
  "(cast$_{object_ptr2document_ptr}$ (cast$_{document_ptr2object_ptr}$ document_ptr) = Some document_ptr)"
  ⟨*proof*⟩

lemma document_ptr_casts_commute3 [simp]:
  assumes "is_document_ptr_kind ptr"
  shows "cast$_{document_ptr2object_ptr}$ (the (cast$_{object_ptr2document_ptr}$ ptr)) = ptr"
  ⟨*proof*⟩

lemma is_document_ptr_kind_obtains:
  assumes "is_document_ptr_kind ptr"
  obtains document_ptr **where** "ptr = cast$_{document_ptr2object_ptr}$ document_ptr"
  ⟨*proof*⟩

lemma is_document_ptr_kind_none:
  assumes "¬is_document_ptr_kind ptr"
  shows "cast$_{object_ptr2document_ptr}$ ptr = None"
  ⟨*proof*⟩

lemma cast$_{document_ptr2object_ptr}$_inject [simp]:
  "cast$_{document_ptr2object_ptr}$ x = cast$_{document_ptr2object_ptr}$ y ⟷ x = y"
  ⟨*proof*⟩

```
lemma cast_object_ptr2document_ptr_ext_none [simp]:
  "cast_object_ptr2document_ptr (object_ptr.Ext (Inr (Inr (Inr object_ext_ptr)))) = None"
  ⟨proof⟩

lemma is_document_ptr_kind_not_element_ptr_kind [dest]:
  "is_document_ptr_kind ptr ⟹ ¬ is_element_ptr_kind ptr"
  ⟨proof⟩
end
```

3.7 ShadowRoot (ShadowRootPointer)

In this theory, we introduce the typed pointers for the class ShadowRoot. Note that, in this document, we will not make use of ShadowRoots nor will we discuss their particular properties. We only include them here, as they are required for future work and they cannot be added alter following the object-oriented extensibility of our data model.

```
theory ShadowRootPointer
  imports
    "DocumentPointer"
begin

datatype 'shadow_root_ptr shadow_root_ptr = Ref (the_ref: ref) | Ext 'shadow_root_ptr
register_default_tvars "'shadow_root_ptr shadow_root_ptr"
type_synonym ('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr,
    'document_ptr, 'shadow_root_ptr) object_ptr
  = "('shadow_root_ptr shadow_root_ptr + 'object_ptr, 'node_ptr, 'element_ptr,
      'character_data_ptr, 'document_ptr) object_ptr"
register_default_tvars "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr,
                         'document_ptr, 'shadow_root_ptr) object_ptr"

definition cast_shadow_root_ptr2shadow_root_ptr :: "(_) shadow_root_ptr ⇒ (_) shadow_root_ptr"
  where
    "cast_shadow_root_ptr2shadow_root_ptr = id"

definition cast_shadow_root_ptr2object_ptr :: "(_)shadow_root_ptr ⇒ (_) object_ptr"
  where
    "cast_shadow_root_ptr2object_ptr ptr = object_ptr.Ext (Inr (Inr (Inl ptr)))"

definition cast_object_ptr2shadow_root_ptr :: "(_) object_ptr ⇒ (_) shadow_root_ptr option"
  where
    "cast_object_ptr2shadow_root_ptr ptr = (case ptr of
              object_ptr.Ext (Inr (Inr (Inl shadow_root_ptr))) ⇒ Some shadow_root_ptr
              | _ ⇒ None)"

adhoc_overloading cast ⇌ cast_shadow_root_ptr2object_ptr cast_object_ptr2shadow_root_ptr cast_shadow_root_ptr2shadow_root_ptr


definition is_shadow_root_ptr_kind :: "(_) object_ptr ⇒ bool"
  where
    "is_shadow_root_ptr_kind ptr = (case cast_object_ptr2shadow_root_ptr ptr of Some _ ⇒ True
                                                                              | None ⇒ False)"

consts is_shadow_root_ptr :: 'a
definition is_shadow_root_ptr_shadow_root_ptr :: "(_) shadow_root_ptr ⇒ bool"
  where
    "is_shadow_root_ptr_shadow_root_ptr ptr = (case ptr of shadow_root_ptr.Ref _ ⇒ True
                                                         | _ ⇒ False)"

abbreviation is_shadow_root_ptr_object_ptr :: "(_) object_ptr ⇒ bool"
  where
    "is_shadow_root_ptr_object_ptr ptr ≡ (case cast_object_ptr2shadow_root_ptr ptr of
      Some shadow_root_ptr ⇒ is_shadow_root_ptr_shadow_root_ptr shadow_root_ptr
```

```
    | None ⇒ False)"
adhoc_overloading is_shadow_root_ptr ⇌ is_shadow_root_ptr_object_ptr is_shadow_root_ptr_shadow_root_ptr
lemmas is_shadow_root_ptr_def = is_shadow_root_ptr_shadow_root_ptr_def

consts is_shadow_root_ptr_ext :: 'a
abbreviation "is_shadow_root_ptr_ext_shadow_root_ptr ptr ≡ ¬ is_shadow_root_ptr_shadow_root_ptr ptr"

abbreviation "is_shadow_root_ptr_ext_object_ptr ptr ≡ (case cast_object_ptr2shadow_root_ptr ptr of
  Some shadow_root_ptr ⇒ is_shadow_root_ptr_ext_shadow_root_ptr shadow_root_ptr
| None ⇒ False)"
adhoc_overloading is_shadow_root_ptr_ext ⇌ is_shadow_root_ptr_ext_object_ptr is_shadow_root_ptr_ext_shadow_root_ptr

instantiation shadow_root_ptr :: (linorder) linorder
begin
definition
  less_eq_shadow_root_ptr :: "(_::linorder) shadow_root_ptr ⇒ (_) shadow_root_ptr ⇒ bool"
  where
    "less_eq_shadow_root_ptr x y ≡ (case x of Ext i ⇒ (case y of Ext j ⇒ i ≤ j | Ref _ ⇒ False)
                                            | Ref i ⇒ (case y of Ext _ ⇒ True | Ref j ⇒ i ≤ j))"
definition less_shadow_root_ptr :: "(_::linorder) shadow_root_ptr ⇒ (_) shadow_root_ptr ⇒ bool"
  where "less_shadow_root_ptr x y ≡ x ≤ y ∧ ¬ y ≤ x"
instance
  ⟨proof⟩
end


lemma is_shadow_root_ptr_ref [simp]: "is_shadow_root_ptr (shadow_root_ptr.Ref n)"
  ⟨proof⟩

lemma is_shadow_root_ptr_not_node_ptr[simp]: "¬is_shadow_root_ptr (cast_node_ptr2object_ptr node_ptr)"
  ⟨proof⟩

lemma cast_shadow_root_ptr_not_node_ptr [simp]:
  "cast_shadow_root_ptr2object_ptr shadow_root_ptr ≠ cast_node_ptr2object_ptr node_ptr"
  "cast_node_ptr2object_ptr node_ptr ≠ cast_shadow_root_ptr2object_ptr shadow_root_ptr"
  ⟨proof⟩

lemma cast_shadow_root_ptr_not_document_ptr [simp]:
  "cast_shadow_root_ptr2object_ptr shadow_root_ptr ≠ cast_document_ptr2object_ptr document_ptr"
  "cast_document_ptr2object_ptr document_ptr ≠ cast_shadow_root_ptr2object_ptr shadow_root_ptr"
  ⟨proof⟩

lemma shadow_root_ptr_no_node_ptr_cast [simp]:
  "¬ is_shadow_root_ptr_kind (cast_node_ptr2object_ptr node_ptr)"
  ⟨proof⟩
lemma node_ptr_no_shadow_root_ptr_cast [simp]:
  "¬ is_node_ptr_kind (cast_shadow_root_ptr2object_ptr shadow_root_ptr)"
  ⟨proof⟩

lemma shadow_root_ptr_no_document_ptr_cast [simp]:
  "¬ is_shadow_root_ptr_kind (cast_document_ptr2object_ptr document_ptr)"
  ⟨proof⟩
lemma document_ptr_no_shadow_root_ptr_cast [simp]:
  "¬ is_document_ptr_kind (cast_shadow_root_ptr2object_ptr shadow_root_ptr)"
  ⟨proof⟩

lemma shadow_root_ptr_shadow_root_ptr_cast [simp]:
  "is_shadow_root_ptr_kind (cast_shadow_root_ptr2object_ptr shadow_root_ptr)"
  ⟨proof⟩

lemma shadow_root_ptr_casts_commute [simp]:
  "cast_object_ptr2shadow_root_ptr ptr = Some shadow_root_ptr ⟷ cast_shadow_root_ptr2object_ptr shadow_root_ptr
= ptr"
```

⟨*proof*⟩

lemma *shadow_root_ptr_casts_commute2 [simp]:*
"(cast$_{object_ptr2shadow_root_ptr}$ (cast$_{shadow_root_ptr2object_ptr}$ shadow_root_ptr) = Some shadow_root_ptr)"
⟨*proof*⟩

lemma *shadow_root_ptr_casts_commute3 [simp]:*
assumes *"is_shadow_root_ptr_kind ptr"*
shows *"cast$_{shadow_root_ptr2object_ptr}$ (the (cast$_{object_ptr2shadow_root_ptr}$ ptr)) = ptr"*
⟨*proof*⟩

lemma *is_shadow_root_ptr_kind_obtains:*
assumes *"is_shadow_root_ptr_kind ptr"*
obtains *shadow_root_ptr* **where** *"ptr = cast$_{shadow_root_ptr2object_ptr}$ shadow_root_ptr"*
⟨*proof*⟩

lemma *is_shadow_root_ptr_kind_none:*
assumes *"¬is_shadow_root_ptr_kind ptr"*
shows *"cast$_{object_ptr2shadow_root_ptr}$ ptr = None"*
⟨*proof*⟩

lemma *cast$_{shadow_root_ptr2object_ptr}$_inject [simp]:*
"cast$_{shadow_root_ptr2object_ptr}$ x = cast$_{shadow_root_ptr2object_ptr}$ y ⟷ x = y"
⟨*proof*⟩

lemma *cast$_{object_ptr2shadow_root_ptr}$_ext_none [simp]:*
"cast$_{object_ptr2shadow_root_ptr}$ (object_ptr.Ext (Inr (Inr (Inr object_ext_ptr)))) = None"
⟨*proof*⟩

lemma *is_shadow_root_ptr_kind_simp1 [dest]: "is_document_ptr_kind ptr ⟹ ¬is_shadow_root_ptr_kind ptr"*
⟨*proof*⟩

lemma *is_shadow_root_ptr_kind_simp2 [dest]: "is_node_ptr_kind ptr ⟹ ¬is_shadow_root_ptr_kind ptr"*
⟨*proof*⟩

end

4 Classes

In this chapter, we introduce the classes of our DOM model. The definition of the class types follows closely the one of the pointer types. Instead of datatypes, we use records for our classes. a generic type for object-oriented references and typed pointers for each class type defined in the DOM standard.

4.1 The Class Infrastructure (BaseClass)

In this theory, we introduce the basic infrastructure for our encoding of classes.

```
theory BaseClass
  imports
    "HOL-Library.Finite_Map"
    "../pointers/Ref"
    "../Core_DOM_Basic_Datatypes"
begin

named_theorems instances

consts get :: 'a
consts put :: 'a
consts delete :: 'a
```

Overall, the definition of the class types follows closely the one of the pointer types. Instead of datatypes, we use records for our classes. This allows us to, first, make use of record inheritance, which is, in addition to the type synonyms of previous class types, the second place where the inheritance relationship of our types manifest. Second, we get a convenient notation to define classes, in addition to automatically generated getter and setter functions.

Along with our class types, we also develop our heap type, which is a finite map at its core. It is important to note that while the map stores a mapping from *object_ptr* to *Object*, we restrict the type variables of the record extension slot of *Object* in such a way that allows down-casting, but requires a bit of taking-apart and re-assembling of our records before they are stored in the heap.

Throughout the theory files, we will use underscore case to reference pointer types, and camel case for class types.

Every class type contains at least one attribute; nothing. This is used for two purposes: first, the record package does not allow records without any attributes. Second, we will use the getter of nothing later to check whether a class of the correct type could be retrieved, for which we will be able to use our infrastructure regarding the behaviour of getters across different heaps.

```
locale l_type_wf = fixes type_wf :: "'heap ⇒ bool"

locale l_known_ptr = fixes known_ptr :: "'ptr ⇒ bool"

end
```

4.2 Object (ObjectClass)

In this theory, we introduce the definition of the class Object. This class is the common superclass of our class model.

```
theory ObjectClass
  imports
    BaseClass
    "../pointers/ObjectPointer"
```

begin

record *RObject =*
  nothing :: unit
register_default_tvars *"'Object RObject_ext"*
type_synonym *'Object Object = "'Object RObject_scheme"*
register_default_tvars *"'Object Object"*

datatype *('object_ptr, 'Object) heap = Heap (the_heap: "((_) object_ptr, (_) Object) fmap")*
register_default_tvars *"('object_ptr, 'Object) heap"*
type_synonym $heap_{final}$ *= "(unit, unit) heap"*

definition *object_ptr_kinds :: "(_) heap ⇒ (_) object_ptr fset"*
  where
    "object_ptr_kinds = fmdom ∘ the_heap"

lemma *object_ptr_kinds_simp [simp]:*
  "object_ptr_kinds (Heap (fmupd object_ptr object (the_heap h)))
         = {|object_ptr|} |∪| object_ptr_kinds h"
  ⟨*proof*⟩

definition get_{Object} *:: "(_) object_ptr ⇒ (_) heap ⇒ (_) Object option"*
  where
    "get_{Object} *ptr h = fmlookup (the_heap h) ptr"*
adhoc_overloading *get* ⇌ get_{Object}

locale $l_type_wf_def_{Object}$
begin
definition *a_type_wf :: "(_) heap ⇒ bool"*
  where
    "a_type_wf h = True"
end
global_interpretation $l_type_wf_def_{Object}$ **defines** *type_wf = a_type_wf* ⟨*proof*⟩
lemmas *type_wf_defs = a_type_wf_def*

locale $l_type_wf_{Object}$ *= l_type_wf type_wf* **for** *type_wf :: "((_) heap ⇒ bool)" +*
  assumes $type_wf_{Object}$*: "type_wf h ⟹ ObjectClass.type_wf h"*

locale $l_get_{Object}_lemmas$ *=* $l_type_wf_{Object}$
begin
lemma $get_{Object}_type_wf:$
  assumes *"type_wf h"*
  shows *"object_ptr |∈| object_ptr_kinds h ⟷* get_{Object} *object_ptr h ≠ None"*
  ⟨*proof*⟩
end

global_interpretation $l_get_{Object}_lemmas$ *type_wf*
  ⟨*proof*⟩

definition put_{Object} *:: "(_) object_ptr ⇒ (_) Object ⇒ (_) heap ⇒ (_) heap"*
  where
    "put_{Object} *ptr obj h = Heap (fmupd ptr obj (the_heap h))"*
adhoc_overloading *put* ⇌ put_{Object}

lemma $put_{Object}_ptr_in_heap:$
  assumes *"*put_{Object} *object_ptr object h = h'"*
  shows *"object_ptr |∈| object_ptr_kinds h'"*
  ⟨*proof*⟩

lemma $put_{Object}_put_ptrs:$
  assumes *"*put_{Object} *object_ptr object h = h'"*
  shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|object_ptr|}"*
  ⟨*proof*⟩

lemma *object_more_extend_id [simp]: "more (extend x y) = y"*
⟨*proof*⟩

lemma *object_empty [simp]: "(|nothing = (), ... = more x|) = x"*
⟨*proof*⟩

locale *l_known_ptr*$_{Object}$
begin
definition *a_known_ptr :: "(_) object_ptr ⇒ bool"*
where
"a_known_ptr ptr = False"

lemma *known_ptr_not_object_ptr:*
"a_known_ptr ptr ⟹ ¬is_object_ptr ptr ⟹ known_ptr ptr"
⟨*proof*⟩
end
global_interpretation *l_known_ptr*$_{Object}$ **defines** *known_ptr = a_known_ptr* ⟨*proof*⟩
lemmas *known_ptr_defs = a_known_ptr_def*

locale *l_known_ptrs = l_known_ptr known_ptr* **for** *known_ptr :: "(_) object_ptr ⇒ bool" +*
fixes *known_ptrs :: "(_) heap ⇒ bool"*
assumes *known_ptrs_known_ptr: "known_ptrs h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptr ptr"*
assumes *known_ptrs_preserved:*
"object_ptr_kinds h = object_ptr_kinds h' ⟹ known_ptrs h = known_ptrs h'"
assumes *known_ptrs_subset:*
"object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ known_ptrs h ⟹ known_ptrs h'"
assumes *known_ptrs_new_ptr:*
"object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
known_ptrs h ⟹ known_ptrs h'"

locale *l_known_ptrs*$_{Object}$ *= l_known_ptr known_ptr* **for** *known_ptr :: "(_) object_ptr ⇒ bool"*
begin
definition *a_known_ptrs :: "(_) heap ⇒ bool"*
where
"a_known_ptrs h = (∀ptr ∈ fset (object_ptr_kinds h). known_ptr ptr)"

lemma *known_ptrs_known_ptr:*
"a_known_ptrs h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptr ptr"
⟨*proof*⟩

lemma *known_ptrs_preserved:*
"object_ptr_kinds h = object_ptr_kinds h' ⟹ a_known_ptrs h = a_known_ptrs h'"
⟨*proof*⟩
lemma *known_ptrs_subset:*
"object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
lemma *known_ptrs_new_ptr:*
"object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
end
global_interpretation *l_known_ptrs*$_{Object}$ *known_ptr* **defines** *known_ptrs = a_known_ptrs* ⟨*proof*⟩
lemmas *known_ptrs_defs = a_known_ptrs_def*

lemma *known_ptrs_is_l_known_ptrs: "l_known_ptrs known_ptr known_ptrs"*
⟨*proof*⟩

lemma *get_object_ptr_simp1 [simp]: "get*$_{Object}$ *object_ptr (put*$_{Object}$ *object_ptr object h) = Some object"*
⟨*proof*⟩
lemma *get_object_ptr_simp2 [simp]:*
"object_ptr ≠ object_ptr'

```
  ⟹ get_Object object_ptr (put_Object object_ptr' object h) = get_Object object_ptr h"
⟨proof⟩
```

4.2.1 Limited Heap Modifications

```
definition heap_unchanged_except :: "(_) object_ptr set ⇒ (_) heap ⇒ (_) heap ⇒ bool"
  where
    "heap_unchanged_except S h h' = (∀ptr ∈ (fset (object_ptr_kinds h)
                                  ∪ (fset (object_ptr_kinds h'))) - S. get ptr h = get ptr h')"

definition delete_Object :: "(_) object_ptr ⇒ (_) heap ⇒ (_) heap option" where
  "delete_Object ptr h = (if ptr |∈| object_ptr_kinds h then Some (Heap (fmdrop ptr (the_heap h)))
                                                        else None)"

lemma delete_Object_pointer_removed:
  assumes "delete_Object ptr h = Some h'"
  shows "ptr |∉| object_ptr_kinds h'"
  ⟨proof⟩

lemma delete_Object_pointer_ptr_in_heap:
  assumes "delete_Object ptr h = Some h'"
  shows "ptr |∈| object_ptr_kinds h"
  ⟨proof⟩

lemma delete_Object_ok:
  assumes "ptr |∈| object_ptr_kinds h"
  shows "delete_Object ptr h ≠ None"
  ⟨proof⟩
```

4.2.2 Code Generator Setup

```
definition "create_heap xs = Heap (fmap_of_list xs)"

code_datatype ObjectClass.heap.Heap create_heap

lemma object_ptr_kinds_code3:
  "fmlookup (the_heap (create_heap xs)) x = map_of xs x"
  ⟨proof⟩

lemma object_ptr_kinds_code5 [code]:
  "the_heap (Heap x) = x"
  ⟨proof⟩

lemma object_ptr_kinds_code4 [code]:
  "the_heap (create_heap xs) = fmap_of_list xs"
  ⟨proof⟩

end
```

4.3 Node (NodeClass)

In this theory, we introduce the types for the Node class.

```
theory NodeClass
  imports
    ObjectClass
    "../pointers/NodePointer"
begin
```

Node

```
record RNode = RObject
  + nothing :: unit
```

```
register_default_tvars "'Node RNode_ext"
type_synonym 'Node Node = "'Node RNode_scheme"
register_default_tvars "'Node Node"
type_synonym ('Object, 'Node) Object = "('Node RNode_ext + 'Object) Object"
register_default_tvars "('Object, 'Node) Object"

type_synonym ('object_ptr, 'node_ptr, 'Object, 'Node) heap
  = "('node_ptr node_ptr + 'object_ptr, 'Node RNode_ext + 'Object) heap"
register_default_tvars
  "('object_ptr, 'node_ptr, 'Object, 'Node) heap"
type_synonym heap_final = "(unit, unit, unit, unit) heap"


definition node_ptr_kinds :: "(_) heap ⇒ (_) node_ptr fset"
  where
    "node_ptr_kinds heap =
    (the |`| (cast_object_ptr2node_ptr |`| (ffilter is_node_ptr_kind (object_ptr_kinds heap))))"

lemma node_ptr_kinds_simp [simp]:
  "node_ptr_kinds (Heap (fmupd (cast node_ptr) node (the_heap h)))
      = {|node_ptr|} |∪| node_ptr_kinds h"
  ⟨proof⟩

definition cast_Object2Node :: "(_) Object ⇒ (_) Node option"
  where
    "cast_Object2Node obj = (case RObject.more obj of Inl node
    ⇒ Some (RObject.extend (RObject.truncate obj) node) | _ ⇒ None)"
adhoc_overloading cast ⇌ cast_Object2Node

definition cast_Node2Object:: "(_) Node ⇒ (_) Object"
  where
    "cast_Node2Object node = (RObject.extend (RObject.truncate node) (Inl (RObject.more node)))"
adhoc_overloading cast ⇌ cast_Node2Object

definition is_node_kind :: "(_) Object ⇒ bool"
  where
    "is_node_kind ptr ⟷ cast_Object2Node ptr ≠ None"

definition get_Node :: "(_) node_ptr ⇒ (_) heap ⇒ (_) Node option"
  where
    "get_Node node_ptr h = Option.bind (get (cast node_ptr) h) cast"
adhoc_overloading get ⇌ get_Node

locale l_type_wf_def_Node
begin
definition a_type_wf :: "(_) heap ⇒ bool"
  where
    "a_type_wf h = (ObjectClass.type_wf h
                  ∧ (∀node_ptr ∈ fset( node_ptr_kinds h). get_Node node_ptr h ≠ None))"
end
global_interpretation l_type_wf_def_Node defines type_wf = a_type_wf ⟨proof⟩
lemmas type_wf_defs = a_type_wf_def

locale l_type_wf_Node = l_type_wf type_wf for type_wf :: "((_) heap ⇒ bool)" +
  assumes type_wf_Node: "type_wf h ⟹ NodeClass.type_wf h"

sublocale l_type_wf_Node ⊆ l_type_wf_Object
  ⟨proof⟩

locale l_get_Node_lemmas = l_type_wf_Node
begin
sublocale l_get_Object_lemmas ⟨proof⟩
lemma get_Node_type_wf:
```

assumes *"type_wf h"*
shows *"node_ptr |∈| node_ptr_kinds h ⟷ get$_{Node}$ node_ptr h ≠ None"*
⟨*proof*⟩
end

global_interpretation *l_get$_{Node}$_lemmas type_wf*
⟨*proof*⟩

definition *put$_{Node}$ :: "(_) node_ptr ⇒ (_) Node ⇒ (_) heap ⇒ (_) heap"*
where
"put$_{Node}$ node_ptr node = put (cast node_ptr) (cast node)"
adhoc_overloading *put ⇌ put$_{Node}$*

lemma *put$_{Node}$_ptr_in_heap:*
assumes *"put$_{Node}$ node_ptr node h = h'"*
shows *"node_ptr |∈| node_ptr_kinds h'"*
⟨*proof*⟩

lemma *put$_{Node}$_put_ptrs:*
assumes *"put$_{Node}$ node_ptr node h = h'"*
shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast node_ptr|}"*
⟨*proof*⟩

lemma *node_ptr_kinds_commutes [simp]:*
"cast node_ptr |∈| object_ptr_kinds h ⟷ node_ptr |∈| node_ptr_kinds h"
⟨*proof*⟩

lemma *node_empty [simp]:*
"(|RObject.nothing = (), RNode.nothing = (), ... = RNode.more node|) = node"
⟨*proof*⟩

lemma *cast$_{Node2Object}$_inject [simp]: "cast$_{Node2Object}$ x = cast$_{Node2Object}$ y ⟷ x = y"*
⟨*proof*⟩

lemma *cast$_{Object2Node}$_none [simp]:*
"cast$_{Object2Node}$ obj = None ⟷ ¬ (∃node. cast$_{Node2Object}$ node = obj)"
⟨*proof*⟩

lemma *cast$_{Object2Node}$_some [simp]: "cast$_{Object2Node}$ obj = Some node ⟷ cast node = obj"*
⟨*proof*⟩

lemma *cast$_{Object2Node}$_inv [simp]: "cast$_{Object2Node}$ (cast$_{Node2Object}$ node) = Some node"*
⟨*proof*⟩

locale *l_known_ptr$_{Node}$*
begin
definition *a_known_ptr :: "(_) object_ptr ⇒ bool"*
where
"a_known_ptr ptr = False"
end
global_interpretation *l_known_ptr$_{Node}$* **defines** *known_ptr = a_known_ptr* ⟨*proof*⟩
lemmas *known_ptr_defs = a_known_ptr_def*

locale *l_known_ptrs$_{Node}$ = l_known_ptr known_ptr* **for** *known_ptr :: "(_) object_ptr ⇒ bool"*
begin
definition *a_known_ptrs :: "(_) heap ⇒ bool"*
where
"a_known_ptrs h = (∀ptr ∈ fset (object_ptr_kinds h). known_ptr ptr)"

lemma *known_ptrs_known_ptr: "a_known_ptrs h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptr ptr"*
⟨*proof*⟩

lemma *known_ptrs_preserved:*
"object_ptr_kinds h = object_ptr_kinds h' ⟹ a_known_ptrs h = a_known_ptrs h'"
⟨*proof*⟩
lemma *known_ptrs_subset:*
"object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
lemma *known_ptrs_new_ptr:*
"object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
end
global_interpretation *l_known_ptrs*$_{Node}$ *known_ptr* **defines** *known_ptrs = a_known_ptrs* ⟨*proof*⟩
lemmas *known_ptrs_defs = a_known_ptrs_def*

lemma *known_ptrs_is_l_known_ptrs: "l_known_ptrs known_ptr known_ptrs"*
⟨*proof*⟩

lemma *get_node_ptr_simp1 [simp]: "get*$_{Node}$ *node_ptr (put*$_{Node}$ *node_ptr node h) = Some node"*
⟨*proof*⟩
lemma *get_node_ptr_simp2 [simp]:*
"node_ptr ≠ node_ptr' ⟹ get$_{Node}$ *node_ptr (put*$_{Node}$ *node_ptr' node h) = get*$_{Node}$ *node_ptr h"*
⟨*proof*⟩

end

4.4 Element (ElementClass)

In this theory, we introduce the types for the Element class.

theory *ElementClass*
imports
"NodeClass"
"ShadowRootPointer"
begin

The type *DOMString* is a type synonym for *string*, define in section 6.

type_synonym *attr_key = DOMString*
type_synonym *attr_value = DOMString*
type_synonym *attrs = "(attr_key, attr_value) fmap"*
type_synonym *tag_name = DOMString*
record *('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr) RElement = RNode +*
nothing :: unit
tag_name :: tag_name
child_nodes :: "('node_ptr, 'element_ptr, 'character_data_ptr) node_ptr list"
attrs :: attrs
shadow_root_opt :: "'shadow_root_ptr shadow_root_ptr option"
type_synonym
('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element) Element
= "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element option)
RElement_scheme"
register_default_tvars
"('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element) Element"
type_synonym
('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Node, 'Element) Node
= "(('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element option) RElement_ext
+ 'Node) Node"
register_default_tvars
"('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Node, 'Element) Node"
type_synonym
('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object, 'Node, 'Element) Object
= "('Object, ('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element option)
RElement_ext + 'Node) Object"
register_default_tvars

```
  "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object, 'Node, 'Element) Object"

type_synonym
  ('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr, 'shadow_root_ptr,
    'Object, 'Node, 'Element) heap
  = "('document_ptr document_ptr + 'shadow_root_ptr shadow_root_ptr + 'object_ptr,
'element_ptr element_ptr + 'character_data_ptr character_data_ptr + 'node_ptr, 'Object,
('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Element option) RElement_ext +
'Node) heap"
register_default_tvars
  "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr, 'shadow_root_ptr,
'Object, 'Node, 'Element) heap"
type_synonym heap_final = "(unit, unit, unit, unit, unit, unit, unit, unit, unit) heap"

definition element_ptr_kinds :: "(_) heap ⇒ (_) element_ptr fset"
  where
    "element_ptr_kinds heap =
the |‘| (cast_node_ptr2element_ptr |‘| (ffilter is_element_ptr_kind (node_ptr_kinds heap)))"

lemma element_ptr_kinds_simp [simp]:
  "element_ptr_kinds (Heap (fmupd (cast element_ptr) element (the_heap h))) =
{|element_ptr|} |∪| element_ptr_kinds h"
  ⟨proof⟩

definition element_ptrs :: "(_) heap ⇒ (_) element_ptr fset"
  where
    "element_ptrs heap = ffilter is_element_ptr (element_ptr_kinds heap)"

definition cast_Node2Element :: "(_) Node ⇒ (_) Element option"
  where
    "cast_Node2Element node =
(case RNode.more node of Inl element ⇒ Some (RNode.extend (RNode.truncate node) element) | _ ⇒ None)"
adhoc_overloading cast ⇌ cast_Node2Element

abbreviation cast_Object2Element :: "(_) Object ⇒ (_) Element option"
  where
    "cast_Object2Element obj ≡ (case cast_Object2Node obj of Some node ⇒ cast_Node2Element node | None ⇒
None)"
adhoc_overloading cast ⇌ cast_Object2Element

definition cast_Element2Node :: "(_) Element ⇒ (_) Node"
  where
    "cast_Element2Node element = RNode.extend (RNode.truncate element) (Inl (RNode.more element))"
adhoc_overloading cast ⇌ cast_Element2Node

abbreviation cast_Element2Object :: "(_) Element ⇒ (_) Object"
  where
    "cast_Element2Object ptr ≡ cast_Node2Object (cast_Element2Node ptr)"
adhoc_overloading cast ⇌ cast_Element2Object

consts is_element_kind :: 'a
definition is_element_kind_Node :: "(_) Node ⇒ bool"
  where
    "is_element_kind_Node ptr ⟷ cast_Node2Element ptr ≠ None"

adhoc_overloading is_element_kind ⇌ is_element_kind_Node
lemmas is_element_kind_def = is_element_kind_Node_def

abbreviation is_element_kind_Object :: "(_) Object ⇒ bool"
  where
    "is_element_kind_Object ptr ≡ cast_Object2Element ptr ≠ None"
adhoc_overloading is_element_kind ⇌ is_element_kind_Object
```

```
lemma element_ptr_kinds_commutes [simp]:
  "cast element_ptr |∈| node_ptr_kinds h ⟷ element_ptr |∈| element_ptr_kinds h"
⟨proof⟩

definition get_Element :: "(_) element_ptr ⇒ (_) heap ⇒ (_) Element option"
  where
    "get_Element element_ptr h = Option.bind (get_Node (cast element_ptr) h) cast"
adhoc_overloading get ⇌ get_Element

locale l_type_wf_def_Element
begin
definition a_type_wf :: "(_) heap ⇒ bool"
  where
    "a_type_wf h = (NodeClass.type_wf h ∧ (∀element_ptr ∈ fset (element_ptr_kinds h).
                                                    get_Element element_ptr h ≠ None))"
end
global_interpretation l_type_wf_def_Element defines type_wf = a_type_wf ⟨proof⟩
lemmas type_wf_defs = a_type_wf_def

locale l_type_wf_Element = l_type_wf type_wf for type_wf :: "((_) heap ⇒ bool)" +
  assumes type_wf_Element: "type_wf h ⟹ ElementClass.type_wf h"

sublocale l_type_wf_Element ⊆ l_type_wf_Node
  ⟨proof⟩

locale l_get_Element_lemmas = l_type_wf_Element
begin
sublocale l_get_Node_lemmas ⟨proof⟩

lemma get_Element_type_wf:
  assumes "type_wf h"
  shows "element_ptr |∈| element_ptr_kinds h ⟷ get_Element element_ptr h ≠ None"
⟨proof⟩

end

global_interpretation l_get_Element_lemmas type_wf
  ⟨proof⟩

definition put_Element :: "(_) element_ptr ⇒ (_) Element ⇒ (_) heap ⇒ (_) heap"
  where
    "put_Element element_ptr element = put_Node (cast element_ptr) (cast element)"
adhoc_overloading put ⇌ put_Element

lemma put_Element_ptr_in_heap:
  assumes "put_Element element_ptr element h = h'"
  shows "element_ptr |∈| element_ptr_kinds h'"
  ⟨proof⟩

lemma put_Element_put_ptrs:
  assumes "put_Element element_ptr element h = h'"
  shows "object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast element_ptr|}"
  ⟨proof⟩



lemma cast_Element2Node_inject [simp]:
  "cast_Element2Node x = cast_Element2Node y ⟷ x = y"
  ⟨proof⟩

lemma cast_Node2Element_none [simp]:
  "cast_Node2Element node = None ⟷ ¬ (∃element. cast_Element2Node element = node)"
  ⟨proof⟩
```

lemma *cast$_{Node2Element}$_some [simp]:*
"cast$_{Node2Element}$ node = Some element ⟷ cast$_{Element2Node}$ element = node"
⟨*proof*⟩

lemma *cast$_{Node2Element}$_inv [simp]: "cast$_{Node2Element}$ (cast$_{Element2Node}$ element) = Some element"*
⟨*proof*⟩

lemma *get_elment_ptr_simp1 [simp]:*
"get$_{Element}$ element_ptr (put$_{Element}$ element_ptr element h) = Some element"
⟨*proof*⟩
lemma *get_elment_ptr_simp2 [simp]:*
"element_ptr ≠ element_ptr'
⟹ get$_{Element}$ element_ptr (put$_{Element}$ element_ptr' element h) = get$_{Element}$ element_ptr h"
⟨*proof*⟩

abbreviation *"create_element_obj tag_name_arg child_nodes_arg attrs_arg shadow_root_opt_arg*
≡ (| RObject.nothing = (), RNode.nothing = (), RElement.nothing = (),
tag_name = tag_name_arg, Element.child_nodes = child_nodes_arg, attrs = attrs_arg,
shadow_root_opt = shadow_root_opt_arg, ... = None |)"

definition *new$_{Element}$ :: "(_) heap ⇒ ((_) element_ptr × (_) heap)"*
where
"new$_{Element}$ h =
(let new_element_ptr = element_ptr.Ref (Suc (fMax (finsert 0 (element_ptr.the_ref
|`| (element_ptrs h)))))
in
(new_element_ptr, put new_element_ptr (create_element_obj '''' [] fmempty None) h))"

lemma *new$_{Element}$_ptr_in_heap:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
shows *"new_element_ptr |∈| element_ptr_kinds h'"*
⟨*proof*⟩

lemma *new_element_ptr_new:*
"element_ptr.Ref (Suc (fMax (finsert 0 (element_ptr.the_ref |`| element_ptrs h)))) |∉| element_ptrs h"
⟨*proof*⟩

lemma *new$_{Element}$_ptr_not_in_heap:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
shows *"new_element_ptr |∉| element_ptr_kinds h"*
⟨*proof*⟩

lemma *new$_{Element}$_new_ptr:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast new_element_ptr|}"*
⟨*proof*⟩

lemma *new$_{Element}$_is_element_ptr:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
shows *"is_element_ptr new_element_ptr"*
⟨*proof*⟩

lemma *new$_{Element}$_get$_{Object}$ [simp]:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
assumes *"ptr ≠ cast new_element_ptr"*
shows *"get$_{Object}$ ptr h = get$_{Object}$ ptr h'"*
⟨*proof*⟩

lemma *new$_{Element}$_get$_{Node}$ [simp]:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
assumes *"ptr ≠ cast new_element_ptr"*

shows "get_{Node} ptr h = get_{Node} ptr h'"
⟨*proof*⟩

lemma $new_{Element}_get_{Element}$ [simp]:
assumes "$new_{Element}$ h = (new_element_ptr, h')"
assumes "ptr ≠ new_element_ptr"
shows "$get_{Element}$ ptr h = $get_{Element}$ ptr h'"
⟨*proof*⟩

locale $l_known_ptr_{Element}$
begin
definition a_known_ptr :: "(_) object_ptr ⇒ bool"
where
"a_known_ptr ptr = (known_ptr ptr ∨ is_element_ptr ptr)"

lemma known_ptr_not_element_ptr: "¬is_element_ptr ptr ⟹ a_known_ptr ptr ⟹ known_ptr ptr"
⟨*proof*⟩
end
global_interpretation $l_known_ptr_{Element}$ **defines** known_ptr = a_known_ptr ⟨*proof*⟩
lemmas known_ptr_defs = a_known_ptr_def

locale $l_known_ptrs_{Element}$ = l_known_ptr known_ptr **for** known_ptr :: "(_) object_ptr ⇒ bool"
begin
definition a_known_ptrs :: "(_) heap ⇒ bool"
where
"a_known_ptrs h = (∀ptr ∈ fset (object_ptr_kinds h). known_ptr ptr)"

lemma known_ptrs_known_ptr:
"ptr |∈| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ known_ptr ptr"
⟨*proof*⟩

lemma known_ptrs_preserved:
"object_ptr_kinds h = object_ptr_kinds h' ⟹ a_known_ptrs h = a_known_ptrs h'"
⟨*proof*⟩
lemma known_ptrs_subset:
"object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
lemma known_ptrs_new_ptr:
"object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
a_known_ptrs h ⟹ a_known_ptrs h'"
⟨*proof*⟩
end
global_interpretation $l_known_ptrs_{Element}$ known_ptr **defines** known_ptrs = a_known_ptrs ⟨*proof*⟩
lemmas known_ptrs_defs = a_known_ptrs_def

lemma known_ptrs_is_l_known_ptrs: "l_known_ptrs known_ptr known_ptrs"
⟨*proof*⟩

end

4.5 CharacterData (CharacterDataClass)

In this theory, we introduce the types for the CharacterData class.

theory CharacterDataClass
imports
ElementClass
begin

CharacterData

The type DOMString is a type synonym for string, defined section 6.

```
record RCharacterData = RNode +
  nothing :: unit
  val :: DOMString
register_default_tvars "'CharacterData RCharacterData_ext"
type_synonym 'CharacterData CharacterData = "'CharacterData option RCharacterData_scheme"
register_default_tvars "'CharacterData CharacterData"
type_synonym ('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Node,
    'Element, 'CharacterData) Node
  = "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr,
      'CharacterData option RCharacterData_ext + 'Node, 'Element) Node"
register_default_tvars "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Node,
                        'Element, 'CharacterData) Node"
type_synonym ('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object, 'Node,
    'Element, 'CharacterData) Object
  = "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object,
      'CharacterData option RCharacterData_ext + 'Node,
      'Element) Object"
register_default_tvars "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object,
                        'Node, 'Element, 'CharacterData) Object"

type_synonym ('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
    'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData) heap
  = "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr, 'shadow_root_ptr,
      'Object, 'CharacterData option RCharacterData_ext + 'Node, 'Element) heap"
register_default_tvars "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
                        'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData) heap"
type_synonym heap_final = "(unit, unit, unit, unit, unit, unit, unit, unit, unit, unit) heap"


definition character_data_ptr_kinds :: "(_) heap ⇒ (_) character_data_ptr fset"
  where
    "character_data_ptr_kinds heap = the |`| (cast |`| (ffilter is_character_data_ptr_kind
                                                          (node_ptr_kinds heap)))"

lemma character_data_ptr_kinds_simp [simp]:
  "character_data_ptr_kinds (Heap (fmupd (cast character_data_ptr) character_data (the_heap h)))
             = {|character_data_ptr|} |∪| character_data_ptr_kinds h"
  ⟨proof⟩

definition character_data_ptrs :: "(_) heap ⇒ _ character_data_ptr fset"
  where
    "character_data_ptrs heap = ffilter is_character_data_ptr (character_data_ptr_kinds heap)"

abbreviation "character_data_ptr_exts heap ≡ character_data_ptr_kinds heap - character_data_ptrs heap"

definition cast_Node2CharacterData :: "(_) Node ⇒ (_) CharacterData option"
  where
    "cast_Node2CharacterData node = (case RNode.more node of
          Inr (Inl character_data) ⇒ Some (RNode.extend (RNode.truncate node) character_data)
        | _ ⇒ None)"
adhoc_overloading cast ⇌ cast_Node2CharacterData

abbreviation cast_Object2CharacterData :: "(_) Object ⇒ (_) CharacterData  option"
  where
    "cast_Object2CharacterData obj ≡ (case cast_Object2Node obj of Some node ⇒ cast_Node2CharacterData node

                                                          | None ⇒ None)"
adhoc_overloading cast ⇌ cast_Object2CharacterData

definition cast_CharacterData2Node :: "(_) CharacterData ⇒ (_) Node"
  where
    "cast_CharacterData2Node character_data = RNode.extend (RNode.truncate character_data)
                                                  (Inr (Inl (RNode.more character_data)))"
```

adhoc_overloading *cast* ⇌ *cast*$_{CharacterData2Node}$

abbreviation *cast*$_{CharacterData2Object}$ *:: "(_) CharacterData ⇒ (_) Object"*
  where
    "cast$_{CharacterData2Object}$ *ptr ≡ cast*$_{Node2Object}$ *(cast*$_{CharacterData2Node}$ *ptr)"*
adhoc_overloading *cast* ⇌ *cast*$_{CharacterData2Object}$

consts *is_character_data_kind :: 'a*
definition *is_character_data_kind*$_{Node}$ *:: "(_) Node ⇒ bool"*
  where
    "is_character_data_kind$_{Node}$ *ptr ⟷ cast*$_{Node2CharacterData}$ *ptr ≠ None"*

adhoc_overloading *is_character_data_kind* ⇌ *is_character_data_kind*$_{Node}$
lemmas *is_character_data_kind_def = is_character_data_kind*$_{Node}$*_def*

abbreviation *is_character_data_kind*$_{Object}$ *:: "(_) Object ⇒ bool"*
  where
    "is_character_data_kind$_{Object}$ *ptr ≡ cast*$_{Object2CharacterData}$ *ptr ≠ None"*
adhoc_overloading *is_character_data_kind* ⇌ *is_character_data_kind*$_{Object}$

lemma *character_data_ptr_kinds_commutes [simp]:*
  "cast character_data_ptr |∈| node_ptr_kinds h
      ⟷ character_data_ptr |∈| character_data_ptr_kinds h"
⟨*proof*⟩

definition *get*$_{CharacterData}$ *:: "(_) character_data_ptr ⇒ (_) heap ⇒ (_) CharacterData option"*
  where
    "get$_{CharacterData}$ *character_data_ptr h = Option.bind (get*$_{Node}$ *(cast character_data_ptr) h) cast"*
adhoc_overloading *get* ⇌ *get*$_{CharacterData}$

locale *l_type_wf_def*$_{CharacterData}$
begin
definition *a_type_wf :: "(_) heap ⇒ bool"*
  where
    "a_type_wf h = (ElementClass.type_wf h
                ∧ (∀ character_data_ptr ∈ fset (character_data_ptr_kinds h).
                    get$_{CharacterData}$ *character_data_ptr h ≠ None))"*
end
global_interpretation *l_type_wf_def*$_{CharacterData}$ **defines** *type_wf = a_type_wf* ⟨*proof*⟩
lemmas *type_wf_defs = a_type_wf_def*

locale *l_type_wf*$_{CharacterData}$ *= l_type_wf type_wf* **for** *type_wf :: "((_) heap ⇒ bool)" +*
  assumes *type_wf*$_{CharacterData}$*: "type_wf h ⟹ CharacterDataClass.type_wf h"*

sublocale *l_type_wf*$_{CharacterData}$ ⊆ *l_type_wf*$_{Element}$
  ⟨*proof*⟩

locale *l_get*$_{CharacterData}$*_lemmas = l_type_wf*$_{CharacterData}$
begin
sublocale *l_get*$_{Element}$*_lemmas* ⟨*proof*⟩

lemma *get*$_{CharacterData}$*_type_wf:*
  assumes *"type_wf h"*
  shows *"character_data_ptr |∈| character_data_ptr_kinds h*
        ⟷ get$_{CharacterData}$ *character_data_ptr h ≠ None"*
  ⟨*proof*⟩
end

global_interpretation *l_get*$_{CharacterData}$*_lemmas type_wf*
  ⟨*proof*⟩

definition *put*$_{CharacterData}$ *:: "(_) character_data_ptr ⇒ (_) CharacterData ⇒ (_) heap ⇒ (_) heap"*
  where

```
    "put_CharacterData character_data_ptr character_data = put_Node (cast character_data_ptr)
                (cast character_data)"
adhoc_overloading put ⇌ put_CharacterData

lemma put_CharacterData_ptr_in_heap:
  assumes "put_CharacterData character_data_ptr character_data h = h'"
  shows "character_data_ptr |∈| character_data_ptr_kinds h'"
  ⟨proof⟩

lemma put_CharacterData_put_ptrs:
  assumes "put_CharacterData character_data_ptr character_data h = h'"
  shows "object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast character_data_ptr|}"
  ⟨proof⟩


lemma cast_CharacterData2Node_inject [simp]: "cast_CharacterData2Node x = cast_CharacterData2Node y ⟷ x
= y"
  ⟨proof⟩

lemma cast_Node2CharacterData_none [simp]:
  "cast_Node2CharacterData node = None ⟷ ¬ (∃ character_data. cast_CharacterData2Node character_data = node)"
  ⟨proof⟩

lemma cast_Node2CharacterData_some [simp]:
  "cast_Node2CharacterData node = Some character_data ⟷ cast_CharacterData2Node character_data = node"
  ⟨proof⟩

lemma cast_Node2CharacterData_inv [simp]:
  "cast_Node2CharacterData (cast_CharacterData2Node character_data) = Some character_data"
  ⟨proof⟩

lemma cast_element_not_character_data [simp]:
  "(cast_Element2Node element ≠ cast_CharacterData2Node character_data)"
  "(cast_CharacterData2Node character_data ≠ cast_Element2Node element)"
  ⟨proof⟩

lemma get_CharacterData_simp1 [simp]:
  "get_CharacterData character_data_ptr (put_CharacterData character_data_ptr character_data h)
      = Some character_data"
  ⟨proof⟩
lemma get_CharacterData_simp2 [simp]:
  "character_data_ptr ≠ character_data_ptr' ⟹ get_CharacterData character_data_ptr
       (put_CharacterData character_data_ptr' character_data h) = get_CharacterData character_data_ptr h"
  ⟨proof⟩

lemma get_CharacterData_simp3 [simp]:
  "get_Element element_ptr (put_CharacterData character_data_ptr f h) = get_Element element_ptr h"
  ⟨proof⟩
lemma get_CharacterData_simp4 [simp]:
  "get_CharacterData element_ptr (put_Element character_data_ptr f h) = get_CharacterData element_ptr h"
  ⟨proof⟩

lemma new_Element_get_CharacterData [simp]:
  assumes "new_Element h = (new_element_ptr, h')"
  shows "get_CharacterData ptr h = get_CharacterData ptr h'"
  ⟨proof⟩



abbreviation "create_character_data_obj val_arg
  ≡ (| RObject.nothing = (), RNode.nothing = (), RCharacterData.nothing = (), val = val_arg, ... = None |)"

definition new_CharacterData :: "(_) heap ⇒ ((_) character_data_ptr × (_) heap)"
```

where
"new$_{CharacterData}$ h =
(let new_character_data_ptr = character_data_ptr.Ref (Suc (fMax (character_data_ptr.the_ref
|‘| (character_data_ptrs h)))) in
(new_character_data_ptr, put new_character_data_ptr (create_character_data_obj ’’’’) h))"

lemma new$_{CharacterData}$_ptr_in_heap:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
shows "new_character_data_ptr |∈| character_data_ptr_kinds h’"
⟨*proof*⟩

lemma new_character_data_ptr_new:
"character_data_ptr.Ref (Suc (fMax (finsert 0 (character_data_ptr.the_ref |‘| character_data_ptrs h))))
|∉| character_data_ptrs h"
⟨*proof*⟩

lemma new$_{CharacterData}$_ptr_not_in_heap:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
shows "new_character_data_ptr |∉| character_data_ptr_kinds h"
⟨*proof*⟩

lemma new$_{CharacterData}$_new_ptr:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
shows "object_ptr_kinds h’ = object_ptr_kinds h |∪| {|cast new_character_data_ptr|}"
⟨*proof*⟩

lemma new$_{CharacterData}$_is_character_data_ptr:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
shows "is_character_data_ptr new_character_data_ptr"
⟨*proof*⟩

lemma new$_{CharacterData}$_get$_{Object}$ [simp]:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
assumes "ptr ≠ cast new_character_data_ptr"
shows "get$_{Object}$ ptr h = get$_{Object}$ ptr h’"
⟨*proof*⟩

lemma new$_{CharacterData}$_get$_{Node}$ [simp]:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
assumes "ptr ≠ cast new_character_data_ptr"
shows "get$_{Node}$ ptr h = get$_{Node}$ ptr h’"
⟨*proof*⟩

lemma new$_{CharacterData}$_get$_{Element}$ [simp]:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
shows "get$_{Element}$ ptr h = get$_{Element}$ ptr h’"
⟨*proof*⟩

lemma new$_{CharacterData}$_get$_{CharacterData}$ [simp]:
assumes "new$_{CharacterData}$ h = (new_character_data_ptr, h’)"
assumes "ptr ≠ new_character_data_ptr"
shows "get$_{CharacterData}$ ptr h = get$_{CharacterData}$ ptr h’"
⟨*proof*⟩

locale l_known_ptr$_{CharacterData}$
begin
definition a_known_ptr :: "(_) object_ptr ⇒ bool"
where
"a_known_ptr ptr = (known_ptr ptr ∨ is_character_data_ptr ptr)"

lemma known_ptr_not_character_data_ptr:

```
  "¬is_character_data_ptr ptr ⟹ a_known_ptr ptr ⟹ known_ptr ptr"
  ⟨proof⟩
end
global_interpretation l_known_ptr_CharacterData defines known_ptr = a_known_ptr ⟨proof⟩
lemmas known_ptr_defs = a_known_ptr_def


locale l_known_ptrs_CharacterData = l_known_ptr known_ptr for known_ptr :: "(_) object_ptr ⇒ bool"
begin
definition a_known_ptrs :: "(_) heap ⇒ bool"
  where
    "a_known_ptrs h = (∀ptr ∈ fset (object_ptr_kinds h). known_ptr ptr)"

lemma known_ptrs_known_ptr: "a_known_ptrs h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptr ptr"
  ⟨proof⟩

lemma known_ptrs_preserved:
  "object_ptr_kinds h = object_ptr_kinds h' ⟹ a_known_ptrs h = a_known_ptrs h'"
  ⟨proof⟩
lemma known_ptrs_subset:
  "object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ a_known_ptrs h'"
  ⟨proof⟩
lemma known_ptrs_new_ptr:
  "object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
a_known_ptrs h ⟹ a_known_ptrs h'"
  ⟨proof⟩
end
global_interpretation l_known_ptrs_CharacterData known_ptr defines known_ptrs = a_known_ptrs ⟨proof⟩
lemmas known_ptrs_defs = a_known_ptrs_def

lemma known_ptrs_is_l_known_ptrs: "l_known_ptrs known_ptr known_ptrs"
  ⟨proof⟩

end
```

