Bondy's Theorem

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Abstract

A proof of Bondy's Theorem following Bollobás [1].

theory *Bondy*
imports *Main*
begin

lemma *card-less-if-surj-not-inj*:
⟦ *finite A*; *f ' A = B*; ¬ *inj-on f A* ⟧ ⟹ *card B < card A*
⟨*proof*⟩

theorem *Bondy* :
  assumes ∀ *A* ∈ *F*. *A* ⊆ *X* **and** *card X ≥ 1* **and** *card F = card X*
  shows ∃ *D*. *D* ⊆ *X* & *card D < card X* & *card (inter D ' F) = card F*
⟨*proof*⟩

end

References

[1] B. Bollobás. *Combinatorics: set systems, hypergraphs, families of vectors and combinatorial probability.* Cambridge University Press, 1986.