4.6 Document (DocumentClass)

In this theory, we introduce the types for the Document class.

```
theory DocumentClass
  imports
    CharacterDataClass
begin
```

The type *doctype* is a type synonym for *string*, defined in section 6.

```
record ('node_ptr, 'element_ptr, 'character_data_ptr) RDocument = RObject +
  nothing :: unit
  doctype :: doctype
  document_element :: "(_) element_ptr option"
  disconnected_nodes :: "('node_ptr, 'element_ptr, 'character_data_ptr) node_ptr list"
type_synonym
  ('node_ptr, 'element_ptr, 'character_data_ptr, 'Document) Document
  = "('node_ptr, 'element_ptr, 'character_data_ptr, 'Document option) RDocument_scheme"
register_default_tvars
  "('node_ptr, 'element_ptr, 'character_data_ptr, 'Document) Document"
type_synonym
  ('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr, 'Object, 'Node,
    'Element, 'CharacterData, 'Document) Object
  = "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr,
       ('node_ptr, 'element_ptr, 'character_data_ptr, 'Document option)
    RDocument_ext + 'Object, 'Node, 'Element, 'CharacterData) Object"
register_default_tvars "('node_ptr, 'element_ptr, 'character_data_ptr, 'shadow_root_ptr,
     'Object, 'Node, 'Element, 'CharacterData, 'Document) Object"
```

```
type_synonym ('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
    'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData, 'Document) heap
  = "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
      'shadow_root_ptr,
      ('node_ptr, 'element_ptr, 'character_data_ptr, 'Document option) RDocument_ext + 'Object, 'Node,
      'Element, 'CharacterData) heap"
register_default_tvars
  "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
  'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData, 'Document) heap"
type_synonym heap_final = "(unit, unit, unit, unit, unit, unit, unit, unit, unit, unit, unit) heap"


definition document_ptr_kinds :: "(_) heap ⇒ (_) document_ptr fset"
  where
    "document_ptr_kinds heap = the |`| (cast_object_ptr2document_ptr |`|
                               (ffilter is_document_ptr_kind (object_ptr_kinds heap)))"

definition document_ptrs :: "(_) heap ⇒ (_) document_ptr fset"
  where
    "document_ptrs heap = ffilter is_document_ptr (document_ptr_kinds heap)"

definition cast_Object2Document :: "(_) Object ⇒ (_) Document option"
  where
    "cast_Object2Document obj = (case RObject.more obj of
      Inr (Inl document) ⇒ Some (RObject.extend (RObject.truncate obj) document)
    | _ ⇒ None)"
adhoc_overloading cast ⇌ cast_Object2Document

definition cast_Document2Object:: "(_) Document ⇒ (_) Object"
  where
    "cast_Document2Object document = (RObject.extend (RObject.truncate document)
                                        (Inr (Inl (RObject.more document))))"
adhoc_overloading cast ⇌ cast_Document2Object

definition is_document_kind :: "(_) Object ⇒ bool"
  where
    "is_document_kind ptr ⟷ cast_Object2Document ptr ≠ None"

lemma document_ptr_kinds_simp [simp]:
  "document_ptr_kinds (Heap (fmupd (cast document_ptr) document (the_heap h)))
         = {|document_ptr|} |∪| document_ptr_kinds h"
  ⟨proof⟩

lemma document_ptr_kinds_commutes [simp]:
  "cast document_ptr |∈| object_ptr_kinds h ⟷ document_ptr |∈| document_ptr_kinds h"
⟨proof⟩

definition get_Document :: "(_) document_ptr ⇒ (_) heap ⇒ (_) Document option"
  where
    "get_Document document_ptr h = Option.bind (get (cast document_ptr) h) cast"
adhoc_overloading get ⇌ get_Document

locale l_type_wf_def_Document
begin
definition a_type_wf :: "(_) heap ⇒ bool"
  where
    "a_type_wf h = (CharacterDataClass.type_wf h ∧
    (∀ document_ptr ∈ fset (document_ptr_kinds h). get_Document document_ptr h ≠ None))"
end
global_interpretation l_type_wf_def_Document defines type_wf = a_type_wf ⟨proof⟩
lemmas type_wf_defs = a_type_wf_def
```

locale *l_type_wf*$_{Document}$ = *l_type_wf type_wf* **for** *type_wf :: "((_) heap ⇒ bool)" +*
  assumes *type_wf*$_{Document}$*: "type_wf h ⟹ DocumentClass.type_wf h"*

sublocale *l_type_wf*$_{Document}$ ⊆ *l_type_wf*$_{CharacterData}$
  ⟨*proof*⟩

locale *l_get*$_{Document}$*_lemmas = l_type_wf*$_{Document}$
begin
sublocale *l_get*$_{CharacterData}$*_lemmas* ⟨*proof*⟩
lemma *get*$_{Document}$*_type_wf:*
  assumes *"type_wf h"*
  shows *"document_ptr |∈| document_ptr_kinds h ⟷ get*$_{Document}$ *document_ptr h ≠ None"*
  ⟨*proof*⟩
end

global_interpretation *l_get*$_{Document}$*_lemmas type_wf* ⟨*proof*⟩

definition *put*$_{Document}$ *:: "(_) document_ptr ⇒ (_) Document ⇒ (_) heap ⇒ (_) heap"*
  where
    "put$_{Document}$ *document_ptr document = put (cast document_ptr) (cast document)"*
adhoc_overloading *put ⇌ put*$_{Document}$

lemma *put*$_{Document}$*_ptr_in_heap:*
  assumes *"put*$_{Document}$ *document_ptr document h = h'"*
  shows *"document_ptr |∈| document_ptr_kinds h'"*
  ⟨*proof*⟩

lemma *put*$_{Document}$*_put_ptrs:*
  assumes *"put*$_{Document}$ *document_ptr document h = h'"*
  shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast document_ptr|}"*
  ⟨*proof*⟩

lemma *cast*$_{Document2Object}$*_inject [simp]: "cast*$_{Document2Object}$ *x = cast*$_{Document2Object}$ *y ⟷ x = y"*
  ⟨*proof*⟩

lemma *cast*$_{Object2Document}$*_none [simp]:*
  "cast$_{Object2Document}$ *obj = None ⟷ ¬ (∃document. cast*$_{Document2Object}$ *document = obj)"*
  ⟨*proof*⟩

lemma *cast*$_{Object2Document}$*_some [simp]:*
  "cast$_{Object2Document}$ *obj = Some document ⟷ cast document = obj"*
  ⟨*proof*⟩

lemma *cast*$_{Object2Document}$*_inv [simp]: "cast*$_{Object2Document}$ *(cast*$_{Document2Object}$ *document) = Some document"*
  ⟨*proof*⟩

lemma *cast_document_not_node [simp]:*
  "cast$_{Document2Object}$ *document ≠ cast*$_{Node2Object}$ *node"*
  "cast$_{Node2Object}$ *node ≠ cast*$_{Document2Object}$ *document"*
  ⟨*proof*⟩

lemma *get_document_ptr_simp1 [simp]:*
  "get$_{Document}$ *document_ptr (put*$_{Document}$ *document_ptr document h) = Some document"*
  ⟨*proof*⟩
lemma *get_document_ptr_simp2 [simp]:*
  "document_ptr ≠ document_ptr'
   ⟹ get$_{Document}$ *document_ptr (put*$_{Document}$ *document_ptr' document h) = get*$_{Document}$ *document_ptr h"*
  ⟨*proof*⟩

lemma *get_document_ptr_simp3 [simp]:*
  "get$_{Element}$ *element_ptr (put*$_{Document}$ *document_ptr f h) = get*$_{Element}$ *element_ptr h"*

⟨proof⟩
lemma *get_document_ptr_simp4 [simp]:*
"get$_{Document}$ document_ptr (put$_{Element}$ element_ptr f h) = get$_{Document}$ document_ptr h"
⟨proof⟩
lemma *get_document_ptr_simp5 [simp]:*
"get$_{CharacterData}$ character_data_ptr (put$_{Document}$ document_ptr f h) = get$_{CharacterData}$ character_data_ptr h"
⟨proof⟩
lemma *get_document_ptr_simp6 [simp]:*
"get$_{Document}$ document_ptr (put$_{CharacterData}$ character_data_ptr f h) = get$_{Document}$ document_ptr h"
⟨proof⟩

lemma *new$_{Element}$_get$_{Document}$ [simp]:*
assumes *"new$_{Element}$ h = (new_element_ptr, h')"*
shows *"get$_{Document}$ ptr h = get$_{Document}$ ptr h'"*
⟨proof⟩

lemma *new$_{CharacterData}$_get$_{Document}$ [simp]:*
assumes *"new$_{CharacterData}$ h = (new_character_data_ptr, h')"*
shows *"get$_{Document}$ ptr h = get$_{Document}$ ptr h'"*
⟨proof⟩

abbreviation
create_document_obj :: "char list ⇒ (_) element_ptr option ⇒ (_) node_ptr list ⇒ (_) Document"
where
"create_document_obj doctype_arg document_element_arg disconnected_nodes_arg
≡ (| RObject.nothing = (), RDocument.nothing = (), doctype = doctype_arg,
document_element = document_element_arg,
disconnected_nodes = disconnected_nodes_arg, ... = None |)"

definition *new$_{Document}$ :: "(_)heap ⇒ ((_) document_ptr × (_) heap)"*
where
"new$_{Document}$ h =
(let new_document_ptr = document_ptr.Ref (Suc (fMax (finsert 0 (document_ptr.the_ref |`| (document_ptrs h)))))
in
(new_document_ptr, put new_document_ptr (create_document_obj '''' None []) h))"

lemma *new$_{Document}$_ptr_in_heap:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"new_document_ptr |∈| document_ptr_kinds h'"*
⟨proof⟩

lemma *new_document_ptr_new:*
"document_ptr.Ref (Suc (fMax (finsert 0 (document_ptr.the_ref |`| document_ptrs h))))
|∉| document_ptrs h"
⟨proof⟩

lemma *new$_{Document}$_ptr_not_in_heap:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"new_document_ptr |∉| document_ptr_kinds h"*
⟨proof⟩

lemma *new$_{Document}$_new_ptr:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast new_document_ptr|}"*
⟨proof⟩

lemma *new$_{Document}$_is_document_ptr:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"is_document_ptr new_document_ptr"*

⟨proof⟩

lemma *new$_{Document}$_get$_{Object}$ [simp]:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
assumes *"ptr ≠ cast new_document_ptr"*
shows *"get$_{Object}$ ptr h = get$_{Object}$ ptr h'"*
⟨proof⟩

lemma *new$_{Document}$_get$_{Node}$ [simp]:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"get$_{Node}$ ptr h = get$_{Node}$ ptr h'"*
⟨proof⟩

lemma *new$_{Document}$_get$_{Element}$ [simp]:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"get$_{Element}$ ptr h = get$_{Element}$ ptr h'"*
⟨proof⟩

lemma *new$_{Document}$_get$_{CharacterData}$ [simp]:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
shows *"get$_{CharacterData}$ ptr h = get$_{CharacterData}$ ptr h'"*
⟨proof⟩

lemma *new$_{Document}$_get$_{Document}$ [simp]:*
assumes *"new$_{Document}$ h = (new_document_ptr, h')"*
assumes *"ptr ≠ new_document_ptr"*
shows *"get$_{Document}$ ptr h = get$_{Document}$ ptr h'"*
⟨proof⟩

locale *l_known_ptr$_{Document}$*
begin
definition *a_known_ptr :: "(_) object_ptr ⇒ bool"*
where
"a_known_ptr ptr = (known_ptr ptr ∨ is_document_ptr ptr)"

lemma *known_ptr_not_document_ptr: "¬is_document_ptr ptr ⟹ a_known_ptr ptr ⟹ known_ptr ptr"*
⟨proof⟩
end
global_interpretation *l_known_ptr$_{Document}$* **defines** *known_ptr = a_known_ptr* ⟨proof⟩
lemmas *known_ptr_defs = a_known_ptr_def*

locale *l_known_ptrs$_{Document}$ = l_known_ptr known_ptr* **for** *known_ptr :: "(_) object_ptr ⇒ bool"*
begin
definition *a_known_ptrs :: "(_) heap ⇒ bool"*
where
"a_known_ptrs h = (∀ptr ∈ fset (object_ptr_kinds h). known_ptr ptr)"

lemma *known_ptrs_known_ptr: "a_known_ptrs h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptr ptr"*
⟨proof⟩

lemma *known_ptrs_preserved:*
"object_ptr_kinds h = object_ptr_kinds h' ⟹ a_known_ptrs h = a_known_ptrs h'"
⟨proof⟩
lemma *known_ptrs_subset:*
"object_ptr_kinds h' |⊆| object_ptr_kinds h ⟹ a_known_ptrs h ⟹ a_known_ptrs h'"
⟨proof⟩
lemma *known_ptrs_new_ptr:*
"object_ptr_kinds h' = object_ptr_kinds h |∪| {|new_ptr|} ⟹ known_ptr new_ptr ⟹
a_known_ptrs h ⟹ a_known_ptrs h'"
⟨proof⟩
end

global_interpretation *l_known_ptrs*$_{Document}$ *known_ptr* **defines** *known_ptrs* = *a_known_ptrs* ⟨*proof*⟩
lemmas *known_ptrs_defs* = *a_known_ptrs_def*

lemma *known_ptrs_is_l_known_ptrs [instances]: "l_known_ptrs known_ptr known_ptrs"*
⟨*proof*⟩

end

5 Monadic Object Constructors and Accessors

In this chapter, we introduce the moandic method definitions for the classes of our DOM formalization. Again the overall structure follows the same structure as for the class types and the pointer types.

5.1 The Monad Infrastructure (BaseMonad)

In this theory, we introduce the basic infrastructure for our monadic class encoding.

```
theory BaseMonad
  imports
    "../classes/BaseClass"
    "../preliminaries/Heap_Error_Monad"
begin
```

5.1.1 Datatypes

```
datatype exception = NotFoundError | HierarchyRequestError | NotSupportedError | SegmentationFault
  | AssertException | NonTerminationException | InvokeError | TypeError

consts put_M :: 'a
consts get_M :: 'a
consts delete_M :: 'a

lemma sorted_list_of_set_eq [dest]:
  "sorted_list_of_set (fset x) = sorted_list_of_set (fset y) ⟹ x = y"
  ⟨proof⟩


locale l_ptr_kinds_M =
  fixes ptr_kinds :: "'heap ⇒ 'ptr::linorder fset"
begin
definition a_ptr_kinds_M :: "('heap, exception, 'ptr list) prog"
  where
    "a_ptr_kinds_M = do {
      h ← get_heap;
      return (sorted_list_of_set (fset (ptr_kinds h)))
    }"

lemma ptr_kinds_M_ok [simp]: "h ⊢ ok a_ptr_kinds_M"
  ⟨proof⟩

lemma ptr_kinds_M_pure [simp]: "pure a_ptr_kinds_M h"
  ⟨proof⟩

lemma ptr_kinds_ptr_kinds_M [simp]: "ptr ∈ set |h ⊢ a_ptr_kinds_M|_r ⟷ ptr |∈| ptr_kinds h"
  ⟨proof⟩

lemma ptr_kinds_M_ptr_kinds [simp]:
  "h ⊢ a_ptr_kinds_M →_r xa ⟷ xa = sorted_list_of_set (fset (ptr_kinds h))"
  ⟨proof⟩
lemma ptr_kinds_M_ptr_kinds_returns_result [simp]:
  "h ⊢ a_ptr_kinds_M ⤜ f →_r x ⟷ h ⊢ f (sorted_list_of_set (fset (ptr_kinds h))) →_r x"
  ⟨proof⟩
lemma ptr_kinds_M_ptr_kinds_returns_heap [simp]:
  "h ⊢ a_ptr_kinds_M ⤜ f →_h h' ⟷ h ⊢ f (sorted_list_of_set (fset (ptr_kinds h))) →_h h'"
  ⟨proof⟩
```

```
end

locale l_get_M =
  fixes get :: "'ptr ⇒ 'heap ⇒ 'obj option"
  fixes type_wf :: "'heap ⇒ bool"
  fixes ptr_kinds :: "'heap ⇒ 'ptr fset"
  assumes "type_wf h ⟹ ptr |∈| ptr_kinds h ⟹ get ptr h ≠ None"
  assumes "get ptr h ≠ None ⟹ ptr |∈| ptr_kinds h"
begin

definition a_get_M :: "'ptr ⇒ ('obj ⇒ 'result) ⇒  ('heap, exception, 'result) prog"
  where
    "a_get_M ptr getter = (do {
      h ← get_heap;
      (case get ptr h of
        Some res ⇒ return (getter res)
      | None ⇒ error SegmentationFault)
    })"

lemma get_M_pure [simp]: "pure (a_get_M ptr getter) h"
  ⟨proof⟩

lemma get_M_ok:
  "type_wf h ⟹ ptr |∈| ptr_kinds h ⟹ h ⊢ ok (a_get_M ptr getter)"
  ⟨proof⟩
lemma get_M_ptr_in_heap:
  "h ⊢ ok (a_get_M ptr getter) ⟹ ptr |∈| ptr_kinds h"
  ⟨proof⟩

end

locale l_put_M = l_get_M get for get :: "'ptr ⇒ 'heap ⇒ 'obj option" +
  fixes put :: "'ptr ⇒ 'obj ⇒ 'heap ⇒ 'heap"
begin
definition a_put_M :: "'ptr ⇒ (('v ⇒ 'v) ⇒ 'obj ⇒ 'obj) ⇒ 'v ⇒ ('heap, exception, unit) prog"
  where
    "a_put_M ptr setter v = (do {
      obj ← a_get_M ptr id;
      h ← get_heap;
      return_heap (put ptr (setter (λ_. v) obj) h)
    })"

lemma put_M_ok:
  "type_wf h ⟹ ptr |∈| ptr_kinds h ⟹ h ⊢ ok (a_put_M ptr setter v)"
  ⟨proof⟩

lemma put_M_ptr_in_heap:
  "h ⊢ ok (a_put_M ptr setter v) ⟹ ptr |∈| ptr_kinds h"
  ⟨proof⟩

end
```

5.1.2 Setup for Defining Partial Functions

```
lemma execute_admissible:
  "ccpo.admissible (fun_lub (flat_lub (Inl (e::'e)))) (fun_ord (flat_ord (Inl e)))
     ((λa. ∀(h::'heap) h2 (r::'result). h ⊢ a = Inr (r, h2) ⟶ P h h2 r) ∘ Prog)"
⟨proof⟩

lemma execute_admissible2:
  "ccpo.admissible (fun_lub (flat_lub (Inl (e::'e)))) (fun_ord (flat_ord (Inl e)))
     ((λa. ∀(h::'heap) h' h2 h2' (r::'result) r'.
                   h ⊢ a = Inr (r, h2) ⟶ h' ⊢ a = Inr (r', h2') ⟶ P h h' h2 h2' r r') ∘ Prog)"
```

⟨*proof*⟩

definition *dom_prog_ord ::*
"('heap, exception, 'result) prog ⇒ ('heap, exception, 'result) prog ⇒ bool" **where**
"dom_prog_ord = img_ord (λa b. execute b a) (fun_ord (flat_ord (Inl NonTerminationException)))"

definition *dom_prog_lub ::*
"('heap, exception, 'result) prog set ⇒ ('heap, exception, 'result) prog" **where**
"dom_prog_lub = img_lub (λa b. execute b a) Prog (fun_lub (flat_lub (Inl NonTerminationException)))"

lemma *dom_prog_lub_empty: "dom_prog_lub {} = error NonTerminationException"*
⟨*proof*⟩

lemma *dom_prog_interpretation: "partial_function_definitions dom_prog_ord dom_prog_lub"*
⟨*proof*⟩

interpretation *dom_prog: partial_function_definitions dom_prog_ord dom_prog_lub*
rewrites *"dom_prog_lub {} ≡ error NonTerminationException"*
⟨*proof*⟩

lemma *admissible_dom_prog:*
"dom_prog.admissible (λf. ∀x h h' r. h ⊢ f x →r r ⟶ h ⊢ f x →h h' ⟶ P x h h' r)"
⟨*proof*⟩

lemma *admissible_dom_prog2:*
"dom_prog.admissible (λf. ∀x h h2 h' h2' r r2. h ⊢ f x →r r ⟶ h ⊢ f x →h h'
⟶ h2 ⊢ f x →r r2 ⟶ h2 ⊢ f x →h h2' ⟶ P x h h2 h' h2' r r2)"
⟨*proof*⟩

lemma *fixp_induct_dom_prog:*
fixes *F :: "'c ⇒ 'c"* **and**
U :: "'c ⇒ 'b ⇒ ('heap, exception, 'result) prog" **and**
C :: "('b ⇒ ('heap, exception, 'result) prog) ⇒ 'c" **and**
P :: "'b ⇒ 'heap ⇒ 'heap ⇒ 'result ⇒ bool"
assumes *mono: "⋀x. monotone (fun_ord dom_prog_ord) dom_prog_ord (λf. U (F (C f)) x)"*
assumes *eq: "f ≡ C (ccpo.fixp (fun_lub dom_prog_lub) (fun_ord dom_prog_ord) (λf. U (F (C f))))"*
assumes *inverse2: "⋀f. U (C f) = f"*
assumes *step: "⋀f x h h' r. (⋀x h h' r. h ⊢ (U f x) →r r ⟹ h ⊢ (U f x) →h h' ⟹ P x h h' r)*
⟹ h ⊢ (U (F f) x) →r r ⟹ h ⊢ (U (F f) x) →h h' ⟹ P x h h' r"
assumes *defined: "h ⊢ (U f x) →r r"* **and** *"h ⊢ (U f x) →h h'"*
shows *"P x h h' r"*
⟨*proof*⟩

⟨*ML*⟩

abbreviation *"mono_dom_prog ≡ monotone (fun_ord dom_prog_ord) dom_prog_ord"*

lemma *dom_prog_ordI:*
assumes *"⋀h. h ⊢ f →e NonTerminationException ∨ h ⊢ f = h ⊢ g"*
shows *"dom_prog_ord f g"*
⟨*proof*⟩

lemma *dom_prog_ordE:*
assumes *"dom_prog_ord x y"*
obtains *"h ⊢ x →e NonTerminationException" | " h ⊢ x = h ⊢ y"*
⟨*proof*⟩

lemma *bind_mono [partial_function_mono]:*
fixes *B :: "('a ⇒ ('heap, exception, 'result) prog) ⇒ ('heap, exception, 'result2) prog"*
assumes *mf: "mono_dom_prog B"* **and** *mg: "⋀y. mono_dom_prog (λf. C y f)"*
shows *"mono_dom_prog (λf. B f ⤜ (λy. C y f))"*

⟨*proof*⟩

lemma *mono_dom_prog1 [partial_function_mono]:*
fixes *g :: "('a ⇒ ('heap, exception, 'result) prog) ⇒ 'b ⇒ ('heap, exception, 'result) prog"*
assumes *"⋀x. (mono_dom_prog (λf. g f x))"*
shows *"mono_dom_prog (λf. map_M (g f) xs)"*
⟨*proof*⟩

lemma *mono_dom_prog2 [partial_function_mono]:*
fixes *g :: "('a ⇒ ('heap, exception, 'result) prog) ⇒ 'b ⇒ ('heap, exception, 'result) prog"*
assumes *"⋀x. (mono_dom_prog (λf. g f x))"*
shows *"mono_dom_prog (λf. forall_M (g f) xs)"*
⟨*proof*⟩

lemma *sorted_list_set_cong [simp]:*
"sorted_list_of_set (fset FS) = sorted_list_of_set (fset FS') ⟷ FS = FS'"
⟨*proof*⟩

end

5.2 Object (ObjectMonad)

In this theory, we introduce the monadic method setup for the Object class.

theory *ObjectMonad*
imports
BaseMonad
"../classes/ObjectClass"
begin

type_synonym *('object_ptr, 'Object, 'result) dom_prog*
= "((_) heap, exception, 'result) prog"
register_default_tvars *"('object_ptr, 'Object, 'result) dom_prog"*

global_interpretation *l_ptr_kinds_M object_ptr_kinds* **defines** *object_ptr_kinds_M = a_ptr_kinds_M* ⟨*proof*⟩
lemmas *object_ptr_kinds_M_defs = a_ptr_kinds_M_def*

global_interpretation *l_dummy* **defines** *get_M$_{Object}$ = "l_get_M.a_get_M get$_{Object}$"* ⟨*proof*⟩
lemma *get_M_is_l_get_M: "l_get_M get$_{Object}$ type_wf object_ptr_kinds"*
⟨*proof*⟩
lemmas *get_M_defs = get_M$_{Object}$_def[unfolded l_get_M.a_get_M_def[OF get_M_is_l_get_M]]*

adhoc_overloading *get_M ⇌ get_M$_{Object}$*

locale *l_get_M$_{Object}$_lemmas = l_type_wf$_{Object}$*
begin
interpretation *l_get_M get$_{Object}$ type_wf object_ptr_kinds*
⟨*proof*⟩
lemmas *get_M$_{Object}$_ok = get_M_ok[folded get_M$_{Object}$_def]*
lemmas *get_M$_{Object}$_ptr_in_heap = get_M_ptr_in_heap[folded get_M$_{Object}$_def]*
end

global_interpretation *l_get_M$_{Object}$_lemmas type_wf*
⟨*proof*⟩

lemma *object_ptr_kinds_M_reads:*
"reads (⋃object_ptr. {preserved (get_M$_{Object}$ object_ptr RObject.nothing)}) object_ptr_kinds_M h h'"
⟨*proof*⟩

global_interpretation *l_put_M type_wf object_ptr_kinds get$_{Object}$ put$_{Object}$*
rewrites *"a_get_M = get_M$_{Object}$"*

defines *put_M$_{Object}$ = a_put_M*
⟨*proof*⟩
lemmas *put_M_defs = a_put_M_def*
adhoc_overloading *put_M ⇌ put_M$_{Object}$*

locale *l_put_M$_{Object}$_lemmas = l_type_wf$_{Object}$*
begin
interpretation *l_put_M type_wf object_ptr_kinds get$_{Object}$ put$_{Object}$*
⟨*proof*⟩
lemmas *put_M$_{Object}$_ok = put_M_ok[folded put_M$_{Object}$_def]*
lemmas *put_M$_{Object}$_ptr_in_heap = put_M_ptr_in_heap[folded put_M$_{Object}$_def]*
end

global_interpretation *l_put_M$_{Object}$_lemmas type_wf*
⟨*proof*⟩

definition *check_in_heap :: "(_) object_ptr ⇒ (_, unit) dom_prog"*
where
"check_in_heap ptr = do {
h ← get_heap;
(if ptr |∈| object_ptr_kinds h then
return ()
else
error SegmentationFault
)}"

lemma *check_in_heap_ptr_in_heap: "ptr |∈| object_ptr_kinds h ⟷ h ⊢ ok (check_in_heap ptr)"*
⟨*proof*⟩
lemma *check_in_heap_pure [simp]: "pure (check_in_heap ptr) h"*
⟨*proof*⟩
lemma *check_in_heap_is_OK [simp]:*
"ptr |∈| object_ptr_kinds h ⟹ h ⊢ ok (check_in_heap ptr ⤜ f) = h ⊢ ok (f ())"
⟨*proof*⟩
lemma *check_in_heap_returns_result [simp]:*
"ptr |∈| object_ptr_kinds h ⟹ h ⊢ (check_in_heap ptr ⤜ f) →$_r$ x = h ⊢ f () →$_r$ x"
⟨*proof*⟩
lemma *check_in_heap_returns_heap [simp]:*
"ptr |∈| object_ptr_kinds h ⟹ h ⊢ (check_in_heap ptr ⤜ f) →$_h$ h' = h ⊢ f () →$_h$ h'"
⟨*proof*⟩

lemma *check_in_heap_reads:*
"reads {preserved (get_M object_ptr nothing)} (check_in_heap object_ptr) h h'"
⟨*proof*⟩

5.2.1 Invoke

fun *invoke_rec :: "(((_) object_ptr ⇒ bool) × ((_) object_ptr ⇒ 'args*
⇒ (_, 'result) dom_prog)) list ⇒ (_) object_ptr ⇒ 'args
⇒ (_, 'result) dom_prog"
where
"invoke_rec ((P, f)#xs) ptr args = (if P ptr then f ptr args else invoke_rec xs ptr args)"
| "invoke_rec [] ptr args = error InvokeError"

definition *invoke :: "(((_) object_ptr ⇒ bool) × ((_) object_ptr ⇒ 'args*
⇒ (_, 'result) dom_prog)) list
⇒ (_) object_ptr ⇒ 'args ⇒ (_, 'result) dom_prog"
where
"invoke xs ptr args = do { check_in_heap ptr; invoke_rec xs ptr args}"

lemma *invoke_split: "P (invoke ((Pred, f) # xs) ptr args) =*
((¬(Pred ptr) ⟶ P (invoke xs ptr args))

```
  ∧ (Pred ptr ⟶ P (do {check_in_heap ptr; f ptr args})))"
```
⟨*proof*⟩

lemma *invoke_split_asm: "P (invoke ((Pred, f) # xs) ptr args) =*
```
    (¬((¬(Pred ptr) ∧ (¬ P (invoke xs ptr args)))
    ∨ (Pred ptr ∧ (¬ P (do {check_in_heap ptr; f ptr args})))))"
```
⟨*proof*⟩
lemmas *invoke_splits = invoke_split invoke_split_asm*

lemma *invoke_ptr_in_heap: "h ⊢ ok (invoke xs ptr args) ⟹ ptr |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *invoke_pure [simp]: "pure (invoke [] ptr args) h"*
⟨*proof*⟩

lemma *invoke_is_OK [simp]:*
```
  "ptr |∈| object_ptr_kinds h ⟹ Pred ptr
    ⟹ h ⊢ ok (invoke ((Pred, f) # xs) ptr args) = h ⊢ ok (f ptr args)"
```
⟨*proof*⟩
lemma *invoke_returns_result [simp]:*
```
  "ptr |∈| object_ptr_kinds h ⟹ Pred ptr
   ⟹ h ⊢ (invoke ((Pred, f) # xs) ptr args) →r x = h ⊢ f ptr args →r x"
```
⟨*proof*⟩
lemma *invoke_returns_heap [simp]:*
```
  "ptr |∈| object_ptr_kinds h ⟹ Pred ptr
   ⟹ h ⊢ (invoke ((Pred, f) # xs) ptr args) →h h' = h ⊢ f ptr args →h h'"
```
⟨*proof*⟩

lemma *invoke_not [simp]: "¬Pred ptr ⟹ invoke ((Pred, f) # xs) ptr args = invoke xs ptr args"*
⟨*proof*⟩

lemma *invoke_empty [simp]: "¬h ⊢ ok (invoke [] ptr args)"*
⟨*proof*⟩

lemma *invoke_empty_reads [simp]: "∀P ∈ S. reflp P ∧ transp P ⟹ reads S (invoke [] ptr args) h h'"*
⟨*proof*⟩

5.2.2 Modified Heaps

lemma *get_object_ptr_simp [simp]:*
```
  "get_Object object_ptr (put_Object ptr obj h) = (if ptr = object_ptr then Some obj else get object_ptr h)"
```
⟨*proof*⟩

lemma *object_ptr_kinds_simp [simp]: "object_ptr_kinds (put_Object ptr obj h) = object_ptr_kinds h |∪| {|ptr|}"*
⟨*proof*⟩

lemma *type_wf_put_I:*
```
  assumes "type_wf h"
  shows "type_wf (put_Object ptr obj h)"
```
⟨*proof*⟩

lemma *type_wf_put_ptr_not_in_heap_E:*
```
  assumes "type_wf (put_Object ptr obj h)"
  assumes "ptr |∉| object_ptr_kinds h"
  shows "type_wf h"
```
⟨*proof*⟩

lemma *type_wf_put_ptr_in_heap_E:*
```
  assumes "type_wf (put_Object ptr obj h)"
  assumes "ptr |∈| object_ptr_kinds h"
  shows "type_wf h"
```
⟨*proof*⟩

5.2.3 Preserving Types

lemma *type_wf_preserved: "type_wf h = type_wf h'"*
⟨*proof*⟩

lemma *object_ptr_kinds_preserved_small:*
assumes *"⋀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"*
shows *"object_ptr_kinds h = object_ptr_kinds h'"*
⟨*proof*⟩

lemma *object_ptr_kinds_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h' w object_ptr. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"
shows *"object_ptr_kinds h = object_ptr_kinds h'"*
⟨*proof*⟩

lemma *reads_writes_preserved2:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h' x. ∀w ∈ SW. h ⊢ w →$_h$ h' ⟶ preserved (get_M$_{Object}$ ptr getter) h h'"*
shows *"preserved (get_M ptr getter) h h'"*
⟨*proof*⟩
end

5.3 Node (NodeMonad)

In this theory, we introduce the monadic method setup for the Node class.

theory *NodeMonad*
imports
ObjectMonad
"../classes/NodeClass"
begin

type_synonym *('object_ptr, 'node_ptr, 'Object, 'Node, 'result) dom_prog*
= "((_) heap, exception, 'result) prog"
register_default_tvars *"('object_ptr, 'node_ptr, 'Object, 'Node, 'result) dom_prog"*

global_interpretation *l_ptr_kinds_M node_ptr_kinds* **defines** *node_ptr_kinds_M = a_ptr_kinds_M* ⟨*proof*⟩
lemmas *node_ptr_kinds_M_defs = a_ptr_kinds_M_def*

lemma *node_ptr_kinds_M_eq:*
assumes *"|h ⊢ object_ptr_kinds_M|$_r$ = |h' ⊢ object_ptr_kinds_M|$_r$"*
shows *"|h ⊢ node_ptr_kinds_M|$_r$ = |h' ⊢ node_ptr_kinds_M|$_r$"*
⟨*proof*⟩

global_interpretation *l_dummy* **defines** *get_M$_{Node}$ = "l_get_M.a_get_M get$_{Node}$"* ⟨*proof*⟩
lemma *get_M_is_l_get_M: "l_get_M get$_{Node}$ type_wf node_ptr_kinds"*
⟨*proof*⟩
lemmas *get_M_defs = get_M$_{Node}$_def[unfolded l_get_M.a_get_M_def[OF get_M_is_l_get_M]]*

adhoc_overloading *get_M ⇌ get_M$_{Node}$*

locale *l_get_M$_{Node}$_lemmas = l_type_wf$_{Node}$*
begin
sublocale *l_get_M$_{Object}$_lemmas* ⟨*proof*⟩

interpretation *l_get_M get$_{Node}$ type_wf node_ptr_kinds*
⟨*proof*⟩
lemmas *get_M$_{Node}$_ok = get_M_ok[folded get_M$_{Node}$_def]*
end

global_interpretation *l_get_M$_{Node}$_lemmas type_wf* ⟨*proof*⟩

lemma *node_ptr_kinds_M_reads:*
"reads (⋃object_ptr. {preserved (get_M$_{Object}$ object_ptr RObject.nothing)}) node_ptr_kinds_M h h'"
⟨*proof*⟩

global_interpretation *l_put_M type_wf node_ptr_kinds get$_{Node}$ put$_{Node}$*
rewrites *"a_get_M = get_M$_{Node}$"*
defines *put_M$_{Node}$ = a_put_M*
⟨*proof*⟩

lemmas *put_M_defs = a_put_M_def*
adhoc_overloading *put_M ⇌ put_M$_{Node}$*

locale *l_put_M$_{Node}$_lemmas = l_type_wf$_{Node}$*
begin
sublocale *l_put_M$_{Object}$_lemmas* ⟨*proof*⟩

interpretation *l_put_M type_wf node_ptr_kinds get$_{Node}$ put$_{Node}$*
⟨*proof*⟩
lemmas *put_M$_{Node}$_ok = put_M_ok[folded put_M$_{Node}$_def]*
end

global_interpretation *l_put_M$_{Node}$_lemmas type_wf* ⟨*proof*⟩

lemma *get_M_Object_preserved1 [simp]:*
"(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x)) ⟹ h ⊢ put_M$_{Node}$ node_ptr setter v →$_h$ h'

⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Object_preserved2 [simp]:*
"cast node_ptr ≠ object_ptr ⟹ h ⊢ put_M$_{Node}$ node_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩
lemma *get_M_Object_preserved3 [simp]:*
"h ⊢ put_M$_{Node}$ node_ptr setter v →$_h$ h' ⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))

⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Object_preserved4 [simp]:*
"cast node_ptr ≠ object_ptr ⟹ h ⊢ put_M$_{Object}$ object_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩

5.3.1 Modified Heaps

lemma *get_node_ptr_simp [simp]:*
"get$_{Node}$ node_ptr (put$_{Object}$ ptr obj h) = (if ptr = cast node_ptr then cast obj else get node_ptr h)"
⟨*proof*⟩

lemma *node_ptr_kinds_simp [simp]:*
"node_ptr_kinds (put$_{Object}$ ptr obj h)
= node_ptr_kinds h |∪| (if is_node_ptr_kind ptr then {|the (cast ptr)|} else {||})"
⟨*proof*⟩

lemma *type_wf_put_I:*
assumes *"type_wf h"*
assumes *"ObjectClass.type_wf (put$_{Object}$ ptr obj h)"*
assumes *"is_node_ptr_kind ptr ⟹ is_node_kind obj"*
shows *"type_wf (put$_{Object}$ ptr obj h)"*
⟨*proof*⟩

lemma *type_wf_put_ptr_not_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∉| object_ptr_kinds h"*
shows *"type_wf h"*
⟨*proof*⟩

lemma *type_wf_put_ptr_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"ObjectClass.type_wf h"*
assumes *"is_node_ptr_kind ptr ⟹ is_node_kind (the (get ptr h))"*
shows *"type_wf h"*
⟨*proof*⟩

5.3.2 Preserving Types

lemma *node_ptr_kinds_small:*
assumes *"⋀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"*
shows *"node_ptr_kinds h = node_ptr_kinds h'"*
⟨*proof*⟩

lemma *node_ptr_kinds_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h'. ∀w ∈ SW. h ⊢ w →$_h$ h'*
⟶ (∀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h')"
shows *"node_ptr_kinds h = node_ptr_kinds h'"*
⟨*proof*⟩

lemma *type_wf_preserved_small:*
assumes *"⋀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"*
assumes *"⋀node_ptr. preserved (get_M$_{Node}$ node_ptr RNode.nothing) h h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩

lemma *type_wf_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ ∀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ ∀node_ptr. preserved (get_M$_{Node}$ node_ptr RNode.nothing) h h'"
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩
end

5.4 Element (ElementMonad)

In this theory, we introduce the monadic method setup for the Element class.

theory *ElementMonad*
imports
NodeMonad
"ElementClass"
begin

type_synonym *('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,*
'shadow_root_ptr, 'Object, 'Node, 'Element,'result) dom_prog
= "((_) heap, exception, 'result) prog"
register_default_tvars *"('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr,*
'document_ptr, 'shadow_root_ptr, 'Object, 'Node, 'Element,'result) dom_prog"

global_interpretation *l_ptr_kinds_M element_ptr_kinds* **defines** *element_ptr_kinds_M = a_ptr_kinds_M* ⟨*proof*⟩
lemmas *element_ptr_kinds_M_defs = a_ptr_kinds_M_def*

lemma *element_ptr_kinds_M_eq:*
assumes *"|h ⊢ node_ptr_kinds_M|$_r$ = |h' ⊢ node_ptr_kinds_M|$_r$"*
shows *"|h ⊢ element_ptr_kinds_M|$_r$ = |h' ⊢ element_ptr_kinds_M|$_r$"*
⟨*proof*⟩

lemma *element_ptr_kinds_M_reads:*
"reads (⋃element_ptr. {preserved (get_M$_{Object}$ element_ptr RObject.nothing)}) element_ptr_kinds_M h h'"
⟨*proof*⟩

global_interpretation *l_dummy* **defines** *get_M$_{Element}$ = "l_get_M.a_get_M get$_{Element}$"* ⟨*proof*⟩
lemma *get_M_is_l_get_M: "l_get_M get$_{Element}$ type_wf element_ptr_kinds"*
⟨*proof*⟩
lemmas *get_M_defs = get_M$_{Element}$_def[unfolded l_get_M.a_get_M_def[OF get_M_is_l_get_M]]*

adhoc_overloading *get_M* ⇌ *get_M$_{Element}$*

locale *l_get_M$_{Element}$_lemmas = l_type_wf$_{Element}$*
begin
sublocale *l_get_M$_{Node}$_lemmas* ⟨*proof*⟩

interpretation *l_get_M get$_{Element}$ type_wf element_ptr_kinds*
⟨*proof*⟩
lemmas *get_M$_{Element}$_ok = get_M_ok[folded get_M$_{Element}$_def]*
lemmas *get_M$_{Element}$_ptr_in_heap = get_M_ptr_in_heap[folded get_M$_{Element}$_def]*
end

global_interpretation *l_get_M$_{Element}$_lemmas type_wf* ⟨*proof*⟩

global_interpretation *l_put_M type_wf element_ptr_kinds get$_{Element}$ put$_{Element}$*
rewrites *"a_get_M = get_M$_{Element}$"*
defines *put_M$_{Element}$ = a_put_M*
⟨*proof*⟩

lemmas *put_M_defs = a_put_M_def*
adhoc_overloading *put_M* ⇌ *put_M$_{Element}$*

locale *l_put_M$_{Element}$_lemmas = l_type_wf$_{Element}$*
begin
sublocale *l_put_M$_{Node}$_lemmas* ⟨*proof*⟩

interpretation *l_put_M type_wf element_ptr_kinds get$_{Element}$ put$_{Element}$*
⟨*proof*⟩

lemmas *put_M$_{Element}$_ok = put_M_ok[folded put_M$_{Element}$_def]*
end

global_interpretation *l_put_M$_{Element}$_lemmas type_wf* ⟨*proof*⟩

lemma *element_put_get [simp]:*
"h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h' ⟹ (⋀x. getter (setter (λ_. v) x) = v)
⟹ h' ⊢ get_M$_{Element}$ element_ptr getter →$_r$ v"
⟨*proof*⟩
lemma *get_M_Element_preserved1 [simp]:*
"element_ptr ≠ element_ptr' ⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Element}$ element_ptr' getter) h h'"
⟨*proof*⟩
lemma *element_put_get_preserved [simp]:*
"(⋀x. getter (setter (λ_. v) x) = getter x) ⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Element}$ element_ptr' getter) h h'"
⟨*proof*⟩
lemma *get_M_Element_preserved3 [simp]:*
"(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h' ⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩
lemma *get_M_Element_preserved4 [simp]:*
"(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h' ⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Element_preserved5 [simp]:*
"cast element_ptr ≠ node_ptr ⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩
lemma *get_M_Element_preserved6 [simp]:*
"h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Element_preserved7 [simp]:*
"cast element_ptr ≠ node_ptr ⟹ h ⊢ put_M$_{Node}$ node_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Element}$ element_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Element_preserved8 [simp]:*
"cast element_ptr ≠ object_ptr ⟹ h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩
lemma *get_M_Element_preserved9 [simp]:*
"h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩

lemma *get_M_Element_preserved10 [simp]:*
"cast element_ptr ≠ object_ptr ⟹ h ⊢ put_M$_{Object}$ object_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Element}$ element_ptr getter) h h'"
⟨*proof*⟩

5.4.1 Creating Elements

definition *new_element :: "(_, (_) element_ptr) dom_prog"*
where
"new_element = do {
h ← get_heap;
(new_ptr, h') ← return (new$_{Element}$ h);
return_heap h';
return new_ptr
}"

lemma *new_element_ok [simp]:*

```
  "h ⊢ ok new_element"
```
⟨*proof*⟩

lemma *new_element_ptr_in_heap:*
```
  assumes "h ⊢ new_element →_h h'"
    and "h ⊢ new_element →_r new_element_ptr"
  shows "new_element_ptr |∈| element_ptr_kinds h'"
```
⟨*proof*⟩

lemma *new_element_ptr_not_in_heap:*
```
  assumes "h ⊢ new_element →_h h'"
    and "h ⊢ new_element →_r new_element_ptr"
  shows "new_element_ptr |∉| element_ptr_kinds h"
```
⟨*proof*⟩

lemma *new_element_new_ptr:*
```
  assumes "h ⊢ new_element →_h h'"
    and "h ⊢ new_element →_r new_element_ptr"
  shows "object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast new_element_ptr|}"
```
⟨*proof*⟩

lemma *new_element_is_element_ptr:*
```
  assumes "h ⊢ new_element →_r new_element_ptr"
  shows "is_element_ptr new_element_ptr"
```
⟨*proof*⟩

lemma *new_element_child_nodes:*
```
  assumes "h ⊢ new_element →_h h'"
  assumes "h ⊢ new_element →_r new_element_ptr"
  shows "h' ⊢ get_M new_element_ptr child_nodes →_r []"
```
⟨*proof*⟩

lemma *new_element_tag_name:*
```
  assumes "h ⊢ new_element →_h h'"
  assumes "h ⊢ new_element →_r new_element_ptr"
  shows "h' ⊢ get_M new_element_ptr tag_name →_r ''''"
```
⟨*proof*⟩

lemma *new_element_attrs:*
```
  assumes "h ⊢ new_element →_h h'"
  assumes "h ⊢ new_element →_r new_element_ptr"
  shows "h' ⊢ get_M new_element_ptr attrs →_r fmempty"
```
⟨*proof*⟩

lemma *new_element_shadow_root_opt:*
```
  assumes "h ⊢ new_element →_h h'"
  assumes "h ⊢ new_element →_r new_element_ptr"
  shows "h' ⊢ get_M new_element_ptr shadow_root_opt →_r None"
```
⟨*proof*⟩

lemma *new_element_get_M$_{Object}$:*
```
  "h ⊢ new_element →_h h' ⟹ h ⊢ new_element →_r new_element_ptr ⟹ ptr ≠ cast new_element_ptr
   ⟹ preserved (get_M_Object ptr getter) h h'"
```
⟨*proof*⟩

lemma *new_element_get_M$_{Node}$:*
```
  "h ⊢ new_element →_h h' ⟹ h ⊢ new_element →_r new_element_ptr ⟹ ptr ≠ cast new_element_ptr
    ⟹ preserved (get_M_Node ptr getter) h h'"
```
⟨*proof*⟩

lemma *new_element_get_M$_{Element}$:*
```
  "h ⊢ new_element →_h h' ⟹ h ⊢ new_element →_r new_element_ptr ⟹ ptr ≠ new_element_ptr
    ⟹ preserved (get_M_Element ptr getter) h h'"
```
⟨*proof*⟩

5.4.2 Modified Heaps

lemma *get_Element_ptr_simp [simp]:*
"get$_{Element}$ element_ptr (put$_{Object}$ ptr obj h)
= (if ptr = cast element_ptr then cast obj else get element_ptr h)"
⟨*proof*⟩

lemma *element_ptr_kinds_simp [simp]:*
"element_ptr_kinds (put$_{Object}$ ptr obj h)
= element_ptr_kinds h |∪| (if is_element_ptr_kind ptr then {|the (cast ptr)|} else {||})"
⟨*proof*⟩

lemma *type_wf_put_I:*
assumes *"type_wf h"*
assumes *"NodeClass.type_wf (put$_{Object}$ ptr obj h)"*
assumes *"is_element_ptr_kind ptr ⟹ is_element_kind obj"*
shows *"type_wf (put$_{Object}$ ptr obj h)"*
⟨*proof*⟩

lemma *type_wf_put_ptr_not_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∉| object_ptr_kinds h"*
shows *"type_wf h"*
⟨*proof*⟩

lemma *type_wf_put_ptr_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"NodeClass.type_wf h"*
assumes *"is_element_ptr_kind ptr ⟹ is_element_kind (the (get ptr h))"*
shows *"type_wf h"*
⟨*proof*⟩

5.4.3 Preserving Types

lemma *new_element_type_wf_preserved [simp]: "h ⊢ new_element →$_h$ h' ⟹ type_wf h = type_wf h'"*
⟨*proof*⟩

locale *l_new_element = l_type_wf +*
assumes *new_element_types_preserved: "h ⊢ new_element →$_h$ h' ⟹ type_wf h = type_wf h'"*

lemma *new_element_is_l_new_element: "l_new_element type_wf"*
⟨*proof*⟩

lemma *put_M$_{Element}$_tag_name_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr tag_name_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_child_nodes_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr child_nodes_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_attrs_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr attrs_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_shadow_root_opt_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr shadow_root_opt_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M_pointers_preserved:*
assumes *"h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'"*

shows *"object_ptr_kinds h = object_ptr_kinds h'"*
⟨*proof*⟩

lemma *element_ptr_kinds_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h'. ∀w ∈ SW. h ⊢ w →$_h$ h'*
⟶ (∀object_ptr. preserved (get_M_{Object} object_ptr RObject.nothing) h h')"
shows *"element_ptr_kinds h = element_ptr_kinds h'"*
⟨*proof*⟩

lemma *element_ptr_kinds_small:*
assumes *"⋀object_ptr. preserved (get_M_{Object} object_ptr RObject.nothing) h h'"*
shows *"element_ptr_kinds h = element_ptr_kinds h'"*
⟨*proof*⟩

lemma *type_wf_preserved_small:*
assumes *"⋀object_ptr. preserved (get_M_{Object} object_ptr RObject.nothing) h h'"*
assumes *"⋀node_ptr. preserved (get_M_{Node} node_ptr RNode.nothing) h h'"*
assumes *"⋀element_ptr. preserved (get_$M_{Element}$ element_ptr RElement.nothing) h h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩

lemma *type_wf_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →$_h$ h'"*
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ ∀object_ptr. preserved (get_M_{Object} object_ptr RObject.nothing) h h'"
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ ∀node_ptr. preserved (get_M_{Node} node_ptr RNode.nothing) h h'"
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →$_h$ h'*
⟹ ∀element_ptr. preserved (get_$M_{Element}$ element_ptr RElement.nothing) h h'"
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩

lemma *type_wf_drop: "type_wf h ⟹ type_wf (Heap (fmdrop ptr (the_heap h)))"*
⟨*proof*⟩

end

5.5 CharacterData (CharacterDataMonad)

In this theory, we introduce the monadic method setup for the CharacterData class.

theory *CharacterDataMonad*
imports
ElementMonad
"../classes/CharacterDataClass"
begin

type_synonym *('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,*
'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData, 'result) dom_prog
= "((_) heap, exception, 'result) prog"
register_default_tvars
"('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr, 'shadow_root_ptr,
'Object, 'Node, 'Element, 'CharacterData, 'result) dom_prog"

global_interpretation *l_ptr_kinds_M character_data_ptr_kinds*
defines *character_data_ptr_kinds_M = a_ptr_kinds_M* ⟨*proof*⟩
lemmas *character_data_ptr_kinds_M_defs = a_ptr_kinds_M_def*

lemma *character_data_ptr_kinds_M_eq:*
assumes "$|h \vdash$ *node_ptr_kinds_M*$|_r$ = $|h' \vdash$ *node_ptr_kinds_M*$|_r$"
shows "$|h \vdash$ *character_data_ptr_kinds_M*$|_r$ = $|h' \vdash$ *character_data_ptr_kinds_M*$|_r$"
⟨*proof*⟩

lemma *character_data_ptr_kinds_M_reads:*
"*reads* ($\bigcup$*node_ptr. {preserved (get_M*$_{Object}$ *node_ptr RObject.nothing)}) character_data_ptr_kinds_M h h'*"
⟨*proof*⟩

global_interpretation *l_dummy* **defines** *get_M*$_{CharacterData}$ = "*l_get_M.a_get_M get*$_{CharacterData}$" ⟨*proof*⟩
lemma *get_M_is_l_get_M: "l_get_M get*$_{CharacterData}$ *type_wf character_data_ptr_kinds"*
⟨*proof*⟩
lemmas *get_M_defs = get_M*$_{CharacterData}$*_def[unfolded l_get_M.a_get_M_def[OF get_M_is_l_get_M]]*

adhoc_overloading *get_M* $\rightleftharpoons$ *get_M*$_{CharacterData}$

locale *l_get_M*$_{CharacterData}$*_lemmas = l_type_wf*$_{CharacterData}$
begin
sublocale *l_get_M*$_{Element}$*_lemmas* ⟨*proof*⟩

interpretation *l_get_M get*$_{CharacterData}$ *type_wf character_data_ptr_kinds*
⟨*proof*⟩
lemmas *get_M*$_{CharacterData}$*_ok = get_M_ok[folded get_M*$_{CharacterData}$*_def]*
end

global_interpretation *l_get_M*$_{CharacterData}$*_lemmas type_wf* ⟨*proof*⟩

global_interpretation *l_put_M type_wf character_data_ptr_kinds get*$_{CharacterData}$ *put*$_{CharacterData}$
rewrites "*a_get_M = get_M*$_{CharacterData}$" **defines** *put_M*$_{CharacterData}$ = *a_put_M*
⟨*proof*⟩

lemmas *put_M_defs = a_put_M_def*
adhoc_overloading *put_M* $\rightleftharpoons$ *put_M*$_{CharacterData}$

locale *l_put_M*$_{CharacterData}$*_lemmas = l_type_wf*$_{CharacterData}$
begin
sublocale *l_put_M*$_{Element}$*_lemmas* ⟨*proof*⟩

interpretation *l_put_M type_wf character_data_ptr_kinds get*$_{CharacterData}$ *put*$_{CharacterData}$
⟨*proof*⟩
lemmas *put_M*$_{CharacterData}$*_ok = put_M_ok[folded put_M*$_{CharacterData}$*_def]*
end

global_interpretation *l_put_M*$_{CharacterData}$*_lemmas type_wf* ⟨*proof*⟩

lemma *CharacterData_simp1 [simp]:*
"($\bigwedge$*x. getter (setter* (λ*_. v) x) = v*) $\Longrightarrow$ *h* $\vdash$ *put_M*$_{CharacterData}$ *character_data_ptr setter v* $\rightarrow_h$ *h'*
$\Longrightarrow$ *h'* $\vdash$ *get_M*$_{CharacterData}$ *character_data_ptr getter* $\rightarrow_r$ *v*"
⟨*proof*⟩
lemma *CharacterData_simp2 [simp]:*
"*character_data_ptr* $\neq$ *character_data_ptr'*
$\Longrightarrow$ *h* $\vdash$ *put_M*$_{CharacterData}$ *character_data_ptr setter v* $\rightarrow_h$ *h'*
$\Longrightarrow$ *preserved (get_M*$_{CharacterData}$ *character_data_ptr' getter) h h'*"
⟨*proof*⟩
lemma *CharacterData_simp3 [simp]: "*
(*$\bigwedge$x. getter (setter* (λ*_. v) x) = getter x*)
$\Longrightarrow$ *h* $\vdash$ *put_M*$_{CharacterData}$ *character_data_ptr setter v* $\rightarrow_h$ *h'*
$\Longrightarrow$ *preserved (get_M*$_{CharacterData}$ *character_data_ptr' getter) h h'*"
⟨*proof*⟩

lemma *CharacterData_simp4 [simp]:*
"h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Element}$ element_ptr getter) h h'"
⟨*proof*⟩
lemma *CharacterData_simp5 [simp]:*
"h ⊢ put_M$_{Element}$ element_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{CharacterData}$ character_data_ptr getter) h h'"
⟨*proof*⟩
lemma *CharacterData_simp6 [simp]:*
"(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩
lemma *CharacterData_simp7 [simp]:*
"(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩

lemma *CharacterData_simp8 [simp]:*
"cast character_data_ptr ≠ node_ptr
⟹ h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩
lemma *CharacterData_simp9 [simp]:*
"h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ preserved (get_M$_{Node}$ node_ptr getter) h h'"
⟨*proof*⟩
lemma *CharacterData_simp10 [simp]:*
"cast character_data_ptr ≠ node_ptr
⟹ h ⊢ put_M$_{Node}$ node_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{CharacterData}$ character_data_ptr getter) h h'"
⟨*proof*⟩

lemma *CharacterData_simp11 [simp]:*
"cast character_data_ptr ≠ object_ptr
⟹ h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩

lemma *CharacterData_simp12 [simp]:*
"h ⊢ put_M$_{CharacterData}$ character_data_ptr setter v →$_h$ h'
⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
⟹ preserved (get_M$_{Object}$ object_ptr getter) h h'"
⟨*proof*⟩

lemma *CharacterData_simp13 [simp]:*
"cast character_data_ptr ≠ object_ptr ⟹ h ⊢ put_M$_{Object}$ object_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{CharacterData}$ character_data_ptr getter) h h'"
⟨*proof*⟩

lemma *new_element_get_M$_{CharacterData}$:*
"h ⊢ new_element →$_h$ h' ⟹ preserved (get_M$_{CharacterData}$ ptr getter) h h'"
⟨*proof*⟩

5.5.1 Creating CharacterData

definition *new_character_data :: "(_, (_) character_data_ptr) dom_prog"*
where
"new_character_data = do {
h ← get_heap;
(new_ptr, h') ← return (new$_{CharacterData}$ h);

```
    return_heap h';
    return new_ptr
  }"

lemma new_character_data_ok [simp]:
  "h ⊢ ok new_character_data"
  ⟨proof⟩

lemma new_character_data_ptr_in_heap:
  assumes "h ⊢ new_character_data →_h h'"
    and "h ⊢ new_character_data →_r new_character_data_ptr"
  shows "new_character_data_ptr |∈| character_data_ptr_kinds h'"
  ⟨proof⟩

lemma new_character_data_ptr_not_in_heap:
  assumes "h ⊢ new_character_data →_h h'"
    and "h ⊢ new_character_data →_r new_character_data_ptr"
  shows "new_character_data_ptr |∉| character_data_ptr_kinds h"
  ⟨proof⟩

lemma new_character_data_new_ptr:
  assumes "h ⊢ new_character_data →_h h'"
    and "h ⊢ new_character_data →_r new_character_data_ptr"
  shows "object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast new_character_data_ptr|}"
  ⟨proof⟩

lemma new_character_data_is_character_data_ptr:
  assumes "h ⊢ new_character_data →_r new_character_data_ptr"
  shows "is_character_data_ptr new_character_data_ptr"
  ⟨proof⟩

lemma new_character_data_child_nodes:
  assumes "h ⊢ new_character_data →_h h'"
  assumes "h ⊢ new_character_data →_r new_character_data_ptr"
  shows "h' ⊢ get_M new_character_data_ptr val →_r ''''"
  ⟨proof⟩

lemma new_character_data_get_M_Object:
  "h ⊢ new_character_data →_h h' ⟹ h ⊢ new_character_data →_r new_character_data_ptr
    ⟹ ptr ≠ cast new_character_data_ptr ⟹ preserved (get_M_Object ptr getter) h h'"
  ⟨proof⟩
lemma new_character_data_get_M_Node:
  "h ⊢ new_character_data →_h h' ⟹ h ⊢ new_character_data →_r new_character_data_ptr
    ⟹ ptr ≠ cast new_character_data_ptr ⟹ preserved (get_M_Node ptr getter) h h'"
  ⟨proof⟩
lemma new_character_data_get_M_Element:
  "h ⊢ new_character_data →_h h' ⟹ h ⊢ new_character_data →_r new_character_data_ptr
    ⟹ preserved (get_M_Element ptr getter) h h'"
  ⟨proof⟩
lemma new_character_data_get_M_CharacterData:
  "h ⊢ new_character_data →_h h' ⟹ h ⊢ new_character_data →_r new_character_data_ptr
    ⟹ ptr ≠ new_character_data_ptr ⟹ preserved (get_M_CharacterData ptr getter) h h'"
  ⟨proof⟩
```

5.5.2 Modified Heaps

```
lemma get_CharacterData_ptr_simp [simp]:
  "get_CharacterData character_data_ptr (put_Object ptr obj h)
     = (if ptr = cast character_data_ptr then cast obj else get character_data_ptr h)"
  ⟨proof⟩

lemma Character_data_ptr_kinds_simp [simp]:
  "character_data_ptr_kinds (put_Object ptr obj h) = character_data_ptr_kinds h |∪|
```

```
                    (if is_character_data_ptr_kind ptr then {|the (cast ptr)|} else {||})"
```
⟨*proof*⟩

lemma *type_wf_put_I:*
assumes *"type_wf h"*
assumes *"ElementClass.type_wf (put$_{Object}$ ptr obj h)"*
assumes *"is_character_data_ptr_kind ptr ⟹ is_character_data_kind obj"*
shows *"type_wf (put$_{Object}$ ptr obj h)"*
⟨*proof*⟩

lemma *type_wf_put_ptr_not_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∉| object_ptr_kinds h"*
shows *"type_wf h"*
⟨*proof*⟩

lemma *type_wf_put_ptr_in_heap_E:*
assumes *"type_wf (put$_{Object}$ ptr obj h)"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"ElementClass.type_wf h"*
assumes *"is_character_data_ptr_kind ptr ⟹ is_character_data_kind (the (get ptr h))"*
shows *"type_wf h"*
⟨*proof*⟩

5.5.3 Preserving Types

lemma *new_element_type_wf_preserved [simp]:*
assumes *"h ⊢ new_element →$_h$ h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩

lemma *new_element_is_l_new_element: "l_new_element type_wf"*
⟨*proof*⟩

lemma *put_M$_{Element}$_tag_name_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr tag_name_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_child_nodes_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr child_nodes_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_attrs_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr attrs_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_shadow_root_opt_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr shadow_root_opt_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *new_character_data_type_wf_preserved [simp]:*
"h ⊢ new_character_data →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

locale *l_new_character_data = l_type_wf +*
assumes *new_character_data_types_preserved: "h ⊢ new_character_data →$_h$ h' ⟹ type_wf h = type_wf h'"*

lemma *new_character_data_is_l_new_character_data: "l_new_character_data type_wf"*
⟨*proof*⟩

lemma *put_M$_{CharacterData}$_val_type_wf_preserved [simp]:*

```
  "h ⊢ put_M character_data_ptr val_update v →_h h' ⟹ type_wf h = type_wf h'"
  ⟨proof⟩

lemma character_data_ptr_kinds_small:
  assumes "⋀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
  shows "character_data_ptr_kinds h = character_data_ptr_kinds h'"
  ⟨proof⟩

lemma character_data_ptr_kinds_preserved:
  assumes "writes SW setter h h'"
  assumes "h ⊢ setter →_h h'"
  assumes "⋀h h'. ∀w ∈ SW. h ⊢ w →_h h'
        ⟶ (∀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h')"
  shows "character_data_ptr_kinds h = character_data_ptr_kinds h'"
  ⟨proof⟩


lemma type_wf_preserved_small:
  assumes "⋀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
  assumes "⋀node_ptr. preserved (get_M_Node node_ptr RNode.nothing) h h'"
  assumes "⋀element_ptr. preserved (get_M_Element element_ptr RElement.nothing) h h'"
  assumes "⋀character_data_ptr. preserved (get_M_CharacterData character_data_ptr
                                        RCharacterData.nothing) h h'"
  shows "type_wf h = type_wf h'"
  ⟨proof⟩

lemma type_wf_preserved:
  assumes "writes SW setter h h'"
  assumes "h ⊢ setter →_h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀node_ptr. preserved (get_M_Node node_ptr RNode.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀element_ptr. preserved (get_M_Element element_ptr RElement.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀character_data_ptr. preserved (get_M_CharacterData character_data_ptr
                                        RCharacterData.nothing) h h'"
  shows "type_wf h = type_wf h'"
⟨proof⟩

lemma type_wf_drop: "type_wf h ⟹ type_wf (Heap (fmdrop ptr (the_heap h)))"
  ⟨proof⟩

end
```

5.6 Document (DocumentMonad)

In this theory, we introduce the monadic method setup for the Document class.

```
theory DocumentMonad
  imports
    CharacterDataMonad
    "../classes/DocumentClass"
begin

type_synonym ('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
    'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData, 'Document, 'result) dom_prog
  = "((_) heap, exception, 'result) prog"
register_default_tvars "('object_ptr, 'node_ptr, 'element_ptr, 'character_data_ptr, 'document_ptr,
          'shadow_root_ptr, 'Object, 'Node, 'Element, 'CharacterData, 'Document, 'result) dom_prog"
```

global_interpretation *l_ptr_kinds_M document_ptr_kinds* **defines** *document_ptr_kinds_M = a_ptr_kinds_M* ⟨*proof*⟩
lemmas *document_ptr_kinds_M_defs = a_ptr_kinds_M_def*

lemma *document_ptr_kinds_M_eq:*
assumes *"|h ⊢ object_ptr_kinds_M|$_r$ = |h' ⊢ object_ptr_kinds_M|$_r$"*
shows *"|h ⊢ document_ptr_kinds_M|$_r$ = |h' ⊢ document_ptr_kinds_M|$_r$"*
⟨*proof*⟩

lemma *document_ptr_kinds_M_reads:*
"reads (⋃object_ptr. {preserved (get_M$_{Object}$ object_ptr RObject.nothing)}) document_ptr_kinds_M h h'"
⟨*proof*⟩

global_interpretation *l_dummy* **defines** *get_M$_{Document}$ = "l_get_M.a_get_M get$_{Document}$"* ⟨*proof*⟩
lemma *get_M_is_l_get_M: "l_get_M get$_{Document}$ type_wf document_ptr_kinds"*
⟨*proof*⟩
lemmas *get_M_defs = get_M$_{Document}$_def[unfolded l_get_M.a_get_M_def[OF get_M_is_l_get_M]]*

adhoc_overloading *get_M ⇌ get_M$_{Document}$*

locale *l_get_M$_{Document}$_lemmas = l_type_wf$_{Document}$*
begin
sublocale *l_get_M$_{CharacterData}$_lemmas* ⟨*proof*⟩

interpretation *l_get_M get$_{Document}$ type_wf document_ptr_kinds*
⟨*proof*⟩
lemmas *get_M$_{Document}$_ok = get_M_ok[folded get_M$_{Document}$_def]*
end

global_interpretation *l_get_M$_{Document}$_lemmas type_wf* ⟨*proof*⟩

global_interpretation *l_put_M type_wf document_ptr_kinds get$_{Document}$ put$_{Document}$*
rewrites *"a_get_M = get_M$_{Document}$"* **defines** *put_M$_{Document}$ = a_put_M*
⟨*proof*⟩

lemmas *put_M_defs = a_put_M_def*
adhoc_overloading *put_M ⇌ put_M$_{Document}$*

locale *l_put_M$_{Document}$_lemmas = l_type_wf$_{Document}$*
begin
sublocale *l_put_M$_{CharacterData}$_lemmas* ⟨*proof*⟩

interpretation *l_put_M type_wf document_ptr_kinds get$_{Document}$ put$_{Document}$*
⟨*proof*⟩
lemmas *put_M$_{Document}$_ok = put_M_ok[folded put_M$_{Document}$_def]*
end

global_interpretation *l_put_M$_{Document}$_lemmas type_wf* ⟨*proof*⟩

lemma *document_put_get [simp]:*
"h ⊢ put_M$_{Document}$ document_ptr setter v →$_h$ h'
⟹ (⋀x. getter (setter (λ_. v) x) = v)
⟹ h' ⊢ get_M$_{Document}$ document_ptr getter →$_r$ v"
⟨*proof*⟩
lemma *get_M_Mdocument_preserved1 [simp]:*
"document_ptr ≠ document_ptr'
⟹ h ⊢ put_M$_{Document}$ document_ptr setter v →$_h$ h'
⟹ preserved (get_M$_{Document}$ document_ptr' getter) h h'"
⟨*proof*⟩
lemma *document_put_get_preserved [simp]:*
"h ⊢ put_M$_{Document}$ document_ptr setter v →$_h$ h'

```
   ⟹ (⋀x. getter (setter (λ_. v) x) = getter x)
   ⟹ preserved (get_M_Document document_ptr' getter) h h'"
  ⟨proof⟩

lemma get_M_Mdocument_preserved2 [simp]:
  "h ⊢ put_M_Document document_ptr setter v →_h h' ⟹ preserved (get_M_Node node_ptr getter) h h'"
  ⟨proof⟩

lemma get_M_Mdocument_preserved3 [simp]:
  "cast document_ptr ≠ object_ptr
   ⟹ h ⊢ put_M_Document document_ptr setter v →_h h'
   ⟹ preserved (get_M_Object object_ptr getter) h h'"
  ⟨proof⟩
lemma get_M_Mdocument_preserved4  [simp]:
  "h ⊢ put_M_Document document_ptr setter v →_h h'
   ⟹ (⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
   ⟹ preserved (get_M_Object object_ptr getter) h h'"
  ⟨proof⟩

lemma get_M_Mdocument_preserved5 [simp]:
  "cast document_ptr ≠ object_ptr
  ⟹ h ⊢ put_M_Object object_ptr setter v →_h h'
  ⟹ preserved (get_M_Document document_ptr getter) h h'"
  ⟨proof⟩

lemma get_M_Mdocument_preserved6 [simp]:
  "h ⊢ put_M_Document document_ptr setter v →_h h' ⟹ preserved (get_M_Element element_ptr getter) h h'"
  ⟨proof⟩
lemma get_M_Mdocument_preserved7 [simp]:
  "h ⊢ put_M_Element element_ptr setter v →_h h' ⟹ preserved (get_M_Document document_ptr getter) h h'"
  ⟨proof⟩
lemma get_M_Mdocument_preserved8 [simp]:
  "h ⊢ put_M_Document document_ptr setter v →_h h'
    ⟹ preserved (get_M_CharacterData character_data_ptr getter) h h'"
  ⟨proof⟩
lemma get_M_Mdocument_preserved9 [simp]:
  "h ⊢ put_M_CharacterData character_data_ptr setter v →_h h'
    ⟹ preserved (get_M_Document document_ptr getter) h h'"
  ⟨proof⟩
lemma get_M_Mdocument_preserved10 [simp]:
  "(⋀x. getter (cast (setter (λ_. v) x)) = getter (cast x))
    ⟹ h ⊢ put_M_Document document_ptr setter v →_h h' ⟹ preserved (get_M_Object object_ptr getter) h
h'"
  ⟨proof⟩

lemma new_element_get_M_Document:
  "h ⊢ new_element →_h h' ⟹ preserved (get_M_Document ptr getter) h h'"
  ⟨proof⟩

lemma new_character_data_get_M_Document:
  "h ⊢ new_character_data →_h h' ⟹ preserved (get_M_Document ptr getter) h h'"
  ⟨proof⟩
```

5.6.1 Creating Documents

```
definition new_document :: "(_, (_) document_ptr) dom_prog"
  where
    "new_document = do {
      h ← get_heap;
      (new_ptr, h') ← return (new_Document h);
      return_heap h';
      return new_ptr
    }"
```

lemma *new_document_ok [simp]:*
"h ⊢ ok new_document"
⟨*proof*⟩

lemma *new_document_ptr_in_heap:*
assumes *"h ⊢ new_document →_h h'"*
and *"h ⊢ new_document →_r new_document_ptr"*
shows *"new_document_ptr |∈| document_ptr_kinds h'"*
⟨*proof*⟩

lemma *new_document_ptr_not_in_heap:*
assumes *"h ⊢ new_document →_h h'"*
and *"h ⊢ new_document →_r new_document_ptr"*
shows *"new_document_ptr |∉| document_ptr_kinds h"*
⟨*proof*⟩

lemma *new_document_new_ptr:*
assumes *"h ⊢ new_document →_h h'"*
and *"h ⊢ new_document →_r new_document_ptr"*
shows *"object_ptr_kinds h' = object_ptr_kinds h |∪| {|cast new_document_ptr|}"*
⟨*proof*⟩

lemma *new_document_is_document_ptr:*
assumes *"h ⊢ new_document →_r new_document_ptr"*
shows *"is_document_ptr new_document_ptr"*
⟨*proof*⟩

lemma *new_document_doctype:*
assumes *"h ⊢ new_document →_h h'"*
assumes *"h ⊢ new_document →_r new_document_ptr"*
shows *"h' ⊢ get_M new_document_ptr doctype →_r ''''"*
⟨*proof*⟩

lemma *new_document_document_element:*
assumes *"h ⊢ new_document →_h h'"*
assumes *"h ⊢ new_document →_r new_document_ptr"*
shows *"h' ⊢ get_M new_document_ptr document_element →_r None"*
⟨*proof*⟩

lemma *new_document_disconnected_nodes:*
assumes *"h ⊢ new_document →_h h'"*
assumes *"h ⊢ new_document →_r new_document_ptr"*
shows *"h' ⊢ get_M new_document_ptr disconnected_nodes →_r []"*
⟨*proof*⟩

lemma *new_document_get_M$_{Object}$:*
"h ⊢ new_document →_h h' ⟹ h ⊢ new_document →_r new_document_ptr
⟹ ptr ≠ cast new_document_ptr ⟹ preserved (get_M$_{Object}$ ptr getter) h h'"
⟨*proof*⟩
lemma *new_document_get_M$_{Node}$:*
"h ⊢ new_document →_h h' ⟹ h ⊢ new_document →_r new_document_ptr
⟹ preserved (get_M$_{Node}$ ptr getter) h h'"
⟨*proof*⟩
lemma *new_document_get_M$_{Element}$:*
"h ⊢ new_document →_h h' ⟹ h ⊢ new_document →_r new_document_ptr
⟹ preserved (get_M$_{Element}$ ptr getter) h h'"
⟨*proof*⟩
lemma *new_document_get_M$_{CharacterData}$:*
"h ⊢ new_document →_h h' ⟹ h ⊢ new_document →_r new_document_ptr
⟹ preserved (get_M$_{CharacterData}$ ptr getter) h h'"
⟨*proof*⟩

lemma *new_document_get_M$_{Document}$:*
"h ⊢ new_document →$_h$ h'
⟹ h ⊢ new_document →$_r$ new_document_ptr ⟹ ptr ≠ new_document_ptr
⟹ preserved (get_M$_{Document}$ ptr getter) h h'"
⟨*proof*⟩

5.6.2 Modified Heaps

lemma *get_document_ptr_simp [simp]:*
"get$_{Document}$ document_ptr (put$_{Object}$ ptr obj h)
= (if ptr = cast document_ptr then cast obj else get document_ptr h)"
⟨*proof*⟩

lemma *document_ptr_kidns_simp [simp]:*
"document_ptr_kinds (put$_{Object}$ ptr obj h)
= document_ptr_kinds h |∪| (if is_document_ptr_kind ptr then {|the (cast ptr)|} else {||})"
⟨*proof*⟩

lemma *type_wf_put_I:*
assumes "type_wf h"
assumes "CharacterDataClass.type_wf (put$_{Object}$ ptr obj h)"
assumes "is_document_ptr_kind ptr ⟹ is_document_kind obj"
shows "type_wf (put$_{Object}$ ptr obj h)"
⟨*proof*⟩

lemma *type_wf_put_ptr_not_in_heap_E:*
assumes "type_wf (put$_{Object}$ ptr obj h)"
assumes "ptr |∉| object_ptr_kinds h"
shows "type_wf h"
⟨*proof*⟩

lemma *type_wf_put_ptr_in_heap_E:*
assumes "type_wf (put$_{Object}$ ptr obj h)"
assumes "ptr |∈| object_ptr_kinds h"
assumes "CharacterDataClass.type_wf h"
assumes "is_document_ptr_kind ptr ⟹ is_document_kind (the (get ptr h))"
shows "type_wf h"
⟨*proof*⟩

5.6.3 Preserving Types

lemma *new_element_type_wf_preserved [simp]:*
"h ⊢ new_element →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *new_element_is_l_new_element [instances]:*
"l_new_element type_wf"
⟨*proof*⟩

lemma *put_M$_{Element}$_tag_name_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr tag_name_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_child_nodes_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr child_nodes_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_attrs_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr attrs_update v →$_h$ h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Element}$_shadow_root_opt_type_wf_preserved [simp]:*
"h ⊢ put_M element_ptr shadow_root_opt_update v →$_h$ h' ⟹ type_wf h = type_wf h'"

⟨*proof*⟩

lemma *new_character_data_type_wf_preserved [simp]:*
"h ⊢ new_character_data →h h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *new_character_data_is_l_new_character_data [instances]:*
"l_new_character_data type_wf"
⟨*proof*⟩

lemma *put_M$_{CharacterData}$_val_type_wf_preserved [simp]:*
"h ⊢ put_M character_data_ptr val_update v →h h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *new_document_type_wf_preserved [simp]: "h ⊢ new_document →h h' ⟹ type_wf h = type_wf h'"*
⟨*proof*⟩

locale *l_new_document = l_type_wf +*
assumes *new_document_types_preserved: "h ⊢ new_document →h h' ⟹ type_wf h = type_wf h'"*

lemma *new_document_is_l_new_document [instances]: "l_new_document type_wf"*
⟨*proof*⟩

lemma *put_M$_{Document}$_doctype_type_wf_preserved [simp]:*
"h ⊢ put_M document_ptr doctype_update v →h h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Document}$_document_element_type_wf_preserved [simp]:*
"h ⊢ put_M document_ptr document_element_update v →h h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *put_M$_{Document}$_disconnected_nodes_type_wf_preserved [simp]:*
"h ⊢ put_M document_ptr disconnected_nodes_update v →h h' ⟹ type_wf h = type_wf h'"
⟨*proof*⟩

lemma *document_ptr_kinds_small:*
assumes *"⋀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"*
shows *"document_ptr_kinds h = document_ptr_kinds h'"*
⟨*proof*⟩

lemma *document_ptr_kinds_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →h h'"*
assumes *"⋀h h'. ∀w ∈ SW. h ⊢ w →h h'*
⟶ (∀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h')"
shows *"document_ptr_kinds h = document_ptr_kinds h'"*
⟨*proof*⟩

lemma *type_wf_preserved_small:*
assumes *"⋀object_ptr. preserved (get_M$_{Object}$ object_ptr RObject.nothing) h h'"*
assumes *"⋀node_ptr. preserved (get_M$_{Node}$ node_ptr RNode.nothing) h h'"*
assumes *"⋀element_ptr. preserved (get_M$_{Element}$ element_ptr RElement.nothing) h h'"*
assumes *"⋀character_data_ptr. preserved*
(get_M$_{CharacterData}$ character_data_ptr RCharacterData.nothing) h h'"
assumes *"⋀document_ptr. preserved (get_M$_{Document}$ document_ptr RDocument.nothing) h h'"*
shows *"DocumentClass.type_wf h = DocumentClass.type_wf h'"*
⟨*proof*⟩

lemma *type_wf_preserved:*
assumes *"writes SW setter h h'"*
assumes *"h ⊢ setter →h h'"*
assumes *"⋀h h' w. w ∈ SW ⟹ h ⊢ w →h h'*

```
        ⟹ ∀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀node_ptr. preserved (get_M_Node node_ptr RNode.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀element_ptr. preserved (get_M_Element element_ptr RElement.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀character_data_ptr. preserved
                (get_M_CharacterData character_data_ptr RCharacterData.nothing) h h'"
  assumes "⋀h h' w. w ∈ SW ⟹ h ⊢ w →_h h'
        ⟹ ∀document_ptr. preserved (get_M_Document document_ptr RDocument.nothing) h h'"
  shows "DocumentClass.type_wf h = DocumentClass.type_wf h'"
⟨proof⟩

lemma type_wf_drop: "type_wf h ⟹ type_wf (Heap (fmdrop ptr (the_heap h)))"
  ⟨proof⟩
end
```

6 The Core DOM

In this chapter, we introduce the formalization of the core DOM, i.e., the most important algorithms for querying or modifying the DOM, as defined in the standard. For more details, we refer the reader to [4].

6.1 Basic Data Types (Core_DOM_Basic_Datatypes)

This theory formalizes the primitive data types used by the DOM standard [1].

theory *Core_DOM_Basic_Datatypes*
  imports
    Main
begin

type_synonym *USVString = string*

In the official standard, the type *USVString* corresponds to the set of all possible sequences of Unicode scalar values. As we are not interested in analyzing the specifics of Unicode strings, we just model *USVString* using the standard type *string* of Isabelle/HOL.

type_synonym *DOMString = string*

In the official standard, the type *DOMString* corresponds to the set of all possible sequences of code units, commonly interpreted as UTF-16 encoded strings. Again, as we are not interested in analyzing the specifics of Unicode strings, we just model *DOMString* using the standard type *string* of Isabelle/HOL.

type_synonym *doctype = DOMString*

Examples **definition** *html :: doctype*
  where *"html = ''<!DOCTYPE html>''"*

hide_const *id*

This dummy locale is used to create scoped definitions by using global interpretations and defines.

locale *l_dummy*
end

6.2 Querying and Modifying the DOM (Core_DOM_Functions)

In this theory, we are formalizing the functions for querying and modifying the DOM.

theory *Core_DOM_Functions*
imports
  "monads/DocumentMonad"
begin

If we do not declare show_variants, then all abbreviations that contain constants that are overloaded by using adhoc_overloading get immediately unfolded.

declare *[[show_variants]]*

6.2.1 Various Functions

lemma *insort_split: "x ∈ set (insort y xs) ⟷ (x = y ∨ x ∈ set xs)"*
  ⟨*proof*⟩

lemma *concat_map_distinct:*
  "distinct (concat (map f xs)) ⟹ y ∈ set (concat (map f xs)) ⟹ ∃!x ∈ set xs. y ∈ set (f x)"

⟨*proof*⟩

lemma *concat_map_all_distinct: "distinct (concat (map f xs)) ⟹ x ∈ set xs ⟹ distinct (f x)"*
⟨*proof*⟩

lemma *distinct_concat_map_I:*
assumes *"distinct xs"*
and *"⋀x. x ∈ set xs ⟹ distinct (f x)"*
and *"⋀x y. x ∈ set xs ⟹ y ∈ set xs ⟹ x ≠ y ⟹ (set (f x)) ∩ (set (f y)) = {}"*
shows *"distinct (concat ((map f xs)))"*
⟨*proof*⟩

lemma *distinct_concat_map_E:*
assumes *"distinct (concat ((map f xs)))"*
shows *"⋀x y. x ∈ set xs ⟹ y ∈ set xs ⟹ x ≠ y ⟹ (set (f x)) ∩ (set (f y)) = {}"*
and *"⋀x. x ∈ set xs ⟹ distinct (f x)"*
⟨*proof*⟩

lemma *bind_is_OK_E3 [elim]:*
assumes *"h ⊢ ok (f ⤜ g)"* **and** *"pure f h"*
obtains *x* **where** *"h ⊢ f →r x"* **and** *"h ⊢ ok (g x)"*
⟨*proof*⟩

6.2.2 Basic Functions

get_child_nodes

locale *l_get_child_nodes$_{Core_DOM}$_defs*
begin

definition *get_child_nodes$_{element_ptr}$ :: "(_) element_ptr ⇒ unit ⇒ (_, (_) node_ptr list) dom_prog"*
where
"get_child_nodes$_{element_ptr}$ element_ptr _ = get_M element_ptr RElement.child_nodes"

definition *get_child_nodes$_{character_data_ptr}$ :: "(_) character_data_ptr ⇒ unit ⇒ (_, (_) node_ptr list) dom_prog"*
where
"get_child_nodes$_{character_data_ptr}$ _ _ = return []"

definition *get_child_nodes$_{document_ptr}$ :: "(_) document_ptr ⇒ unit ⇒ (_, (_) node_ptr list) dom_prog"*
where
"get_child_nodes$_{document_ptr}$ document_ptr _ = do {
doc_elem ← get_M document_ptr document_element;
(case doc_elem of
Some element_ptr ⇒ return [cast element_ptr]
| None ⇒ return [])
}"

definition *a_get_child_nodes_tups :: "(((_) object_ptr ⇒ bool) × ((_) object_ptr ⇒ unit*
⇒ (_, (_) node_ptr list) dom_prog)) list"
where
"a_get_child_nodes_tups = [
(is_element_ptr, get_child_nodes$_{element_ptr}$ ∘ the ∘ cast),
(is_character_data_ptr, get_child_nodes$_{character_data_ptr}$ ∘ the ∘ cast),
(is_document_ptr, get_child_nodes$_{document_ptr}$ ∘ the ∘ cast)
]"

definition *a_get_child_nodes :: "(_) object_ptr ⇒ (_, (_) node_ptr list) dom_prog"*
where
"a_get_child_nodes ptr = invoke a_get_child_nodes_tups ptr ()"

definition *a_get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
where
"a_get_child_nodes_locs ptr ≡

```
      (if is_element_ptr_kind ptr then {preserved (get_M (the (cast ptr)) RElement.child_nodes)} else {})
∪
      (if is_document_ptr_kind ptr then {preserved (get_M (the (cast ptr)) RDocument.document_element)}
else {}) ∪
      {preserved (get_M ptr RObject.nothing)}"

definition first_child :: "(_) object_ptr ⇒ (_, (_) node_ptr option) dom_prog"
  where
    "first_child ptr = do {
      children ← a_get_child_nodes ptr;
      return (case children of [] ⇒ None | child#_ ⇒ Some child)}"
end

locale l_get_child_nodes_defs =
  fixes get_child_nodes :: "(_) object_ptr ⇒ (_, (_) node_ptr list) dom_prog"
  fixes get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"

locale l_get_child_nodes_Core_DOM =
  l_type_wf type_wf +
  l_known_ptr known_ptr +
  l_get_child_nodes_defs get_child_nodes get_child_nodes_locs +
  l_get_child_nodes_Core_DOM_defs
  for type_wf :: "(_) heap ⇒ bool"
  and known_ptr :: "(_) object_ptr ⇒ bool"
  and get_child_nodes :: "(_) object_ptr ⇒ (_, (_) node_ptr list) dom_prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set" +
  assumes known_ptr_impl: "known_ptr = DocumentClass.known_ptr"
  assumes type_wf_impl: "type_wf = DocumentClass.type_wf"
  assumes get_child_nodes_impl: "get_child_nodes = a_get_child_nodes"
  assumes get_child_nodes_locs_impl: "get_child_nodes_locs = a_get_child_nodes_locs"
begin
lemmas get_child_nodes_def = get_child_nodes_impl[unfolded a_get_child_nodes_def]
lemmas get_child_nodes_locs_def = get_child_nodes_locs_impl[unfolded a_get_child_nodes_locs_def]

lemma get_child_nodes_split:
  "P (invoke (a_get_child_nodes_tups @ xs) ptr ()) =
    ((known_ptr ptr ⟶ P (get_child_nodes ptr))
    ∧ (¬(known_ptr ptr) ⟶ P (invoke xs ptr ())))"
  ⟨proof⟩

lemma get_child_nodes_split_asm:
  "P (invoke (a_get_child_nodes_tups @ xs) ptr ()) =
    (¬((known_ptr ptr ∧ ¬P (get_child_nodes ptr))
    ∨ (¬(known_ptr ptr) ∧ ¬P (invoke xs ptr ()))))"
  ⟨proof⟩

lemmas get_child_nodes_splits = get_child_nodes_split get_child_nodes_split_asm

lemma get_child_nodes_ok [simp]:
  assumes "known_ptr ptr"
  assumes "type_wf h"
  assumes "ptr |∈| object_ptr_kinds h"
  shows "h ⊢ ok (get_child_nodes ptr)"
  ⟨proof⟩

lemma get_child_nodes_ptr_in_heap [simp]:
  assumes "h ⊢ get_child_nodes ptr →r children"
  shows "ptr |∈| object_ptr_kinds h"
  ⟨proof⟩

lemma get_child_nodes_pure [simp]:
  "pure (get_child_nodes ptr) h"
  ⟨proof⟩
```

lemma *get_child_nodes_reads: "reads (get_child_nodes_locs ptr) (get_child_nodes ptr) h h'"*
⟨*proof*⟩
end

locale *l_get_child_nodes = l_type_wf + l_known_ptr + l_get_child_nodes_defs +*
assumes *get_child_nodes_reads: "reads (get_child_nodes_locs ptr) (get_child_nodes ptr) h h'"*
assumes *get_child_nodes_ok: "type_wf h ⟹ known_ptr ptr ⟹ ptr |∈| object_ptr_kinds h*
⟹ h ⊢ ok (get_child_nodes ptr)"
assumes *get_child_nodes_ptr_in_heap: "h ⊢ ok (get_child_nodes ptr) ⟹ ptr |∈| object_ptr_kinds h"*
assumes *get_child_nodes_pure [simp]: "pure (get_child_nodes ptr) h"*

global_interpretation *l_get_child_nodes$_{Core_DOM}$_defs* **defines**
get_child_nodes = l_get_child_nodes$_{Core_DOM}$_defs.a_get_child_nodes **and**
get_child_nodes_locs = l_get_child_nodes$_{Core_DOM}$_defs.a_get_child_nodes_locs
⟨*proof*⟩

interpretation
i_get_child_nodes?: l_get_child_nodes$_{Core_DOM}$ type_wf known_ptr get_child_nodes get_child_nodes_locs
⟨*proof*⟩
declare *l_get_child_nodes$_{Core_DOM}$_axioms[instances]*

lemma *get_child_nodes_is_l_get_child_nodes [instances]:*
"l_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

new_element **locale** *l_new_element_get_child_nodes$_{Core_DOM}$ =*
l_get_child_nodes$_{Core_DOM}$ type_wf known_ptr get_child_nodes get_child_nodes_locs
for *type_wf :: "(_) heap ⇒ bool"*
and *known_ptr :: "(_) object_ptr ⇒ bool"*
and *get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *get_child_nodes_new_element:*
"ptr' ≠ cast new_element_ptr ⟹ h ⊢ new_element →$_r$ new_element_ptr ⟹ h ⊢ new_element →$_h$ h'
⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
⟨*proof*⟩

lemma *new_element_no_child_nodes:*
"h ⊢ new_element →$_r$ new_element_ptr ⟹ h ⊢ new_element →$_h$ h'
⟹ h' ⊢ get_child_nodes (cast new_element_ptr) →$_r$ []"
⟨*proof*⟩
end

locale *l_new_element_get_child_nodes = l_new_element + l_get_child_nodes +*
assumes *get_child_nodes_new_element:*
"ptr' ≠ cast new_element_ptr ⟹ h ⊢ new_element →$_r$ new_element_ptr
⟹ h ⊢ new_element →$_h$ h' ⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
assumes *new_element_no_child_nodes:*
"h ⊢ new_element →$_r$ new_element_ptr ⟹ h ⊢ new_element →$_h$ h'
⟹ h' ⊢ get_child_nodes (cast new_element_ptr) →$_r$ []"

interpretation *i_new_element_get_child_nodes?:*
l_new_element_get_child_nodes$_{Core_DOM}$ type_wf known_ptr get_child_nodes get_child_nodes_locs
⟨*proof*⟩
declare *l_new_element_get_child_nodes$_{Core_DOM}$_axioms[instances]*

lemma *new_element_get_child_nodes_is_l_new_element_get_child_nodes [instances]:*
"l_new_element_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

new_character_data **locale** *l_new_character_data_get_child_nodes$_{Core_DOM}$ =*
l_get_child_nodes$_{Core_DOM}$ type_wf known_ptr get_child_nodes get_child_nodes_locs

for *type_wf :: "(_) heap ⇒ bool"*
and *known_ptr :: "(_) object_ptr ⇒ bool"*
and *get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *get_child_nodes_new_character_data:*
"ptr' ≠ cast new_character_data_ptr ⟹ h ⊢ new_character_data →r new_character_data_ptr
⟹ h ⊢ new_character_data →h h' ⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
⟨*proof*⟩

lemma *new_character_data_no_child_nodes:*
"h ⊢ new_character_data →r new_character_data_ptr ⟹ h ⊢ new_character_data →h h'
⟹ h' ⊢ get_child_nodes (cast new_character_data_ptr) →r []"
⟨*proof*⟩
end

locale *l_new_character_data_get_child_nodes = l_new_character_data + l_get_child_nodes +*
assumes *get_child_nodes_new_character_data:*
"ptr' ≠ cast new_character_data_ptr ⟹ h ⊢ new_character_data →r new_character_data_ptr
⟹ h ⊢ new_character_data →h h' ⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
assumes *new_character_data_no_child_nodes:*
"h ⊢ new_character_data →r new_character_data_ptr ⟹ h ⊢ new_character_data →h h'
⟹ h' ⊢ get_child_nodes (cast new_character_data_ptr) →r []"

interpretation *i_new_character_data_get_child_nodes?:*
l_new_character_data_get_child_nodes$_{Core_DOM}$ *type_wf known_ptr get_child_nodes get_child_nodes_locs*
⟨*proof*⟩
declare *l_new_character_data_get_child_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *new_character_data_get_child_nodes_is_l_new_character_data_get_child_nodes [instances]:*
"l_new_character_data_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

new_document **locale** *l_new_document_get_child_nodes*$_{Core_DOM}$ *=*
l_get_child_nodes$_{Core_DOM}$ *type_wf known_ptr get_child_nodes get_child_nodes_locs*
for *type_wf :: "(_) heap ⇒ bool"*
and *known_ptr :: "(_) object_ptr ⇒ bool"*
and *get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *get_child_nodes_new_document:*
"ptr' ≠ cast new_document_ptr ⟹ h ⊢ new_document →r new_document_ptr
⟹ h ⊢ new_document →h h' ⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
⟨*proof*⟩

lemma *new_document_no_child_nodes:*
"h ⊢ new_document →r new_document_ptr ⟹ h ⊢ new_document →h h'
⟹ h' ⊢ get_child_nodes (cast new_document_ptr) →r []"
⟨*proof*⟩
end

locale *l_new_document_get_child_nodes = l_new_document + l_get_child_nodes +*
assumes *get_child_nodes_new_document:*
"ptr' ≠ cast new_document_ptr ⟹ h ⊢ new_document →r new_document_ptr
⟹ h ⊢ new_document →h h' ⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"
assumes *new_document_no_child_nodes:*
"h ⊢ new_document →r new_document_ptr ⟹ h ⊢ new_document →h h'
⟹ h' ⊢ get_child_nodes (cast new_document_ptr) →r []"

interpretation *i_new_document_get_child_nodes?:*
l_new_document_get_child_nodes$_{Core_DOM}$ *type_wf known_ptr get_child_nodes get_child_nodes_locs*
⟨*proof*⟩
declare *l_new_document_get_child_nodes*$_{Core_DOM}$*_axioms[instances]*

```
lemma new_document_get_child_nodes_is_l_new_document_get_child_nodes [instances]:
  "l_new_document_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs"
  ⟨proof⟩
```

set_child_nodes

```
locale l_set_child_nodes_Core_DOM_defs
begin
definition set_child_nodes_element_ptr ::
  "(_) element_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  where
  "set_child_nodes_element_ptr element_ptr children = put_M element_ptr RElement.child_nodes_update children"

definition set_child_nodes_character_data_ptr ::
  "(_) character_data_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  where
    "set_child_nodes_character_data_ptr _ _ =  error HierarchyRequestError"

definition set_child_nodes_document_ptr :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  where
    "set_child_nodes_document_ptr document_ptr children = do {
      (case children of
        [] ⇒ put_M document_ptr document_element_update None
      | child # [] ⇒ (case cast child of
          Some element_ptr ⇒ put_M document_ptr document_element_update (Some element_ptr)
        | None ⇒ error HierarchyRequestError)
      | _ ⇒ error HierarchyRequestError)
    }"

definition a_set_child_nodes_tups ::
  "(((_) object_ptr ⇒ bool) × ((_) object_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog)) list"
  where
    "a_set_child_nodes_tups ≡ [
        (is_element_ptr, set_child_nodes_element_ptr ∘ the ∘ cast),
        (is_character_data_ptr, set_child_nodes_character_data_ptr ∘ the ∘ cast),
        (is_document_ptr, set_child_nodes_document_ptr ∘ the ∘ cast)
     ]"

definition a_set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  where
    "a_set_child_nodes ptr children = invoke a_set_child_nodes_tups ptr (children)"
lemmas set_child_nodes_defs = a_set_child_nodes_def

definition a_set_child_nodes_locs :: "(_) object_ptr ⇒ (_, unit) dom_prog set"
  where
    "a_set_child_nodes_locs ptr ≡
      (if is_element_ptr_kind ptr
          then all_args (put_M_Element (the (cast ptr)) RElement.child_nodes_update) else {}) ∪
      (if is_document_ptr_kind ptr
          then all_args (put_M_Document (the (cast ptr)) document_element_update) else {})"
end

locale l_set_child_nodes_defs =
  fixes set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  fixes set_child_nodes_locs :: "(_) object_ptr ⇒ (_, unit) dom_prog set"

locale l_set_child_nodes_Core_DOM =
  l_type_wf type_wf +
  l_known_ptr known_ptr +
  l_set_child_nodes_defs set_child_nodes set_child_nodes_locs +
  l_set_child_nodes_Core_DOM_defs
  for type_wf :: "(_) heap ⇒ bool"
```

```
  and known_ptr :: "(_) object_ptr ⇒ bool"
  and set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"
  and set_child_nodes_locs :: "(_) object_ptr ⇒ (_, unit) dom_prog set" +
  assumes known_ptr_impl: "known_ptr = DocumentClass.known_ptr"
  assumes type_wf_impl: "type_wf = DocumentClass.type_wf"
  assumes set_child_nodes_impl: "set_child_nodes = a_set_child_nodes"
  assumes set_child_nodes_locs_impl: "set_child_nodes_locs = a_set_child_nodes_locs"
begin
lemmas set_child_nodes_def = set_child_nodes_impl[unfolded a_set_child_nodes_def]
lemmas set_child_nodes_locs_def = set_child_nodes_locs_impl[unfolded a_set_child_nodes_locs_def]

lemma set_child_nodes_split:
  "P (invoke (a_set_child_nodes_tups @ xs) ptr (children)) =
    ((known_ptr ptr ⟶ P (set_child_nodes ptr children))
    ∧ (¬(known_ptr ptr) ⟶ P (invoke xs ptr (children))))"
  ⟨proof⟩

lemma set_child_nodes_split_asm:
  "P (invoke (a_set_child_nodes_tups @ xs) ptr (children)) =
    (¬((known_ptr ptr ∧ ¬P (set_child_nodes ptr children))
    ∨ (¬(known_ptr ptr) ∧ ¬P (invoke xs ptr (children)))))"
  ⟨proof⟩
lemmas set_child_nodes_splits = set_child_nodes_split set_child_nodes_split_asm

lemma set_child_nodes_writes: "writes (set_child_nodes_locs ptr) (set_child_nodes ptr children) h h'"
  ⟨proof⟩

lemma set_child_nodes_pointers_preserved:
  assumes "w ∈ set_child_nodes_locs object_ptr"
  assumes "h ⊢ w →_h h'"
  shows "object_ptr_kinds h = object_ptr_kinds h'"
  ⟨proof⟩

lemma set_child_nodes_typess_preserved:
  assumes "w ∈ set_child_nodes_locs object_ptr"
  assumes "h ⊢ w →_h h'"
  shows "type_wf h = type_wf h'"
  ⟨proof⟩
end

locale l_set_child_nodes = l_type_wf + l_set_child_nodes_defs +
  assumes set_child_nodes_writes:
   "writes (set_child_nodes_locs ptr) (set_child_nodes ptr children) h h'"
 assumes set_child_nodes_pointers_preserved:
   "w ∈ set_child_nodes_locs object_ptr ⟹ h ⊢ w →_h h' ⟹ object_ptr_kinds h = object_ptr_kinds h'"
 assumes set_child_nodes_types_preserved:
   "w ∈ set_child_nodes_locs object_ptr ⟹ h ⊢ w →_h h' ⟹ type_wf h = type_wf h'"

global_interpretation l_set_child_nodes_Core_DOM_defs defines
  set_child_nodes = l_set_child_nodes_Core_DOM_defs.a_set_child_nodes and
  set_child_nodes_locs = l_set_child_nodes_Core_DOM_defs.a_set_child_nodes_locs ⟨proof⟩

interpretation
  i_set_child_nodes?: l_set_child_nodes_Core_DOM type_wf known_ptr set_child_nodes set_child_nodes_locs
  ⟨proof⟩
declare l_set_child_nodes_Core_DOM_axioms[instances]


lemma set_child_nodes_is_l_set_child_nodes [instances]:
  "l_set_child_nodes type_wf set_child_nodes set_child_nodes_locs"
  ⟨proof⟩
```

get_child_nodes locale l_set_child_nodes_get_child_nodes_Core_DOM = l_get_child_nodes_Core_DOM + l_set_child_nod

begin

lemma *set_child_nodes_get_child_nodes:*
assumes *"known_ptr ptr"*
assumes *"type_wf h"*
assumes *"h ⊢ set_child_nodes ptr children →$_h$ h'"*
shows *"h' ⊢ get_child_nodes ptr →$_r$ children"*
⟨*proof*⟩

lemma *set_child_nodes_get_child_nodes_different_pointers:*
assumes *"ptr ≠ ptr'"*
assumes *"w ∈ set_child_nodes_locs ptr"*
assumes *"h ⊢ w →$_h$ h'"*
assumes *"r ∈ get_child_nodes_locs ptr'"*
shows *"r h h'"*
⟨*proof*⟩

lemma *set_child_nodes_element_ok [simp]:*
assumes *"known_ptr ptr"*
assumes *"type_wf h"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"is_element_ptr_kind ptr"*
shows *"h ⊢ ok (set_child_nodes ptr children)"*
⟨*proof*⟩

lemma *set_child_nodes_document1_ok [simp]:*
assumes *"known_ptr ptr"*
assumes *"type_wf h"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"is_document_ptr_kind ptr"*
assumes *"children = []"*
shows *"h ⊢ ok (set_child_nodes ptr children)"*
⟨*proof*⟩

lemma *set_child_nodes_document2_ok [simp]:*
assumes *"known_ptr ptr"*
assumes *"type_wf h"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"is_document_ptr_kind ptr"*
assumes *"children = [child]"*
assumes *"is_element_ptr_kind child"*
shows *"h ⊢ ok (set_child_nodes ptr children)"*
⟨*proof*⟩
end

locale *l_set_child_nodes_get_child_nodes = l_get_child_nodes + l_set_child_nodes +*
assumes *set_child_nodes_get_child_nodes:*
"type_wf h ⟹ known_ptr ptr
⟹ h ⊢ set_child_nodes ptr children →$_h$ h' ⟹ h' ⊢ get_child_nodes ptr →$_r$ children"
assumes *set_child_nodes_get_child_nodes_different_pointers:*
"ptr ≠ ptr' ⟹ w ∈ set_child_nodes_locs ptr ⟹ h ⊢ w →$_h$ h'
⟹ r ∈ get_child_nodes_locs ptr' ⟹ r h h'"

interpretation
i_set_child_nodes_get_child_nodes?: l_set_child_nodes_get_child_nodes$_{Core_DOM}$ type_wf
known_ptr get_child_nodes get_child_nodes_locs set_child_nodes set_child_nodes_locs
⟨*proof*⟩
declare *l_set_child_nodes_get_child_nodes$_{Core_DOM}$_axioms[instances]*

lemma *set_child_nodes_get_child_nodes_is_l_set_child_nodes_get_child_nodes [instances]:*
"l_set_child_nodes_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs
set_child_nodes set_child_nodes_locs"
⟨*proof*⟩

get_attribute

locale *l_get_attribute$_{Core_DOM}$_defs*
begin
definition *a_get_attribute :: "(_) element_ptr ⇒ attr_key ⇒ (_, attr_value option) dom_prog"*
  where
    "a_get_attribute ptr k = do {m ← get_M ptr attrs; return (fmlookup m k)}"
lemmas *get_attribute_defs = a_get_attribute_def*

definition *a_get_attribute_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
  where
    "a_get_attribute_locs element_ptr = {preserved (get_M element_ptr attrs)}"
end

locale *l_get_attribute_defs =*
  fixes *get_attribute :: "(_) element_ptr ⇒ attr_key ⇒ (_, attr_value option) dom_prog"*
  fixes *get_attribute_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*

locale *l_get_attribute$_{Core_DOM}$ =*
  l_type_wf type_wf +
  l_get_attribute_defs get_attribute get_attribute_locs +
  l_get_attribute$_{Core_DOM}$_defs
  for *type_wf :: "(_) heap ⇒ bool"*
  and *get_attribute :: "(_) element_ptr ⇒ attr_key ⇒ (_, attr_value option) dom_prog"*
  and *get_attribute_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set" +*
  assumes *type_wf_impl: "type_wf = DocumentClass.type_wf"*
  assumes *get_attribute_impl: "get_attribute = a_get_attribute"*
  assumes *get_attribute_locs_impl: "get_attribute_locs = a_get_attribute_locs"*
begin
lemma *get_attribute_pure [simp]: "pure (get_attribute ptr k) h"*
  ⟨*proof*⟩

lemma *get_attribute_ok:*
  "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (get_attribute element_ptr k)"
  ⟨*proof*⟩

lemma *get_attribute_ptr_in_heap:*
  "h ⊢ ok (get_attribute element_ptr k) ⟹ element_ptr |∈| element_ptr_kinds h"
  ⟨*proof*⟩

lemma *get_attribute_reads:*
  "reads (get_attribute_locs element_ptr) (get_attribute element_ptr k) h h'"
  ⟨*proof*⟩
end

locale *l_get_attribute = l_type_wf + l_get_attribute_defs +*
assumes *get_attribute_reads:*
  "reads (get_attribute_locs element_ptr) (get_attribute element_ptr k) h h'"
assumes *get_attribute_ok:*
  "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (get_attribute element_ptr k)"
assumes *get_attribute_ptr_in_heap:*
  "h ⊢ ok (get_attribute element_ptr k) ⟹ element_ptr |∈| element_ptr_kinds h"
assumes *get_attribute_pure [simp]: "pure (get_attribute element_ptr k) h"*

global_interpretation *l_get_attribute$_{Core_DOM}$_defs* **defines**
  get_attribute = l_get_attribute$_{Core_DOM}$_defs.a_get_attribute **and**
  get_attribute_locs = l_get_attribute$_{Core_DOM}$_defs.a_get_attribute_locs ⟨*proof*⟩

interpretation
  i_get_attribute?: l_get_attribute$_{Core_DOM}$ type_wf get_attribute get_attribute_locs
  ⟨*proof*⟩
declare *l_get_attribute$_{Core_DOM}$_axioms[instances]*

lemma *get_attribute_is_l_get_attribute [instances]:*
"l_get_attribute type_wf get_attribute get_attribute_locs"
⟨*proof*⟩

set_attribute

locale *l_set_attribute*$_{Core_DOM}$*_defs*
begin

definition
a_set_attribute :: "(_) element_ptr ⇒ attr_key ⇒ attr_value option ⇒ (_, unit) dom_prog"
where
"a_set_attribute ptr k v = do {
m ← get_M ptr attrs;
put_M ptr attrs_update (if v = None then fmdrop k m else fmupd k (the v) m)
}"

definition *a_set_attribute_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set"*
where
"a_set_attribute_locs element_ptr ≡ all_args (put_M element_ptr attrs_update)"
end

locale *l_set_attribute_defs =*
fixes *set_attribute :: "(_) element_ptr ⇒ attr_key ⇒ attr_value option ⇒ (_, unit) dom_prog"*
fixes *set_attribute_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set"*

locale *l_set_attribute*$_{Core_DOM}$ *=*
l_type_wf type_wf +
l_set_attribute_defs set_attribute set_attribute_locs +
l_set_attribute$_{Core_DOM}$*_defs*
for *type_wf :: "(_) heap ⇒ bool"*
and *set_attribute :: "(_) element_ptr ⇒ attr_key ⇒ attr_value option ⇒ (_, unit) dom_prog"*
and *set_attribute_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set" +*
assumes *type_wf_impl: "type_wf = DocumentClass.type_wf"*
assumes *set_attribute_impl: "set_attribute = a_set_attribute"*
assumes *set_attribute_locs_impl: "set_attribute_locs = a_set_attribute_locs"*
begin
lemmas *set_attribute_def = set_attribute_impl[folded a_set_attribute_def]*
lemmas *set_attribute_locs_def = set_attribute_locs_impl[unfolded a_set_attribute_locs_def]*

lemma *set_attribute_ok: "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (set_attribute element_ptr k v)"*
⟨*proof*⟩

lemma *set_attribute_writes:*
"writes (set_attribute_locs element_ptr) (set_attribute element_ptr k v) h h'"
⟨*proof*⟩
end

locale *l_set_attribute = l_type_wf + l_set_attribute_defs +*
assumes *set_attribute_writes:*
"writes (set_attribute_locs element_ptr) (set_attribute element_ptr k v) h h'"
assumes *set_attribute_ok:*
"type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (set_attribute element_ptr k v)"

global_interpretation *l_set_attribute*$_{Core_DOM}$*_defs* **defines**
set_attribute = l_set_attribute$_{Core_DOM}$*_defs.a_set_attribute* **and**
set_attribute_locs = l_set_attribute$_{Core_DOM}$*_defs.a_set_attribute_locs* ⟨*proof*⟩
interpretation
i_set_attribute?: l_set_attribute$_{Core_DOM}$ *type_wf set_attribute set_attribute_locs*
⟨*proof*⟩
declare *l_set_attribute*$_{Core_DOM}$*_axioms[instances]*

lemma *set_attribute_is_l_set_attribute [instances]:*
"l_set_attribute type_wf set_attribute set_attribute_locs"
⟨*proof*⟩

get_attribute **locale** *l_set_attribute_get_attribute*$_{Core_DOM}$ =
l_get_attribute$_{Core_DOM}$ +
l_set_attribute$_{Core_DOM}$
begin

lemma *set_attribute_get_attribute:*
"h ⊢ set_attribute ptr k v →$_h$ h' ⟹ h' ⊢ get_attribute ptr k →$_r$ v"
⟨*proof*⟩
end

locale *l_set_attribute_get_attribute = l_get_attribute + l_set_attribute +*
assumes *set_attribute_get_attribute:*
"h ⊢ set_attribute ptr k v →$_h$ h' ⟹ h' ⊢ get_attribute ptr k →$_r$ v"

interpretation
i_set_attribute_get_attribute?: l_set_attribute_get_attribute$_{Core_DOM}$ *type_wf*
get_attribute get_attribute_locs set_attribute set_attribute_locs
⟨*proof*⟩
declare *l_set_attribute_get_attribute*$_{Core_DOM}$*_axioms[instances]*

lemma *set_attribute_get_attribute_is_l_set_attribute_get_attribute [instances]:*
"l_set_attribute_get_attribute type_wf get_attribute get_attribute_locs set_attribute set_attribute_locs"
⟨*proof*⟩

get_child_nodes **locale** *l_set_attribute_get_child_nodes*$_{Core_DOM}$ =
l_set_attribute$_{Core_DOM}$ +
l_get_child_nodes$_{Core_DOM}$
begin
lemma *set_attribute_get_child_nodes:*
"∀w ∈ set_attribute_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"
⟨*proof*⟩
end

locale *l_set_attribute_get_child_nodes =*
l_set_attribute +
l_get_child_nodes +
assumes *set_attribute_get_child_nodes:*
"∀w ∈ set_attribute_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"

interpretation
i_set_attribute_get_child_nodes?: l_set_attribute_get_child_nodes$_{Core_DOM}$ *type_wf*
set_attribute set_attribute_locs known_ptr get_child_nodes get_child_nodes_locs
⟨*proof*⟩
declare *l_set_attribute_get_child_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *set_attribute_get_child_nodes_is_l_set_attribute_get_child_nodes [instances]:*
"l_set_attribute_get_child_nodes type_wf set_attribute set_attribute_locs known_ptr
get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

get_disconnected_nodes

locale *l_get_disconnected_nodes*$_{Core_DOM}$*_defs*
begin
definition *a_get_disconnected_nodes :: "(_) document_ptr*
⇒ (_, (_) node_ptr list) dom_prog"
where
"a_get_disconnected_nodes document_ptr = get_M document_ptr disconnected_nodes"
lemmas *get_disconnected_nodes_defs = a_get_disconnected_nodes_def*

definition *a_get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
where
"a_get_disconnected_nodes_locs document_ptr = {preserved (get_M document_ptr disconnected_nodes)}"
end

locale *l_get_disconnected_nodes_defs =*
fixes *get_disconnected_nodes :: "(_) document_ptr ⇒ (_, (_) node_ptr list) dom_prog"*
fixes *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*

locale *l_get_disconnected_nodes*$_{Core_DOM}$ =
l_type_wf type_wf +
l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
l_get_disconnected_nodes$_{Core_DOM}$*_defs*
for *type_wf :: "(_) heap ⇒ bool"*
and *get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set" +*
assumes *type_wf_impl: "type_wf = DocumentClass.type_wf"*
assumes *get_disconnected_nodes_impl: "get_disconnected_nodes = a_get_disconnected_nodes"*
assumes *get_disconnected_nodes_locs_impl: "get_disconnected_nodes_locs = a_get_disconnected_nodes_locs"*
begin
lemmas
get_disconnected_nodes_def = get_disconnected_nodes_impl[unfolded a_get_disconnected_nodes_def]
lemmas
get_disconnected_nodes_locs_def = get_disconnected_nodes_locs_impl[unfolded a_get_disconnected_nodes_locs_def]

lemma *get_disconnected_nodes_ok:*
"type_wf h ⟹ document_ptr |∈| document_ptr_kinds h ⟹ h ⊢ ok (get_disconnected_nodes document_ptr)"
⟨*proof*⟩

lemma *get_disconnected_nodes_ptr_in_heap:*
"h ⊢ ok (get_disconnected_nodes document_ptr) ⟹ document_ptr |∈| document_ptr_kinds h"
⟨*proof*⟩

lemma *get_disconnected_nodes_pure [simp]: "pure (get_disconnected_nodes document_ptr) h"*
⟨*proof*⟩

lemma *get_disconnected_nodes_reads:*
"reads (get_disconnected_nodes_locs document_ptr) (get_disconnected_nodes document_ptr) h h'"
⟨*proof*⟩
end

locale *l_get_disconnected_nodes = l_type_wf + l_get_disconnected_nodes_defs +*
assumes *get_disconnected_nodes_reads:*
"reads (get_disconnected_nodes_locs document_ptr) (get_disconnected_nodes document_ptr) h h'"
assumes *get_disconnected_nodes_ok:*
"type_wf h ⟹ document_ptr |∈| document_ptr_kinds h ⟹ h ⊢ ok (get_disconnected_nodes document_ptr)"
assumes *get_disconnected_nodes_ptr_in_heap:*
"h ⊢ ok (get_disconnected_nodes document_ptr) ⟹ document_ptr |∈| document_ptr_kinds h"
assumes *get_disconnected_nodes_pure [simp]:*
"pure (get_disconnected_nodes document_ptr) h"

global_interpretation *l_get_disconnected_nodes*$_{Core_DOM}$*_defs* **defines**
get_disconnected_nodes = l_get_disconnected_nodes$_{Core_DOM}$*_defs.a_get_disconnected_nodes* **and**
get_disconnected_nodes_locs = l_get_disconnected_nodes$_{Core_DOM}$*_defs.a_get_disconnected_nodes_locs* ⟨*proof*⟩
interpretation
i_get_disconnected_nodes?: l_get_disconnected_nodes$_{Core_DOM}$ *type_wf get_disconnected_nodes*
get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_get_disconnected_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *get_disconnected_nodes_is_l_get_disconnected_nodes [instances]:*
"l_get_disconnected_nodes type_wf get_disconnected_nodes get_disconnected_nodes_locs"

⟨*proof*⟩

set_child_nodes **locale** *l_set_child_nodes_get_disconnected_nodes*$_{Core_DOM}$ =
l_set_child_nodes$_{Core_DOM}$ +
CD: l_get_disconnected_nodes$_{Core_DOM}$
begin
lemma *set_child_nodes_get_disconnected_nodes:*
"∀*w* ∈ *a_set_child_nodes_locs ptr. (h* ⊢ *w* →$_h$ *h'* ⟶ *(*∀*r* ∈ *a_get_disconnected_nodes_locs ptr'. r h h'))"
⟨*proof*⟩
end

locale *l_set_child_nodes_get_disconnected_nodes = l_set_child_nodes + l_get_disconnected_nodes +*
assumes *set_child_nodes_get_disconnected_nodes:*
"∀*w* ∈ *set_child_nodes_locs ptr. (h* ⊢ *w* →$_h$ *h'* ⟶ *(*∀*r* ∈ *get_disconnected_nodes_locs ptr'. r h h'))"

interpretation
i_set_child_nodes_get_disconnected_nodes?: l_set_child_nodes_get_disconnected_nodes$_{Core_DOM}$ *type_wf*
known_ptr set_child_nodes set_child_nodes_locs
get_disconnected_nodes get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_set_child_nodes_get_disconnected_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *set_child_nodes_get_disconnected_nodes_is_l_set_child_nodes_get_disconnected_nodes [instances]:*
"*l_set_child_nodes_get_disconnected_nodes type_wf set_child_nodes set_child_nodes_locs*
get_disconnected_nodes get_disconnected_nodes_locs"
⟨*proof*⟩

set_attribute **locale** *l_set_attribute_get_disconnected_nodes*$_{Core_DOM}$ =
l_set_attribute$_{Core_DOM}$ +
l_get_disconnected_nodes$_{Core_DOM}$
begin
lemma *set_attribute_get_disconnected_nodes:*
"∀*w* ∈ *a_set_attribute_locs ptr. (h* ⊢ *w* →$_h$ *h'* ⟶ *(*∀*r* ∈ *a_get_disconnected_nodes_locs ptr'. r h h'))"
⟨*proof*⟩
end

locale *l_set_attribute_get_disconnected_nodes = l_set_attribute + l_get_disconnected_nodes +*
assumes *set_attribute_get_disconnected_nodes:*
"∀*w* ∈ *set_attribute_locs ptr. (h* ⊢ *w* →$_h$ *h'* ⟶ *(*∀*r* ∈ *get_disconnected_nodes_locs ptr'. r h h'))"

interpretation
i_set_attribute_get_disconnected_nodes?: l_set_attribute_get_disconnected_nodes$_{Core_DOM}$ *type_wf*
set_attribute set_attribute_locs get_disconnected_nodes get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_set_attribute_get_disconnected_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *set_attribute_get_disconnected_nodes_is_l_set_attribute_get_disconnected_nodes [instances]:*
"*l_set_attribute_get_disconnected_nodes type_wf set_attribute set_attribute_locs*
get_disconnected_nodes get_disconnected_nodes_locs"
⟨*proof*⟩

new_element **locale** *l_new_element_get_disconnected_nodes*$_{Core_DOM}$ =
l_get_disconnected_nodes$_{Core_DOM}$ *type_wf get_disconnected_nodes get_disconnected_nodes_locs*
for *type_wf :: "(_) heap* ⇒ *bool"*
and *get_disconnected_nodes :: "(_) document_ptr* ⇒ *((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr* ⇒ *((_) heap* ⇒ *(_) heap* ⇒ *bool) set"*
begin
lemma *get_disconnected_nodes_new_element:*
"h ⊢ *new_element* →$_r$ *new_element_ptr* ⟹ *h* ⊢ *new_element* →$_h$ *h'*
⟹ *r* ∈ *get_disconnected_nodes_locs ptr'* ⟹ *r h h'"*
⟨*proof*⟩
end

locale *l_new_element_get_disconnected_nodes = l_get_disconnected_nodes_defs +*
assumes *get_disconnected_nodes_new_element:*
"h ⊢ new_element →r new_element_ptr ⟹ h ⊢ new_element →h h' ⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"

interpretation *i_new_element_get_disconnected_nodes?:*
l_new_element_get_disconnected_nodes$_{Core_DOM}$ type_wf get_disconnected_nodes get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_new_element_get_disconnected_nodes$_{Core_DOM}$_axioms[instances]*

lemma *new_element_get_disconnected_nodes_is_l_new_element_get_disconnected_nodes [instances]:*
"l_new_element_get_disconnected_nodes get_disconnected_nodes_locs"
⟨*proof*⟩

new_character_data **locale** *l_new_character_data_get_disconnected_nodes$_{Core_DOM}$ =*
l_get_disconnected_nodes$_{Core_DOM}$ type_wf get_disconnected_nodes get_disconnected_nodes_locs
for *type_wf :: "(_) heap ⇒ bool"*
and *get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *get_disconnected_nodes_new_character_data:*
"h ⊢ new_character_data →r new_character_data_ptr ⟹ h ⊢ new_character_data →h h' ⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"
⟨*proof*⟩
end

locale *l_new_character_data_get_disconnected_nodes = l_get_disconnected_nodes_defs +*
assumes *get_disconnected_nodes_new_character_data:*
"h ⊢ new_character_data →r new_character_data_ptr ⟹ h ⊢ new_character_data →h h' ⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"

interpretation *i_new_character_data_get_disconnected_nodes?:*
l_new_character_data_get_disconnected_nodes$_{Core_DOM}$ type_wf get_disconnected_nodes get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_new_character_data_get_disconnected_nodes$_{Core_DOM}$_axioms[instances]*

lemma *new_character_data_get_disconnected_nodes_is_l_new_character_data_get_disconnected_nodes [instances]:*
"l_new_character_data_get_disconnected_nodes get_disconnected_nodes_locs"
⟨*proof*⟩

new_document **locale** *l_new_document_get_disconnected_nodes$_{Core_DOM}$ =*
l_get_disconnected_nodes$_{Core_DOM}$ type_wf get_disconnected_nodes get_disconnected_nodes_locs
for *type_wf :: "(_) heap ⇒ bool"*
and *get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *get_disconnected_nodes_new_document_different_pointers:*
"new_document_ptr ≠ ptr' ⟹ h ⊢ new_document →r new_document_ptr ⟹ h ⊢ new_document →h h' ⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"
⟨*proof*⟩

lemma *new_document_no_disconnected_nodes:*
"h ⊢ new_document →r new_document_ptr ⟹ h ⊢ new_document →h h' ⟹ h' ⊢ get_disconnected_nodes new_document_ptr →r []"
⟨*proof*⟩

end

interpretation *i_new_document_get_disconnected_nodes?:*
l_new_document_get_disconnected_nodes$_{Core_DOM}$ type_wf get_disconnected_nodes get_disconnected_nodes_locs

⟨*proof*⟩
declare *l_new_document_get_disconnected_nodes$_{Core_DOM}$_axioms[instances]*

locale *l_new_document_get_disconnected_nodes = l_get_disconnected_nodes_defs +*
assumes *get_disconnected_nodes_new_document_different_pointers:*
"new_document_ptr ≠ ptr' ⟹ h ⊢ new_document →$_r$ new_document_ptr ⟹ h ⊢ new_document →$_h$ h'
⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"
assumes *new_document_no_disconnected_nodes:*
"h ⊢ new_document →$_r$ new_document_ptr ⟹ h ⊢ new_document →$_h$ h'
⟹ h' ⊢ get_disconnected_nodes new_document_ptr →$_r$ []"

lemma *new_document_get_disconnected_nodes_is_l_new_document_get_disconnected_nodes [instances]:*
"l_new_document_get_disconnected_nodes get_disconnected_nodes get_disconnected_nodes_locs"
⟨*proof*⟩

set_disconnected_nodes

locale *l_set_disconnected_nodes$_{Core_DOM}$_defs*
begin

definition *a_set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"*
where
"a_set_disconnected_nodes document_ptr disc_nodes =
put_M document_ptr disconnected_nodes_update disc_nodes"
lemmas *set_disconnected_nodes_defs = a_set_disconnected_nodes_def*

definition *a_set_disconnected_nodes_locs :: "(_) document_ptr ⇒ (_, unit) dom_prog set"*
where
"a_set_disconnected_nodes_locs document_ptr ≡ all_args (put_M document_ptr disconnected_nodes_update)"
end

locale *l_set_disconnected_nodes_defs =*
fixes *set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"*
fixes *set_disconnected_nodes_locs :: "(_) document_ptr ⇒ (_, unit) dom_prog set"*

locale *l_set_disconnected_nodes$_{Core_DOM}$ =*
l_type_wf type_wf +
l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs +
l_set_disconnected_nodes$_{Core_DOM}$_defs
for *type_wf :: "(_) heap ⇒ bool"*
and *set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ (_, unit) dom_prog"*
and *set_disconnected_nodes_locs :: "(_) document_ptr ⇒ (_, unit) dom_prog set" +*
assumes *type_wf_impl: "type_wf = DocumentClass.type_wf"*
assumes *set_disconnected_nodes_impl: "set_disconnected_nodes = a_set_disconnected_nodes"*
assumes *set_disconnected_nodes_locs_impl: "set_disconnected_nodes_locs = a_set_disconnected_nodes_locs"*
begin
lemmas *set_disconnected_nodes_def = set_disconnected_nodes_impl[unfolded a_set_disconnected_nodes_def]*
lemmas *set_disconnected_nodes_locs_def =*
set_disconnected_nodes_locs_impl[unfolded a_set_disconnected_nodes_locs_def]
lemma *set_disconnected_nodes_ok:*
"type_wf h ⟹ document_ptr |∈| document_ptr_kinds h ⟹
h ⊢ ok (set_disconnected_nodes document_ptr node_ptrs)"
⟨*proof*⟩

lemma *set_disconnected_nodes_ptr_in_heap:*
"h ⊢ ok (set_disconnected_nodes document_ptr disc_nodes) ⟹ document_ptr |∈| document_ptr_kinds h"
⟨*proof*⟩

lemma *set_disconnected_nodes_writes:*
"writes (set_disconnected_nodes_locs document_ptr) (set_disconnected_nodes document_ptr disc_nodes) h
h'"
⟨*proof*⟩

lemma *set_disconnected_nodes_pointers_preserved:*
assumes *"w ∈ set_disconnected_nodes_locs object_ptr"*
assumes *"h ⊢ w →_h h'"*
shows *"object_ptr_kinds h = object_ptr_kinds h'"*
⟨*proof*⟩

lemma *set_disconnected_nodes_typess_preserved:*
assumes *"w ∈ set_disconnected_nodes_locs object_ptr"*
assumes *"h ⊢ w →_h h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩
end

locale *l_set_disconnected_nodes = l_type_wf + l_set_disconnected_nodes_defs +*
assumes *set_disconnected_nodes_writes:*
"writes (set_disconnected_nodes_locs document_ptr)
(set_disconnected_nodes document_ptr disc_nodes) h h'"
assumes *set_disconnected_nodes_ok:*
"type_wf h ⟹ document_ptr |∈| document_ptr_kinds h ⟹
h ⊢ ok (set_disconnected_nodes document_ptr disc_noded)"
assumes *set_disconnected_nodes_ptr_in_heap:*
"h ⊢ ok (set_disconnected_nodes document_ptr disc_noded) ⟹
document_ptr |∈| document_ptr_kinds h"
assumes *set_disconnected_nodes_pointers_preserved:*
"w ∈ set_disconnected_nodes_locs document_ptr ⟹ h ⊢ w →_h h' ⟹
object_ptr_kinds h = object_ptr_kinds h'"
assumes *set_disconnected_nodes_types_preserved:*
"w ∈ set_disconnected_nodes_locs document_ptr ⟹ h ⊢ w →_h h' ⟹ type_wf h = type_wf h'"

global_interpretation *l_set_disconnected_nodes_Core_DOM_defs* **defines**
set_disconnected_nodes = l_set_disconnected_nodes_Core_DOM_defs.a_set_disconnected_nodes **and**
set_disconnected_nodes_locs = l_set_disconnected_nodes_Core_DOM_defs.a_set_disconnected_nodes_locs ⟨*proof*⟩
interpretation
i_set_disconnected_nodes?: l_set_disconnected_nodes_Core_DOM type_wf set_disconnected_nodes
set_disconnected_nodes_locs
⟨*proof*⟩
declare *l_set_disconnected_nodes_Core_DOM_axioms[instances]*

lemma *set_disconnected_nodes_is_l_set_disconnected_nodes [instances]:*
"l_set_disconnected_nodes type_wf set_disconnected_nodes set_disconnected_nodes_locs"
⟨*proof*⟩

get_disconnected_nodes **locale** *l_set_disconnected_nodes_get_disconnected_nodes_Core_DOM = l_get_disconnected_node*

+ l_set_disconnected_nodes_Core_DOM
begin
lemma *set_disconnected_nodes_get_disconnected_nodes:*
assumes *"h ⊢ a_set_disconnected_nodes document_ptr disc_nodes →_h h'"*
shows *"h' ⊢ a_get_disconnected_nodes document_ptr →_r disc_nodes"*
⟨*proof*⟩

lemma *set_disconnected_nodes_get_disconnected_nodes_different_pointers:*
assumes *"ptr ≠ ptr'"*
assumes *"w ∈ a_set_disconnected_nodes_locs ptr"*
assumes *"h ⊢ w →_h h'"*
assumes *"r ∈ a_get_disconnected_nodes_locs ptr'"*
shows *"r h h'"*
⟨*proof*⟩
end

locale *l_set_disconnected_nodes_get_disconnected_nodes = l_get_disconnected_nodes*
+ l_set_disconnected_nodes +
assumes *set_disconnected_nodes_get_disconnected_nodes:*

```
  "h ⊢ set_disconnected_nodes document_ptr disc_nodes →_h h'
     ⟹ h' ⊢ get_disconnected_nodes document_ptr →_r disc_nodes"
  assumes set_disconnected_nodes_get_disconnected_nodes_different_pointers:
  "ptr ≠ ptr' ⟹ w ∈ set_disconnected_nodes_locs ptr ⟹ h ⊢ w →_h h'
    ⟹ r ∈ get_disconnected_nodes_locs ptr' ⟹ r h h'"

interpretation i_set_disconnected_nodes_get_disconnected_nodes?:
  l_set_disconnected_nodes_get_disconnected_nodes_Core_DOM type_wf get_disconnected_nodes
                    get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs
  ⟨proof⟩
declare l_set_disconnected_nodes_get_disconnected_nodes_Core_DOM_axioms[instances]

lemma set_disconnected_nodes_get_disconnected_nodes_is_l_set_disconnected_nodes_get_disconnected_nodes
  [instances]:
  "l_set_disconnected_nodes_get_disconnected_nodes  type_wf get_disconnected_nodes get_disconnected_nodes_locs

                                                    set_disconnected_nodes set_disconnected_nodes_locs"
  ⟨proof⟩
```

get_child_nodes

```
locale l_set_disconnected_nodes_get_child_nodes_Core_DOM =
  l_set_disconnected_nodes_Core_DOM +
  l_get_child_nodes_Core_DOM
begin
lemma set_disconnected_nodes_get_child_nodes:
  "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"
  ⟨proof⟩
end

locale l_set_disconnected_nodes_get_child_nodes = l_set_disconnected_nodes_defs + l_get_child_nodes_defs
+
  assumes set_disconnected_nodes_get_child_nodes [simp]:
    "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"

interpretation
  i_set_disconnected_nodes_get_child_nodes?: l_set_disconnected_nodes_get_child_nodes_Core_DOM
                                             type_wf
                                             set_disconnected_nodes set_disconnected_nodes_locs
                                             known_ptr get_child_nodes get_child_nodes_locs
  ⟨proof⟩
declare l_set_disconnected_nodes_get_child_nodes_Core_DOM_axioms[instances]

lemma set_disconnected_nodes_get_child_nodes_is_l_set_disconnected_nodes_get_child_nodes [instances]:
  "l_set_disconnected_nodes_get_child_nodes set_disconnected_nodes_locs get_child_nodes_locs"
  ⟨proof⟩
```

get_tag_name

```
locale l_get_tag_name_Core_DOM_defs
begin
definition a_get_tag_name :: "(_) element_ptr ⇒ (_, tag_name) dom_prog"
  where
    "a_get_tag_name element_ptr = get_M element_ptr tag_name"

definition a_get_tag_name_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  where
    "a_get_tag_name_locs element_ptr ≡ {preserved (get_M element_ptr tag_name)}"
end

locale l_get_tag_name_defs =
  fixes get_tag_name :: "(_) element_ptr ⇒ (_, tag_name) dom_prog"
  fixes get_tag_name_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"

locale l_get_tag_name_Core_DOM =
```

```
  l_type_wf type_wf +
  l_get_tag_name_defs get_tag_name get_tag_name_locs +
  l_get_tag_name_Core_DOM_defs
  for type_wf :: "(_) heap ⇒ bool"
    and get_tag_name :: "(_) element_ptr ⇒ (_, tag_name) dom_prog"
  and get_tag_name_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set" +
  assumes type_wf_impl: "type_wf = DocumentClass.type_wf"
  assumes get_tag_name_impl: "get_tag_name = a_get_tag_name"
  assumes get_tag_name_locs_impl: "get_tag_name_locs = a_get_tag_name_locs"
begin
lemmas get_tag_name_def = get_tag_name_impl[unfolded a_get_tag_name_def]
lemmas get_tag_name_locs_def = get_tag_name_locs_impl[unfolded a_get_tag_name_locs_def]



lemma get_tag_name_ok:
  "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (get_tag_name element_ptr)"
  ⟨proof⟩

lemma get_tag_name_pure [simp]: "pure (get_tag_name element_ptr) h"
  ⟨proof⟩

lemma get_tag_name_ptr_in_heap [simp]:
  assumes "h ⊢ get_tag_name element_ptr →r children"
  shows "element_ptr |∈| element_ptr_kinds h"
  ⟨proof⟩

lemma get_tag_name_reads: "reads (get_tag_name_locs element_ptr) (get_tag_name element_ptr) h h'"
  ⟨proof⟩
end

locale l_get_tag_name = l_type_wf + l_get_tag_name_defs +
  assumes get_tag_name_reads:
    "reads (get_tag_name_locs element_ptr) (get_tag_name element_ptr) h h'"
  assumes get_tag_name_ok:
    "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (get_tag_name element_ptr)"
  assumes get_tag_name_ptr_in_heap:
    "h ⊢ ok (get_tag_name element_ptr) ⟹ element_ptr |∈| element_ptr_kinds h"
  assumes get_tag_name_pure [simp]:
    "pure (get_tag_name element_ptr) h"


global_interpretation l_get_tag_name_Core_DOM_defs defines
  get_tag_name = l_get_tag_name_Core_DOM_defs.a_get_tag_name and
  get_tag_name_locs = l_get_tag_name_Core_DOM_defs.a_get_tag_name_locs ⟨proof⟩

interpretation
  i_get_tag_name?: l_get_tag_name_Core_DOM type_wf get_tag_name get_tag_name_locs
  ⟨proof⟩
declare l_get_tag_name_Core_DOM_axioms[instances]

lemma get_tag_name_is_l_get_tag_name [instances]:
  "l_get_tag_name type_wf get_tag_name get_tag_name_locs"
  ⟨proof⟩

set_disconnected_nodes locale l_set_disconnected_nodes_get_tag_name_Core_DOM =
  l_set_disconnected_nodes_Core_DOM +
  l_get_tag_name_Core_DOM
begin
lemma set_disconnected_nodes_get_tag_name:
  "∀w ∈ a_set_disconnected_nodes_locs ptr. (h ⊢ w →h h' ⟶ (∀r ∈ a_get_tag_name_locs ptr'. r h h'))"
  ⟨proof⟩
end
```

locale *l_set_disconnected_nodes_get_tag_name = l_set_disconnected_nodes + l_get_tag_name +*
  assumes *set_disconnected_nodes_get_tag_name:*
  "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_tag_name_locs ptr'. r h h'))"

interpretation
  i_set_disconnected_nodes_get_tag_name?: l_set_disconnected_nodes_get_tag_name$_{Core_DOM}$ type_wf
  set_disconnected_nodes set_disconnected_nodes_locs
  get_tag_name get_tag_name_locs
  ⟨*proof*⟩
declare *l_set_disconnected_nodes_get_tag_name$_{Core_DOM}$_axioms[instances]*

lemma *set_disconnected_nodes_get_tag_name_is_l_set_disconnected_nodes_get_tag_name [instances]:*
  "l_set_disconnected_nodes_get_tag_name type_wf set_disconnected_nodes set_disconnected_nodes_locs
  get_tag_name get_tag_name_locs"
  ⟨*proof*⟩

set_child_nodes **locale** *l_set_child_nodes_get_tag_name$_{Core_DOM}$ =*
  l_set_child_nodes$_{Core_DOM}$ +
  l_get_tag_name$_{Core_DOM}$
begin
lemma *set_child_nodes_get_tag_name:*
  "∀w ∈ set_child_nodes_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_tag_name_locs ptr'. r h h'))"
  ⟨*proof*⟩
end

locale *l_set_child_nodes_get_tag_name = l_set_child_nodes + l_get_tag_name +*
  assumes *set_child_nodes_get_tag_name:*
    "∀w ∈ set_child_nodes_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_tag_name_locs ptr'. r h h'))"

interpretation
  i_set_child_nodes_get_tag_name?: l_set_child_nodes_get_tag_name$_{Core_DOM}$ type_wf known_ptr
  set_child_nodes set_child_nodes_locs get_tag_name get_tag_name_locs
  ⟨*proof*⟩
declare *l_set_child_nodes_get_tag_name$_{Core_DOM}$_axioms[instances]*

lemma *set_child_nodes_get_tag_name_is_l_set_child_nodes_get_tag_name [instances]:*
  "l_set_child_nodes_get_tag_name type_wf set_child_nodes set_child_nodes_locs get_tag_name get_tag_name_locs"
  ⟨*proof*⟩

set_tag_type

locale *l_set_tag_name$_{Core_DOM}$_defs*
begin

definition *a_set_tag_name :: "(_) element_ptr ⇒ tag_name ⇒ (_, unit) dom_prog"*
  where
    "a_set_tag_name ptr tag = do {
      m ← get_M ptr attrs;
      put_M ptr tag_name_update tag
    }"
lemmas *set_tag_name_defs = a_set_tag_name_def*

definition *a_set_tag_name_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set"*
  where
    "a_set_tag_name_locs element_ptr ≡ all_args (put_M element_ptr tag_name_update)"
end

locale *l_set_tag_name_defs =*
  fixes *set_tag_name :: "(_) element_ptr ⇒ tag_name ⇒ (_, unit) dom_prog"*
  fixes *set_tag_name_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set"*

locale *l_set_tag_name$_{Core_DOM}$ =*

```
  l_type_wf type_wf +
  l_set_tag_name_defs set_tag_name set_tag_name_locs +
  l_set_tag_name_Core_DOM_defs
  for type_wf :: "(_) heap ⇒ bool"
    and set_tag_name :: "(_) element_ptr ⇒ char list ⇒ (_, unit) dom_prog"
    and set_tag_name_locs :: "(_) element_ptr ⇒ (_, unit) dom_prog set" +
  assumes type_wf_impl: "type_wf = DocumentClass.type_wf"
  assumes set_tag_name_impl: "set_tag_name = a_set_tag_name"
  assumes set_tag_name_locs_impl: "set_tag_name_locs = a_set_tag_name_locs"
begin

lemma set_tag_name_ok:
  "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (set_tag_name element_ptr tag)"
  ⟨proof⟩

lemma set_tag_name_writes:
  "writes (set_tag_name_locs element_ptr) (set_tag_name element_ptr tag) h h'"
  ⟨proof⟩

lemma set_tag_name_pointers_preserved:
  assumes "w ∈ set_tag_name_locs element_ptr"
  assumes "h ⊢ w →_h h'"
  shows "object_ptr_kinds h = object_ptr_kinds h'"
  ⟨proof⟩

lemma set_tag_name_typess_preserved:
  assumes "w ∈ set_tag_name_locs element_ptr"
  assumes "h ⊢ w →_h h'"
  shows "type_wf h = type_wf h'"
  ⟨proof⟩
end

locale l_set_tag_name = l_type_wf + l_set_tag_name_defs +
  assumes set_tag_name_writes:
    "writes (set_tag_name_locs element_ptr) (set_tag_name element_ptr tag) h h'"
  assumes set_tag_name_ok:
    "type_wf h ⟹ element_ptr |∈| element_ptr_kinds h ⟹ h ⊢ ok (set_tag_name element_ptr tag)"
  assumes set_tag_name_pointers_preserved:
    "w ∈ set_tag_name_locs element_ptr ⟹ h ⊢ w →_h h' ⟹ object_ptr_kinds h = object_ptr_kinds h'"
  assumes set_tag_name_types_preserved:
    "w ∈ set_tag_name_locs element_ptr ⟹ h ⊢ w →_h h' ⟹ type_wf h = type_wf h'"


global_interpretation l_set_tag_name_Core_DOM_defs defines
  set_tag_name = l_set_tag_name_Core_DOM_defs.a_set_tag_name and
  set_tag_name_locs = l_set_tag_name_Core_DOM_defs.a_set_tag_name_locs ⟨proof⟩
interpretation
  i_set_tag_name?: l_set_tag_name_Core_DOM type_wf set_tag_name set_tag_name_locs
  ⟨proof⟩
declare l_set_tag_name_Core_DOM_axioms[instances]

lemma set_tag_name_is_l_set_tag_name [instances]:
  "l_set_tag_name type_wf set_tag_name set_tag_name_locs"
  ⟨proof⟩

get_child_nodes locale l_set_tag_name_get_child_nodes_Core_DOM =
  l_set_tag_name_Core_DOM +
  l_get_child_nodes_Core_DOM
begin
lemma set_tag_name_get_child_nodes:
  "∀w ∈ set_tag_name_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"
  ⟨proof⟩
end
```

locale *l_set_tag_name_get_child_nodes = l_set_tag_name + l_get_child_nodes +*
  assumes *set_tag_name_get_child_nodes:*
    "∀w ∈ set_tag_name_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"

interpretation
  i_set_tag_name_get_child_nodes?: l_set_tag_name_get_child_nodes$_{Core_DOM}$ type_wf
  set_tag_name set_tag_name_locs known_ptr
                                get_child_nodes get_child_nodes_locs
  ⟨*proof*⟩
declare *l_set_tag_name_get_child_nodes$_{Core_DOM}$_axioms[instances]*

lemma *set_tag_name_get_child_nodes_is_l_set_tag_name_get_child_nodes [instances]:*
  "l_set_tag_name_get_child_nodes type_wf set_tag_name set_tag_name_locs known_ptr get_child_nodes
                                get_child_nodes_locs"
  ⟨*proof*⟩

get_disconnected_nodes **locale** *l_set_tag_name_get_disconnected_nodes$_{Core_DOM}$ =*
  l_set_tag_name$_{Core_DOM}$ +
  l_get_disconnected_nodes$_{Core_DOM}$
begin
lemma *set_tag_name_get_disconnected_nodes:*
  "∀w ∈ set_tag_name_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_disconnected_nodes_locs ptr'. r h h'))"
  ⟨*proof*⟩
end

locale *l_set_tag_name_get_disconnected_nodes = l_set_tag_name + l_get_disconnected_nodes +*
  assumes *set_tag_name_get_disconnected_nodes:*
    "∀w ∈ set_tag_name_locs ptr. (h ⊢ w →$_h$ h' ⟶ (∀r ∈ get_disconnected_nodes_locs ptr'. r h h'))"

interpretation
  i_set_tag_name_get_disconnected_nodes?: l_set_tag_name_get_disconnected_nodes$_{Core_DOM}$ type_wf
  set_tag_name set_tag_name_locs get_disconnected_nodes
                                    get_disconnected_nodes_locs
  ⟨*proof*⟩
declare *l_set_tag_name_get_disconnected_nodes$_{Core_DOM}$_axioms[instances]*

lemma *set_tag_name_get_disconnected_nodes_is_l_set_tag_name_get_disconnected_nodes [instances]:*
  "l_set_tag_name_get_disconnected_nodes type_wf set_tag_name set_tag_name_locs get_disconnected_nodes
                                    get_disconnected_nodes_locs"
  ⟨*proof*⟩

get_tag_type **locale** *l_set_tag_name_get_tag_name$_{Core_DOM}$ = l_get_tag_name$_{Core_DOM}$*
  + l_set_tag_name$_{Core_DOM}$
begin
lemma *set_tag_name_get_tag_name:*
  assumes *"h ⊢ a_set_tag_name element_ptr tag →$_h$ h'"*
  shows *"h' ⊢ a_get_tag_name element_ptr →$_r$ tag"*
  ⟨*proof*⟩

lemma *set_tag_name_get_tag_name_different_pointers:*
  assumes *"ptr ≠ ptr'"*
  assumes *"w ∈ a_set_tag_name_locs ptr"*
  assumes *"h ⊢ w →$_h$ h'"*
  assumes *"r ∈ a_get_tag_name_locs ptr'"*
  shows *"r h h'"*
  ⟨*proof*⟩
end

locale *l_set_tag_name_get_tag_name = l_get_tag_name + l_set_tag_name +*
  assumes *set_tag_name_get_tag_name:*
    "h ⊢ set_tag_name element_ptr tag →$_h$ h'
     ⟹ h' ⊢ get_tag_name element_ptr →$_r$ tag"

```
  assumes set_tag_name_get_tag_name_different_pointers:
    "ptr ≠ ptr' ⟹ w ∈ set_tag_name_locs ptr ⟹ h ⊢ w →_h h'
    ⟹ r ∈ get_tag_name_locs ptr' ⟹ r h h'"

interpretation i_set_tag_name_get_tag_name?:
  l_set_tag_name_get_tag_name_Core_DOM type_wf get_tag_name
  get_tag_name_locs set_tag_name set_tag_name_locs
  ⟨proof⟩
declare l_set_tag_name_get_tag_name_Core_DOM_axioms[instances]

lemma set_tag_name_get_tag_name_is_l_set_tag_name_get_tag_name [instances]:
  "l_set_tag_name_get_tag_name  type_wf get_tag_name get_tag_name_locs
                                                    set_tag_name set_tag_name_locs"
  ⟨proof⟩
```

set_val

```
locale l_set_val_Core_DOM_defs
begin

definition a_set_val :: "(_) character_data_ptr ⇒ DOMString ⇒ (_, unit) dom_prog"
  where
    "a_set_val ptr v = do {
      m ← get_M ptr val;
      put_M ptr val_update v
    }"
lemmas set_val_defs = a_set_val_def

definition a_set_val_locs :: "(_) character_data_ptr ⇒ (_, unit) dom_prog set"
  where
    "a_set_val_locs character_data_ptr ≡ all_args (put_M character_data_ptr val_update)"
end

locale l_set_val_defs =
  fixes set_val :: "(_) character_data_ptr ⇒ DOMString ⇒ (_, unit) dom_prog"
  fixes set_val_locs :: "(_) character_data_ptr ⇒ (_, unit) dom_prog set"

locale l_set_val_Core_DOM =
  l_type_wf type_wf +
  l_set_val_defs set_val set_val_locs +
  l_set_val_Core_DOM_defs
  for type_wf :: "(_) heap ⇒ bool"
  and set_val :: "(_) character_data_ptr ⇒ char list ⇒ (_, unit) dom_prog"
  and set_val_locs :: "(_) character_data_ptr ⇒ (_, unit) dom_prog set"  +
  assumes type_wf_impl: "type_wf = DocumentClass.type_wf"
  assumes set_val_impl: "set_val = a_set_val"
  assumes set_val_locs_impl: "set_val_locs = a_set_val_locs"
begin

lemma set_val_ok:
  "type_wf h ⟹ character_data_ptr |∈| character_data_ptr_kinds h ⟹ h ⊢ ok (set_val character_data_ptr
tag)"
  ⟨proof⟩

lemma set_val_writes: "writes (set_val_locs character_data_ptr) (set_val character_data_ptr tag) h h'"
  ⟨proof⟩

lemma set_val_pointers_preserved:
  assumes "w ∈ set_val_locs character_data_ptr"
  assumes "h ⊢ w →_h h'"
  shows "object_ptr_kinds h = object_ptr_kinds h'"
  ⟨proof⟩
```

lemma *set_val_typess_preserved:*
assumes *"w ∈ set_val_locs character_data_ptr"*
assumes *"h ⊢ w →_h h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩
end

locale *l_set_val = l_type_wf + l_set_val_defs +*
assumes *set_val_writes:*
"writes (set_val_locs character_data_ptr) (set_val character_data_ptr tag) h h'"
assumes *set_val_ok:*
"type_wf h ⟹ character_data_ptr |∈| character_data_ptr_kinds h ⟹ h ⊢ ok (set_val character_data_ptr tag)"
assumes *set_val_pointers_preserved:*
"w ∈ set_val_locs character_data_ptr ⟹ h ⊢ w →_h h' ⟹ object_ptr_kinds h = object_ptr_kinds h'"
assumes *set_val_types_preserved:*
"w ∈ set_val_locs character_data_ptr ⟹ h ⊢ w →_h h' ⟹ type_wf h = type_wf h'"

global_interpretation *l_set_val*$_{Core_DOM}$*_defs* **defines**
set_val = l_set_val$_{Core_DOM}$*_defs.a_set_val* **and**
set_val_locs = l_set_val$_{Core_DOM}$*_defs.a_set_val_locs* ⟨*proof*⟩
interpretation
i_set_val?: l_set_val$_{Core_DOM}$ *type_wf set_val set_val_locs*
⟨*proof*⟩
declare *l_set_val*$_{Core_DOM}$*_axioms[instances]*

lemma *set_val_is_l_set_val [instances]: "l_set_val type_wf set_val set_val_locs"*
⟨*proof*⟩

get_child_nodes **locale** *l_set_val_get_child_nodes*$_{Core_DOM}$ =
l_set_val$_{Core_DOM}$ +
l_get_child_nodes$_{Core_DOM}$
begin
lemma *set_val_get_child_nodes:*
"∀w ∈ set_val_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"
⟨*proof*⟩
end

locale *l_set_val_get_child_nodes = l_set_val + l_get_child_nodes +*
assumes *set_val_get_child_nodes:*
"∀w ∈ set_val_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_child_nodes_locs ptr'. r h h'))"

interpretation
i_set_val_get_child_nodes?: l_set_val_get_child_nodes$_{Core_DOM}$ *type_wf set_val set_val_locs known_ptr*
get_child_nodes get_child_nodes_locs
⟨*proof*⟩
declare *l_set_val_get_child_nodes*$_{Core_DOM}$*_axioms[instances]*

lemma *set_val_get_child_nodes_is_l_set_val_get_child_nodes [instances]:*
"l_set_val_get_child_nodes type_wf set_val set_val_locs known_ptr get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

get_disconnected_nodes **locale** *l_set_val_get_disconnected_nodes*$_{Core_DOM}$ =
l_set_val$_{Core_DOM}$ +
l_get_disconnected_nodes$_{Core_DOM}$
begin
lemma *set_val_get_disconnected_nodes:*
"∀w ∈ set_val_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_disconnected_nodes_locs ptr'. r h h'))"
⟨*proof*⟩
end

```
locale l_set_val_get_disconnected_nodes = l_set_val + l_get_disconnected_nodes +
  assumes set_val_get_disconnected_nodes:
   "∀w ∈ set_val_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_disconnected_nodes_locs ptr'. r h h'))"

interpretation
  i_set_val_get_disconnected_nodes?: l_set_val_get_disconnected_nodes_Core_DOM type_wf set_val
                                     set_val_locs get_disconnected_nodes get_disconnected_nodes_locs
  ⟨proof⟩
declare l_set_val_get_disconnected_nodes_Core_DOM_axioms[instances]

lemma set_val_get_disconnected_nodes_is_l_set_val_get_disconnected_nodes [instances]:
  "l_set_val_get_disconnected_nodes type_wf set_val set_val_locs get_disconnected_nodes
get_disconnected_nodes_locs"
  ⟨proof⟩
```

get_parent

```
locale l_get_parent_Core_DOM_defs =
  l_get_child_nodes_defs get_child_nodes get_child_nodes_locs
  for get_child_nodes :: "(_::linorder) object_ptr ⇒ (_, (_) node_ptr list) dom_prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
definition a_get_parent :: "(_) node_ptr ⇒ (_, (_::linorder) object_ptr option) dom_prog"
  where
    "a_get_parent node_ptr = do {
      check_in_heap (cast node_ptr);
      parent_ptrs ← object_ptr_kinds_M ⤜ filter_M (λptr. do {
        children ← get_child_nodes ptr;
        return (node_ptr ∈ set children)
      });
      (if parent_ptrs = []
        then return None
        else return (Some (hd parent_ptrs)))
    }"

definition
  "a_get_parent_locs ≡ (⋃ptr. get_child_nodes_locs ptr ∪ {preserved (get_M_Object  ptr RObject.nothing)})"
end

locale l_get_parent_defs =
  fixes get_parent :: "(_) node_ptr ⇒ (_, (_::linorder) object_ptr option) dom_prog"
  fixes get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"

locale l_get_parent_Core_DOM =
  l_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs +
  l_known_ptrs known_ptr known_ptrs +
  l_get_parent_Core_DOM_defs get_child_nodes get_child_nodes_locs +
  l_get_parent_defs get_parent get_parent_locs
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
  and type_wf :: "(_) heap ⇒ bool"
  and get_child_nodes
  and get_child_nodes_locs
  and known_ptrs :: "(_) heap ⇒ bool"
  and get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"
  and get_parent_locs  +
  assumes get_parent_impl: "get_parent = a_get_parent"
  assumes get_parent_locs_impl: "get_parent_locs = a_get_parent_locs"
begin
lemmas get_parent_def = get_parent_impl[unfolded a_get_parent_def]
lemmas get_parent_locs_def = get_parent_locs_impl[unfolded a_get_parent_locs_def]

lemma get_parent_pure [simp]: "pure (get_parent ptr) h"
  ⟨proof⟩
```

lemma *get_parent_ok [simp]:*
assumes *"type_wf h"*
assumes *"known_ptrs h"*
assumes *"ptr |∈| node_ptr_kinds h"*
shows *"h ⊢ ok (get_parent ptr)"*
⟨*proof*⟩

lemma *get_parent_ptr_in_heap [simp]: "h ⊢ ok (get_parent node_ptr) ⟹ node_ptr |∈| node_ptr_kinds h"*
⟨*proof*⟩

lemma *get_parent_parent_in_heap:*
assumes *"h ⊢ get_parent child_node →r Some parent"*
shows *"parent |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *get_parent_child_dual:*
assumes *"h ⊢ get_parent child →r Some ptr"*
obtains *children* **where** *"h ⊢ get_child_nodes ptr →r children"* **and** *"child ∈ set children"*
⟨*proof*⟩

lemma *get_parent_reads: "reads get_parent_locs (get_parent node_ptr) h h'"*
⟨*proof*⟩

lemma *get_parent_reads_pointers: "preserved (get_M$_{Object}$ ptr RObject.nothing) ∈ get_parent_locs"*
⟨*proof*⟩
end

locale *l_get_parent = l_type_wf + l_known_ptrs + l_get_parent_defs + l_get_child_nodes +*
assumes *get_parent_reads:*
"reads get_parent_locs (get_parent node_ptr) h h'"
assumes *get_parent_ok:*
"type_wf h ⟹ known_ptrs h ⟹ node_ptr |∈| node_ptr_kinds h ⟹ h ⊢ ok (get_parent node_ptr)"
assumes *get_parent_ptr_in_heap:*
"h ⊢ ok (get_parent node_ptr) ⟹ node_ptr |∈| node_ptr_kinds h"
assumes *get_parent_pure [simp]:*
"pure (get_parent node_ptr) h"
assumes *get_parent_parent_in_heap:*
"h ⊢ get_parent child_node →r Some parent ⟹ parent |∈| object_ptr_kinds h"
assumes *get_parent_child_dual:*
"h ⊢ get_parent child →r Some ptr ⟹ (⋀children. h ⊢ get_child_nodes ptr →r children
⟹ child ∈ set children ⟹ thesis) ⟹ thesis"
assumes *get_parent_reads_pointers:*
"preserved (get_M$_{Object}$ ptr RObject.nothing) ∈ get_parent_locs"

global_interpretation *l_get_parent$_{Core_DOM}$_defs get_child_nodes get_child_nodes_locs* **defines**
get_parent = "l_get_parent$_{Core_DOM}$_defs.a_get_parent get_child_nodes" **and**
get_parent_locs = "l_get_parent$_{Core_DOM}$_defs.a_get_parent_locs get_child_nodes_locs" ⟨*proof*⟩

interpretation
i_get_parent?: l_get_parent$_{Core_DOM}$ known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs

get_parent get_parent_locs
⟨*proof*⟩
declare *l_get_parent$_{Core_DOM}$_axioms[instances]*

lemma *get_parent_is_l_get_parent [instances]:*
"l_get_parent type_wf known_ptr known_ptrs get_parent get_parent_locs get_child_nodes get_child_nodes_locs"
⟨*proof*⟩

set_disconnected_nodes **locale** *l_set_disconnected_nodes_get_parent$_{Core_DOM}$ =*
l_set_disconnected_nodes_get_child_nodes
set_disconnected_nodes set_disconnected_nodes_locs get_child_nodes get_child_nodes_locs

```
  + l_set_disconnected_nodes_Core_DOM
    type_wf set_disconnected_nodes set_disconnected_nodes_locs
  + l_get_parent_Core_DOM
    known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  for known_ptr :: "('_::linorder) object_ptr ⇒ bool"
  and type_wf :: "('_) heap ⇒ bool"
  and set_disconnected_nodes :: "('_) document_ptr ⇒ ('_) node_ptr list ⇒ (('_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "('_) document_ptr ⇒ (('_) heap, exception, unit) prog set"
  and get_child_nodes :: "('_) object_ptr ⇒ (('_) heap, exception, ('_) node_ptr list) prog"
  and get_child_nodes_locs :: "('_) object_ptr ⇒ (('_) heap ⇒ ('_) heap ⇒ bool) set"
  and known_ptrs :: "('_) heap ⇒ bool"
  and get_parent :: "('_) node_ptr ⇒ (('_) heap, exception, ('_) object_ptr option) prog"
  and get_parent_locs :: "(('_) heap ⇒ ('_) heap ⇒ bool) set"
begin
lemma set_disconnected_nodes_get_parent [simp]:
  "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_parent_locs. r h h'))"
  ⟨proof⟩
end

locale l_set_disconnected_nodes_get_parent = l_set_disconnected_nodes_defs + l_get_parent_defs +
  assumes set_disconnected_nodes_get_parent [simp]:
    "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →_h h' ⟶ (∀r ∈ get_parent_locs. r h h'))"

interpretation i_set_disconnected_nodes_get_parent?:
  l_set_disconnected_nodes_get_parent_Core_DOM known_ptr type_wf set_disconnected_nodes
  set_disconnected_nodes_locs get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  ⟨proof⟩
declare l_set_disconnected_nodes_get_parent_Core_DOM_axioms[instances]

lemma set_disconnected_nodes_get_parent_is_l_set_disconnected_nodes_get_parent [instances]:
  "l_set_disconnected_nodes_get_parent set_disconnected_nodes_locs get_parent_locs"
  ⟨proof⟩
```

get_root_node

```
locale l_get_root_node_Core_DOM_defs =
  l_get_parent_defs get_parent get_parent_locs
  for get_parent :: "('_) node_ptr ⇒ (('_) heap, exception, ('_::linorder) object_ptr option) prog"
  and get_parent_locs :: "(('_) heap ⇒ ('_) heap ⇒ bool) set"
begin
partial_function (dom_prog)
  a_get_ancestors :: "('_::linorder) object_ptr ⇒ ('_, ('_) object_ptr list) dom_prog"
  where
    "a_get_ancestors ptr = do {
      check_in_heap ptr;
      ancestors ← (case cast_object_ptr2node_ptr ptr of
        Some node_ptr ⇒ do {
          parent_ptr_opt ← get_parent node_ptr;
          (case parent_ptr_opt of
            Some parent_ptr ⇒ a_get_ancestors parent_ptr
          | None ⇒ return [])
        }
      | None ⇒ return []);
      return (ptr # ancestors)
    }"

definition "a_get_ancestors_locs = get_parent_locs"

definition a_get_root_node :: "('_) object_ptr ⇒ ('_, ('_) object_ptr) dom_prog"
  where
    "a_get_root_node ptr = do {
      ancestors ← a_get_ancestors ptr;
      return (last ancestors)
```

```
    }"
definition "a_get_root_node_locs = a_get_ancestors_locs"
end

locale l_get_ancestors_defs =
  fixes get_ancestors :: "(_::linorder) object_ptr ⇒ (_, (_) object_ptr list) dom_prog"
  fixes get_ancestors_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"

locale l_get_root_node_defs =
  fixes get_root_node :: "(_) object_ptr ⇒ (_, (_) object_ptr) dom_prog"
  fixes get_root_node_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"

locale l_get_root_node_Core_DOM =
  l_get_parent +
  l_get_root_node_Core_DOM_defs +
  l_get_ancestors_defs +
  l_get_root_node_defs +
  assumes get_ancestors_impl: "get_ancestors = a_get_ancestors"
  assumes get_ancestors_locs_impl: "get_ancestors_locs = a_get_ancestors_locs"
  assumes get_root_node_impl: "get_root_node = a_get_root_node"
  assumes get_root_node_locs_impl: "get_root_node_locs = a_get_root_node_locs"
begin
lemmas get_ancestors_def = a_get_ancestors.simps[folded get_ancestors_impl]
lemmas get_ancestors_locs_def = a_get_ancestors_locs_def[folded get_ancestors_locs_impl]
lemmas get_root_node_def = a_get_root_node_def[folded get_root_node_impl get_ancestors_impl]
lemmas get_root_node_locs_def = a_get_root_node_locs_def[folded get_root_node_locs_impl
                                                          get_ancestors_locs_impl]

lemma get_ancestors_pure [simp]:
  "pure (get_ancestors ptr) h"
⟨proof⟩


lemma get_root_node_pure [simp]: "pure (get_root_node ptr) h"
  ⟨proof⟩


lemma get_ancestors_ptr_in_heap:
  assumes "h ⊢ ok (get_ancestors ptr)"
  shows "ptr |∈| object_ptr_kinds h"
  ⟨proof⟩

lemma get_ancestors_ptr:
  assumes "h ⊢ get_ancestors ptr →r ancestors"
  shows "ptr ∈ set ancestors"
  ⟨proof⟩

lemma get_ancestors_not_node:
  assumes "h ⊢ get_ancestors ptr →r ancestors"
  assumes "¬is_node_ptr_kind ptr"
  shows "ancestors = [ptr]"
  ⟨proof⟩

lemma get_root_node_no_parent:
  "h ⊢ get_parent node_ptr →r None ⟹ h ⊢ get_root_node (cast node_ptr) →r cast node_ptr"
  ⟨proof⟩

end

locale l_get_ancestors = l_get_ancestors_defs +
  assumes get_ancestors_pure [simp]: "pure (get_ancestors node_ptr) h"
  assumes get_ancestors_ptr_in_heap: "h ⊢ ok (get_ancestors ptr) ⟹ ptr |∈| object_ptr_kinds h"
  assumes get_ancestors_ptr: "h ⊢ get_ancestors ptr →r ancestors ⟹ ptr ∈ set ancestors"
```

locale *l_get_root_node = l_get_root_node_defs + l_get_parent_defs +*
  assumes *get_root_node_pure[simp]:*
   "pure (get_root_node ptr) h"
  assumes *get_root_node_no_parent:*
   "h ⊢ get_parent node_ptr →r None ⟹ h ⊢ get_root_node (cast node_ptr) →r cast node_ptr"

global_interpretation *l_get_root_node$_{Core_DOM}$_defs get_parent get_parent_locs*
  defines *get_root_node = "l_get_root_node$_{Core_DOM}$_defs.a_get_root_node get_parent"*
      and *get_root_node_locs = "l_get_root_node$_{Core_DOM}$_defs.a_get_root_node_locs get_parent_locs"*
      and *get_ancestors = "l_get_root_node$_{Core_DOM}$_defs.a_get_ancestors get_parent"*
      and *get_ancestors_locs = "l_get_root_node$_{Core_DOM}$_defs.a_get_ancestors_locs get_parent_locs"*
  ⟨*proof*⟩
declare *a_get_ancestors.simps [code]*

interpretation
  i_get_root_node?: l_get_root_node$_{Core_DOM}$ type_wf known_ptr known_ptrs get_parent get_parent_locs
                  get_child_nodes get_child_nodes_locs get_ancestors get_ancestors_locs
                  get_root_node get_root_node_locs
  ⟨*proof*⟩
declare *l_get_root_node$_{Core_DOM}$_axioms[instances]*

lemma *get_ancestors_is_l_get_ancestors [instances]: "l_get_ancestors get_ancestors"*
  ⟨*proof*⟩

lemma *get_root_node_is_l_get_root_node [instances]: "l_get_root_node get_root_node get_parent"*
  ⟨*proof*⟩

set_disconnected_nodes **locale** *l_set_disconnected_nodes_get_ancestors$_{Core_DOM}$ =*
  l_set_disconnected_nodes_get_parent
    set_disconnected_nodes set_disconnected_nodes_locs get_parent get_parent_locs
  + l_get_root_node$_{Core_DOM}$
    type_wf known_ptr known_ptrs get_parent get_parent_locs get_child_nodes get_child_nodes_locs
    get_ancestors get_ancestors_locs get_root_node get_root_node_locs
  + l_set_disconnected_nodes$_{Core_DOM}$
     type_wf set_disconnected_nodes set_disconnected_nodes_locs
  for *known_ptr :: "(_::linorder) object_ptr ⇒ bool"*
  and *set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"*
  and *set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"*
  and *get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
  and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
  and *get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"*
  and *get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"*
  and *type_wf :: "(_) heap ⇒ bool"*
  and *known_ptrs :: "(_) heap ⇒ bool"*
  and *get_ancestors :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"*
  and *get_ancestors_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"*
  and *get_root_node :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr) prog"*
  and *get_root_node_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"*
begin
lemma *set_disconnected_nodes_get_ancestors:*
  "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →h h' ⟶ (∀r ∈ get_ancestors_locs. r h h'))"
  ⟨*proof*⟩
end

locale *l_set_disconnected_nodes_get_ancestors = l_set_disconnected_nodes_defs + l_get_ancestors_defs +*
  assumes *set_disconnected_nodes_get_ancestors:*
    "∀w ∈ set_disconnected_nodes_locs ptr. (h ⊢ w →h h' ⟶ (∀r ∈ get_ancestors_locs. r h h'))"

interpretation
  i_set_disconnected_nodes_get_ancestors?: l_set_disconnected_nodes_get_ancestors$_{Core_DOM}$ known_ptr
                                        set_disconnected_nodes set_disconnected_nodes_locs
                                        get_child_nodes get_child_nodes_locs get_parent

```
                                        get_parent_locs type_wf known_ptrs get_ancestors
                                        get_ancestors_locs get_root_node get_root_node_locs
  ⟨proof⟩
declare l_set_disconnected_nodes_get_ancestors_Core_DOM_axioms[instances]


lemma set_disconnected_nodes_get_ancestors_is_l_set_disconnected_nodes_get_ancestors [instances]:
  "l_set_disconnected_nodes_get_ancestors set_disconnected_nodes_locs get_ancestors_locs"
  ⟨proof⟩
```

get_owner_document

```
locale l_get_owner_document_Core_DOM_defs =
  l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
  l_get_root_node_defs get_root_node get_root_node_locs
  for get_root_node :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) object_ptr) prog"
  and get_root_node_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin

definition a_get_owner_document_node_ptr :: "(_) node_ptr ⇒ unit ⇒ (_, (_) document_ptr) dom_prog"
  where
    "a_get_owner_document_node_ptr node_ptr _ = do {
      root ← get_root_node (cast node_ptr);
      (case cast root of
        Some document_ptr ⇒ return document_ptr
      | None ⇒ do {
        ptrs ← document_ptr_kinds_M;
        candidates ← filter_M (λdocument_ptr. do {
          disconnected_nodes ← get_disconnected_nodes document_ptr;
          return (root ∈ cast ` set disconnected_nodes)
        }) ptrs;
        return (hd candidates)
      })
    }"

definition
  a_get_owner_document_document_ptr :: "(_) document_ptr ⇒ unit ⇒ (_, (_) document_ptr) dom_prog"
  where
    "a_get_owner_document_document_ptr document_ptr _ = do {
      document_ptrs ← document_ptr_kinds_M;
      (if document_ptr ∈ set document_ptrs then return document_ptr else error SegmentationFault)}"

definition
  a_get_owner_document_tups :: "(((_) object_ptr ⇒ bool) × ((_) object_ptr ⇒ unit
                                          ⇒ (_, (_) document_ptr) dom_prog)) list"
  where
     "a_get_owner_document_tups = [
        (is_element_ptr, a_get_owner_document_node_ptr ∘ the ∘ cast),
        (is_character_data_ptr, a_get_owner_document_node_ptr ∘ the ∘ cast),
        (is_document_ptr, a_get_owner_document_document_ptr ∘ the ∘ cast)
      ]"

definition a_get_owner_document :: "(_) object_ptr ⇒ (_, (_) document_ptr) dom_prog"
  where
    "a_get_owner_document ptr = invoke a_get_owner_document_tups ptr ()"
end

locale l_get_owner_document_defs =
  fixes get_owner_document :: "(_::linorder) object_ptr ⇒ (_, (_) document_ptr) dom_prog"

locale l_get_owner_document_Core_DOM =
```

```
  l_known_ptr known_ptr +
  l_get_disconnected_nodes type_wf get_disconnected_nodes get_disconnected_nodes_locs +
  l_get_root_node get_root_node get_root_node_locs +
  l_get_owner_document_Core_DOM_defs get_root_node get_root_node_locs get_disconnected_nodes
                                get_disconnected_nodes_locs +
  l_get_owner_document_defs get_owner_document
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
  and type_wf :: "(_) heap ⇒ bool"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_root_node :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr) prog"
  and get_root_node_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_owner_document :: "(_) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog" +
  assumes known_ptr_impl: "known_ptr = a_known_ptr"
  assumes get_owner_document_impl: "get_owner_document = a_get_owner_document"
begin
lemmas known_ptr_def = known_ptr_impl[unfolded a_known_ptr_def]
lemmas get_owner_document_def = a_get_owner_document_def[folded get_owner_document_impl]

lemma get_owner_document_split:
  "P (invoke (a_get_owner_document_tups @ xs) ptr ()) =
    ((known_ptr ptr ⟶ P (get_owner_document ptr))
    ∧ (¬(known_ptr ptr) ⟶ P (invoke xs ptr ())))"
  ⟨proof⟩

lemma get_owner_document_split_asm:
  "P (invoke (a_get_owner_document_tups @ xs) ptr ()) =
    (¬((known_ptr ptr ∧ ¬P (get_owner_document ptr))
    ∨ (¬(known_ptr ptr) ∧ ¬P (invoke xs ptr ()))))"
  ⟨proof⟩
lemmas get_owner_document_splits = get_owner_document_split get_owner_document_split_asm

lemma get_owner_document_pure [simp]:
  "pure (get_owner_document ptr) h"
⟨proof⟩

lemma get_owner_document_ptr_in_heap:
  assumes "h ⊢ ok (get_owner_document ptr)"
  shows "ptr |∈| object_ptr_kinds h"
  ⟨proof⟩
end

locale l_get_owner_document = l_get_owner_document_defs +
  assumes get_owner_document_ptr_in_heap:
    "h ⊢ ok (get_owner_document ptr) ⟹ ptr |∈| object_ptr_kinds h"
  assumes get_owner_document_pure [simp]:
    "pure (get_owner_document ptr) h"

global_interpretation l_get_owner_document_Core_DOM_defs get_root_node get_root_node_locs
                                                         get_disconnected_nodes get_disconnected_nodes_locs
  defines get_owner_document_tups =
          "l_get_owner_document_Core_DOM_defs.a_get_owner_document_tups get_root_node get_disconnected_nodes"
    and get_owner_document =
        "l_get_owner_document_Core_DOM_defs.a_get_owner_document get_root_node get_disconnected_nodes"
    and get_owner_document_node_ptr =
        "l_get_owner_document_Core_DOM_defs.a_get_owner_document_node_ptr get_root_node get_disconnected_nodes"
  ⟨proof⟩
interpretation
  i_get_owner_document?: l_get_owner_document_Core_DOM get_parent get_parent_locs known_ptr type_wf
                         get_disconnected_nodes get_disconnected_nodes_locs get_root_node
                         get_root_node_locs get_owner_document
  ⟨proof⟩
declare l_get_owner_document_Core_DOM_axioms[instances]
```

lemma *get_owner_document_is_l_get_owner_document [instances]:*
"l_get_owner_document get_owner_document"
⟨*proof*⟩

remove_child

locale *l_remove_child*$_{Core_DOM}$*_defs =*
l_get_child_nodes_defs get_child_nodes get_child_nodes_locs +
l_set_child_nodes_defs set_child_nodes set_child_nodes_locs +
l_get_parent_defs get_parent get_parent_locs +
l_get_owner_document_defs get_owner_document +
l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs
for *get_child_nodes :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
and *set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"*
and *set_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap, exception, unit) prog set"*
and *get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"*
and *get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"*
and *get_owner_document :: "(_) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog"*
and *get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
and *set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"*
and *set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"*
begin
definition *a_remove_child :: "(_) object_ptr ⇒ (_) node_ptr ⇒ (_, unit) dom_prog"*
where
"a_remove_child ptr child = do {
children ← get_child_nodes ptr;
if child ∉ set children then
error NotFoundError
else do {
owner_document ← get_owner_document (cast child);
disc_nodes ← get_disconnected_nodes owner_document;
set_disconnected_nodes owner_document (child # disc_nodes);
set_child_nodes ptr (remove1 child children)
}
}"

definition *a_remove_child_locs :: "(_) object_ptr ⇒ (_) document_ptr ⇒ (_, unit) dom_prog set"*
where
"a_remove_child_locs ptr owner_document = set_child_nodes_locs ptr
∪ set_disconnected_nodes_locs owner_document"

definition *a_remove :: "(_) node_ptr ⇒ (_, unit) dom_prog"*
where
"a_remove node_ptr = do {
parent_opt ← get_parent node_ptr;
(case parent_opt of
Some parent ⇒ do {
a_remove_child parent node_ptr;
return ()
}
| None ⇒ return ())
}"
end

locale *l_remove_child_defs =*
fixes *remove_child :: "(_::linorder) object_ptr ⇒ (_) node_ptr ⇒ (_, unit) dom_prog"*
fixes *remove_child_locs :: "(_) object_ptr ⇒ (_) document_ptr ⇒ (_, unit) dom_prog set"*

locale *l_remove_defs =*

fixes *remove :: "(_) node_ptr ⇒ (_, unit) dom_prog"*

locale *l_remove_child*$_{Core_DOM}$ =
l_remove_child$_{Core_DOM}$*_defs +*
l_remove_child_defs +
l_remove_defs +
l_get_parent +
l_get_owner_document +
l_set_child_nodes_get_child_nodes +
l_set_child_nodes_get_disconnected_nodes +
l_set_disconnected_nodes_get_disconnected_nodes +
l_set_disconnected_nodes_get_child_nodes +
assumes *remove_child_impl: "remove_child = a_remove_child"*
assumes *remove_child_locs_impl: "remove_child_locs = a_remove_child_locs"*
assumes *remove_impl: "remove = a_remove"*
begin
lemmas *remove_child_def = a_remove_child_def[folded remove_child_impl]*
lemmas *remove_child_locs_def = a_remove_child_locs_def[folded remove_child_locs_impl]*
lemmas *remove_def = a_remove_def[folded remove_child_impl remove_impl]*

lemma *remove_child_ptr_in_heap:*
assumes *"h ⊢ ok (remove_child ptr child)"*
shows *"ptr |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *remove_child_child_in_heap:*
assumes *"h ⊢ remove_child ptr' child →*$_h$ *h'"*
shows *"child |∈| node_ptr_kinds h"*
⟨*proof*⟩

lemma *remove_child_in_disconnected_nodes:*

assumes *"h ⊢ remove_child ptr child →*$_h$ *h'"*
assumes *"h ⊢ get_owner_document (cast child) →*$_r$ *owner_document"*
assumes *"h' ⊢ get_disconnected_nodes owner_document →*$_r$ *disc_nodes"*
shows *"child ∈ set disc_nodes"*
⟨*proof*⟩

lemma *remove_child_writes [simp]:*
"writes (remove_child_locs ptr |h ⊢ get_owner_document (cast child)|$_r$*) (remove_child ptr child) h h'"*
⟨*proof*⟩

lemma *remove_writes:*
"writes (remove_child_locs (the |h ⊢ get_parent child|$_r$*) |h ⊢ get_owner_document (cast child)|*$_r$*)*
(remove child) h h'"
⟨*proof*⟩

lemma *remove_child_children_subset:*
assumes *"h ⊢ remove_child parent child →*$_h$ *h'"*
and *"h ⊢ get_child_nodes ptr →*$_r$ *children"*
and *"h' ⊢ get_child_nodes ptr →*$_r$ *children'"*
and *known_ptrs: "known_ptrs h"*
and *type_wf: "type_wf h"*
shows *"set children' ⊆ set children"*
⟨*proof*⟩

lemma *remove_child_pointers_preserved:*
assumes *"w ∈ remove_child_locs ptr owner_document"*
assumes *"h ⊢ w →*$_h$ *h'"*
shows *"object_ptr_kinds h = object_ptr_kinds h'"*

⟨*proof*⟩

lemma *remove_child_types_preserved:*
assumes *"w ∈ remove_child_locs ptr owner_document"*
assumes *"h ⊢ w →$_h$ h'"*
shows *"type_wf h = type_wf h'"*
⟨*proof*⟩
end

locale *l_remove_child = l_type_wf + l_known_ptrs + l_remove_child_defs + l_get_owner_document_defs*
+ l_get_child_nodes_defs + l_get_disconnected_nodes_defs +
assumes *remove_child_writes:*
"writes (remove_child_locs object_ptr |h ⊢ get_owner_document (cast child)|$_r$)
(remove_child object_ptr child) h h'"
assumes *remove_child_pointers_preserved:*
"w ∈ remove_child_locs ptr owner_document ⟹ h ⊢ w →$_h$ h' ⟹ object_ptr_kinds h = object_ptr_kinds h'"
assumes *remove_child_types_preserved:*
"w ∈ remove_child_locs ptr owner_document ⟹ h ⊢ w →$_h$ h' ⟹ type_wf h = type_wf h'"
assumes *remove_child_in_disconnected_nodes:*
"known_ptrs h ⟹ h ⊢ remove_child ptr child →$_h$ h'
⟹ h ⊢ get_owner_document (cast child) →$_r$ owner_document
⟹ h' ⊢ get_disconnected_nodes owner_document →$_r$ disc_nodes
⟹ child ∈ set disc_nodes"
assumes *remove_child_ptr_in_heap: "h ⊢ ok (remove_child ptr child) ⟹ ptr |∈| object_ptr_kinds h"*
assumes *remove_child_child_in_heap: "h ⊢ remove_child ptr' child →$_h$ h' ⟹ child |∈| node_ptr_kinds h"*
assumes *remove_child_children_subset:*
"known_ptrs h ⟹ type_wf h ⟹ h ⊢ remove_child parent child →$_h$ h'
⟹ h ⊢ get_child_nodes ptr →$_r$ children
⟹ h' ⊢ get_child_nodes ptr →$_r$ children'
⟹ set children' ⊆ set children"

locale *l_remove*

global_interpretation *l_remove_child$_{Core_DOM}$_defs get_child_nodes get_child_nodes_locs set_child_nodes*
set_child_nodes_locs get_parent get_parent_locs
get_owner_document get_disconnected_nodes
get_disconnected_nodes_locs set_disconnected_nodes
set_disconnected_nodes_locs
defines *remove =*
"l_remove_child$_{Core_DOM}$_defs.a_remove get_child_nodes set_child_nodes get_parent get_owner_document
get_disconnected_nodes set_disconnected_nodes"
and *remove_child =*
"l_remove_child$_{Core_DOM}$_defs.a_remove_child get_child_nodes set_child_nodes get_owner_document
get_disconnected_nodes set_disconnected_nodes"
and *remove_child_locs =*
"l_remove_child$_{Core_DOM}$_defs.a_remove_child_locs set_child_nodes_locs set_disconnected_nodes_locs"
⟨*proof*⟩
interpretation
i_remove_child?: l_remove_child$_{Core_DOM}$ get_child_nodes get_child_nodes_locs set_child_nodes
set_child_nodes_locs get_parent get_parent_locs get_owner_document
get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
set_disconnected_nodes_locs remove_child remove_child_locs remove type_wf
known_ptr known_ptrs
⟨*proof*⟩
declare *l_remove_child$_{Core_DOM}$_axioms[instances]*

lemma *remove_child_is_l_remove_child [instances]:*
"l_remove_child type_wf known_ptr known_ptrs remove_child remove_child_locs get_owner_document

```
                get_child_nodes get_disconnected_nodes"
```
⟨*proof*⟩

adopt_node

```
locale l_adopt_node_Core_DOM_defs =
  l_get_owner_document_defs get_owner_document +
  l_get_parent_defs get_parent get_parent_locs +
  l_remove_child_defs remove_child remove_child_locs +
  l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
  l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs
  for get_owner_document :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog"
  and get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"
  and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and remove_child :: "(_) object_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and remove_child_locs :: "(_) object_ptr ⇒ (_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
begin
definition a_adopt_node :: "(_) document_ptr ⇒ (_) node_ptr ⇒ (_, unit) dom_prog"
  where
    "a_adopt_node document node = do {
      old_document ← get_owner_document (cast node);
      parent_opt ← get_parent node;
      (case parent_opt of
        Some parent ⇒ do {
          remove_child parent node
        } | None ⇒ do {
          return ()
        });
      (if document ≠ old_document then do {
        old_disc_nodes ← get_disconnected_nodes old_document;
        set_disconnected_nodes old_document (remove1 node old_disc_nodes);
        disc_nodes ← get_disconnected_nodes document;
        set_disconnected_nodes document (node # disc_nodes)
      } else do {
        return ()
      })
    }"

definition
  a_adopt_node_locs :: "(_) object_ptr option ⇒ (_) document_ptr ⇒ (_) document_ptr ⇒ (_, unit) dom_prog
set"
  where
    "a_adopt_node_locs parent owner_document document_ptr =
    ((if parent = None
      then {}
      else remove_child_locs (the parent) owner_document) ∪ set_disconnected_nodes_locs document_ptr
                                                          ∪ set_disconnected_nodes_locs owner_document)"
end

locale l_adopt_node_defs =
  fixes
  adopt_node :: "(_) document_ptr ⇒ (_) node_ptr ⇒ (_, unit) dom_prog"
  fixes
  adopt_node_locs :: "(_) object_ptr option ⇒ (_) document_ptr ⇒ (_) document_ptr ⇒ (_, unit) dom_prog
set"

global_interpretation l_adopt_node_Core_DOM_defs get_owner_document get_parent get_parent_locs remove_child
                                                 remove_child_locs get_disconnected_nodes
```

```
                                        get_disconnected_nodes_locs set_disconnected_nodes
                                        set_disconnected_nodes_locs
  defines adopt_node = "l_adopt_node_Core_DOM_defs.a_adopt_node get_owner_document get_parent remove_child

                                                          get_disconnected_nodes set_disconnected_nodes"
      and adopt_node_locs = "l_adopt_node_Core_DOM_defs.a_adopt_node_locs
                                                    remove_child_locs set_disconnected_nodes_locs"
  ⟨proof⟩

locale l_adopt_node_Core_DOM =
  l_adopt_node_Core_DOM_defs
    get_owner_document get_parent get_parent_locs remove_child remove_child_locs get_disconnected_nodes

                        get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs
  + l_adopt_node_defs
    adopt_node adopt_node_locs
  + l_get_owner_document
    get_owner_document
  + l_get_parent_Core_DOM
    known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  + l_remove_child_Core_DOM
    get_child_nodes get_child_nodes_locs set_child_nodes set_child_nodes_locs get_parent
    get_parent_locs get_owner_document get_disconnected_nodes get_disconnected_nodes_locs
    set_disconnected_nodes set_disconnected_nodes_locs remove_child remove_child_locs remove type_wf
    known_ptr known_ptrs
  + l_set_disconnected_nodes_get_disconnected_nodes
    type_wf get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
    set_disconnected_nodes_locs
  for get_owner_document :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog"
  and get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"
  and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and remove_child :: "(_) object_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and remove_child_locs :: "(_) object_ptr ⇒ (_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and adopt_node :: "(_) document_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and adopt_node_locs :: "(_) object_ptr option ⇒ (_) document_ptr ⇒ (_) document_ptr
                          ⇒ ((_) heap, exception, unit) prog set"
  and known_ptr :: "(_) object_ptr ⇒ bool"
  and type_wf :: "(_) heap ⇒ bool"
  and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and known_ptrs :: "(_) heap ⇒ bool"
  and set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap, exception, unit) prog set"
  and remove :: "(_) node_ptr ⇒ ((_) heap, exception, unit) prog" +
  assumes adopt_node_impl: "adopt_node = a_adopt_node"
  assumes adopt_node_locs_impl: "adopt_node_locs = a_adopt_node_locs"
begin
lemmas adopt_node_def = a_adopt_node_def[folded adopt_node_impl]
lemmas adopt_node_locs_def = a_adopt_node_locs_def[folded adopt_node_locs_impl]

lemma adopt_node_writes:
  shows "writes (adopt_node_locs |h ⊢ get_parent node|_r |h
        ⊢ get_owner_document (cast node)|_r document_ptr) (adopt_node document_ptr node) h h'"
  ⟨proof⟩

lemma adopt_node_children_subset:
  assumes "h ⊢ adopt_node owner_document node →_h h'"
    and "h ⊢ get_child_nodes ptr →_r children"
    and "h' ⊢ get_child_nodes ptr →_r children'"
```

```
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "set children' ⊆ set children"
⟨proof⟩

lemma adopt_node_child_in_heap:
  assumes "h ⊢ ok (adopt_node document_ptr child)"
  shows "child |∈| node_ptr_kinds h"
  ⟨proof⟩

lemma adopt_node_pointers_preserved:
  assumes "w ∈ adopt_node_locs parent owner_document document_ptr"
  assumes "h ⊢ w →_h h'"
  shows "object_ptr_kinds h = object_ptr_kinds h'"
  ⟨proof⟩

lemma adopt_node_types_preserved:
  assumes "w ∈ adopt_node_locs parent owner_document document_ptr"
  assumes "h ⊢ w →_h h'"
  shows "type_wf h = type_wf h'"
  ⟨proof⟩
end

locale l_adopt_node = l_type_wf + l_known_ptrs + l_get_parent_defs + l_adopt_node_defs +
  l_get_child_nodes_defs + l_get_owner_document_defs +
  assumes adopt_node_writes:
  "writes (adopt_node_locs |h ⊢ get_parent node|_r
           |h ⊢ get_owner_document (cast node)|_r document_ptr) (adopt_node document_ptr node) h h'"
  assumes adopt_node_pointers_preserved:
  "w ∈ adopt_node_locs parent owner_document document_ptr
   ⟹ h ⊢ w →_h h' ⟹ object_ptr_kinds h = object_ptr_kinds h'"
  assumes adopt_node_types_preserved:
  "w ∈ adopt_node_locs parent owner_document document_ptr
   ⟹ h ⊢ w →_h h' ⟹ type_wf h = type_wf h'"
  assumes adopt_node_child_in_heap:
  "h ⊢ ok (adopt_node document_ptr child) ⟹ child |∈| node_ptr_kinds h"
  assumes adopt_node_children_subset:
  "h ⊢ adopt_node owner_document node →_h h' ⟹ h ⊢ get_child_nodes ptr →_r children
     ⟹ h' ⊢ get_child_nodes ptr →_r children'
     ⟹ known_ptrs h ⟹ type_wf h ⟹ set children' ⊆ set children"

interpretation
  i_adopt_node?: l_adopt_node_Core_DOM get_owner_document get_parent get_parent_locs remove_child
                 remove_child_locs get_disconnected_nodes get_disconnected_nodes_locs
                 set_disconnected_nodes set_disconnected_nodes_locs adopt_node adopt_node_locs
                 known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs set_child_nodes
                 set_child_nodes_locs remove
  ⟨proof⟩
declare l_adopt_node_Core_DOM_axioms[instances]


lemma adopt_node_is_l_adopt_node [instances]:
  "l_adopt_node type_wf known_ptr known_ptrs get_parent adopt_node adopt_node_locs get_child_nodes
               get_owner_document"
  ⟨proof⟩
```

insert_before

```
locale l_insert_before_Core_DOM_defs =
  l_get_parent_defs get_parent get_parent_locs
  + l_get_child_nodes_defs get_child_nodes get_child_nodes_locs
  + l_set_child_nodes_defs set_child_nodes set_child_nodes_locs
  + l_get_ancestors_defs get_ancestors get_ancestors_locs
```

```
  + l_adopt_node_defs adopt_node adopt_node_locs
  + l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs
  + l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs
  + l_get_owner_document_defs get_owner_document
  for get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_::linorder) object_ptr option) prog"
  and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_ancestors :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
  and get_ancestors_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and adopt_node :: "(_) document_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and adopt_node_locs :: "(_) object_ptr option ⇒ (_) document_ptr ⇒ (_) document_ptr
                                                ⇒ ((_) heap, exception, unit) prog set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_owner_document :: "(_) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog"
begin

definition a_next_sibling :: "(_) node_ptr ⇒ (_, (_) node_ptr option) dom_prog"
  where
    "a_next_sibling node_ptr = do {
      parent_opt ← get_parent node_ptr;
      (case parent_opt of
        Some parent ⇒ do {
          children ← get_child_nodes parent;
          (case (dropWhile (λptr. ptr = node_ptr) (dropWhile (λptr. ptr ≠ node_ptr) children)) of
            x#_ ⇒ return (Some x)
          | [] ⇒ return None)}
      | None ⇒ return None)
    }"

fun insert_before_list :: "'xyz ⇒ 'xyz option ⇒ 'xyz list ⇒ 'xyz list"
  where
    "insert_before_list v (Some reference) (x#xs) = (if reference = x
      then v#x#xs else x # insert_before_list v (Some reference) xs)"
    | "insert_before_list v (Some _) [] = [v]"
    | "insert_before_list v None xs = xs @ [v]"

definition a_insert_node :: "(_) object_ptr ⇒ (_) node_ptr ⇒ (_) node_ptr option
  ⇒ (_, unit) dom_prog"
  where
    "a_insert_node ptr new_child reference_child_opt = do {
      children ← get_child_nodes ptr;
      set_child_nodes ptr (insert_before_list new_child reference_child_opt children)
    }"

definition a_ensure_pre_insertion_validity :: "(_) node_ptr ⇒ (_) object_ptr
  ⇒ (_) node_ptr option ⇒ (_, unit) dom_prog"
  where
    "a_ensure_pre_insertion_validity node parent child_opt = do {
      (if is_character_data_ptr_kind parent
        then error HierarchyRequestError else return ());
      ancestors ← get_ancestors parent;
      (if cast node ∈ set ancestors then error HierarchyRequestError else return ());
      (case child_opt of
        Some child ⇒ do {
          child_parent ← get_parent child;
          (if child_parent ≠ Some parent then error NotFoundError else return ())}
      | None ⇒ return ());
```

```
      children ← get_child_nodes parent;
      (if children ≠ [] ∧ is_document_ptr parent
        then error HierarchyRequestError else return ());
      (if is_character_data_ptr node ∧ is_document_ptr parent
        then error HierarchyRequestError else return ())
    }"

definition a_insert_before :: "(_) object_ptr ⇒ (_) node_ptr
  ⇒ (_) node_ptr option ⇒ (_, unit) dom_prog"
  where
    "a_insert_before ptr node child = do {
      a_ensure_pre_insertion_validity node ptr child;
      reference_child ← (if Some node = child
        then a_next_sibling node
        else return child);
      owner_document ← get_owner_document ptr;
      adopt_node owner_document node;
      disc_nodes ← get_disconnected_nodes owner_document;
      set_disconnected_nodes owner_document (remove1 node disc_nodes);
      a_insert_node ptr node reference_child
    }"

definition a_insert_before_locs :: "(_) object_ptr ⇒ (_) object_ptr option ⇒ (_) document_ptr
                                                  ⇒ (_) document_ptr ⇒ (_, unit) dom_prog set"
  where
    "a_insert_before_locs ptr old_parent child_owner_document ptr_owner_document =
      adopt_node_locs old_parent child_owner_document ptr_owner_document ∪
      set_child_nodes_locs ptr ∪
      set_disconnected_nodes_locs ptr_owner_document"
end

locale l_insert_before_defs =
  fixes insert_before :: "(_) object_ptr ⇒ (_) node_ptr ⇒ (_) node_ptr option ⇒ (_, unit) dom_prog"
  fixes insert_before_locs :: "(_) object_ptr ⇒ (_) object_ptr option ⇒ (_) document_ptr
                                            ⇒ (_) document_ptr ⇒ (_, unit) dom_prog set"

locale l_append_child_Core_DOM_defs =
  l_insert_before_defs
begin
definition "a_append_child ptr child = insert_before ptr child None"
end

locale l_append_child_defs =
  fixes append_child :: "(_) object_ptr ⇒ (_) node_ptr ⇒ (_, unit) dom_prog"

locale l_insert_before_Core_DOM =
  l_insert_before_Core_DOM_defs
    get_parent get_parent_locs get_child_nodes get_child_nodes_locs set_child_nodes
    set_child_nodes_locs get_ancestors get_ancestors_locs adopt_node adopt_node_locs
    set_disconnected_nodes set_disconnected_nodes_locs get_disconnected_nodes
    get_disconnected_nodes_locs get_owner_document
  + l_insert_before_defs
    insert_before insert_before_locs
  + l_append_child_defs
    append_child
  + l_set_child_nodes_get_child_nodes
    type_wf known_ptr get_child_nodes get_child_nodes_locs set_child_nodes set_child_nodes_locs
  + l_get_ancestors
    get_ancestors get_ancestors_locs
  + l_adopt_node
    type_wf known_ptr known_ptrs get_parent get_parent_locs adopt_node adopt_node_locs
    get_child_nodes get_child_nodes_locs get_owner_document
  + l_set_disconnected_nodes
```

```
    type_wf set_disconnected_nodes set_disconnected_nodes_locs
  + l_get_disconnected_nodes
    type_wf get_disconnected_nodes get_disconnected_nodes_locs
  + l_get_owner_document
    get_owner_document
  + l_get_parent_Core_DOM
    known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  + l_set_disconnected_nodes_get_child_nodes
    set_disconnected_nodes set_disconnected_nodes_locs get_child_nodes get_child_nodes_locs
  for get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_::linorder) object_ptr option) prog"
  and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_child_nodes :: "(_) object_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_ancestors :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
  and get_ancestors_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
  and adopt_node :: "(_) document_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and adopt_node_locs :: "(_) object_ptr option ⇒ (_) document_ptr ⇒ (_) document_ptr
                                          ⇒ ((_) heap, exception, unit) prog set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and get_owner_document :: "(_) object_ptr ⇒ ((_) heap, exception, (_) document_ptr) prog"
    and insert_before ::
    "(_) object_ptr ⇒ (_) node_ptr ⇒ (_) node_ptr option ⇒ ((_) heap, exception, unit) prog"
  and insert_before_locs :: "(_) object_ptr ⇒ (_) object_ptr option ⇒ (_) document_ptr
                                        ⇒ (_) document_ptr ⇒ (_, unit) dom_prog set"
  and append_child :: "(_) object_ptr ⇒ (_) node_ptr ⇒ ((_) heap, exception, unit) prog"
  and type_wf :: "(_) heap ⇒ bool"
  and known_ptr :: "(_) object_ptr ⇒ bool"
  and known_ptrs :: "(_) heap ⇒ bool" +
  assumes insert_before_impl: "insert_before = a_insert_before"
  assumes insert_before_locs_impl: "insert_before_locs = a_insert_before_locs"
begin
lemmas insert_before_def = a_insert_before_def[folded insert_before_impl]
lemmas insert_before_locs_def = a_insert_before_locs_def[folded insert_before_locs_impl]

lemma next_sibling_pure [simp]:
  "pure (a_next_sibling new_child) h"
  ⟨proof⟩

lemma insert_before_list_in_set: "x ∈ set (insert_before_list v ref xs) ⟷ x = v ∨ x ∈ set xs"
  ⟨proof⟩

lemma insert_before_list_distinct: "x ∉ set xs ⟹ distinct xs ⟹ distinct (insert_before_list x ref
xs)"
  ⟨proof⟩

lemma insert_before_list_subset: "set xs ⊆ set (insert_before_list x ref xs)"
  ⟨proof⟩

lemma insert_before_list_node_in_set: "x ∈ set (insert_before_list x ref xs)"
  ⟨proof⟩

lemma insert_node_writes:
  "writes (set_child_nodes_locs ptr) (a_insert_node ptr new_child reference_child_opt) h h'"
  ⟨proof⟩

lemma ensure_pre_insertion_validity_pure [simp]:
  "pure (a_ensure_pre_insertion_validity node ptr child) h"
  ⟨proof⟩
```

lemma *insert_before_reference_child_not_in_children:*
  assumes *"h ⊢ get_parent child →r Some parent"*
    and *"ptr ≠ parent"*
    and *"¬is_character_data_ptr_kind ptr"*
    and *"h ⊢ get_ancestors ptr →r ancestors"*
    and *"cast node ∉ set ancestors"*
  shows *"h ⊢ insert_before ptr node (Some child) →e NotFoundError"*
⟨*proof*⟩

lemma *insert_before_ptr_in_heap:*
  assumes *"h ⊢ ok (insert_before ptr node reference_child)"*
  shows *"ptr |∈| object_ptr_kinds h"*
  ⟨*proof*⟩

lemma *insert_before_child_in_heap:*
  assumes *"h ⊢ ok (insert_before ptr node reference_child)"*
  shows *"node |∈| node_ptr_kinds h"*
  ⟨*proof*⟩

lemma *insert_node_children_remain_distinct:*
    assumes *insert_node: "h ⊢ a_insert_node ptr new_child reference_child_opt →h h2"*
    and *"h ⊢ get_child_nodes ptr →r children"*
    and *"new_child ∉ set children"*
    and *"⋀ptr children. h ⊢ get_child_nodes ptr →r children ⟹ distinct children"*
    and *known_ptr: "known_ptr ptr"*
    and *type_wf: "type_wf h"*
  shows *"⋀ptr children. h2 ⊢ get_child_nodes ptr →r children ⟹ distinct children"*
⟨*proof*⟩

lemma *insert_before_writes:*
  "writes (insert_before_locs ptr |h ⊢ get_parent child|r
      |h ⊢ get_owner_document (cast child)|r |h ⊢ get_owner_document ptr|r) (insert_before ptr child ref) h h'"
  ⟨*proof*⟩
end

locale *l_append_child*$_{Core_DOM}$ =
  l_append_child_defs +
  l_append_child$_{Core_DOM}$*_defs +*
  assumes *append_child_impl: "append_child = a_append_child"*
begin

lemmas *append_child_def = a_append_child_def[folded append_child_impl]*
end

locale *l_insert_before = l_insert_before_defs*

locale *l_append_child = l_append_child_defs*

global_interpretation *l_insert_before*$_{Core_DOM}$*_defs get_parent get_parent_locs get_child_nodes get_child_nodes_locs set_child_nodes set_child_nodes_locs get_ancestors get_ancestors_locs adopt_node adopt_node_locs set_disconnected_nodes set_disconnected_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs get_owner_document*
  defines
    next_sibling = "l_insert_before$_{Core_DOM}$*_defs.a_next_sibling get_parent get_child_nodes"* **and**
    insert_node = "l_insert_before$_{Core_DOM}$*_defs.a_insert_node get_child_nodes set_child_nodes"* **and**
    ensure_pre_insertion_validity = "l_insert_before$_{Core_DOM}$*_defs.a_ensure_pre_insertion_validity get_parent get_child_nodes get_ancestors"* **and**
    insert_before = "l_insert_before$_{Core_DOM}$*_defs.a_insert_before get_parent get_child_nodes set_child_nodes get_ancestors adopt_node set_disconnected_nodes get_disconnected_nodes get_owner_document"* **and**

```
    insert_before_locs = "l_insert_before_Core_DOM_defs.a_insert_before_locs set_child_nodes_locs
                                                        adopt_node_locs set_disconnected_nodes_locs"
  ⟨proof⟩

global_interpretation l_append_child_Core_DOM_defs insert_before
  defines append_child = "l_append_child_Core_DOM_defs.a_append_child insert_before"
  ⟨proof⟩

interpretation
  i_insert_before?: l_insert_before_Core_DOM get_parent get_parent_locs get_child_nodes
  get_child_nodes_locs set_child_nodes set_child_nodes_locs get_ancestors get_ancestors_locs
  adopt_node adopt_node_locs set_disconnected_nodes set_disconnected_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs get_owner_document insert_before insert_before_locs append_child
  type_wf known_ptr known_ptrs
  ⟨proof⟩
declare l_insert_before_Core_DOM_axioms[instances]

interpretation i_append_child?: l_append_child_Core_DOM append_child insert_before insert_before_locs
  ⟨proof⟩
declare l_append_child_Core_DOM_axioms[instances]
```

create_element

```
locale l_create_element_Core_DOM_defs =
  l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
  l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs +
  l_set_tag_name_defs set_tag_name set_tag_name_locs
  for get_disconnected_nodes ::
      "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs ::
      "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes ::
      "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs ::
      "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
    and set_tag_name ::
      "(_) element_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
    and set_tag_name_locs ::
      "(_) element_ptr ⇒ ((_) heap, exception, unit) prog set"
begin
definition a_create_element :: "(_) document_ptr ⇒ tag_name ⇒ (_, (_) element_ptr) dom_prog"
  where
    "a_create_element document_ptr tag = do {
      new_element_ptr ← new_element;
      set_tag_name new_element_ptr tag;
      disc_nodes ← get_disconnected_nodes document_ptr;
      set_disconnected_nodes document_ptr (cast new_element_ptr # disc_nodes);
      return new_element_ptr
    }"
end

locale l_create_element_defs =
  fixes create_element :: "(_) document_ptr ⇒ tag_name ⇒ (_, (_) element_ptr) dom_prog"

global_interpretation l_create_element_Core_DOM_defs get_disconnected_nodes get_disconnected_nodes_locs

                                                set_disconnected_nodes set_disconnected_nodes_locs
  set_tag_name set_tag_name_locs
  defines
  create_element = "l_create_element_Core_DOM_defs.a_create_element get_disconnected_nodes
                                                              set_disconnected_nodes set_tag_name"
  ⟨proof⟩
```

```
locale l_create_element_Core_DOM =
  l_create_element_Core_DOM_defs get_disconnected_nodes get_disconnected_nodes_locs
  set_disconnected_nodes set_disconnected_nodes_locs set_tag_name set_tag_name_locs +
  l_get_disconnected_nodes type_wf get_disconnected_nodes get_disconnected_nodes_locs +
  l_set_tag_name type_wf set_tag_name set_tag_name_locs +
  l_create_element_defs create_element +
  l_known_ptr known_ptr
  for get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
    and set_tag_name :: "(_) element_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
    and set_tag_name_locs :: "(_) element_ptr ⇒ ((_) heap, exception, unit) prog set"
  and type_wf :: "(_) heap ⇒ bool"
  and create_element :: "(_) document_ptr ⇒ char list ⇒ ((_) heap, exception, (_) element_ptr) prog"
  and known_ptr :: "(_) object_ptr ⇒ bool" +
  assumes known_ptr_impl: "known_ptr = a_known_ptr"
  assumes create_element_impl: "create_element = a_create_element"
begin
lemmas create_element_def = a_create_element_def[folded create_element_impl]

lemma create_element_document_in_heap:
  assumes "h ⊢ ok (create_element document_ptr tag)"
  shows "document_ptr |∈| document_ptr_kinds h"
⟨proof⟩

lemma create_element_known_ptr:
  assumes "h ⊢ create_element document_ptr tag →_r new_element_ptr"
  shows "known_ptr (cast new_element_ptr)"
⟨proof⟩
end

locale l_create_element = l_create_element_defs

interpretation
  i_create_element?: l_create_element_Core_DOM get_disconnected_nodes get_disconnected_nodes_locs
  set_disconnected_nodes set_disconnected_nodes_locs set_tag_name set_tag_name_locs type_wf
  create_element known_ptr
  ⟨proof⟩
declare l_create_element_Core_DOM_axioms[instances]
```

create_character_data

```
locale l_create_character_data_Core_DOM_defs =
  l_set_val_defs set_val set_val_locs +
  l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs +
  l_set_disconnected_nodes_defs set_disconnected_nodes set_disconnected_nodes_locs
  for set_val :: "(_) character_data_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
  and set_val_locs :: "(_) character_data_ptr ⇒ ((_) heap, exception, unit) prog set"
  and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
begin
definition a_create_character_data :: "(_) document_ptr ⇒ string ⇒ (_, (_) character_data_ptr) dom_prog"
  where
    "a_create_character_data document_ptr text = do {
      new_character_data_ptr ← new_character_data;
      set_val new_character_data_ptr text;
      disc_nodes ← get_disconnected_nodes document_ptr;
      set_disconnected_nodes document_ptr (cast new_character_data_ptr # disc_nodes);
      return new_character_data_ptr
    }"
```

```
end

locale l_create_character_data_defs =
  fixes create_character_data :: "(_) document_ptr ⇒ string ⇒ (_, (_) character_data_ptr) dom_prog"

global_interpretation l_create_character_data_Core_DOM_defs set_val set_val_locs get_disconnected_nodes
                      get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs
  defines create_character_data = "l_create_character_data_Core_DOM_defs.a_create_character_data
                                     set_val get_disconnected_nodes set_disconnected_nodes"
  ⟨proof⟩

locale l_create_character_data_Core_DOM =
  l_create_character_data_Core_DOM_defs set_val set_val_locs get_disconnected_nodes
  get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs +
  l_get_disconnected_nodes type_wf get_disconnected_nodes get_disconnected_nodes_locs +
  l_set_val type_wf set_val set_val_locs +
  l_create_character_data_defs create_character_data +
  l_known_ptr known_ptr
  for get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
  and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
  and set_val :: "(_) character_data_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
  and set_val_locs :: "(_) character_data_ptr ⇒ ((_) heap, exception, unit) prog set"
  and type_wf :: "(_) heap ⇒ bool"
  and create_character_data :: "(_) document_ptr ⇒ char list ⇒ ((_) heap, exception, (_) character_data_ptr)
prog"
  and known_ptr :: "(_) object_ptr ⇒ bool" +
  assumes known_ptr_impl: "known_ptr = a_known_ptr"
  assumes create_character_data_impl: "create_character_data = a_create_character_data"
begin
lemmas create_character_data_def = a_create_character_data_def[folded create_character_data_impl]

lemma create_character_data_document_in_heap:
  assumes "h ⊢ ok (create_character_data document_ptr text)"
  shows "document_ptr |∈| document_ptr_kinds h"
⟨proof⟩

lemma create_character_data_known_ptr:
  assumes "h ⊢ create_character_data document_ptr text →_r new_character_data_ptr"
  shows "known_ptr (cast new_character_data_ptr)"
⟨proof⟩
end

locale l_create_character_data = l_create_character_data_defs

interpretation
  i_create_character_data?: l_create_character_data_Core_DOM get_disconnected_nodes
  get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs set_val set_val_locs
  type_wf create_character_data known_ptr
  ⟨proof⟩
declare l_create_character_data_Core_DOM_axioms [instances]
```

create_character_data

```
locale l_create_document_Core_DOM_defs
begin
definition a_create_document :: "(_, (_) document_ptr) dom_prog"
  where
    "a_create_document = new_document"
end
```

```
locale l_create_document_defs =
  fixes create_document :: "(_, (_) document_ptr) dom_prog"

global_interpretation l_create_document_Core_DOM_defs
  defines create_document = "l_create_document_Core_DOM_defs.a_create_document"
  ⟨proof⟩

locale l_create_document_Core_DOM =
  l_create_document_Core_DOM_defs +
  l_create_document_defs +
  assumes create_document_impl: "create_document = a_create_document"
begin
lemmas
  create_document_def = create_document_impl[unfolded create_document_def, unfolded a_create_document_def]
end

locale l_create_document = l_create_document_defs

interpretation
  i_create_document?: l_create_document_Core_DOM create_document
  ⟨proof⟩
declare l_create_document_Core_DOM_axioms [instances]
```

tree_order

```
locale l_to_tree_order_Core_DOM_defs =
  l_get_child_nodes_defs get_child_nodes get_child_nodes_locs
  for get_child_nodes :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
partial_function (dom_prog) a_to_tree_order :: "(_) object_ptr ⇒ (_, (_) object_ptr list) dom_prog"
  where
    "a_to_tree_order ptr = (do {
      children ← get_child_nodes ptr;
      treeorders ← map_M a_to_tree_order (map (cast) children);
      return (ptr # concat treeorders)
    })"
end

locale l_to_tree_order_defs =
  fixes to_tree_order :: "(_) object_ptr ⇒ (_, (_) object_ptr list) dom_prog"

global_interpretation l_to_tree_order_Core_DOM_defs get_child_nodes get_child_nodes_locs defines
  to_tree_order = "l_to_tree_order_Core_DOM_defs.a_to_tree_order get_child_nodes" ⟨proof⟩
declare a_to_tree_order.simps [code]

locale l_to_tree_order_Core_DOM =
  l_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs +
  l_to_tree_order_Core_DOM_defs get_child_nodes get_child_nodes_locs +
  l_to_tree_order_defs to_tree_order
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
  and type_wf :: "(_) heap ⇒ bool"
  and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
  and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
  and to_tree_order :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog" +
  assumes to_tree_order_impl: "to_tree_order = a_to_tree_order"
begin
lemmas to_tree_order_def = a_to_tree_order.simps[folded to_tree_order_impl]

lemma to_tree_order_pure [simp]: "pure (to_tree_order ptr) h"
⟨proof⟩
end
```

```
locale l_to_tree_order =
  fixes to_tree_order :: "(_) object_ptr ⇒ (_, (_) object_ptr list) dom_prog"
  assumes to_tree_order_pure [simp]: "pure (to_tree_order ptr) h"

interpretation
  i_to_tree_order?: l_to_tree_order_Core_DOM known_ptr type_wf get_child_nodes get_child_nodes_locs
                    to_tree_order
  ⟨proof⟩
declare l_to_tree_order_Core_DOM_axioms[instances]

lemma to_tree_order_is_l_to_tree_order [instances]: "l_to_tree_order to_tree_order"
  ⟨proof⟩
```

first_in_tree_order

```
locale l_first_in_tree_order_Core_DOM_defs =
  l_to_tree_order_defs to_tree_order
  for to_tree_order :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
begin
definition a_first_in_tree_order :: "(_) object_ptr ⇒ ((_) object_ptr
                                ⇒ (_, 'result option) dom_prog) ⇒ (_, 'result option) dom_prog"
  where
    "a_first_in_tree_order ptr f = (do {
      tree_order ← to_tree_order ptr;
      results ← map_filter_M f tree_order;
      (case results of
        [] ⇒ return None
      | x#_⇒ return (Some x))
    })"
end

locale l_first_in_tree_order_defs =
  fixes first_in_tree_order :: "(_) object_ptr ⇒ ((_) object_ptr ⇒ (_, 'result option) dom_prog)
                                            ⇒ (_, 'result option) dom_prog"

global_interpretation l_first_in_tree_order_Core_DOM_defs to_tree_order defines
  first_in_tree_order = "l_first_in_tree_order_Core_DOM_defs.a_first_in_tree_order to_tree_order" ⟨proof⟩

locale l_first_in_tree_order_Core_DOM =
  l_first_in_tree_order_Core_DOM_defs to_tree_order +
  l_first_in_tree_order_defs first_in_tree_order
  for to_tree_order :: "(_) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
  and first_in_tree_order :: "(_) object_ptr ⇒ ((_) object_ptr ⇒ ((_) heap, exception, 'result option)
prog)
                                            ⇒ ((_) heap, exception, 'result option) prog" +
assumes first_in_tree_order_impl: "first_in_tree_order = a_first_in_tree_order"
begin
lemmas first_in_tree_order_def = first_in_tree_order_impl[unfolded a_first_in_tree_order_def]
end

locale l_first_in_tree_order

interpretation i_first_in_tree_order?:
  l_first_in_tree_order_Core_DOM to_tree_order first_in_tree_order
  ⟨proof⟩
declare l_first_in_tree_order_Core_DOM_axioms[instances]
```

get_element_by

```
locale l_get_element_by_Core_DOM_defs =
  l_first_in_tree_order_defs first_in_tree_order +
  l_to_tree_order_defs to_tree_order +
  l_get_attribute_defs get_attribute get_attribute_locs
```

```
  for to_tree_order :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
  and first_in_tree_order :: "(_) object_ptr ⇒ ((_) object_ptr
                              ⇒ ((_) heap, exception, (_) element_ptr option) prog)
                              ⇒ ((_) heap, exception, (_) element_ptr option) prog"
  and get_attribute :: "(_) element_ptr ⇒ char list ⇒ ((_) heap, exception, char list option) prog"
  and get_attribute_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
definition a_get_element_by_id :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr option) dom_prog"
  where
    "a_get_element_by_id ptr iden = first_in_tree_order ptr (λptr. (case cast ptr of
      Some element_ptr ⇒ do {
        id_opt ← get_attribute element_ptr ''id'';
        (if id_opt = Some iden then return (Some element_ptr) else return None)
      }
    | _ ⇒ return None
    ))"

definition a_get_elements_by_class_name :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr list) dom_prog"
  where
    "a_get_elements_by_class_name ptr class_name = to_tree_order ptr ≫=
      map_filter_M (λptr. (case cast ptr of
        Some element_ptr ⇒ do {
          class_name_opt ← get_attribute element_ptr ''class'';
          (if class_name_opt = Some class_name then return (Some element_ptr) else return None)
        }
      | _ ⇒ return None))"

definition a_get_elements_by_tag_name :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr list) dom_prog"
  where
    "a_get_elements_by_tag_name ptr tag = to_tree_order ptr ≫=
      map_filter_M (λptr. (case cast ptr of
        Some element_ptr ⇒ do {
          this_tag_name ← get_M element_ptr tag_name;
          (if this_tag_name = tag then return (Some element_ptr) else return None)
        }
      | _ ⇒ return None))"
end

locale l_get_element_by_defs =
  fixes get_element_by_id :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr option) dom_prog"
  fixes get_elements_by_class_name :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr list) dom_prog"
  fixes get_elements_by_tag_name :: "(_) object_ptr ⇒ attr_value ⇒ (_, (_) element_ptr list) dom_prog"

global_interpretation
l_get_element_by_Core_DOM_defs to_tree_order first_in_tree_order get_attribute get_attribute_locs
defines
  get_element_by_id = "l_get_element_by_Core_DOM_defs.a_get_element_by_id first_in_tree_order get_attribute"

and
    get_elements_by_class_name = "l_get_element_by_Core_DOM_defs.a_get_elements_by_class_name
to_tree_order get_attribute"
and
  get_elements_by_tag_name = "l_get_element_by_Core_DOM_defs.a_get_elements_by_tag_name to_tree_order" ⟨proof⟩

locale l_get_element_by_Core_DOM =
  l_get_element_by_Core_DOM_defs to_tree_order first_in_tree_order get_attribute get_attribute_locs +
  l_get_element_by_defs get_element_by_id get_elements_by_class_name get_elements_by_tag_name +
  l_first_in_tree_order_Core_DOM to_tree_order first_in_tree_order +
  l_to_tree_order to_tree_order +
  l_get_attribute type_wf get_attribute get_attribute_locs
  for to_tree_order :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) object_ptr list) prog"
    and first_in_tree_order ::
    "(_) object_ptr ⇒ ((_) object_ptr ⇒ ((_) heap, exception, (_) element_ptr option) prog)
```

```
                                        ⇒ ((_) heap, exception, (_) element_ptr option) prog"
  and get_attribute :: "(_) element_ptr ⇒ char list ⇒ ((_) heap, exception, char list option) prog"
  and get_attribute_locs :: "(_) element_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and get_element_by_id ::
    "(_) object_ptr ⇒ char list ⇒ ((_) heap, exception, (_) element_ptr option) prog"
    and get_elements_by_class_name ::
    "(_) object_ptr ⇒ char list ⇒ ((_) heap, exception, (_) element_ptr list) prog"
    and get_elements_by_tag_name ::
    "(_) object_ptr ⇒ char list ⇒ ((_) heap, exception, (_) element_ptr list) prog"
  and type_wf :: "(_) heap ⇒ bool" +
  assumes get_element_by_id_impl: "get_element_by_id = a_get_element_by_id"
  assumes get_elements_by_class_name_impl: "get_elements_by_class_name = a_get_elements_by_class_name"
  assumes get_elements_by_tag_name_impl: "get_elements_by_tag_name = a_get_elements_by_tag_name"
begin
lemmas
  get_element_by_id_def = get_element_by_id_impl[unfolded a_get_element_by_id_def]
lemmas
  get_elements_by_class_name_def = get_elements_by_class_name_impl[unfolded a_get_elements_by_class_name_def]
lemmas
  get_elements_by_tag_name_def = get_elements_by_tag_name_impl[unfolded a_get_elements_by_tag_name_def]

lemma get_element_by_id_result_in_tree_order:
  assumes "h ⊢ get_element_by_id ptr iden →r Some element_ptr"
  assumes "h ⊢ to_tree_order ptr →r to"
  shows "cast element_ptr ∈ set to"
  ⟨proof⟩

lemma get_elements_by_class_name_result_in_tree_order:
  assumes "h ⊢ get_elements_by_class_name ptr name →r results"
  assumes "h ⊢ to_tree_order ptr →r to"
  assumes "element_ptr ∈ set results"
  shows "cast element_ptr ∈ set to"
  ⟨proof⟩

lemma get_elements_by_tag_name_result_in_tree_order:
  assumes "h ⊢ get_elements_by_tag_name ptr name →r results"
  assumes "h ⊢ to_tree_order ptr →r to"
  assumes "element_ptr ∈ set results"
  shows "cast element_ptr ∈ set to"
  ⟨proof⟩

lemma get_elements_by_tag_name_pure [simp]: "pure (get_elements_by_tag_name ptr tag) h"
  ⟨proof⟩
end

locale l_get_element_by = l_get_element_by_defs + l_to_tree_order_defs +
  assumes get_element_by_id_result_in_tree_order:
          "h ⊢ get_element_by_id ptr iden →r Some element_ptr ⟹ h ⊢ to_tree_order ptr →r to
               ⟹ cast element_ptr ∈ set to"
  assumes get_elements_by_tag_name_pure [simp]: "pure (get_elements_by_tag_name ptr tag) h"

interpretation
  i_get_element_by?: l_get_element_by_Core_DOM to_tree_order first_in_tree_order get_attribute
                     get_attribute_locs get_element_by_id get_elements_by_class_name
                     get_elements_by_tag_name type_wf
  ⟨proof⟩
declare l_get_element_by_Core_DOM_axioms[instances]

lemma get_element_by_is_l_get_element_by [instances]:
  "l_get_element_by get_element_by_id get_elements_by_tag_name to_tree_order"
  ⟨proof⟩
end
```

6.3 Wellformedness (Core_DOM_Heap_WF)

In this theory, we discuss the wellformedness of the DOM. First, we define wellformedness and, second, we show for all functions for querying and modifying the DOM to what extend they preserve wellformendess.

```
theory Core_DOM_Heap_WF
  imports
    "Core_DOM_Functions"
begin

locale l_heap_is_wellformed_Core_DOM_defs =
  l_get_child_nodes_defs get_child_nodes get_child_nodes_locs +
  l_get_disconnected_nodes_defs get_disconnected_nodes get_disconnected_nodes_locs
  for get_child_nodes :: "(_::linorder) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
definition a_owner_document_valid :: "(_) heap ⇒ bool"
  where
    "a_owner_document_valid h ⟷ (∀ node_ptr ∈ fset (node_ptr_kinds h).
      ((∃ document_ptr. document_ptr |∈| document_ptr_kinds h
         ∧ node_ptr ∈ set |h ⊢ get_disconnected_nodes document_ptr|_r)
    ∨ (∃ parent_ptr. parent_ptr |∈| object_ptr_kinds h
          ∧ node_ptr ∈ set |h ⊢ get_child_nodes parent_ptr|_r)))"

lemma a_owner_document_valid_code [code]: "a_owner_document_valid h ⟷ node_ptr_kinds h |⊆|
  fset_of_list (concat (map (λparent. |h ⊢ get_child_nodes parent|_r)
(sorted_list_of_fset (object_ptr_kinds h)) @ map (λparent. |h ⊢ get_disconnected_nodes parent|_r)
(sorted_list_of_fset (document_ptr_kinds h))))
"
  ⟨proof⟩

definition a_parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"
  where
    "a_parent_child_rel h = {(parent, child). parent |∈| object_ptr_kinds h
                           ∧ child ∈ cast ‘ set |h ⊢ get_child_nodes parent|_r}"

lemma a_parent_child_rel_code [code]: "a_parent_child_rel h = set (concat (map
  (λparent. map
    (λchild. (parent, cast_node_ptr2object_ptr child))
    |h ⊢ get_child_nodes parent|_r)
  (sorted_list_of_fset (object_ptr_kinds h)))
)"
  ⟨proof⟩

definition a_acyclic_heap :: "(_) heap ⇒ bool"
  where
    "a_acyclic_heap h = acyclic (a_parent_child_rel h)"

definition a_all_ptrs_in_heap :: "(_) heap ⇒ bool"
  where
    "a_all_ptrs_in_heap h ⟷
      (∀ ptr ∈ fset (object_ptr_kinds h). set |h ⊢ get_child_nodes ptr|_r ⊆ fset (node_ptr_kinds h)) ∧
      (∀ document_ptr ∈ fset (document_ptr_kinds h).
set |h ⊢ get_disconnected_nodes document_ptr|_r ⊆ fset (node_ptr_kinds h))"

definition a_distinct_lists :: "(_) heap ⇒ bool"
  where
    "a_distinct_lists h = distinct (concat (
      (map (λptr. |h ⊢ get_child_nodes ptr|_r) |h ⊢ object_ptr_kinds_M|_r)
   @ (map (λdocument_ptr. |h ⊢ get_disconnected_nodes document_ptr|_r) |h ⊢ document_ptr_kinds_M|_r)
    ))"
```

```
definition a_heap_is_wellformed :: "(_) heap ⇒ bool"
  where
    "a_heap_is_wellformed h ⟷
      a_acyclic_heap h ∧ a_all_ptrs_in_heap h ∧ a_distinct_lists h ∧ a_owner_document_valid h"
end

locale l_heap_is_wellformed_defs =
  fixes heap_is_wellformed :: "(_) heap ⇒ bool"
  fixes parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"

global_interpretation l_heap_is_wellformed_Core_DOM_defs get_child_nodes get_child_nodes_locs
  get_disconnected_nodes get_disconnected_nodes_locs
  defines heap_is_wellformed = "l_heap_is_wellformed_Core_DOM_defs.a_heap_is_wellformed get_child_nodes
                  get_disconnected_nodes"
    and parent_child_rel = "l_heap_is_wellformed_Core_DOM_defs.a_parent_child_rel get_child_nodes"
    and acyclic_heap = a_acyclic_heap
    and all_ptrs_in_heap = a_all_ptrs_in_heap
    and distinct_lists = a_distinct_lists
    and owner_document_valid = a_owner_document_valid
  ⟨proof⟩

locale l_heap_is_wellformed_Core_DOM =
  l_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs
  + l_heap_is_wellformed_Core_DOM_defs get_child_nodes get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs
  + l_heap_is_wellformed_defs heap_is_wellformed parent_child_rel
  + l_get_disconnected_nodes type_wf get_disconnected_nodes get_disconnected_nodes_locs
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
    and type_wf :: "(_) heap ⇒ bool"
    and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and heap_is_wellformed :: "(_) heap ⇒ bool"
    and parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set" +
  assumes heap_is_wellformed_impl: "heap_is_wellformed = a_heap_is_wellformed"
  assumes parent_child_rel_impl: "parent_child_rel = a_parent_child_rel"
begin
lemmas heap_is_wellformed_def = heap_is_wellformed_impl[unfolded a_heap_is_wellformed_def]
lemmas parent_child_rel_def = parent_child_rel_impl[unfolded a_parent_child_rel_def]
lemmas acyclic_heap_def = a_acyclic_heap_def[folded parent_child_rel_impl]

lemma parent_child_rel_node_ptr:
  "(parent, child) ∈ parent_child_rel h ⟹ is_node_ptr_kind child"
  ⟨proof⟩

lemma parent_child_rel_child_nodes:
  assumes "known_ptr parent"
    and "h ⊢ get_child_nodes parent →r children"
    and "child ∈ set children"
  shows "(parent, cast child) ∈ parent_child_rel h"
  ⟨proof⟩

lemma parent_child_rel_child_nodes2:
  assumes "known_ptr parent"
    and "h ⊢ get_child_nodes parent →r children"
    and "child ∈ set children"
    and "cast_node_ptr2object_ptr child = child_obj"
  shows "(parent, child_obj) ∈ parent_child_rel h"
  ⟨proof⟩
```

lemma *parent_child_rel_finite: "finite (parent_child_rel h)"*
⟨*proof*⟩

lemma *distinct_lists_no_parent:*
assumes *"a_distinct_lists h"*
assumes *"h ⊢ get_disconnected_nodes document_ptr →$_r$ disc_nodes"*
assumes *"node_ptr ∈ set disc_nodes"*
shows *"¬(∃parent_ptr. parent_ptr |∈| object_ptr_kinds h*
∧ node_ptr ∈ set |h ⊢ get_child_nodes parent_ptr|$_r$)"
⟨*proof*⟩

lemma *distinct_lists_disconnected_nodes:*
assumes *"a_distinct_lists h"*
and *"h ⊢ get_disconnected_nodes document_ptr →$_r$ disc_nodes"*
shows *"distinct disc_nodes"*
⟨*proof*⟩

lemma *distinct_lists_children:*
assumes *"a_distinct_lists h"*
and *"known_ptr ptr"*
and *"h ⊢ get_child_nodes ptr →$_r$ children"*
shows *"distinct children"*
⟨*proof*⟩

lemma *heap_is_wellformed_children_in_heap:*
assumes *"heap_is_wellformed h"*
assumes *"h ⊢ get_child_nodes ptr →$_r$ children"*
assumes *"child ∈ set children"*
shows *"child |∈| node_ptr_kinds h"*
⟨*proof*⟩

lemma *heap_is_wellformed_one_parent:*
assumes *"heap_is_wellformed h"*
assumes *"h ⊢ get_child_nodes ptr →$_r$ children"*
assumes *"h ⊢ get_child_nodes ptr' →$_r$ children'"*
assumes *"set children ∩ set children' ≠ {}"*
shows *"ptr = ptr'"*
⟨*proof*⟩

lemma *parent_child_rel_child:*
"h ⊢ get_child_nodes ptr →$_r$ children ⟹
child ∈ set children ⟷ (ptr, cast child) ∈ parent_child_rel h"
⟨*proof*⟩

lemma *parent_child_rel_acyclic: "heap_is_wellformed h ⟹ acyclic (parent_child_rel h)"*
⟨*proof*⟩

lemma *heap_is_wellformed_disconnected_nodes_distinct:*
"heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →$_r$ disc_nodes ⟹
distinct disc_nodes"
⟨*proof*⟩

lemma *parent_child_rel_parent_in_heap:*
"(parent, child_ptr) ∈ parent_child_rel h ⟹ parent |∈| object_ptr_kinds h"
⟨*proof*⟩

lemma *parent_child_rel_child_in_heap:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptr parent
⟹ (parent, child_ptr) ∈ parent_child_rel h ⟹ child_ptr |∈| object_ptr_kinds h"
⟨*proof*⟩

lemma *heap_is_wellformed_disc_nodes_in_heap:*

```
  "heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
   ⟹ node ∈ set disc_nodes ⟹ node |∈| node_ptr_kinds h"
  ⟨proof⟩

lemma heap_is_wellformed_one_disc_parent:
  "heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
   ⟹ h ⊢ get_disconnected_nodes document_ptr' →r disc_nodes'
   ⟹ set disc_nodes ∩ set disc_nodes' ≠ {} ⟹ document_ptr = document_ptr'"
  ⟨proof⟩

lemma heap_is_wellformed_children_disc_nodes_different:
  "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children
    ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
   ⟹ set children ∩ set disc_nodes = {}"
  ⟨proof⟩

lemma heap_is_wellformed_children_disc_nodes:
  "heap_is_wellformed h ⟹ node_ptr |∈| node_ptr_kinds h
   ⟹ ¬(∃parent ∈ fset (object_ptr_kinds h). node_ptr ∈ set |h ⊢ get_child_nodes parent|r)
   ⟹ (∃document_ptr ∈ fset (document_ptr_kinds h). node_ptr ∈ set |h ⊢ get_disconnected_nodes document_ptr|r)"
  ⟨proof⟩

lemma heap_is_wellformed_children_distinct:
  "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children ⟹ distinct children"
  ⟨proof⟩
end

locale l_heap_is_wellformed = l_type_wf + l_known_ptr + l_heap_is_wellformed_defs
  + l_get_child_nodes_defs + l_get_disconnected_nodes_defs +
  assumes heap_is_wellformed_children_in_heap:
    "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children ⟹ child ∈ set children
                          ⟹ child |∈| node_ptr_kinds h"
  assumes heap_is_wellformed_disc_nodes_in_heap:
    "heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
                          ⟹ node ∈ set disc_nodes ⟹ node |∈| node_ptr_kinds h"
  assumes heap_is_wellformed_one_parent:
    "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children
                          ⟹ h ⊢ get_child_nodes ptr' →r children'
                          ⟹ set children ∩ set children' ≠ {} ⟹ ptr = ptr'"
  assumes heap_is_wellformed_one_disc_parent:
    "heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
                          ⟹ h ⊢ get_disconnected_nodes document_ptr' →r disc_nodes'
                          ⟹ set disc_nodes ∩ set disc_nodes' ≠ {} ⟹ document_ptr = document_ptr'"
  assumes heap_is_wellformed_children_disc_nodes_different:
    "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children
                          ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
                          ⟹ set children ∩ set disc_nodes = {}"
  assumes heap_is_wellformed_disconnected_nodes_distinct:
    "heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes document_ptr →r disc_nodes
                          ⟹ distinct disc_nodes"
  assumes heap_is_wellformed_children_distinct:
    "heap_is_wellformed h ⟹ h ⊢ get_child_nodes ptr →r children ⟹ distinct children"
  assumes heap_is_wellformed_children_disc_nodes:
    "heap_is_wellformed h ⟹ node_ptr |∈| node_ptr_kinds h
   ⟹ ¬(∃parent ∈ fset (object_ptr_kinds h). node_ptr ∈ set |h ⊢ get_child_nodes parent|r)
   ⟹ (∃document_ptr ∈ fset (document_ptr_kinds h). node_ptr ∈ set |h ⊢ get_disconnected_nodes document_ptr|r)"
  assumes parent_child_rel_child:
    "h ⊢ get_child_nodes ptr →r children
   ⟹ child ∈ set children ⟷ (ptr, cast child) ∈ parent_child_rel h"
  assumes parent_child_rel_finite:
    "heap_is_wellformed h ⟹ finite (parent_child_rel h)"
  assumes parent_child_rel_acyclic:
    "heap_is_wellformed h ⟹ acyclic (parent_child_rel h)"
```

```
  assumes parent_child_rel_node_ptr:
    "(parent, child_ptr) ∈ parent_child_rel h ⟹ is_node_ptr_kind child_ptr"
  assumes parent_child_rel_parent_in_heap:
    "(parent, child_ptr) ∈ parent_child_rel h ⟹ parent |∈| object_ptr_kinds h"
  assumes parent_child_rel_child_in_heap:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptr parent
   ⟹ (parent, child_ptr) ∈ parent_child_rel h ⟹ child_ptr |∈| object_ptr_kinds h"

interpretation i_heap_is_wellformed?: l_heap_is_wellformed_Core_DOM known_ptr type_wf get_child_nodes
  get_child_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs
  heap_is_wellformed parent_child_rel
  ⟨proof⟩
declare l_heap_is_wellformed_Core_DOM_axioms[instances]


lemma heap_is_wellformed_is_l_heap_is_wellformed [instances]:
  "l_heap_is_wellformed type_wf known_ptr heap_is_wellformed parent_child_rel get_child_nodes
                        get_disconnected_nodes"
  ⟨proof⟩
```

6.3.1 get_parent

```
locale l_get_parent_wf_Core_DOM =
  l_get_parent_Core_DOM
  known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  + l_heap_is_wellformed
  type_wf known_ptr heap_is_wellformed parent_child_rel get_child_nodes get_child_nodes_locs
  get_disconnected_nodes get_disconnected_nodes_locs
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
    and type_wf :: "(_) heap ⇒ bool"
    and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and known_ptrs :: "(_) heap ⇒ bool"
    and get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"
    and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
    and heap_is_wellformed :: "(_) heap ⇒ bool"
    and parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
lemma child_parent_dual:
  assumes heap_is_wellformed: "heap_is_wellformed h"
  assumes "h ⊢ get_child_nodes ptr →_r children"
  assumes "child ∈ set children"
  assumes "known_ptrs h"
  assumes type_wf: "type_wf h"
  shows "h ⊢ get_parent child →_r Some ptr"
⟨proof⟩

lemma parent_child_rel_parent:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_parent child_node →_r Some parent"
  shows "(parent, cast child_node) ∈ parent_child_rel h"
  ⟨proof⟩

lemma heap_wellformed_induct [consumes 1, case_names step]:
  assumes "heap_is_wellformed h"
    and step: "⋀parent. (⋀children child. h ⊢ get_child_nodes parent →_r children
              ⟹ child ∈ set children ⟹ P (cast child)) ⟹ P parent"
  shows "P ptr"
⟨proof⟩

lemma heap_wellformed_induct2 [consumes 3, case_names not_in_heap empty_children step]:
```

```
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
    and not_in_heap: "⋀parent. parent |∉| object_ptr_kinds h ⟹ P parent"
    and empty_children: "⋀parent. h ⊢ get_child_nodes parent →r [] ⟹ P parent"
    and step: "⋀parent children child. h ⊢ get_child_nodes parent →r children
               ⟹ child ∈ set children ⟹ P (cast child) ⟹ P parent"
  shows "P ptr"
⟨proof⟩

lemma heap_wellformed_induct_rev [consumes 1, case_names step]:
  assumes "heap_is_wellformed h"
    and step: "⋀child. (⋀parent child_node. cast child_node = child
               ⟹ h ⊢ get_parent child_node →r Some parent ⟹ P parent) ⟹ P child"
  shows "P ptr"
⟨proof⟩
end

interpretation i_get_parent_wf?: l_get_parent_wf_Core_DOM known_ptr type_wf get_child_nodes
  get_child_nodes_locs known_ptrs get_parent get_parent_locs heap_is_wellformed
  parent_child_rel get_disconnected_nodes
  ⟨proof⟩
declare l_get_parent_wf_Core_DOM_axioms[instances]


locale l_get_parent_wf2_Core_DOM =
  l_get_parent_wf_Core_DOM
  known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
  heap_is_wellformed parent_child_rel get_disconnected_nodes get_disconnected_nodes_locs
  + l_heap_is_wellformed_Core_DOM
  known_ptr type_wf get_child_nodes get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs heap_is_wellformed parent_child_rel
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
    and type_wf :: "(_) heap ⇒ bool"
    and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and known_ptrs :: "(_) heap ⇒ bool"
    and get_parent :: "(_) node_ptr ⇒ ((_) heap, exception, (_) object_ptr option) prog"
    and get_parent_locs :: "((_) heap ⇒ (_) heap ⇒ bool) set"
    and heap_is_wellformed :: "(_) heap ⇒ bool"
    and parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
begin
lemma preserves_wellformedness_writes_needed:
  assumes heap_is_wellformed: "heap_is_wellformed h"
    and "h ⊢ f →h h'"
    and "writes SW f h h'"
    and preserved_get_child_nodes:
    "⋀h h' w. w ∈ SW ⟹ h ⊢ w →h h'
                    ⟹ ∀object_ptr. ∀r ∈ get_child_nodes_locs object_ptr. r h h'"
    and preserved_get_disconnected_nodes:
    "⋀h h' w. w ∈ SW ⟹ h ⊢ w →h h'
                    ⟹ ∀document_ptr. ∀r ∈ get_disconnected_nodes_locs document_ptr. r h h'"
    and preserved_object_pointers:
    "⋀h h' w. w ∈ SW ⟹ h ⊢ w →h h'
                   ⟹ ∀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
  shows "heap_is_wellformed h'"
⟨proof⟩
end

interpretation i_get_parent_wf2?: l_get_parent_wf2_Core_DOM known_ptr type_wf get_child_nodes
  get_child_nodes_locs known_ptrs get_parent get_parent_locs
  heap_is_wellformed parent_child_rel get_disconnected_nodes
  get_disconnected_nodes_locs
```

⟨*proof*⟩

declare *l_get_parent_wf2*$_{Core_DOM}$*_axioms[instances]*
locale *l_get_parent_wf = l_type_wf + l_known_ptrs + l_heap_is_wellformed_defs*
+ l_get_child_nodes_defs + l_get_parent_defs +
assumes *child_parent_dual:*
"heap_is_wellformed h
⟹ type_wf h
⟹ known_ptrs h
⟹ h ⊢ get_child_nodes ptr →$_r$ children
⟹ child ∈ set children
⟹ h ⊢ get_parent child →$_r$ Some ptr"
assumes *heap_wellformed_induct [consumes 1, case_names step]:*
"heap_is_wellformed h
⟹ (⋀parent. (⋀children child. h ⊢ get_child_nodes parent →$_r$ children
⟹ child ∈ set children ⟹ P (cast child)) ⟹ P parent)
⟹ P ptr"
assumes *heap_wellformed_induct_rev [consumes 1, case_names step]:*
"heap_is_wellformed h
⟹ (⋀child. (⋀parent child_node. cast child_node = child
⟹ h ⊢ get_parent child_node →$_r$ Some parent ⟹ P parent) ⟹ P child)
⟹ P ptr"
assumes *parent_child_rel_parent: "heap_is_wellformed h*
⟹ h ⊢ get_parent child_node →$_r$ Some parent
⟹ (parent, cast child_node) ∈ parent_child_rel h"

lemma *get_parent_wf_is_l_get_parent_wf [instances]:*
"l_get_parent_wf type_wf known_ptr known_ptrs heap_is_wellformed parent_child_rel
get_child_nodes get_parent"
⟨*proof*⟩

6.3.2 get_disconnected_nodes

6.3.3 set_disconnected_nodes

get_disconnected_nodes

locale *l_set_disconnected_nodes_get_disconnected_nodes_wf*$_{Core_DOM}$ =
l_set_disconnected_nodes_get_disconnected_nodes
type_wf get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
set_disconnected_nodes_locs
+ l_heap_is_wellformed
type_wf known_ptr heap_is_wellformed parent_child_rel get_child_nodes get_child_nodes_locs
get_disconnected_nodes get_disconnected_nodes_locs
for *known_ptr :: "(_) object_ptr ⇒ bool"*
and *type_wf :: "(_) heap ⇒ bool"*
and *get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
and *set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit)*
prog"
and *set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"*
and *heap_is_wellformed :: "(_) heap ⇒ bool"*
and *parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"*
and *get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"*
and *get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"*
begin

lemma *remove_from_disconnected_nodes_removes:*
assumes *"heap_is_wellformed h"*
assumes *"h ⊢ get_disconnected_nodes ptr →$_r$ disc_nodes"*
assumes *"h ⊢ set_disconnected_nodes ptr (remove1 node_ptr disc_nodes) →$_h$ h'"*
assumes *"h' ⊢ get_disconnected_nodes ptr →$_r$ disc_nodes'"*
shows *"node_ptr ∉ set disc_nodes'"*

⟨*proof*⟩
end

locale *l_set_disconnected_nodes_get_disconnected_nodes_wf = l_heap_is_wellformed*
+ l_set_disconnected_nodes_get_disconnected_nodes +
assumes *remove_from_disconnected_nodes_removes:*
"heap_is_wellformed h ⟹ h ⊢ get_disconnected_nodes ptr →r disc_nodes
⟹ h ⊢ set_disconnected_nodes ptr (remove1 node_ptr disc_nodes) →h h'
⟹ h' ⊢ get_disconnected_nodes ptr →r disc_nodes'
⟹ node_ptr ∉ set disc_nodes'"

interpretation *i_set_disconnected_nodes_get_disconnected_nodes_wf$_{Core_DOM}$?:*
l_set_disconnected_nodes_get_disconnected_nodes_wf$_{Core_DOM}$ known_ptr type_wf get_disconnected_nodes
get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs heap_is_wellformed
parent_child_rel get_child_nodes
⟨*proof*⟩
declare *l_set_disconnected_nodes_get_disconnected_nodes_wf$_{Core_DOM}$_axioms[instances]*

lemma *set_disconnected_nodes_get_disconnected_nodes_wf_is_l_set_disconnected_nodes_get_disconnected_nodes_wf*
[instances]:
"l_set_disconnected_nodes_get_disconnected_nodes_wf type_wf known_ptr heap_is_wellformed parent_child_rel
get_child_nodes get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
set_disconnected_nodes_locs"
⟨*proof*⟩

6.3.4 get_root_node

locale *l_get_root_node_wf$_{Core_DOM}$ =*
l_heap_is_wellformed
type_wf known_ptr heap_is_wellformed parent_child_rel get_child_nodes get_child_nodes_locs
get_disconnected_nodes get_disconnected_nodes_locs
+ l_get_parent$_{Core_DOM}$
known_ptr type_wf get_child_nodes get_child_nodes_locs known_ptrs get_parent get_parent_locs
+ l_get_parent_wf
type_wf known_ptr known_ptrs heap_is_wellformed parent_child_rel get_child_nodes
get_child_nodes_locs get_parent get_parent_locs
+ l_get_root_node$_{Core_DOM}$
type_wf known_ptr known_ptrs get_parent get_parent_locs get_child_nodes get_child_nodes_locs
get_ancestors get_ancestors_locs get_root_node get_root_node_locs
for *known_ptr :: "('_::linorder) object_ptr ⇒ bool"*
and *type_wf :: "('_) heap ⇒ bool"*
and *known_ptrs :: "('_) heap ⇒ bool"*
and *heap_is_wellformed :: "('_) heap ⇒ bool"*
and *parent_child_rel :: "('_) heap ⇒ (('_) object_ptr × ('_) object_ptr) set"*
and *get_child_nodes :: "('_) object_ptr ⇒ (('_) heap, exception, ('_) node_ptr list) prog"*
and *get_child_nodes_locs :: "('_) object_ptr ⇒ (('_) heap ⇒ ('_) heap ⇒ bool) set"*
and *get_disconnected_nodes :: "('_) document_ptr ⇒ (('_) heap, exception, ('_) node_ptr list) prog"*
and *get_disconnected_nodes_locs :: "('_) document_ptr ⇒ (('_) heap ⇒ ('_) heap ⇒ bool) set"*
and *get_parent :: "('_) node_ptr ⇒ (('_) heap, exception, ('_) object_ptr option) prog"*
and *get_parent_locs :: "(('_) heap ⇒ ('_) heap ⇒ bool) set"*
and *get_ancestors :: "('_) object_ptr ⇒ (('_) heap, exception, ('_) object_ptr list) prog"*
and *get_ancestors_locs :: "(('_) heap ⇒ ('_) heap ⇒ bool) set"*
and *get_root_node :: "('_) object_ptr ⇒ (('_) heap, exception, ('_) object_ptr) prog"*
and *get_root_node_locs :: "(('_) heap ⇒ ('_) heap ⇒ bool) set"*

begin
lemma *get_ancestors_reads:*
assumes *"heap_is_wellformed h"*
shows *"reads get_ancestors_locs (get_ancestors node_ptr) h h'"*
⟨*proof*⟩

lemma *get_ancestors_ok:*
assumes *"heap_is_wellformed h"*

```
    and "ptr |∈| object_ptr_kinds h"
    and "known_ptrs h"
    and type_wf: "type_wf h"
  shows "h ⊢ ok (get_ancestors ptr)"
```
⟨*proof*⟩

```
lemma get_root_node_ptr_in_heap:
  assumes "h ⊢ ok (get_root_node ptr)"
  shows "ptr |∈| object_ptr_kinds h"
```
⟨*proof*⟩

```
lemma get_root_node_ok:
  assumes "heap_is_wellformed h" "known_ptrs h" "type_wf h"
    and "ptr |∈| object_ptr_kinds h"
  shows "h ⊢ ok (get_root_node ptr)"
```
⟨*proof*⟩

```
lemma get_ancestors_parent:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_parent child →r Some parent"
  shows "h ⊢ get_ancestors (cast child) →r (cast child) # parent # ancestors
     ⟷ h ⊢ get_ancestors parent →r parent # ancestors"
```
⟨*proof*⟩

```
lemma get_ancestors_never_empty:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_ancestors child →r ancestors"
  shows "ancestors ≠ []"
```
⟨*proof*⟩

```
lemma get_ancestors_subset:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_ancestors ptr →r ancestors"
    and "ancestor ∈ set ancestors"
    and "h ⊢ get_ancestors ancestor →r ancestor_ancestors"
    and type_wf: "type_wf h"
    and known_ptrs: "known_ptrs h"
  shows "set ancestor_ancestors ⊆ set ancestors"
```
⟨*proof*⟩

```
lemma get_ancestors_also_parent:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_ancestors some_ptr →r ancestors"
    and "cast child ∈ set ancestors"
    and "h ⊢ get_parent child →r Some parent"
    and type_wf: "type_wf h"
    and known_ptrs: "known_ptrs h"
  shows "parent ∈ set ancestors"
```
⟨*proof*⟩

```
lemma get_ancestors_obtains_children:
  assumes "heap_is_wellformed h"
    and "ancestor ≠ ptr"
    and "ancestor ∈ set ancestors"
    and "h ⊢ get_ancestors ptr →r ancestors"
    and type_wf: "type_wf h"
    and known_ptrs: "known_ptrs h"
  obtains children ancestor_child where "h ⊢ get_child_nodes ancestor →r children"
```

```
    and "ancestor_child ∈ set children" and "cast ancestor_child ∈ set ancestors"
⟨proof⟩

lemma get_ancestors_parent_child_rel:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_ancestors child →_r ancestors"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "(ptr, child) ∈ (parent_child_rel h)* ⟷ ptr ∈ set ancestors"
⟨proof⟩

lemma get_root_node_parent_child_rel:
  assumes "heap_is_wellformed h"
    and "h ⊢ get_root_node child →_r root"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "(root, child) ∈ (parent_child_rel h)*"
  ⟨proof⟩


lemma get_ancestors_eq:
  assumes "heap_is_wellformed h"
    and "heap_is_wellformed h'"
    and "⋀object_ptr w. object_ptr ≠ ptr ⟹ w ∈ get_child_nodes_locs object_ptr ⟹ w h h'"
    and pointers_preserved: "⋀object_ptr. preserved (get_M_Object object_ptr RObject.nothing) h h'"
    and known_ptrs: "known_ptrs h"
    and known_ptrs': "known_ptrs h'"
    and "h ⊢ get_ancestors ptr →_r ancestors"
    and type_wf: "type_wf h"
    and type_wf': "type_wf h'"
  shows "h' ⊢ get_ancestors ptr →_r ancestors"
⟨proof⟩

lemma get_ancestors_remains_not_in_ancestors:
  assumes "heap_is_wellformed h"
    and "heap_is_wellformed h'"
    and "h ⊢ get_ancestors ptr →_r ancestors"
    and "h' ⊢ get_ancestors ptr →_r ancestors'"
    and "⋀p children children'. h ⊢ get_child_nodes p →_r children
        ⟹ h' ⊢ get_child_nodes p →_r children' ⟹ set children' ⊆ set children"
    and "node ∉ set ancestors"
    and object_ptr_kinds_eq3: "object_ptr_kinds h = object_ptr_kinds h'"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
    and type_wf': "type_wf h'"
  shows "node ∉ set ancestors'"
⟨proof⟩

lemma get_ancestors_ptrs_in_heap:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_ancestors ptr →_r ancestors"
  assumes "ptr' ∈ set ancestors"
  shows "ptr' |∈| object_ptr_kinds h"
⟨proof⟩


lemma get_ancestors_prefix:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_ancestors ptr →_r ancestors"
  assumes "ptr' ∈ set ancestors"
  assumes "h ⊢ get_ancestors ptr' →_r ancestors'"
  shows "∃pre. ancestors = pre @ ancestors'"
⟨proof⟩
```

lemma *get_ancestors_same_root_node:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_ancestors ptr →r ancestors"*
assumes *"ptr' ∈ set ancestors"*
assumes *"ptr'' ∈ set ancestors"*
shows *"h ⊢ get_root_node ptr' →r root_ptr ⟷ h ⊢ get_root_node ptr'' →r root_ptr"*
⟨*proof*⟩

lemma *get_root_node_parent_same:*
assumes *"h ⊢ get_parent child →r Some ptr"*
shows *"h ⊢ get_root_node (cast child) →r root ⟷ h ⊢ get_root_node ptr →r root"*
⟨*proof*⟩

lemma *get_root_node_same_no_parent:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_root_node ptr →r cast child"*
shows *"h ⊢ get_parent child →r None"*
⟨*proof*⟩

lemma *get_root_node_not_node_same:*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"¬is_node_ptr_kind ptr"*
shows *"h ⊢ get_root_node ptr →r ptr"*
⟨*proof*⟩

lemma *get_root_node_root_in_heap:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_root_node ptr →r root"*
shows *"root |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *get_root_node_same_no_parent_parent_child_rel:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_root_node ptr' →r ptr'"*
shows *"¬(∃p. (p, ptr') ∈ (parent_child_rel h))"*
⟨*proof*⟩

end

locale *l_get_ancestors_wf = l_heap_is_wellformed_defs + l_known_ptrs + l_type_wf + l_get_ancestors_defs*
+ l_get_child_nodes_defs + l_get_parent_defs +
assumes *get_ancestors_never_empty:*
"heap_is_wellformed h ⟹ h ⊢ get_ancestors child →r ancestors ⟹ ancestors ≠ []"
assumes *get_ancestors_ok:*
"heap_is_wellformed h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptrs h ⟹ type_wf h
⟹ h ⊢ ok (get_ancestors ptr)"
assumes *get_ancestors_reads:*
"heap_is_wellformed h ⟹ reads get_ancestors_locs (get_ancestors node_ptr) h h'"
assumes *get_ancestors_ptrs_in_heap:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
⟹ h ⊢ get_ancestors ptr →r ancestors ⟹ ptr' ∈ set ancestors
⟹ ptr' |∈| object_ptr_kinds h"
assumes *get_ancestors_remains_not_in_ancestors:*
"heap_is_wellformed h ⟹ heap_is_wellformed h' ⟹ h ⊢ get_ancestors ptr →r ancestors
⟹ h' ⊢ get_ancestors ptr →r ancestors'
⟹ (⋀p children children'. h ⊢ get_child_nodes p →r children
⟹ h' ⊢ get_child_nodes p →r children'
⟹ set children' ⊆ set children)

```
                        ⟹ node ∉ set ancestors
                        ⟹ object_ptr_kinds h = object_ptr_kinds h' ⟹ known_ptrs h
                        ⟹ type_wf h ⟹ type_wf h' ⟹ node ∉ set ancestors'"
  assumes get_ancestors_also_parent:
    "heap_is_wellformed h ⟹ h ⊢ get_ancestors some_ptr →r ancestors
                        ⟹ cast child_node ∈ set ancestors
                        ⟹ h ⊢ get_parent child_node →r Some parent ⟹ type_wf h
                        ⟹ known_ptrs h ⟹ parent ∈ set ancestors"
  assumes get_ancestors_obtains_children:
    "heap_is_wellformed h ⟹ ancestor ≠ ptr ⟹ ancestor ∈ set ancestors
                        ⟹ h ⊢ get_ancestors ptr →r ancestors ⟹ type_wf h ⟹ known_ptrs h
                        ⟹ (⋀children ancestor_child . h ⊢ get_child_nodes ancestor →r children
                             ⟹ ancestor_child ∈ set children
                             ⟹ cast ancestor_child ∈ set ancestors
                             ⟹ thesis)
                        ⟹ thesis"
  assumes get_ancestors_parent_child_rel:
    "heap_is_wellformed h ⟹ h ⊢ get_ancestors child →r ancestors ⟹ known_ptrs h ⟹ type_wf h
                        ⟹ (ptr, child) ∈ (parent_child_rel h)* ⟷ ptr ∈ set ancestors"

locale l_get_root_node_wf = l_heap_is_wellformed_defs + l_get_root_node_defs + l_type_wf
  + l_known_ptrs + l_get_ancestors_defs + l_get_parent_defs +
  assumes get_root_node_ok:
    "heap_is_wellformed h ⟹ known_ptrs h ⟹ type_wf h ⟹ ptr |∈| object_ptr_kinds h
                        ⟹ h ⊢ ok (get_root_node ptr)"
  assumes get_root_node_ptr_in_heap:
    "h ⊢ ok (get_root_node ptr) ⟹ ptr |∈| object_ptr_kinds h"
  assumes get_root_node_root_in_heap:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
                        ⟹ h ⊢ get_root_node ptr →r root ⟹ root |∈| object_ptr_kinds h"
  assumes get_ancestors_same_root_node:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
                        ⟹ h ⊢ get_ancestors ptr →r ancestors ⟹ ptr' ∈ set ancestors
                        ⟹ ptr'' ∈ set ancestors
                        ⟹ h ⊢ get_root_node ptr' →r root_ptr ⟷ h ⊢ get_root_node ptr'' →r root_ptr"
  assumes get_root_node_same_no_parent:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
                        ⟹ h ⊢ get_root_node ptr →r cast child ⟹ h ⊢ get_parent child →r None"
  assumes get_root_node_parent_same:
    "h ⊢ get_parent child →r Some ptr
        ⟹ h ⊢ get_root_node (cast child) →r root ⟷ h ⊢ get_root_node ptr →r root"

interpretation i_get_root_node_wf?:
  l_get_root_node_wf_Core_DOM known_ptr type_wf known_ptrs heap_is_wellformed parent_child_rel
  get_child_nodes get_child_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs
  get_parent get_parent_locs get_ancestors get_ancestors_locs get_root_node get_root_node_locs
  ⟨proof⟩
declare l_get_root_node_wf_Core_DOM_axioms[instances]

lemma get_ancestors_wf_is_l_get_ancestors_wf [instances]:
  "l_get_ancestors_wf heap_is_wellformed parent_child_rel known_ptr known_ptrs type_wf get_ancestors
  get_ancestors_locs get_child_nodes get_parent"
  ⟨proof⟩

lemma get_root_node_wf_is_l_get_root_node_wf [instances]:
  "l_get_root_node_wf heap_is_wellformed get_root_node type_wf known_ptr known_ptrs
   get_ancestors get_parent"
  ⟨proof⟩
```

6.3.5 to_tree_order

```
locale l_to_tree_order_wf_Core_DOM =
  l_to_tree_order_Core_DOM +
```

```
  l_get_parent +
  l_get_parent_wf +
  l_heap_is_wellformed
```

begin

lemma *to_tree_order_ptr_in_heap:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ ok (to_tree_order ptr)"*
shows *"ptr |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *to_tree_order_either_ptr_or_in_children:*
assumes *"h ⊢ to_tree_order ptr →r nodes"*
and *"node ∈ set nodes"*
and *"h ⊢ get_child_nodes ptr →r children"*
and *"node ≠ ptr"*
obtains *child child_to* **where** *"child ∈ set children"*
and *"h ⊢ to_tree_order (cast child) →r child_to"* **and** *"node ∈ set child_to"*
⟨*proof*⟩

lemma *to_tree_order_ptrs_in_heap:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ to_tree_order ptr →r to"*
assumes *"ptr' ∈ set to"*
shows *"ptr' |∈| object_ptr_kinds h"*
⟨*proof*⟩

lemma *to_tree_order_ok:*
assumes *wellformed: "heap_is_wellformed h"*
and *"ptr |∈| object_ptr_kinds h"*
and *"known_ptrs h"*
and *type_wf: "type_wf h"*
shows *"h ⊢ ok (to_tree_order ptr)"*
⟨*proof*⟩

lemma *to_tree_order_child_subset:*
assumes *"heap_is_wellformed h"*
and *"h ⊢ to_tree_order ptr →r nodes"*
and *"h ⊢ get_child_nodes ptr →r children"*
and *"node ∈ set children"*
and *"h ⊢ to_tree_order (cast node) →r nodes'"*
shows *"set nodes' ⊆ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_ptr_in_result:*
assumes *"h ⊢ to_tree_order ptr →r nodes"*
shows *"ptr ∈ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_subset:*
assumes *"heap_is_wellformed h"*
and *"h ⊢ to_tree_order ptr →r nodes"*
and *"node ∈ set nodes"*
and *"h ⊢ to_tree_order node →r nodes'"*
and *"known_ptrs h"*
and *type_wf: "type_wf h"*
shows *"set nodes' ⊆ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_parent:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*

assumes *"h ⊢ to_tree_order ptr →r nodes"*
assumes *"h ⊢ get_parent child →r Some parent"*
assumes *"parent ∈ set nodes"*
shows *"cast child ∈ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_child:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ to_tree_order ptr →r nodes"*
assumes *"h ⊢ get_child_nodes parent →r children"*
assumes *"cast child ≠ ptr"*
assumes *"child ∈ set children"*
assumes *"cast child ∈ set nodes"*
shows *"parent ∈ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_node_ptrs:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ to_tree_order ptr →r nodes"*
assumes *"ptr' ≠ ptr"*
assumes *"ptr' ∈ set nodes"*
shows *"is_node_ptr_kind ptr'"*
⟨*proof*⟩

lemma *to_tree_order_child2:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ to_tree_order ptr →r nodes"*
assumes *"cast child ≠ ptr"*
assumes *"cast child ∈ set nodes"*
obtains *parent* **where** *"h ⊢ get_parent child →r Some parent"* **and** *"parent ∈ set nodes"*
⟨*proof*⟩

lemma *to_tree_order_parent_child_rel:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ to_tree_order ptr →r to"*
shows *"(ptr, child) ∈ (parent_child_rel h)* ⟷ child ∈ set to"*
⟨*proof*⟩
end

interpretation *i_to_tree_order_wf?: l_to_tree_order_wf*$_{Core_DOM}$ *known_ptr type_wf get_child_nodes*
get_child_nodes_locs to_tree_order known_ptrs get_parent
get_parent_locs heap_is_wellformed parent_child_rel
get_disconnected_nodes get_disconnected_nodes_locs
⟨*proof*⟩
declare *l_to_tree_order_wf*$_{Core_DOM}$*_axioms [instances]*

locale *l_to_tree_order_wf = l_heap_is_wellformed_defs + l_type_wf + l_known_ptrs*
+ l_to_tree_order_defs
+ l_get_parent_defs + l_get_child_nodes_defs +
assumes *to_tree_order_ok:*
"heap_is_wellformed h ⟹ ptr |∈| object_ptr_kinds h ⟹ known_ptrs h ⟹ type_wf h
⟹ h ⊢ ok (to_tree_order ptr)"
assumes *to_tree_order_ptrs_in_heap:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r to
⟹ ptr' ∈ set to ⟹ ptr' |∈| object_ptr_kinds h"
assumes *to_tree_order_parent_child_rel:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r to
⟹ (ptr, child_ptr) ∈ (parent_child_rel h) ⟷ child_ptr ∈ set to"*
assumes *to_tree_order_child2:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r nodes
⟹ cast child ≠ ptr ⟹ cast child ∈ set nodes
⟹ (⋀parent. h ⊢ get_parent child →r Some parent
⟹ parent ∈ set nodes ⟹ thesis)

```
                        ⟹ thesis"
  assumes to_tree_order_node_ptrs:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r nodes
                        ⟹ ptr' ≠ ptr ⟹ ptr' ∈ set nodes ⟹ is_node_ptr_kind ptr'"
  assumes to_tree_order_child:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r nodes
                        ⟹ h ⊢ get_child_nodes parent →r children ⟹ cast child ≠ ptr
                        ⟹ child ∈ set children ⟹ cast child ∈ set nodes
                        ⟹ parent ∈ set nodes"
  assumes to_tree_order_ptr_in_result:
    "h ⊢ to_tree_order ptr →r nodes ⟹ ptr ∈ set nodes"
  assumes to_tree_order_parent:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r nodes
                        ⟹ h ⊢ get_parent child →r Some parent ⟹ parent ∈ set nodes
                        ⟹ cast child ∈ set nodes"
  assumes to_tree_order_subset:
    "heap_is_wellformed h ⟹ h ⊢ to_tree_order ptr →r nodes ⟹ node ∈ set nodes
                        ⟹ h ⊢ to_tree_order node →r nodes' ⟹ known_ptrs h
                        ⟹ type_wf h ⟹ set nodes' ⊆ set nodes"

lemma to_tree_order_wf_is_l_to_tree_order_wf [instances]:
  "l_to_tree_order_wf heap_is_wellformed parent_child_rel type_wf known_ptr known_ptrs
                      to_tree_order get_parent get_child_nodes"
  ⟨proof⟩
```

get_root_node

```
locale l_to_tree_order_wf_get_root_node_wf_Core_DOM =
  l_get_root_node_wf_Core_DOM
  + l_to_tree_order_wf
begin
lemma to_tree_order_get_root_node:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ to_tree_order ptr →r to"
  assumes "ptr' ∈ set to"
  assumes "h ⊢ get_root_node ptr' →r root_ptr"
  assumes "ptr'' ∈ set to"
  shows "h ⊢ get_root_node ptr'' →r root_ptr"
⟨proof⟩

lemma to_tree_order_same_root:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_root_node ptr →r root_ptr"
  assumes "h ⊢ to_tree_order root_ptr →r to"
  assumes "ptr' ∈ set to"
  shows "h ⊢ get_root_node ptr' →r root_ptr"
⟨proof⟩
end

interpretation i_to_tree_order_wf_get_root_node_wf?: l_to_tree_order_wf_get_root_node_wf_Core_DOM
  known_ptr type_wf known_ptrs heap_is_wellformed parent_child_rel get_child_nodes
  get_child_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs get_parent get_parent_locs
  get_ancestors get_ancestors_locs get_root_node get_root_node_locs to_tree_order
  ⟨proof⟩

locale l_to_tree_order_wf_get_root_node_wf = l_type_wf + l_known_ptrs + l_to_tree_order_defs
  + l_get_root_node_defs + l_heap_is_wellformed_defs +
  assumes to_tree_order_get_root_node:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ to_tree_order ptr →r to
                        ⟹ ptr' ∈ set to ⟹ h ⊢ get_root_node ptr' →r root_ptr
                        ⟹ ptr'' ∈ set to ⟹ h ⊢ get_root_node ptr'' →r root_ptr"
  assumes to_tree_order_same_root:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
```

```
                                ⟹ h ⊢ get_root_node ptr →r root_ptr
                                ⟹ h ⊢ to_tree_order root_ptr →r to ⟹ ptr' ∈ set to
                                ⟹ h ⊢ get_root_node ptr' →r root_ptr"
```

lemma *to_tree_order_wf_get_root_node_wf_is_l_to_tree_order_wf_get_root_node_wf [instances]:*
```
  "l_to_tree_order_wf_get_root_node_wf type_wf known_ptr known_ptrs to_tree_order
                                       get_root_node heap_is_wellformed"
```
⟨*proof*⟩

6.3.6 get_owner_document

locale *l_get_owner_document_wf*$_{Core_DOM}$ =
```
  l_known_ptrs
  + l_heap_is_wellformed
  + l_get_root_node_Core_DOM
  + l_get_ancestors
  + l_get_ancestors_wf
  + l_get_parent
  + l_get_parent_wf
  + l_get_root_node_wf
  + l_get_owner_document_Core_DOM
```
begin

lemma *get_owner_document_disconnected_nodes:*
```
  assumes "heap_is_wellformed h"
  assumes "h ⊢ get_disconnected_nodes document_ptr →r disc_nodes"
  assumes "node_ptr ∈ set disc_nodes"
  assumes known_ptrs: "known_ptrs h"
  assumes type_wf: "type_wf h"
  shows "h ⊢ get_owner_document (cast node_ptr) →r document_ptr"
```
⟨*proof*⟩

lemma *in_disconnected_nodes_no_parent:*
```
  assumes "heap_is_wellformed h"
    and "h ⊢ get_parent node_ptr →r None"
    and "h ⊢ get_owner_document (cast node_ptr) →r owner_document"
    and "h ⊢ get_disconnected_nodes owner_document →r disc_nodes"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "node_ptr ∈ set disc_nodes"
```
⟨*proof*⟩

lemma *get_owner_document_owner_document_in_heap:*
```
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_owner_document ptr →r owner_document"
  shows "owner_document |∈| document_ptr_kinds h"
```
⟨*proof*⟩

lemma *get_owner_document_ok:*
```
  assumes "heap_is_wellformed h"  "known_ptrs h" "type_wf h"
  assumes "ptr |∈| object_ptr_kinds h"
  shows "h ⊢ ok (get_owner_document ptr)"
```
⟨*proof*⟩

lemma *get_owner_document_child_same:*
```
  assumes "heap_is_wellformed h"  "known_ptrs h" "type_wf h"
  assumes "h ⊢ get_child_nodes ptr →r children"
  assumes "child ∈ set children"
  shows "h ⊢ get_owner_document ptr →r owner_document ⟷ h ⊢ get_owner_document (cast child) →r owner_documen
```
⟨*proof*⟩

end

locale *l_get_owner_document_wf = l_heap_is_wellformed_defs + l_type_wf + l_known_ptrs*
+ l_get_disconnected_nodes_defs + l_get_owner_document_defs
+ l_get_parent_defs +
assumes *get_owner_document_disconnected_nodes:*
"heap_is_wellformed h ⟹
known_ptrs h ⟹
type_wf h ⟹
h ⊢ get_disconnected_nodes document_ptr →r disc_nodes ⟹
node_ptr ∈ set disc_nodes ⟹
h ⊢ get_owner_document (cast node_ptr) →r document_ptr"
assumes *in_disconnected_nodes_no_parent:*
"heap_is_wellformed h ⟹
h ⊢ get_parent node_ptr →r None⟹
h ⊢ get_owner_document (cast node_ptr) →r owner_document ⟹
h ⊢ get_disconnected_nodes owner_document →r disc_nodes ⟹
known_ptrs h ⟹
type_wf h⟹
node_ptr ∈ set disc_nodes"
assumes *get_owner_document_owner_document_in_heap:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹
h ⊢ get_owner_document ptr →r owner_document ⟹
owner_document |∈| document_ptr_kinds h"
assumes *get_owner_document_ok:*
"heap_is_wellformed h ⟹ known_ptrs h ⟹ type_wf h ⟹ ptr |∈| object_ptr_kinds h
⟹ h ⊢ ok (get_owner_document ptr)"

interpretation *i_get_owner_document_wf?: l_get_owner_document_wf*$_{Core_DOM}$
known_ptr known_ptrs type_wf heap_is_wellformed parent_child_rel get_child_nodes
get_child_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs get_parent get_parent_locs
get_ancestors get_ancestors_locs get_root_node get_root_node_locs get_owner_document
⟨*proof*⟩
declare *l_get_owner_document_wf*$_{Core_DOM}$*_axioms [instances]*

lemma *get_owner_document_wf_is_l_get_owner_document_wf [instances]:*
"l_get_owner_document_wf heap_is_wellformed type_wf known_ptr known_ptrs get_disconnected_nodes
get_owner_document get_parent"
⟨*proof*⟩

get_root_node

locale *l_get_owner_document_wf_get_root_node_wf*$_{Core_DOM}$ *=*
l_get_root_node$_{Core_DOM}$ *+*
l_get_root_node_wf +
l_get_owner_document$_{Core_DOM}$ *+*
l_get_owner_document_wf
begin

lemma *get_root_node_document:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_root_node ptr →r root"*
assumes *"is_document_ptr_kind root"*
shows *"h ⊢ get_owner_document ptr →r the (cast root)"*
⟨*proof*⟩

lemma *get_root_node_same_owner_document:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_root_node ptr →r root"*
shows *"h ⊢ get_owner_document ptr →r owner_document ⟷ h ⊢ get_owner_document root →r owner_document"*
⟨*proof*⟩
end

interpretation *get_owner_document_wf_get_root_node_wf?: l_get_owner_document_wf_get_root_node_wf*$_{Core_DOM}$
type_wf known_ptr known_ptrs get_parent get_parent_locs get_child_nodes get_child_nodes_locs

```
  get_ancestors get_ancestors_locs get_root_node get_root_node_locs heap_is_wellformed parent_child_rel
  get_disconnected_nodes get_disconnected_nodes_locs get_owner_document
  ⟨proof⟩
declare l_get_owner_document_wf_get_root_node_wf_Core_DOM_axioms [instances]

locale l_get_owner_document_wf_get_root_node_wf = l_heap_is_wellformed_defs + l_type_wf +
  l_known_ptrs + l_get_root_node_defs + l_get_owner_document_defs +
  assumes get_root_node_document:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ get_root_node ptr →r root ⟹
is_document_ptr_kind root ⟹ h ⊢ get_owner_document ptr →r the (cast root)"
  assumes get_root_node_same_owner_document:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ get_root_node ptr →r root ⟹
h ⊢ get_owner_document ptr →r owner_document ⟷ h ⊢ get_owner_document root →r owner_document"

lemma get_owner_document_wf_get_root_node_wf_is_l_get_owner_document_wf_get_root_node_wf [instances]:
  "l_get_owner_document_wf_get_root_node_wf heap_is_wellformed type_wf known_ptr known_ptrs
get_root_node get_owner_document"
  ⟨proof⟩
```

6.3.7 Preserving heap-wellformedness

6.3.8 set_attribute

```
locale l_set_attribute_wf_Core_DOM =
  l_get_parent_wf2_Core_DOM +
  l_set_attribute_Core_DOM +
  l_set_attribute_get_disconnected_nodes +
  l_set_attribute_get_child_nodes
begin
lemma set_attribute_preserves_wellformedness:
  assumes "heap_is_wellformed h"
    and "h ⊢ set_attribute element_ptr k v →h h'"
  shows "heap_is_wellformed h'"
  thm preserves_wellformedness_writes_needed
  ⟨proof⟩
end
```

6.3.9 remove_child

```
locale l_remove_child_wf_Core_DOM =
  l_remove_child_Core_DOM +
  l_get_parent_wf_Core_DOM +
  l_heap_is_wellformed +
  l_set_disconnected_nodes_get_child_nodes
begin
lemma remove_child_removes_parent:
  assumes wellformed: "heap_is_wellformed h"
    and remove_child: "h ⊢ remove_child ptr child →h h2"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "h2 ⊢ get_parent child →r None"
⟨proof⟩
end

locale l_remove_child_wf2_Core_DOM =
  l_remove_child_wf_Core_DOM +
  l_heap_is_wellformed_Core_DOM
begin

lemma remove_child_parent_child_rel_subset:
  assumes "heap_is_wellformed h"
    and "h ⊢ remove_child ptr child →h h'"
    and "known_ptrs h"
```

```
    and type_wf: "type_wf h"
  shows "parent_child_rel h' ⊆ parent_child_rel h"
⟨proof⟩


lemma remove_child_heap_is_wellformed_preserved:
  assumes "heap_is_wellformed h"
    and "h ⊢ remove_child ptr child →_h h'"
    and "known_ptrs h"
    and type_wf: "type_wf h"
  shows "type_wf h'" and "known_ptrs h'" and "heap_is_wellformed h'"
⟨proof⟩

lemma remove_heap_is_wellformed_preserved:
  assumes "heap_is_wellformed h"
    and "h ⊢ remove child →_h h'"
    and "known_ptrs h"
    and type_wf: "type_wf h"
  shows "type_wf h'" and "known_ptrs h'" and "heap_is_wellformed h'"
  ⟨proof⟩

lemma remove_child_removes_child:
  assumes wellformed: "heap_is_wellformed h"
    and remove_child: "h ⊢ remove_child ptr' child →_h h'"
    and children: "h' ⊢ get_child_nodes ptr →_r children"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "child ∉ set children"
⟨proof⟩

lemma remove_child_removes_first_child:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_child_nodes ptr →_r node_ptr # children"
  assumes "h ⊢ remove_child ptr node_ptr →_h h'"
  shows "h' ⊢ get_child_nodes ptr →_r children"
⟨proof⟩

lemma remove_removes_child:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_child_nodes ptr →_r node_ptr # children"
  assumes "h ⊢ remove node_ptr →_h h'"
  shows "h' ⊢ get_child_nodes ptr →_r children"
⟨proof⟩

lemma remove_for_all_empty_children:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "h ⊢ get_child_nodes ptr →_r children"
  assumes "h ⊢ forall_M remove children →_h h'"
  shows "h' ⊢ get_child_nodes ptr →_r []"
  ⟨proof⟩
end

locale l_remove_child_wf2 = l_type_wf + l_known_ptrs + l_remove_child_defs + l_heap_is_wellformed_defs
  + l_get_child_nodes_defs + l_remove_defs +
  assumes remove_child_preserves_type_wf:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ remove_child ptr child →_h h'
                          ⟹ type_wf h'"
  assumes remove_child_preserves_known_ptrs:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ remove_child ptr child →_h h'
                          ⟹ known_ptrs h'"
  assumes remove_child_heap_is_wellformed_preserved:
    "type_wf h ⟹ known_ptrs h ⟹ heap_is_wellformed h ⟹ h ⊢ remove_child ptr child →_h h'
               ⟹ heap_is_wellformed h'"
```

assumes *remove_preserves_type_wf:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ remove child →h h'
⟹ type_wf h'"
assumes *remove_preserves_known_ptrs:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ remove child →h h'
⟹ known_ptrs h'"
assumes *remove_heap_is_wellformed_preserved:*
"type_wf h ⟹ known_ptrs h ⟹ heap_is_wellformed h ⟹ h ⊢ remove child →h h'
⟹ heap_is_wellformed h'"
assumes *remove_child_removes_child:*
"heap_is_wellformed h ⟹ h ⊢ remove_child ptr' child →h h' ⟹ h' ⊢ get_child_nodes ptr →r children
⟹ known_ptrs h ⟹ type_wf h
⟹ child ∉ set children"
assumes *remove_child_removes_first_child:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
⟹ h ⊢ get_child_nodes ptr →r node_ptr # children
⟹ h ⊢ remove_child ptr node_ptr →h h'
⟹ h' ⊢ get_child_nodes ptr →r children"
assumes *remove_removes_child:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
⟹ h ⊢ get_child_nodes ptr →r node_ptr # children
⟹ h ⊢ remove node_ptr →h h' ⟹ h' ⊢ get_child_nodes ptr →r children"
assumes *remove_for_all_empty_children:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ get_child_nodes ptr →r children
⟹ h ⊢ forall_M remove children →h h' ⟹ h' ⊢ get_child_nodes ptr →r []"

interpretation *i_remove_child_wf2?: l_remove_child_wf2$_{Core_DOM}$ get_child_nodes get_child_nodes_locs*
set_child_nodes set_child_nodes_locs get_parent get_parent_locs get_owner_document
get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
set_disconnected_nodes_locs remove_child remove_child_locs remove type_wf known_ptr known_ptrs
heap_is_wellformed parent_child_rel
⟨*proof*⟩

lemma *remove_child_wf2_is_l_remove_child_wf2 [instances]:*
"l_remove_child_wf2 type_wf known_ptr known_ptrs remove_child heap_is_wellformed get_child_nodes remove"
⟨*proof*⟩

6.3.10 adopt_node

locale *l_adopt_node_wf$_{Core_DOM}$ =*
l_adopt_node$_{Core_DOM}$ +
l_get_parent_wf +
l_get_owner_document_wf +
l_remove_child_wf2 +
l_heap_is_wellformed
begin
lemma *adopt_node_removes_first_child:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ adopt_node owner_document node →h h'"*
assumes *"h ⊢ get_child_nodes ptr' →r node # children"*
shows *"h' ⊢ get_child_nodes ptr' →r children"*
⟨*proof*⟩

lemma *adopt_node_document_in_heap:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ ok (adopt_node owner_document node)"*
shows *"owner_document |∈| document_ptr_kinds h"*
⟨*proof*⟩
end

locale *l_adopt_node_wf2$_{Core_DOM}$ =*
l_adopt_node_wf$_{Core_DOM}$ +

```
  l_adopt_node_Core_DOM +
  l_get_parent_wf_Core_DOM +
  l_get_root_node +
  l_get_owner_document_wf +
  l_remove_child_wf2 +
  l_heap_is_wellformed_Core_DOM
begin

lemma adopt_node_removes_child_step:
  assumes wellformed: "heap_is_wellformed h"
    and adopt_node: "h ⊢ adopt_node owner_document node_ptr →_h h2"
    and children: "h2 ⊢ get_child_nodes ptr →_r children"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "node_ptr ∉ set children"
⟨proof⟩

lemma adopt_node_removes_child:
  assumes "heap_is_wellformed h" and "known_ptrs h" and "type_wf h"
  assumes "h ⊢ adopt_node owner_document node_ptr →_h h'"
  shows "⋀ptr' children'.
  h' ⊢ get_child_nodes ptr' →_r children' ⟹ node_ptr ∉ set children'"
  ⟨proof⟩

lemma adopt_node_preserves_wellformedness:
  assumes "heap_is_wellformed h"
    and "h ⊢ adopt_node document_ptr child →_h h'"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "heap_is_wellformed h'" and "known_ptrs h'" and "type_wf h'"
⟨proof⟩

lemma adopt_node_node_in_disconnected_nodes:
  assumes wellformed: "heap_is_wellformed h"
    and adopt_node: "h ⊢ adopt_node owner_document node_ptr →_h h'"
    and "h' ⊢ get_disconnected_nodes owner_document →_r disc_nodes"
    and known_ptrs: "known_ptrs h"
    and type_wf: "type_wf h"
  shows "node_ptr ∈ set disc_nodes"
⟨proof⟩
end

interpretation i_adopt_node_wf?: l_adopt_node_wf_Core_DOM get_owner_document get_parent get_parent_locs
  remove_child remove_child_locs get_disconnected_nodes get_disconnected_nodes_locs
  set_disconnected_nodes set_disconnected_nodes_locs adopt_node adopt_node_locs known_ptr
  type_wf get_child_nodes get_child_nodes_locs known_ptrs set_child_nodes set_child_nodes_locs
  remove heap_is_wellformed parent_child_rel
  ⟨proof⟩
declare l_adopt_node_wf_Core_DOM_axioms[instances]

interpretation i_adopt_node_wf2?: l_adopt_node_wf2_Core_DOM get_owner_document get_parent get_parent_locs
  remove_child remove_child_locs get_disconnected_nodes get_disconnected_nodes_locs
  set_disconnected_nodes set_disconnected_nodes_locs adopt_node adopt_node_locs known_ptr
  type_wf get_child_nodes get_child_nodes_locs known_ptrs set_child_nodes set_child_nodes_locs
  remove heap_is_wellformed parent_child_rel get_root_node get_root_node_locs
  ⟨proof⟩
declare l_adopt_node_wf2_Core_DOM_axioms[instances]


locale l_adopt_node_wf = l_heap_is_wellformed + l_known_ptrs + l_type_wf + l_adopt_node_defs
  + l_get_child_nodes_defs + l_get_disconnected_nodes_defs +
  assumes adopt_node_preserves_wellformedness:
    "heap_is_wellformed h ⟹ h ⊢ adopt_node document_ptr child →_h h' ⟹ known_ptrs h
```

```
                    ⟹ type_wf h ⟹ heap_is_wellformed h'"
  assumes adopt_node_removes_child:
    "heap_is_wellformed h ⟹ h ⊢ adopt_node owner_document node_ptr →_h h2
                    ⟹ h2 ⊢ get_child_nodes ptr →_r children ⟹ known_ptrs h
                    ⟹ type_wf h ⟹ node_ptr ∉ set children"
  assumes adopt_node_node_in_disconnected_nodes:
    "heap_is_wellformed h ⟹ h ⊢ adopt_node owner_document node_ptr →_h h'
                    ⟹ h' ⊢ get_disconnected_nodes owner_document →_r disc_nodes
                    ⟹ known_ptrs h ⟹ type_wf h ⟹ node_ptr ∈ set disc_nodes"
  assumes adopt_node_removes_first_child: "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h
                    ⟹ h ⊢ adopt_node owner_document node →_h h'
                    ⟹ h ⊢ get_child_nodes ptr' →_r node # children
                    ⟹ h' ⊢ get_child_nodes ptr' →_r children"
  assumes adopt_node_document_in_heap: "heap_is_wellformed h ⟹ known_ptrs h ⟹ type_wf h
                   ⟹ h ⊢ ok (adopt_node owner_document node)
                   ⟹ owner_document |∈| document_ptr_kinds h"
  assumes adopt_node_preserves_type_wf:
    "heap_is_wellformed h ⟹ h ⊢ adopt_node document_ptr child →_h h' ⟹ known_ptrs h
                    ⟹ type_wf h ⟹ type_wf h'"
  assumes adopt_node_preserves_known_ptrs:
    "heap_is_wellformed h ⟹ h ⊢ adopt_node document_ptr child →_h h' ⟹ known_ptrs h
                    ⟹ type_wf h ⟹ known_ptrs h'"


lemma adopt_node_wf_is_l_adopt_node_wf [instances]:
  "l_adopt_node_wf type_wf known_ptr heap_is_wellformed parent_child_rel get_child_nodes
                   get_disconnected_nodes known_ptrs adopt_node"
  ⟨proof⟩
```

6.3.11 insert_before

```
locale l_insert_before_wf_Core_DOM =
  l_insert_before_Core_DOM +
  l_adopt_node_wf +
  l_set_disconnected_nodes_get_child_nodes +
  l_heap_is_wellformed
begin
lemma insert_before_removes_child:
  assumes "heap_is_wellformed h" and "type_wf h" and "known_ptrs h"
  assumes "ptr ≠ ptr'"
  assumes "h ⊢ insert_before ptr node child →_h h'"
  assumes "h ⊢ get_child_nodes ptr' →_r node # children"
  shows "h' ⊢ get_child_nodes ptr' →_r children"
⟨proof⟩
end

locale l_insert_before_wf = l_heap_is_wellformed_defs + l_type_wf + l_known_ptrs
  + l_insert_before_defs + l_get_child_nodes_defs +
  assumes insert_before_removes_child:
    "heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ ptr ≠ ptr'
                    ⟹ h ⊢ insert_before ptr node child →_h h'
                    ⟹ h ⊢ get_child_nodes ptr' →_r node # children
                    ⟹ h' ⊢ get_child_nodes ptr' →_r children"

interpretation i_insert_before_wf?: l_insert_before_wf_Core_DOM get_parent get_parent_locs
  get_child_nodes get_child_nodes_locs set_child_nodes
  set_child_nodes_locs get_ancestors get_ancestors_locs
  adopt_node adopt_node_locs set_disconnected_nodes
  set_disconnected_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs get_owner_document insert_before
  insert_before_locs append_child type_wf known_ptr known_ptrs
  heap_is_wellformed parent_child_rel
  ⟨proof⟩
```

declare *l_insert_before_wf$_{Core_DOM}$_axioms [instances]*

lemma *insert_before_wf_is_l_insert_before_wf [instances]:*
"l_insert_before_wf heap_is_wellformed type_wf known_ptr known_ptrs insert_before get_child_nodes"
⟨*proof*⟩

locale *l_insert_before_wf2$_{Core_DOM}$ =*
l_insert_before_wf$_{Core_DOM}$ +
l_set_child_nodes_get_disconnected_nodes +
l_remove_child +
l_get_root_node_wf +
l_set_disconnected_nodes_get_disconnected_nodes_wf +
l_set_disconnected_nodes_get_ancestors +
l_get_ancestors_wf +
l_get_owner_document +
l_heap_is_wellformed$_{Core_DOM}$ +
l_get_owner_document_wf
begin

lemma *insert_before_preserves_acyclitity:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ insert_before ptr node child →$_h$ h'"*
shows *"acyclic (parent_child_rel h')"*
⟨*proof*⟩

lemma *insert_before_heap_is_wellformed_preserved:*
assumes *wellformed: "heap_is_wellformed h"*
and *insert_before: "h ⊢ insert_before ptr node child →$_h$ h'"*
and *known_ptrs: "known_ptrs h"*
and *type_wf: "type_wf h"*
shows *"heap_is_wellformed h'"* **and** *"type_wf h'"* **and** *"known_ptrs h'"*
⟨*proof*⟩

lemma *adopt_node_children_remain_distinct:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ adopt_node owner_document node_ptr →$_h$ h'"*
shows *"⋀ptr' children'.*
h' ⊢ get_child_nodes ptr' →$_r$ children' ⟹ distinct children'"
⟨*proof*⟩

lemma *insert_node_children_remain_distinct:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ a_insert_node ptr new_child reference_child_opt →$_h$ h'"*
assumes *"h ⊢ get_child_nodes ptr →$_r$ children"*
assumes *"new_child ∉ set children"*
shows *"⋀children'.*
h' ⊢ get_child_nodes ptr →$_r$ children' ⟹ distinct children'"
⟨*proof*⟩

lemma *insert_before_children_remain_distinct:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ insert_before ptr new_child child_opt →$_h$ h'"*
shows *"⋀ptr' children'.*
h' ⊢ get_child_nodes ptr' →$_r$ children' ⟹ distinct children'"
⟨*proof*⟩

lemma *insert_before_removes_child:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ insert_before ptr node child →$_h$ h'"*
assumes *"ptr ≠ ptr'"*
shows *"⋀children'. h' ⊢ get_child_nodes ptr' →$_r$ children' ⟹ node ∉ set children'"*

⟨*proof*⟩

lemma *ensure_pre_insertion_validity_ok:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"ptr |∈| object_ptr_kinds h"*
assumes *"¬is_character_data_ptr_kind parent"*
assumes *"cast node ∉ set |h ⊢ get_ancestors parent|$_r$"*
assumes *"h ⊢ get_parent ref →$_r$ Some parent"*
assumes *"is_document_ptr parent ⟹ h ⊢ get_child_nodes parent →$_r$ []"*
assumes *"is_document_ptr parent ⟹ ¬is_character_data_ptr_kind node"*
shows *"h ⊢ ok (a_ensure_pre_insertion_validity node parent (Some ref))"*
⟨*proof*⟩
end

locale *l_insert_before_wf2 = l_type_wf + l_known_ptrs + l_insert_before_defs*
+ l_heap_is_wellformed_defs + l_get_child_nodes_defs + l_remove_defs +
assumes *insert_before_preserves_type_wf:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ insert_before ptr child ref →$_h$ h' ⟹ type_wf h'"
assumes *insert_before_preserves_known_ptrs:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ insert_before ptr child ref →$_h$ h' ⟹ known_ptrs h'"
assumes *insert_before_heap_is_wellformed_preserved:*
"type_wf h ⟹ known_ptrs h ⟹ heap_is_wellformed h ⟹ h ⊢ insert_before ptr child ref →$_h$ h' ⟹ heap_is_wellformed h'"

interpretation *i_insert_before_wf2?: l_insert_before_wf2$_{Core_DOM}$ get_parent get_parent_locs*
get_child_nodes get_child_nodes_locs set_child_nodes
set_child_nodes_locs get_ancestors get_ancestors_locs
adopt_node adopt_node_locs set_disconnected_nodes
set_disconnected_nodes_locs get_disconnected_nodes
get_disconnected_nodes_locs get_owner_document insert_before
insert_before_locs append_child type_wf known_ptr known_ptrs
heap_is_wellformed parent_child_rel remove_child
remove_child_locs get_root_node get_root_node_locs
⟨*proof*⟩
declare *l_insert_before_wf2$_{Core_DOM}$_axioms [instances]*

lemma *insert_before_wf2_is_l_insert_before_wf2 [instances]:*
"l_insert_before_wf2 type_wf known_ptr known_ptrs insert_before heap_is_wellformed"
⟨*proof*⟩

locale *l_insert_before_wf3$_{Core_DOM}$ =*
l_insert_before_wf2$_{Core_DOM}$ +
l_adopt_node$_{Core_DOM}$ +
l_set_child_nodes_get_child_nodes$_{Core_DOM}$ +
l_remove_child_wf2
begin

lemma *next_sibling_ok:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"node_ptr |∈| node_ptr_kinds h"*
shows *"h ⊢ ok (a_next_sibling node_ptr)"*
⟨*proof*⟩

lemma *remove_child_ok:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"h ⊢ get_child_nodes ptr →$_r$ children"*
assumes *"child ∈ set children"*
shows *"h ⊢ ok (remove_child ptr child)"*
⟨*proof*⟩

lemma *adopt_node_ok:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"document_ptr |∈| document_ptr_kinds h"*
assumes *"child |∈| node_ptr_kinds h"*
shows *"h ⊢ ok (adopt_node document_ptr child)"*
⟨*proof*⟩

lemma *insert_node_ok:*
assumes *"known_ptr parent"* **and** *"type_wf h"*
assumes *"parent |∈| object_ptr_kinds h"*
assumes *"¬is_character_data_ptr_kind parent"*
assumes *"is_document_ptr parent ⟹ h ⊢ get_child_nodes parent →r []"*
assumes *"is_document_ptr parent ⟹ ¬is_character_data_ptr_kind node"*
assumes *"known_ptr (cast node)"*
shows *"h ⊢ ok (a_insert_node parent node ref)"*
⟨*proof*⟩

lemma *insert_before_ok:*
assumes *"heap_is_wellformed h"* **and** *"known_ptrs h"* **and** *"type_wf h"*
assumes *"parent |∈| object_ptr_kinds h"*
assumes *"node |∈| node_ptr_kinds h"*
assumes *"¬is_character_data_ptr_kind parent"*
assumes *"cast node ∉ set |h ⊢ get_ancestors parent|r"*
assumes *"h ⊢ get_parent ref →r Some parent"*
assumes *"is_document_ptr parent ⟹ h ⊢ get_child_nodes parent →r []"*
assumes *"is_document_ptr parent ⟹ ¬is_character_data_ptr_kind node"*
shows *"h ⊢ ok (insert_before parent node (Some ref))"*
⟨*proof*⟩
end

interpretation *i_insert_before_wf3?: l_insert_before_wf3*$_{Core_DOM}$
get_parent get_parent_locs get_child_nodes get_child_nodes_locs set_child_nodes set_child_nodes_locs
get_ancestors get_ancestors_locs adopt_node adopt_node_locs set_disconnected_nodes
set_disconnected_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs get_owner_document
insert_before insert_before_locs append_child type_wf known_ptr known_ptrs heap_is_wellformed
parent_child_rel remove_child remove_child_locs get_root_node get_root_node_locs remove
⟨*proof*⟩
declare *l_insert_before_wf3*$_{Core_DOM}$*_axioms [instances]*

locale *l_append_child_wf*$_{Core_DOM}$ =
l_adopt_node$_{Core_DOM}$ +
l_insert_before$_{Core_DOM}$ +
l_append_child$_{Core_DOM}$ +
l_insert_before_wf +
l_insert_before_wf2 +
l_get_child_nodes
begin

lemma *append_child_heap_is_wellformed_preserved:*
assumes *wellformed: "heap_is_wellformed h"*
and *append_child: "h ⊢ append_child ptr node →h h'"*
and *known_ptrs: "known_ptrs h"*
and *type_wf: "type_wf h"*
shows *"heap_is_wellformed h'"* **and** *"type_wf h'"* **and** *"known_ptrs h'"*
⟨*proof*⟩

lemma *append_child_children:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_child_nodes ptr →r xs"*
assumes *"h ⊢ append_child ptr node →h h'"*

assumes *"node ∉ set xs"*
shows *"h' ⊢ get_child_nodes ptr →r xs @ [node]"*
⟨*proof*⟩

lemma *append_child_for_all_on_children:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_child_nodes ptr →r xs"*
assumes *"h ⊢ forall_M (append_child ptr) nodes →h h'"*
assumes *"set nodes ∩ set xs = {}"*
assumes *"distinct nodes"*
shows *"h' ⊢ get_child_nodes ptr →r xs@nodes"*
⟨*proof*⟩

lemma *append_child_for_all_on_no_children:*
assumes *"heap_is_wellformed h"* **and** *"type_wf h"* **and** *"known_ptrs h"*
assumes *"h ⊢ get_child_nodes ptr →r []"*
assumes *"h ⊢ forall_M (append_child ptr) nodes →h h'"*
assumes *"distinct nodes"*
shows *"h' ⊢ get_child_nodes ptr →r nodes"*
⟨*proof*⟩
end

locale *l_append_child_wf = l_type_wf + l_known_ptrs + l_append_child_defs + l_heap_is_wellformed_defs +*
assumes *append_child_preserves_type_wf:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ append_child ptr child →h h'
⟹ type_wf h'"
assumes *append_child_preserves_known_ptrs:*
"heap_is_wellformed h ⟹ type_wf h ⟹ known_ptrs h ⟹ h ⊢ append_child ptr child →h h'
⟹ known_ptrs h'"
assumes *append_child_heap_is_wellformed_preserved:*
"type_wf h ⟹ known_ptrs h ⟹ heap_is_wellformed h ⟹ h ⊢ append_child ptr child →h h'
⟹ heap_is_wellformed h'"

interpretation *i_append_child_wf?: l_append_child_wf*$_{Core_DOM}$ *get_owner_document get_parent*
get_parent_locs remove_child remove_child_locs
get_disconnected_nodes get_disconnected_nodes_locs
set_disconnected_nodes set_disconnected_nodes_locs
adopt_node adopt_node_locs known_ptr type_wf get_child_nodes
get_child_nodes_locs known_ptrs set_child_nodes
set_child_nodes_locs remove get_ancestors get_ancestors_locs
insert_before insert_before_locs append_child heap_is_wellformed
parent_child_rel
⟨*proof*⟩

lemma *append_child_wf_is_l_append_child_wf [instances]: "l_append_child_wf type_wf known_ptr known_ptrs append_child heap_is_wellformed"*
⟨*proof*⟩

6.3.12 create_element

locale *l_create_element_wf*$_{Core_DOM}$ =
l_heap_is_wellformed$_{Core_DOM}$ *known_ptr type_wf get_child_nodes get_child_nodes_locs*
get_disconnected_nodes get_disconnected_nodes_locs
heap_is_wellformed parent_child_rel +
l_new_element_get_disconnected_nodes get_disconnected_nodes get_disconnected_nodes_locs +
l_set_tag_name_get_disconnected_nodes type_wf set_tag_name set_tag_name_locs
get_disconnected_nodes get_disconnected_nodes_locs +
l_create_element$_{Core_DOM}$ *get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes*
set_disconnected_nodes_locs set_tag_name set_tag_name_locs type_wf create_element known_ptr +
l_new_element_get_child_nodes type_wf known_ptr get_child_nodes get_child_nodes_locs +
l_set_tag_name_get_child_nodes type_wf set_tag_name set_tag_name_locs known_ptr
get_child_nodes get_child_nodes_locs +

```
  l_set_disconnected_nodes_get_child_nodes set_disconnected_nodes set_disconnected_nodes_locs
  get_child_nodes get_child_nodes_locs +
  l_set_disconnected_nodes type_wf set_disconnected_nodes set_disconnected_nodes_locs +
  l_set_disconnected_nodes_get_disconnected_nodes  type_wf get_disconnected_nodes
  get_disconnected_nodes_locs set_disconnected_nodes set_disconnected_nodes_locs +
  l_new_element type_wf +
  l_known_ptrs known_ptr known_ptrs
  for known_ptr :: "('_::linorder) object_ptr ⇒ bool"
    and known_ptrs :: "('_) heap ⇒ bool"
    and type_wf :: "('_) heap ⇒ bool"
    and get_child_nodes :: "('_) object_ptr ⇒ (('_) heap, exception, ('_) node_ptr list) prog"
    and get_child_nodes_locs :: "('_) object_ptr ⇒ (('_) heap ⇒ ('_) heap ⇒ bool) set"
    and get_disconnected_nodes :: "('_) document_ptr ⇒ (('_) heap, exception, ('_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "('_) document_ptr ⇒ (('_) heap ⇒ ('_) heap ⇒ bool) set"
    and heap_is_wellformed :: "('_) heap ⇒ bool"
    and parent_child_rel :: "('_) heap ⇒ (('_) object_ptr × ('_) object_ptr) set"
    and set_tag_name :: "('_) element_ptr ⇒ char list ⇒ (('_) heap, exception, unit) prog"
    and set_tag_name_locs :: "('_) element_ptr ⇒ (('_) heap, exception, unit) prog set"
    and set_disconnected_nodes :: "('_) document_ptr ⇒ ('_) node_ptr list ⇒ (('_) heap, exception, unit)
prog"
    and set_disconnected_nodes_locs :: "('_) document_ptr ⇒ (('_) heap, exception, unit) prog set"
    and create_element :: "('_) document_ptr ⇒ char list ⇒ (('_) heap, exception, ('_) element_ptr) prog"
begin
lemma create_element_preserves_wellformedness:
  assumes "heap_is_wellformed h"
    and "h ⊢ create_element document_ptr tag →_h h'"
    and "type_wf h"
    and "known_ptrs h"
  shows "heap_is_wellformed h'" and "type_wf h'" and "known_ptrs h'"
⟨proof⟩
end

interpretation i_create_element_wf?: l_create_element_wf_Core_DOM known_ptr known_ptrs type_wf
  get_child_nodes get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs heap_is_wellformed parent_child_rel
  set_tag_name set_tag_name_locs
  set_disconnected_nodes set_disconnected_nodes_locs create_element
  ⟨proof⟩
declare l_create_element_wf_Core_DOM_axioms [instances]
```

6.3.13 create_character_data

```
locale l_create_character_data_wf_Core_DOM =
  l_heap_is_wellformed_Core_DOM
  known_ptr type_wf get_child_nodes get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs heap_is_wellformed parent_child_rel
  + l_new_character_data_get_disconnected_nodes
  get_disconnected_nodes get_disconnected_nodes_locs
  + l_set_val_get_disconnected_nodes
  type_wf set_val set_val_locs get_disconnected_nodes get_disconnected_nodes_locs
  + l_create_character_data_Core_DOM
  get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
  set_disconnected_nodes_locs set_val set_val_locs type_wf create_character_data known_ptr
  + l_new_character_data_get_child_nodes
  type_wf known_ptr get_child_nodes get_child_nodes_locs
  + l_set_val_get_child_nodes
  type_wf set_val set_val_locs known_ptr get_child_nodes get_child_nodes_locs
  + l_set_disconnected_nodes_get_child_nodes
  set_disconnected_nodes set_disconnected_nodes_locs get_child_nodes get_child_nodes_locs
  + l_set_disconnected_nodes
  type_wf set_disconnected_nodes set_disconnected_nodes_locs
  + l_set_disconnected_nodes_get_disconnected_nodes
  type_wf get_disconnected_nodes get_disconnected_nodes_locs set_disconnected_nodes
```

```
  set_disconnected_nodes_locs
  + l_new_character_data
  type_wf
  + l_known_ptrs
  known_ptr known_ptrs
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
    and type_wf :: "(_) heap ⇒ bool"
    and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and heap_is_wellformed :: "(_) heap ⇒ bool"
    and parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"
    and set_val :: "(_) character_data_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
    and set_val_locs :: "(_) character_data_ptr ⇒ ((_) heap, exception, unit) prog set"
    and set_disconnected_nodes ::
    "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit) prog"
    and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
    and create_character_data ::
    "(_) document_ptr ⇒ char list ⇒ ((_) heap, exception, (_) character_data_ptr) prog"
    and known_ptrs :: "(_) heap ⇒ bool"
begin

lemma create_character_data_preserves_wellformedness:
  assumes "heap_is_wellformed h"
    and "h ⊢ create_character_data document_ptr text →_h h'"
    and "type_wf h"
    and "known_ptrs h"
  shows "heap_is_wellformed h'" and "type_wf h'" and "known_ptrs h'"
⟨proof⟩
end

interpretation i_create_character_data_wf?: l_create_character_data_wf_Core_DOM known_ptr type_wf
  get_child_nodes get_child_nodes_locs get_disconnected_nodes get_disconnected_nodes_locs
  heap_is_wellformed parent_child_rel set_val set_val_locs set_disconnected_nodes
  set_disconnected_nodes_locs create_character_data known_ptrs
  ⟨proof⟩
declare l_create_character_data_wf_Core_DOM_axioms [instances]
```

6.3.14 create_document

```
locale l_create_document_wf_Core_DOM =
  l_heap_is_wellformed_Core_DOM
  known_ptr type_wf get_child_nodes get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs heap_is_wellformed parent_child_rel
  + l_new_document_get_disconnected_nodes
  get_disconnected_nodes get_disconnected_nodes_locs
  + l_create_document_Core_DOM
  create_document
  + l_new_document_get_child_nodes
  type_wf known_ptr get_child_nodes get_child_nodes_locs
  + l_new_document
  type_wf
  + l_known_ptrs
  known_ptr known_ptrs
  for known_ptr :: "(_::linorder) object_ptr ⇒ bool"
    and type_wf :: "(_) heap ⇒ bool"
    and get_child_nodes :: "(_) object_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_child_nodes_locs :: "(_) object_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and get_disconnected_nodes :: "(_) document_ptr ⇒ ((_) heap, exception, (_) node_ptr list) prog"
    and get_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap ⇒ (_) heap ⇒ bool) set"
    and heap_is_wellformed :: "(_) heap ⇒ bool"
    and parent_child_rel :: "(_) heap ⇒ ((_) object_ptr × (_) object_ptr) set"
```

```
    and set_val :: "(_) character_data_ptr ⇒ char list ⇒ ((_) heap, exception, unit) prog"
    and set_val_locs :: "(_) character_data_ptr ⇒ ((_) heap, exception, unit) prog set"
    and set_disconnected_nodes :: "(_) document_ptr ⇒ (_) node_ptr list ⇒ ((_) heap, exception, unit)
prog"
    and set_disconnected_nodes_locs :: "(_) document_ptr ⇒ ((_) heap, exception, unit) prog set"
    and create_document :: "((_) heap, exception, (_) document_ptr) prog"
    and known_ptrs :: "(_) heap ⇒ bool"
begin

lemma create_document_preserves_wellformedness:
  assumes "heap_is_wellformed h"
    and "h ⊢ create_document →_h h'"
    and "type_wf h"
    and "known_ptrs h"
  shows "heap_is_wellformed h'"
⟨proof⟩
end

interpretation i_create_document_wf?: l_create_document_wf_Core_DOM known_ptr type_wf get_child_nodes
  get_child_nodes_locs get_disconnected_nodes
  get_disconnected_nodes_locs heap_is_wellformed parent_child_rel
  set_val set_val_locs set_disconnected_nodes
  set_disconnected_nodes_locs create_document known_ptrs
  ⟨proof⟩
declare l_create_document_wf_Core_DOM_axioms [instances]


end
```

6.4 The Core DOM (Core_DOM)

This theory is the main entry point of our formalization of the core DOM.

```
theory Core_DOM
imports
  "Core_DOM_Heap_WF"
begin


end
```

7 Test Suite

In this chapter, we present the formalized compliance test cases for the core DOM. As our formalization is executable, we can (symbolically) execute the test cases on top of our model. Executing these test cases successfully shows that our model is compliant to the official DOM standard. As future work, we plan to generate test cases from our formal model (e.g., using [6, 8]) to improve the quality of the official compliance test suite. For more details on the relation of test and proof in the context of web standards, we refer the reader to [5].

7.1 Common Test Setup (Core_DOM_BaseTest)

This theory provides the common test setup that is used by all formalized test cases.

```
theory Core_DOM_BaseTest
  imports

  "../Core_DOM"
begin

definition "assert_throws e p = do {
  h ← get_heap;
  (if (h ⊢ p →_e e) then return () else error AssertException)
}"
notation assert_throws (<assert'_throws'(_, _')>)

definition "test p h ⟷ h ⊢ ok p"


definition field_access :: "(string ⇒ (_, (_) object_ptr option) dom_prog) ⇒ string
                                    ⇒ (_, (_) object_ptr option) dom_prog"  (infix <.> 80)
  where
    "field_access m field = m field"

definition assert_equals :: "'a ⇒ 'a ⇒ (_, unit) dom_prog"
  where
    "assert_equals l r = (if l = r then return () else error AssertException)"
definition assert_equals_with_message :: "'a ⇒ 'a ⇒ 'b ⇒ (_, unit) dom_prog"
  where
    "assert_equals_with_message l r _ = (if l = r then return () else error AssertException)"
notation assert_equals (<assert'_equals'(_, _')>)
notation assert_equals_with_message (<assert'_equals'(_, _, _')>)
notation assert_equals (<assert'_array'_equals'(_, _')>)
notation assert_equals_with_message (<assert'_array'_equals'(_, _, _')>)

definition assert_not_equals :: "'a ⇒ 'a ⇒ (_, unit) dom_prog"
  where
    "assert_not_equals l r = (if l ≠ r then return () else error AssertException)"
definition assert_not_equals_with_message :: "'a ⇒ 'a ⇒ 'b ⇒ (_, unit) dom_prog"
  where
    "assert_not_equals_with_message l r _ = (if l ≠ r then return () else error AssertException)"
notation assert_not_equals (<assert'_not'_equals'(_, _')>)
notation assert_not_equals_with_message (<assert'_not'_equals'(_, _, _')>)
notation assert_not_equals (<assert'_array'_not'_equals'(_, _')>)
notation assert_not_equals_with_message (<assert'_array'_not'_equals'(_, _, _')>)

definition removeWhiteSpaceOnlyTextNodes :: "((_) object_ptr option) ⇒ (_, unit) dom_prog"
```

```
  where
    "removeWhiteSpaceOnlyTextNodes _ = return ()"
```

7.1.1 Making the functions under test compatible with untyped languages such as JavaScript

```
fun set_attribute_with_null :: "((_) object_ptr option) ⇒ attr_key ⇒ attr_value ⇒ (_, unit) dom_prog"
  where
    "set_attribute_with_null (Some ptr) k v = (case cast ptr of
      Some element_ptr ⇒ set_attribute element_ptr k (Some v))"
fun set_attribute_with_null2 :: "((_) object_ptr option) ⇒ attr_key ⇒ attr_value option ⇒ (_, unit) dom_prog"
  where
    "set_attribute_with_null2 (Some ptr) k v = (case cast ptr of
      Some element_ptr ⇒ set_attribute element_ptr k v)"
notation set_attribute_with_null (‹_ . setAttribute'(_, _')›)
notation set_attribute_with_null2 (‹_ . setAttribute'(_, _')›)

fun get_child_nodes_Core_DOM_with_null :: "((_) object_ptr option) ⇒ (_, (_) object_ptr option list) dom_prog"
  where
    "get_child_nodes_Core_DOM_with_null (Some ptr) = do {
      children ← get_child_nodes ptr;
      return (map (Some ∘ cast) children)
    }"
notation get_child_nodes_Core_DOM_with_null (‹_ . childNodes›)

fun create_element_with_null :: "((_) object_ptr option) ⇒ string ⇒ (_, ((_) object_ptr option)) dom_prog"
  where
    "create_element_with_null (Some owner_document_obj) tag = (case cast owner_document_obj of
      Some owner_document ⇒ do {
        element_ptr ← create_element owner_document tag;
        return (Some (cast element_ptr))})"
notation create_element_with_null (‹_ . createElement'(_')›)

fun create_character_data_with_null :: "((_) object_ptr option) ⇒ string ⇒ (_, ((_) object_ptr option))
dom_prog"
  where
    "create_character_data_with_null (Some owner_document_obj) tag = (case cast owner_document_obj of
      Some owner_document ⇒ do {
        character_data_ptr ← create_character_data owner_document tag;
        return (Some (cast character_data_ptr))})"
notation create_character_data_with_null (‹_ . createTextNode'(_')›)

definition create_document_with_null :: "string ⇒ (_, ((_::linorder) object_ptr option)) dom_prog"
  where
    "create_document_with_null title = do {
      new_document_ptr ← create_document;
      html ← create_element new_document_ptr ''html'';
      append_child (cast new_document_ptr) (cast html);
      heap ← create_element new_document_ptr ''heap'';
      append_child (cast html) (cast heap);
      body ← create_element new_document_ptr ''body'';
      append_child (cast html) (cast body);
      return (Some (cast new_document_ptr))
    }"
abbreviation "create_document_with_null2 _ _ _ ≡ create_document_with_null ''''"
notation create_document_with_null (‹createDocument'(_')›)
notation create_document_with_null2 (‹createDocument'(_, _, _')›)

fun get_element_by_id_with_null :: "((_::linorder) object_ptr option) ⇒ string ⇒ (_, ((_) object_ptr option))
dom_prog"
  where
    "get_element_by_id_with_null (Some ptr) id' = do {
        element_ptr_opt ← get_element_by_id ptr id';
```

```
        (case element_ptr_opt of
          Some element_ptr ⇒ return (Some (cast_element_ptr2object_ptr element_ptr))
        | None ⇒ return None)}"
  | "get_element_by_id_with_null _ _ = error SegmentationFault"
notation get_element_by_id_with_null (<_ . getElementById'(_')>)

fun get_elements_by_class_name_with_null ::
"((_::linorder) object_ptr option) ⇒ string ⇒ (_, ((_) object_ptr option) list) dom_prog"
  where
    "get_elements_by_class_name_with_null (Some ptr) class_name =
      get_elements_by_class_name ptr class_name ≫= map_M (return ∘ Some ∘ cast_element_ptr2object_ptr)"
notation get_elements_by_class_name_with_null (<_ . getElementsByClassName'(_')>)

fun get_elements_by_tag_name_with_null ::
"((_::linorder) object_ptr option) ⇒ string ⇒ (_, ((_) object_ptr option) list) dom_prog"
  where
    "get_elements_by_tag_name_with_null (Some ptr) tag =
      get_elements_by_tag_name ptr tag ≫= map_M (return ∘ Some ∘ cast_element_ptr2object_ptr)"
notation get_elements_by_tag_name_with_null (<_ . getElementsByTagName'(_')>)

fun insert_before_with_null ::
"((_::linorder) object_ptr option) ⇒ ((_) object_ptr option) ⇒ ((_) object_ptr option) ⇒
(_, ((_) object_ptr option)) dom_prog"
  where
    "insert_before_with_null (Some ptr) (Some child_obj) ref_child_obj_opt = (case cast child_obj of
      Some child ⇒ do {
        (case ref_child_obj_opt of
          Some ref_child_obj ⇒ insert_before ptr child (cast ref_child_obj)
        | None ⇒ insert_before ptr child None);
        return (Some child_obj)}
    | None ⇒ error HierarchyRequestError)"
notation insert_before_with_null (<_ . insertBefore'(_, _')>)

fun append_child_with_null :: "((_::linorder) object_ptr option) ⇒ ((_) object_ptr option) ⇒
(_, unit) dom_prog"
  where
    "append_child_with_null (Some ptr) (Some child_obj) = (case cast child_obj of
      Some child ⇒ append_child ptr child
    | None ⇒ error SegmentationFault)"
notation append_child_with_null (<_ . appendChild'(_')>)

fun get_body :: "((_::linorder) object_ptr option) ⇒ (_, ((_) object_ptr option)) dom_prog"
  where
    "get_body ptr = do {
       ptrs ← ptr . getElementsByTagName(''body'');
       return (hd ptrs)
    }"
notation get_body (<_ . body>)

fun get_document_element_with_null :: "((_::linorder) object_ptr option) ⇒
(_, ((_) object_ptr option)) dom_prog"
  where
    "get_document_element_with_null (Some ptr) = (case cast_object_ptr2document_ptr ptr of
    Some document_ptr ⇒ do {
      element_ptr_opt ← get_M document_ptr document_element;
      return (case element_ptr_opt of
        Some element_ptr ⇒ Some (cast_element_ptr2object_ptr element_ptr)
      | None ⇒ None)})"
notation get_document_element_with_null (<_ . documentElement>)

fun get_owner_document_with_null :: "((_::linorder) object_ptr option) ⇒
(_, ((_) object_ptr option)) dom_prog"
  where
```

```
    "get_owner_document_with_null (Some ptr) = (do {
        document_ptr ← get_owner_document ptr;
        return (Some (cast_document_ptr2object_ptr document_ptr))})"
notation get_owner_document_with_null (‹_ . ownerDocument›)

fun remove_with_null :: "((_::linorder) object_ptr option) ⇒ ((_) object_ptr option) ⇒
(_, ((_) object_ptr option)) dom_prog"
  where
    "remove_with_null (Some ptr) (Some child) = (case cast child of
      Some child_node ⇒ do {
        remove child_node;
        return (Some child)}
    | None ⇒ error NotFoundError)"
  | "remove_with_null None _ = error TypeError"
  | "remove_with_null _ None = error TypeError"
notation remove_with_null (‹_ . remove'(')›)

fun remove_child_with_null :: "((_::linorder) object_ptr option) ⇒ ((_) object_ptr option) ⇒
(_, ((_) object_ptr option)) dom_prog"
  where
    "remove_child_with_null (Some ptr) (Some child) = (case cast child of
      Some child_node ⇒ do {
        remove_child ptr child_node;
        return (Some child)}
    | None ⇒ error NotFoundError)"
  | "remove_child_with_null None _ = error TypeError"
  | "remove_child_with_null _ None = error TypeError"
notation remove_child_with_null (‹_ . removeChild›)

fun get_tag_name_with_null :: "((_) object_ptr option) ⇒ (_, attr_value) dom_prog"
  where
    "get_tag_name_with_null (Some ptr) = (case cast ptr of
      Some element_ptr ⇒ get_M element_ptr tag_name)"
notation get_tag_name_with_null (‹_ . tagName›)

abbreviation "remove_attribute_with_null ptr k ≡ set_attribute_with_null2 ptr k None"
notation remove_attribute_with_null (‹_ . removeAttribute'(_')›)

fun get_attribute_with_null :: "((_) object_ptr option) ⇒ attr_key ⇒ (_, attr_value option) dom_prog"
  where
    "get_attribute_with_null (Some ptr) k = (case cast ptr of
      Some element_ptr ⇒ get_attribute element_ptr k)"
fun get_attribute_with_null2 :: "((_) object_ptr option) ⇒ attr_key ⇒ (_, attr_value) dom_prog"
  where
    "get_attribute_with_null2 (Some ptr) k = (case cast ptr of
      Some element_ptr ⇒ do {
        a ← get_attribute element_ptr k;
        return (the a)})"
notation get_attribute_with_null (‹_ . getAttribute'(_')›)
notation get_attribute_with_null2 (‹_ . getAttribute'(_')›)

fun get_parent_with_null :: "((_::linorder) object_ptr option) ⇒ (_, (_) object_ptr option) dom_prog"
  where
    "get_parent_with_null (Some ptr) = (case cast ptr of
      Some node_ptr ⇒ get_parent node_ptr)"
notation get_parent_with_null (‹_ . parentNode›)

fun first_child_with_null :: "((_) object_ptr option) ⇒ (_, ((_) object_ptr option)) dom_prog"
  where
    "first_child_with_null (Some ptr) = do {
      child_opt ← first_child ptr;
      return (case child_opt of
        Some child ⇒ Some (cast child)
```

```
      | None ⇒ None)}"
notation first_child_with_null (<_ . firstChild>)

fun adopt_node_with_null ::
"((_::linorder) object_ptr option) ⇒ ((_) object_ptr option) ⇒(_, ((_) object_ptr option)) dom_prog"
  where
    "adopt_node_with_null (Some ptr) (Some child) = (case cast ptr of
      Some document_ptr ⇒ (case cast child of
        Some child_node ⇒ do {
          adopt_node document_ptr child_node;
          return (Some child)}))"
notation adopt_node_with_null (<_ . adoptNode'(_')>)


definition createTestTree ::
"((_::linorder) object_ptr option) ⇒ (_, (string ⇒  (_, ((_) object_ptr option)) dom_prog)) dom_prog"
  where
   "createTestTree ref = return (λid. get_element_by_id_with_null ref id)"

end
```

7.2 Testing Document_adoptNode (Document_adoptNode)

This theory contains the test cases for Document_adoptNode.

```
theory Document_adoptNode
imports
  "Core_DOM_BaseTest"
begin

definition Document_adoptNode_heap :: heap_final where
  "Document_adoptNode_heap = create_heap [(cast (document_ptr.Ref 1), cast (create_document_obj html (Some
(cast (element_ptr.Ref 1))) [])),
    (cast (element_ptr.Ref 1), cast (create_element_obj ''html'' [cast (element_ptr.Ref 2), cast (element_ptr.Ref
8)] fmempty None)),
    (cast (element_ptr.Ref 2), cast (create_element_obj ''head'' [cast (element_ptr.Ref 3), cast (element_ptr.Ref
4), cast (element_ptr.Ref 5), cast (element_ptr.Ref 6), cast (element_ptr.Ref 7)] fmempty None)),
    (cast (element_ptr.Ref 3), cast (create_element_obj ''meta'' [] (fmap_of_list [(''charset'', ''utf-8'')])
None)),
    (cast (element_ptr.Ref 4), cast (create_element_obj ''title'' [cast (character_data_ptr.Ref 1)] fmempty
None)),
    (cast (character_data_ptr.Ref 1), cast (create_character_data_obj ''Document.adoptNode'')),
    (cast (element_ptr.Ref 5), cast (create_element_obj ''link'' [] (fmap_of_list [(''rel'', ''help''),
(''href'', ''https://dom.spec.whatwg.org/#dom-document-adoptnode'')]) None)),
    (cast (element_ptr.Ref 6), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 7), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 8), cast (create_element_obj ''body'' [cast (element_ptr.Ref 9), cast (element_ptr.Ref
10), cast (element_ptr.Ref 11)] fmempty None)),
    (cast (element_ptr.Ref 9), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''log'')]) None)),
    (cast (element_ptr.Ref 10), cast (create_element_obj ''x<'' [cast (character_data_ptr.Ref 2)] fmempty
None)),
    (cast (character_data_ptr.Ref 2), cast (create_character_data_obj ''x'')),
    (cast (element_ptr.Ref 11), cast (create_element_obj ''script'' [cast (character_data_ptr.Ref 3)] fmempty
None)),
    (cast (character_data_ptr.Ref 3), cast (create_character_data_obj ''%3C%3Cscript%3E%3E''))]"

definition Document_adoptNode_document :: "(unit, unit, unit, unit, unit, unit) object_ptr option" where
"Document_adoptNode_document = Some (cast (document_ptr.Ref 1))"
```

"Adopting an Element called 'x<' should work."

```
lemma "test (do {
```

```
  tmp0 ← Document_adoptNode_document . getElementsByTagName(''x<'');
  y ← return (tmp0 ! 0);
  child ← y . firstChild;
  tmp1 ← y . parentNode;
  tmp2 ← Document_adoptNode_document . body;
  assert_equals(tmp1, tmp2);
  tmp3 ← y . ownerDocument;
  assert_equals(tmp3, Document_adoptNode_document);
  tmp4 ← Document_adoptNode_document . adoptNode(y);
  assert_equals(tmp4, y);
  tmp5 ← y . parentNode;
  assert_equals(tmp5, None);
  tmp6 ← y . firstChild;
  assert_equals(tmp6, child);
  tmp7 ← y . ownerDocument;
  assert_equals(tmp7, Document_adoptNode_document);
  tmp8 ← child . ownerDocument;
  assert_equals(tmp8, Document_adoptNode_document);
  doc ← createDocument(None, None, None);
  tmp9 ← doc . adoptNode(y);
  assert_equals(tmp9, y);
  tmp10 ← y . parentNode;
  assert_equals(tmp10, None);
  tmp11 ← y . firstChild;
  assert_equals(tmp11, child);
  tmp12 ← y . ownerDocument;
  assert_equals(tmp12, doc);
  tmp13 ← child . ownerDocument;
  assert_equals(tmp13, doc)
}) Document_adoptNode_heap"
  ⟨proof⟩
```

"Adopting an Element called ':good:times:' should work."

```
lemma "test (do {
  x ← Document_adoptNode_document . createElement('':good:times:'');
  tmp0 ← Document_adoptNode_document . adoptNode(x);
  assert_equals(tmp0, x);
  doc ← createDocument(None, None, None);
  tmp1 ← doc . adoptNode(x);
  assert_equals(tmp1, x);
  tmp2 ← x . parentNode;
  assert_equals(tmp2, None);
  tmp3 ← x . ownerDocument;
  assert_equals(tmp3, doc)
}) Document_adoptNode_heap"
  ⟨proof⟩
```

end

7.3 Testing Document_getElementById (Document_getElementById)

This theory contains the test cases for Document_getElementById.

```
theory Document_getElementById
imports
  "Core_DOM_BaseTest"
begin

definition Document_getElementById_heap :: heap_final where
  "Document_getElementById_heap = create_heap [(cast (document_ptr.Ref 1), cast (create_document_obj html
(Some (cast (element_ptr.Ref 1))) []),
```

```
    (cast (element_ptr.Ref 1), cast (create_element_obj ''html'' [cast (element_ptr.Ref 2), cast (element_ptr.Ref
9)] fmempty None)),
    (cast (element_ptr.Ref 2), cast (create_element_obj ''head'' [cast (element_ptr.Ref 3), cast (element_ptr.Ref
4), cast (element_ptr.Ref 5), cast (element_ptr.Ref 6), cast (element_ptr.Ref 7), cast (element_ptr.Ref
8)] fmempty None)),
    (cast (element_ptr.Ref 3), cast (create_element_obj ''meta'' [] (fmap_of_list [(''charset'', ''utf-8'')])
None)),
    (cast (element_ptr.Ref 4), cast (create_element_obj ''title'' [cast (character_data_ptr.Ref 1)] fmempty
None)),
    (cast (character_data_ptr.Ref 1), cast (create_character_data_obj ''Document.getElementById'')),
    (cast (element_ptr.Ref 5), cast (create_element_obj ''link'' [] (fmap_of_list [(''rel'', ''author''),
(''title'', ''Tetsuharu OHZEKI''), (''href'', ''mailto:saneyuki.snyk@gmail.com'')]) None)),
    (cast (element_ptr.Ref 6), cast (create_element_obj ''link'' [] (fmap_of_list [(''rel'', ''help''),
(''href'', ''https://dom.spec.whatwg.org/#dom-document-getelementbyid'')]) None)),
    (cast (element_ptr.Ref 7), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 8), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 9), cast (create_element_obj ''body'' [cast (element_ptr.Ref 10), cast (element_ptr.Ref
11), cast (element_ptr.Ref 12), cast (element_ptr.Ref 13), cast (element_ptr.Ref 16), cast (element_ptr.Ref
19)] fmempty None)),
    (cast (element_ptr.Ref 10), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''log'')]) None)),
    (cast (element_ptr.Ref 11), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', '''')]) None)),
    (cast (element_ptr.Ref 12), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''test1'')])
None)),
    (cast (element_ptr.Ref 13), cast (create_element_obj ''div'' [cast (element_ptr.Ref 14), cast (element_ptr.Ref
15)] (fmap_of_list [(''id'', ''test5''), (''data-name'', ''1st'')]) None)),
    (cast (element_ptr.Ref 14), cast (create_element_obj ''p'' [cast (character_data_ptr.Ref 2)] (fmap_of_list
[(''id'', ''test5''), (''data-name'', ''2nd'')]) None)),
    (cast (character_data_ptr.Ref 2), cast (create_character_data_obj ''P'')),
    (cast (element_ptr.Ref 15), cast (create_element_obj ''input'' [] (fmap_of_list [(''id'', ''test5''),
(''type'', ''submit''), (''value'', ''Submit''), (''data-name'', ''3rd'')]) None)),
    (cast (element_ptr.Ref 16), cast (create_element_obj ''div'' [cast (element_ptr.Ref 17)] (fmap_of_list
[(''id'', ''outer'')]) None)),
    (cast (element_ptr.Ref 17), cast (create_element_obj ''div'' [cast (element_ptr.Ref 18)] (fmap_of_list
[(''id'', ''middle'')]) None)),
    (cast (element_ptr.Ref 18), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''inner'')])
None)),
    (cast (element_ptr.Ref 19), cast (create_element_obj ''script'' [cast (character_data_ptr.Ref 3)] fmempty
None)),
    (cast (character_data_ptr.Ref 3), cast (create_character_data_obj ''%3C%3Cscript%3E%3E''))]"

definition Document_getElementById_document :: "(unit, unit, unit, unit, unit, unit) object_ptr option" where
"Document_getElementById_document = Some (cast (document_ptr.Ref 1))"
```

"Document.getElementById with a script-inserted element"

```
lemma "test (do {
  gBody ← Document_getElementById_document . body;
  TEST_ID ← return ''test2'';
  test ← Document_getElementById_document . createElement(''div'');
  test . setAttribute(''id'', TEST_ID);
  gBody . appendChild(test);
  result ← Document_getElementById_document . getElementById(TEST_ID);
  assert_not_equals(result, None, ''should not be null.'');
  tmp0 ← result . tagName;
  assert_equals(tmp0, ''div'', ''should have appended element's tag name'');
  gBody . removeChild(test);
  removed ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(removed, None, ''should not get removed element.'')
}) Document_getElementById_heap"
```

⟨*proof*⟩

"update ‘id‘ attribute via setAttribute/removeAttribute"

lemma *"test (do {*
gBody ← Document_getElementById_document . body;
TEST_ID ← return ''test3'';
test ← Document_getElementById_document . createElement(''div'');
test . setAttribute(''id'', TEST_ID);
gBody . appendChild(test);
UPDATED_ID ← return ''test3-updated'';
test . setAttribute(''id'', UPDATED_ID);
e ← Document_getElementById_document . getElementById(UPDATED_ID);
assert_equals(e, test, ''should get the element with id.'');
old ← Document_getElementById_document . getElementById(TEST_ID);
assert_equals(old, None, ''shouldn't get the element by the old id.'');
test . removeAttribute(''id'');
e2 ← Document_getElementById_document . getElementById(UPDATED_ID);
assert_equals(e2, None, ''should return null when the passed id is none in document.'')
}) Document_getElementById_heap"
⟨*proof*⟩

"Ensure that the id attribute only affects elements present in a document"

lemma *"test (do {*
TEST_ID ← return ''test4-should-not-exist'';
e ← Document_getElementById_document . createElement(''div'');
e . setAttribute(''id'', TEST_ID);
tmp0 ← Document_getElementById_document . getElementById(TEST_ID);
assert_equals(tmp0, None, ''should be null'');
tmp1 ← Document_getElementById_document . body;
tmp1 . appendChild(e);
tmp2 ← Document_getElementById_document . getElementById(TEST_ID);
assert_equals(tmp2, e, ''should be the appended element'')
}) Document_getElementById_heap"
⟨*proof*⟩

"in tree order, within the context object's tree"

lemma *"test (do {*
gBody ← Document_getElementById_document . body;
TEST_ID ← return ''test5'';
target ← Document_getElementById_document . getElementById(TEST_ID);
assert_not_equals(target, None, ''should not be null'');
tmp0 ← target . getAttribute(''data-name'');
assert_equals(tmp0, ''1st'', ''should return the 1st'');
element4 ← Document_getElementById_document . createElement(''div'');
element4 . setAttribute(''id'', TEST_ID);
element4 . setAttribute(''data-name'', ''4th'');
gBody . appendChild(element4);
target2 ← Document_getElementById_document . getElementById(TEST_ID);
assert_not_equals(target2, None, ''should not be null'');
tmp1 ← target2 . getAttribute(''data-name'');
assert_equals(tmp1, ''1st'', ''should be the 1st'');
tmp2 ← target2 . parentNode;
tmp2 . removeChild(target2);
target3 ← Document_getElementById_document . getElementById(TEST_ID);
assert_not_equals(target3, None, ''should not be null'');
tmp3 ← target3 . getAttribute(''data-name'');
assert_equals(tmp3, ''4th'', ''should be the 4th'')
}) Document_getElementById_heap"
⟨*proof*⟩

"Modern browsers optimize this method with using internal id cache. This test checks that their optimization should effect only append to ‘Document‘, not append to ‘Node‘."

lemma *"test (do {*
TEST_ID ← return ''test6'';
s ← Document_getElementById_document . createElement(''div'');
s . setAttribute(''id'', TEST_ID);

```
  tmp0 ← Document_getElementById_document . createElement(''div'');
  tmp0 . appendChild(s);
  tmp1 ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(tmp1, None, ''should be null'')
}) Document_getElementById_heap"
```
⟨*proof*⟩

"changing attribute's value via ‘Attr‘ gotten from ‘Element.attribute‘."

lemma *"test (do {*
```
  gBody ← Document_getElementById_document . body;
  TEST_ID ← return ''test7'';
  element ← Document_getElementById_document . createElement(''div'');
  element . setAttribute(''id'', TEST_ID);
  gBody . appendChild(element);
  target ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(target, element, ''should return the element before changing the value'');
  element . setAttribute(''id'', (TEST_ID @ ''-updated''));
  target2 ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(target2, None, ''should return null after updated id via Attr.value'');
  target3 ← Document_getElementById_document . getElementById((TEST_ID @ ''-updated''));
  assert_equals(target3, element, ''should be equal to the updated element.'')
}) Document_getElementById_heap"
```
⟨*proof*⟩

"update ‘id‘ attribute via element.id"

lemma *"test (do {*
```
  gBody ← Document_getElementById_document . body;
  TEST_ID ← return ''test12'';
  test ← Document_getElementById_document . createElement(''div'');
  test . setAttribute(''id'', TEST_ID);
  gBody . appendChild(test);
  UPDATED_ID ← return (TEST_ID @ ''-updated'');
  test . setAttribute(''id'', UPDATED_ID);
  e ← Document_getElementById_document . getElementById(UPDATED_ID);
  assert_equals(e, test, ''should get the element with id.'');
  old ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(old, None, ''shouldn't get the element by the old id.'');
  test . setAttribute(''id'', '''');
  e2 ← Document_getElementById_document . getElementById(UPDATED_ID);
  assert_equals(e2, None, ''should return null when the passed id is none in document.'')
}) Document_getElementById_heap"
```
⟨*proof*⟩

"where insertion order and tree order don't match"

lemma *"test (do {*
```
  gBody ← Document_getElementById_document . body;
  TEST_ID ← return ''test13'';
  container ← Document_getElementById_document . createElement(''div'');
  container . setAttribute(''id'', (TEST_ID @ ''-fixture''));
  gBody . appendChild(container);
  element1 ← Document_getElementById_document . createElement(''div'');
  element1 . setAttribute(''id'', TEST_ID);
  element2 ← Document_getElementById_document . createElement(''div'');
  element2 . setAttribute(''id'', TEST_ID);
  element3 ← Document_getElementById_document . createElement(''div'');
  element3 . setAttribute(''id'', TEST_ID);
  element4 ← Document_getElementById_document . createElement(''div'');
  element4 . setAttribute(''id'', TEST_ID);
  container . appendChild(element2);
  container . appendChild(element4);
  container . insertBefore(element3, element4);
  container . insertBefore(element1, element2);
  test ← Document_getElementById_document . getElementById(TEST_ID);
```

```
  assert_equals(test, element1, ''should return 1st element'');
  container . removeChild(element1);
  test ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(test, element2, ''should return 2nd element'');
  container . removeChild(element2);
  test ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(test, element3, ''should return 3rd element'');
  container . removeChild(element3);
  test ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(test, element4, ''should return 4th element'');
  container . removeChild(element4)
}) Document_getElementById_heap"
```

⟨*proof*⟩

"Inserting an id by inserting its parent node"

lemma *"test (do {*

```
  gBody ← Document_getElementById_document . body;
  TEST_ID ← return ''test14'';
  a ← Document_getElementById_document . createElement(''a'');
  b ← Document_getElementById_document . createElement(''b'');
  a . appendChild(b);
  b . setAttribute(''id'', TEST_ID);
  tmp0 ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(tmp0, None);
  gBody . appendChild(a);
  tmp1 ← Document_getElementById_document . getElementById(TEST_ID);
  assert_equals(tmp1, b)
}) Document_getElementById_heap"
```

⟨*proof*⟩

"Document.getElementById must not return nodes not present in document"

lemma *"test (do {*

```
  TEST_ID ← return ''test15'';
  outer ← Document_getElementById_document . getElementById(''outer'');
  middle ← Document_getElementById_document . getElementById(''middle'');
  inner ← Document_getElementById_document . getElementById(''inner'');
  tmp0 ← Document_getElementById_document . getElementById(''middle'');
  outer . removeChild(tmp0);
  new_el ← Document_getElementById_document . createElement(''h1'');
  new_el . setAttribute(''id'', ''heading'');
  inner . appendChild(new_el);
  tmp1 ← Document_getElementById_document . getElementById(''heading'');
  assert_equals(tmp1, None)
}) Document_getElementById_heap"
```

⟨*proof*⟩

end

7.4 Testing Node_insertBefore (Node_insertBefore)

This theory contains the test cases for Node_insertBefore.

theory *Node_insertBefore*
imports
  "Core_DOM_BaseTest"
begin

definition *Node_insertBefore_heap :: heap*$_{final}$ **where**

```
  "Node_insertBefore_heap = create_heap [(cast (document_ptr.Ref 1), cast (create_document_obj html (Some
(cast (element_ptr.Ref 1))) [])),
    (cast (element_ptr.Ref 1), cast (create_element_obj ''html'' [cast (element_ptr.Ref 2), cast (element_ptr.Ref
6)] fmempty None)),
```

```
    (cast (element_ptr.Ref 2), cast (create_element_obj ''head'' [cast (element_ptr.Ref 3), cast (element_ptr.Ref
4), cast (element_ptr.Ref 5)] fmempty None)),
    (cast (element_ptr.Ref 3), cast (create_element_obj ''title'' [cast (character_data_ptr.Ref 1)] fmempty
None)),
    (cast (character_data_ptr.Ref 1), cast (create_character_data_obj ''Node.insertBefore'')),
    (cast (element_ptr.Ref 4), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 5), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar
None)),
    (cast (element_ptr.Ref 6), cast (create_element_obj ''body'' [cast (element_ptr.Ref 7), cast (element_ptr.Ref
8)] fmempty None)),
    (cast (element_ptr.Ref 7), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''log'')]) None)),
    (cast (element_ptr.Ref 8), cast (create_element_obj ''script'' [cast (character_data_ptr.Ref 2)] fmempty
None)),
    (cast (character_data_ptr.Ref 2), cast (create_character_data_obj ''%3C%3Cscript%3E%3E''))]"
```

definition *Node_insertBefore_document :: "(unit, unit, unit, unit, unit, unit) object_ptr option"* **where** *"Node_insertBefore_document = Some (cast (document_ptr.Ref 1))"*

"Calling insertBefore an a leaf node Text must throw HIERARCHY_REQUEST_ERR."

lemma *"test (do {*
* node ← Node_insertBefore_document . createTextNode(''Foo'');*
* tmp0 ← Node_insertBefore_document . createTextNode(''fail'');*
* assert_throws(HierarchyRequestError, node . insertBefore(tmp0, None))*
}) Node_insertBefore_heap"
⟨*proof*⟩

"Calling insertBefore with an inclusive ancestor of the context object must throw HIERARCHY_REQUEST_ERR."

lemma *"test (do {*
* tmp1 ← Node_insertBefore_document . body;*
* tmp2 ← Node_insertBefore_document . getElementById(''log'');*
* tmp0 ← Node_insertBefore_document . body;*
* assert_throws(HierarchyRequestError, tmp0 . insertBefore(tmp1, tmp2));*
* tmp4 ← Node_insertBefore_document . documentElement;*
* tmp5 ← Node_insertBefore_document . getElementById(''log'');*
* tmp3 ← Node_insertBefore_document . body;*
* assert_throws(HierarchyRequestError, tmp3 . insertBefore(tmp4, tmp5))*
}) Node_insertBefore_heap"
⟨*proof*⟩

"Calling insertBefore with a reference child whose parent is not the context node must throw a NotFoundError."

lemma *"test (do {*
* a ← Node_insertBefore_document . createElement(''div'');*
* b ← Node_insertBefore_document . createElement(''div'');*
* c ← Node_insertBefore_document . createElement(''div'');*
* assert_throws(NotFoundError, a . insertBefore(b, c))*
}) Node_insertBefore_heap"
⟨*proof*⟩

"If the context node is a document, inserting a document or text node should throw a HierarchyRequestError."

lemma *"test (do {*
* doc ← createDocument(''title'');*
* doc2 ← createDocument(''title2'');*
* tmp0 ← doc . documentElement;*
* assert_throws(HierarchyRequestError, doc . insertBefore(doc2, tmp0));*
* tmp1 ← doc . createTextNode(''text'');*
* tmp2 ← doc . documentElement;*
* assert_throws(HierarchyRequestError, doc . insertBefore(tmp1, tmp2))*
}) Node_insertBefore_heap"
⟨*proof*⟩

"Inserting a node before itself should not move the node"

lemma *"test (do {*

```
  a ← Node_insertBefore_document . createElement(''div'');
  b ← Node_insertBefore_document . createElement(''div'');
  c ← Node_insertBefore_document . createElement(''div'');
  a . appendChild(b);
  a . appendChild(c);
  tmp0 ← a . childNodes;
  assert_array_equals(tmp0, [b, c]);
  tmp1 ← a . insertBefore(b, b);
  assert_equals(tmp1, b);
  tmp2 ← a . childNodes;
  assert_array_equals(tmp2, [b, c]);
  tmp3 ← a . insertBefore(c, c);
  assert_equals(tmp3, c);
  tmp4 ← a . childNodes;
  assert_array_equals(tmp4, [b, c])
}) Node_insertBefore_heap"
```
⟨*proof*⟩

end

7.5 Testing Node_removeChild (Node_removeChild)

This theory contains the test cases for Node_removeChild.

theory *Node_removeChild*
imports
"Core_DOM_BaseTest"
begin

definition *Node_removeChild_heap :: heap*$_{final}$ **where**
"Node_removeChild_heap = create_heap [(cast (document_ptr.Ref 1), cast (create_document_obj html (Some (cast (element_ptr.Ref 1))) [])),
(cast (element_ptr.Ref 1), cast (create_element_obj ''html'' [cast (element_ptr.Ref 2), cast (element_ptr.Ref 7)] fmempty None)),
(cast (element_ptr.Ref 2), cast (create_element_obj ''head'' [cast (element_ptr.Ref 3), cast (element_ptr.Ref 4), cast (element_ptr.Ref 5), cast (element_ptr.Ref 6)] fmempty None)),
(cast (element_ptr.Ref 3), cast (create_element_obj ''title'' [cast (character_data_ptr.Ref 1)] fmempty None)),
(cast (character_data_ptr.Ref 1), cast (create_character_data_obj ''Node.removeChild'')),
(cast (element_ptr.Ref 4), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar None)),
(cast (element_ptr.Ref 5), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''/resources/testhar None)),
(cast (element_ptr.Ref 6), cast (create_element_obj ''script'' [] (fmap_of_list [(''src'', ''creators.js'')]) None)),
(cast (element_ptr.Ref 7), cast (create_element_obj ''body'' [cast (element_ptr.Ref 8), cast (element_ptr.Ref 9), cast (element_ptr.Ref 10)] fmempty None)),
(cast (element_ptr.Ref 8), cast (create_element_obj ''div'' [] (fmap_of_list [(''id'', ''log'')]) None)),
(cast (element_ptr.Ref 9), cast (create_element_obj ''iframe'' [] (fmap_of_list [(''src'', ''about:blank'')]) None)),
(cast (element_ptr.Ref 10), cast (create_element_obj ''script'' [cast (character_data_ptr.Ref 2)] fmempty None)),
(cast (character_data_ptr.Ref 2), cast (create_character_data_obj ''%3C%3Cscript%3E%3E''))]"

definition *Node_removeChild_document :: "(unit, unit, unit, unit, unit, unit) object_ptr option"* **where** *"Node_remov = Some (cast (document_ptr.Ref 1))"*

"Passing a detached Element to removeChild should not affect it."

lemma *"test (do {*
doc ← return Node_removeChild_document;
s ← doc . createElement(''div'');
tmp0 ← s . ownerDocument;

```
  assert_equals(tmp0, doc);
  tmp1 ← Node_removeChild_document . body;
  assert_throws(NotFoundError, tmp1 . removeChild(s));
  tmp2 ← s . ownerDocument;
  assert_equals(tmp2, doc)
}) Node_removeChild_heap"
```
⟨*proof*⟩

"Passing a non-detached Element to removeChild should not affect it."

lemma *"test (do {*
```
  doc ← return Node_removeChild_document;
  s ← doc . createElement(''div'');
  tmp0 ← doc . documentElement;
  tmp0 . appendChild(s);
  tmp1 ← s . ownerDocument;
  assert_equals(tmp1, doc);
  tmp2 ← Node_removeChild_document . body;
  assert_throws(NotFoundError, tmp2 . removeChild(s));
  tmp3 ← s . ownerDocument;
  assert_equals(tmp3, doc)
}) Node_removeChild_heap"
```
⟨*proof*⟩

"Calling removeChild on an Element with no children should throw NOT_FOUND_ERR."

lemma *"test (do {*
```
  doc ← return Node_removeChild_document;
  s ← doc . createElement(''div'');
  tmp0 ← doc . body;
  tmp0 . appendChild(s);
  tmp1 ← s . ownerDocument;
  assert_equals(tmp1, doc);
  assert_throws(NotFoundError, s . removeChild(doc))
}) Node_removeChild_heap"
```
⟨*proof*⟩

"Passing a detached Element to removeChild should not affect it."

lemma *"test (do {*
```
  doc ← createDocument('''');
  s ← doc . createElement(''div'');
  tmp0 ← s . ownerDocument;
  assert_equals(tmp0, doc);
  tmp1 ← Node_removeChild_document . body;
  assert_throws(NotFoundError, tmp1 . removeChild(s));
  tmp2 ← s . ownerDocument;
  assert_equals(tmp2, doc)
}) Node_removeChild_heap"
```
⟨*proof*⟩

"Passing a non-detached Element to removeChild should not affect it."

lemma *"test (do {*
```
  doc ← createDocument('''');
  s ← doc . createElement(''div'');
  tmp0 ← doc . documentElement;
  tmp0 . appendChild(s);
  tmp1 ← s . ownerDocument;
  assert_equals(tmp1, doc);
  tmp2 ← Node_removeChild_document . body;
  assert_throws(NotFoundError, tmp2 . removeChild(s));
  tmp3 ← s . ownerDocument;
  assert_equals(tmp3, doc)
}) Node_removeChild_heap"
```
⟨*proof*⟩

"Calling removeChild on an Element with no children should throw NOT_FOUND_ERR."

lemma *"test (do {*
doc ← createDocument('''');
s ← doc . createElement(''div'');
tmp0 ← doc . body;
tmp0 . appendChild(s);
tmp1 ← s . ownerDocument;
assert_equals(tmp1, doc);
assert_throws(NotFoundError, s . removeChild(doc))
}) Node_removeChild_heap"
⟨*proof*⟩

"Passing a value that is not a Node reference to removeChild should throw TypeError."

lemma *"test (do {*
tmp0 ← Node_removeChild_document . body;
assert_throws(TypeError, tmp0 . removeChild(None))
}) Node_removeChild_heap"
⟨*proof*⟩

end

7.6 Core DOM Test Cases (Core_DOM_Tests)

This theory aggregates the individual test cases for the core DOM.

theory *Core_DOM_Tests*
imports
"tests/Document_adoptNode"
"tests/Document_getElementById"
"tests/Node_insertBefore"
"tests/Node_removeChild"
begin
end

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